2.3 WORK DONE BY GAS

Consider a system consisting of a fixed amount of gas in a cylinder with a movable piston of area A. Let pressure of the gas inside the system be p. The piston is held in place by an external force $\vec{F}$ that balances the force, whose magnitude is equal to pA, by the gas molecules on the piston (Fig. 2.3).

How much work will the gas do if the gas expands by an infinitesimal volume dV? To accomplish this, we can reduce the external force $\vec{F}$ by an infinitesimal amount by pulling on the piston. This would create an infinitesimal imbalance of forces on the piston which will cause the piston to accelerate outward.

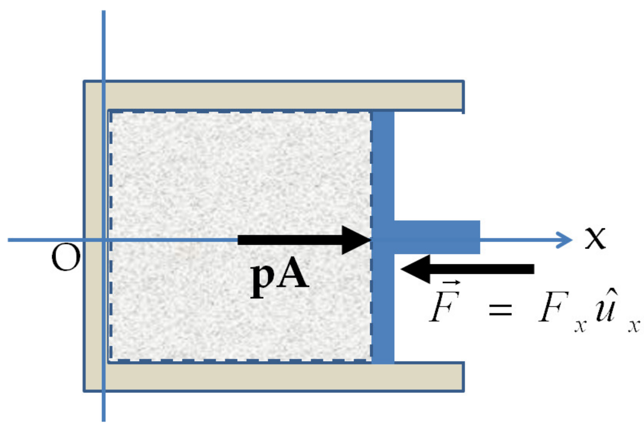

Figure 2.3: Calculating work done by a gas against external agent applying a force $\vec{F}$.

The amount of work done by the gas is equal to the negative of the work by the external force when the piston moves an infinitesimal distance dx.

$$dW = -\vec{F} \cdot d\vec{r} = -F_x dx \tag{2.13}$$

Since the magnitude of the external force $\vec{F}$ is assumed to be only infinitesimally different than the force by the gas and opposite in direction, we can replace F_x in Eq. 2.13 by $-pA$ to obtain the following for the infinitesimal work done by the gas when the piston moves by a distance dx.

$$dW = pAdx \tag{2.14}$$

Here, $dx > 0$ for an expansion and $dx < 0$ for a compression of the gas. The work by the gas is positive when it expands and negative when it contracts. Positive work by the gas transfers energy from the gas to the external agent and negative work transfers energy to the system.

Note that Adx is the change in volume of the gas. Therefore, work done by the gas against an external agent is

$$\boxed{dW = pdV. \quad \text{(against force } \vec{F}\text{)}} \tag{2.15}$$

In some disciplines, such as chemistry, the standard treatment is in terms of the work done on the system, in which case there will be a minus sign on the right side of Eq. 2.18. Choosing p and V as independent variables, we can represent the path of a quasi-static process in the pV plane. This gives p as a function of V on the path representing the particular way state of the gas changes during the process.

Work on gas: $-pdV$
Work by gas: $+pdV$

$$\boxed{p(V) = \text{Path in pV plane corresponding to the process.}} \tag{2.16}$$

Integrating over the path $p(V)$ from an initial volume to the final volume gives the total work done by the system.

$$\boxed{W = \int_{V_i}^{V_f} p(V)dV.} \tag{2.17}$$

There is another useful way of looking at the integral in Eq. 2.17. The work between initial state (p_i, V_i) and final state (p_f, V_f) is a definite integral over one variable. A definite integral over one variable is also equal to the area under the curve. Therefore, the work integral is also equal to the area under the $p(V)$ curve between the initial and final states when p is plotted along y axis and V along x axis as shown in Fig. 2.4.

While utilizing the area under the curve $p(V)$ for calculating the work integral, care must be exercised as to the sign of the work in a particular process. If the process is an compression, you need to multiply the area under the curve by minus one to properly reflect the gain in the energy of the system as a result of compression. If you perform the integral from the given $p(V)$, the sign would be automatically included in your calculation based on the limits on the integration. But in the case of calculation by the area under the curve you will need to put the sign by hand.

Further Remarks

What happens if there is a vacuum outside the gas container and the container of the gas is punctured so that the gas can freely flow into the vacuum? In this situation, the gas would not be in a mechanical equilibrium. Hence, you cannot assign any pressure to the entire gas. However, since, there would be no force opposing the movement of

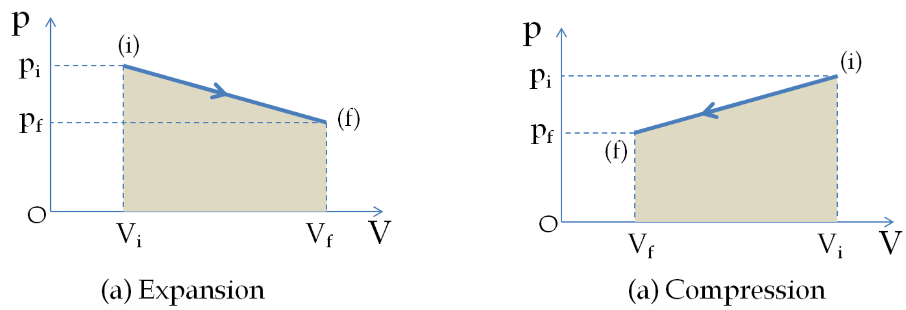

Figure 2.4: Work for a quasi-static process is equal to the area under the curve above $p = 0$ line in the pV plane. The work by the gas for an expansion (a) is equal to the area under the curve and for a compression (b) is equal to the negative of the area.

the gas molecules and the expansion of the gas will not be used to accelerate any material body, the gas molecules would not do any work on any outside body. The molecules will, of course do work on each other if interactions between molecules are significant, which we will ignore for the moment. This gives the following for a gas that is 'freely" expanding in vacuum.

$$dW = 0. \quad \text{(expanding in vacuum.)} \tag{2.18}$$

The puncturing of the container to let the gas out into the vacuum is different from letting the piston of the gas cylinder move as a result of the pressure differential between inside and outside while keeping the gas inside the enclosed container. Suppose, you create vacuum in the space outside the cylinder of gas while keeping the piston fixed in place. Now, when you release the piston, there would be an imbalance of forces on the piston. Let the mass of the piston be M. Then, over the course of expansion of the gas by the amount dV, the piston would have moved dx under the force pA, where A is the are of cross-section of the cylinder. This would mean that gas would do the work on the piston of the amount pdV causing the piston to accelerate. The energy lost by the gas would go into changing the kinetic energy of the piston.