7.9 EXERCISES

Impulse and Momentum

Ex 7.9.1. A ball of mass 0.5 kg is thrown in the air. What is the magnitude and direction of the impulse of the weight of the ball during the 5 sec interval the ball is in flight? Ans: 24.6 N.s pointed down.

Ex 7.9.2. A man pushes a shopping cart with a constant force of 5 N pointed towards North for 3 sec. What is the magnitude and direction of the impulse of the push? Ans: 15 N.s North.

Ex 7.9.3. A large crate is pushed by two men. One of them applies a constant force of 100 N due North and the other applies a constant force of magnitude 120 N and direction due East. What is the net impulse over a period of 3 sec? Ans: 469 N.s 39.8° North of East.

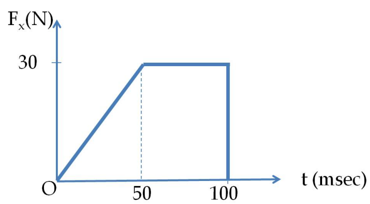

Figure 7.28: Exercise 7.9.4.

Ex 7.9.4. The x-component of a force on a 46-gram golf ball by a 7-iron versus time is plotted in Fig. 7.28. Find the x-component of the impulse during the intervals (i) [0, 50 msec], and (ii) [50 msec, 100 msec]. (b) Find the change in the x-component of the momentum during the intervals (i) [0, 50 msec], and (ii) [50 msec, 100 msec]. Ans: (a) (i) 0.75 N.s, (ii) 1.5 N.s. (b) (i) 0.75 kg.m/s, (ii) 1.5 kg.m/s.

Ex 7.9.5. The y-component of a force on 2-kg box varies with time as shown in the Fig. 7.29. Find the y-component of the impulse during the intervals (i) [0, 2 sec], (ii) [2 sec, 4 sec], (iii) [4 sec, 8 sec], (iv) [8 sec, 10 sec], and (v) [0, 10 sec]. (b) Find the change in the y-component of the velocity during the intervals in part (a).

Ans: (a)(i)3 N.s, (ii)6 N.s, (iii) 2 N.s, (iv) -2 N.s, (v) 9 N.s.

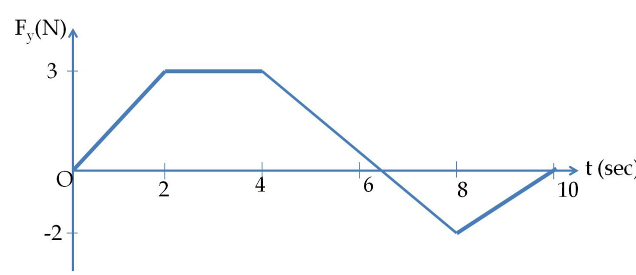

Figure 7.29: Exercise 7.9.5.

Ex 7.9.6. A batter strikes a ball of mass 300 grams with a bat of mass 3 kg. Before the hit the ball is moving horizontally towards the bat at a speed of 50 m/s. After the hit, the velocity of the ball changes direction by 180° and leaves the bat at 60 m/s speed. The impact of the bat on the ball lasts only 2 msec. (a) Find the magnitude and direction of the change in momentum of the ball. (b) What is the magnitude and direction of the impulse of the bat on the ball? (c) What is the magnitude and direction of the impulse of the ball on the bat? (d) What is the magnitude and direction of the average force of the bat on the ball? (e) What is the magnitude and direction of the average force of the ball on the bat?

Ans: (a) 33 kg.m/s pointed from the bat towards the ball, (d) 16,500 N, towards ball.

Ex 7.9.7. A batter strikes a ball of mass 300 grams with a bat of mass 3 kg. Before the hit the ball is moving horizontally towards the bat at a speed of 50 m/s. After the hit, the velocity of the ball changes direction by 120° but still pointed in the horizontal plane containing the velocity before the hit and velocity after the hit, and leaves the bat at 60 m/s speed. The impact of the bat on the ball lasts only 2 msec. (a) Find the magnitude and direction of the change in momentum of the ball. (b) What is the magnitude and direction of the impulse of the bat on the ball? (c) What is the magnitude and direction of the impulse of the ball on the bat? (d) What is the magnitude and direction of the average force of the bat on the ball? (e) What is the magnitude and direction of the average force of the ball on the bat?

