

2.5 APPLICATIONS OF FIRST LAW

2.5.1 Calorimetry

When two systems at different temperatures come in a thermal contact, energy is exchanged between them through heat and work. The energy conservation says that if the two systems together are isolated from everything else, then their combined energy will remain the same, and any gain in the energy of one will be accompanied by loss of energy by the other. Labeling the two systems as A and B we say that

$$\Delta E_A = -\Delta E_B. \quad (2.24)$$

We classify the transfer of energy from one to the other as heat Q and work W . Let Q_A be the amount of heat into A and W_A be the amount of work by A. Then, the change of energy of A is

$$\Delta E_A = Q_A - W_A.$$

Similarly, suppose Q_B be the amount of heat into B and W_B be the amount of work by B. Then,

$$\Delta E_B = Q_B - W_B.$$

Writing Eq. 2.24 in terms of heat and work we obtain

$$Q_A - W_A = -(Q_B - W_B). \quad (2.25)$$

Now, if the work done by A and by B are negligible, then this balancing of energy leads to the balancing of heat into A and heat into B.

$$Q_A = -Q_B. \quad (2.26)$$

On the right side we have the negative of the heat into B, which is equal to the heat out of B. Therefore, we can say that if neither of the two interacting systems do work, then the heat into one of the systems is equal to the heat out of the other system. We write this result in more colloquial language as

$$\boxed{|\text{Heat Lost}| \approx |\text{Heat Gained}| \quad (\text{since, } W_{\text{on either system}} \approx 0.)} \quad (2.27)$$

While working out the heat gained or the heat lost, you must pay particular attention to any phase change(s) and not only to the changes in the temperature. At each phase change you will need to account for the latent heat involved in the change of phase. I will illustrate these cautionary remarks in the examples below.

A more serious limitation in the use of Eq. 2.27 is that this equation is valid only when no work is done by either system. Therefore, we normally use Eq. 2.27 for experiments with either liquids or solids since they are somewhat incompressible. This equation can also be applied to a system of gas in a rigid container to prevent the volume of the gas from changing. If volume of the system changes much, then Eq. 2.27 cannot be used since energy conservation will not work if you take into only heat, you must also include the amount in work.

A device using the principles of heat exchange in the absence of any work can be used to measure the heat released in a particular process. Such a device is called a **calorimeter**, and the technique of measuring heat this way is called **calorimetry**. We will illustrate the idea with examples.

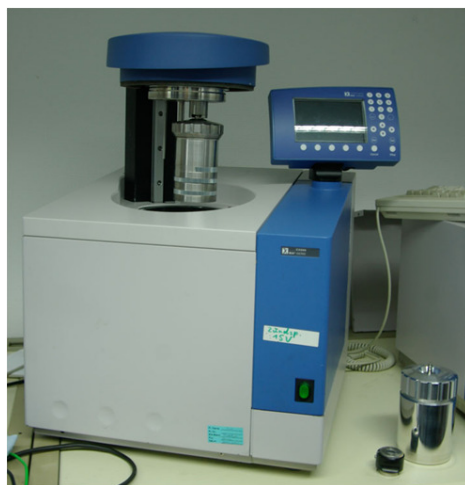


Figure 2.12: A **bomb calorimeter**. The container shown to the side is the bomb in which chemical to be tested is placed and ignited so that the thermal energy released in the explosion can be measured. Photo credit: Harbor1 via Wikimedia Commons.

Example 2.5.1. Mixing Water at Different Temperatures. An insulated 20-kg copper container contains 5 kg water at 10°C and 1 atm pressure. Two kg of water at 90°C is poured into the insulated container with water. The mixture comes to equilibrium while the pressure is maintained at 1 atm throughout the process. What is the final temperature? Assume negligible expansion or contraction during mixing.

