8.2 Luminosity of Stars

The total amount of energy radiated by a star per unit time is called its **luminosity** L, absolute luminosity. Luminosity is an intrinsic property of a star. The emitted radiation spreads out in space. Suppose the spreading of energy happens uniformly in all directions. Let F_* be the **flux**, i.e., the amount of radiant energy per unit time per unit area, at the surface of the star of radius r_*. Then, the total energy leaving the star in unit time will be

$$L = 4\pi {r_*}^2 F_*.$$

This energy will pass a larger spherical surface at a later time. Let F be the intensity of the radiation at a distance r from the star. Since the total energy per unit time emitted from the star is same regardless of where the energy is observed, we must also have

$$\boxed{L = 4\pi r^2 F,}$$

at any distance r from the star. Therefore, the flux observed at a distance r from the star must drop as the square of the distance.

$$F = \frac{L}{4\pi r^2} \tag{8.7}$$

Now, if F is measured on Earth and the distance from Earth to the star r is obtained from some other method, such as the parallax method we have discussed above, we can deduce L of the star. Now, think about it: just by making appropriate measurements from Earth we can know something intrinsic to a star! For instance, the average flux of sunlight observed on Earth, called the **solar constant**, is $F = 1.361\,\text{kW/m}^2$, and the distance to the sun is 1.5×10^{11} m. Therefore, the sun must have the following luminosity.

$$L = 4\pi r^2 F = 3.8 \times 10^{26}\ \text{W}.$$

That is, the Sun is putting out 3.8×10^{26} Joules of energy per second. Of course, if you can figure out L somehow and measure F then you can use Eq. 8.7 to deduce the distance to the star.

$$r = \sqrt{L/4\pi F}. \tag{8.8}$$

The flux F of a star measured on Earth is called its **apparent brightness**. Sometimes, we designate the apparent brightness by F_m to distinguish it from a related quantity called **absolute brightness**, which is the brightness of the star if the said star were 10 pc away instead of r away. We will denote the absolute brightness by F_M.

$$F_m = \frac{L}{4\pi r^2}; \quad F_M = \frac{L}{4\pi (10\ \text{pc})^2}; \tag{8.9}$$

Here F_m is the measured quantity, not F_M. The ratio of the fluxes gives

$$\boxed{\frac{F_M}{F_m} = \left(\frac{r\ [\text{pc}]}{10\ \text{pc}}\right)^2.} \tag{8.10}$$

Astronomer's Way

Although, the flux F_m would provide a reasonable scale to classify apparent brightness of stars, the system used by Astronomers is based on a logarithmic scale because our visual perception is actually a logarithmic detector. The designation brightness of stars dates back to **Hipparchus of Nicaea** who cataloged 1000 stars in about 130 B.C. and introduced a system of six **magnitudes** for classifying them. Hipparchus assigned magnitude 1 to the brightest stars, 2 to the next group, and so on, finally 6 to the stars that were barely visible to the naked eye without the aid of a telescope.

The system of Hipparchus was systematized and made more quantitative by **Norman Pogson** in 1856, who gave the brightness of the brightest star a factor of 100 compared to the brightness of the faintest star and then divided the factor 100 into equal factors for each magnitude. This gave him a factor of 2.512 for each magnitude.

$$100^{1/5} = 2.512.$$

Thus, a star of magnitude 1 was 2.512 times as bright as a star of magnitude 2, which in turn was 2.512 times as bright as a start of magnitude 3, and so on. Thus, we have five factors of 2.512 between magnitudes 1 and 6.

$$(2.512)^5 = 100.$$

The bright stable star Vega is taken to be the reference zero-point of the magnitude scale. This gives the following formula for apparent magnitude m of a star with respect to the apparent magnitude $m_0 = 0$ for Vega.

$$m - m_0 = -2.512\ \log_{10}\left(\frac{F_m}{F_0}\right). \tag{8.11}$$

Of course, you can write this equation in terms of the luminosities of the two stars by multiplying the two fluxes by a common factor of $4\pi r$.

