1.5 PROBLEMS

Problem 1.5.1. Equal charges $+Q$ each are located at the corners of a cube of side a. Find the electric force experienced by any one of the charges. Ans: Magnitude: $\frac{1}{4\pi\epsilon_0}\left(\frac{Q^2}{a^2}\right)\left(\frac{1}{3}+\sqrt{3}+\sqrt{\frac{3}{2}}\right)$, direction varies.

Problem 1.5.2. An electron is moving horizontally with a speed of 5×10^4 m/s and enters a region where a uniform electric field of magnitude 2000 N/C is pointed upwards. (a) Find the acceleration of electron, both magnitude and direction. (b) If the electron moves in the xy-plane of a Cartesian coordinate with the x-axis along the horizontal direction and the y-axis in the vertical direction, find either $x(y)$ or $y(x)$ that would give the trajectory and sketch the trajectory. (c) How much time will the electron take to move 5 cm horizontally? (d) How much distance vertically the electron will move when it has moved 5 cm horizontally? (Ignore effects of gravity). Ans: (a) 1.8×10^{14} m/s^2, (c) 1 μs, (d) 90 m.

Problem 1.5.3. Two charged objects with charges $+q$ and $-2q$ are fixed at a distance d between them. Find a place where electric field will be zero.

Problem 1.5.4. Two charges of magnitude Q each are located along the x-axis at $x = R$ and $x = -R$ respectively. Electric fields at points on the y-axis from these charges will be pointed along the $\pm y$-axis. (a) Find the electric field at an arbitrary point on the y-axis. (b) Where on the y-axis does the electric field have the largest magnitude? (c) What is the magnitude of the electric field on the y-axis where the electric field has the largest magnitude? Ans: (b) $y = 2.1$ cm.

Problem 1.5.5. A thin plastic rod of length L is sprayed with an electrically charged paint in such a manner that electric charge density varies along the length, being zero in the middle and highest at the ends. If the rod is placed along the x-axis with the middle point coinciding with the origin, the line charge density on the rod is a linear function of coordinate x, $\lambda = a|x|$, where a is a positive constant. (a) How much is the total charge on the rod? (b) Find the electric field at an arbitrary point on the y-axis. Ans: (a) 5×10^{-8} C if $a =$ 5 C/m^2 and $L = 20$ cm.

Problem 1.5.6. A thin plastic ring of radius R is sprayed with electrically charged paint in a manner that half of the ring has a constant line charge density of $+\lambda$ and the other half has constant line charge density of $-\lambda$. Find the electric field at the center of the ring. (Note: E is not zero even though the total charge on the ring is clearly zero.)

Ans: Magnitude: $E = \lambda/\pi\epsilon_0 R$, direction away from + and towards −.

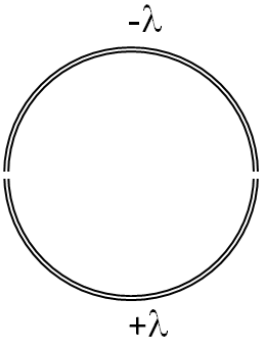
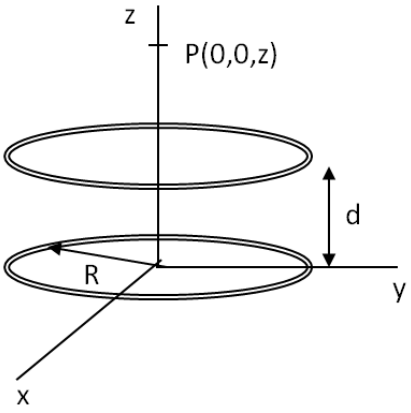


Figure 1.37: Problem 1.5.6.

Problem 1.5.7. Two parallel rings of radius R have their centers on the z-axis separated by a distance d. Consider one ring in the xy-plane centered at the origin and the other above it at $z = d$. The rings carry uniform line charge density λ. (a) Find the electric field at an arbitrary point P on the z-axis. (b) What will be the electric field if the ring in the xy-plane has a negative charge and the ring above has a positive charge density.



Problem 1.5.8. Consider two hemispherical shells of radius R, one with uniform surface charge density $+\sigma$ and the other with surface charge density $-\sigma$. Find the electric field at the center of the sphere. Ans: Magnitude: $\sigma/3\epsilon_0$.

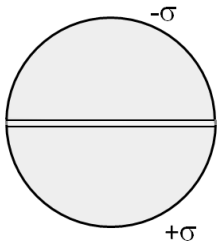


Figure 1.38: Problem 1.5.8.

Problem 1.5.9. An annulus with inner radius R_1 and outer radius R_2 is uniformly charged on one side with a surface charge density $+\sigma$. (a) Find the electric field at the point P at a height h above the center. (b) What would you expect in the limit $R_1 \to 0$? Why? Perform the limit and verify. (c) Find the result of the limit $R_2 \to \infty$.

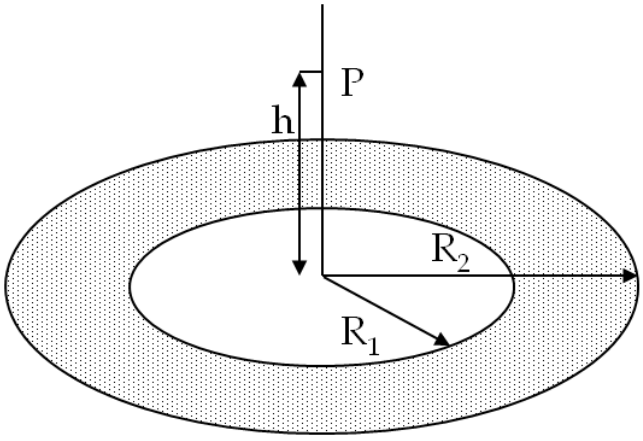


Figure 1.39: Problem 1.5.9.

Problem 1.5.10. A negatively charged particle of mass m and charge $-Q$ is placed near the center on the symmetry axis (z-axis) of a uniformly charged ring of total charge $+q$ and radius R. (a) Show that,

if $|z| << R$, the particle will execute a simple harmonic motion, and find the period of the simple harmonic motion. (c) Find the numerical value of the frequency of oscillation if the particle is an electron, the charge on the ring $q = 1$ μC, and the radius of the ring $R = 1$ cm. Ans: (b) 6.4 GHz.

Problem 1.5.11. Consider a one-dimensional lattice of ions having charges $+e$ and $-e$, where all the positive ions are located at $x = -a$, $-2a$, $-3a$, $\cdots$, and all the negative ions are at $x = +a$, $+2a$, $+3a$, $\cdots$. An ion of charge $+3e$ is placed at $x = 0$. Find the electric force on the ion at $x = 0$. Ans: $\frac{3e^2}{2\pi\epsilon_0 a^2}\zeta(2)$, where $\zeta(2)$ is the Riemann zeta function evaluated at 2.