Solution. In this problem, the two systems exchanging energy are: system A consisting of the container and 5 kg water, and system B consisting of 2 kg water. Here, the system A gains energy and the system B loses energy. Since, we have assumed that no work is done,

we will have

$$|\text{Heat gained by A}| = |\text{Heat lost by B}|.$$

We can convert this observation into an equation for the unknown final temperature t by using specific heat of water and change in temperature. Since no phase change of water occurs between 10°C and 90°C , we need not worry about the latent heat here.

It is important to note that $10^\circ\text{C} < t < 90^\circ\text{C}$ here, and the two sides of heat gained and heat lost must both be positive number. Therefore, for the heat lost side you must use $90 - t$ for the change in temperature and for the heat gained you need to use $t - 10$. We obtain the following equation for the unknown t .

$$\begin{aligned} 2\text{kg} \times 4186 \frac{\text{J}}{\text{kg}\cdot\text{C}} \times (90 - t)\text{C} &= 5 \text{ kg} \times 4186 \frac{\text{J}}{\text{kg}\cdot\text{C}} \times (t - 10)\text{C} \\ &+ 20 \text{ kg} \times 390 \frac{\text{J}}{\text{kg}\cdot\text{C}} \times (t - 10)\text{C} \end{aligned}$$

Now, we solve the equation for the unknown t .

$$t = 28^\circ\text{C}.$$

Does this answer make sense? Think about it. Why is the temperature not half-way between the hot water and the cold water?

Example 2.5.2. Mixing Water With Ice An insulated 60-kg Aluminum container has a 50 kg ice block at temperature -10°C and pressure 1 atm. Two kg of warm water at 50°C is poured into the container and the lid is closed quickly so that heat loss is negligible. Find the final state, i.e. the final temperature and the amounts of ice and water of the mixture in the Aluminum container. Assume negligible expansion or contraction of any part of the systems, and that the pressure is kept at 1 atm at all time.

Solution. Here, we do not know if the final temperature is below 0°C or above 0°C or right at 0°C . So, there are three scenarios to check out. The final temperature will go below 0°C if all of liquid water comes to 0°C , freezes, and then loses some more heat so that the total heat lost will equal that gained by ice and the container. The final temperature will be above 0°C if ice and container come to 0°C , all ice melts, and then container and ice gain some more heat so that the total heat gained would equal the heat lost by water. The final temperature will be 0°C if all heat available in the water from 50°C to 0°C minus necessary to raise the temperature of the container plus ice from -10°C to 0°C is not enough to melt the ice, OR if all the heat available in the water from 50°C to 0°C minus necessary to

raise the temperature of the container plus ice from -10°C to 0°C is negative but not more than released when all the water freezes.

We first check out the $t = 0^\circ\text{C}$ scenarios: Let us check first if heat released in simply lowering the temperature of water at 50°C to 0°C is enough to raise the temperature of ice/container to 0°C without part or all of liquid water needing to freeze. Heat available when water goes for 50°C to 0°C is $2 \times 4186 \times 50 = 418600 \text{ J}$, and heat needed to raise the temperature of the container and ice is $50 \times 2100 \times 10 + 60 \times 900 \times 10$ or $1,590,000 \text{ J}$. This says that there is not enough heat released to bring the ice/container to 0°C .

Now the questions are: (a) How much heat will be released if all water froze? and (b) Would the total heat released this way is greater than the heat needed to raise the temperature of ice/container to 0°C ? We take up these questions next.

Heat released if all water freezes at 0°C is $2 \times 333,000$, which is $666,000 \text{ J}$.

We see that total heat released $418600 + 666,000 = 1,084,600$ is less than what is needed to raise the temperature of the ice/container to 0°C . Hence, we conclude that all water freezes and the final temperature is somewhere between -10°C and 0°C . Let the final temperature be $t^\circ\text{C}$. $|\text{Heat gained}| = |\text{Heat lost}|$ equation gives:

$$\begin{aligned} \left[50 \text{ kg} \times 2100 \frac{\text{J}}{\text{kg} \cdot \text{C}} + 60 \text{ kg} \times 900 \frac{\text{J}}{\text{kg} \cdot \text{C}} \right] \times [t - (-10)] \\ = 1,084,600 \text{ J} + 2 \text{ kg} \times 2100 \frac{\text{J}}{\text{kg} \cdot \text{C}} (0 - t) \end{aligned}$$

Solving for t we find $t = -3.1^\circ\text{C}$.

