

3.8 EXERCISES

Displacement, Average Velocity and Speed

Ex 3.8.1. Figure 3.42 shows three displacement vectors. (a) Read off the x and y -components of each displacement vector. (b) Use a ruler and a protractor to find the magnitude and directions of the displacement vectors. The scale is given in the axes. (c) Determine the magnitude and direction of each vector from its x and y -components. (d) Check if the magnitude and direction of the vectors found from the x and y -components agree with the magnitude and direction you found by graphical methods. Give a reason for any discrepancies you find.

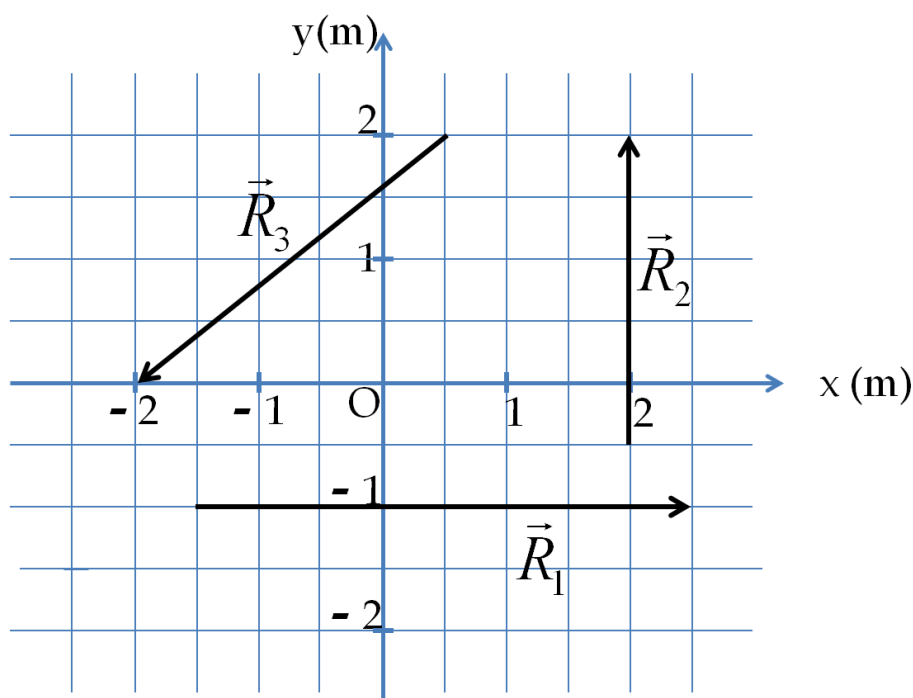


Figure 3.42: Exercise 3.8.1.

Ex 3.8.2. Figure 3.43 shows twelve displacement vectors which has only three unequal vectors. (a) Indicate which vectors are equal. (b) Using the scale given in the drawing, read the magnitude and directions of the displacement vectors. (c) Find the x and y -components of each displacement vector by either reading off from the figure or some other way. State your method. (d) Determine the magnitude and direction of each vector from its x and y -components. (e) Check if the magnitude and direction of the vectors found from the x and y -components agree with the magnitude and direction you found by graphical methods. Give a reason for any discrepancies.

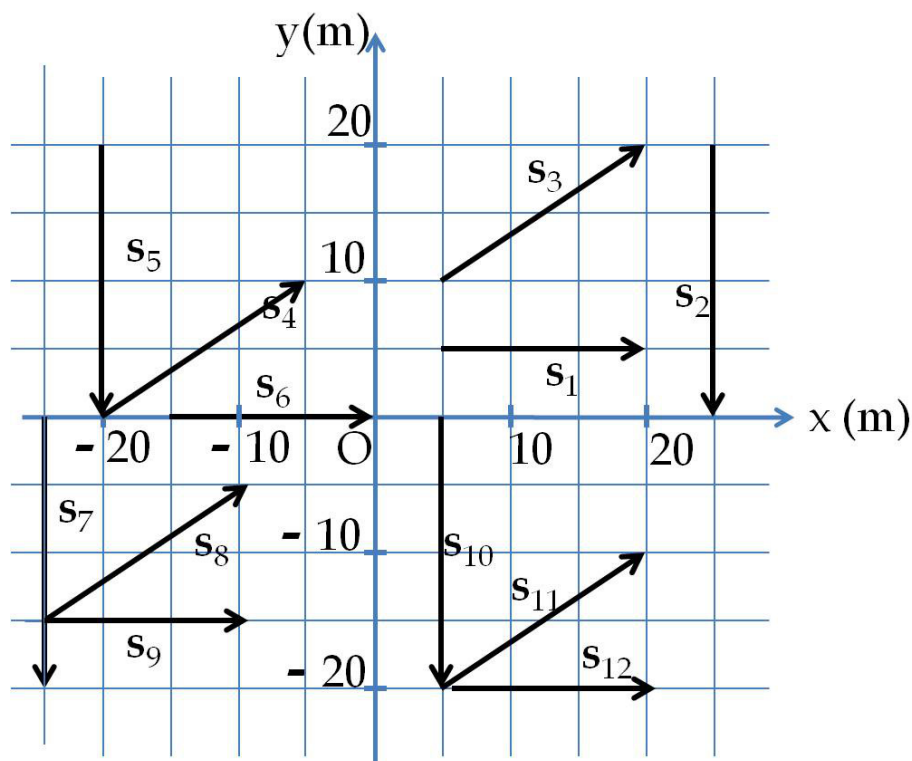


Figure 3.43: Exercise 3.8.2.

Ex 3.8.3. Add the three displacement vectors in Figure 3.42 by two methods: (a) graphically, and (b) analytically. In each case give the magnitude and direction of the sum.

