Lunar Based Maneuvers for Applications in Space Transit

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The objective was to look into using the Moon as a device to make space based transit more efficient with the motivation for satellites which could orbit the lunar surface. Through research and simulations the use of the Moon in orbital maneuvers was found to offer a more efficient route, along with an area for future commercial and scientific development.

I. Nomenclature

e	=	eccentricity
h	=	specific relative angular momentum
R_a	=	radius of apoapsis
R_p	=	radius of periapsis
$T_{p,s}$	=	orbital period of spacecraft
V_a	=	velocity at apoapsis
V_p	=	velocity at periapsis
V_∞	=	escape velocity
Δv	=	change in velocity
μ	=	standard gravitational parameter

II. Introduction

Space based transit is an important part of today's society as space applications span the entire world, bringing near instant communication across the entire planet. These constant streams of data include those in atmospheric observation, commercial investments, scientific observations, as well as many other elements.

Due to the nature of space applications it is important to make the transit in space as efficient as possible. One way of increasing this efficiency is through the use of time tested and applied orbital maneuvers, which saves resources, time and money on transits from the Earth to near Earth space and beyond. One class of such maneuvers uses the Moon as a pivotal tool to increase overall efficiency.

These maneuvers have been successful since the late 1950's to propel spacecraft beyond the influence of Earth while using less fuel to achieve the required escape velocity when compared to a traditional non-assisted burn.

The Moon has now been used multiple times to do the same thing with a multitude of satellites, which showcases one major application of using the Moon for these types of flybys. Many other applications can range from lunar free return, which was used as a contingency plan on Apollo 8, Apollo 10, and Apollo 11, large inclination changes with better efficiency, satellite parking orbits for observation of Earth, and even the use of flybys to make lunar commercialization a profitable enterprise [1].

This report will cover the applications of these lunar maneuvers. A brief history of how these maneuvers have been used will introduce the topic and lead to applications for modern day maneuvers. The discussion will continue on how the Moon can be used as an asset in an ever growing era of interest in deep space exploration and scientific development. Such analysis will lead into a brief description of how these maneuvers could be executed using simplified tools to simulate the transfer of these orbits. The conclusion of this report will consider the plans to use these orbital maneuvers around the Moon in the present day and future.

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III. Historical Uses of Lunar Based Maneuvers

There have been numerous missions which have taken advantage of the Moon for its ability to provide essentially "free" delta-v for spacecraft in the form of gravitational assists. A few examples of such missions are STEREO; a space mission focused on the observation of the Sun, PAS-22 [2]; a Hong Kong based satellite that used the Moon's gravity to recover from an unstable geosynchronous transfer orbit, and Geotail [3]; a satellite that observes the Earth's magnetosphere and used lunar flybys to keep apogees inside the far reaching radius of the magnetosphere .

A. About the STEREO Mission

The solar based mission STEREO, consists of two almost identical satellites that were launched from Earth into a leading and trailing orbit, with respect to the Sun. The satellites are named STEREO-A, where the A indicates it is in the ahead orbit, and STEREO-B, where the B indicates it is in a trailing orbit behind the Earth. In order to place the spacecraft into their correct orbits, a flight plan consisting of a lunar flyby was needed. The spacecrafts initially orbited in a highly elliptical geocentric orbit before reaching the Moon's orbit and being influenced by the gravity of the Moon. STEREO-A was put into a heliocentric orbit inside Earth's orbit, and STEREO-B orbited the Earth one more time before being ejected into a heliocentric orbit outside the Earth's orbit [4].

B. STEREO Mission Trajectories

Figure 1 shows the different trajectories of the two satellites plotted with a time interval of one hour from 10/26/2006 to 2/4/2007 using data made available by the HORIZONS system from NASA [5]. As shown, the interaction between STEREO-A with the Moon's gravitational field causes the spacecraft to leave the Earth-Moon system. However, STEREO-B can be seen orbiting back around before encountering the Moon again and then being sent into its trailing heliocentric orbit.

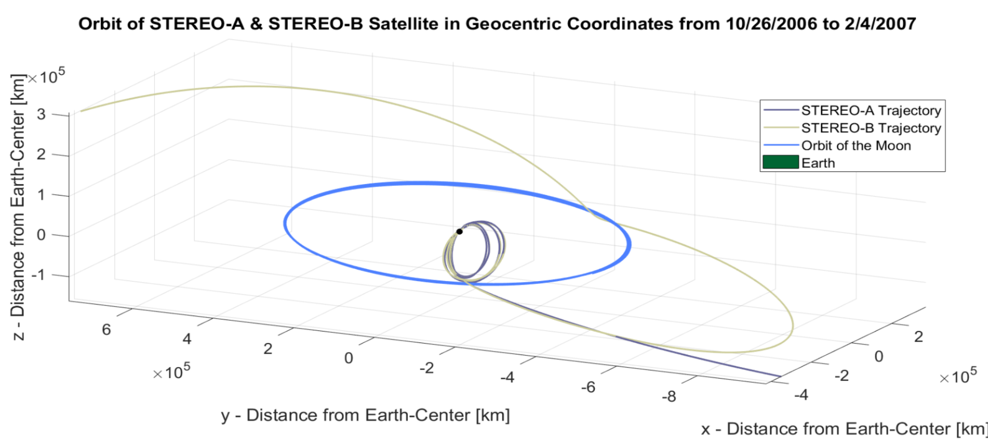

Fig. 1 MATLAB generated 3D plot of Earth, STEREO-A and STEREO-B in lunar flyby.

