2.7 PROBLEMS

Problem 2.7.1. [Skip this problem if you have not taken Calculus III yet. Use this problem as statements of fact when solving other problems.] Prove that the gravitational force on a point particle of mass m placed outside a spherical shell of mass M is equal to the gravitational force on m by a point mass of mass M placed at the center of the shell.

Problem 2.7.2. [Skip this problem if you have not taken Calculus III yet. Use this problem as statements of fact when solving other problems.] Prove that the gravitational force on a point particle of mass m inside a spherical shell of mass M is zero.

Problem 2.7.3. A tunnel is dug through the center of Earth. Show that a particle of mass m dropped in the tunnel will execute a simple harmonic motion, and deduce the frequency of oscillation of m.

Ans: $f = 1.98 \times 10^{-4}$ Hz.

Problem 2.7.4. A planet of mass m moves around a star of mass M. For an inertial observer the planet and the star appear to move in circles of radii r and R respectively. For $m = M/4$ and $r = 1.0 \times 10^{10}$ m, find radius R of the orbit of the star.

Ans: $2.5 \times 10^9\ m$.

Problem 2.7.5. A satellite of mass 1000 kg is in circular orbit about Earth. The radius of the orbit of the satellite is equal to two times the radius of Earth. (a) How far away is the satellite? (b) Find the mechanical energy of the satellite. (c) Find the angular momentum of the satellite?

Ans: (a) $1.3 \times 10^7\ m$; (b) $-1.56 \times 10^10\ J$; (c) $7.12 \times 10^13\ kg.m^2/s$.

Problem 2.7.6. A spaceship of mass 3000 kg is to be sent from Earth to Venus. Assume the orbits of the Earth and the Venus around the Sun to be approximately circular. (a) What is the minimum energy required for the transfer? (b) The most efficient transfer from one circular orbit to another circular orbit about the Sun is achieved by what is called **Hohmann transfer**, which uses an elliptical orbit around the Sun as an intermediary with the smaller and larger circles located at perihelion and aphelion of an ellipse as shown in Fig. 2.21. The speed of the satellite must be increased at point Q and reduced at point P for the transfer. Find the changes in speed required at points P and Q. Note that the spaceship will have speeds of Earth and Venus while in their orbits. Ignore the gravitational pulls of the Earth and the Venus on the spaceship.

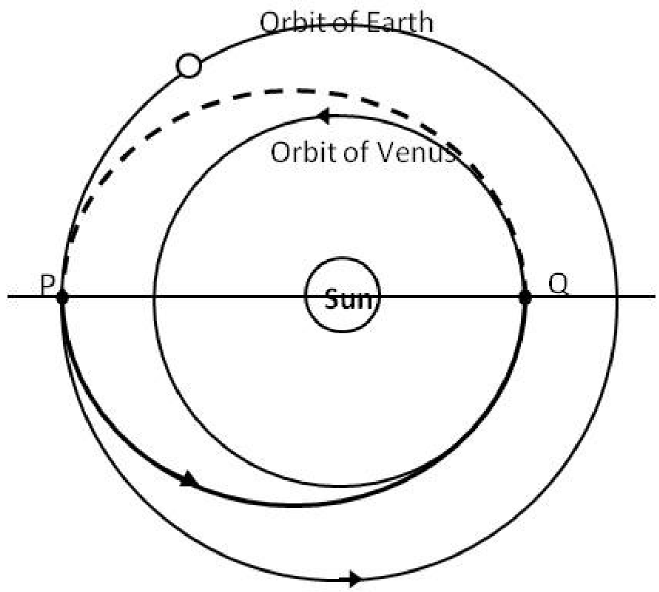

Figure 2.21: Problem 2.7.6.

Ans: (a) $-G_N Mm\left(\frac{1}{2r_V}-\frac{1}{2r_E}\right)$. Numerical value, $\Delta E=-5.16\times 10^{11}$ J.

Problem 2.7.7. A satellite is in an elliptical orbit about the Earth with a minimum altitude of 1500 km and a maximum altitude of 8000 km. In one of the flights, its engine is fired when it is at the closest approach point so that its speed increases by 10%. You can assume the speed increased over a very short period of time. What is the maximum altitude reached in its new elliptical orbit?

Ans: 2.5×10^7 m.

Problem 2.7.8. Two objects of masses m and M interact with a central force that varies as $1/r^4$ with proportionality constant k. Derive a formula for the potential energy function. State the location of the reference for your formula of potential energy?

Ans: $U(r)=-\frac{k}{3r^3}$.

Problem 2.7.9. A space station of mass m moves in a circular orbit of radius R around Jupiter (mass M). (a) A rocket of mass Δm is fired radially inward towards the center of Jupiter from the space station which causes the space station to "instantaneously" acquire a radial velocity v_r in addition to the angular velocity v_θ it had before the rocket firing. As a result the space station is thrown into an elliptical orbit. Find the semimajor axis and eccentricity of the elliptical orbit. (b) What would have happened if the rocket was fired tangentially to the circular orbit with relative speed u towards the front?

Ans Hint: (a) During the process of firing of the rocket the angular momentum of the satellite is conserved but energy is not conserved. This will give you conditions on the r_{min} and r_{max} of the new elliptical orbit. From these you can determine the expression for the eccentricity of the orbit. (b) In this case both angular momentum and energy change.