2.4 EXERCISES

Geometric Picture of Vectors

Ex 2.4.1. In this exercise you will draw two vectors to scale on a graph paper, and then add them graphically. Two forces act on an object: a force of 3 Newton(N) in some direction and 4 N force in the direction 90° from the direction of the 3 N force such that the two forces fall in a plane. (a) Choose a scale for drawing, such as a 1 *cm* line for 1 N, and draw the vectors on the same graph paper using your scale. Make sure that you give your scale with the figure. (b) Draw or redraw the vectors so that the tail of the second vector is at the tip of the first vector. Use tip-to-tail method to draw the vector that is equal to the sum of the two vectors. (c) Use the scale for the drawing and convert the length of the sum vector on the graph to determine the magnitude of the sum. (d) Use a protractor and read off the direction of the sum vector with respect to the force whose magnitude is 3 N.

Ex 2.4.2. Three forces act at three points on a ring such that their directions are in one plane. We will use one of the forces to indicate the directions of the other two. The forces are: 3 N force in some direction in the plane, 6 N force in a direction that is 60° counter-clockwise from the direction of the 3-N force and 4.5 N force that is acting 90° clockwise from the direction of the 3-N force. Graphically add the given force vectors by drawing them in a tip-to-tail way, and find the magnitude and direction of the sum force, also called the net force.

Ex 2.4.3. Three forces act at three points on a ring such that their directions are in one plane. We will use one of the forces to indicate the directions of the other two. The forces are: 20 N force in some direction in the plane, 60 N force in a direction that is 135° counter-clockwise from the direction of the 20-N force and 45 N force that is acting 30° clockwise from the direction of the 20-N force. By adding these forces graphically find out the fourth force needed to make the net force on the ring zero.

Ex 2.4.4. Write vector equations from the vector diagrams given in Fig. 2.33.

Ex 2.4.5. Draw vector diagrams for the following vector equations. (a) $\vec{A}+\vec{B}-\vec{C}=0$; (b) $\vec{A}-\vec{B}-\vec{C}=0$; (c) $\vec{A}-\vec{B}-\vec{C}+\vec{D}=0$; (d) $\vec{A}+\vec{B}+\vec{C}-\vec{D}=0$; (e) $\vec{A}+\vec{B}+\vec{C}+\vec{D}-\vec{E}=0$.

Ex 2.4.6. A force $\vec{F}$ of magnitude 10 N and a displacement $\vec{s}$ of magnitude 10 cm are shown to their corresponding scales in Fig. 2.34.

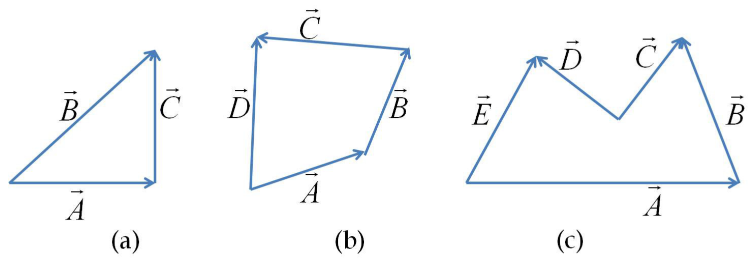

Figure 2.33: Exercise 2.4.4.

To work graphically on the given vectors, it may be helpful to transfer the figure to another paper using the same scale. (a) Determine the scales for the two vectors that was used for the given drawings, i.e., state the length of the force arrow that represents 1 N of force and the length of the displacement arrow that represents 1 cm of displacement. Since the figure shows two different properties we have two different scales, one for each property. (b) Draw the projection of the force vector on the displacement vector, and determine the value of the projection of the force vector onto the displacement vector in the unit of N. (c) Draw the projection of the displacement vector on the force vector, and determine the value of the projection of the displacement vector onto the force vector in the unit of cm. (d) Verify that the product of the projection of $\vec{F}$ on $\vec{s}$ with the magnitude of $\vec{s}$ is equal to the product of the projection of $\vec{s}$ on $\vec{F}$ with the magnitude of $\vec{F}$ within the margin of error of your measurements of the projections.