The three-dimensional view is useful as it shows the inclination of the orbits with respect to spherically represented Earth. It also allows for an easy reference to the inclination of the Moon and how this tilt changes the final heliocentric orbits of both STEREO-A and STEREO-B.

C. Purpose of the Lunar Flyby

The purpose of using the Moon for the gravitational assist was to limit the needed delta-v to put the STEREO satellites into a stable orbit around the Sun with both having a similar orbital period to Earth. However, the slight difference between the orbital period of A and B allows for the satellites to eventually observe both sides of the Sun, the main goal of the mission. This observation is able to take place since STEREO-A will move farther and farther from STEREO-B due to differing periods.

IV. Modern Day Uses of Lunar Based Maneuvers

The applications of modern day lunar based maneuvers are nearly unlimited and can be used to project and increase the efficiency of human based space transit. The applications cover a broad range in use, from the basic concept of using the Moon as a place to park satellites to make scientific and commercial observations to the use of lunar based flybys for efficient propulsion to deep space.

A. Large Inclination Changes

Efficiency is important due to the nature of large degree inclination changes being intensive on satellite resources. Such efficiency is essential as many times it is less resource exhaustive to leave the gravitational well and return compared to that of a full inclination burn.

With proper timing and positioning the Moon can be used to execute these changes while utilizing less resources of the satellite. These applications are based on a lunar flyby which alters the direction of the satellite as it leaves the lunar orbit. In the next section, simplified simulations will show large inclination changes and their given total delta-v versus two alternate maneuvers to change inclination for similar orbits.

B. Lunar Slingshot

Transitions from Earth orbit to deep space have always been about a large acceleration to escape velocity at a required angle. The use of lunar flybys could act as a way to more efficiently leave the Earth's gravitational field without using a complete burn solution to reach escape velocity.

By executing a flyby at the correct point in relation to the lunar surface the Moon's orbital velocity can be used as a slingshot into deep space by reducing the total burn needed to leave Earth. Using this maneuver, most of the burn can occur in deep space. Further simulations will show how using the Moon to project a satellite at escape velocity into deep space is more efficient than just a single large delta-v burn.

C. Lunar Parking Orbits

The Moon can be used as an efficient tool to observe the Earth and relay data from the surface of the Moon back to Earth [6]. Given the large distance and natural inclination of lunar parking orbits they can be used to fully observe the Earth. Changing orbital parameters of a parking satellite in a lunar orbit could be used to modify the lunar and Earth ground track in an effort to establish reliable communication between the Earth and Moon. Lunar space stations and lunar deep space radio relays are also based on the core idea of orbiting satellites around the Moon. Simulations and descriptions of lunar parking orbits will show the practicality of these concepts.

D. Lunar Economic Possibilities

The development of more efficient rockets and more economic possibilities in space extends to the application of such technologies on the Moon. Acting as a safer, and closer neighbor to Earth, the Moon could be the first outreach into commercial businesses in space. The Moon offers materials and a landscape not offered on Earth which could be advantageous to businesses seeking expansion into space.

An efficient way to get from the Moon to Earth, and vice versa would be of utmost importance due to the cost of moving equipment taking up the majority of the total costs of lunar economic prospects. Maneuvers from previous historical missions will aid in the development of the highest efficiency for transit from and to the Moon, thereby making it a more viable option in the future.

V. Orbital Simulations

The simulations will go into developing the ideas discussed in both the history of lunar maneuvers and the modern day applications. The main software used is NASA developed General Mission Analysis Tool, or GMAT, to generate data which is used in conjunction with MATLAB to develop plots and diagrams to aid in topics [7] [8].

When using GMAT a few assumptions will be made, the first of which is that the only masses in the system will be the Moon, Earth, and the Sun. These masses will be represented by point masses to stop the perturbations caused by the variance in shape of the Moon and the Earth.

The second assumption concerns the transfers from Earth to the Moon in the large inclinations and lunar slingshot maneuvers where the starting position will be from zero inclination and a parking orbit 300 km above the surface of

Earth. For transfers between the Moon and Earth, the satellite's orbit and the Moon's orbit will be considered co-planar.

The final assumption will be that all simulations and models will exclusively use impulsive burns. The impulsive burn describes an instantaneous change in acceleration of the spacecraft.

A. Lunar Maneuver Inclination Change

The concept that a lunar maneuver would be more efficient at certain angles to increase the inclination of a parking orbit around the Earth was the motivation behind this premise. Multiple simulations modified the angle at which the approach hyperbola came in relative to direction of the motion between the Moon and Earth. Two plots were generated that detail the total increase in velocity in relation to inclination change. Figure 2 shows the comparison between a scalene maneuver, an escape maneuver, and the theoretical lunar maneuver.

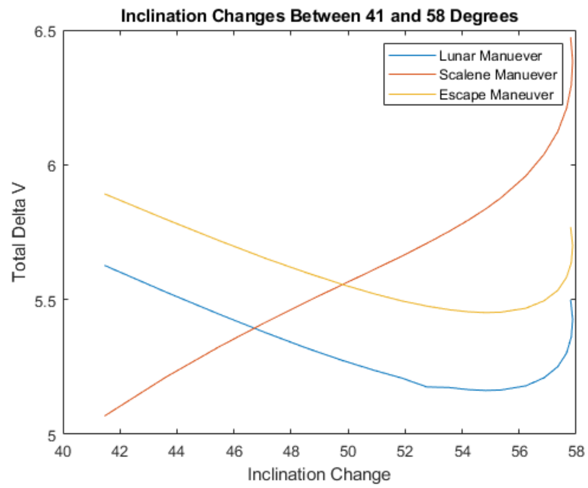

Fig. 2 MATLAB generated plot comparing inclination change to delta-v required.

