6.7 SURFACE TENSION

If you place a stainless steel needle carefully upon a calm surface of water, it will float even though the density of steel is considerably more than the density of water and the needle is a compact solid (Fig. 6.21). The needle floats not due to the buoyancy as you can easily demonstrate by placing the needle infinitesimally under the surface in which case the needle will sink. Instead, the needle floats because there is an upward force on the needle from the surface molecules.

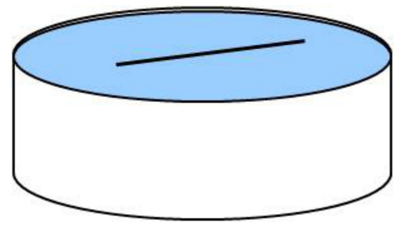

Figure 6.21: The weight of the floating needle is balanced by the force from the surface tension of water.

The reason for the different behavior at the surface can be found in the special situation of the surface molecules as compared to the molecules in the volume. A molecule inside the volume of the liquid has other molecules of the fluid all around it so that on average there is no net intermolecular force on the molecules in the bulk. But, a molecule on the surface, on the other hand, has molecules of the liquid only on one side, which results in a net intermolecular force pointed inwards from the surface in the liquid (Fig. 6.22). The intermolecular force on a surface molecule is balanced by the net upward impulse from movement of molecules below the surface.

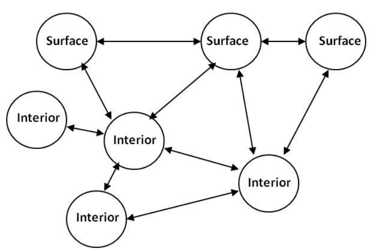

Figure 6.22: Intermolecular forces on a surface molecule are not balanced.

When we place the needle at the surface, the weight of the needle pushes the surface molecules aside and increases the surface area of the fluid. To expand the surface area of a liquid, requires bringing additional molecules from the inside to the surface against the net attractive force of the other molecules. Therefore, it costs energy to create a larger surface area.

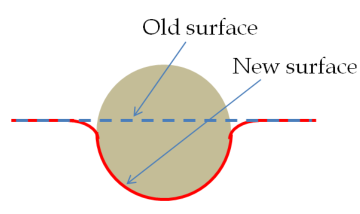

Figure 6.23: The weight of the floating needle stretches the surface of the fluid, which leads to an increase in upward force from the surface that balances the weight of the needle.

The **surface tension** of a liquid is defined to take into account the energy needed to increase the surface area. Thus, if it takes an energy ΔU to increase the surface area of a fluid by ΔA, then the surface tension (γ) is defined as:

$$\boxed{\gamma = \frac{\Delta U}{\Delta A}} \tag{6.27}$$

In the absence of external forces, a liquid will tend to assume the least surface area so as to minimize the surface energy. Since a spherical shape has the least area per unit volume, a liquid forms spherical droplets in the absence of external forces such as in the zero gravity situation of outer space (Fig. 6.24).

Figure 6.24: Marble-sized drop of water filled with tiny bubbles floating in International Space Station. Photo credit: Ken Bowersox, ISS Expedition 6 Commander, NASA, Apri1 2003.

The dimension of surface tension can be obtained from the dimensions of energy and area.

$$[\gamma] = \frac{[M][L]^2/[T]^2}{[L]^2} = \frac{[M]}{[T]^2}.$$

The unit of surface tension in SI system is J/m^2, or equivalently N/m or kg/s^2. More commonly used unit of surface tension, however, is the unit dyne/cm that belongs to the cgs-system. Since 1 N equals 10^5 dyne and 1 m equals 10^2 cm, therefore 1 N/m would equal 10^3 dyne/cm.

$$1\ \frac{N}{m} = 10^3\ \frac{\text{dyne}}{\text{cm}}.$$

Pure water at 20°C has a surface tension of 72.8 dyne/cm compared to 22.3 dyne/cm for ethyl alcohol, 32 dyne/cm for olive oil, 63.1 dunes/cm for glycerin and 465 dyne/cm for mercury.

Table 6.3: Surface Tension (dyne/cm)

(Divide by 1000 to obtain N/m) (Source: various)

Liquid	Temp (0°C)	Surface Tension γ (dyne/cm)
Water	0	75.6
	20	72.8
	100	58.8
Soapy water	20	$\sim$ 40
Blood	37	$\sim$ 60
Ethyl alcohol	20	22.4
Glycerin (Glycerol)	20	63.4
Mercury	20	480
Olive oil	20	32.0

The soap lowers the surface tension of water. With the reduced surface tension, the water molecules can squeeze through microscopic spaces between the fibers of the fabric, and remove any trapped dirt particles. The detergent molecules themselves may not be able to pass through the space between fabric fibers, but help water molecules to pass by lowering the surface tension of the mixture.