Ex 3.8.4. A steel ball rolls in straight track as shown in Fig. 3.44. Six positions on the track are marked $A - F$. The distance on the track are drawn according to the scale shown in the figure. (a) Draw the displacement vectors for displacements from A to B , from B to C , from C to D , from D to E , and from E to F . (b) Find the magnitudes and directions of the displacement vectors by using graphical or analytic method, whichever you find convenient. (c) If the ball takes 10 seconds between successively marked points in the figure, what are the values of the average speed in m/s between A and B , B and C , C and D , D and E , and E and F ? (d) What would be the average velocity vectors between A and B , B and C , C and D , D and E , and E and F . Don't forget about the directions of the vectors.

Ex 3.8.5. A truck travels north at a constant speed on a straight North-South road covering a distance of 35 km in 30 min. The driver realizes that he forgot to pick up a package, and turns the truck around, and heads straight back to the original place. It took him 25 min on the return trip. (a) Draw two displacement vectors, one

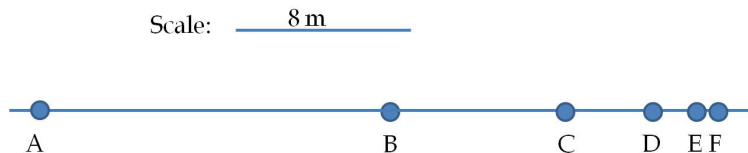


Figure 3.44: Exercise 3.8.4.

for the Northward motion and the other for the return motion. (b) Find the displacement, average velocity and average speed for the first part of the motion to the north. (c) Find the displacement, average velocity and average speed for the second part of the motion. (d) Find the displacement, average speed and average velocity for the entire trip. Express your speed and velocity in km/h.

Ex 3.8.6. A sprinter runs 100 meters on a straight track in 10.1 seconds. Then he walks back to the starting place taking 5 minutes. After reaching the starting place he runs again on the same track, this time taking only 10.0 sec. (a) Find the displacement, the average velocity and the average speed of the athlete during the following time intervals given in seconds. (i) $[0, 10.1]$, (ii) $[10.1, 310.1]$, (iii) $[0, 310.1]$, (iv) $[310.1, 320.1]$, and (v) $[0, 320.1]$. (b) Draw the displacement vectors for these time intervals using the same scale. (c) Draw the average velocity vectors for these time intervals using the same scale.

Ex 3.8.7. A subway train travels on a straight rail between two stations. Taking one of the stations as the origin, placing the x -axis on the track, the x -coordinate of the train is measured at different times. The result is displayed in Fig 3.45. (a) Draw displacement vectors for the following intervals given in minutes: (i) $[0, 5]$, (ii) $[5, 15]$, (iii) $[15, 25]$, (iv) $[25, 45]$, and (v) $[45, 50]$. (b) Estimate the displacement, average velocity and average speed for these intervals and for the $[0, 50 \text{ min}]$ interval.

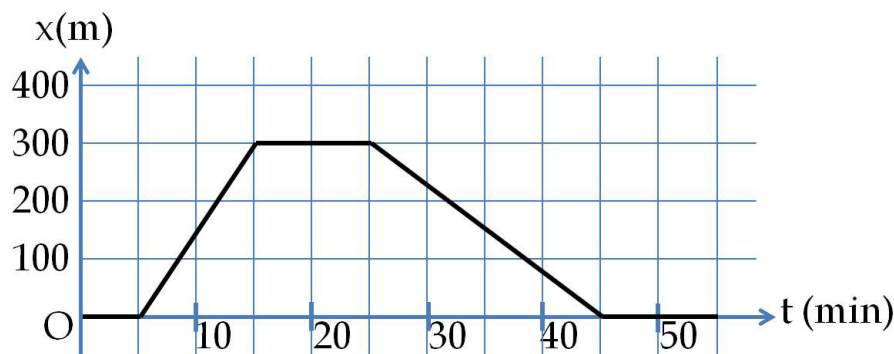


Figure 3.45: Exercise 3.8.7.

Ex 3.8.8. A train runs on a straight track all day repeating its route every 60 minutes, stopping at only the designated stations. Placing the origin at one of the stations and the x -axis on the track, the position of the train is recorded as its x -coordinate. Fig. 3.46 shows a plot of the x -coordinate of the train with time. (a) On a separate sheet of paper, draw displacement vectors for each successive 5 minute interval. (b) Assuming the train stops only at the stations, how many stations are there in the train's route? (c) For how long does the train stop at each station? (d) What are the average velocities of the train between different stations? Give the magnitude and velocity between pairs of the stations which are visited successively. (e) What is the average velocity over any 60-minute interval? (f) What is the average speed over any 60-minute interval?

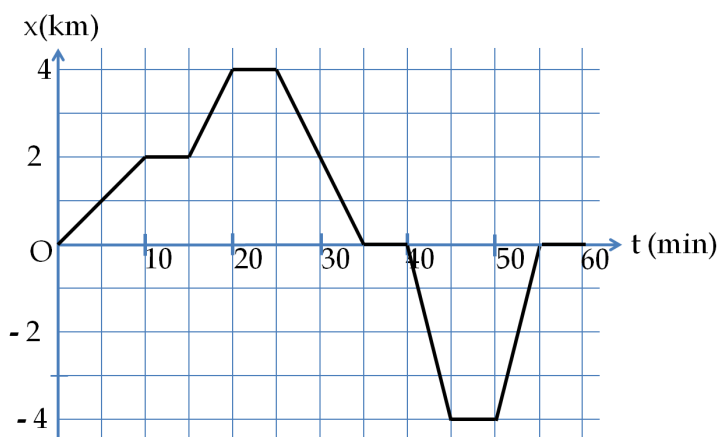


Figure 3.46: Exercise 3.8.8.

