6.5 CIRCULAR APERTURE AND RESOLUTION

So far we have studied the diffraction of light through a rectangular slit which may be horizontal or vertical opening in an opaque body. To keep the math simple we have kept the slit to have a width but be of an infinite length so that we didn't need to perform an integration over the length dimension of the slit. In this section we will study the diffraction through a finite size hole in the shape of a circle.

The diffraction through a circular aperture has many applications. The circular aperture may be just a circular hole in a opaque object. The circular aperture may also be a circular lens through which light could pass such as in the viewing tube of a telescope or microscope.

The diffraction through a circular aperture gives rise to a central bright spot which is surrounded by a dark ring which itself is surrounded by a bright ring and so forth as illustrated in Fig. 6.13. The circular rings in the diffraction through a circular aperture are also called **Airy rings** since the intensity of the diffraction function turns out to be a special function called the Airy function.

To obtain the mathematical expression for the intensity of the diffraction pattern we proceed in the same way as we have done in the case of diffraction through a slit. Specifically, we seek the superposition of the secondary wavelets of the wavefront emerging from the circular slit. The integration is now done over the domain of the slit. We will not present the derivation here and refer the student to more advanced textbooks in optics. We do however wish to discuss some important aspects of the result.

Suppose the circular aperture has a diameter D and we wish to study the diffraction pattern observed on a screen a distance L away. We find that the radius R of the central bright spot on the screen is not necessarily equal to the diameter of the aperture but depends on the wavelength λ and the distance L also.

$$R \approx 1.22\frac{L\lambda}{D}, \tag{6.35}$$

where λ is the wavelength of the monochromatic light. Sometimes it is more convenient to write this equation for the central bright spot in terms of the angle θ subtended at the slit. As the distance to the screen is far greater than the radius of the bright spot, we use small angle approximation and write

$$\theta \approx \tan\theta = \frac{R}{L} = 1.22\frac{\lambda}{D}, \tag{6.36}$$

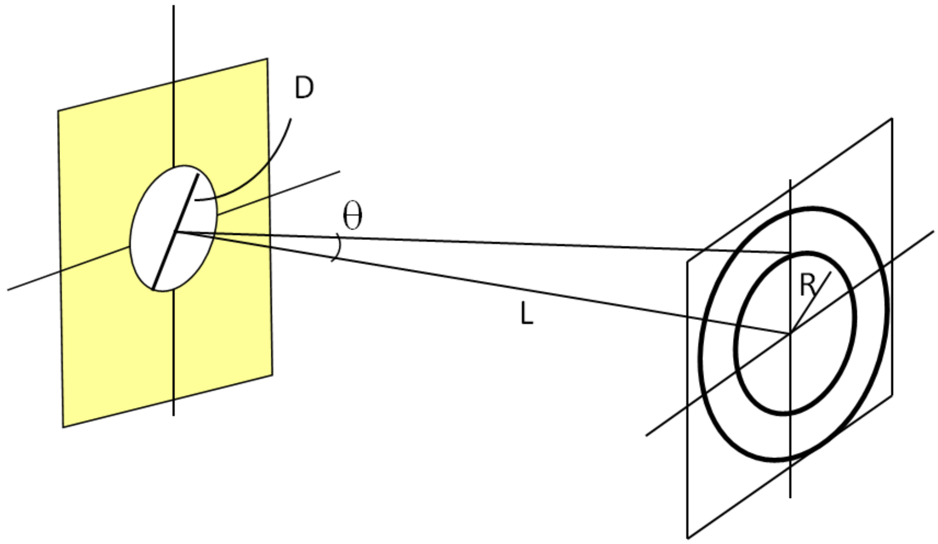


Figure 6.13: The circular aperture geometry.

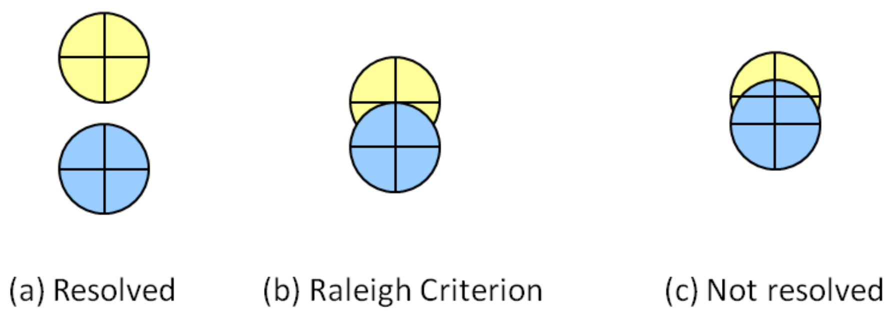


Figure 6.14: Raleigh criterion of resolution

which gives us a relation purely in terms of the wavelength of the light and the diameter of the slit.

