## 5.3 FORCES ON DIELECTRICS

When you place a dielectric in a uniform electric field, the net force on the dielectric must be zero because the sum of forces on each dipole is zero. But, if you put a dielectric in a non-uniform electric field, there will be a net force on each dipole due to the changing electric field and the dielectric will be pulled towards the region of increasing electric field. We found that the force on a dipole in a non-homogeneous electric field is given by

$$\vec{F} = p_x \frac{\partial \vec{E}}{\partial x} + p_y \frac{\partial \vec{E}}{\partial y} + p_z \frac{\partial \vec{E}}{\partial z}. \tag{5.23}$$

One way to find the force on a dielectric will be to sum up forces on every microscopic dipole. This brute-force method turns out to be rather difficult to implement in practice since we usually do not know the orientation of various microscopic dipoles in the material.

An alternative method based on the following relation between a force $\vec{F}$ and the work done by this force is more useful as we will now illustrate.

$$dW = \vec{F}_{\text{applied}} \cdot d\vec{r}. \tag{5.24}$$

To find the energy associated with the electric force on the dielectric we envision applying a force that would balance the electric force as we have done when deducing the change in the potential energy. The required applied force would act in the opposite direction to the electric force $\vec{F}_e$ on the dielectric and therefore they are related by

$$\vec{F}_{\text{applied}} = -\vec{F}_e. \tag{5.25}$$

The work done by an applied force $\vec{F}_{\text{applied}}$ would change the potential energy energy $U$ of the dielectric body.

$$dW = \vec{F}_{\text{applied}} \cdot d\vec{r}. - \vec{F} \cdot d\vec{r}. \tag{5.26}$$

This can be written in terms of the electric force as

$$dW = -\vec{F}_e \cdot d\vec{r}. \tag{5.27}$$

The work done by the applied force changes the potential energy by amount $dU$. Hence the change in the potential energy of the dielectric $dU$ is related to the electric force on the dielectric by the following equation.

$$dU = -\vec{F}_e \cdot d\vec{r}. \tag{5.28}$$

For a concrete application of this result consider the displacement along the $x$ axis of a coordinate system. In that case we will find that

$$dU = -\vec{F}_e \cdot d\vec{r} = -F_x dx, \quad (x\text{-axis displacement only}), \tag{5.29}$$

where $F_x$ is the $x$-component of the electric force $\vec{F}_e$ on the dielectric. Therefore, the $x$-component of the electric force will be related to the way the potential energy changes with the position.

$$\boxed{F_x = -\frac{dU}{dx} \quad (x\text{-axis displacement only})} \tag{5.30}$$

Thus if there is a way to find the potential energy of a dielectric in a nonuniform electric field as a function of position, then we can obtain the force from this function. We now illustrate the use of this approach in a simple example of a dielectric rectangular slab being pulled between two charged parallel plates.

**Example 5.3.1. Force on a Dielectric.** Consider an isolated parallel plate capacitor that has charges $\pm Q$ on the plates separated by a distance $d$. Each of the plates has area $A$. A dielectric slab of thickness infinitesimally less than $d$ is partially inserted inside such that it does not strip any charges from the plates. What is the force on the dielectric slab of dielectric constant $\epsilon_r$ when the dielectric is partially inside the space between the plates as shown in the figure?

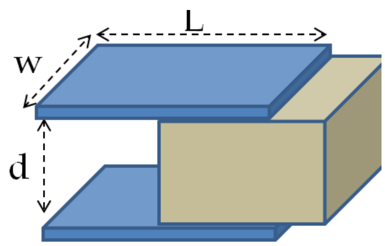


Figure 5.11: A dielectric between parallel plate capacitor. Dielectric is in a distance x between the plates.

**Solution.** To find the electric force on the dielectric slab, we need to find the expression of the potential energy as a function of the position of the slab so that we can fin dhow the energy changes with its position. Let us use a coordinate system with the $x$ along horizontal. Let us use the $x$-coordinate of the left end of the slab for the position indicator of the slab as shown in Fig. 5.12.

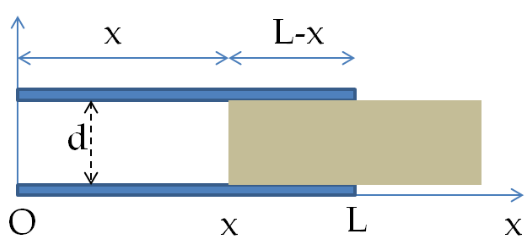


Figure 5.12: The choice of coordinates for calculations (Example 5.3.1.)

Then, we have noting in between the parallel plates up to a length $x$ and then the rest of the distance $L - x$ is filled with the dielectric. The capcitance of the capacitor will depend upon $x$ but the charge of the capacitor is fixed here to be $\pm Q$. Therefore, the energy will be

$$U(x) = \frac{1}{2}\frac{Q^2}{C(x)}$$

This says that if we can figure out $C(x)$ we can then figure out the force by taking the derivative of $U(x)$.

$$F_x = -\frac{dU}{dx} = \frac{1}{2}\frac{Q^2}{C^2(x)}\,\frac{dC}{dx}.$$

So, the problem boils down to figuring out $C(x)$. A trick helps here. We note that the that the capacitor is equivalent to two capacitors side by side so that the capacitance of the full capacitor will be the sum of the parts.

$$C(x) = C_1[w \times x \times d \text{ capacitor}] + C_2[w \times (L-x) \times d \text{ capacitor}]. \quad (5.31)$$

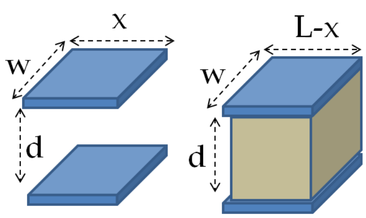


Figure 5.13: The partially filled parallel plate capacitor is equivalent to two capacitors in parallel.

The capacitances are

$$C_1 = \frac{\epsilon_0 w.x}{d}$$
$$C_2 = \frac{\epsilon_r \epsilon_0 w.(L-x)}{d}$$

Therefore,

$$C(x) = \frac{\epsilon_0 w.x}{d} + \frac{\epsilon_r \epsilon_0 w.(L-x)}{d}. \quad (5.32)$$

Using $C(x)$, we can write $F_x$ explicitly.

$$F_x = -\frac{1}{2}\,\frac{Q^2 d}{\epsilon_0 w}\,\frac{\epsilon_r - 1}{\left[x + \epsilon_r (L-x)\right]^2}. \quad (5.33)$$

Since $\epsilon_r > 1$ for dielectrics, $F_x < 0$, therefore force on the dielectric will tend to pull the dielectric inside the space between the parallel plates. Note the net force on the dipoles of the dielectric must come from the inhomogeneous fringing fields since the force on a dipole in a uniform field is zero.

The present method based on the energy consideration avoids the calculation of fringing fields which are difficult to compute. Once the dielectric is in the uniform field region between the plates, it can be moved about without any work since there is no net electric force on a dielectric in a uniform electric field.