

Programming

Date _____
Page _____

Assignment

⇒ Team:

- Dhruv Shah (B20EE017)
- Sudhir Singh (B20ME082)

⇒ Problem:

Given ~~noised~~ noised and blurred signal $y[n]$. Our task is to denoise and deblur it to get the original signal $x[n]$.

- 1) First denoise then deblur.
- 2) First deblur then denoise.

⇒ Theoretical Explanation.

$$y[n] = x[n] * h[n] + d[n]$$

$$h[n] = \frac{1}{16} [1 \ 4 \ 6 \ 4 \ 1] \quad (\text{Blurring matrix})$$

$d[n] \rightarrow$ Noise signal.

1) First denoise than deblur

$$z[n] = x[n] * h[n] = y[n] - d[n]$$

\hookrightarrow Denoised Signal.

For denoising apply averaging method

$$\text{i.e. } y[n] * d'[n]$$

where

$$d'[n] = \frac{1}{3} [1 \ 1 \ 1] \quad \text{or} \quad \frac{1}{5} [1 \ 1 \ 1 \ 1 \ 1]$$

or in general

$$\frac{1}{k} [1 \ 1 \ 1 \ \dots \ (k \text{ times})]$$

k is odd

Here middle value corresponds to $n=0$.

$$\therefore Z[n] = Y[n] * d'[n]$$

By fourier transform,

$$Z(e^{j\omega}) = Y(e^{j\omega}) \times D(e^{j\omega})$$

$Z[n]$ is denoised

→ Step 1

$$Z[n] = x[n] * h[n]$$

$$\therefore Z(e^{j\omega}) = X(e^{j\omega}) \times H(e^{j\omega})$$

$$\therefore X(e^{j\omega}) = \frac{Z(e^{j\omega})}{H(e^{j\omega})} \rightarrow \text{Step 2}$$

Now,

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

For computing integral we use summation

$$d\omega = \frac{2\pi}{1000}$$

$$\begin{aligned} \text{lower limit : } -\pi &\rightarrow -500 \left(\frac{2\pi}{1000} \right) \\ \text{upper limit : } \pi &\rightarrow 500 \left(\frac{2\pi}{1000} \right) \end{aligned}$$

$$\therefore \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \sum_{k=-499}^{499} X(e^{j\omega_k}) e^{j\omega_k n} \left(\frac{2\pi}{1000} \right)$$

$$= \frac{1}{1000} \sum_{k=-499}^{499} X(e^{j\omega_k}) e^{j\omega_k n}$$

$$\left(\omega_k = k \times \frac{2\pi}{1000} = \frac{k\pi}{500} \right)$$

$$\therefore x[n] = \frac{1}{1000} \sum_{k=-499}^{499} X(e^{j\omega_k}) e^{j\omega_k n}$$

— Step 3

$$Y(e^{j\omega}) \longrightarrow Z(e^{j\omega}) = \frac{Y(e^{j\omega})}{H(e^{j\omega})}$$

$$Z(e^{j\omega}) \longrightarrow X(e^{j\omega}) = Z(e^{j\omega}) \times D(e^{j\omega})$$

$$X(e^{j\omega}) \longrightarrow x_1[n]$$

2) First deblur than denoise

$$Z(e^{j\omega}) = \frac{Y(e^{j\omega})}{H(e^{j\omega})}$$

$z[n]$ is deblurred — Step 1

$$Z(e^{j\omega}) \times D(e^{j\omega}) = X(e^{j\omega})$$

— Step 2

$$x[n] = \frac{1}{1000} \sum_{k=-499}^{499} X(e^{j\omega_k}) e^{j\omega_k n}$$

$$(\omega_k = \frac{k\pi}{500})$$

— Step 3

Also theoretically

$$X_1(e^{j\omega}) = \frac{[Y(e^{j\omega}) * D(e^{j\omega})]}{H(e^{j\omega})}$$

$$X_2(e^{j\omega}) = \left[\frac{Y(e^{j\omega})}{H(e^{j\omega})} \right] \times D(e^{j\omega})$$

$$\therefore X_1(e^{j\omega}) = X_2(e^{j\omega})$$

$$\therefore x_1[n] = x_2[n]$$

Contribution

Group Members :-

- Dhruv Shah (B20EE017)
- Sudhir Singh (B20ME082)

- Interactions/discussions on google meet
- Theoretically problem was discussed by both.
- Concept and code of denoising by averaging method worked by Dhruv.
- Concept and code of deblurring and improving fourier transform to get better result worked by Sudhir.
- Code for fourier transform and reverse fourier done by Sudhir.
- Using Riemann integral for integration ideated by Dhruv.
- Use of graphs for better outputs ideated by Sudhir.
- Coding part was looked by both.