Ex 3.8.9. A steel ball rolls in a circular track as shown in Fig. 3.47. Six positions on the track are marked $A - F$. (a) In the figure, draw the displacement vectors for the displacements from A to B , from B to C , from C to D , from D to E , and from E to F . (b) Find the magnitudes and directions of the displacement vectors by using the graphical or the analytic method, whichever you find convenient. (c) If the ball takes 2 sec to complete one full revolution and if we assume that that ball rolls at constant speed, what is the value of the average speed in m/s? (d) What would be the average velocity vectors between A and B , B and C , C and D , D and E , and E and F ? (e) Even though the speed is constant, why would you have five different average velocities for your answer?

Ex 3.8.10. A race car is being driven on an elliptical track as shown in Fig. 3.48. Six positions on the track are marked $A - F$. (a) In the figure draw the displacement vectors for displacements from A to B , from B to C , from C to D , from D to E , and from E to F . (b) Find the magnitudes and directions of the displacement vectors by using

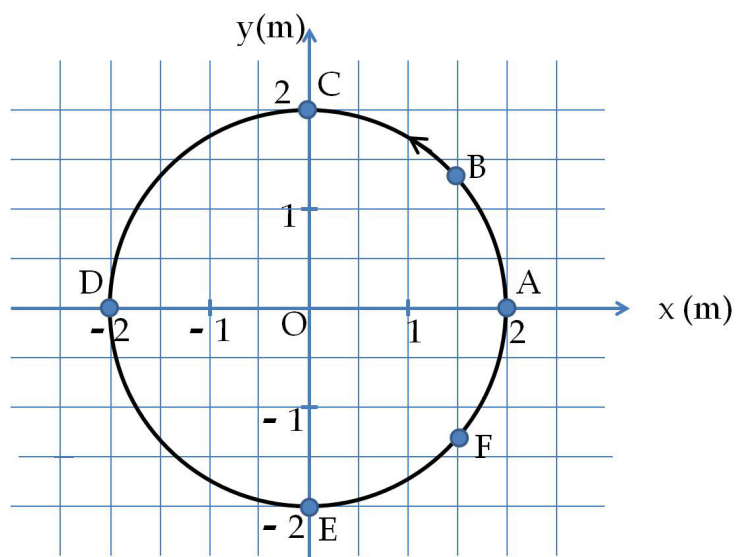


Figure 3.47: Exercise 3.8.9.

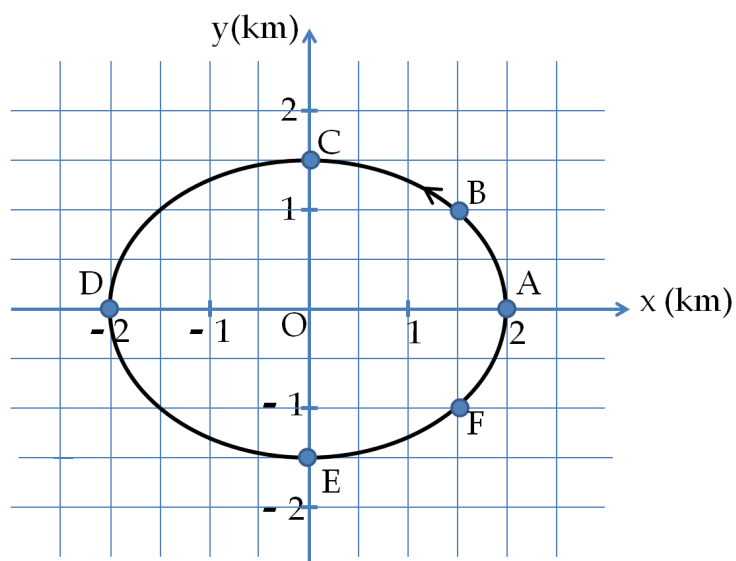


Figure 3.48: Exercise 3.8.10.

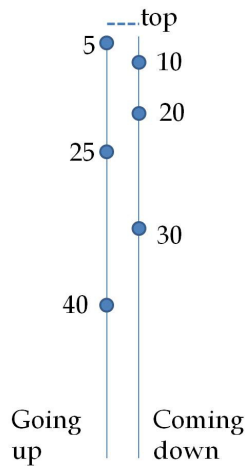


Figure 3.49: Exercise 3.8.11. Speeds in m/s.

the graphical or the analytic method, whichever you find convenient. (c) If the car takes 3 min to complete one full revolution and if we assume that that ball rolls at constant speed, what is the value of average speed in km/hr and in m/s? (d) What would be the average velocities vectors between A and B , B and C , C and D , D and E , and E and F ? (e) Even though the speed is constant, why would you have five different average velocities for your answer?

Instantaneous velocity and speed

Ex 3.8.11. Fig. 3.49 shows instantaneous speeds in m/s at various times when a ball is thrown vertically up. Decide on a scale for the velocity vectors and draw the velocity vectors at those instants. You may copy the figure on a larger piece of paper to show your drawings more clearly.

Ex 3.8.12. Refer to Fig. 3.47 for a ball rolling in a circular track. Suppose the ball rolls with a constant speed of 10 m/s. Decide on a scale for velocity and draw velocity vectors at points marked A , B , C , D , E , and F on the figure.

Ex 3.8.13. Refer to Fig. 3.48 for a car moving in an elliptical race-track. Suppose the car moves with a constant speed of 250 km/h. Decide on a scale for the velocity and draw the velocity vectors at points marked A , B , C , D , E , and F on the figure.

Ex 3.8.14. Fig. 3.50 shows three positions of a car rounding a bend. The speedometer of the car has readings of 40 mph, 15 mph, and 30 mph when the car is at A , B , and C , respectively. Here, mph means miles per hour. Decide on a scale for the velocity vectors and draw the velocity vectors at those instants. Show your scale in the drawing.

