10.9 NONCONSERVATIVE FIELDS AND KIRCHHOFF'S LOOP RULE

A conservative field $\vec{G}$ is a field whose line integral around any loop in space is zero.

$$\oint_{\text{any loop}} \vec{G} \cdot d\vec{l} = 0. \tag{10.41}$$

We have seen that electric fields of static charges obey such a rule.

$$\oint_{\text{any loop}} \vec{E} \cdot d\vec{l} = 0 \quad \text{(static)} \tag{10.42}$$

Therefore, electric fields of static charges are conservative fields. Since the line integral of a conservative field vanishes for an arbitrary loop, the work integral for the corresponding force would be independent of path. The independence of work integral on the path between any two positions makes it possible to introduce the concept of conserved scalar potential energy. Kirchhoff's loop rule states the consequence of the conservative nature of electric field for an electric circuit where current is steady.

As an example of the consequences of the conservative electric field consider the simple circuit containing a battery of EMF, $\mathcal{E}_{\text{bat}}$, and a resistor of resistance, R, as shown in Fig. 10.31. Notice that, in this circuit, the electric field of charges at the terminals goes from the positive terminal to the negative terminal of the battery, which is same as the direction of current in the resistor, but inside the battery, the electric field is in the opposite direction to the current.

Now, let us perform a line integral of the electric field around the loop of the circuit. When a line integral is done around a loop of the circuit, say in the direction of current all the way around, there are two contributions in this circuit, one in the resistor connected to the two terminals of the battery, which is positive, and one inside the battery, which is negative.

$$\int_{a\text{-}b} \vec{E} \cdot d\vec{l} = IR \quad \text{(Ohm's law)}$$
$$\int_{b\text{-}c\text{-}d\text{-}a} \vec{E} \cdot d\vec{l} = -\mathcal{E}_{\text{bat}} \quad \text{(Definition of battery EMF.)}$$

Therefore, the integral around a-b-c-d-a loop is found to be:

$$\oint \vec{E} \cdot d\vec{l} = IR - \mathcal{E}_{\text{bat}}.$$

Since the electric field in this situation is conservative, we set this to zero to obtain the relation between the voltage of the battery and

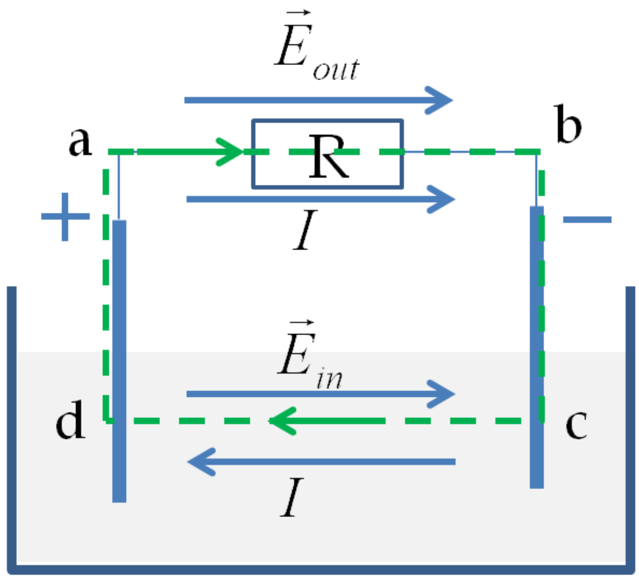

Figure 10.31: Kirchhoff's rule is based on the conservative nature of the electric field generated by charges at the terminals of a battery. The electric field is directed from positive end to negative terminal, both inside the resistor and inside the battery. The electric current has the same direction and magnitude through out the loop. Since electric field and current are in the same direction in a resistor, this means electric field is in the opposite direction to current in the battery. The integral $\oint \vec{E}\cdot d\vec{l} = 0$ around the loop since magnetic flux through the loop does not change with time.

the current in the circuit.

$$\oint \vec{E}\cdot d\vec{l} = 0 \quad \Longrightarrow IR - \mathcal{E}_{\text{bat}} = 0.$$

We have learned from Faraday that electric field does not always obey the conservative field condition, Eq. 10.42. Instead, if the loop, around which we conduct the line integral, contains a time-dependent magnetic flux through the area of the loop, then the line integral around such a loop is not zero. That means that electric field is a non-conservative field when time-dependent magnetic fluxes are around.

$$\oint \vec{E}\cdot d\vec{l} \neq 0 \quad \text{if } d\Phi_B/dt \neq 0. \tag{10.43}$$

Since Kirchhoff's loop rule is true only when electric field is conservative, Kirchhoff's loop rule is not applicable in situations where electric field acts as a non-conservative field. We replace Kirkchhoff's rule by Faraday's law around the loop of interest.

$$\text{Replace Kirchhoff's loop rule by:} \quad \oint_{\text{loop}} \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}. \tag{10.44}$$

To give an illustration of the loop equation that would result in the resistor/battery circuit shown in Fig. 10.31, let us consider placing the circuit in an external magnetic field that is pointed out-of-page and increasing in magnitude as shown in Fig. 10.32. Here we replace

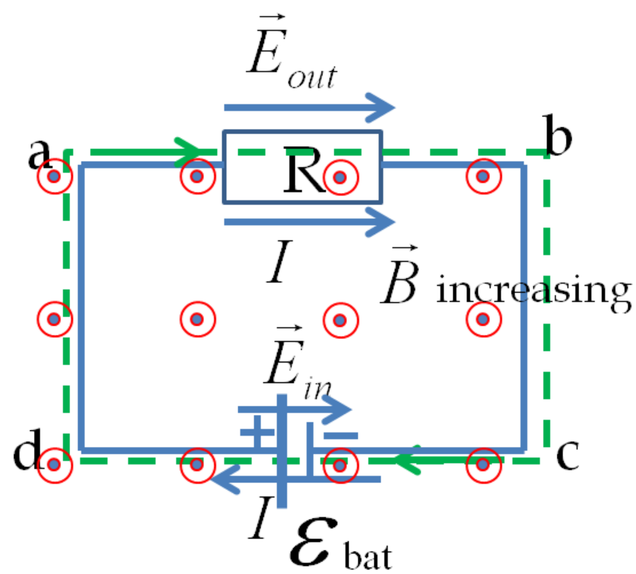

Figure 10.32: Faraday's equation replaces Kirchhoff's loop rule when magnetic flux though a loop varies with time. The equation for loop a-b-c-d-a is: $IR - \mathcal{E}_{\rm bat} = -d\Phi_B/dt$.

the battery with its symbol for simplicity. Going around the loop in the direction of the current we find that the equation for the loop will now be

$$IR - \mathcal{E}_{\rm bat} = -\frac{d\Phi_B}{dt}. \tag{10.45}$$

The right hand side of this equation is the induced EMF, $\mathcal{E}_{\rm ind}$. Therefore, current in this circuit will depend on both the permanent EMF of the battery and the induced EMF of the changing magnetic flux.

$$I = \frac{\mathcal{E}_{\rm bat} + \mathcal{E}_{\rm ind}}{R}. \tag{10.46}$$

So, *is electric field a conservative field or a non-conservative field? The answer to this question is not simple - it depends on the physical situation.* To determine the answer to this question, you will need to ask if a time-dependent magnetic flux exists in your system. If no time-dependent magnetic flux exists, then electric field will be a conservative field and you can define and work with scalar electric potential. But, if a time-dependent magnetic flux exists, then, electric field will be non-conservative and you cannot define an electric potential simply from electric field. In this physical situation, it is best to work with the Faraday's law.