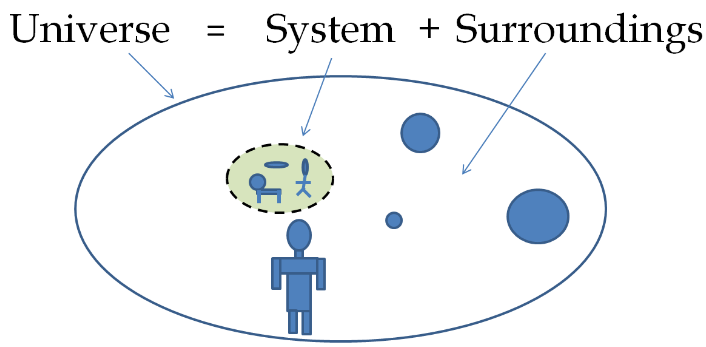

Figure 7.17: Division of the universe into system and surroundings takes place when we focus on a part of the universe for study. The objects of interest make up the system and everything else is the surroundings or external to the system. We usually draw a dashed boundary line to indicate the objects that are within the system from those which are external to the system.

7.5 ISOLATED SYSTEMS AND CONSERVATION OF MOMENTUM

We found above that the change in the total momentum of any system, whether it contains a single particle or several particles, is caused only by the external forces on the system, and the rate of that change at any instant is exactly equal to the net external force on all parts of the system.

$$\frac{d\vec{P}_{system}}{dt} = \vec{F}_{net}^{ext}. \tag{7.47}$$

Therefore, if the net external force on a system is zero, then the total momentum cannot change. That is, the magnitude and direction of the momentum of a system that does not interact with anything else will remain fixed in time, i.e. conserved. We will call such systems **isolated systems**. This result is also called **the principle of conservation of momentum**.

By a system we usually mean any physical body of interest. A choice of a physical body for study leads to an artificial division of the entire universe into a system and the rest, called the surroundings or the external world, as illustrated in Fig. 7.17. We study the overall motion of the system by examining the forces on the system by the external world. The surroundings is external to the system and influences the changes in momentum of the system. Similarly, the system is external to the surroundings and therefore, influences the changes in the momentum of the surroundings. The system and surroundings will obey their own equations of motion which can be written as:

$$\text{System:} \quad \frac{d\vec{P}_{\text{system}}}{dt} = \vec{F}_{\text{net}}^{\text{on system}} \tag{7.48}$$

$$\text{Surroundings:} \quad \frac{d\vec{P}_{\text{surroundings}}}{dt} = \vec{F}_{\text{net}}^{\text{on surroundings}} \tag{7.49}$$

Now, when we add the two equations we obtain the equations of motion of the universe. Since the forces from the surroundings on the system are equal in magnitude but opposite in directions to the forces from the system on the surroundings, the net external force on the universe is zero.

$$\boxed{\frac{d\vec{P}_{\text{Univese}}}{dt} = 0.} \tag{7.50}$$

This makes sense, since the larger system, which we have called the universe above, is an isolated system. Since there is nothing external to the larger system, the total momentum of the system and surroundings together would not change with time.

Principle of conservation of momentum

The total momentum of a system and the rest of the universe does not change in time. We will call the rest of the universe “the surroundings” or “the environment”. The force on the system by the objects in the environment changes the momentum of the system and the force on the objects of the environement by the system changes the momentum of the environment. But, since the forces are equal in magnitude and opposite in directions, the impulse on the system imparted by the environment in any interval must be equal in magnitude and opposite in direction to the impulse imparted to the environment by the system. Consequently, any change in the momentum of the system is accompanied by a change in the momentum of the environment of an equal magnitude and of opposite direction.

$$\boxed{\Delta\vec{P}_{\text{System}} = -\Delta\vec{P}_{\text{Environment}}.} \tag{7.51}$$

Since, momentum is a vector quantity, the conservation of momentum applies independently for each direction in space. Decomposing these vectors in x, y and z-components, we find that changes in different components of the momenta of the system are accompanied by equal changes of opposite sign in the changes in corresponding components of the momenta of the surroundings.

$$\boxed{\Delta P_x^{\text{System}} = -\Delta P_x^{\text{Environment}}} \tag{7.52}$$

$$\boxed{\Delta P_y^{\text{System}} = -\Delta P_y^{\text{Environment}}} \tag{7.53}$$

$$\boxed{\Delta P_z^{\text{System}} = -\Delta P_z^{\text{Environment}}} \tag{7.54}$$

When you bring the changes in momenta of the system and the environment on one side of the equation, we find that the net change in any component of the total momentum of the system and the environment together is always zero, i.e the total momentum is conserved component-by-component. The decomposition in three Cartesian components makes it clear that the momentum component in any direction is independently conserved from the momentum component in any other direction that is perpendicular to it.