Ex 3.8.15. Fig. 3.51 shows five points on the trajectory of motion of a pendulum bob. The pendulum bob goes from A to E and back to A . The speed of the bob is 0, 4.2 m/s, 6.4 m/s, 4.2 m/s, and 0 when it is at A , B , C , D , and E respectively. Decide on a scale for the velocity vectors and draw the velocity vectors at each location for all instants in one full swing. Note that the velocity vectors at some locations will be different for different instants. Show your scale in the drawing.

Ex 3.8.16. A boy throws a ball straight up. The vertical position of the ball is given by the y -coordinate in a coordinate system in which the positive y -axis is pointed up and the origin is at the point where the ball leaves the hand. The x and z -coordinates of the ball do not

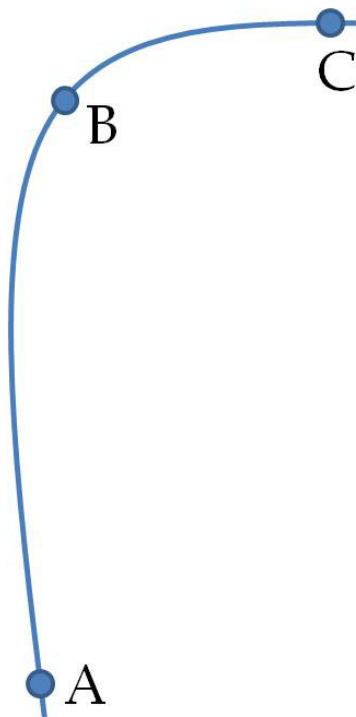


Figure 3.50: Exercise 3.8.14.

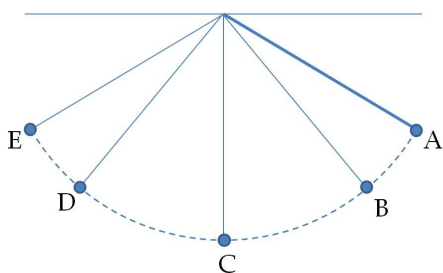


Figure 3.51: Exercise 3.8.15.

change with time. The y -coordinate of the ball as a function of time is shown in Fig. 3.52. (a) From the graph, estimate the instantaneous velocity of the ball at the following instants in time. (i) $t = 0$, (ii) $t = 0.4$ sec, (iii) $t = 1$ sec, (iv) $t = 1.2$ sec, and (v) $t = 2$ sec. (b)

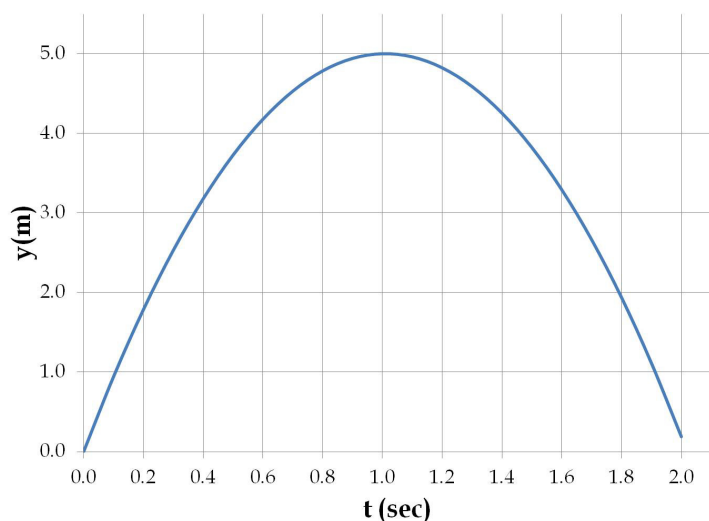


Figure 3.52: Exercise 3.8.16.

Draw these velocity vectors on a sketch of the vertical motion of the ball. (c) From the graph, estimate the average velocity between the following intervals. (i) $[0, 0.4 \text{ sec}]$, (ii) $[0, 1 \text{ sec}]$, (iii) $[0.4 \text{ sec}, 1 \text{ sec}]$, (iv) $[1 \text{ sec}, 1.2 \text{ sec}]$, and (v) $[0.4 \text{ sec}, 2 \text{ sec}]$. (b) How would your answer be affected if the x and z -coordinates also changed with time? Could you still determine the instantaneous velocities from the given data in Fig. 3.52?

Ex 3.8.17. The x -coordinate of a moving object varies in time according to the graph shown in Fig. 3.53, while the y and z -coordinates do not change with time. Assume the corners in the figure to be smooth. Determine the instantaneous velocity at the following instants (i) $t = 0$, (ii) $t = 5 \text{ min}$, (iii) $t = 15 \text{ min}$, (iv) $t = 35 \text{ min}$, and (v) $t = 45 \text{ min}$.

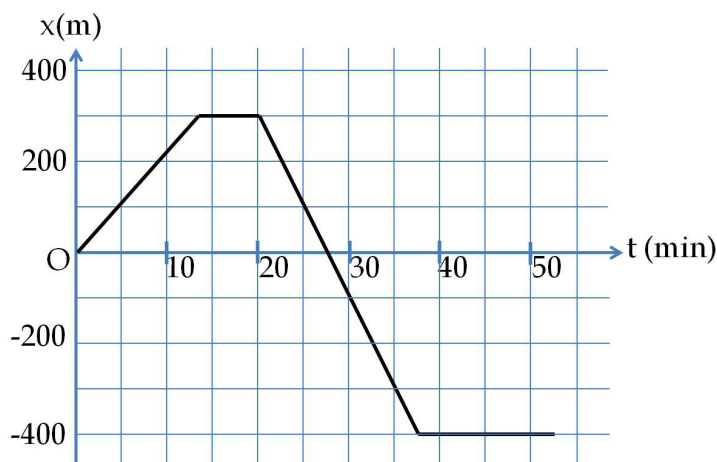
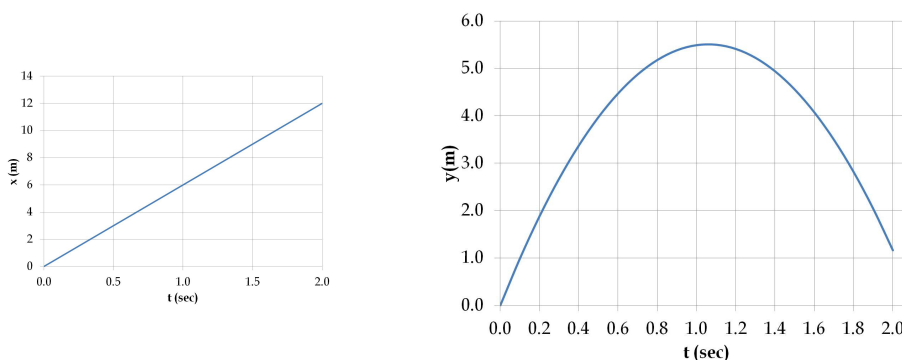