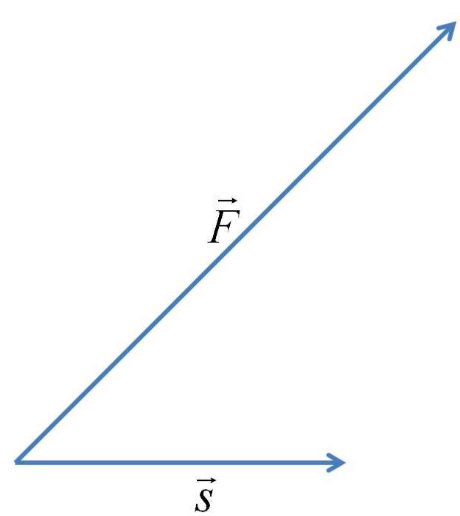

Figure 2.34: Exercise 2.4.6.

Ex 2.4.7. Repeat the exercises given in Exercise 2.4.6 for the force and displacement vectors given in Fig. 2.35 with the magnitude of force equal to 40 N and the magnitude of displacement equal to 3 m.

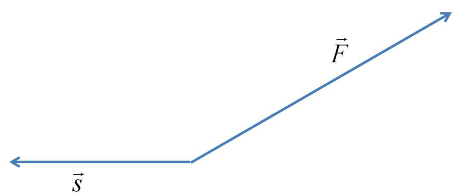

Figure 2.35: Exercise 2.4.7.

Ex 2.4.8. Determine graphically the magnitude and direction of the vector product $\vec{s} \times \vec{F}$ of the two vectors given in Fig. 2.34.

Ex 2.4.9. Determine graphically the magnitude and direction of the vector product $\vec{s} \times \vec{F}$ of the two vectors given in Fig. 2.35.

Analytic Picture of Vectors

Note: Although I have included units when giving values of physical vectors that have dimensions, it is cumbersome to write units all the time in calculations. A regular practice is to convert all quantities in the same system of units before using those numbers in the calculations. If you follow this advice, you do not need to write units all the time in your calculations. You can do the calculations without the units, and then put the units at the end in your final answer.

Ex 2.4.10. Draw the following vectors written in component form. Here m is the unit meter. (a) (2 m) $\hat{u}_x$ + (4 m) $\hat{u}_y$. (b) (3 m) $\hat{u}_x$ + (−5 m) $\hat{u}_z$. (c) (−1 m) $\hat{u}_y$ + (4 m) $\hat{u}_z$.

Ex 2.4.11. (a) Find the components of the vector given in Fig. 2.36 with respect to the axes $Oxyz$ and $Ox'y'z'$. The z and z'-axes are perpendicular to the given figure. Assume the given vector is entirely in the xy-plane of the two coordinate systems. (b) Write the vector using the unit vectors along axes in the two different coordinates - you should have two answers, one for each coordinate system. (c) Since there are two different representations for the same vector in the two different coordinates, and since the choice of coordinate system is arbitrary, what physical content do the values of components have for the vector?

Ex 2.4.12. Determine the x and y-components of the following vectors in the xy-plane of a particular coordinate system. Here, the terms counterclockwise and clockwise refer to the rotation direction as you look from the side of the positive z-axis. (a) A force of magnitude 10 N in the direction of 30° counterclockwise from the positive x-axis direction. (b) A force of magnitude 10 N in the direction of 30° clockwise from the negative x-axis direction. (c) A force of magni-

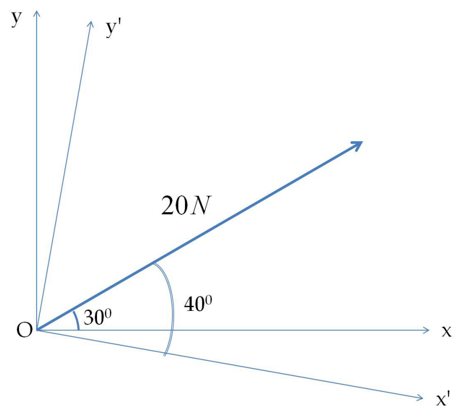

Figure 2.36: Exercise 2.4.11.

tude 10 N in the direction of 30° counter-clockwise from the positive y-axis direction. (d) A force of magnitude 10 N in the direction of 30° clockwise from the negative y-axis direction.