For instance, when a football is kicked at an angle to the ground, the momentum in the horizontal direction is conserved independently of whatever happens in the momentum component in the vertical direction. Actually, although the horizontal component of the momentum of a football in free flight does not change, the absolute value of the vertical component changes with time, decreasing in the upward part of the flight, becoming momentarily zero at the top of the flight, and increasing afterwards until the ball hits the ground. The change in the vertical component of the momentum happens because of the external force on the ball in that direction, which is the gravitational force of the Earth on the ball.

The principle of conservation of momentum provides a powerful tool for solving problems by giving us a conservation equation for the direction in which there are no external forces. It is easy to peel off that component of the motion that will have conserved momentum and depending upon the question to be investigated, that may be enough for the problem.

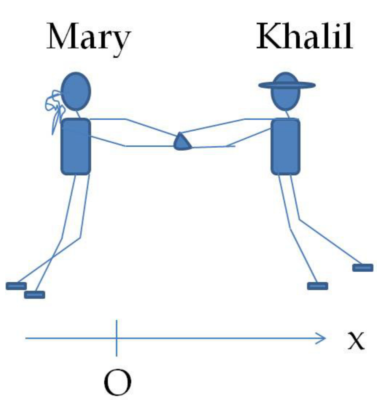

Figure 7.18: Example 7.5.1.

Example 7.5.1. Momentum of ice skaters. Khalil (mass 60 kg) and Mary (50 kg) are ice skating on a smooth surface. They start from rest by pushing on each other with an average force of magnitude 30 N and directed horizontally for 0.5 sec. Assume the skating surface to be horizontal and frictionless. Find (a) the total momentum of the two skaters at $t = 0.2$ sec, (b) the momentum of each skater at $t = 0.5$ sec and the total momentum, (c) the velocity of each skater at $t = 0.5$ sec, and (d) Mary's momentum at $t = 0.6$ *sec*.

Solution. Since the value of momentum depends upon the reference frame, we will pick a reference frame before starting on the solution of the problem. Furthermore, since the principle of conservation of momentum requires an inertial reference frame, we will look for an inertial frame in the physical setting of the problem. In the present case, a frame attached to the ice surface would be convenient choice for an inertial frame.

(a) Now, if we consider the two skaters together as our multiparticle system. Then, their forces on each other will be an internal force of the system. The only other forces on the two bodies consisting of the system are the gravitational pull of the Earth and the normal force from Earth's surface. Both of these external forces are in the vertical direction. Hence, from the conservation of momentum, the horizontal component of the total momentum of the skaters will not change. As for the vertical component of the momentum, we note that since there is no acceleration in that direction, the vertical forces must be balanced. Therefore, the vertical component of the total momentum is also fixed in time. Since, the two skaters started out with zero momentum and their momentum cannot change by the argument presented, therefore their net momentum will remain zero.

(b) Although the total momentum of the two skaters together remains zero, the momentum of each skater can change since individually they are not isolated systems. **To find the change in the momentum of a particular skater we focus on that skater alone as a new system.** The vertical forces, i.e. weight of the skater and the normal force are still balanced, but the horizontal force from the other skater is not zero any more.

To be specific, let us calculate the change in momentum of Khalil over the time interval [0, 0.5 s] using the analytic approach for vectors. Let Mary and Khalil move on the x-axis so that the force from Mary is towards positive x-axis as shown in the Fig. 7.18. Let P_x be the x-component of Khalil's momentum at time $t = 0.5$ sec and P_{0x} be his momentum at $t = 0$. Then the x momentum equation for Khalil's momentum will be

$$\text{Khalil:}\;\; P_x - P_{0x} = 30\text{ N} \times 0.5\text{ s} \quad\Longrightarrow\quad P_x = 15\text{ kg.m/s.}$$

The change in momentum of Mary must be equal in magnitude but opposite in direction.

$$\text{Mary:}\;\; P_x = -15\text{ kg.m/s.}$$

The total momentum of the two skaters together, of course, does not change since the net external force on the two skaters together as one system is zero.

(c) The velocity of each skater can be obtained by dividing the corresponding momentum by the appropriate mass. Just as the momentum, the only non-zero component of the velocity is the x-component here.

$$\text{Khalil:}\;\; v_x(0.5\text{s}) = \frac{15\text{ kg.m/s}}{60\text{ kg}} = 0.25\text{ m/s.}$$

$$\text{Mary:}\;\; v_x(0.5\text{s}) = -\frac{15\text{ kg.m/s}}{50\text{ kg}} = -0.30\text{ m/s.}$$

(d) Since there is no net force on Mary during [0.5 s, 0.6 s] interval, Mary herself is an isolated system during this interval with zero net external force. Therefore, Mary's momentum at $t = 0.6$ s will be equal to her momentum at $t = 0.5$ s, which is already given in part (b).