6.8 EXERCISES

Single-slit diffraction

Ex 6.8.1. Monochromatic light of wavelength 530 nm passes through a horizontal single slit of width 1.5 μm in an opaque plate. A screen of dimensions 2 m $\times$ 2 m is 1.2 m away from the slit. (a) Which way is the diffraction pattern spread out on the screen? (b) What are the directions of the minima? (c) What are the directions of the maxima? (d) How wide is the central bright fringe on the screen? (e) How wide is the next bright fringe on the screen? Ans: (a) Vertically, (b) $\pm 21°$, $\pm 45°$, (c) 0, $\pm 30°$, $\pm 60°$, (d) 92 cm, (e) 74 cm.

Ex 6.8.2. Light of wavelength 630 nm passes through a vertical single slit of width 1.5 μm in an opaque plate. A screen of dimensions 2 m $\times$ 2 m is 1.2 m away from the slit. (a) Which way is the diffraction pattern spread out on the screen? (b) Where on the screen are the minima from the slit? (c) Where on the screen are the maxima? (d) How wide is the central bright fringe on the screen? (e) How wide is the next bright fringe on the screen?

Ex 6.8.3. A monochromatic light of wavelength 632.8 nm incident on a single slit has central bright spot of width 5 cm at a screen 1.6 m away. Find the width of the slit. Ans: 40.5 μm.

Ex 6.8.4. A monochromatic light of unknown wavelength is incident a slit of width 20 μm. A diffraction pattern is seen at a screen 2.5 m away where the central maximum is spread over a distance of 10 cm. Find the wavelength. Ans: 400 nm.

Ex 6.8.5. A source of light having two wavelength 550 nm and 600 nm of equal intensity is incident on a slit of width 1.8 μm. Find the separation of the $m = 1$ bright spots of the two wavelengths on a screen 30 cm away. Ans: 2.1 cm.

Ex 6.8.6. Solve the following equations graphically. (a) $x^2 = \sin(x)$, (b) $x \tan(x) = 1$. Ans: (a) $x = 0, 0.875$.

Ex 6.8.7. A monochromatic light of wavelength 600 nm is incident on a slit of width 1.6 μm. Find the angular width of the central peak. Ans: 0.77 rad.

Ex 6.8.8. A single slit of width 2100 nm is illuminated normally by a wave of wavelength 632.8 nm. Find the phase difference between waves from the top and 1/3-rd from the bottom of the slit to a point P on a screen at a horizontal distance of 2 meters and vertical distance of 10 cm from the center. Ans: $\Delta\phi = 0.695$ rad.

Ex 6.8.9. A single slit of width 3.0 μm is illuminated by a sodium yellow light of wavelength 589 nm. Find the intensity at a 15° angle to the axis in terms of the intensity of the central maximum which can be taken to be 1 unit. Ans: $I/I_0 = 0.04$.

Ex 6.8.10. A single slit of width 0.1 mm is illuminated by a mercury light of wavelength 576 nm. Find the intensity at a 10° angle to the axis in terms of the intensity of the central maximum which can be taken to be 1 unit.

Double-slit diffraction

Ex 6.8.11. Two slits of width 2 μm each in an opaque material are separated by a center-to-center distance of 6 μm. A monochromatic light of wavelength 450 nm is incident on the double-slit. One finds a combined interference and diffraction pattern on the screen.

(a) How many peaks of the interference will be observed in the central maximum of the diffraction pattern?

(b) How many peaks of the interference will be observed if the slit width is doubled while keeping the distance between the slits same?

(c) How many peaks of interference will be observed if the slits are separated by twice the distance, i.e., 12 μm, while keeping the widths of the slits same?

(d) What will happen in (a) if instead of 450-nm light another light of wavelength 680 nm was used?

(e) What is the value of the ratio of the intensity of the central peak to the intensity of the next bright peak in (a)?

(f) Does this ratio depend on the wavelength of the light?

(g) Does this ratio depend on the width or separation of the slits?

Ans: (a) 7, (b) 3, (c) 13, (d) Similar to part (a), (e) 0.684, (f) Yes, (g) Yes.

Ex 6.8.12. A monochromatic light of wavelength 589 nm incident on a double-slit with slit width 2.5 μm and unknown separation results in a diffraction pattern containing nine interference peaks inside the central maximum. Find the separation of the slits. Ans: 10 .0 μm to 12.5 μm.

Ex 6.8.13. When a monochromatic light of wavelength 430 nm incident on a double slit of slit separation 5 μm, there are 11 interference fringes in its central maximum. How many interference fringes will be in the central maximum of a light of wavelength 632.8 nm for the same double-slit? Ans: 9.

Ex 6.8.14. Determine the intensities of two interference peaks other than the central peak in the central maximum of the diffraction, if possible, when a light of wavelength 628 nm is incident on a double slit of width 500 nm and separation 1500 nm. Use the intensity of the central spot to be 1 mW/cm^2.

