1.2 FUNDAMENTAL UNITS

In order to understand a physical phenomenon, we make careful measurements of the relevant physical quantities. To facilitate communication among people and to build devices that would work with each other, the measurements of physical quantities are expressed in terms of agreed-upon standards. Length, time, and mass are fundamental quantities in mechanics. Other fundamental quantities will be added to this list when we study heat, electricity and optics.

1.2.1 Length

Length is a measure of the distance between two points in space. Before, the French Revolution (1790), different standards of length were used in different countries, and even different localities in the same country; for instance, a Greek foot was approximately 1.012 times the English foot, a Roman foot was approximately 0.97 of an English foot, etc. Furthermore, to make matters worse, the multipliers between different units in common use were not uniform; for instance, there are three feet in a yard, and twelve inches in a foot.

In 1792, the government of France after the French revolution adopted a new system of weights and measures with meter as the fundamental unit of length. The name meter comes from the Latin word metrum and the Greek word metron, both meaning "measure". The meter was defined as 10^{-7} times or one ten-millionth of the distance on the meridian from the North Pole to the equator passing through Paris. The factor was chosen to get a size close to the "human scale". Later it was found that prototypes based on earth-based definition were 0.2 mm too short due to the flattening of earth due to its rotational motion.

In 1889, the first meeting of International Committee for Weights and Measures (CIPM for Comité International des Poids et Mesures) replaced the earth-based meter to the distance between two fine markings on a Platinum-Iridium rod kept at zero degrees Celsius temperature and standard pressure at the International Bureau of Weights and Measures (BIPM, Bureau International des Poids et Mesures) near Paris, France. Accurate copies of the original rod were made and distributed to other standard-keeping laboratories throughout the world. These secondary standards were used to produce more accessible copies such as meter rulers for the general public.

With the advancement in optical technology, it became possible to measure lengths more precisely than the fine markings on the standard meter rod. In 1960 a new standard for meter based on the wavelength of orange-red light emitted by Krypton-86 in a gas discharge tube was adopted. The meter was redefined to be equal to $1,650,763.73$ wavelengths of this light.

To reduce further uncertainty in measurements, in 1983 General Conference on Weights and Measures (CGPM, Conférence Générale des Poids et Mesures) replaced the definition of meter based on Krypton-86 to the one based on the measurement of the speed of light. An exact value of $299,792,458$ m/s for the speed of light is assumed which is then used to define the meter.

One **meter** is the length of the path travelled by light in vacuum during a time interval of $1/299,792,458$ of a second.

Note that this way of defining a meter uses the unit of time (second) to define the unit of length. With this definition of the unit of length, the precision of length is now tied with the precision of time measurements. We will see below that the time measurements have become extremely accurate and therefore a better unit to serve as a base unit for other units.

1.2.2 Time

To define time we think of either a natural phenomenon that repeats itself or an experiment that can be performed repeatedly. For instance, oscillations of a pendulum provides a basis for time in an experimental setting while earth's rotation provides a naturally occurring phenomenon that has been used for time measurements.

Figure 1.2: Sundial in thyme garden at Minnesota Landscape Arboretum. Photographed June 17, 2007 at 12:21 solar time. Photocredit: S. E. Wilco, via Wikimedia Commons.

The SI unit of time is one second. How one second came to be a standard of time is a fascinating story unto itself in the history of science. It is known that ancient civilizations used the apparent motion of celestial bodies across the sky - Sun, Moon, planets and stars - for keeping track of the passage of time and seasons. It is known that Egyptians made time keeping devices and used a similar system as our own.

For instance, the current division of a year into 365 days seems to have come from Egyptian calendar as far back as 3100 BCE (Before Common Era) based on the rising of the "Dog Star" in Canis Major, now called Sirius, next to the sun every 365 days, which coincided with the flooding of Nile. Egyptians had built obelisks (slender, tapering, four-sided monuments) as far back as 3500 BCE whose shadow was used to determine time during the day. The obelisks were like primitive sundials and had markings at the base to indicate the shadows corresponding to the shortest and longest days of the year.