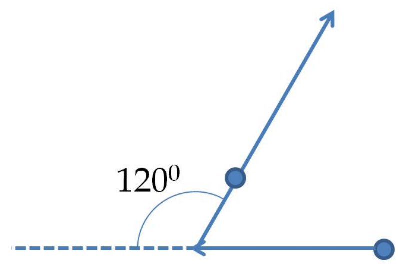

Figure 7.30: Exercise 7.9.7.

Ans: (a)28.6 kg.m/ at 33° with respect to the positive x-axis. (b) Same as (a). (c) Same as (b) but in the opposite direction. (d) 14,300 N in the same direction as (a). (e) Same as (d) but in opposite direction.

Ex 7.9.8. A car of mass $2,800$ kg moving at 40 m/s strikes a stationary truck of mass $8,000$ kg. After the collision, the car comes to rest within 2.5 seconds of the hit. (a) What is the magnitude and direction of the average force on the car? (b) Compare this force to the force on the truck by the car.

Ans: (a) 44,800 N in the opposite direction of the initial momentum of the car. (b) 44,800 N in the direction of the initial momentum of the car.

Ex 7.9.9. Two cars, of mass 3000 kg each, moving in the opposite directions on a straight road collide. Before the collision, each car had a speed of 65 miles per hour with respect to the ground, and after the collision, they move in the backward direction with a speed of 20 miles per hour. If the cars were in contact for 1.5 sec, what was the average force on each car? Hint: Find the change in momentum of one of the cars.

Ans: Magnitude of change in momentum 114,000 kg.m/s. Magnitude of average force 76,000 N.

Ex 7.9.10. A car of mass $2,000$ kg and a truck of mass $10,000$ kg start from rest at an intersection of two perpendicular roads. The car has a constant velocity of 10 m/s due East while the truck has a constant velocity of 25 m/s due North. Find the total momentum of the two vehicles at $t = 0$ and $t = 1$ sec.

Ans: Zero at $t = 0$ and magnitude 251,000 kg.m/s direction 85° N of E at $t = 10$ sec.

Center of Mass

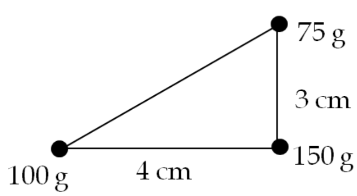

Figure 7.31: Exercise 7.9.11.

Ex 7.9.11. Three point masses are placed at the corners of a triangle as shown in Fig. 7.31. Find the center of mass of the three-mass system.

Ans: Xcm = -1.23 cm and Ycm = 0.69 cm with 150 gram mass at the origin.

Ex 7.9.12. Two particles of masses m_1 and m_2 separated by a horizontal distance D are let go from the same height h at the same time. Find the vertical position of the center of mass at a time before the two particles strike the ground. Assume no air resistance. If you like numbers try: $m_1 = (0.2 \text{ kg})$, $m_2 = (0.25 \text{ kg})$, $D = (10 \text{ cm})$, and $h = (30 \text{ m})$.

Ans: $X_{cm} = 5.6\ cm$; $Y_{cm} = 30m - 4.9(m/s^2)t^2$.

Ex 7.9.13. Two particles of masses m_1 and m_2 separated by a horizontal distance D are let go from the same height h at different times. The particle 1 starts at $t = 0$, and the particle 2 is let go at $t = T$. Find the vertical position of the center of mass at a time before the first particle strikes the ground. Assume no air resistance.