Ex 6.8.15. Determine the intensities of three interference peaks other than the central peak in the central maximum of the diffraction, if possible, when a light of wavelength 500 nm is incident normally on a double slit of width 1000 nm and separation 1500 nm. Use the intensity of the central spot to be 1 mW/cm^2.

Diffraction by a circular aperture

Ex 6.8.16. A laser light of wavelength 628 nm is reflected from a point object on the far corner of a room. The reflected light is then incident on a converging lens of diameter 5 cm and focal length 10 cm so that a real image is formed in the focal plane. Find the size of the bright central spot corresponding to the image. Ignore the effect of spherical aberration. Ans: 1.532 μm.

Ex 6.8.17. A converging lens of diameter 8 cm and focal length 20 cm is used to focus the image of a star. Find the radius of the central bright spot by using 550 nm for the wavelength of light from the star. Ignore the effect of spherical aberration. Ans: 1.68 μm.

Ex 6.8.18. What is the minimum distance between two points on the Moon that the 40-in refractor telescope at Yerkes observatory in Wisconsin can resolve if the resolution was limited due to the diffraction only? Use 540 nm as the wavelength of light and a distance of 4×10^8 m from the Earth to the Moon. Ans: 260 m.

Ex 6.8.19. Two lamps producing light of frequency 589 nm are fixed 1-meter apart on a wooden plank. How far the plank can be moved from the eye so that the eye can still resolve them if the resolution is affected solely by the diffraction of light in the eye? Assume light enters the eye through a pupil of diameter 4.5 mm. Ans: 6.3 km.

Ex 6.8.20. On a bright clear day you are at the top of a mountain and looking at a city 12 km away. There are two tall towers 20 meters apart in the city. Can your eye resolve the two towers if the diameter of the pupil is 4 mm? If not, what should be the minimum magnification power of the telescope needed to resolve the two towers? In your calculations use 550 nm for the wavelength of the light.

Diffraction grating

Ex 6.8.21. A transmission diffraction grating contains 500 lines per mm. (a) What will be the angle of separation between the $m = 1$ peaks of a yellow light of wavelength 590 nm and a red light of wavelength 630 nm? (b) What is the distance of separation of the $m = 1$ peaks on a screen 1.2 m away? Ans: (a) 1.2°, (b) 2.53 cm.

Ex 6.8.22. How many lines per mm are there in the diffraction grating if the second order principal maximum for a light of wavelength 536 nm occurs at an angle of 24 degrees with respect to the line from the grating to the center of the diffraction pattern? Ans: 379.4 lines per mm.

Ex 6.8.23. A diffraction grating produces a first order peak at 18 degrees for a yellow light of wavelength 580 nm. Find the spacing between the slits. Ans: 1.9 μm.

Ex 6.8.24. The spectrum of mercury has a pair of yellow orange light at 576.959 nm and 579.065 nm. A diffraction grating with 200 lines per mm is available in the lab. Determine if this grating will do the job of resolving the two mercury yellow orange lines in $m = 1$ order if the beam diameter is 3 mm. If not, what is the minimum number of lines per mm is needed for this task if it needs to be resolved in the first-order? Ans: 96 lines/mm.

Ex 6.8.25. A more powerful diffraction grating with 1000 lines/mm is tried for the separation of yellow orange light pair of wavelength 576.959 nm and 579.065 nm in the mercury spectrum. Find (a) the maximum number of bright lines that can be observed, (b) the angular separation of the two wavelengths for $m = 1$, and (c) the half-width of each mercury line.

Ex 6.8.26. A light beam of an unknown wavelength is incident over 5 mm of a diffraction grating that has 400 lines per mm, and the first-order peak is observed at an angle of 20°. Find (a) the wavelength, and (b) the half-width of the peak. Ans: (a) 0.855 μm, (b) $\theta_{\text{hw}} = 0.01°$.

Ex 6.8.27. A light beam of an unknown wavelength is incident over 3 cm of a diffraction grating that has 400 lines per mm, and the first-order peak is observed at an angle of 20°. Find (a) the wavelength, and (b) the half-width of the peak.

Ex 6.8.28. A laser light of wave length 632.8 nm is incident perpendicularly on a 2.5-cm wide diffraction grating over the entire face. It is seen that the second-order peak is at 30° angle. (a) What is the resolving power of the grating for the second-order? (b) What is the dispersion for the second-order? Ans: (a) 20,000, (b) 910 rad/mm^{-1}.

Ex 6.8.29. A laser light of wave length 539 nm is incident perpendicularly on a 2-cm wide diffraction grating over the entire face. It is seen that the first-order peak is at an 18-degrees angle. (a) What is the half-width of the first-order peak? (b) What is the resolving power for the first-order?