$$\boxed{m - m_0 = -2.512\ \log_{10}\left(\frac{L_m}{L_0}\right).} \tag{8.12}$$

Using this equation you can compare the magnitudes m_1 and m_2 of any two stars. Subtracting the formulas of $m = m_1$ and $m = m_2$ cancels out any reference to the reference star and we find

$$m_1 - m_2 = -2.512\ \log_{10}\left(\frac{F_1}{F_2}\right). \tag{8.13}$$

Thus, if we have a star of magnitude 1 and another of magnitude 6, then by definition, the ratio of their fluxes should be 100.

$$\frac{F_1}{F_6} = 10^{(5/2.512)} \approx 100.$$

We can also use Eq.8.13 to write the absolute magnitude M and apparent magnitude m of a star with M being the magnitude at a distance of 10 pc from the star. To convert the ratio of the fluxes we use Eq. 8.10. Thus,

$$\boxed{m - M = 5.024\ \log_{10}\left(\frac{r\ [\mathrm{pc}]}{10\ \mathrm{pc}}\right).} \tag{8.14}$$

The quantitative method provides us with a more precise way to classify the apparent brightness. With improvement in observation technology, we can make measurements on extremely dim stars and use the magnitude scale to refer to them. For instance, Hubble telescope could observe stars with magnitude 30, which $2.5^{24} = 4 \times 10^9$ times fainter than the faintest star visible to the naked eye. On this scale, the moon has $m = -12.74$ and the sun $m = -26.71$. The brightest star in the sky, Sirius has $m = -1.4$.

Example 8.2. Star A has an apparent magnitude of 2 and its flux is 1000 times more than the flux by star B. What is the apparent magnitude of star B?

Solution.

The fluxes differ by a factor of 1000 and the magnitudes go as log base 10. For each power of 10 in flux, the magnitude changes by 2.415 and the brighter objects have lower magnitude than the dimmer objects. Therefore, the apparent magnitude of B will be $2.512 \times 3 = 7.536$ more than that of A.

$$m_B = m_A + 7.536 = 9.536.$$

Example 8.3. The Sun has an apparent magnitude of -26.8 and Betelgeuse has magnitude $+0.41$. The flux from the Sun on Earth is $1.361\ \mathrm{kW/m^2}$, what is the flux of Betelgeuse?

Solution.

We can Eq. 8.13 to set up the equation for the unknown flux and solve.

$$m_1 - m_2 = -2.512\ \log_{10}\left(\frac{F_1}{F_2}\right),$$

with 1 for the Betelgeuse and 2 for the Sun.

$$0.41 - (-26.8) = -2.512\ \log_{10}\left(\frac{F_1}{1.361\ \mathrm{kW/m^2}}\right),$$

Therefore,

$$F_1 = 1.361\ \mathrm{kW/m^2} \times 10^{-27.21/2.512} = 2.0 \times 10^{-11}\ \mathrm{kW/m^2}.$$

Example 8.4. The brightest star in the sky is Sirius which has an apparent magnitude of $m = -1.41$ and that of the Sun is $m = -26.8$. By using parallax its distance is determined to be 2.61 pc. What is the luminosity of the star? Data: $d_\odot = 1\ \mathrm{AU} = 4.76 \times 10^{-6}\ \mathrm{pc}$, $L_\odot = 3.8 \times 10^{26}\ \mathrm{J.s^{-1}}$

Solution.

The absolute magnitude of the star and the Sun are easily computed to be

$$M = 1.52, \quad M_\odot = 4.96.$$

Now, the absolute magnitudes will be related to the absolute luminosities by

$$M - M_\odot = -2.512 \log_{10}\left(\frac{L}{L_\odot}\right).$$

Therefore,

$$L = L_\odot\ 10^{(M_\odot - M)/10}.$$

Putting in the numbers we get

$$L = 3.8 \times 10^{26}\ \text{ J.s}^{-1} \times 10^{(4.96-1.52)/10} = 8.9 \times 10^{27}\ \text{ J.s}^{-1}.$$

8.2.1 Temperature of Stars

In the early 1900s, stars were grouped into a series of spectral types based on the types and strengths of the spectral absorption lines. The spectral types are labeled by the letters, O, B, A, F, G, K, M. Often a mnemonic is used to remember this sequence: "Oh Be A Fine Girl/Guy, Kiss Me!" Our Sun is a G type star. Much finer classification is also used, but we will not discuss them here. The absorption lines are primarily determined by the surface temperature of the star, with O type star having the highest temperature, B the next hot, and so on.