Ans: if $m_1 = (0.2 \text{ kg})$, $m_2 = (0.25 \text{ kg})$, $D = (10 \text{ cm})$, and $h = (30 \text{ m})$, $T = 0.2$ sec, then $Y_{cm}(t) - Y_{cm}(0.2sec) = (0.87)(t - 0.2) - (9.8)(t - 0.2)^2$.

Ex 7.9.14. Two particles of masses m_1 and m_2 move uniformly in different circles of radii R_1 and R_2 about origin in the xyplane. The x- and y-coordinates of the center of mass and that of particle #1 are given as follows where length is in meters and t in seconds.

$$X_{\text{CM}}(t) = 3\cos(2t); Y_{\text{CM}}(t) = 3\sin(2t)$$
$$x_1(t) = 4\cos(2t); y_1(t) = 4\sin(2t)$$

(a) Find the radius of the circle in which particle #1 moves. (b) Find the x and y-coordinates of particle #2 and the radius of the circle this particle moves.

Ans: (a) 4 m. (b) $R_2 = \dfrac{1}{m_2}\sqrt{9M^2 + 16m_1^2 + 24m_1 M}$ with $M = m_1 + m_2$.

Ex 7.9.15. Two particles of masses m_1 and m_2 move uniformly in different circles of radii R_1 and R_2 about the origin in xy plane. The coordinates of the two particles in meters are given as follows ($z = 0$ for both). Here t is in seconds.

$$x_1(t) = 4\cos(2t); y_1(t) = 4\sin(2t)$$
$$x_2(t) = 2\cos\left(3t - \frac{\pi}{2}\right); y_2(t) = 2\sin\left(3t - \frac{\pi}{2}\right)$$

(a) Find the radii of the circles of motion of particles #1 and #2. (b) Find the x and y-coordinates of the center of mass. (c) Decide if the center of mass moves in a circle by plotting the trajectory of CM.

Ans: (a) $R_1 = 4$ m and $R_2 = 2$ m. (b) $X_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$, $Y_{cm} = \frac{m_1y_1+m_2y_2}{m_1+m_2}$. (c)Yes, moves in a circle of radius $R = \frac{1}{M}\sqrt{16m_1^2 + 4m_2^2}$.

Ex 7.9.16. Find the center of mass of a one-meter long rod, made of 50-cm long iron (density 8 g/cc) and 50-cm long aluminum (density 2.7 g/cc) rods.

Ans: 12.6 *cm* from the center of the iron rod towards the aluminum rod.

Ex 7.9.17. Find the center of mass of a rod of length L whose mass density changes from one end to the other quadratically. That is, if the rod is laid out along the x-axis with one end at the origin and the other end at $x = L$, the density is given by $\rho(x) = \rho_0 + (\rho_1 - \rho_0)\,(x/L)^2$, where ρ_0 and ρ_1 are constant values.

Ans: $\frac{3}{4}L\ \left(\frac{\rho_1+\rho_0}{\rho_1+2\rho_0}\right)$.

Ex 7.9.18. Find the center of mass of a rectangular block of length a and width b that has a non-uniform density such that when the rectangle is placed in the xy-plane with one corner at the origin and the block placed in the first quadrant with the two edges along the x and y-axes, the density is given by $\rho(x, y) = \rho_0 x$, where ρ_0 is a constant.

Ans: $(2/3a, b/2)$.

Ex 7.9.19. Find the center of mass of a rectangular material of length a and width b made up of a material of non-uniform density. The density is such that when the rectangle is placed in the xy-plane as in Exercise 7.9.18, the density is given by $\rho(x, y) = \rho_0 xy$.

Ans: $(2a/3, 2b/3)$.

Ex 7.9.20. Find the center of mass of a sphere of mass M and radius R and a cylinder of mass m, radius r and height h arranged as shown

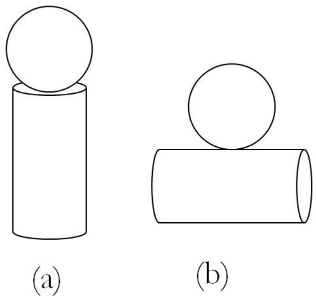

Figure 7.32: Exercise 7.9.20.

in Fig 7.32(a) and (b). Express your answer in a coordinate system that has the origin at the center of the cylinder.