Ex 2.4.13. Determine the magnitudes and directions of the following vectors given in the component form with respect to a particular Cartesian coordinate system, where the positive x-axis points to the East, the positive y-axis points to the North and the positive z-axis points vertically up. Make sure you give the units for the magnitudes of the vectors if appropriate. From any angle(s) you calculate, state the physical direction in space for each vector. Here m is the unit meter. (a) $(3\text{ m})\ \hat{u}_x + (4\text{ m})\ \hat{u}_y$. (b) $(-3\text{ m})\ \hat{u}_x + (4\text{ m})\ \hat{u}_y$. (c) $(-3\text{ m})\ \hat{u}_x + (-4\text{ m})\ \hat{u}_y$. (d) $(3\text{ m})\ \hat{u}_x + (-4\text{ m})\ \hat{u}_y$.

Ex 2.4.14. Determine the magnitudes and directions of the following vectors given in component form with respect to a particular Cartesian coordinate system, where the positive x-axis points to the East, the positive y-axis points to the North and the positive z-axis points vertically up. Make sure you give units for the magnitudes of the vectors if appropriate. From any angle(s) you calculate, state the physical direction in space for each vector. Here m is the unit meter. (a) $(3\text{ m})\ \hat{u}_x+(4\text{ m})\ \hat{u}_y+(12\text{ m})\ \hat{u}_z$. (b) $(-3\text{ m})\ \hat{u}_x+(4\text{ m})\ \hat{u}_y+(12\text{ m})\ \hat{u}_z$. (c) $(3\text{ m})\ \hat{u}_x + (-4\text{ m})\ \hat{u}_y + (12\text{ m})\ \hat{u}_z$. (d) $(3\text{ m})\ \hat{u}_x + (4\text{ m})\ \hat{u}_y + (-12\text{ m})\ \hat{u}_z$. (e) $(-3\text{ m})\ \hat{u}_x + (-4\text{ m})\ \hat{u}_y + (-12\text{ m})\ \hat{u}_z$.

Ex 2.4.15. Three velocity vectors fall in the xy-plane of a coordinate system and have the following representations: $\vec{V}_1 = (3\text{ m/s})\hat{u}_x + (2\text{ m/s})\hat{u}_y$, $\vec{V}_2 = (9\text{ m/s})\hat{u}_x + (11\text{ m/s})\hat{u}_y$, and $\vec{V}_3 = (-8\text{ m/s})\hat{u}_y$. (a) Find the x and y-components of the sum of the three vectors. (b) Determine the magnitude and direction of the sum.

Ex 2.4.16. Three force vectors fall in the yz-plane of a coordinate system and have the following representations: $\vec{V}_1 = (100\text{ N})\hat{u}_y +$

(200 N)$\hat{u}_z$, $\vec{V}_2 = (-500\ \text{N})\hat{u}_y + (100\ \text{N})\hat{u}_z$, and $\vec{V}_3 = (-400\ \text{N})\hat{u}_y +$ (500 N)$\hat{u}_z$. (a) Find the y and z-components of the sum of the three vectors. (b) Determine the magnitude and direction of the sum.

Ex 2.4.17. (a) Find the x and y-components of the two forces given in Fig. 2.37 with respect to the coordinates given in the figure. (b) Add the two vectors and find the magnitude and direction of the resultant vector.

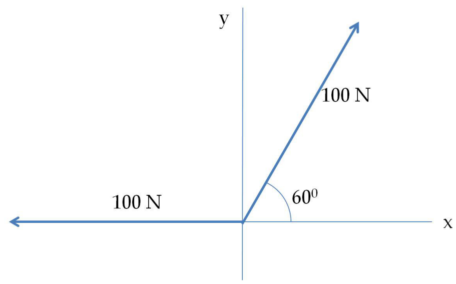

Figure 2.37: Exercise 2.4.17.

