1.1 Galilean Relativity

1.1.1 Motion is Relative

Suppose you are inside a windowless room in a ship that is coasting on a calm sea at a constant speed in a fixed direction. Can you tell whether the ship is moving without looking outside the closed room? According to the experiments of Galileo, the answer to this question is "No" as long as the ship is not accelerating.

To find whether the ship on a calm sea is moving at all you would have to observe some object outside the ship, such as a tree at the shore. If the ship is moving relative to the shore, the tree will appear to be moving with respect to you. Then, assuming the tree to be fixed to the shore and not moving, you can conclude that the ship is moving relative to the tree. This type of observations convinced Galileo that the motion itself is relative.

Motion is relative!

The object relative to which we observe motion serves as a reference for the motion. The object serving as the reference for the motion can be placed at the origin of a Cartesian coordinate system which is used to assign positions to other objects. Thus, if we take the tree at the shore at the origin with x, y on the surface of the Earth and the positive z-axis pointed up, the ship will be seen to move approximately in the xy-plane with respect to the tree. We say that the ship is moving in the Tree-frame, i.e. the reference frame in which the tree is at rest.

Any object can be used this way to define a reference frame. Even a point in the stationary space can be used to define a reference point point for this purpose. As a concrete example, suppose there are two ships, both moving towards East with speed 5 m/s with respect to a woman standing at the shore. Then, in the frames of either of the ship the other ship will not be moving at all. If ship #1 is moving at 5 m/s and ship #2 at 2 m/s both towards East with respect to the person at the shore. Then, you will find that in the frame of ship #1, ship #2 will me moving towards West with speed 3 m/s and the woman herself will be moving [sometimes people say, "appear to be moving" instead of actually moving] towards West with speed 5 m/s. These observations show that the velocity of an object is also relative, being different relative to different reference frames.

Velocity is relative!

In general, consider three bodies A, B, and C as shown in Fig. 1.1. Let $\vec{v}_{BA}$ and $\vec{v}_{CA}$ be the velocities of bodies B and C with respect to body A, and $\vec{v}_{CB}$ be the velocity of body C with respect to body B. Then, you will find that

$$\boxed{\vec{v}_{CA} = \vec{v}_{CB} + \vec{v}_{BA}} \quad \Longrightarrow \quad \boxed{\vec{v}_{CB} = \vec{v}_{CA} - \vec{v}_{BA}}. \tag{1.1}$$

You have encountered this relation when you studied relative velocity in an

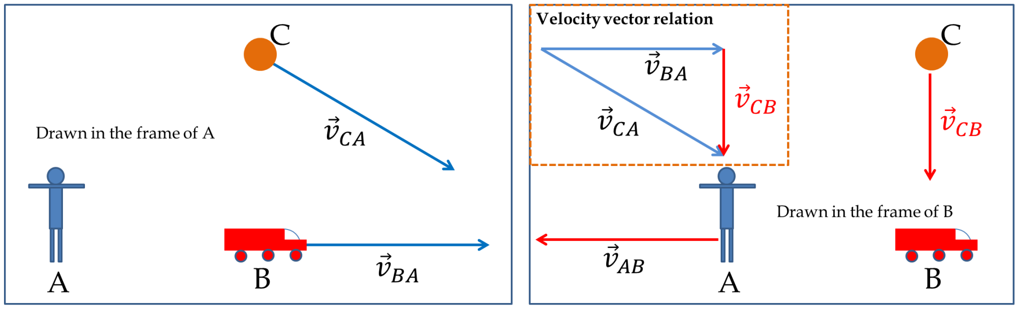

Figure 1.1: Velocities of B and C in the frame of A and the velocities of A and C in the frame of B are shown here. The inset shows the relations among velocities with respect to the two frames.

earlier chapter. We often state this relation in terms of the language of frames. Let us denote the velocities with respect to frame B with a prime on the symbol and with respect to frame A without a prime, and drop the subscripts altogether. Also, let us denote the velocity of the frame B relative to the frame A by the symbol $\vec{V}$.

$$\boxed{\vec{v}\,' = \vec{v} - \vec{V}.} \tag{1.2}$$

This relation is called the Galilean relativity of velocity.

1.1.2 Inertial Reference Frames

To write his laws of motion Newton assumed that there was a special reference frame of absolute rest. Other reference frames would be either moving uniformly with constant velocity or accelerating with respect to this special frame. All frames that have a constant velocity with respect to this special frame are called inertial frames.

Galileo found that physics experiments such as finding the period of a pendulum determined at the shore and inside a ship at constant velocity gave the same result. Based on several experiments Galileo concluded that there was no experimental way to know whether someone was in a uniform motion (in the ship) or was at rest (at the shore). This conclusion has led to an important principle of physics, called Principle of Relativity:

Principle of Relativity: The laws of physics are same for all inertial observers.

We will see below that although the velocity of an object depends on a frame, the second law of motion, $\vec{F} = m\vec{a}$, takes the same mathematical form in all inertial frames. Thus, if we denote the quantities in one inertial frame with primes and in the other inertial frame without primes we will have the law taking the same mathematical form in the two frames.

$$\vec{F} = m\vec{a}; \qquad \vec{F}' = m'\vec{a}'. \tag{1.3}$$

1.1.3 The Galilean Transformations

The Galilean transformations relate the same event in two different frames that are in uniform relative motion with respect to each other. An **event** is some incident that occurs at a particular place and time. The location of a particle at a particular time is an example of an event that says that the particle was at "that" location at "that" time. In the following we will deduce the analytic expressions for Galilean transformations.