2.5.2 Free Expansion of a Gas

Imagine doing the following experiment. Connect two thermally insulated vessels with a tube and a stopcock as shown in Fig. 2.13. Create vacuum in one vessel and fill the other vessel with air at high pressure. Put a thermometer in the chamber that has gas in it. Now, open the stopcock so that the gas flows into the empty chamber. What does the temperature read after the equilibrium has been re-established?

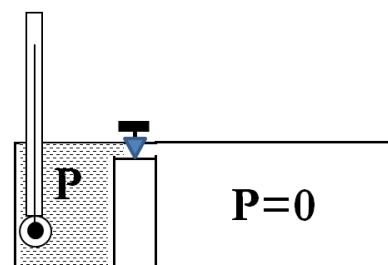


Figure 2.13: Free expansion into vacuum.

As the gas expands against zero pressure, no work is done on it even though its volume changes in the process.

$$W = 0 \quad (p = 0 \text{ for free expansion})$$

Since the vessels are insulated the heat Q into the vessels must be zero.

$$Q = 0 \quad (\text{since insulated})$$

Hence, the change in the internal energy is also zero for the free expansion of the gas kept in an insulated environment.

$$\Delta U = 0 \quad (\text{insulated free expansion})$$

Even though the internal energy will not change in an insulated free expansion, the temperature may not remain the same. This is due to the fact that the molecules of real gases have a net attraction towards each other. When the separation between the molecules increases, this leads to an increase in the potential energy of the molecules, as you have seen for attractive interactions such as gravitation. If the total energy remains constant, the kinetic energy of the molecules must decrease in this process. Since, the temperature of a gas is a measure of the thermal motion, we would expect the temperature to decrease in an insulated free expansion.

$$Q = 0, W = 0, \Delta U = 0 \implies \Delta T < 0 \text{ if } \Delta V > 0. \text{ Real Gases} \quad (2.28)$$

This does not happen for an ideal gas since the interaction between molecules is ignored to arrive at the ideal gas model. The constancy of energy in an ideal gas expanding adiabatically will imply that the kinetic energy of the molecules of an ideal gas will not change in this process. That would mean that, if you expand an ideal gas into vacuum in an insulated process, the temperature of the gas would be unchanged.

$$Q = 0, W = 0, \Delta U = 0 \implies \Delta T = 0 \text{ even when } \Delta V > 0. \text{ Real Gases} \quad (2.29)$$

We will show in the chapter on Kinetic Theory, that the internal energy of ideal gases does not depend on the volume or pressure but only on the temperature consistent with the observation here on the free expansion.

$$\begin{aligned} \text{Ideal Gas: } U \text{ is a function of temperature } T \text{ only.} \\ \text{Real Gases: } U \text{ is a function of } (T, V) \text{ or } (T, p) \text{ or } (p, V). \end{aligned} \quad (2.30)$$

Kinetic theory of gases predicts that the internal energy of ideal gas depends on the **degrees of freedom** of the motion of each molecule and the temperature of the gas. For instance, if molecules of a gas consists of only one atom per molecule, such as the Helium or Argon gas. Then each molecule has only three degrees of freedom, which are the three translational directions. These gases are called

monatomic gas. The internal energy of n moles of a monatomic ideal gas is given by the following formula.

$$U = 3 \times \frac{1}{2}nRT, \quad (\text{monatomic ideal gas}) \quad (2.31)$$

where the multiplier 3 is for the three degrees of freedom. A diatomic molecule has six degrees of freedom, which are three translational directions of motion, two axes of rotation perpendicular to the bond, and vibration of the atoms along the chemical bond between the two atoms. At ordinary temperatures, the energy of the molecules is not sufficient to be excited in the vibrational mode, and very few molecules if any are vibrating along the bond. Therefore, we say that at ordinary temperatures, a diatomic molecule has only five active degrees of freedom with each active degree having energy $\frac{1}{2}nRT$. Therefore, n moles of a diatomic gas treated as an ideal gas will have the following internal energy at temperatures when vibrational mode is not active.

