3.7 PROBLEMS

Problem 3.7.1. The mass of a body on Earth is easily found by using a spring balance which measures weight mg of the body. In outer space this method is not available for determining mass of a body. (a) Design a method for finding mass based on simple harmonic oscillator. (b) Describe how you will find the mass of an astronaut in space.

Ans: (a) Hint: A spring of known k can be attached to the body and the other end of the spring attached to the fixed support.

Problem 3.7.2. A plank of mass 10-kg rests on four identical springs of spring constant k. (a) What can be the largest spring constant if the plank is not to vibrate at greater than 1 Hz frequency? (b) If 20 kg of lead is put on the plank, what will be the new frequency of vibration?

Ans: (a) 99 N/m; (b) 0.58 Hz.

Problem 3.7.3. A disk of mass m and radius R is suspended from a thin string of torsion constant κ. The string is fixed to the center of disk and oriented perpendicular to the disk. When the disk is rotated by a small angle about the equilibrium, there is a restoring torque due to twist in the string, which tends to bring the disk back to the equilibrium. Find the frequency of small oscillations about the equilibrium. Ans: $\frac{1}{2\pi R}\sqrt{2\kappa M}$.

Problem 3.7.4. A bullet of mass m and speed v_0 is fired horizontally on a block of mass M attached to a spring of spring constant k whose other end is fixed to a wall. The direction of the velocity of the bullet is along the length of the spring. Upon impact, the bullet is embedded in the block. (a) Ignoring damping, find the amplitude of the resulting harmonic motion. (b) Compare the combined mechanical energy of the bullet and the block together after the impact to that of their mechanical energy before the impact.

Ans: (a) $\frac{mv_0}{\sqrt{k(M+m)}}$; (b) $\frac{E_{\text{after}}}{E_{\text{before}}} = \frac{m}{M+m} < 1$.

Problem 3.7.5. A block of mass m rests over a block of mass M which rests on a frictionless table. A spring of spring constant k is attached to M. The two-mass assembly is pulled so that the spring stretches by a distance A and then released from rest. If the stretching is greater than a critical value A_0, the block m slides on the block M. What is the coefficient of static friction between m and M?

Ans: $\mu_s = \frac{kA_0}{(M+m)g}$.

Problem 3.7.6. Two springs of spring constants k_1 and k_2 are attached on the two opposite sides of a block of mass m and the free ends of the springs are attached to two fixed supports such that the springs are taut and stretched. The block rests on a frictionless flat horizontal surface and can move along the line of the two springs. When the block is pulled a little from the equilibrium position in the line of the springs and released from rest, the block executes a simple harmonic motion whose frequency depends on the spring constants of the two springs and the mass of the block. (a) By looking at forces on the block at an arbitrary point in time, find the equation of motion of the block. (b) From the equation of motion, show that the angular frequency of oscillation is given by $\omega = \sqrt{\omega_1^2 + \omega_2^2}$, where $\omega_1^2 = k_1/m$ and $\omega_2^2 = k_2/m$. (c) Deduce the formula for the frequency by examining the expression for the energy of the system.

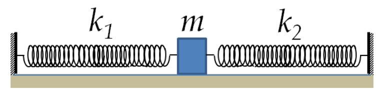

Figure 3.27: Problem 3.7.6

Ans: (a) $m\dfrac{d^2x}{dt^2} = -\left(k_1 + k_2\right)\ x.$

Problem 3.7.7. Show that the system of two springs of spring constants k_1 and k_2 connected in parallel to an object has an effective spring constant equal to $k_{\text{eff}} = k_1 + k_2$.

Hint: $F_{net,x} = -(k_1 + k_2)x$.

Problem 3.7.8. Two springs of spring constants k_1 and k_2 are glued with a light but strong glue. One end of the combination is attached to a fixed wall and a block of mass m is attached to the other end and the block is placed on a frictionless table as shown in Fig. 3.28. (a) Write the equation of motion of the block at an arbitrary time. (b) Write the equation of motion of the glue at the junction of the two springs and simplify the equation by setting the mass of the glue to zero. Assume $m\vec{a}$ of the glue to be zero. (c) Combine the two equations to show that the frequency of oscillation of the block is given by $\omega = \frac{\omega_1\omega_2}{\sqrt{\omega_1^2+\omega_2^2}}$, where $\omega_1^2 = k_1/m$ and $\omega_2^2 = k_2/m$. (d) Deduce the formula for the frequency by examining the expression for the energy of the system.

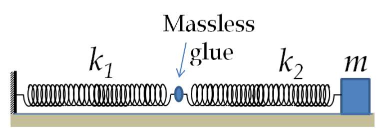

Figure 3.28: Problem 3.7.8

Hint: If springs 1 and 2 stretch by x_1 and x_2, then you will have $k_1x_1 = k_2x_2$ also.

Problem 3.7.9. A U-tube of uniform cross-section of area A is filled with a liquid of density ρ up to a height h_0. The total length of water in the U-tube is l. At equilibrium the levels of the liquid on the two sides are at the same height h_0 from the ground. Now, the liquid on one side is pushed in by a small height Δh and let go. It is then observed that the liquid executes a simple harmonic motion about the equilibrium level h_0. For this problem ignore the effects of resistance and viscosity. Use a coordinate system whose y-axis is pointed up and the origin is at the equilibrium level of the fluid in the

U-tube to answer the following questions. (a) Find an expression for the potential energy of water at an arbitrary time. Express your answer in terms of the y-coordinate of the level in one of the arms. (b) Find an expression for the kinetic energy of the liquid at an arbitrary time. Note that all liquid in the U-tube will be moving. Express your answer in terms of dy/dt, the rate at which the level will be changing. (c) Find the frequency of oscillation.

