6.4 Nuclear Reactions

A nuclear reaction is a process in which one or more nucleus changes to form some other nucleus or nuclei. In 1919 Rutherford noticed that when alpha particles were projected into air peculiar radiation came out. He then experimented with pure nitrogen and verified that alpha particles actually converted nitrogen nuclei to oxygen nuclei and released protons in the process.

$$ {}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}. \tag{6.53}$$

This was the first observed deliberate transmutation of one element to another. Rutherford suggested that nuclear reactions can occur if a heavy particle of sufficient energy is incident on a nucleus so that the particle can penetrate the nucleus and form another nucleus with different A, Z, and/or N values. The product nucleus may or may not be stable. Most often the resulting nucleus is unstable and undergoes further nuclear transformations.

Every nuclear reaction must obey several fundamental conservation laws.

1. Conservation of energy
2. Conservantion of momentum
3. Conservation of angular momentum
4. Conservation of electric charge
5. Conservation of number of nucleon

The conservations of energy, momentum, angular momentum, and charge are familiar from before. The additional criteria now is the conservation of the nucleon number, which is A of the species. That is the sum of A values of particles before the reaction must equal the sum after the reaction. For instance, consider the nuclear reaction observed by Rutherford given in Eq. 6.53.

$$A_{\text{before}} = 14 + 4 = 18; \quad A_{\text{after}} = 17 + 1 = 18.$$

Therefore,

$$A_{\text{before}} = A_{\text{after}}.$$

Many nuclear reactions occur when a target nucleus X at rest is struck by a particle u. The target nucleus is then tranformed into another nucleus Y and a particle w is emitted.

$$ {}^{A_1}_{Z_1}\mathrm{X} + {}^{A_2}_{Z_2}\mathrm{u} \longrightarrow {}^{A_3}_{Z_3}\mathrm{Y} + {}^{A_4}_{Z_4}\mathrm{w}. \tag{6.54}$$

This reaction can be written in two short-hand notations:

$$\boxed{\mathrm{X} + \mathrm{u} \longrightarrow \mathrm{Y} + \mathrm{w} \quad \text{or} \quad \mathrm{X(u,w)Y.}} \tag{6.55}$$

Table 6.4: Some nuclear reactions (from C. W. Li, W. Whaling, W.A. Fowler, and C. C. Lauritson, Physical Review, vol 83, p 512 (1951).

Reaction	Reaction-simpler notation	Measured Q (MeV)
^{2}H + n $\longrightarrow$ ^{3}H + γ	^{2}H(n,γ)^{3}H	6.257
^{6}Li + p $\longrightarrow$ ^{3}H + ^{4}He	^{6}Li(p,^{4}He)^{3}H	4.016
^{7}Li + p $\longrightarrow$ ^{4}He + α	^{7}Li(p,α)^{4}He	17.337
^{9}Be + ^{2}H $\longrightarrow$ ^{10}Be + p	^{9}Be(^{2}H,p)^{10}Be	4.585
^{10}B + n $\longrightarrow$ ^{7}Li + α	^{10}B(n,α)^{7}Li	2.793
^{14}N + n $\longrightarrow$ ^{14}C + p	^{14}N(n,p)^{14}C	-0.627
^{18}O + p $\longrightarrow$ ^{18}F + n	^{18}O(p,n)^{18}F	-2.453

The conservation of the nucleon number means that we must have

$$A_1 + A_2 = A_3 + A_4.$$

Similar to the disintegration energy we define **reaction energy** from the difference of the rest energies of the reactants [i.e. particles before the reaction] and the products [i.e. particles after the reaction].

$$Q = [(m_X + m_u) - (m_Y + m_w)]\ c^2. \tag{6.56}$$

Applying the conservation of energy we find that the difference in the kinetic energies must equal Q also.

$$\begin{aligned} &E_{\text{reactants}} = E_{\text{products}} \text{ implies} \\ &\qquad Q = [(m_X + m_u) - (m_Y + m_w)]\ c^2 = K_Y + K_w - K_u, \end{aligned} \tag{6.57}$$

with $K_X = 0$ since X is at rest in the experiment. The reaction is called **exoergic** [in analogy to exothermic for chemical reactions] if $Q > 0$ and **endoergic** [in analogy to endothermic for chemical reactions] if $Q < 0$. In an exoergic reaction, mass energy is converted in the process as net increase in the kinetic energy. For endoergic reaction to proceed, the particle u must have sufficient kinetic energy to overcome a **threshold energy**, K_{th}. By applying the conservation of energy and momentum to a endoergic reaction where the target nucleus X is at rest gives

$$K_{\text{th}} = -Q\left(1 + \frac{m_u}{m_X}\right). \quad (K_X = 0,\ Q < 0). \tag{6.58}$$

Some examples of nuclear reactions are listed in Table 6.4.

Example 6.12. A 1-MeV proton is incident on 7Li at rest and causes the following reaction.

$$^7\text{Li} +\ ^1\text{H} \longrightarrow\ ^4\text{He} +\ ^4\text{He}.$$

(a) Compute the Q of the reaction. (b) Assuming the two product particles move along the same line as the incident proton, find the kinetic energies of the product particles. The data needed: M_{Li7} = 7.016004 u, M_{He4} = 4.002 603 u, M_{H1} = 1.007825 u.