The spectra of stars contain evidence of the chemical content of the stars as well as the temperature at the surface of the stars. The light coming from stars has absorption and emission lines corresponding to elements in the star. Elements are excited at different temperatures and lines from different elements become strongest at different temperatures. Thus, the absoption and emissions lines of a star are indicative of the temperature of the star. Table 8.1 lists some characteristics of different types of stars.

We can assign a surface temperature to a star if we assume that the emission of the star closely resembles that of a blackbody spectrum. The nuclear processes at the outer layer of a star tends to thermalize the photons before they are emitted and are close to the blackbody spectum. Thus, the flux F_* at the star surface would be given by Stefan-Boltzmann law.

$$\boxed{F_* = \sigma T^4,} \tag{8.15}$$

where T is the temperature of the blackbody, $\sigma = 5.67 \times 10^8 \text{W m}^2\text{K}^4$, the Stefan-Boltzmann constant, and F_* is related to the luminosity and radius r_* of the star as

$$F_* = \frac{L}{4\pi r_*^2}. \tag{8.16}$$

Table 8.1: Some Properties of Stars

Spectral Type	Temperature (K)	Color	Elements	Example spectral lines
O	28,000 - 50,000	Blue	Ionized helium	He^+: 4400 Å
B	10,000 - 28,000	Blue-white	Helium, some hydrogen	He^+: 4400 Å, He: 4200 Å
A	7500 - 10,000	White	Strong hydrogen, some ionized metals	H_α: 6600 Å, H_β: 4800 Å, H_γ: 4350 Å
F	6000 - 7500	Yellow-white	Hydrogen, ionized calcium (H and K), iron	Ca^+: 3800 - 4000 Å Fe: 4200 - 4500 Å
G	5000 - 6000	Yellowish	Strong G band, neutral and ionized metals, especially calcium,	G band of CH molecule: 4250 Å Ca^+: 3800 - 4000 Å
K	3500 - 5000	Orange	Neutral metals, sodium	Na: 5900 Å
M	2500 - 3500	Reddish	Strong titanium oxide, very strong sodium	TiO_2: 4900 - 5200 Å, 5400 - 5700 Å, TiO_2: 6200 - 6300 Å, 6700 - 6900Å

From Eqs. 8.15 and 8.16

$$T = \left[\frac{L}{4\pi\sigma r_*^2}\right]^{1/4}. \tag{8.17}$$

The temperature of a star can also be obtained from the wavelength λ_{max} at which the spectrum of the star has the maximum intensity by applying Wien's displacement law.

$$\boxed{T = \frac{2.897772 \times 10^3 \text{ K.m}}{\lambda_{\text{max}}}.} \tag{8.18}$$

From Eqs. 8.15 and 8.18, we find that a determination of the maximum of the spectrum can be used to determine the flux at the surface of the star.

$$F_* = \left(\frac{0.04}{\lambda_{\text{max}}[\text{in m}]}\right)^4 \text{ W/m}^2.$$

Example 8.5. The Sun has $L = 3.8 \times 10^{26}$ W and $r_* = 6.96 \times 10^8$ m. Find the temperature at the surface of the Sun.

Solution.

From Eq. 8.17, we get

$$T = \left[\frac{L}{4\pi\sigma r_*^2}\right]^{1/4} = 5,760\,\text{K}.$$

8.2.2 The Hertzsprung-Russell Diagram

During 1905-1913, the Danish astronomer Ejnar Hertzsprung and the American astronomer Henry Norris Russell independently introduced a plot of stars according to their luminosity and spectral type, or equivalently, the surface temperature which has turned out to be extremely valuable tool for thinking about stars of various types. The diagram is called Hertzsprung-Russell or H-R diagram.