Ans: (a) $U = (\rho y A) g y$, (b) $\frac{1}{2}(\rho A l)(\dot{y})^2$, (c) $\omega = \sqrt{\frac{2g}{l}}$.

Problem 3.7.10. The power input to an oscillator by an applied force $\vec{F}$ depends on the velocity $\vec{v}$ of the oscillator and the applied force at that instant as given by the defining equation for the instantaneous power, $P(t) = \vec{F}\cdot\vec{v}$. By integrating over a period and dividing by the period, we obtain an average power of the force. (a) Find a formula for the average power delivered to a sinusoidally driven oscillator in the steady state. (b) From the formula in (a), deduce the formula for the resonance frequency of average power.

Ans: (a) $P_{\text{ave}} = \frac{1}{2}\omega F_0 A \sin\delta$, where A and δ are the amplitude and phase lag of the displacement of the oscillator, and F_0 is the amplitude of the force. (b) $\omega_R = \omega_0$. This problem is about showing the steps in the derivation of this result.

Problem 3.7.11. The front suspension of a car has a natural frequency of 0.5 Hz. At the time the car was built, according to specifications, the shock absorbers of the car are set to provide damping at the critical damping. With time, the shock absorbers get worn out so that they no longer provide the critical damping. The car then acts as an under-damped oscillator: when the car goes over a bump, it oscillates through many cycles. From measurements on the damped oscillations of the car, you calculate that $\beta = 0.4\ rad/sec$. When the car is driven on a bumpy road with bumps placed at regular intervals of 50 m, the car shakes violently when driven at a particular speed. Find this critical speed that the driver must avoid.

Ans: $v = 24.8$ m/s.

Problem 3.7.12. To determine the value of acceleration due to gravity from a physical pendulum one needs to determine the time period and the moment of inertia of the pendulum. However, the moment of inertia is usually difficult to determine experimentally. The Kater's pendulum is a special physical pendulum that removes the necessity of knowing the moment of inertia by using the periods from two suspension points.

The **Kater's pendulum** consists of a long metallic rod with a slideable weight on the rod. The pendulum has two knife edges at two ends that are used to suspend the pendulum from a support with a hole over which the knife edge rests. By adjusting the pivot points and the mass distribution over the rod, it is possible to obtain a configuration such that the time periods of oscillations about the two suspension points O_1 and O_2 are equal. Let k be radius of gyration of the pendulum, l_1 and l_2 the distances from the center of gravity to the suspensions O_1 and O_1 respectively. Let T_1 and T_2 be the time periods of oscillations when Kater's pendulum is hung from knife edges O_1 and O_2 respectively.

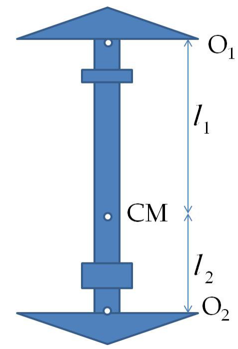

Figure 3.29: A Kater's pendulum.

(a) Find the time periods T_1 and T_2 in terms of l_1, l_2, k, and g. (b) Prove that when the time periods are equal, the period is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where $L = l_1 + l_2$, the distance between the two knife edges.

Ans: (a) $T_1 = 2\pi\sqrt{\frac{k^2+l_1^2}{gl_1}}$, $T_2 = 2\pi\sqrt{\frac{k^2+l_2^2}{gl_2}}$, (b) $T = 2\pi\sqrt{\frac{L}{g}}$.

Problem 3.7.13. The potential energy of a particle of mass m moving along x-axis is given as $U(x) = x(x-1)^2$. (a) Plot the potential energy as a function of x. Discuss the types of motion that a particle will execute for various values of energy of the particle. (b) Find the location of the minimum of the potential energy function. (c) Find the angular frequency of small oscillation about the minimum of the potential. (d) Are there any restrictions on the displacement for the particle to oscillate about the minimum? Explain.

Ans: (c) If the energy of the particle is between 0 and $\frac{4a}{27}$ and $0.33 < x < 1.33$ then the particle will oscillate about $x = 1$. Frequency of oscillations about the minimum $\omega = \sqrt{\frac{2a}{m}}$.

Problem 3.7.14. The potential energy of a particle of mass m moving along x-axis is given as $U(x) = x^2(x-1)^2$. (a) Plot the potential energy as a function of x. Discuss the types of motion that a particle will execute for various values of energy of the particle. (b) Find the location of the minima of the potential energy function. (c) Find the angular frequency of small oscillation about the minimum of the potential. (d) Are there any restrictions on the displacement for the particle to oscillate about the minimum? Explain.

Problem 3.7.15. A block of mass m attached to a spring of stiffness constant k is hung vertically in a fluid which damps the motion of the mass. The damping force is proportional to the speed with a constant of proportionality b. Ignore the force of buoyancy. The block is pulled a distance A from the equilibrium position so that the spring stretches by A, and released from rest. Find the subsequent motion by deriving the expressions for the displacement and velocity at an arbitrary time.

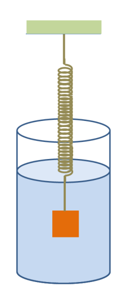
Figure 3.30: Exercise ??

Hint: $m\dfrac{d^2y}{dt^2} = -ky - b\dfrac{dy}{dt}$.