Solution.

(a) We compute Q from the rest energies of the particles in the reaction.

$$Q = (M_{Li7} + M_{H1} - 2 \times M_{He4})\ c^2.$$

This gives

$$Q = (0.018623\,\text{u})c^2 \times 931.494\,\frac{\text{MeV}}{c^2} = 17.34\,\text{MeV}.$$

(b) Let K_1 and K_2 be the kinetic energies of the two product particles. Let us denote the kinetic energy of the incident proton by K_p. We assume that speeds of the particles are not in the relativistic domain so that

$$p_1 = \sqrt{2M_{He4}K_1}, \quad p_2 = \sqrt{2M_{He4}K_2},$$

The conservation of energy gives

$$Q + K_p = K_1 + K_2. \tag{6.59}$$

Let us label the helium particle with 1 that is in the same direction as the proton. The other one labeled 2 will be moving in the opposite direction. The conservation of momentum gives

$$\sqrt{2M_{H1}K_p} = \sqrt{2M_{He4}K_1} - \sqrt{2M_{He4}K_2}. \tag{6.60}$$

Let $x = \sqrt{K_1}$ and $y = \sqrt{K_2}$. Simplifying and putting the numerical values we get

$$x^2 + y^2 = 18.34, \quad x - y = 0.5.$$

Eliminating y from the first of these equations we get

$$x^2 + (x - 0.5)^2 = 18.35.$$

Solving for x we get

$$x = 3.27\,\sqrt{\text{MeV}}.$$

Squaring we get

$$K_1 = 10.7\ \text{ MeV}.$$

Use this K_1 to get

$$K_2 = 7.64\ \text{ MeV}.$$

Let us verify if the non-relativistic assumption was correct.

$$v_1 = \sqrt{2K_1/M_{He4}} = c\sqrt{2K_1/M_{He4}c^2} = c\sqrt{2 \times 10.7\ \text{ MeV}/3728.4\,\text{MeV}} = 0.076\,c.$$

This speed is only 7.6% of the speed of light, which can be safely treated as non-relativistic.

Nuclear reaction cross-section

The rate at which a nuclear reaction occurs is often indicated by a quantity called **reaction cross-section**. Suppose N_0 particles of u are incident on a foil of X and N of these suffer the reaction given in Eq. 6.54 while the rest are unaffected. Let there be n atoms of X per unit volume, ΔA the area of the foil, and Δx be the thickness of the foil. Suppose each nucleus has an area of cross-section σ such that if a particle of u enters within this area, it will be captured and nuclear reaction will take place. Then, the fraction of ΔA that is within the reaction area will be equal to the N/N_0, the fraction that suffered reaction.

$$\frac{N}{N_0} = \frac{\sigma \times (\text{number of X atoms in the foil})}{\Delta A} = \frac{\sigma \times (n\Delta A \Delta x)}{\Delta A} = \sigma n \Delta x.$$

The quantity σ is called reaction cross-section because it represents the effective area over which the reaction occurs. The typical values of the nuclear reaction cross-section are in the range of square of the radius of a nucleus. Since radius of a nucleus $\sim 10^{-14}$ m, the reaction cross-section $\sim 10^{-28}$ m^2. The nuclear reaction cross-section is given in units of 10^{-28} m^2, which are also called a **barn (b)**.

$$\boxed{1 \text{ barn} = 10^{-28} \text{ m}^2.} \tag{6.61}$$

Reaction cross-section not only depends on the nuclide X and the bombarding particle u but also on the kinetic energy of the bombarding particle u. For instance, the cross-section for ^{15}N(p,α)^{12}C is $\sigma = 0.5$ b at $E_p = 2$ MeV.

Nuclear reaction with neutron

Nuclear reactions involving capture of neutrons play important role in the study of fission and nuclear reactors. Since neutron is a neutral particle, it interacts very little with electrons. When you send a neutron beam into a target, most of the target appears empty to neutrons. The neutron scatters mostly elastically with nuclei. At each collision with the nuclei the neutron loses some kinetic energy to the nucleus, and eventually the neutron reaches a thermal equilibrium with the material. When the neutron has reached thermal equilibrium its kinetic energy is of the order of $k_B T$, which is about 0.025 eV at room temperature, and is called a **thermal neutron**. When thermal neutrons encounter a nucleus they get bound to the nucleus by the strong force in a process called **neutron capture**.

$${}^{A}_{Z}X + \text{n} \longrightarrow {}^{A+1}_{Z}X^* \longrightarrow {}^{A+1}_{Z}X + \gamma. \tag{6.62}$$

The nuclear cross-section for this reaction depends on the energy of the neutron and the nature of the nuclide X. The nuclear cross-sections for neutron capture by graphite and some hydrogen rich materials, such as water and paraffin, are very small for fast neutrons. These materials are used in a nuclear reactor to slow down the neutron so that the slow neutron can be captured by the fissile material.