The data was normalized such that all eccentricities, inclinations, and semi-major axis remained the same in all maneuvers. All three data trends curve due to the lunar maneuver being used to create the other two maneuvers as well as the lunar maneuver being limited to the angle based on various geometries. Figure 2 shows that a lunar maneuver for angles above 46.5 degrees is more efficient than the scalene maneuver, and in all cases is more efficient than the escape and return maneuver.

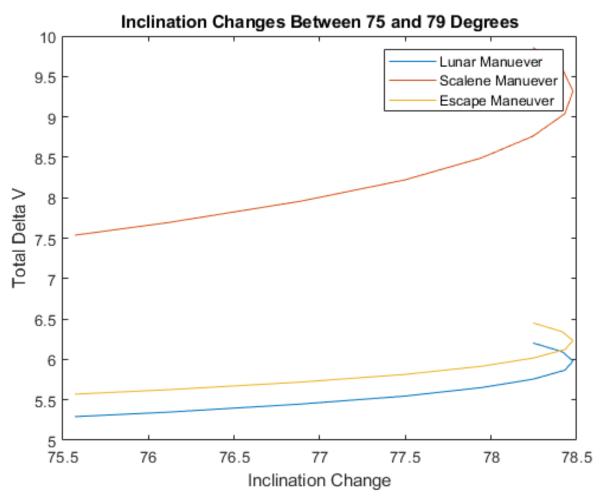

Fig. 3 MATLAB generated plot comparing inclination change to delta-v required.

To ensure this relation was not an anomaly, the geometric constraint that manages the maximum angle was increased to between 75 and 79 degrees. As seen in Figure 3, the same relation is observed where the lunar maneuver is more efficient at all times compared to an escape maneuver. Based on interpolation it can be assumed that for inclination changes larger than 46.5 degrees using the Moon is more efficient than with other similar two burn systems. Seen in Figure 4 is a visual representation of the effect post lunar maneuver.

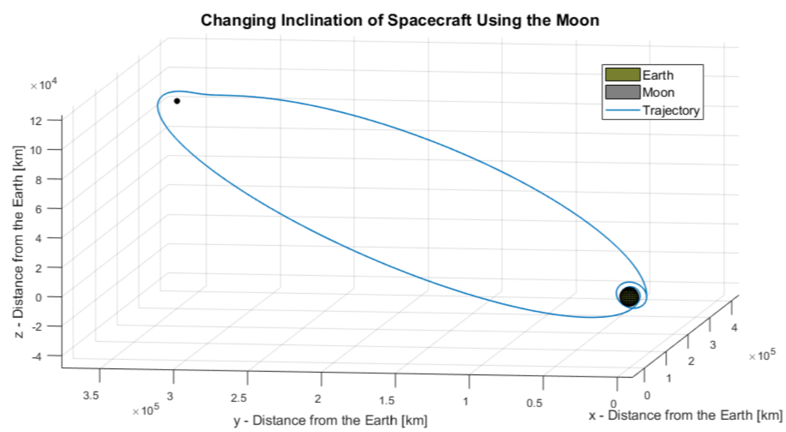

Fig. 4 MATLAB generated 3D plot of inclination change.

B. Lunar Slingshot

Another lunar maneuver to increase the overall efficiency of space transit is reaching the escape velocity of Earth with a less expensive burn by using a lunar slingshot. In order to perform this maneuver a system consisting of a single burn to get from LEO to escape velocity is established. From this application, the total delta-v burn required to reach the Moon is the majority of the overall velocity change.

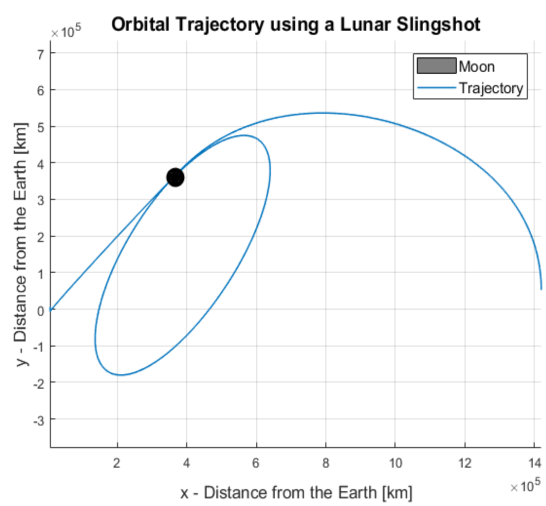

Fig. 5 MATLAB generated 2D plot of lunar slingshot.

Using GMAT simulations the average delta-v needed to reach escape velocity from LEO was 3.06 km/s when using the gravitational assist of the Moon. Without this assistance, the estimated single burn directly to escape velocity requires a delta-v of 3.21 km/s from LEO. In comparison, a burn to escape velocity using a lunar slingshot decreased the required delta-v by 0.15 km/s or a 4.6% difference. This seemingly small decrease offers a moderate enough difference to be considered applicable as it saves on the necessary mass budget of the spacecraft and allows for more payload to be inserted into space. Figure 5 shows a visual representation of this slingshot maneuver.

C. Lunar Parking Orbits

A lunar parking orbit has various applications in regards to data acquisition and transfer and can be used as an important tool in the relay of data from a potential Moon base. By setting a spacecraft in a parking orbit around the Moon, it becomes possible to form a consistent ground track in regards to both the lunar surface and the surface of the Earth. When dealing with orbits, eccentricity is one of many orbital parameters that can be modified as needed.