Around 1500 BCE the Egyptians invented the sundial that divided the day from sunrise to sunset into ten parts plus two "twilight hours". Similarly they also divided nighttime into 12 hours thus making a total of 24 hours in a full day. The divisions of an hour into sixty minutes and a minute into sixty seconds is said to have come from the Sumerian culture, which had a sexagesimal system that was based on number 60.

Egyptians also invented water clock or clepsydra before 1500 BCE, the earliest time keeping device not dependent on the motion of celestial objects. Greeks started using water clocks around 325 BCE and built even more impressive and elaborate water clocks. The complexities were added to make the flow of water as steady as possible. Despite these efforts it was very difficult to control the water flow with high accuracy and new mechanical clocks were needed.

Little progress after the Egyptian inventions seems to have been made in time keeping until Galileo Galilei (1564 - 1642) who suggested using the natural period of a pendulum. Although Galileo sketched a design of a pendulum clock, he never constructed it. The first pendulum clock was built by Christian Huygens of Netherlands in 1656. It had an error of less than 1 minute a day and was the most accurate clock to date. Christian Huygens also invented the balance wheel and spring assembly in 1675, which led to the construction of more accurate clocks. The oscillations of the balance wheel, which oscillates at around 5 cycles per second, provide the time standard for the mechanical watch. In 1889 Sigmund Riefler's clock was made that kept time with an error of less than a hundredth's of a second a day and became a standard fixture for astronomers.

With the discovery of piezoelectricity in 1880 by Pierre and Jacques Curie, the Curie brothers, it was found that piezo-crystals of quartz vibrate at a definite frequency when one applies voltage upon them. A vibrating quartz crystal generates an oscillating current of constant frequency that can be determined quite accurately with appropriate electrical circuitry. In 1927 a Canadian-born telecommunications engineer Warren Morrison (1896-1980) invented the quartz watch. Morrison and others demonstrated that the accuracy of time based on quartz crystals far exceeded clocks based on balance wheel and spring assembly. Today inexpensive electronic clocks based on quartz vibration are commonplace.

Figure 1.3: A quartz watch by

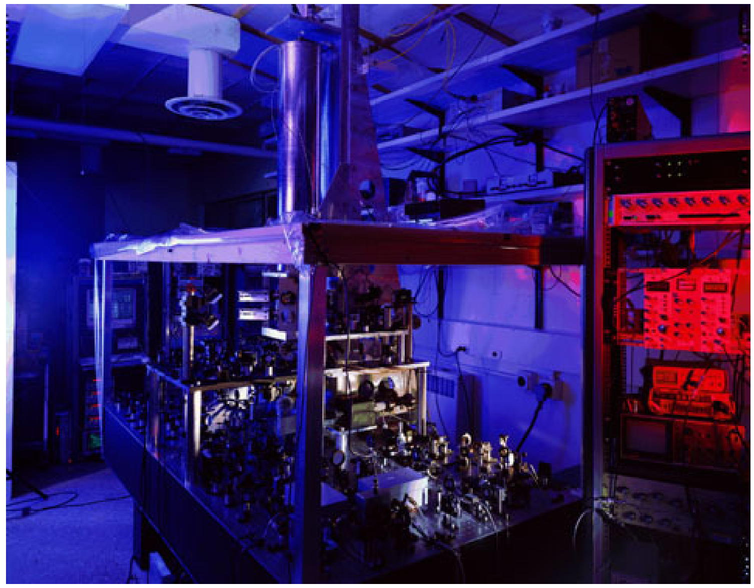


Figure 1.4: Atomic clock NIST-F1 at National Institute of Standards and Technology, USA.

Despite a better performance of quartz crystals, they are no match for atomic clocks developed in 1940's and 50's. The possibility of atomic clock based on atomic beam magnetic resonance was first suggested in 1945 by I. Rabi of Columbia University (New York). In 1949 the National Bureau of Standards of United States (now called the National Institute of Standards and Technology or NIST) developed the first atomic clock using ammonia molecule. However, the atomic clock based on ammonia molecule was not much better than the existing quartz clocks. In 1955 Louis Essen at the National Physical Laboratory in United Kingdom constructed the **world's first atomic clock** based on the atomic transitions of cesium atoms that had an accuracy of 1 sec in 300 years. The measured time using the atomic clock was compared with the time based on the rotation of earth and found to be much more accurate and stable. Therefore, in 1967 the 13th General Conference of Weights and Measures decided to replace the definition of a second by the following.