Similarly, hot water is better at cleaning than the cold water since with rise in temperature the surface tension of water drops, e.g. the surface tension of water is 72.8 dyne/cm at 20°C and 58.9 dynes/cm at 100°C. Table 6.3 provides surface tension of some common substances. Note that surface tension is not a force but force per unit length as will be illustrated in the following example.

Example 6.7.1. Stretching a Planar Soap Film.

Consider a thin film of soapy water produced on a wire frame where one end is movable as shown in Fig. 6.25.

The force needed to move the wire outward by a distance Δx depends on the width w of the film. Furthermore, there are two sides of the film that need to be stretched together. Surface energy will increase due to the work done by the applied force F.

$$\Delta U = F\Delta x \quad \text{(since force } \vec{F} \text{ is parallel to displacement.)}$$

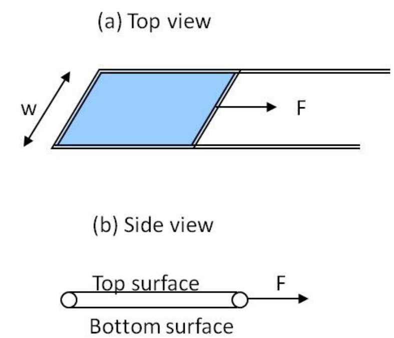

Figure 6.25: Stretching of a soap film.

The film has two surfaces, one top and the other bottom. Therefore, the amount of increase in area would be twice the area on one side.

$$\Delta A = 2 \times w\Delta x \quad \text{(two surfaces, each stretching } \Delta x.\text{)}$$

Therefore, the surface tension will be

$$\gamma = \frac{\Delta U}{\Delta A} = \frac{F}{2w}.$$

Example 6.7.2. Surface Energy of a Water Droplet

Assuming surface tension to be independent of the size of the bubble, find the energy stored in the surface energy of a spherical water droplet of 1-mm radius at $20°C$.

Integrating the equation $dU/dA = \gamma$ from $r = 0$ to $r = R$, we find that the surface energy is equal to the product of surface tension and surface area of the spherical droplet.

$$U(R) = 4\pi\gamma R^2.$$

Note that in the droplet there is only one surface as opposed to the soap film on a wire frame where there were two surfaces. We have used $U(0) = 0$ since if there is no droplet there will be no surface energy. Putting the numerical values we obtain.

$$U(R) = 4\pi \times (72.8 \times 10^{-3}\text{N/m}) \times (10^{-3}\text{m})^2 = 9.1 \times 10^{-7}\text{J}.$$

Capillary action

Recall from our discussion above that a liquid has the same level in all open tubes connected by freely flowing pathways. As a result

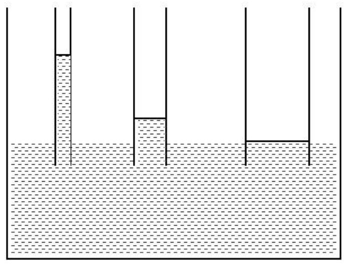

Figure 6.26: Illustration of the capillary action.

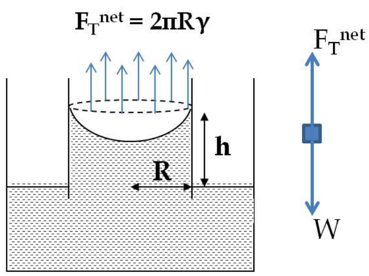

Figure 6.27: Balancing forces on the fluid column.

when you dip a wide tube in a fluid, the level of the fluid in the tube is same inside the tube as outside. However, when very thin tubes are dipped in a liquid, this is not the case. Instead, liquid rises to various heights depending on the diameter of the opening as illustrated in Fig. 6.26. The effect is called the capillary action.

Let us consider an open tube of radius R dipped in a fluid of density ρ. The height h to which the fluid will rise in the tube can be found by balancing forces on the column of the fluid in the capillary shown in Fig. 6.27.

The capillary action is due to the adhesion of liquid molecules to the tube material and the surface tension. Due to the surface tension along the surface, the surface molecules of the fluid pull on the wall, which in turn, pulls on the fluid upward. The pull of the wall, called adhesive force, supports the weight of the fluid. Therefore, the net upward force from the adhesion force has the magnitude:

$$F_T^{net} = 2\pi R\gamma$$

where γ is the surface tension of the liquid, which must balance the weight of the liquid in the tube given by

$$W = \pi R^2 h\rho g.$$

Equating the two we obtain the height the fluid will rise in a capillary as

$$h = \frac{2\gamma}{\rho g R}.$$

We find that height h is inversely proportional to the tube radius R. If we use the surface tension of water at 20°C in this relation, we find that water will rise 3 cm in a tube of inner diameter 1 mm.