$$U = 5 \times \frac{1}{2}nRT \quad (\text{diatomic ideal gas}) \quad (2.32)$$

In 1843 James Joule conducted experiments to determine the change in temperature of a gas when expanded in vacuum. He constructed a rigid container with two chambers connected by a short tube with a stopcock (Fig. 2.14). One chamber was filled with air at high pressure and the other evacuated. The apparatus was then placed in an insulated tank containing water. The change in temperature of the water was measured when the stopcock was opened to let the gas expand into the empty chamber.

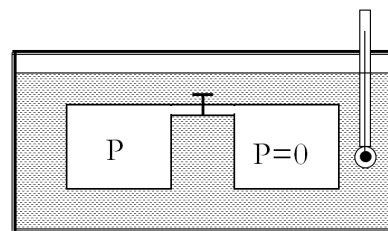


Figure 2.14: Joule experiment.

He expected to find the change in the internal energy of the gas by finding the heat exchange with the water bath which would be reflected in the change in the thermometer readings. He did not find any change in the temperature of the water bath in his experiments. Although the temperature change of air in Joule experiment should have been detectable, he failed to observe it because the change in the temperature of water was too small due to the large specific heat of water compared to that of air.

2.5.3 Specific Heat of an Ideal Gas

The specific heat of an ideal gas for constant pressure and constant volume processes have a very simple relation. We will derive this relation from first principles, using only the definitions of the specific heat and the fact that the internal energy of an ideal gas is only

a function of temperature. We will work with molar specific heats. The definitions of molar specific heats are based on heat for changing temperature by one degree for a particular process. Thus, C_V and C_p refer to constant volume and constant pressure respectively. Let n be the number of moles in the gas. Let heat into the system when the temperature of the gas changes by ΔT be Q_V the process occurs at constant volume and Q_p when the process occurs at constant pressure. I have placed subscripts to Q to indicate the heat in the two situations. Then, the definition of C_V and C_p says that

$$C_V = \frac{1}{n} \frac{Q_V}{\Delta T} \quad (2.33)$$

$$C_p = \frac{1}{n} \frac{Q_p}{\Delta T} \quad (2.34)$$

Now, we can use first law of thermodynamics for gas to replace Q by $\Delta U + p\Delta V$. For constant pressure process we have $\Delta V \neq 0$. Therefore,

$$C_V = \frac{1}{n} \left(\frac{\Delta U}{\Delta T} \right)_V \quad (2.35)$$

$$C_p = \frac{1}{n} \left(\frac{\Delta U}{\Delta T} \right)_p + \frac{p}{n} \left(\frac{\Delta V}{\Delta T} \right)_p \quad (2.36)$$

where I have placed subscripts on the right side to indicate the type of process for each change. We have stated above the change in the internal energy of an ideal gas is the same for both processes since U of an ideal gas only depends on the temperature. Therefore, the first term on the right side of Eq. 2.36 is same as C_V . Therefore, we have the following relation between the two specific heats.

$$C_p = C_V + \frac{p}{n} \left(\frac{\Delta V}{\Delta T} \right)_p \quad (2.37)$$

The second term on the right side can be simplified by using the ideal gas equation since we are dealing with ideal gas with the following result.

$$\boxed{C_p = C_v + R} \quad (\text{Ideal Gas}) \quad (2.38)$$

If the molecules of an ideal gas contain only one atom per molecule, such as helium gas, then by kinetic theory in a later chapter we show that the internal energy is given by:

$$U = \frac{3}{2} nRT \quad (\text{monatomic ideal gas}) \quad (2.39)$$

This gives the following value for C_v for a monatomic ideal gas.