One **second** is the duration of $9,192,631,770$ periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

Although there have been improvements in atomic clocks, the definition of a second today is the one adopted in 1967. Atomic clocks are getting better everyday, and in 2010 the reported uncertainty for NIST-F1 clock shown in Fig. 1.4 was merely 3×10^{-16} second in one second, i.e. 1 second in 3 million years - a fantastic precision!

1.2.3 Mass

Mass is a measure of mechanical response of an object. Two objects of equal mass, regardless of their chemical content, shape or size, are accelerated equally when subjected to the same force, and two objects of different masses have different accelerations when subjected to the same force.

The SI unit of mass, the kilogram (kg), is the only base quantity now that is still defined by a physical artifact. The original sample was an alloy made up of 90% platinum and 10% iridium by mass in the shape of a cylinder of height 39 mm and diameter 39 mm in 1879 by George Matthey of Johnson Matthey and stored at the atmospheric pressure in a special triple-bell jar at BIPM, the International Bureau of Weights and Measures near Paris. The alloy was chosen for its non-corrosive properties and the shape was chosen to correspond to the minimum area for a given volume of a cylinder; the spherical shape would be better for minimizing the surface exposed, but since spheres roll off easily it was decided that a cylinder would serve better. The definition of a kilogram can be given as follows.

One **kilogram** is the mass of the prototype of the kilogram kept at the International Bureau of Weights and Measures.

Forty copies of the original prototype were made in 1882 and distributed to various countries. Copies of these secondary standards were made widely available to the tradesmen and general public. Thus all 1-kg samples are traceable to the international prototype kept at BIPM.

The unit of kilogram defined by an artefact has some intrinsic problems. For instance, the prototype may be damaged or corrode due to oxidation or other wear and tear due to the environment. As a result of these problems, the prototype is said to be gaining approximately 1 micro-gram per year. Therefore, it is hard to keep the standard kilogram constant to a very high degree of precision. Presently, several new methods for defining kilogram are being investigated. A particularly attractive possibility is to define the kilogram based on the mass of a fixed number of molecules of a substance that can be made with high purity. In another method developed at National Institute of Standards and Technology (NIST), force of gravity on a standard kilogram is balanced by magnetic force between two coils in a **Watt balance** (Fig. 1.5).

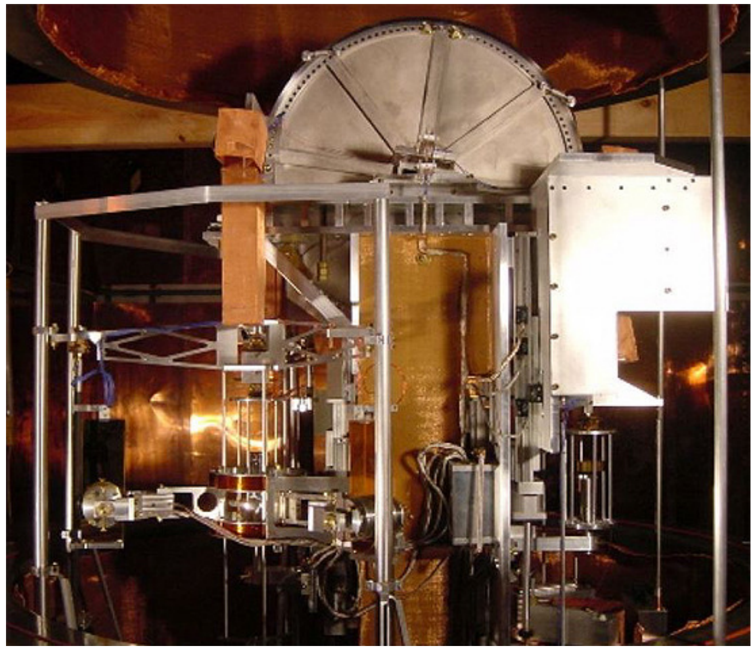


Figure 1.5: The Watt balance at National Institute of Standards and Technology, USA. (Photo by Richard Steiner)

1.2.4 Relation Between Metric and Imperial Units

Although metric units are preferred in scientific circles, the Imperial units, also called the English units, are also used commonly at least in the United States. Therefore, it is important to note their relations.