In the H-R diagram, shown in Fig. 8.2 the vertical axis corresponds to increasing luminosity and the horizontal axis to the decreasing temperature following Russell's choice. The top left corner corresponds to high luminosity and high temperature and the bottom right corner to the lowest luminosity and lowest temperature. Each star is placed on the diagram according to its luminosity and temperature. As more and more stars are placed in the diagram, only certain areas of the diagram fill up while other areas remain empty with very few stars if any. Different parts of the diagram that fill up correspond to different types of stars.

The Sun is near the middle of the H-R diagram in a band of stars that goes roughly from the top left corner to the bottom right corner. These stars are called the **main-sequence stars**. The main-sequence stars get their energy from burning hydrogen into helium similar to the process in the Sun. The Red Giants and Super Giants are very massive stars. They occupy a region above the main-sequence in the H-R diagram. The burnt out stars, so-called white dwarfs, occupy the bottom left side of the diagram.

8.2.3 Masses of Astronomical Objects

How do we ascertain the masses of the large mass objects, such as planets, stars, and galaxies? A general method for determining the mass of a celestial object is based on observing the motion of another body that is in a motion about the celestial object of interest. For instance, to determine the mass of Earth, we can observe a satellite, say the Moon, around Earth and use the equation of motion for the moon or a satellite.

$$G_N \frac{M_E m}{r^2} = ma, \tag{8.19}$$

where M_E is the mass of the Earth, m is the mass of the satellite, a its acceleration, and r the distance to the satellite from the center of Earth. The mass of the satellite cancels out in this equation and we get the mass of the Earth from directly observable quantities.

$$M_E = \frac{a\, r^2}{G_N}. \tag{8.20}$$

Assuming the orbit the satellite to be circular of radius r, and the speed of the satellite to be constant v, then the acceleration happens to be related to the speed v and the radius r of the orbit.

$$a = \frac{v^2}{r}. \tag{8.21}$$

Thus, we get the following for the mass of Earth from measuring v and r of the satellite.

