5.1 STATIC EQUILIBRIUM

We say that a system is in a static equilibrium if the net force on every particle of the system is zero. Let there be N particles in a system, and let $\vec{F}_i$ be the net force on the i^{th} particle. Then the **static equilibrium** corresponds to the following set of conditions.

$$\boxed{\left.\begin{array}{ll} \vec{F}_1 = 0 & \text{net force on particle \#1} \\ \vec{F}_2 = 0 & \text{net force on particle \#2} \\ \vdots & \\ \vec{F}_N = 0 & \text{net force on particle \#N} \end{array}\right\}} \tag{5.1}$$

Recall that the forces on each particle can be classified based on which agent applies that force: if the force is from another particle in the system, we call that force an **internal force** and if the force is by some object outside the system of N particles, then we call that force an **external force**. Suppose we sum up all the internal forces on part 1 and call it $\vec{F}_1^{int}$ and sum up all the external forces on particle 1, call it $\vec{F}_1^{ext}$, then we can write the force $\vec{F}_1$ on particle 1 as

$$\vec{F}_1 = \vec{F}_1^{int} + \vec{F}_1^{ext}.$$

Similarly for the forces on the other particles. Using this separation, Eq. 5.1 should actually be written as

$$\left.\begin{array}{ll} \vec{F}_1^{int} + \vec{F}_1^{ext} = 0 & \text{net force on particle \#1} \\ \vec{F}_2^{int} + \vec{F}_2^{ext} = 0 & \text{net force on particle \#2} \\ \vdots & \\ \vec{F}_N^{int} + \vec{F}_N^{ext} = 0 & \text{net force on particle \#N} \end{array}\right\} \tag{5.2}$$

Ideal nondeformable objects and static equilibrium

All physical bodies are deformable and are deformed when a force is applied on them. Even when an object does not appear deformed, there must be compression or stretching at the microscopic level, otherwise we would not be able to explain the development of reaction forces such as the normal force and the frictional force. For instance, when you place a book on the table, neither the book nor the table appears deformed, but they must be a little compressed at the contact surface so that a normal force can develop there.

For the sake of mathematical simplicity, we will first study "ideal" nondeformable objects defined as follows. An ideal nondeformable object applies reaction forces such as the normal and friction forces and action-at-a-distance forces on the external bodies but its shape does not change under the prevailing conditions. Most solids and liquids will fall into this category.

Suppose you have an ideally strong nondeformable object. Then, the requirement of vanishing of forces on all parts can be shown to simplify to a consideration of only the external forces. When you sum up the equations for force on each particle given in Eq.5.2, the internal forces will cancel out since they are all paired up as demanded by the Newton's third law of motion.

$$\boxed{\vec{F}_1^{\,ext} + \vec{F}_2^{\,ext} + \cdots + \vec{F}_N^{\,ext} = 0} \tag{5.3}$$

Now, if you pick an arbitrary pivot point, and calculate the net torque about that point, you will find that the net torque is also zero if the internal forces between two particles act along the line joining them.

$$\boxed{\vec{\tau}_1^{\,ext} + \vec{\tau}_2^{\,ext} + \cdots + \vec{\tau}_N^{\,ext} = 0} \tag{5.4}$$

Therefore, for a nondeformable object, the vanishing of net external force and net external torque assures static equilibrium.

For simplicity, all the problems considered in this chapter will have forces in one plane only, which we will take to be the xy-plane. Therefore these two conditions for the static equilibrium would yield at most three equations for an analysis: two for the x and y-components of the net external force and one for the net torque about an axis parallel to the z-axis passing through an arbitrarily chosen pivot point.

$$F_{1x}^{ext} + \vec{F}_{2x}^{ext} + \cdots + \vec{F}_{Nx}^{ext} = 0 \tag{5.5}$$

$$F_{1y}^{ext} + \vec{F}_{2y}^{ext} + \cdots + \vec{F}_{Ny}^{ext} = 0 \tag{5.6}$$

$$\tau_{1z}^{ext} + \tau_{2z}^{ext} + \cdots + \tau_{Nz}^{ext} = 0 \tag{5.7}$$

Deformable objects and static equilibrium

For an arbitrary structure to be in a static equilibrium, the vanishing of the net external force does not ensure static equilibrium. For instance, if you push on a piece of foam from two opposite sides, the net force on the foam is zero, but the foam gets deformed because of an imbalance between the external and internal forces on the particles at the surface.

While there is no limit on external forces, the internal forces, just like the static frictional force, have an upper limit and can be overcome by a large enough external force. Therefore, to study the equilibrium of a deformable object, we need the complete condition for the static equilibrium given in Eq. 5.2, viz., the net force on each and every particle of the system must vanish independently.

To be in static equilibrium, the external forces must balance the internal forces at each point. In this way, the analysis of the static equilibrium of deformable systems provides us with an understanding of the strength of the material. In this chapter, we will study equilibrium with regard to the external forces as well as internal forces.

But, for simplicity, we will look at the problems for the ideal undeformable objects first. This would be a review of our treatment of the static equilibrium in the chapter on forces.