The Imperial unit of length such as inch (in), foot (ft), yard(yd) and mile(mi) do not have independent standards. Instead they are now defined in terms of the metric unit of meter. The unit of inch is taken to be exactly 2.54 cm, and the conversion of other units into the metric units are made by first converting them into inches, and then using this exact conversion of inches into centimeters.

$$\boxed{1\ \text{in} = 2.54\ \text{cm} \qquad (\textit{Exactly}).} \tag{1.1}$$

The other units of length in the English system have the following relations to inch.

$$\begin{aligned} 1\ \text{ft} &= 12\ \text{in} \\ 1\ \text{yd} &= 3\ \text{ft} \\ 1\ \text{mi} &= 1760\ \text{yd} \end{aligned}$$

The English unit of mass is a pound (lb) or pound-mass, which is approximately 1 lb = 453.59237 grams $\approx$ 453.6 grams. The English unit of time is same as the metric unit of time, namely, second.

1.2.5 Other Metric Units

Often a meter, a kilogram, or a second is too large or too small a unit for the physical phenomena under study. In the metric system, multiples of positive and negative powers of 10 are then used to simplify the numerical values to ordinary sizes and unit names are given to various multiples of the fundamental units meter, kilogram, and second in a uniform way by adding prefixes to the names of units (Table 1.1). For instance, one hundredth of a meter is called a centimeter, one thousand meters is a kilometer, and a millionth of a second is called a microsecond.

Table 1.1: SI prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^1	deka-	da	10^{-1}	deci-	d
10^2	hecto-	h	10^{-2}	centi-	c
10^3	kilo-	k	10^{-3}	milli-	m
10^6	mega-	M	10^{-6}	micro-	μ
10^9	giga-	G	10^{-9}	nano-	n
10^{12}	tera-	T	10^{-12}	pico-	p
10^{15}	peta-	P	10^{-15}	femto-	f
10^{18}	exa-	E	10^{-18}	atto-	a
10^{21}	zetta-	Z	10^{-21}	zepto-	z
10^{24}	yotta-	Y	10^{-24}	yocto-	y

1.2.6 The SI AND CGS Systems of Units

There are two systems of units based on the metric units that are commonly used in science - the International System (SI) and the centimeter-gram-second (cgs) systems. In the SI system all units are expressed in meter, kilogram and second while in the cgs system they are in centimeter, gram and second. The SI system is dominant in textbooks while cgs is more convenient for laboratory experiments. The SI system for mechanical properties is also called MKS or meter-kilogram-second system.

All mechanical properties can be expressed in terms of length, mass and time. Therefore, their units can be related to the units of length, mass and time. For this reason, the units meter, kilogram and second are called fundamental units while other units are said to be derived from them and are called derived units. There are other properties that are not related to motion of objects, such as an electric charge or temperature of an object and therefore, the set of fundamental units or base units contains additional quantities. Presently, there are seven base standards in the SI system of units as shown in Table 1.2. In the initial part of the book we will be using only the units of length, time and mass.

Table 1.2: Base SI units

Quantity	SI Unit Name	Unit Symbol
Length	meter	m
Time	mecond	s
Mass	kilogram	kg
Electric current	ampere	A
(Thermodynamic) Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

1.2.7 Conversion of Units

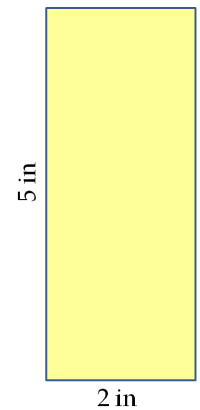
Conversion of units is frequently needed in physics and you should make sure you have a method of calculation that works consistently for you. A particularly useful method is to multiply the given number by a fraction whose value is 1. The fraction is chosen so that numerator and denominator are different units for the same physical quantity. For instance, if we need to change minutes to hours, we will need a fraction of hours to minutes or minutes to hours depending on whether minute to be changed is in the numerator or the denominator respectively. We now illustrate this method of changing units with examples.