$$C_v = \frac{3}{2} R \quad (\text{monatomic ideal gas}) \quad (2.40)$$

Hence, C_p of a monatomic ideal gas is

$$C_p = \frac{5}{2}R \quad (\text{monatomic ideal gas}) \quad (2.41)$$

For diatomic ideal gas at temperatures when vibration mode is not excited, the specific heats will be

$$C_v = \frac{5}{2}R \quad (\text{diatomic ideal gas}) \quad (2.42)$$

$$C_p = \frac{7}{2}R \quad (\text{diatomic ideal gas}) \quad (2.43)$$

Example 2.5.3. Heating an Ideal Gas. Find heat needed to raise the temperature of 2 kg of helium, treated as a monatomic ideal gas, from 20°C to 200°C at constant pressure of 1 atmosphere.

Solution. The number of moles of helium in 2-kg of nitrogen gas is:

$$n = 2 \text{ kg} \times \frac{1 \text{ mole}}{4 \text{ g}} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 500 \text{ moles.}$$

Using the expression for the molar specific heat at constant pressure of a monatomic ideal gas we find the heat needed:

$$\begin{aligned} Q &= nC_p\Delta T = \frac{5}{2}nR\Delta T \\ &= \frac{5}{2} \times 500 \text{ moles} \times \frac{8.31 \text{ J}}{\text{mole.K}} \times 180 \text{ K} = 1.87 \times 10^6 \text{ J.} \end{aligned}$$

2.5.4 Joule-Thomson Effect

Imagine pumping a gas through a tube that has a porous plug such that the pressure on the two sides of the plug is different. Let p_1 be the pressure on one side of the plug and a different pressure p_2 on the other side as shown in Fig. 2.15. We will take $p_1 > p_2$ for the discussion below. Let the pressure on each side be maintained steady by a pump and the entire system be thermally insulated. The process of pumping a gas this way is called **Joule-Thomson process** or **Joule-Thomson throttling**. In this subsection we will find a relation between the properties of the gas on the two sides of the porous plug. In the process we will discover the existence of another useful thermodynamic property called enthalpy.

Suppose that in some time duration Δt a quantity of gas, say n moles, occupying a volume V_1 on the p_1 side pass into the other side where the gas occupies a volume V_2 at pressure p_2 . We say that the process takes the gas from a state (p_1, V_1) to another state (p_2, V_2) .

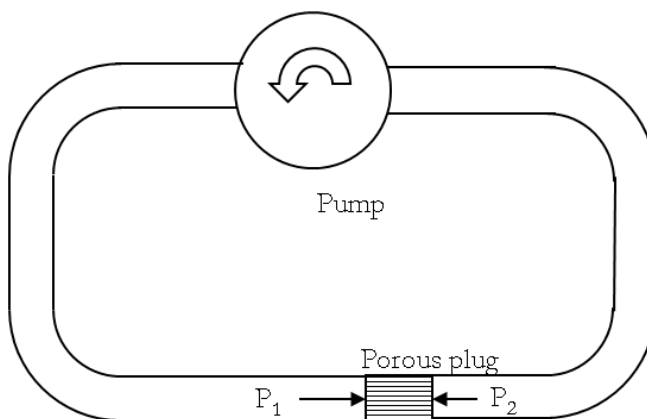


Figure 2.15: Performing a Joule-Thomson throttling experiment.

The throttling process does not occur slowly or quasi-statically. However, we can still apply the first law of thermodynamics to obtain important conclusions about this process. Since the system is thermally insulated, the change in internal energy $U_2 - U_1$ of the gas will be equal to the negative of the amount of work done by the gas in this nonquasistatic process. How do we calculate the work done by the gas here since we do not have $p(V)$ for the process? We take a different approach as follows.