Figure 3.53: Exercise 3.8.17.

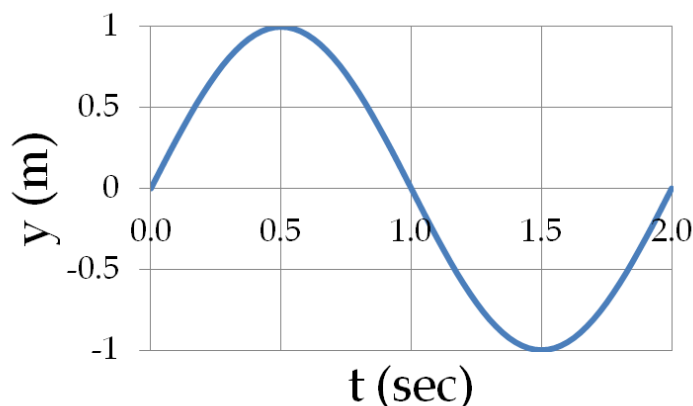
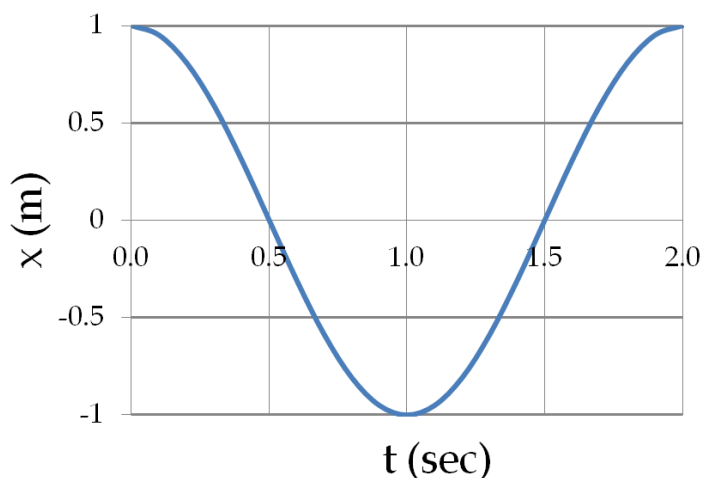
Figure 3.54: Exercise 3.8.18. Plot of x and y -coordinates of a projectile.

Ex 3.8.18. A golf ball is shot and its motion is recorded with respect to a Cartesian coordinate system. The z -coordinate of the ball does not change with time. The change in the x and y -coordinates of the ball with time is displayed in Fig. 3.54.

(a) From the graphs, estimate the instantaneous velocity of the ball at the following instants in time giving both the magnitude and direction: (i) $t = 0$, (ii) $t = 0.4$ sec, (iii) $t = 1$ sec, (iv) $t = 1.4$ sec, and (v) $t = 2$ sec. (b) Draw these velocity vectors in the xy -plane using a convenient scale so that all vectors can be drawn on the same graph. Give the scale for your drawing.

Ex 3.8.19. The motion of a steel ball rolling in a fixed track is recorded with respect to a Cartesian coordinate system. The z -coordinate of the ball does not change with time. The change in the x and y -coordinates of the ball with time is displayed in figures below. (a) From the graphs, estimate the instantaneous velocity of the ball at the following instants in time giving both the magnitude and direction: (i) $t = 0$, (ii) $t = 0.5$ sec, (iii) $t = 1$ sec, (iv) $t = 1.5$

sec, and (v) $t = 2$ sec. (b) Draw these velocity vectors in the xy -plane using a convenient scale so that all vectors can be drawn on the same graph. Give the scale for your drawing. (c) Can you guess the shape of the track? Why do you think so?



Ex 3.8.20. The position of a box sliding in a straight line is given by the x -coordinate of one of its corners. It is found that, while the y and z -coordinates do not change, the x -coordinate varies as a function of time as $x(t) = 4 + 18t - 5t^2$, where t is in seconds and x in meters. (a) Determine the instantaneous velocity and speed of the box at following instants in time. (i) $t = 0$, (ii) $t = 1$ sec, (iii) $t = 2$ sec, and (iv) $t = 3$ sec. (b) Find the average velocity of the box during the following intervals. (i) $[0, 1]$ sec, (ii) $[0, 2]$ sec, (iii) $[1, 2]$ sec, (iv) $[0, 3]$ sec, and (v) $[1, 3]$ sec.

Ex 3.8.21. A block of copper is attached to a spring that executes a one-dimensional motion along the direction of the spring. A Cartesian coordinate system is chosen so that the motion of the block occurs along the x -axis only. In this coordinate system, the position of the center of the block varies with time according to the following functions, $x(t) = 5.0 \cos(2\pi t)$, $y(t) = 0$, $z(t) = 0$, where t is in

sec and x in meters. (a) Plot x vs t for the time domain $(0, 3 \text{ sec})$. (b) Determine the instantaneous velocity and speed at the following instants: (i) $t = 0$, (ii) $t = \frac{1}{4} \text{ sec}$, (iii) $t = \frac{1}{2} \text{ sec}$, (iv) $t = \frac{3}{4} \text{ sec}$, and (v) $t = 1 \text{ sec}$. (c) Find the average velocity during the following intervals. (i) $[0, \frac{1}{4} \text{ sec}]$, (ii) $[\frac{1}{4} \text{ sec}, \frac{1}{2} \text{ sec}]$, (iii) $[0, \frac{1}{2} \text{ sec}]$, (iv) $[0, \frac{3}{4} \text{ sec}]$, and (v) $[0, 1 \text{ sec}]$.

