2.8 Problems

Problem 2.1. An oven is heated to a high temperature and the electromagnetic radiation coming out of the oven through a tiny hole in the oven is analyzed for radiance $R_T(\lambda)$, which is the power content per unit wavelength range per unit cross-section area of the hole. The data obtained at five wavelengths are: $(0.3\ \mu\text{m}, 1.1 \times 10^{13}\ \text{W/m}^3)$, $(0.4\ \mu\text{m}, 2.7 \times 10^{13}\ \text{W/m}^3)$, $(0.5\ \mu\text{m}, 3.8 \times 10^{13}\ \text{W/m}^3)$, $(0.6\ \mu\text{m}, 4.0 \times 10^{13}\ \text{W/m}^3)$, $(0.7\ \mu\text{m}, 3.7 \times 10^{13}\ \text{W/m}^3)$, $(0.8\ \mu\text{m}, 3.2 \times 10^{13}\ \text{W/m}^3)$.

(a) Plot R_T versus λ. (b) From the data find the temperature of the oven. (c) Find the total power radiated per unit area of cross-section of the hole.

Problem 2.2. A photon of energy hf collides head-on with a nearly free electron at rest. Let E_0 be the rest energy of an electron. Show that the kinetic energy of the recoiled electron is given by

$$K = \frac{2h^2f^2}{2hf + E_0}.$$

Problem 2.3. (a) Prove that in the Compton scattering, a photon cannot transfer all of its energy to an electron. (b) Is there a maximum percentage of energy that a photon can transfer to an electron at rest? If so, what is it? If not, why not?

Problem 2.4. (a) Treating $R_T(\lambda)$ as a function of λ, prove that the maximum of the radiance occurs at a wavelength λ_{max} whose product with temperature is a constant.

$$\lambda_{\text{max}} = \text{constant}.$$

(b) Find the value of the constant.

Problem 2.5. Integrate $R_T(\lambda)$ over all values of $0 \le \lambda \le \infty$ to deduce the Stefan-Boltzmann law.

$$I = \int_0^\infty R_T(\lambda)d\lambda = \sigma T^4.$$

Hint: Let $x = hc/\lambda k_B T$.