In order to study the effects of orbit shape on the lunar ground track, as well as the ground track on Earth, simulations were run using the open source General Mission Analysis Tool (GMAT) software from NASA. Figure 6 shows the mentioned eccentricities and the orbital trajectory of the simulated spacecraft. Simulations were bounded with a lunar parking orbit stemming from a transfer orbit from a Earth based parked position, a spacecraft of consistent mass, and impulsive maneuvers only.

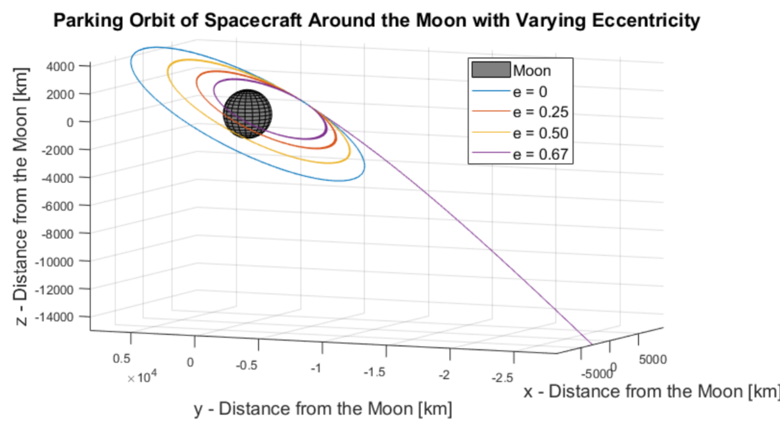

Fig. 6 MATLAB generated 3D plot various orbit eccentricities around the Moon.

Four different simulations were used to compare the consequence of various eccentricities on both the Earth ground track and the Moon ground track. As shown by Figure 7, all eccentricities resulted in nearly the same ground track on both the Earth and lunar surface. All four simulations produced tracking data that could prove to be useful in the event of a lunar base with telecommunication to the Earth. By not having to worry about the effects of varying eccentricity, it would allow for the development of a relay system with consistent ground track allowing for effective two-way communication between a Moon base and Earth.

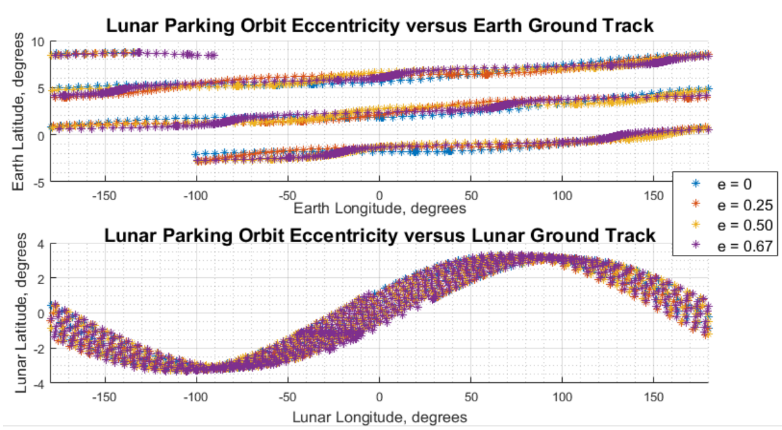

Fig. 7 MATLAB generated plot of lunar and Earth ground tracks.

One important note to make in regards to varying eccentricity in a lunar parking orbit is the impact on other parameters. The simulation showcased how raising the eccentricity of an orbit leads to a shorter period, smaller perigee radius, lower apogee velocity, and higher perigee velocity as shown in Table 1. Since it was shown that the ground tracks remain unchanged as the eccentricity varies, having the ability to modify these parameters with no issue to the prospective communication relay between the lunar surface and Earth is advantageous.

Table 1 Normalized Parameters as a Function of Eccentricity

Eccentricity	$T_{p,s}$	R_p	R_a	V_p	V_a
e = 0	22.1251 days	9197.1 km	9246.1 km	0.7311 km/s	0.7272 km/s
e = 0.25	15.8264 days	5500.3 km	9254.8 km	1.0574 km/s	0.6285 km/s
e = 0.50	12.0358 days	3039.3 km	9257.6 km	1.5585 km/s	0.5117 km/s
e = 0.67	10.2495 days	1792.6 km	9254.9 km	2.1407 km/s	0.4146 km/s

D. Lunar Economic Possibilities

The full application of lunar economic possibilities lies in the ability to send and retrieve materials to and from the Moon while using minimal amounts of fuel. Being able to complete such a mission with a single burn would be the most optimal method. This single burn is where lunar free return can be used. Figure 8 shows an example of the mentioned lunar free return. Such a process could be used for ensuring efficiency in applications of sending and retrieving materials on a large transport vessel from a parking orbit around Earth to lunar perigee. The materials are dropped off at the Moon and the spacecraft is returned to LEO where the process repeats. A procedure such as this would make for an efficient gate between the Moon and Earth as specially designed spacecraft would never have to return to Earth's surface, only operating in free space.

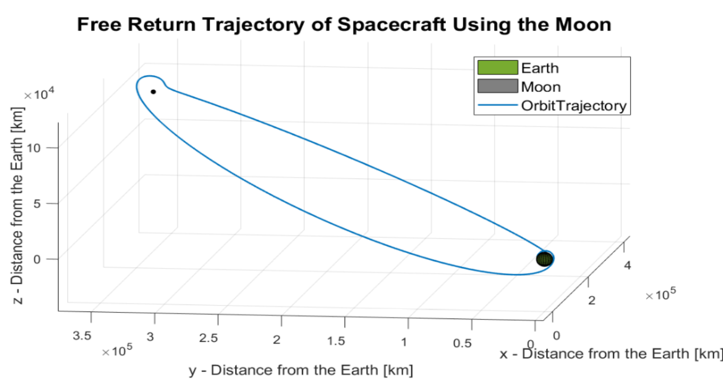

Fig. 8 MATLAB generated plot of lunar free return trajectory.

