1.11 Massless Particles

Massless particles refer to particle whose rest mass m_0 is zero.

$$\text{Massless particles: } m_0 = 0. \tag{1.147}$$

The momentum of a massless particle will be zero unless it has speed equal to the speed of light.

$$\vec{p} = \lim_{m_0 \to 0} \frac{m_0 \vec{u}}{\sqrt{1 - u^2/c^2}} = 0, \quad \text{unless} \quad u = c. \tag{1.148}$$

When $u = c$, i.e., when a massless particle is miving at the speed of light we use Eq. 1.146 to write the momentum in terms of the energy of the particle.

$$\lim_{m_0 \to 0} \left[E^2 = p^2 c^2 + m_0^2 c^4 \right] \implies E^2 = p^2 c^2. \tag{1.149}$$

Therefore, keeping the positive root we get the energy-momentum relation of a massless particle to be

$$\boxed{E = p\, c.} \tag{1.150}$$

The particles of light are called photons. They are massless particles. The energy of each photon depends on the frequency of the light. Each photon of light of frequency f has energy equal to hf, where h is Planck constant, whose value is equal to 6.627×10^{-34} J.s.

$$E = hf. \tag{1.151}$$

Therefore, momentum a photon will be related to its frequency as

$$p = \frac{E}{c} = \frac{hf}{c}. \tag{1.152}$$

Writing the ratio of c to f as the wavelength λ of light,

$$\lambda = \frac{c}{f}, \tag{1.153}$$

we obtain an interesting result relating the momentum and wavelength of light.

$$p\lambda = h. \tag{1.154}$$

Example 1.14. Decay of neutral pion. A neutral pion, π^0, decays into two light particles, called photons. The rest mass of neutral pion is 135 MeV/c^2. That is, if you multiply 135 MeV/c^2 by c^2 you will get the rest energy in MeV unit. Find the energy and momenta of the two photons released when a neutral pion at rest decays into two photons.

Solution. Let E be the energy of one of the photons. Balancing the energy in the decay reaction we find

$$2E = 135\,\text{MeV}.$$

Therefore, the energy of each photon is

$$E = 67.5\,\mathrm{MeV} = 67.5\,\mathrm{MeV} \times 10^6 \times 1.67 \times 10^{-19}\,\mathrm{J/MeV} = 1.13 \times 10^{-11}\,\mathrm{J}.$$

The momentum of each photon

$$p = \frac{E}{c} = \frac{1.13 \times 10^{-11}\,\mathrm{J}}{3.0 \times 10^{8}\,\mathrm{m/s}} = 3.77 \times 10^{-20}\,\mathrm{kg.m/s}.$$