Ans: (a) $Y_{cm} = \frac{M\left(\frac{1}{2}h+R\right)}{m+M}$. (b) $Y_{cm} = \frac{M(r+R)}{m+M}$.

Ex 7.9.21. Find the center of mass of the structure given in Fig. 7.33. Assume uniform thickness of 20 cm, and density to uniform having magnitude 1 g/cm^3.

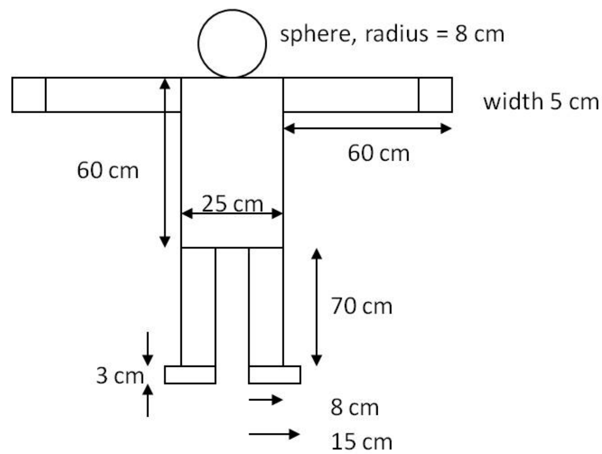

Figure 7.33: Exercise 7.9.21.

Conservation of Momentum - Single Body

Ex 7.9.22. A car of mass 2,000 kg is moving with a constant velocity of 10 m/s due East. What is the momentum of the car at $t = 0$ and $t = 1$ sec? Ans: 20,000 kg.m/s due East.

Ex 7.9.23. A skater of mass 40 kg is carrying a box of mass 5 kg. The skater has a speed of 5 m/s with respect to the floor and is gliding without any friction on a smooth surface. (a) Find the momentum of the box with respect to the floor. (b) Find the momentum of the box with respect to the floor after she puts the box down on the frictionless skating surface.

Ans: (a) Magnitude: 25 kg.m/s. (b) Same as (a).

Ex 7.9.24. A hockey puck mass 150 gram is sliding horizontally on a frictionless table with a speed of 10 m/s. Suddenly a constant force of magnitude 5 N and direction due North is applied vertical to its initial direction, which was due East, for 1.5 seconds. (a) Which component(s) of momentum is(are) conserved during the 1.5 second interval and which is(are) not conserved? (b) Find the components of the momentum along its original direction and in the direction vertical to it at the end of 1.5 second duration.

Ans: (a) Only the component of momentum in the direction of North (or in the direction of South) is conserved. (b) Let positive x-axis be in the direction of the original momentum. Then $p_x = 1.5$ kg.m/s and p_y =7.5 kg.m/s.

Ex 7.9.25. A ball of mass 250 grams is thrown with an initial velocity of 25 m/s at an angle of 30° with the horizontal direction. Ignore air resistance. (a) After the ball leaves the hand of the thrower, which component(s) of the momentum will be conserved and which will not be conserved? (b) What is the momentum of the ball after 0.2 seconds? Do this problem by finding the components of the momentum first, and then constructing the magnitude and direction of the momentum vector from the components.

Ans: (a) horizontal component conserved. (b) 6 kg.m/s at 26°.

Conservation of Momentum - Collision

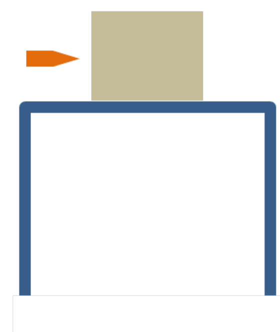

Figure 7.34: Exercise 7.9.26.