The trick is to imagine a process in which we empty n moles of gas occupying a volume V_1 while keeping the pressure at p_1 all the time. Let W_1 be the work done by the gas in this process. And, then we fill a volume v_2 of space with n moles of gas at pressure p_2 . Let the work done by the gas in this process by W_2 . The net work by the gas will be

$$W_{\text{net}} = W_1 + W_2.$$

The works W_1 and W_2 are readily calculated since they are constant pressure processes with the following results.

$$W_1 = -p_1 V_1 \quad (2.44)$$

$$w_2 = p_2 V_2 \quad (2.45)$$

Hence the change in the internal energy of the gas under consideration is

$$U_2 - U_1 = p_1 V_1 - p_2 V_2. \quad (\text{thermally insulated})$$

Rearranging, we find that in the Joule-Thompson process the combination, $U + pV$, is conserved.

$$\boxed{U_2 + p_2 V_2 = U_1 + p_1 V_1 \quad (\text{Joule-Thomson process})} \quad (2.46)$$

The quantity $U + pV$ is called **enthalpy** or **heat function** H . It is a state function similar in status to the internal energy U .

$$\boxed{H = U + pV. \text{ Definition of enthalpy}} \quad (2.47)$$

You can think of H as energy needed to “create” a system of internal energy U and volume V at constant pressure p . A process that conserves enthalpy is called iso-enthalpic process. The Joule-Thompson process conserves enthalpy. Comparing the final enthalpy and initial enthalpy we can conclude that if pressure times volume goes up, then internal energy must go down and vice-versa. Joule-Thompson throttling is often used in cooling and **liquefying gases**.

2.5.5 Enthalpy and First Law

The first law of thermodynamics can also be written in terms of enthalpy change instead of the internal energy change. In this section we will work out the first law in the infinitesimal form in terms of the enthalpy. We will use $p dV$ for the work associated with the infinitesimal process.

From the definition of the enthalpy, $H = U + pV$, we find that the change in enthalpy for an infinitesimal process will be the difference,

$$(H + dH) - H = [(U + dU) + (p + dp)(V + dV)] - (U + pV)$$

Expanding and dropping quadratic terms in infinitesimals, we find the following.

$$dH = dU + p dV + V dp \quad (2.48)$$

Putting dQ for $dU + p dV$ from the expression for the first law of thermodynamics we find an alternative statement of the first law.

$$\boxed{dH = dQ + V dp.} \quad (2.49)$$

This relation says that, for a constant-pressure quasi-static process, the enthalpy change is equal to the heat exchanged.

$$dH|_{const\ p} = dQ|_{const\ p} \quad (2.50)$$

Many engineering processes and chemical reactions are conducted at constant pressures. For these systems enthalpy is a more appropriate quantity for analysis. For instance, if a chemical reaction that releases 10 kJ of energy takes place at atmospheric pressure, we say that the enthalpy of the products is less than those of the reactants by 10 kJ. The latent heat is another example of enthalpy since latent heat is usually measured at atmospheric pressure.

Internal energy is equal to the heat exchanged when a process is conducted at constant volume while enthalpy is the heat exchanged at constant pressure. Hence, internal energy is related to the specific heat at constant volume C_v and enthalpy is related to specific heat at constant pressure C_p . First law of thermodynamics yields the following results for internal energy and enthalpy of constant volume and constant pressure processes respectively.

$$\boxed{dU = nC_v dT} \quad (2.51)$$

$$\boxed{dH = nC_p dT} \quad (2.52)$$

2.5.6 Quasi-static Adiabatic Expansion of an Ideal Gas

We have defined an adiabatic process above as a process in which the system is kept insulated. A quasistatic adiabatic process of an ideal gas system occurs in many useful situations as we will see in the next chapter. In this subsection we will study the quasi-static adiabatic process for an ideal gas in a little more depth than done before.

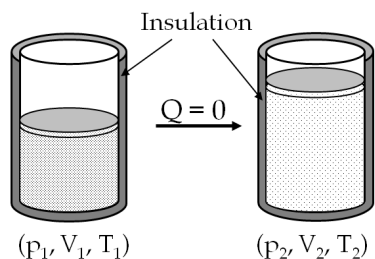


Figure 2.16: Expansion under insulating environment.

