4.14 PROBLEMS

Problem 4.14.1. A bat emits an ultrasound of frequency 50,000 Hz which bounces off a wall and returns to the bat in 1 ms. (a) Ignoring the speed of the bat, how far away is the bat from the wall? (b) What is the wavelength of the ultrasound wave. Use density of air 1.3 kg/m^3 and bulk modulus 1×10^5 Pa.

Ans: (a) 0.239 m, (b) 0 5.54 mm.

Problem 4.14.2. A continuous traveling wave of amplitude 2 cm and frequency 10 Hz rides towards the positive x-axis on a string of mass per unit length 0.1 kg/m and the tension 100 N. At $t = 0$, the displacement at $x = 0$ is found to be 1 cm. Represent the wave by a wave function.

Ans: $\psi(x,t) = (2 \text{ cm}) \cos\left[(1.99 \text{ m}^{-1})x - (20\pi \text{ s}^{-1})t + \pi/3\right]$.

Problem 4.14.3. A traveling wave on a string of mass 0.15 kg/m is given by the following wave function.

$$\psi(x,t) = (3 \text{ cm}) \cos\left(0.5x + 1200t + \frac{\pi}{6}\right)$$

where x is in m and t in sec. (a) What is the speed of the wave, and in which direction the wave is traveling? (b) How much is the tension in the string? (c) What is the amplitude of the wave? (d) What is the amplitude of the wave at the origin at $t = 1$ ms?

Ans: (a) 2,400 m/s, (b) 864,000 N, (c) 3 cm, (d) 2.72 cm.

Problem 4.14.4. Two stones are dropped in a pond 3 m apart at the same time. They produce waves of wavelength $\frac{1}{2}$ m. The two waves meet at the rectangular edge of the pond 20 m away, and interfere constructively and destructively. At the symmetric middle point the interference is constructive. Where are the next three constructive interference sites, P_1, P_2, and P_3 as shown in 4.31?

Ans: $P_0P_1 = 3.35$ m, P_0P_2 6.80 m, $P_0P_3 = 10.5$ m.

Problem 4.14.5. A guitar string of length 60 cm and mass per unit length 10 g/m is tuned to play the note A of frequency 440 Hz at the fundamental. (a) What is the wavelength of the wave on the string? (b) What is the tension in the string? (c) Assuming sound in air travels at speed 344 m/s, what is the wavelength of the sound produced?

Ans: (a) 120 cm. (b) 2,800 N. (c) 78 cm.

Problem 4.14.6. A string of length 108 cm is held fixed at one end and the other end is attached to a motor that moves it up and down

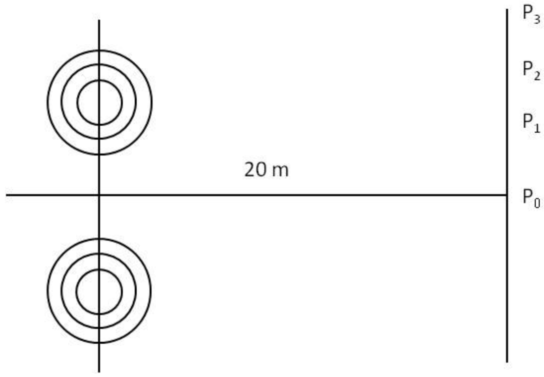

Figure 4.31: Problem 4.14.4.

by 0.3 cm at a set frequency. The tension in the string has a value of 90 N, and it has a mass density of 150 grams per meter. What must be the frequencies of the motor to excite the fundamental mode and the next three harmonics?

Ans: $f_0 = 11.3$ Hz, $f_1 = 22.6$ Hz, $f_2 = 33.9$ Hz, $f_3 = 45.2$ Hz.

Problem 4.14.7. Light wave acts similar to the mechanical waves when it comes to interference. Consider a light source that shines on a screen far away. When a mirror is brought, as shown in the Fig. 4.32, then a direct ray to the screen interferes with the reflected ray. Note that due to change in the phase of the wave upon reflection by π

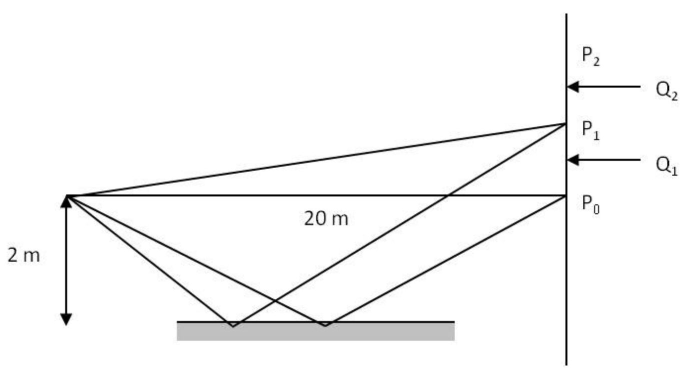

Figure 4.32: Problem 4.14.7

radians, two waves will be completely out of phase and a destructive interference will result at a point there the difference in distances traveled is an integral multiple of wavelength. That is, the conditions for constructive and destructive interferences will be opposite of the Young's double-slit formulas. The screen is positioned so that the two rays are in phase at the straight point P_0. Find the location of two dark spots (Q_1 and Q_2) and two other bright spots (P_1 and P_2) on the screen.

Ans: Dark spots at $y = n\ \lambda\ D/2h$, n = 0, 1, 2, 3, $\cdots$. Bright spots when $y = m\,\lambda\, D/4h$, $m = 1, 3, 5, \cdots$.