Ex 7.9.26. A bullet of mass 200 grams traveling horizontally towards East with speed 400 m/s strikes a block of mass 1.5 kg at rest on a frictionless table. After striking the block the bullet is imbedded in the block and the block and the bullet move together as one unit. (a) What is the magnitude and direction of the velocity of the block/bullet combination immediately after the impact? (b) What is the magnitude and direction of the impulse by the block on the bullet. (c) What is the magnitude and direction of the impulse from the bullet on the block? (d) If it took 3 msec for the bullet to change the speed from 400 m/s to the final speed after impact, what is the average force between the block and the bullet during this time?

Ans: (a) 47 m/s in the bullet to block direction, (b) 70.6 $N.s$, towards bullet, (c) 70.6 $N.s$, towards block, (d) magnitude: $23,500$ N.

Ex 7.9.27. A billiard ball, labeled 1, moving horizontally strikes another ball, labeled 2, at rest. Before the impact, ball 1 was moving at a speed of 3 m/s and after the impact it is moving at 1 m/s at 150° from the original direction. If the two balls have equal masses of 300 grams, what is the velocity of the ball 2 after the impact.

Ans: 3.9 m/s, 7° below the horizontal direction.

Ex 7.9.28. A projectile of mass 2 kg is fired in the air at an angle of 40° to the horizon at a speed of 50 m/s. At the highest point in its flight the projectile breaks into three parts of mass 1 kg, 0.7 kg and 0.3 kg. The 1-kg part falls straight down after breakup with an initial speed of 10 m/s, the 0.7-kg part moves in the original forward direction, and the 0.3-kg part goes straight up. (a) Find the speeds of 0.3-kg and 0.7-kg pieces immediately after the break-up. (b) How high from the break-up point does the 0.3-kg piece go before coming to rest? (c) Where does the 0.7-kg piece land relative to where it was fired from?

Ans: (a) 33.3 m/s and 109.4 m/s, (b) 56.5 m, (c) 482 m.

Ex 7.9.29. Two projectiles of mass m_1 and m_2 are fired in the opposite directions from two launch sites separated by a distance D. They both reach the same spot in their highest point and strike there. As a result of the impact they stick together and move as a single body afterwards. Find the place they will land.

Ans: $|m_1 - m_2|D/[2(m_1 + m_2)]$ from the mid point.

Time-Dependent Mass

Ex 7.9.30. Grains from a grain hopper falls at a rate of 10 kg/sec vertically onto a freight car that is moving horizontally at a constant speed 2 m/s on a straight track. What force is needed to keep the freight car moving at a constant velocity.

Ans: 20 N, horizontal.

Ex 7.9.31. A crate of mass 50 kg containing bananas is sliding in a straight line on a frictionless surface at a constant velocity of 10 m/s. A monkey of mass 45 kg hanging from the ceiling times it correctly and jumps vertically down landing in the crate. At the time monkey lands in the crate his velocity was 12 m/s vertically downward. (a) What is the velocity of the crate with monkey afterwards? (b) Compare the momenta of the care plus monkey afterwards to that of the crate plus the monkey separately. Is the total momentum of the crate plus monkey system conserved? Why or why not?

Ans: 5.3 m/s horizontally in the original direction.

Ex 7.9.32. In the previous problem, the monkey starts to consume banana and throws away the peels each having mass of 200-grams at a constant rate of one banana per 40 seconds and a speed of 5 m/s with respect to the crate. The peels are being thrown in the forward direction of the motion of the cart. If there are 100 bananas, what will be the final velocity of the monkey and crate?

Ans: $v_N = v_0 + m_b u \sum_{j=1}^{N} \left[\frac{1}{M+(N-j+1)m_b} \right]$.

Ex 7.9.33. Find $v_x(t)$ from the following equation if $v_x(0) = v_{x0}$. Here, M, u, and α are constants.

$$\frac{dv_x}{dt} = \frac{u\alpha}{M - \alpha t}.$$

Ans: $u \ln\left(\frac{M}{M-\alpha t}\right)$.