To be concrete, consider n moles of an ideal gas in a thermally insulated cylinder (Fig. 2.16). Let the pressure, volume and temperature of the gas be p_1 , V_1 , and T_1 respectively. These variables are related by the ideal gas law.

$$p_1 V_1 = nRT_1 \quad (2.53)$$

Imagine an adiabatic process that changes the pressure, volume and temperature to p_2 , V_2 , and T_2 respectively. These variables must also be related by the ideal gas law.

$$p_2 V_2 = nRT_2 \quad (2.54)$$

We wish to find the path in the (p, V) plane for this adiabatic process. The path would be given by a function $p(V)$ which we determine in this section. We can use this function to find an expression for work by an ideal gas in a quasi-static adiabatic process by performing the integral $W = \int p(V) dV$.

Now, on to the derivation. Note that according to the first law we have the following for an infinitesimal adiabatic process.

$$dU = -pdV \quad (\text{since adiabatic, } Q = 0) \quad (2.55)$$

We have seen that the internal energy change can also be written in terms of the specific heat at constant volume.

$$dU = nC_v dT. \quad (2.56)$$

Therefore, for an adiabatic process of a gas we will have the following relation.

$$nC_V dT = -pdV \quad (\text{General}) \quad (2.57)$$

Now, for an ideal gas we can use the ideal gas equation, $pV = nRT$, to change this equation to

$$\frac{dT}{T} = -\frac{R}{C_v} \frac{dV}{V} \quad (\text{Ideal Gas}) \quad (2.58)$$

This equation can also be written in terms of the ratio of C_p and C_v , $\gamma \equiv C_p/C_v$ (read gamma) we find the following relation.

$$\frac{dT}{T} = -(\gamma - 1) \frac{dV}{V} \quad (2.59)$$

where $\gamma = C_p/C_v$. Integrating this equation, we find that

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad (2.60)$$

This means that $TV^{\gamma-1}$ in an adiabatic process of an ideal gas is constant.

$$\boxed{TV^{\gamma-1} = \text{constant} \quad (\text{quasi-static adiabatic process})} \quad (2.61)$$

Using the equation of state for an ideal gas we can rewrite Eq. 2.60 in other forms. The form of this equation in terms of pressure and volume of the initial and final states connected by a quasi-static adiabatic process is

$$p_1 V_1^\gamma = p_2 V_2^\gamma \quad (2.62)$$

This implies that in the pV -plane the adiabatic path will be such that $pV^\gamma = \text{constant}$.

$$\boxed{pV^\gamma = \text{constant.}} \quad (2.63)$$

A quasi-static adiabatic process can be drawn in the pV diagram. As shown in Fig. 2.17 a quasi-static adiabatic process forms a steeper curve than the isotherms and mediates between two isotherm curves. Analytically, while $p \sim 1/V$ for an isotherm, $p \sim 1/V^\gamma$ for an adiabatic process with $\gamma > 1$. In the pV plane an adiabatic process will therefore take the system from one isotherm to another: the temperature of gas will decrease if expanded adiabatically and go up if contracted adiabatically.

Example 2.5.4. Adiabatic Expansion of a Monatomic Ideal Gas. A cylindrical container containing 1.5 moles of Argon (ideal monatomic gas) at a temperature of 27°C has a piston that can slide freely so that volume can adjust freely making the pressure outside

