Euler 145

For this problem, we can write down a closed form solution for $R(n)$, the number of reversible numbers with n digits. The main observations come from keeping track of carries. By propogating the information that there is no incoming carry to the 1s place in the reverse-sum, it is easy to see that if n is even, then there can be no carries in the addition. Now, counting shows that there are 30 ways to assign a pair of digits so that the result is an odd number with no carries, and 20 ways to assign so that the leading digit isn't 0. So, if n is even, $R(n) = 30^{\frac{n}{2}-1} \cdot 20$. If n is odd, then the middle digit requires a carry to become odd, and propogating this information gives us that there must be alternating carry and non-carries in the digits. The number of ways of assigning so there is a carry out within a carry in is 20, the number of choices with a carry in but no out is 25. Also, this works if $N \equiv 3 \mod 4$, but if $N \equiv 1 \mod 4$, then there is actually no way of finding a reversible number. Also, there are 54 ways to choose the middle digit. Combining all of this information, we find that

$$R(n) = \begin{cases} 30^{n/2-1} & n \text{ even} \\ 5 \cdot 20^{\lceil \frac{n-1}{4} \rceil} 25^{\lfloor \frac{n-1}{4} \rfloor} & n \equiv 3 \mod 4 \\ 0 & n \equiv 1 \mod 4 \end{cases}$$

Also we note that $R(0) = R(1) = 0$. Using this, the solution to this problem is

$$\sum_{n=2}^{8} R(n) = 20 + 20 \cdot 30 + 20 \cdot 30^2 + 20 \cdot 30^3 + 5 \cdot 20 + 5 \cdot 20^2 \cdot 25 = 608720$$