Example 1.2.1. Changing one unit only. How many seconds are in 20 minutes?

Solution. You can start with 20 minutes and then multiply with a fraction of second [the unit desired] to minute [the unit to be converted] whose value is 1. The fraction is obtained by the conversion of 60 seconds in 1 minute.

$$\begin{aligned} 20 \text{ min} &= 20\text{min} \times \frac{60 \text{ sec}}{1 \text{ min}} \\ &= 20\cancel{\text{min}} \times \frac{60 \text{ sec}}{1 \cancel{\text{min}}} \\ &= 1200 \text{ sec.} \end{aligned}$$

Example 1.2.2. Changing product of units. A rectangular board is 5 in by 2 in. The are of the board is 10 square inches. How many square centimeters in 10 square inch?



Solution. A square inch is a unit of area. Note that square inches stands for inches times inches. It is usually helpful to write out square inches this way before become proficient at conversions and can skip this step. We need a fraction for each inch to be converted. The fraction needed has *cm* at the numerator and *in* at the denominator, which we can construct from the fact that there are 2.54 *cm* in 1 *in*.

The calculation is presented below. You should note that when I used the calculator I got way too many digits than the three digits in 2.54 that went into the calculation, so I put in another step where the final answer was rounded off to three digits. This happens a lot in calculations and you should learn to round off numbers to correct number of digits. I will show you later a consistent way of rounding off that is commonly practised.

$$\begin{aligned}
10\ \text{in}^2 &= 10\ \text{in} \times \text{in} \\
&= 10\ \text{in} \times \text{in} \times \frac{2.54\ \text{cm}}{1\ \text{in}} \times \frac{2.54\ \text{cm}}{1\ \text{in}} \quad \text{(no change)} \\
&= 10\ \cancel{\text{in}} \times \cancel{\text{in}} \times \frac{2.54\ \text{cm}}{1\ \cancel{\text{in}}} \times \frac{2.54\ \text{cm}}{1\ \cancel{\text{in}}} \\
&= 10 \times 2.54\ \text{cm} \ \times 2.54 \times\ \text{cm} \\
&= 64.516\ \text{[from calculator]}\ \text{cm}^2 \\
&= 64.5\ \text{cm}^2.
\end{aligned}$$

Example 1.2.3. Changing product of units. Convert 1.5 lb mi/h^2 into kg m/s^2?

Solution. This example is definitely messier than the previous ones. We will need four fractions, one for each factor of the units; we will combine the two factor of hours into one to save space. We will also need to round off the final answer. Here is the gory details of the calculation.

$$\begin{aligned}
1.5\ \text{lb mi/h}^2 &= 1.5\ \text{lb mi/h}^2 \times \frac{0.4536\ \text{kg}}{1\ \text{lb}} \times \frac{1.6093\ \text{km}}{1\ \text{mi}} \times \frac{1\ \text{h}^2}{3600^2\ \text{s}^2} \\
&= 1.5\ \cancel{\text{lb}}\ \cancel{\text{mi}}/\cancel{\text{h}}^2 \times \frac{0.4536\ \text{kg}}{1\ \cancel{\text{lb}}} \times \frac{1609\ \text{m}}{1\ \cancel{\text{mi}}} \times \frac{1\ \cancel{\text{h}}^2}{3600^2\ \text{s}^2} \\
&= 8.4488...\ \text{[from calculator] kg m/s}^2 \\
&= 8.4 \times 10^{-5}\text{kg m/s}^2.\ \text{[Rounding off to two digits.]}
\end{aligned}$$

Note that I rounded off the final number to two digits - I did that because the lease precise number was 1.5 which has two significant digits.