2.1 KEPLER'S LAWS OF PLANETARY MOTION

Johannes Kepler (1571-1630) was an assistant to the great observational astronomer Tycho Brahe (1546-1601) who meticulously recorded the positions of planets for over twenty years at the observatory built on the island of Hveen with the financial support of the Danish king Frederik II. After Brahe's death in 1601, Kepler persuaded Brahe's widow to let him study the voluminous data collected by Tycho Brahe. Kepler originally believed that the motion of planets must be based on the perfect symmetry of the platonic solids and expected to find circular paths for the planetary motion. However, when he found that the observed orbit of Mars was not a circular motion, he had to abandon his belief, and instead, try to understand the data directly. With the new mindset, he discovered that planets follow an elliptic path rather than a circular path, which is called Kepler's first law. He also deduced that the planets did not travel at constant speed around the ellipse, but the radius vector from the Sun to the planet sweeps out equal area in equal amount of time. This is called Kepler's second law. The first two laws were discovered in 1609. It took nine more years for Kepler to figure out the relation between the orbits of different planets, as formulated in the law of harmonies. The first two laws were not accepted for several decades, but the scientific community adopted the third law almost immediately after its publication in 1618. The three laws are as follows.

First Law or the Law of Elliptical Orbits - Planets travel in elliptical orbits about the Sun with the Sun at one focus.

Second Law or the Law of Equal Areas - The line joining the Sun and a planet covers equal area in equal time. Thus when a planet is nearer to the Sun, it has a higher speed than when it is further out. Figure 2.1 illustrates this law.

Third Law or the Law of Harmonies - The ratio of square of the period of revolution about the Sun to the cube of the semi major axis of the elliptical orbit of two planets are equal to each other. Thus if T_1 and T_2 are periods, and a_1 and a_2 the semi-major axes of two planets, we have the following equality.

$$\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3} \tag{2.1}$$

Kepler's three laws have played critical role in the discovery of the universal law of gravitation. By 1666 Newton had early versions of his three laws of motion, but yet did not have the law of gravitation.

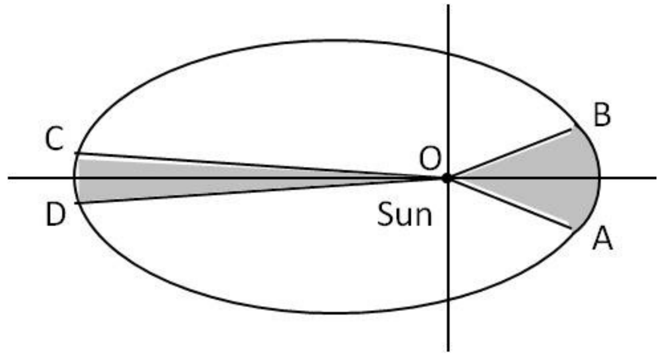

Figure 2.1: The area OAB covered by a planet in a duration Δt is equal to the area OCD covered in the same duration Δt.

In that year he had an insight that Earth's gravity extended also to the Moon and was counterbalanced by the centrifugal force. Newton used the balance of the two forces to find that the gravitational force must decrease as inverse square of the distance. The calculation was, however, only for a circular motion.

Newton did not work on the planetary motion problem any further till 1679 when another physicist, Robert Hooke, which you have met in the Hooke's law, went to see him about the elliptical orbit problem of the planets. Hooke had conjectured that a planet moving in an ellipse must be acted on by a central force by the Sun. Hooke had also come to the conclusion that this force must vary as the square of the inverse of the distance of the planet from the Sun, but he could not prove his conjectures mathematically.

After Hooke's visit, Newton went back to work on the planetary motion problem. First he showed mathematically that, if a body obeys Kepler's second law, then the force on the body must be central. Isaac Newton also showed that the angular momentum of a body is conserved if the body is acted upon by a central force. This finding demonstrated the physical basis of Kepler's second law as we have seen in the chapter on rotation.

Next, Newton showed that if a body is in an elliptical path, then the force must be pointed towards one of the foci and must vary as the square of the inverse of the distance from the focus. However, Newton did not publish any of these results until after the great Astronomer, Edmund Halley (1656-1742), asked him in 1684 if he could prove Hooke's conjecture. To Haley's surprise, Newton immediately replied that he had proved it five years earlier. Had Hooke been successful at proving his conjecture mathematically, we might be calling the law of gravitation the Hooke's law of gravitation instead of Newton's law of gravitation.

Halley used Newtonian calculations to ascertain that comets appearing in the sky in 1531, 1607 and 1682 were the same object and it was regularly appearing every 76 years. Sure enough the comet, now called Halley's comet, arrived on Christmas day in 1758 as predicted by Edmund Halley. The last time Halley's comet observable on Earth was in 1986 shown in Fig. 2.2. Upon Halley's insistence

Figure 2.2: Halley's Comet. Picture by NASA (1986).

Newton wrote up his treatise on mechanics and its applications to the celestial mechanics called the Philosophiae naturalis principia mathematica, or *Principia*, as commonly known, which was published in 1687. You have already encountered a mention of this seminal work of Isaac Newton in the chapter on Newton's laws of motion.