6.9 PROBLEMS

Problem 6.9.1. A U-tube has a uniform cross-sectional area of 3 cm^2. First 2 kg mercury is poured in, and then, one end is sealed. The closed end has air of volume 60 cm^3. Now, 0.5 kg of mercury is poured in the tube from the open end. Find the rise in the mercury level in the close end. Assume air in the closed end obeys Boyle's law $p_1V_1 = p_2V_2$ where p_1, V_1 and p_2, V_2 are pressures and volumes of air at two instances. Density of Hg = 13,500 kg/m^3.

Ans Hint: The rise x in the closed end will be a solution of $(\rho g)\, x^2 - (P_{atm} + \rho g h + \rho g H_0)\, x + \rho g h H_0 = 0$ where h is the height of additional mercury poured in the open end and H_0 is the original height of air in the closed end.

Problem 6.9.2. A U-tube open at both ends has water on one side and olive oil on the other. The two liquids meet exactly at the bottom. If the height on the oil side is 10 cm, what is the height on the water side?

Ans: 9.2 cm.

Problem 6.9.3. A tank is filled with water to $\frac{1}{4}$ of the height, and the rest is filled with oil of density 0.7 g/cc. How does the pressure in the tank vary with depth?

Ans Hint: It will be a step-wise function.

Problem 6.9.4. The density of a fluid changes with depth according to the following formula.

$$\rho(h) = \rho_0 + ch,$$

where h is the height from the top, ρ_0 the density at the top, and c a small constant. Find a formula for the variation of pressure with depth if the pressure at the top is p_0.

Ans: $p(y) = g\left(\rho_0 y + \frac{1}{2}cy^2\right) + p_0$, where y is the depth from the top.

Problem 6.9.5. A tube filled with a fluid of uniform density ρ is put in an elevator. What is the pressure at depth h if the elevator has an upward acceleration of magnitude a?

Ans: $p = p_0 + \rho(g + a)h$.

Problem 6.9.6. A steel ball of mass 2 kg is hung from a support, and dipped in water as shown. The arrangement is put in an elevator. (a) What will be the tension in the string when the elevator has an upward accelerating of magnitude 2 m/s^2? (b) What will be the tension in the string when the elevator has a downward acceleration of magnitude 2 m/s^2? (c) What is the tension in the string when the elevator is in free fall?

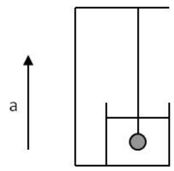

Figure 6.28: Problem 6.9.6.

Problem 6.9.7. A piece of plastic floats in water with 90% of its volume submerged. An oil of density 0.7 g/cc is poured over the water. It is found that less volume of plastic piece is submerged in water. Find the fraction of volume of the plastic piece under water when it is completely submerged in oil.

Ans: 2/3.

Problem 6.9.8. One way of finding surface tension of a liquid is based on the measurement of the magnitude of the force F needed to lift a ring from the surface of a liquid. Let R be the radius of the ring. Derive a formula for the surface tension γ in terms of F and R.

Ans: $\gamma = (F - Mg)/4\pi R$.

Problem 6.9.9. The pressure inside a soap bubble is higher than the pressure outside. Derive a formula for the difference in the pressure in terms of the surface tension γ and the radius R of the bubble.

Ans: $\Delta P = 4\gamma/R$.

Problem 6.9.10. How much weight can be supported by a 4-in diameter suction cup stuck to a flat glass ceiling in such a way that there is a perfect vacuum inside the suction cup.

Ans: 3290 N.

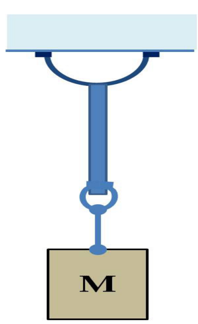

Figure 6.29: Problem 6.9.10.