Problem 4.14.8. A rectangular pulse is given by the following function at time $t = 0$.

$$\psi(x,0) = \begin{cases} 0 & x \leq -1 \\ \text{A} & -1 < x < 1 \\ 0 & x \geq 1 \end{cases}$$

The wave is traveling towards the positive x-axis with speed v. (a) What is the wave function at some later time t? (b) Determine the instantaneous power in the wave.

Problem 4.14.9. A wave pulse is given by the following function at time $t = 0$.

$$\psi(x,0) = A \exp(-x).$$

The wave travels with speed v towards the negative x-axis. What is the wave function at some later time t?

Problem 4.14.10. Two strings on two cellos are tuned to the same frequency of 247 Hz. One of the strings is then tightened further increasing its tension by 30%. When the two cellos are played a beat is heard. Find the beat frequency.

Ans: 34.6 Hz.

Problem 4.14.11. In a valley an echo is heard after 1.5 second of the scream. How far away are the mountains? Use 343 m/s for the speed of sound.

Ans: 257 m.

Problem 4.14.12. A piano key is to be tuned by using a 440 Hz tuning fork. The key is loose and produces a sound that is lower that 440 Hz. The piano tuner hears 4 beats per second when the key is struck at the same time as the tuning fork. How different is the tension in the cord now compared to what it should be to produce the right frequency?

Ans: 0.98

Problem 4.14.13. Two loudspeakers are connected to a 2000 Hz signal generator. A person while standing between the two speakers does not hear any beats, but when he runs from one speaker to another he hears a 25 Hz beat. How fast must he be moving away from one speaker and towards the other speaker? Use 343 m/s for the speed of sound.

Ans. 2.14 m/s.

Problem 4.14.14. An aluminum rod can be vibrated to produce squealing sound in a demonstration called the "singing rod". You can buy an aluminum rod of length approximately 1 m and diameter 6 mm or 3 mm from a hardware store. Balance the rod at the mid-point, and while holding it there, stroke the rod between your thumb and the index finger back and forth from the middle to one end. If you do it right, you will produce a high pitch squeal. Try it. Consider one such rod of length 1.2 m and the area of cross-section 3 mm. (a) What would be the lowest frequency of sound produced? (b) What would be the wavelength of the sound on the aluminum rod corresponding to the lowest frequency sound produced? (c) What is the frequency of the same sound in the air? (d) What is the wavelength of the same sound in the air? Assume the speed of the sound in air to be 343 m/s.

Ans: (a) 1.2 m. (b) 4,400 Hz. (c) 4,400 Hz. (d) 0.078 m.

Problem 4.14.15. Two speakers connected to the same signal generator are placed 2 m apart. When the signal generator sends sinusoidally varying signal of frequency 500 Hz, the speakers produce pure tone of 500 Hz. The waves coming out of the speakers are in sync with each other. A microphone, which can move parallel to the line joining the speakers, is placed 6 meters from the speakers as shown. An intensity maximum is found when the microphone is in the middle. As the microphone is moved, we find intensity minima and maxima alternately. Find two other intensity maxima (P_1 and P_2) and three intensity minima.

Ans: Constructive: $y_{\pm 1} = = \pm$ 2.06 m. Destructive: $y'_{\pm 1} = \pm$ 1.03 m.

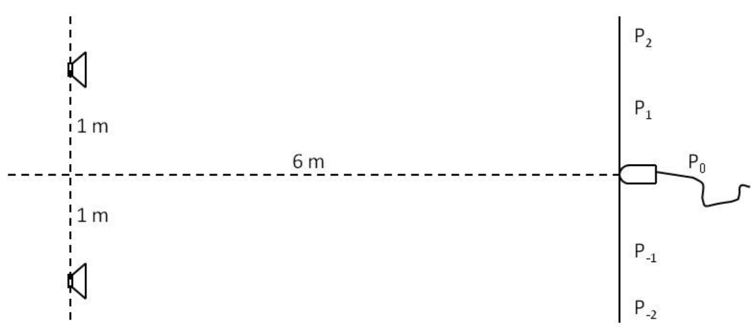

Figure 4.33: Problem 4.14.15.

Problem 4.14.16. Doppler flow meters work with the principle of the Doppler shift. In these instruments, when sound wave is incident on a moving particle, the reflected wave has an altered frequency because the reflected wave is emitted at the original frequency but the source now is moving with respect to the detector. By detecting the change in the frequency observed by the detector, one can deduce

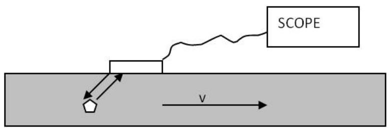

Figure 4.34: Problem 4.14.16.

the speed of the particle in the fluid. In a certain experiment, a wave is sent by a transducer, which is a source of ultrasound wave, at 45 degrees angle to the flow. The reflected waves are detected by the same transducer that emitted the original wave. When a frequency of 10 MHz is employed, a shift of 1000 Hz is observed. What is the speed of flow? Use the speed of sound in water to be 1500 m/s. [Note the speed of sound in materials is much greater than the speed of sound in the air.]

Ans: 21 cm/s.

Problem 4.14.17. A police car emits siren at a frequency of 1000 Hz when at rest. The police car rides past you at a constant speed of 10 m/s with respect to you. Your speed with respect to the road is 31 m/s. What will be the frequencies you will hear (a) as the police car approaches you and (b) when it is moving away from you? Assume 344 m/s for sound.

Hint: Its the relative speed that matters.