Retrieving materials could use the same maneuver, except they would launch from the lunar surface en route to Earth. The spacecraft would follow the same path as it uses Earth's gravitational potential as a tool to slow down without using fuel, thus creating another single burn system. These two systems in conjunction would be the most efficient way to send and retrieve materials from the lunar surface or from lunar orbit.

VI. Conclusion

The purpose of this paper was to address the concepts of developing more efficient maneuvers through the use of the Moon and addressing applications for such maneuvers. The discussion only addressed a few of these maneuvers and their effects such as changing inclination, reaching escape velocity, and transporting materials to and from the Moon. These applications were compared to preexisting maneuvers to analyze the efficiency of the velocity changes.

Lunar maneuvers offered a more efficient device for changing to large inclinations, and also reaching escape velocities compared to pre-existing maneuvers. A few ways to implement different lunar maneuvers and positions to place satellites in advantageous locations around the moon were also identified. These simulations and research have shown that using the Moon and characteristics of a three body system allows for more efficient and more economic maneuvers to make space transit more affordable and feasible as time advances.

References

[1] Tate, K., "The Apollo Moon Landings: How They Worked," *Space.com*, 2014.

[2] Skidmore, M., "An alternative view of the HGS-1 salvage mission," *The Space Review*, 2013.

[3] "GEOTAIL Magnetosphere Tail Observation Satellite," *JAXA*, 2003.

[4] Kucera, T., "About the STEREO Mission," *STEREO Website*, 2013.

[5] "HORIZONS Web-Interface," *Solar System Dynamics*, 2018.

[6] Tirró, S. (ed.), *Satellite Communication Systems Design*, 2nd ed., Plenum Press, New York, 1993, p. 738.

[7] "MATLAB - MathWorks - MATLAB Simulink," *MATLAB*, 2018.

[8] "NASA Technology Transfer Program," *General Mission Analysis Tool (GMAT)*, 2016.

Appendix

In order to produce the plots in this report MATLAB was used to organize and display the data generated by NASA's GMAT software. This was made possible by utilizing GMAT's ability to output a text file with data points at each time step iteration with respect to a user defined coordinate system. The collected data was manipulated and plotted together to form a visual basis for the reader, as well as assisting in the numerical calculation using the appended equations.

A. Equations

1. Large Inclination Changes

For the Large Inclination Changes section the above process was used. The data gathered from GMAT was as follows, total inclination change, magnitude of periapsis post inclination change, and total change in velocity. From this data set all values were set equal and solved for to get the total change in velocity for the scalene and escape maneuvers. Equations (1 - 7) were used to complete this calculation.

Equations for Scalene Maneuver:

$$R_{park} = \frac{h^2}{\mu} * \frac{1}{1+e} \tag{1}$$

$$R_{apoapsis} = \frac{h^2}{\mu} * \frac{1}{1-e} \tag{2}$$

$$\Delta V_1 = \frac{h}{R_{park}} - \sqrt{\frac{\mu}{R_{park}}} \tag{3}$$

$$\Delta V_2 = \sqrt{\sqrt{\frac{\mu}{R_{apoapsis}}}^2 + \frac{h}{R_{apoapsis}}^2 - 2 * \frac{h}{R_{apoapsis}} * \sqrt{\frac{\mu}{R_{apoapsis}}} * cos(\theta)} \tag{4}$$

Equations for Escape Maneuver:

$$\Delta V_1 = \sqrt{\frac{\mu}{R_{park}}} * (\sqrt{2} - 1) \tag{5}$$

$$\Delta V_2 = \sqrt{\frac{\mu}{R_{apoapsis}}} * (\sqrt{2} - 1) \tag{6}$$

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 \tag{7}$$

2. Lunar Slingshot

The comparison for the escape velocity from Earth required for the lunar maneuver versus a single burn required the use of two equations. Equations (8 - 9) were used to find the escape velocity of an unassisted maneuver from LEO.

$$v_e = \sqrt{\frac{2\mu}{r}} \tag{8}$$

$$v_\infty = \sqrt{V^2 - v_e^2} \tag{9}$$

3. Lunar Parking Orbits

The simulations for the lunar parking orbits did not explicitly use equations; however, back of the envelope calculations were used to verify the simulation was producing valid results. As shown by Eq. (10), eccentricity can be expressed in terms of only apoapsis and periapsis radii. This expression allows a quick double check of the simulated outcome. The assumption that angular momentum is conserved in closed orbits was also used to verify the simulated model with varying eccentricity as described by Eqs. (11 - 12).

$$e = \frac{r_a - r_p}{r_a + r_p} = 1 - \frac{2}{\frac{r_a}{r_p} + 1} \tag{10}$$

$$\vec{h} = \vec{r} \times \vec{v} \tag{11}$$

$$|\vec{h}| = r_p v_p = r_a v_a \tag{12}$$

B. MATLAB Scripts

Attached on the following pages is the MATLAB scripts that were used to generate the plots using data from GMAT.