Light from a point object passing through a circular aperture will spread out in accordance with Eq. 6.35 producing a bright circle on the screen instead of a point along with larger circular rings around the circle. The circular images of two close by points, such as two stars, may overlap making it difficult or impossible to distinguish them. The resolvability of two objects are once again given by the **Raleigh criterion** which states that two images are resolvable if the center of one is at the edge of the other circle (Fig. 6.14).

In terms of angular separation of the centers of the two images, the Raleigh criterion states that the angle of separation of the centers

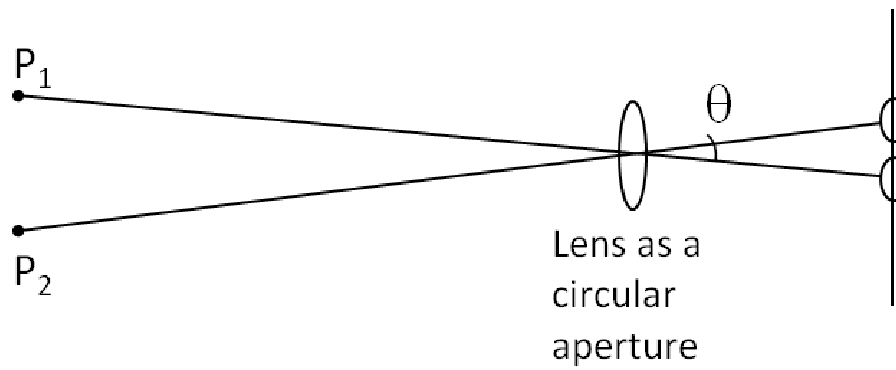


Figure 6.15: Angular separation of two images. For the stars to be resolvable the angle θ must be larger than θ_R given by $1.22\lambda/D$, where D is the diameter of the lens serving as the aperture.

of the images must greater than a minimum angle given by the first dark ring of the diffraction pattern of one of the objects.

$$\text{Angular separation,}\quad \theta > \theta_R = \frac{1.22\lambda}{D} \quad \text{(Raleigh Criterion)} \tag{6.37}$$

Thus, according to the Raleigh criterion, two stars cannot be resolved by a telescope of aperture D operating at the wavelength λ, if the stars are not far enough apart (Fig. 6.15). Since the loss of resolution cannot be eliminated by grinding a better lens or adding additional optical elements we say that the resolution is **diffraction-limited**. An image that is diffraction-limited can be improved by changing the aperture or observing in a different part of the electromagnetic spectrum.

Example 6.5.1. Resolution of images by lens.

A circular converging lens of diameter 100 mm and focal length 50 cm is to be used to project the images of two far away point sources of wavelength 632.8 nm. The image distance can be taken to be equal to the focal length and the image assumed to be on the focal plane of the lens. (a) Looking from the lens's center, what should be the minimum angular separation of the two objects so as to satisfy the Raleigh criterion? (b) How far apart will the central bright spots be on the screen at the Raleigh criterion?

Solution. (a) The figure for this example will be same as Fig. 6.15. The angular separation θ must be larger than the angle for the Raleigh criterion.

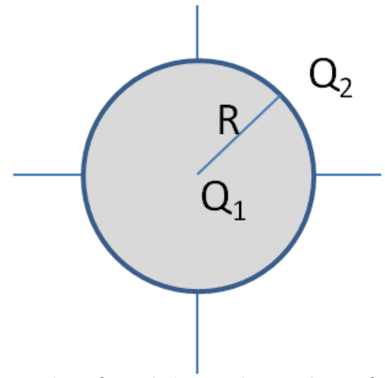
$$\theta \approx \tan\theta = \frac{R}{L} = 1.22\frac{\lambda}{D}.$$

Putting in the numerical values appropriate for this problem we get

$$\theta_R = \frac{1.22 \times 632 \times 10^{-9}\ \mathrm{m}}{0.1\ \mathrm{m}} = 7.7 \times 10^{-6}\ \mathrm{rad}.$$

Note that we need to express both λ and D in the same units.

(b) At the Raleigh criterion, the distance between the centers of the images will be equal to the radius of the central ring about one of the images. That is the distance Q_1Q_2 will be equal to R of one of the images.



$$Q_1Q_2 = R.$$

The distance from the lens is equal to the focal length. Therefore, the distance betwee the images will be

$$Q_1Q_2 = R = L\theta = f\theta.$$

Putting in the numbers we obtain

$$Q_1Q_2 = 50\ \mathrm{cm} \times 7.7 \times 10^{-6}\ \mathrm{rad} = 3.85\ \mu\mathrm{m}.$$