Ex 2.4.18. Calculate the scalar product between the following pairs of vectors: (a) (10 N) $\hat{u}_x$ + (−5 N) $\hat{u}_y$ and (3 m) $\hat{u}_x$ + (4 m) $\hat{u}_y$, (b) (40 N) $\hat{u}_x$ + (−10 N) $\hat{u}_y$ and (3 m) $\hat{u}_x$ + (4 m) $\hat{u}_z$, (c) (100 N) $\hat{u}_x$ + (−10 N) $\hat{u}_y$ and (30 m) $\hat{u}_y$ + (4 m) $\hat{u}_z$, (d) (2 N) $\hat{u}_x$ + (3 N) $\hat{u}_y$ + (4 N) $\hat{u}_z$ and (4 m/s) $\hat{u}_x$ + (3 m/s) $\hat{u}_y$ + (2 m/s) $\hat{u}_z$, (e) (100 N) $\hat{u}_x$ + (−10 N) $\hat{u}_y$ + (100 N) $\hat{u}_z$ and (30 m/s) $\hat{u}_y$ + (4 m/s) $\hat{u}_z$.

Ex 2.4.19. Find the angle between each pair of vectors given in Ex. 2.4.18.

Ex 2.4.20. Find the angles each vector makes with positive x, y and z-axes. (a) $3\hat{u}_x+2\hat{u}_y+4\hat{u}_z$, (b) $-3\hat{u}_x+2\hat{u}_y+4\hat{u}_z$, (c) $3\hat{u}_x-2\hat{u}_y+4\hat{u}_z$, (d) $3\hat{u}_x+2\hat{u}_y-4\hat{u}_z$, (e) $3\hat{u}_x-2\hat{u}_y-4\hat{u}_z$, (f) $-3\hat{u}_x-2\hat{u}_y-4\hat{u}_z$,

Ex 2.4.21. Find unit vectors in the directions of the following vectors. (a) (10 N) $\hat{u}_x$ + (−5 N) $\hat{u}_y$, (b) (3 m) $\hat{u}_x$ + (4 m) $\hat{u}_z$, (c) (30 m) $\hat{u}_y$ + (4 m) $\hat{u}_z$, (d) (2 N) $\hat{u}_x$ + (3 N) $\hat{u}_y$ + (4 N) $\hat{u}_z$, (e) (100 N) $\hat{u}_x$ + (−10 N) $\hat{u}_y$ + (100 N) $\hat{u}_z$.

Ex 2.4.22. For each vector in the following list find two unit vectors in the xy-plane perpendicular to it. (a) $\hat{u}_x$, (b) $\hat{u}_y$, (c) $\hat{u}_x+\hat{u}_y$, (d) $\hat{u}_x-\hat{u}_y$, (d) $\frac{1}{2}\hat{u}_x-\frac{\sqrt{3}}{2}\hat{u}_y$, (f) $a\hat{u}_x+b\hat{u}_y$, (g) $\cos(\theta)\hat{u}_x+\sin(\theta)\hat{u}_y$.

Ex 2.4.23. Evaluate the vector products:

(a)[(3 m) $\hat{u}_x$ + (4 m) $\hat{u}_y$] × [(10 N) $\hat{u}_x$ + (−5 N) $\hat{u}_y$], (b) [(3 m) $\hat{u}_x$ + (4 m) $\hat{u}_z$] × [(40 N) $\hat{u}_x$ + (−10 N) $\hat{u}_y$], (c)[(30 m) $\hat{u}_y$ + (4 m) $\hat{u}_z$] × [(100 N) $\hat{u}_x$ + (−10 N) $\hat{u}_y$], (d) [(4 m/s) $\hat{u}_x$ + (3 m/s) $\hat{u}_y$ + (2 m/s) $\hat{u}_z$] × [(2 N) $\hat{u}_x$ + (3 N) $\hat{u}_y$ + (4 N) $\hat{u}_z$], (e) [(30 m/s) $\hat{u}_y$ + (4 m/s) $\hat{u}_z$] × [(100 N) $\hat{u}_x$ + (−10 N) $\hat{u}_y$ + (100 N) $\hat{u}_z$].