Contents


```
%STEREO Ahead Simulation

clc;
clear;
close all;
```

Stereo Orbit Plot

```
Earth = dlmread('EARTH.txt');
Earth = Earth(1:end,3:5) * 1.496e+8;

STEREOA = dlmread('STEREOA.txt');
STEREOA = STEREOA(1:end,3:5) * 1.496e+8;

STEREOB = dlmread('STEREOB.txt');
STEREOB = STEREOB(1:end,3:5) * 1.496e+8;

%Orbit Plots
h = plot3(Earth(1:end,1),Earth(1:end,2),Earth(1:end,3),'color',[0 0.4 0.2]);
set(h,'LineWidth', 1.4);
hold on
h = plot3(STEREOA(1:end,1),STEREOA(1:end,2),STEREOA(1:end,3),'color',[0.4,0.4,0.6]);
set(h,'LineWidth', 1.4);
h = plot3(STEREOB(1:end,1),STEREOB(1:end,2),STEREOB(1:end,3),'color',[0.8 0.8 0.6]);
set(h,'LineWidth', 1.4);
title('Sun-Centric Orbits of Earth, STEREO-A, and STEREO-B from Day 1 - 2007 to Day 1 - 20
09');

%Sun, Earth, STEREO A & B Plots
[x, y, z] = sphere;

sun = 10000000;
surf(sun*x,sun*y,sun*z,'FaceColor', [0.93 .8 0.25])

e = 7500000;
surf(e*x+Earth(end,1),e*y+Earth(end,2),e*z+Earth(end,3),'FaceColor', [0 0.4 0.2])

sat = 5000000;
surf(sat*x+STEREOA(end,1),sat*y+STEREOA(end,2),sat*z+STEREOA(end,3),'FaceColor', [.4 .4 0.
6])
surf(sat*x+STEREOB(end,1),sat*y+STEREOB(end,2),sat*z+STEREOB(end,3),'FaceColor', [0.8 0.8
0.6])

legend('Earth Orbit', 'STEREO-A Orbit', 'STEREO-B Orbit','Sun (Not to Scale)','Earth (Not
to Scale)','STEREO-A (Not to Scale)','STEREO-B (Not to Scale)');
xlabel('x - Distance from Sun [km]');
ylabel('y - Distance from Sun [km]');
zlabel('z - Distance from Sun [km]');
```

```
grid on
axis equal

%Uncomment for 2D Plot
view(0,90);
```

STEREO A & B Departure Trajectories

```
figure

%Load data and clean for plotting
load('STEREOAdepart.mat')
A = table2array(STEREOAdepart);
A = A(2:4:end,:) * 1.496e8;

load('STEREOBdepart.mat')
B = table2array(STEREOBdepart);
B = B(2:4:end,:) * 1.496e8;

load('moonDepart.mat')
C = table2array(moonDepart);
C = C(2:4:end,:) * 1.496e8;

%Stereo-A Trajectory
h = plot3(A(1:end,1),A(1:end,2),A(1:end,3),'color',[.4 .4 0.6]);
set(h,'LineWidth', 1.4);
axis equal
grid on
hold on

%Stereo-B Trajectory
h = plot3(B(1:end,1),B(1:end,2),B(1:end,3),'color',[0.8 0.8 0.6]);
set(h,'LineWidth', 1.4);

%Moon Trajectory
h = plot3(C(1:end,1),C(1:end,2),C(1:end,3),'color',[0.3 0.5 1]);
set(h,'LineWidth', 1.4);

%Earth Centric Frame
[x, y, z] = sphere;
e = 6378;
surf(e*x,e*y,e*z,'FaceColor', [0 0.4 0.2]);

title('Orbit of STEREO-A & STEREO-B Satellite in Geocentric Coordinates from 10/26/2006 to
 2/4/2007');
legend({'STEREO-A Trajectory', 'STEREO-B Trajectory', 'Orbit of the Moon','Earth'},'Locati
on','northeast');
xlabel('x - Distance from Earth-Center [km]');
ylabel('y - Distance from Earth-Center [km]');
zlabel('z - Distance from Earth-Center [km]');

%Uncomment for 2D Plot
%view(90,0);
```

```
clear;
VoBase=3.063599;

K=[ 196.6,75.57,12286,1,2.22883457;
    196.75,76.1154,11735,1,2.28436069;
    197,76.8856,10874,1,2.38406;
    197.25,77.493,10097,1,2.4824;
    197.5,77.943,9355,1,2.5879;
    197.75,78.245,8689,1,2.6922;
    198,78.431,8068,1,2.803648;
    198.25,78.478,7492,1,2.9167;
    198.5,78.4156,6974,1,3.031;
    198.75,78.245,6513.75,1,3.14;
];

[n,m]=size(K);
L=1;
Kprime=zeros(n,2);
while L<=n
Kprime(L,:)=[K(L,2),K(L,5)+VoBase];
    L=L+1;
end

%-----------------------------------------------------------------------------
%Straight Burn
mu=3.986*10^5;
Vcircular=7.753166;

syms h e
L=1;
Kappa=zeros(n,2);
while L<=n
    eqnt=[6631==(h^2)/mu*(1/(1+e)),K(L,3)==(h^2)/mu*(1/(1-e))];
    [ha,ea]=solve(eqnt,[h,e]);
    ha=double(ha);
    ea=double(ea);
    Kappa(L,:)=[ha(2),ea(1)];
    L=L+1;
end

Vp(:,1)=Kappa(:,1)./6631;
Va(:,1)=Kappa(:,1)./K(:,3);
Vc2(:,1)=sqrt(mu./K(:,3));
DV2(:,1)=sqrt(Va.^2+Vc2.^2-2.*Va.*Vc2.*cos(Kprime(:,1).*3.14159./180));
DV1(:,1)=Vp(:,1)-Vcircular;
DVtot=DV2+DV1;
%----------------------------------------------------------------------------
DV1B=zeros(n,1);
DV1B(:,1)=(sqrt(2)-1)*Vcircular;
DV2B(:,1)=(sqrt(2)-1).*Vc2(:,1);
DVTB=DV1B+DV2B;

%----------------------------------------------------------------------------

plot(Kprime(:,1),Kprime(:,2),Kprime(:,1),DVtot(:),Kprime(:,1),DVTB(:));
```

```
xlabel('Inclination Change');
ylabel('Total Delta V');
title('Inclination Changes Between 75 and 79 Degrees');
legend('Lunar Manuever','Scalene Manuever','Escape Maneuver');
```