$$M_E = \frac{v^2\, r}{G_N}. \tag{8.22}$$

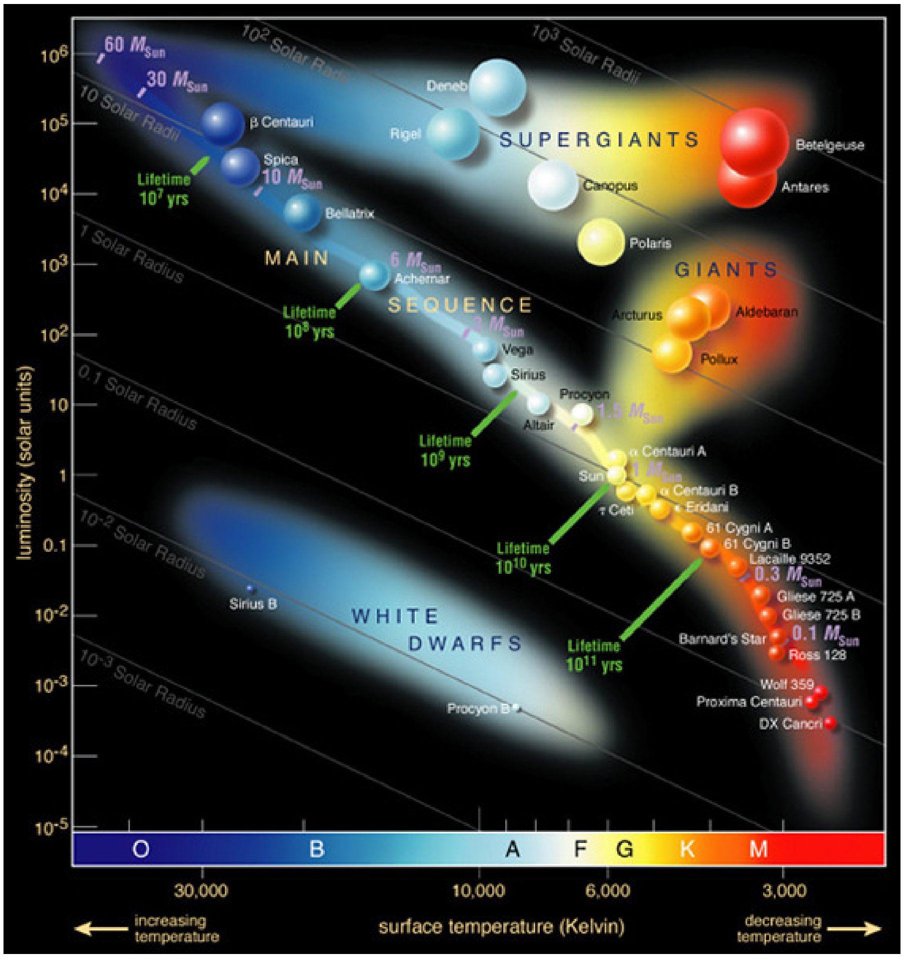

Figure 8.2: The Hertzsprung-Russel Diagram identifying many well known stars in the Milky Way galaxy. Credits: This photograph was produced by European Southern Observatory (ESO) and released under Creative Commons.

We can write the constant speed v in terms of r and period T of the motion of the satellite by noting that the satellite will go around once, covering a distance $2\pi r$ in time T.

$$v = \frac{2\pi r}{T}. \tag{8.23}$$

Therefore, the satellite has the following acceleration.

$$a = \frac{4\pi^2 r}{T^2}. \tag{8.24}$$

Putting this for a in Eq. 8.20, we obtain the formula for the mass of Earth completely in terms of measurable quantities.

$$M_E = \frac{4\pi^2 r^3}{G_N T^2}. \tag{8.25}$$

Let us see how this works out for values for the values of r and T for Moon as the test body. We have

$$r = 3.84 \times 10^8 \text{ m}, \quad \text{Orbital period } \; T = 27.321582 \text{ days} = 2.36 \times 10^6 \text{ sec}. \tag{8.26}$$

These values in Eq. 8.25 give the following value of the mass of Earth.

$$M_E = \frac{4\pi^2 \times (3.84 \times 10^8)^3}{6.67 \times 10^{-11} \times (2.36 \times 10^6)^2} = 6.02 \times 10^{24}\ \text{kg}. \tag{8.27}$$

The satellite method illustrated above can be applied also to the planets as test bodies to determine the mass of the Sun. This is actually how the mass of the Sun is determined from a planet orbit and period of revolution giving the following value.

$$M_{\text{sun}} = \frac{v^2\, r}{G_N} = \frac{4\pi^2 r^3}{G_N T^2}, \tag{8.28}$$

where r is the radius of a planet's orbit about the sun and T its orbital period. Most stars are binaries, meaning part of a two-star system, which gravitate around each other. The dynamics of binary stars can be used to determine the mass of star in a binary system. You can apply Eq. 8.28 to the orbit of a star as the test body at the edge of a galaxy to determine the mass of the galaxy.

$$M_{\text{galaxy}} = \frac{v^2\, r}{G_N} = \frac{4\pi^2 r^3}{G_N T^2}, \tag{8.29}$$

where r is the distance of the test star from the center of the galaxy, v its orbital speed, and T the orbital period.

8.2.4 The Energy Source of the Sun

The energy for most main-sequence stars come from nuclear fusion of protons into helium. For instance, it is believed that the following chain of fusion reactions, called the proton-proton chain - I, occur in the Sun.

$$\boxed{\begin{array}{l} \mathrm{p} + \mathrm{p} \rightarrow {}^2\mathrm{H} + e^- + \nu_e \\ {}^2\mathrm{H} + \mathrm{p} \rightarrow {}^3\mathrm{He} + \gamma \end{array}} \qquad \boxed{\begin{array}{l} \mathrm{p} + \mathrm{p} \rightarrow {}^2\mathrm{H} + e^- + \nu_e \\ {}^2\mathrm{H} + \mathrm{p} \rightarrow {}^3\mathrm{He} + \gamma \end{array}}$$

$$\boxed{{}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^3\mathrm{He} + \mathrm{p} + \mathrm{p}}$$

In the first reaction, two protons are fused to make a deuteron, a positron, and a neutrino. In the second reaction, the deuteron fuses with another proton with the production of ^{3}He. Two ^{3}He then combine to form ^{4}He and release two protons. Thus, two times the first two reactions and one time the third equation forms a full cycle. In the full chain, four protons combine in the first reaction and two in the second reaction, and two protons are released in the third reaction. Thus, a total of four protons combine to yield one ^{4}He, two positrons, two neutrino, and two photons.