Consider two inertial frames $Oxyz$ and $O'x'y'z'$. For brevity let us call these frames S and S' respectively. Suppose at time $t = 0$ the two frames completely coincide and thereafter frame S' is moving towards the positive x-axis with speed V relative to S as shown in Fig. 1.2. Newton assumed that the time was absolute

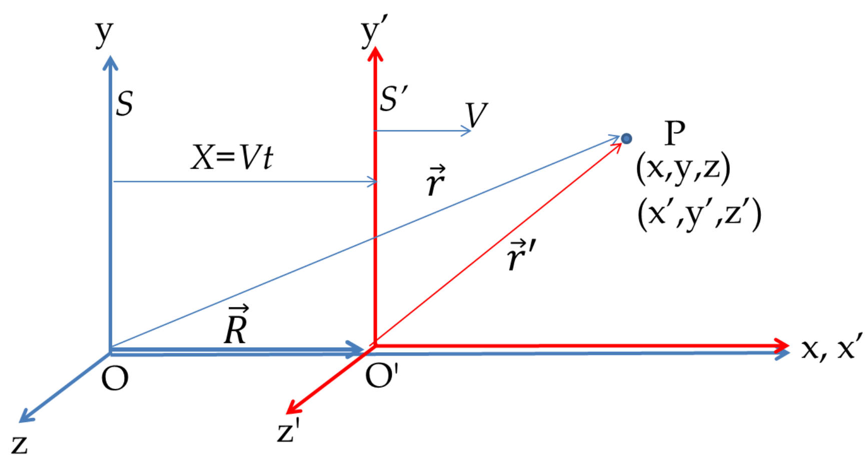

Figure 1.2: Coordinates of a particle P with respect to two frames S and S'.

such that the time elapsed is same in all inertial frames. Therefore, a later time t in frame S will equal the time t' in frame S'.

$$t' = t. \quad \text{(Galilean relativity of time)} \tag{1.4}$$

Fig 1.2 shows the situation of the two frames at time t and the position of a particle P with respect to the origins of the two frames. Let the position of a particle P at time t be $\vec{r} = (x, y, z)$ in frame S and $\vec{r}' = (x', y', z')$ in frame S'. Let the position of the origin of S' frame will be $\vec{R} = (X, Y, Z)$. In the figure we have $\vec{R} = (X, 0, 0)$. From the triangle of position vectors $\vec{r}$, $\vec{r}'$, and $\vec{R}$ we find that these are related as follows.

$$\vec{r} = \vec{r}' + \vec{R}. \tag{1.5}$$

The position vector $\vec{r}\,'$ is said to be with respect to the "moving frame" while the position vector $\vec{r}$ is said to be with respect to the "stationary frame". We often write Eq. 1.6 with the quantities relative to the moving frame on the left of the equation and the rest on the right.

$$\vec{r}\,' = \vec{r} - \vec{R}. \tag{1.6}$$

From the velocity of the frame S' with respect to S, we find that the position of the origin of S' frame will be

$$X = Vt, \quad Y = 0, \quad Z = 0. \tag{1.7}$$

Therefore, we have the following relations between the coordinates of P is frames S and S' when frame S' is moving towards the positive x-axis with speed V.

$$\text{Relativity of position:} \qquad \begin{cases} x' = x - V\,t \\ y' = y \\ z' = z. \end{cases} \tag{1.8}$$

These equations are called **Galilean transformations** for two frames that have a relative motion along the x-axis. The more general case, in which the frame S' has the velocity $\vec{V}$ with respect to frame S will be given by

$$\boxed{\text{Relativity of position:} \qquad \vec{r}\,' = \vec{r} - \vec{V}\,t.} \tag{1.9}$$

By taking derivatives with respect to time t we obtain the relativity of velocity and acceleration in the two frames.

$$\vec{v}\,' = \vec{v} - \vec{V} \tag{1.10}$$
$$\vec{a}\,' = \vec{a}. \tag{1.11}$$

The acceleration with respect to two inertial frames have identical values. Thus, if mass of the particle is not dependent on the frame,

$$\text{Galilean relativity of mass:} \qquad m' = m, \tag{1.12}$$

then, Newton's law of motion in the two frames will take identical mathematical form.

$$\boxed{\text{Frame } S: \quad \vec{F} = m\,\vec{a}; \qquad \text{Frame } S': \quad \vec{F}\,' = m'\,\vec{a}\,'.} \tag{1.13}$$

This is **the Principle of Relativity** as applied to Newton's second law of motion. Einstein demanded that the Principle of Relativity must also apply to the laws of electricity and magnetism, and by extension to all laws of physics. It turns out that when Galilean transformations are applied to the laws of electricity and magnetism, they take different forms in different inertial frames. Therefore, *Galilean transformations are not fundamental and must be replaced by other set of transformations, called Lorentz Transformations.* The laws of electricity and magnetism take the same form in different inertial frames if they are related by Lorentz Transformations, but Newton's laws of mechanics have to be modified as we will see below.

We will find below that the Galilean transformations given in Eq. 1.8 depend crucially on the assumption of a universal absolute time t. If this assumption is somehow wrong, then all these relations will turn out to be wrong.