Published with MATLAB® R2018a

```
clear;
VoBase=3.063599;

K=[   10,31.4597,9517,1,2.563;
    10.5,33.0931,10218,1,2.465;
    11,34.4523,10871,1,2.383;
    11.5,35.5922,11469.1,1,2.314;
    12,36.5233,11986,1,2.257;
    12.5,37.2797,12437,1,2.211;
    13,37.9065,12817,1,2.174;
    13.5,38.3824,13116,1,2.145;
    14,38.76,13341,1,2.112;
    14.5,39.0385,13500,1,2.110;
    15,39.22162,13595,1,2.102;
    15.5,39.324,13631,1,2.099;
    16,39.3561,13612,1,2.1006;
    17,39.229,13435,1,2.116;
    18,38.87,13113,1,2.1459;
    19,38.354,12679,1,2.1874;
    20,37.662,12164,1,2.238;
    21,36.827,11617,1,2.298;
    22,35.874,11032,1,2.363;
    23,34.803,10458,1,2.435;

];

[n,m]=size(K);
L=1;
Kprime=zeros(n,2);
while L<=n
Kprime(L,:)=[K(L,1)+K(L,2),K(L,5)+VoBase];
    L=L+1;
end

%----------------------------------------------------------------------------
%Straight Burn
mu=3.986*10^5;
Vcircular=7.753166;

syms h e
L=1;
Kappa=zeros(n,2);
while L<=n
    eqnt=[6631==(h^2)/mu*(1/(1+e)),K(L,3)==(h^2)/mu*(1/(1-e))];
    [ha,ea]=solve(eqnt,[h,e]);
    ha=double(ha);
    ea=double(ea);
    Kappa(L,:)=[ha(2),ea(1)];
    L=L+1;
end

Vp(:,1)=Kappa(:,1)./6631;
Va(:,1)=Kappa(:,1)./K(:,3);
Vc2(:,1)=sqrt(mu./K(:,3));
DV2(:,1)=sqrt(Va.^2+Vc2.^2-2.*Va.*Vc2.*cos(Kprime(:,1).*3.14159./180));
```

```
DV1(:,1)=Vp(:,1)-Vcircular;
DVtot=DV2+DV1;
%--------------------------------------------------------------------------
DV1B=zeros(n,1);
DV1B(:,1)=(sqrt(2)-1)*Vcircular;
DV2B(:,1)=(sqrt(2)-1).*Vc2(:,1);
DVTB=DV1B+DV2B;

%--------------------------------------------------------------------------

plot(Kprime(:,1),Kprime(:,2),Kprime(:,1),DVtot(:),Kprime(:,1),DVTB(:));
xlabel('Inclination Change');
ylabel('Total Delta V');
title('Inclination Changes Between 41 and 58 Degrees');
legend('Lunar Manuever','Scalene Manuever','Escape Maneuver');
```

```
%Inclination Change Simulation Plot

clear;
clc;
close all;

load('InclinationChange.mat')

M = InclinationChange;
X = M(1:end,1);
Y = M(1:end,2);
Z = M(1:end,3);

hold on
[x, y, z] = sphere;
earth = 6371;
surf(earth*x,earth*y,earth*z,'FaceColor', [0.47 0.5 0.19])

moon = 1737;
surf(moon*x + 3.67e5, moon*y + 3.6e5,moon*z + 8.5e4,'FaceColor', [0.5 0.5 0.5])

plot3(X,Y,Z,'Color',[0 0.45 0.74],'LineWidth',1.2);
grid on
axis equal
legend('Earth','Moon','Trajectory');

title('Changing Inclination of Spacecraft Using the Moon','FontSize',12);
xlabel('x - Distance from the Earth [km]');
ylabel('y - Distance from the Earth [km]');
zlabel('z - Distance from the Earth [km]');
view(-82,9);
```

```
%Lunar Sling Shot Simulation Plot

clear;
clc;
close all;

load('LunarSling.mat')

M = LunarSling;
X = M(1:end,1);
Y = M(1:end,2);
Z = M(1:end,3);

hold on
[x, y, z] = sphere;

moon = 25000;
surf(moon*x + 3.67e5, moon*y + 3.6e5,moon*z + 8.5e4,'FaceColor', [0.5 0.5 0.5])

plot3(X,Y,Z,'Color',[0 0.45 0.74],'LineWidth',1.2);
grid on
axis equal
legend('Moon','Trajectory');

title('Orbital Trajectory using a Lunar Slingshot','FontSize',12);
xlabel('x - Distance from the Earth [km]');
ylabel('y - Distance from the Earth [km]');
zlabel('z - Distance from the Earth [km]');
```

```
%Parking Orbit from Earth to Moon

clc;
clear;
close all;

%Read in data for each eccentricity
%e = 0
M_0 = dlmread('ReportFile0.txt');
t_0 = M_0(165:end,2);
t_0 = norm(t_0,inf) / 3600;

lat_0_m = M_0(150:end,7);
long_0_m = M_0(150:end,8);
lat_0_e = M_0(150:end,12);
long_0_e = M_0(150:end,13);
X_0 = M_0(140:end,9);
Y_0 = M_0(140:end,10);
Z_0 = M_0(140:end,11);