Ex 3.8.22. After a football is thrown in air, the ball follows a parabolic trajectory that lies entirely in one plane. A coordinate system is chosen such that z -coordinate of the ball is always equal to zero. In this coordinate system, the x -axis is horizontal and the y -axis is pointed up. The changing x and y -coordinates of the ball are given by the following functions: $x(t) = 5t$ and $y(t) = 5t - 5t^2$, where t is in sec and x and y in meters. (a) Find the expressions for the x and y -components of the instantaneous velocity. (b) Determine the instantaneous velocity of the ball at the following instants in time. (i) $t = 0$, (ii) $t = 0.25 \text{ sec}$, (iii) $t = 0.5 \text{ sec}$, and (iv) $t = 1 \text{ sec}$. (c) Draw the velocity vectors on a graph paper with an appropriate scale. Show the scale on your graph.

Ex 3.8.23. A bug is flying in a helical path. The position of the bug in a Cartesian coordinate is given by its coordinates (x, y, z) . These coordinates vary with time according to: $x(t) = 2 \cos(\frac{\pi}{4}t)$, $y(t) = 2 \sin(\frac{\pi}{4}t)$ and $z = 0.1 t$, where t is in sec and the coordinates are in meters. (a) Find the expressions for the x , y , and z -components of the instantaneous velocity. (b) Determine the instantaneous velocity of the bug at following instants in time. (i) $t = 0$, (ii) $t = 10 \text{ sec}$, (iii) $t = 20 \text{ sec}$, and (iv) $t = 30 \text{ sec}$. (c) Draw the velocity vectors at these instants with an appropriate scale.

Acceleration

Ex 3.8.24. The velocity vectors of a car at three instants are shown in Fig. 3.55. Use the arrow for the velocity at $t = 0$ for the scale for vectors in the figure. Use the geometric approach of vectors to find the average acceleration in the following intervals: (i) from $t = 0$ to $t = 50 \text{ sec}$, (ii) from $t = 50 \text{ sec}$ to $t = 100 \text{ sec}$, (iii) from $t = 0$ to $t = 100 \text{ sec}$.

Ex 3.8.25. The velocity vectors of a rocket at two instants are shown in Fig. 3.56. (a) Use the geometric approach of vectors to find the average acceleration during an interval from $t = 0$ to $t = 2 \text{ sec}$. (b) Choose a coordinate system so that the two velocity vectors fall in the xy -plane, and determine the x and y -components of the two velocity vectors. (c) Find the average acceleration using the rate of change

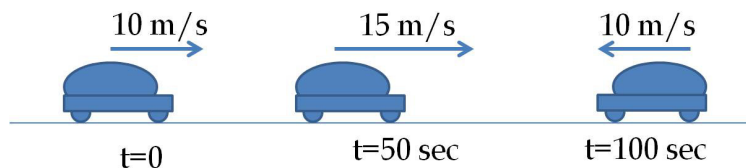


Figure 3.55: Exercise 3.8.24.

of the components of the velocity during the interval from $t = 0$ to $t = 2$ sec.

Ex 3.8.26. A car is moving at a constant speed of 5 m/s on a curved road as shown in Fig. 3.57. Suppose the time interval between successive points A , B , C , D , and E is 30 sec. (a) Draw the velocity vectors to scale at points A , B , C , D , and E . State the scale for your drawings. (b) Find the average acceleration between successive points marked on the trajectory. Give both the magnitude and the direction.

Ex 3.8.27. A motion detector is used to measure the velocity of a cart that moves along a straight track. The data is recorded as the x -component of the velocity by placing the x -coordinate on the track. The y and z -components of the velocity are zero here. The recorded x -component of the velocity is shown the Table 3.3. Plot the data, and, from the graph, determine the acceleration at the following instants: (i) $t = 0$, (ii) $t = 5$ sec, and (iii) $t = 10$ sec.

Ex 3.8.28. The position of a glider on a straight track is determined at various times. The x -coordinate of the glider at various times is shown in Table 3.4, and $y = z = 0$ at all times. (a) From the given table or a plot of x vs t from the data in the table, determine the instantaneous velocities at sufficient number of instants [10 may be enough] to draw a smooth curve for the v_x vs t plot. (b) Draw the v_x vs t plot from the data generated in part (a). (c) From the v_x vs t plot you generated, estimate the values of the instantaneous acceleration at the following instants: (i) $t = 2$ sec, (ii) $t = 8$ sec, and (iii) $t = 16$ sec.

Ex 3.8.29. The data for the velocity of a projectile in space is shown in Fig. 3.58. The axes are chosen so that the x -axis is horizontal and the y -axis vertically up. The z -coordinate of the projectile is not changing with time. (a) Find the instantaneous velocity of the projectile at the following instants: (i) $t = 0$, (ii) $t = 0.5$ sec, (iii) $t = 1$ sec, (iv) $t = 1.5$ sec, and (v) $t = 2$ sec. (b) Find the instantaneous acceleration of the projectile at the following instants: (i) $t = 0$, (ii) $t = 0.5$ sec, (iii) $t = 1$ sec, (iv) $t = 1.5$ sec, and (v) $t = 2$ sec.

