6.4 PRESSURE MEASUREMENTS

The variation of pressure with height provides a simple way to measure pressure. This principle was utilized in the invention of mercury barometer by the Italian mathematician and physicist Evangelista Torricelli (1608-1647) in 1643. Since then, many other techniques of pressure measurement have been developed.

Any property that changes with pressure in a known way can be used to construct pressure-measuring devices called **pressure gauges**. The most common ones are the strain gauges which use the change in shape of a material with pressure, the capacitance pressure gauges which use the change in capacitance due to shape change with pressure, the piezoelectric pressure gauge which generates a voltage difference across a piezoelectric material under pressure difference between the two sides, and the ion gauges which measures pressure by ionizing molecules in highly evacuated chambers. Different pressure gauges are useful in different pressure ranges and under different physical situations. Here I will describe only two instruments, namely the barometer and a related instrument called the manometer.

Barometer

A mercury barometer is a tube with an opening at one end. The tube is filled with mercury and the open end covered so that no air is trapped in the tube when the tube is inverted. The filled tube is then inverted and opened in a mercury tank such that no air can enter the tube. Torricelli found that mercury in the tube empties out to a height of approximately 29.9 inches. Originally, when the same experiment was done with water in the inverted tube, it was found that the tube was filled with water to increasing heights in longer and longer tubes. As a matter of fact atmospheric pressure can support approximately 34 feet of water column. It was interpreted to indicate that nature "abhors" vacuum. Torricelli was Galileo's secretary in the last three months of latter's life. Galileo apparently suggested to Torricelli to try mercury since mercury was a much denser substance and a smaller tube might be sufficient as found later by Torricelli. Torricelli correctly interpreted that the empty space above mercury in the tube was vacuum, and in this way he was able to create sustained vacuum for the first time in science.

Using modern analysis we can understand how the height of mercury in the tube is directly related to the atmospheric pressure. Recall that the pressure at the same horizontal level in one fluid is same whether in the tube or outside as long as the fluid can flow in and

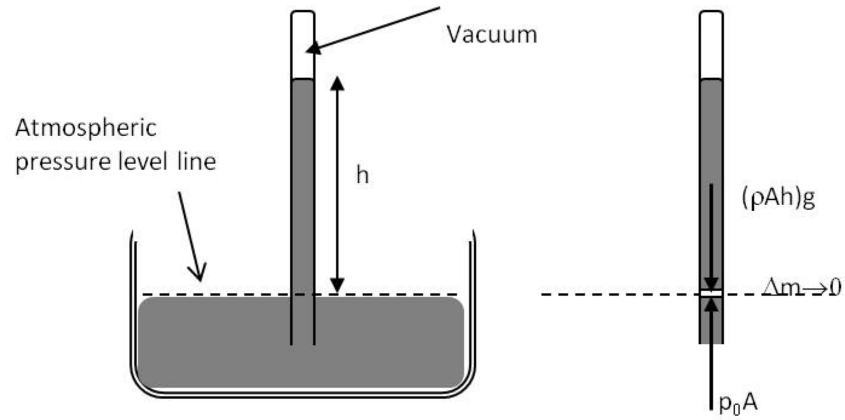

Figure 6.8: Torricellis experiment.

out of the tube freely. Consider an infinitesimally small element of mercury at the level point (Fig. 6.8). Since the mass element is not accelerating the net force on the element must be zero. From the free-body diagram the equation for vertical direction is found to be

$$p_0 A - \rho A h g - \Delta m g = 0 \quad (\Delta m \to 0) \tag{6.14}$$

With $\Delta m \to 0$, we find the following for atmospheric pressure p_0.

$$p_0 = \rho g h. \tag{6.15}$$

Hence, the height of the liquid in the tube is directly proportional to the atmospheric pressure in the barometer. Our analysis also shows that higher the density of the fluid, the lower the height h for the same pressure p_0. Since water is approximately 13.6 times less dense, you will need $13.6 \times 29.9 = 407$ inches tall tube to measure atmospheric pressure if you use water in place of mercury!

Closed-tube Manometer

The pressure of a gas can be measured by making a modification to the barometer. The inverted tube of Torricelli's experiment is made into a U-shaped tube with the open end connected to the gas container whose pressure we wish to measure (Fig. 6.9). The instrument is called a **manometer**.

Applying Newton's second law of motion on an infinitesimal fluid element Δm leads to the following equation.

$$pA - \rho A h g - \Delta m g = 0 \quad (\Delta m \to 0) \tag{6.16}$$

where p is the pressure of the gas, A the area of the cross-section of the tube, h the height difference on the two sides of the U-tube and ρ the density of the fluid in the tube (not the density of gas). Simplifying

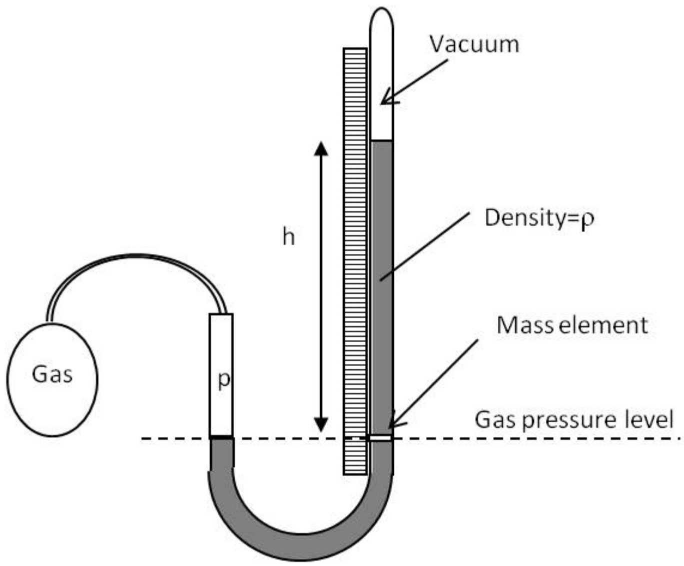

Figure 6.9: Manometer.

the equation we find the following for the pressure reading for the gas.

$$p = \rho g h. \tag{6.17}$$

The closed end of a manometer are calibrated and marked for directly reading off the pressure.