The net energy released as positron, neutrino, and gamma rays over one proton-proton chain will be

$$Q_{\text{net}} = \left(4m_p - m[\,^4\text{He}]\right)\, c^2 \approx 25\,\text{MeV}.$$

When the two positrons collide with two electrons in the Sun, they annihilate each other and produce gamma rays with additional energy of approximately 2 MeV. This makes the total energy released from the proton-proton chain plus annihialtion of positron to be about 27 MeV per four proton. The proton-proton chain reaction occurs deep inside the Sun where the temperature is quite high, $\sim$ 15 MK.

8.2.5 Life Cycle of Stars

A star is born when gravity pulls together clouds of material in the interstellar medium. The force of gravity tries to collapse the star while the pressure of the gas and radiation inside the star counteracts the force of gravity. The two forces remain in a hydrostatic equilibrium for much of the life of a star. The heat and radiation in the interior of a star is provide by various nuclear reactions. Initially, stars get their energy from fusing hydrogen into helium, as is presently done by the Sun. The stars where this is the dominant source of energy are in the main-sequence line in the H-R diagram.

When there is not much hydrogen left in the center, the star contracts which increases the pressure sufficiently to cause the fusion of helium into carbon at the center and that of hydrogen into helium in a shell around the center. The star then moves up and right in the H-R diagram as it evolves away from the main sequence. Subsequent nuclear burning produces an iron-rich core. Recall that the energy per nucleon is highest near iron and adding more nucleons to iron requires more energy rather than release of energy.

Eventually, the star runs out of energy sources and it begins to collapse. The initial collapse in the core is preveneted from collapsing the star completely due to the degeneracy pressure caused by the quantum requirement of Pauli's exclusion principle. Pauli's exclusion principle states that no two electrons (or fermions) can occupy the same quantum state.

Collapsing the star brings fermions into the same physical area where no more than two electrons, one with up spin and the other with down spin can exist. This causes an upward pressure called the degenracy pressure which supports the gravitational force up to a limit called, the **Chandrasekhar limit**. If the mass of the collapsing material is more than about 1.4 solar mass, the Chandrasekhar limit, the degeneracy pressure cannot prevent collapse. In that case, the star continues to collapse. If the mass of the star is less than the Chandrasekhar's limit, the star just dies after collapsing into a small ball of electrons. This type of star is called **white dwarf**.

For more massive stars than 1.4 solar masses, the pressure from gravity is more than the degeneracy pressure and the star continues to collapse. With increase in pressure a;nd temperature in the core it becomes possible to cause reaction between protons and electrons to make neutrons and antineutrinos

$$\mathrm{p} + \mathrm{e}^- \rightarrow \ \mathrm{n} + \nu_e.$$

The core of the star turns into neutrons. The collapsing material releases enormous amount of gravitational energy which causes a spectacular explosion, called **supernova**. The explosion of supernova releases neutrinos and light. The light from one such supernova in a galaxy masks all other stars in the host galaxy. The pressure at the center in a supernova explosion is so great that all the elements above iron are formed during these explosions and with the explosion they are dispersed in the interstellar space. All the other elements found on Earth and else where were made in supernovae.

Neutron Stars

At the end of the supernova explosion, the core left is a very dense star which consists mostly of neutrons. The remnant is called a neutron star whose gravitational collapse is supported by the degeneracy pressure of neutrons, just as white dwarf is supported by the degeneracy pressure of electrons. The mass of a neutron star is comparable to that of the Sun, but its radius is only about 10 km or so. Due to the conservation of angular momentum and due to the fact of explosion being radially out, the angular momentum of the original star ends up as the angular momentum of the neutron star. Since the moment of inertia of the neutron star is considerably smaller than the original star, it rotates at enormous angular speed.

$$I_0\omega_0 = I_{NS}\omega_{NS}, \quad I_{NS} << I_0 \Longrightarrow \quad \omega_{NS} >> \omega_0.$$