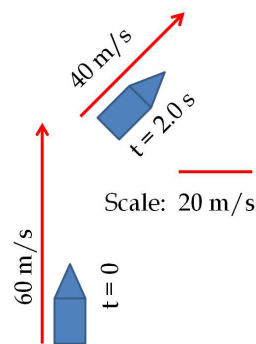


Figure 3.56: Exercise 3.8.25.

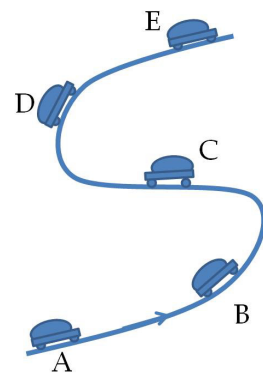


Figure 3.57: Exercise 3.8.26.

Table 3.3: Exercise 3.8.27

$t(s)$	$v_x(m/s)$	$t(s)$	$v_x(m/s)$
0	6.0	10	11.0
2	6.2	12	13.2
4	6.8	14	15.8
6	7.8	16	18.8
8	9.2		

Table 3.4: Exercise 3.8.28

$t(s)$	$x(m)$	$t(s)$	$x(m)$
0	0	9	-3

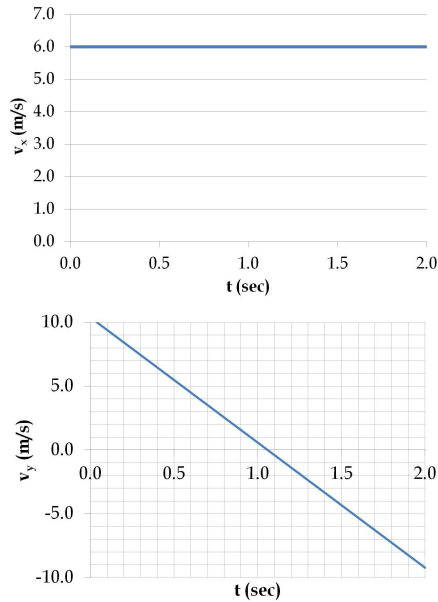


Figure 3.58: Exercise 3.8.29. Plot of v_x and v_y of projectile fired in space.

Ex 3.8.30. The data for the velocity of a soccer player in a field is shown in Fig. 3.59. The axes are chosen so that the x -axis is pointed from one goal post to the other and the y -axis from side to side. The z -coordinate of the player does not change during the time interval shown. (a) Find the instantaneous velocity of the soccer player at the following instants: (i) $t = 0$, (ii) $t = 5$ sec, (iii) $t = 25$ sec, (iv) $t = 35$ sec, and (v) $t = 45$ sec. (b) Find the instantaneous acceleration of the player at the following instants: (i) $t = 0$, (ii) $t = 5$ sec, (iii) $t = 12$ sec, (iv) $t = 25$ sec, (v) $t = 35$ sec, and $t = 45$ sec.

Ex 3.8.31. The x -coordinate of a particle moving in space is given by $x(t) = 3 + 5t + 9t^2$, where t is in sec and x in meter. (a) Determine the x -components of velocity and acceleration at (i) $t = 0$, (ii) $t = 1$ sec and (iii) $t = 2$ sec. (b) Determine the average velocity for the following intervals: (i) $[0, 1]$ sec, (ii) $[1, 2]$ sec, (iii) $[0, 2]$ sec. (c) Determine the average acceleration for the following intervals: (i) $[0, 1]$ sec, (ii) $[1, 2]$ sec, (iii) $[0, 2]$ sec.

Ex 3.8.32. A car moves on a straight road such that its position from a reference point is given as $x(t) = 3 + 5t + 2t^2 + 0.4t^3$, $y = 0$, $z = 0$, where t is in sec and x in meter. (a) Determine the velocity and acceleration at (i) $t = 0$, (ii) $t = 0.5$ sec and (iii) $t = 1$ sec. (b) Determine the average velocity between $t = 0$ and $t = 1$ sec. (c) Determine the average acceleration between $t = 0$ and $t = 1$ sec.

Ex 3.8.33. The position of a projectile in the air is monitored with respect to a reference point in a Cartesian coordinate system. The x -axis of the Cartesian coordinate system is in the horizontal direction

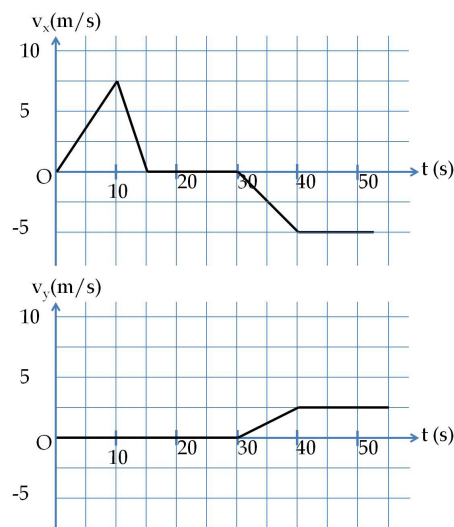


Figure 3.59: Exercise 3.8.30. Plot of v_x and v_y of a soccer player.

and the y -axis vertically up. The z -coordinate of the projectile does not change with time. The data for the x and y -coordinates are plotted and the following functions fit the x vs t and y vs t curves: $x(t) = 40t$ and $y(t) = 60t - 5t^2$. (a) Find the instantaneous velocity of the projectile at the following instants: (i) $t = 0$, (ii) $t = 4$ sec, (iii) $t = 8$ sec, and (iv) $t = 10$ sec. (b) Find the instantaneous acceleration of the projectile at the following instants: (i) $t = 0$, (ii) $t = 4$ sec, (iii) $t = 8$ sec, and (iv) $t = 10$ sec.