V_p_0 = mean(M_0(165:end,15));
V_a_0 = mean(M_0(165:end,14));
R_p_0 = mean(M_0(165:end,6));
R_a_0 = mean(M_0(165:end,5));

%e = 0.25
M_25 = dlmread('ReportFile25.txt');
t_25 = M_25(165:end,2);
t_25 = norm(t_25,inf) / 3600;

lat_25_m = M_25(150:end,7);
long_25_m = M_25(150:end,8);
lat_25_e = M_25(150:end,12);
long_25_e = M_25(150:end,13);
X_25 = M_25(140:end,9);
Y_25 = M_25(140:end,10);
Z_25 = M_25(140:end,11);

V_p_25 = mean(M_25(165:end,15));
V_a_25 = mean(M_25(165:end,14));
R_p_25 = mean(M_25(165:end,6));
R_a_25 = mean(M_25(165:end,5));

%e = 0.50
M_5 = dlmread('ReportFile5.txt');
t_5 = M_5(165:end,2);
t_5 = norm(t_5,inf) / 3600;

lat_5_m = M_5(150:end,7);
long_5_m = M_5(150:end,8);
lat_5_e = M_5(150:end,12);
long_5_e = M_5(150:end,13);
X_5 = M_5(140:end,9);
Y_5 = M_5(140:end,10);
Z_5 = M_5(140:end,11);
```

```
V_p_5 = mean(M_5(165:end,15));
V_a_5 = mean(M_5(165:end,14));
R_p_5 = mean(M_5(165:end,6));
R_a_5 = mean(M_5(165:end,5));

%e = 0.67
M_67 = dlmread('ReportFile67.txt');
t_67 = M_67(165:end,2);
t_67 = norm(t_67,inf) / 3600;

lat_67_m = M_67(150:end,7);
long_67_m = M_67(150:end,8);
lat_67_e = M_67(150:end,12);
long_67_e = M_67(150:end,13);
X_67 = M_67(140:end,9);
Y_67 = M_67(140:end,10);
Z_67 = M_67(140:end,11);

V_p_67 = mean(M_67(165:end,15));
V_a_67 = mean(M_67(165:end,14));
R_p_67 = mean(M_67(165:end,6));
R_a_67 = mean(M_67(165:end,5));

%Lunar Ground Track
subplot(2,1,2);
hold on
plot(long_0_m,lat_0_m,'*');
plot(long_25_m,lat_25_m,'*');
plot(long_5_m,lat_5_m,'*');
plot(long_67_m,lat_67_m,'*');

xlim([-180 180])
title('Lunar Parking Orbit Eccentricity versus Lunar Ground Track');
ylabel('Lunar Latitude, degrees')
xlabel('Lunar Longitude, degrees')
grid on
grid minor
legend('e = 0', 'e = 0.25', 'e = 0.50', 'e = 0.67');

%Earth Ground Track
subplot(2,1,1);
hold on
plot(long_0_e,lat_0_e,'*');
plot(long_25_e,lat_25_e,'*');
plot(long_5_e,lat_5_e,'*');
plot(long_67_e,lat_67_e,'*');

xlim([-180 180])
title('Lunar Parking Orbit Eccentricity versus Earth Ground Track');
ylabel('Earth Latitude, degrees')
xlabel('Earth Longitude, degrees')
grid on
grid minor
legend('e = 0', 'e = 0.25', 'e = 0.50', 'e = 0.67');

%3D Coordinate Plot
```

```
figure
[x, y, z] = sphere;
moon = 1737;
surf(moon*x,moon*y,moon*z,'FaceColor', [0.5 0.5 0.5])
hold on
plot3(X_0,Y_0,Z_0)
plot3(X_5,Y_5,Z_5)
plot3(X_25,Y_25,Z_25)
plot3(X_67,Y_67,Z_67)
title('Parking Orbit of Spacecraft Around the Moon with Varying Eccentricity','FontSize',1
2);
legend('Moon', 'e = 0', 'e = 0.25', 'e = 0.50', 'e = 0.67','Location','northeast');
grid on
axis equal
xlabel('x - Distance from the Moon [km]');
ylabel('y - Distance from the Moon [km]');
zlabel('z - Distance from the Moon [km]');
view(-45,12);

%Period Table
T = [t_0, t_25, t_5, t_67];
```

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```
%Free Return Simulation Plot

clear;
clc;
close all;

load('FreeReturn.mat')

M = FreeReturnEdit;
X = M(1:end,1);
Y = M(1:end,2);
Z = M(1:end,3);

hold on
[x, y, z] = sphere;
earth = 6371;
surf(earth*x,earth*y,earth*z,'FaceColor', [0.47 0.67 0.19])

moon = 1737;
surf(moon*x + 3.67e5, moon*y + 3.6e5,moon*z + 8.5e4,'FaceColor', [0.5 0.5 0.5])

plot3(X,Y,Z,'Color',[0 0.45 0.74],'LineWidth',1.2);
grid on
axis equal
legend('Earth','Moon','Trajectory');
title('Free Return Trajectory of Spacecraft Using the Moon','FontSize',12);
xlabel('x - Distance from the Earth [km]');
ylabel('y - Distance from the Earth [km]');
zlabel('z - Distance from the Earth [km]');
view(-82,9);
```