Open-end manometer

Manometers can also be constructed to read the difference of pressure of the gas and the atmospheric pressure, called the gauge pressure, if both ends of the U-tube are open as I will show now. One end of the U-tube is opened to air and the other to the gas as shown in the Fig. 6.10.

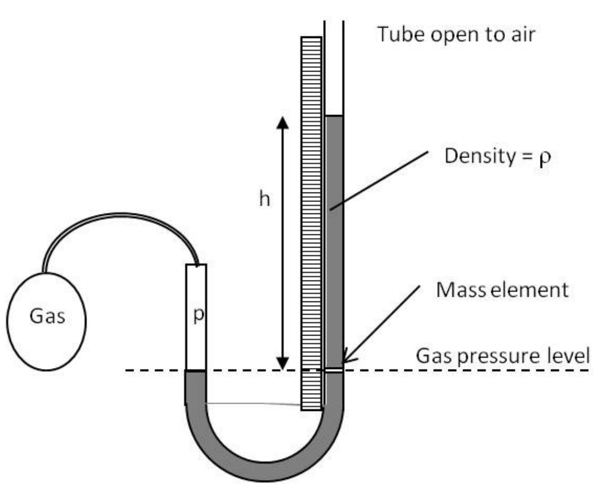

Figure 6.10: Open-end Manometer.

The competition between the pressures of gas and atmosphere results in the difference in height of the liquid in the U-tube. The height difference of the liquid in the two arms of the U-tube is the **gauge pressure** of the gas. If the gas pressure is higher than atmospheric pressure then the level of fluid in the gas side will be lower than the one exposed to the atmosphere, while if the gas pressure is less than the atmospheric pressure then the level on the gas side will be higher.

Figure 6.11: Gas tank pressure gauges.

Apply Newton's second law of motion on an infinitesimal fluid element at the lower fluid level in the arm with higher fluid level. In the figure, the fluid element is on the atmosphere side. Let p be the pressure of the gas, p_0 the atmospheric pressure, A the area of the cross-section of the tube, h the height the fluid and ρ its density (Fig. 6.10). Free-body diagram for the forces on the infinitesimal mass element leads to the following equation.

$$pA - (\rho h g + p_0)\, A - \Delta m g = 0 \quad (\Delta m \to 0) \tag{6.18}$$

Simplifying we get the pressure of the gas to be

$$p = p_0 + \rho g h \tag{6.19}$$


Figure 6.12: Tire pressure gauge.

The difference of actual or absolute pressure p and the atmospheric pressure p_0 is also called the gauge pressure.

$$\begin{aligned} p_0 &= \text{Atmospheric pressure} \\ p &= \text{Absolute pressure} \\ p - p_0 &= \text{Gauge pressure} \end{aligned}$$

Pressure gauges often measure the difference of the absolute pressure and the ambient atmospheric pressure and hence the name gauge pressure. Open manometer is one kind of pressure gauge. Many mechanical and electrical pressure gauges are available commercially which are more convenient to use than the open manometer (see margin for examples of pressure gauges).


Figure 6.13: Ionization gauge.

Example 6.4.1. Fluid Heights in an Open U-tube.

A U-tube with both ends open is filled a liquid of density ρ_1 to a height h on both sides. A liquid of density $rho_2 < \rho_1$ is poured on one side. We find that the liquid 2 settles on top of the liquid. The heights on the two sides are different. The height to the top of the liquid 2 from the interface is h_2 and the height to the top of liquid from the level of the interface is h_1. Deduce a formula for the height difference.

Solution. The pressure at the same height on the two sides of a U-tube must be same as long as the two points are in the same liquid. Therefore, we consider two points at the same level in the two arms of the tube: One point will be the interface on the side of the liquid 2 and the other will be a point in the arm with liquid 1 that is at the same level as the interface in the other arm.

$$\text{Pressure on the side with liquid 1} = p_0 + \rho_1 g h_1.$$

$$\text{Pressure on the side with liquid 2} = p_0 + \rho_2 g h_2.$$

Since, the two points are in the liquid 1 and are at the same height, the pressure at the two points must be the same pressure. Therefore, we have

$$p_0 + \rho_1 g h_1 = p_0 + \rho_2 g h_2.$$

Hence,

$$\rho_1 h_1 = \rho_2 h_2.$$

This says that the difference in heights of the two sides on the U-tube would be

$$h_2 - h_1 = \left(1 - \frac{\rho_1}{\rho_2}\right) h_2.$$

The result makes sense if we set $\rho_2 = \rho_1$, which gives $h_2 = h_1$. If the two sides have the same density, they would have the same height.