Neutrons have magnetic dipole moment also. As a result, a neutron star has an enormous magnetic field, about a trillion times stronger than Earth's magnetic. In 1967 Franco Pacini suggested that if the neutron stars were spinning and had large magnetic fields, then electromagnetic waves would be emitted. A search of intense electromagnetic source in radio waves led to the discovery of pulsars by Jocelyn Bell and Anthony Hewish in England in 1967. **Pulsars** are neutron stars whose axis of rotation is misaligned with respect to the magnetic poles such that the electromagnetic waves emitted from the magnetic poles come in the view of the Earth and appear as if some one is turning light on and off with a definite period, akin to the light from the light house. The pulsar detected by Bell and Hewish had a period of 1.337302088331 second and pulse width of 0.04 second. A schematic view of the rotation of neutron star leading to observation of pulses on Earth is illustrated in Fig. 8.3.

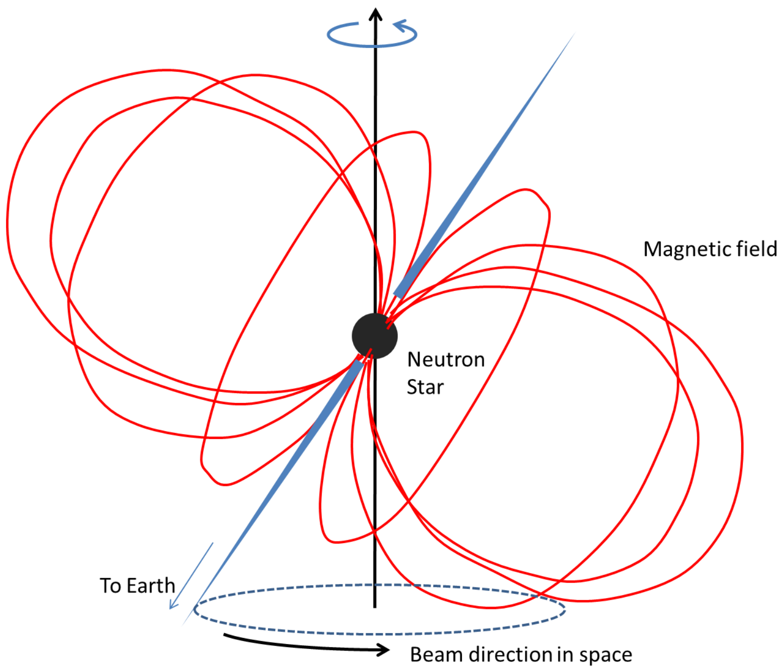

Figure 8.3: Schematic view of a pulsar/neutron star. The emission from the neutron star is at an angle to the rotation axis so that the beam from the star comes towards the once per period.

Black Holes

If the supernova was from the evolutionary end point of a star with mass more than 10 times the mass of the Sun, the degenracy pressure from neutrons may not be enough to support the gravitational collapse of the remnant star. The star continues to collapse to a very dense compact object of nearly zero volume. The gravity around such a compact object is so strong that even light cannot escape its pull. Such an object is called a black hole. The center of the Milky Way galaxy is supposed to be a black hole. The pull of the black hole is especially strong within a radius of the black hole, called event horizon. The distance from the center of the black hole to the event horizon is equal to the **Schwarzscild radius** of the black hole. For a black hole of mass M the Schwarzschild radius is give by

$$r_s = \frac{2G_N M}{c^2}.$$

Einstein's theory of gravity shows that no information from within the event horizon, $r < r_S$, of a black hole can reach outside world. For an object of the mass of the mass of the Sun the Schwarzschild radius is about 3 km. Although the r_s of the Sun is much less than the radius of the Sun, the r_s of a black hole is larger than the radius of the blackhole due to the collapse to almost zero size. If a black hole is formed near another star, then the material from the companion star will be sucked into the balck hole. The falling material will be ionized and heated to millions of degrees and therefore will emit X rays. The source of X ray binaries have yielded several candidates for black holes. One such candidate is the V404 Cygni binary star system which has a black hole of mass about 12 solar masses with the companion star slightly less mass than the mass of the Sun from which the black hole is accreting matter. As of 2014 several black hole candidates have been identified.