Ex 3.8.34. A car moves on a circular track. A coordinate system is chosen whose origin coincides with the center of the circle and the circle falls in the xy -plane. The position of the car with time varies according to the following functions of time: $x(t) = 2 \cos\left(\frac{\pi}{100}t\right)$, $y(t) = 2 \sin\left(\frac{\pi}{100}t\right)$, $z = 0$. Determine the velocity and acceleration of the car at (i) $t = 0$, (ii) $t = 50$ sec, (iii) $t = 100$ sec, (iii) $t = 150$ sec, and (iv) $t = 200$ sec.

Polar Coordinates

Ex 3.8.35. Convert the following Cartesian coordinates (x, y) into polar coordinates. (i) $(0, 1)$; (ii) $(1, 0)$; (iii) $(-1, 0)$; (iv) $(0, -1)$; (v) $(3, 4)$; (vi) $(-3, 4)$; (vii) $(-3, -4)$; (vii) $(3, -4)$.

Ex 3.8.36. Convert the following polar coordinates (r, θ) into Cartesian coordinates. The angle is given in radians counterclockwise from the positive x -axis as seen from the side of the positive z -axis. (i) $(1, 0)$; (ii) $(1, \pi/3)$; (iii) $(1, 2\pi/3)$; (iv) $(1, 3\pi/3)$; (v) $(1, 4\pi/3)$; (vi) $(1, 5\pi/3)$; (vii) $(1, 6\pi/3)$; (viii) $(2, 29\pi/3)$.

Ex 3.8.37. (a) Draw the unit vectors \hat{u}_r and \hat{u}_θ for the following rays. The rays are specified by the angles they make with the positive x -axis. (i) $\theta = 0$, (ii) $\theta = \pi/3$, (iii) $\theta = 2\pi/3$, (iv) $\theta = 3\pi/3$, (v) $\theta = 4\pi/3$, (vi) $\theta = 5\pi/3$, (vii) $\theta = 6\pi/3$. (b) Give the expression of these unit vectors in terms of the unit vectors $\{\hat{u}_x, \hat{u}_y\}$ along the Cartesian axes.

Ex 3.8.38. A displacement vector is given in the xy -plane by its magnitude 2 m and direction of 30° with respect to the x -axis. Write the displacement vector in terms of the unit vectors (i) $\{\hat{u}_x, \hat{u}_y\}$, (ii) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 30^\circ$, (iii) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 45^\circ$, and (iv) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 90^\circ$.

Ex 3.8.39. A velocity vector is given in the xy -plane by its magnitude 10 m/s and direction of 60° with respect to the x -axis. Write the velocity vector in terms of the unit vectors (i) $\{\hat{u}_x, \hat{u}_y\}$, (ii) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 60^\circ$, (iii) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 120^\circ$, (iv) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 180^\circ$, (v) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 240^\circ$, and (vi) $\{\hat{u}_r, \hat{u}_\theta\}$ for $\theta = 300^\circ$.

Ex 3.8.40. The position of a particle moving in the xy -plane is given by $\vec{r}(t) = a \cos(\pi t) \hat{u}_x + a \sin(\pi t) \hat{u}_y$, where t is in *sec* and a in *cm*. (a) Show that the motion is a circular motion, and find the value of the radius of the circle, and find the Cartesian coordinates of the center of the circle. (b) Find an expression for the velocity of the particle at an arbitrary instant t . (b) Find an expression for the angular speed of the particle at an arbitrary time t . (c) Find an expression of the acceleration of the particle at an arbitrary instant t .

Ex 3.8.41. A ball is rolling in a circular track of radius R which is in the xy -plane but the center of the track is not at the origin of the Cartesian coordinate system in use. Instead, the center of the track is at (x_0, y_0) . The ball has a constant speed of v_s . (a) Give an expression of the position vector as a function of time $\vec{r}(t)$ in terms of the unit vectors $\{\hat{u}_x, \hat{u}_y\}$ of the Cartesian coordinates. (b) Use your expression for $\vec{r}(t)$ to find an expression for the velocity of the particle at an arbitrary time t . (c) Use your expression for $\vec{r}(t)$ to find an expression of the acceleration of the particle at an arbitrary instant t .

Ex 3.8.42. A particle moves in a circle of radius 20 cm and centered at $(x = 1 \text{ m}, y = 0)$ in the xy -plane with a constant angular speed of 50 rad/sec . At $t = 0$, the particle is at $(x = 1.2 \text{ m}, y = 0)$. (a) For the particle moving in a counterclockwise sense, find the velocity at $t = 0$? (b) Find the position of the particle at an arbitrary time t ?

Ex 3.8.43. A particle moves in a circular motion in the xy -plane centered at the origin. At $t = 0$ it is at $(x = 30 \text{ cm}, y = 0)$. Its angular speed is given by $\omega = 2 + 3t$, where ω is in rad/sec^2 and

t in seconds. (a) What is the radius of the circle? (b) Find the change in the angular coordinate θ between $t = 0$ and $t = 1$ sec. (c) Find the change in the angular coordinate θ between $t = 1$ sec and $t = 2$ sec. (d) Find the velocity of the particle at $t = 0$. (e) Find the velocity of the particle at $t = 1$ sec. (f) Find the angular component a_θ of the acceleration at $t = 1$ sec. (g) Find the radial component of acceleration at $t = 1$ sec (h) Find the acceleration at $t = 1$ sec. (h) Find the angular component, radial component of the acceleration at $t = 2$ sec (i) Find the acceleration at $t = 2$ sec.

Ex 3.8.44. A bicycle with wheels of radius R is rolling without slipping. The center of the wheel moves with a constant speed v_s in a straight line towards the East. Write the position, velocity and acceleration vectors of a point on the outer edge of the wheel assuming the point is at $x = 0$ at $t = 0$ and the x -axis is pointed towards the East.

Ex 3.8.45. A platform is rotating about its center at constant angular speed ω_0 . A man is standing at a distance R from the center of the platform and rotating with the platform. Find the position, velocity and acceleration vectors of the man.