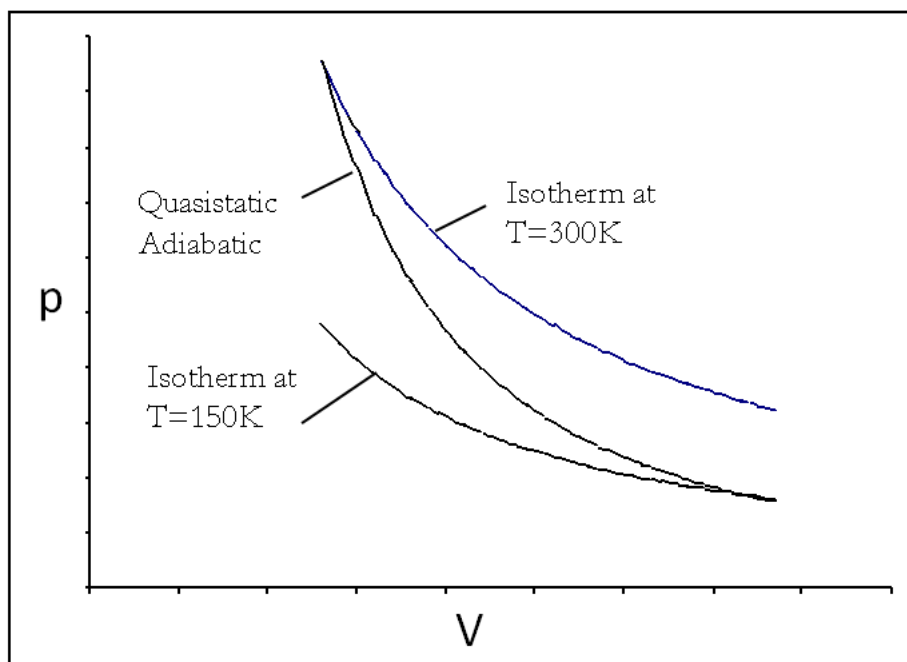


Figure 2.17: An adiabatic process mediates between isotherms.

the same as inside. Initially the volume occupied by the gas is $4.0 \times 10^{-3} \text{ m}^3$. The cylinder and piston are insulated, and therefore no heat enters or leaves the cylinder. The piston is then pushed in slowly till the volume becomes half as much. What are the final pressure and temperature of the gas?

Solution. Note that the process is adiabatic. From the above we have the following relations between two states of an ideal gas connected by an adiabatic process.

$$\begin{aligned} (1) \quad p_1 V_1^\gamma &= p_2 V_2^\gamma \\ (2) \quad T_1 V_1^{\gamma-1} &= T_2 V_2^{\gamma-1} \end{aligned}$$

We recall that the ratio of specific heats γ for a monatomic ideal gas is equal to $5/3$.

$$\gamma = \frac{5}{3} \quad (\text{monatomic ideal gas})$$

Now, we can work out the numbers for the initial state to be used in these equations.

$$\begin{aligned} T_1 &= 27 + 273.15 = 300.15 \text{ K} \\ V_1 &= 4.0 \times 10^{-3} \text{ m}^3 \\ p_1 &= \frac{nRT_1}{V_1} = \frac{1.5 \text{ moles} \times 8.31 \text{ J.K}^{-1} \text{ mole}^{-1} \times 300.15 \text{ K}}{4.0 \times 10^{-3} \text{ m}^3} \\ &= 9.35 \times 10^5 \text{ Pa.} \end{aligned}$$

The volume of the final state is also given here.

$$V_2 = 2.0 \times 10^{-3} \text{m}^3.$$

Using $pV^\gamma = \text{constant}$ we can easily find the pressure of the final state.

$$\begin{aligned} p_2 &= p_1 \left(\frac{V_1}{V_2} \right)^\gamma = 9.35 \times 10^5 \text{ Pa} \times 2^{5/3} \\ &= 2.97 \times 10^6 \text{ Pa}. \end{aligned} \tag{2.64}$$

Now, we can use the equation of state for initial or final state to calculate the final temperature. For the data of the final state the calculation goes as

$$\begin{aligned} p_2 V_2 &= nRT_2 \implies \\ T_2 &= \frac{p_2 V_2}{nR} = \frac{2.97 \times 10^6 \text{ Pa} \times 2.0 \times 10^{-3} \text{m}^3}{1.5 \text{ moles} \times 8.31 \text{ J.K}^{-1} \text{mole}^{-1}} = 477 \text{ K}. \end{aligned}$$

This temperature in degrees Celsius is 204°C .