## 4.1 ELECTROMAGNETIC WAVE

The wave nature of light is essential for understanding many phenomena exhibited by light. Maxwell identified light as inter-twined waves of electric and magnetic fields. The two waves together are known as “electromagnetic (EM) waves”. Maxwell’s studies on electromagnetic waves unified two separate parts of physics, namely, electromagnetism and optics. According to Maxwell’s equations of electricity and magnetism, an oscillating electric field induces an oscillating magnetic field which, in turn, induces an oscillating electric field. The mutual induction of oscillating electric and magnetic fields makes the propagation of electromagnetic waves self-sustaining even in vacuum. Maxwell also showed that the electric and magnetic fields of an electromagnetic wave propagating vacuum and the propagation direction of the waves are mutually perpendicular.

A particularly simple form of electromagnetic wave is a plane harmonic wave in which electric field oscillates along a single axis, and magnetic field in an axis perpendicular to that as shown in Fig. 4.1. For instance, an electromagnetic wave traveling towards the positive $x$-axis with electric field pointing along the $y$-axis and the magnetic field pointing along the $z$-axis is an example of a plane wave.

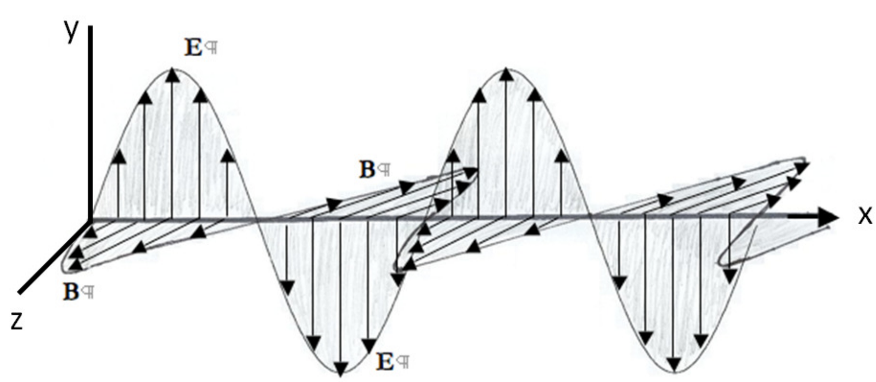


Figure 4.1: Plane Electromagnetic wave.

The **plane electromagnetic wave** shown in Fig. 4.1 can be represented analytically by the following functions, called **wavefunctions**.

$$\vec{E} = E_y \hat{u}_y = \hat{u}_y E_0 \cos(kx - \omega t + \phi), \tag{4.1}$$

$$\vec{B} = B_z \hat{u}_z = \hat{u}_z B_0 \cos(kx - \omega t + \phi), \tag{4.2}$$

where $\hat{u}_y$ and $\hat{u}_z$ are unit vectors pointed towards the positive $y$- and $z$-axes respectively, and the amplitudes $E_0$ and $B_0$ of the electric and magnetic field waves, respectively, are related as

$$B_0 = \frac{E_0}{c}. \tag{4.3}$$

Here $v$ is the speed of the electromagnetic wave, $\omega$ is the **angular frequency**, $k$ the **wave number**, and $\phi$ the **phase constant**. .

The argument of the trigonometric functions on the right of Eqs. 4.1 and 4.2 is called the **phase** of the wave. As shown in these formulas for the electric and magnetic fields, the waves of these fields march with the same phase: the crest of the electric field coincides with the crest of the magnetic field wave and the trough with the trough.

The angular frequency $\omega$ (radians per second) and wave number $k$ are related to the frequency $f$ (cycles per second) and wavelength $\lambda$ respectively.

$$\omega = 2\pi f. \tag{4.4}$$

$$k = \frac{2\pi}{\lambda} \tag{4.5}$$

The ratio $\omega/k$ is therefore equal to the speed $c$ of the light wave in the medium, which is vacuum here.

$$\frac{\omega}{k} = \lambda f = c. \tag{4.6}$$

Maxwell showed that the speed of electromagnetic wave in the vacuum is simply related to the electric susceptibility $\epsilon_0$ and the magnetic permeability $\mu_0$ of vacuum.

$$\text{Speed of light in vacuum,} \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}. \tag{4.7}$$

The speed of light in vacuum is a universal constant, and given an exact value of 299,792,458 m/s. The speed of light in vacuum is denoted by the letter $c$.

$$c = 299,792,458 \text{ m/s} \quad \text{(Exact)} \tag{4.8}$$

Since the electric and magnetic fields of an electromagnetic wave are interdependent, it often suffices to examine only the electric field of the wave. You should, however, keep in mind that magnetic field wave is also present.

The points in space with equal phase form a surface called the **wavefront**. Evidently, the wave function has the same value at all points of the wavefront. We see that the wavefronts of the plane waves

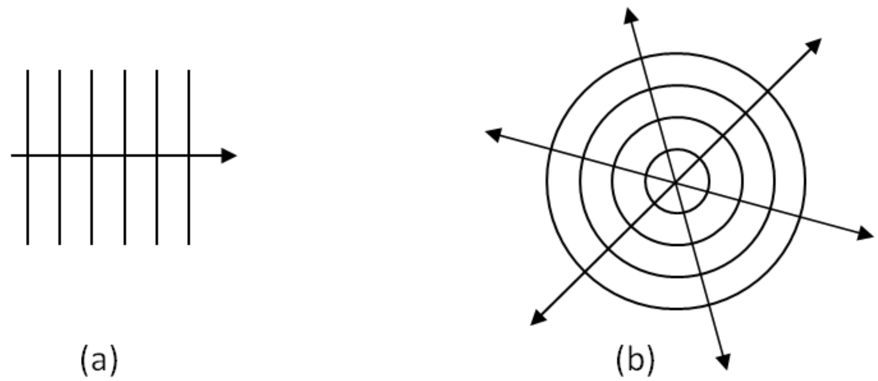


Figure 4.2: Wave fronts are perpendicular to the waves: (a) plane wave, (b) spherical wave.

given in Eq. 4.1 and 4.2 are planes perpendicular to the direction of the wave. That is why these waves are also called **plane waves**.

Let us examine the crest of the waves at $t = 0$. The crests will occur when the argument of the cosine is equal to $2n\pi$ for $n$ an integer.

$$-kx + \phi = 2n\pi \quad \Longrightarrow \quad x = \text{constant}$$

Thus, a plane wave moving along the $x$-axis crests simultaneously at all points of all planes parallel to the $yz$-plane separated by one wavelength. Not all waves are plane waves. For instance, an isotropic light emitted by a point source will spread out as spherical-shaped wave fronts centered at the source. These waves are called **spherical waves**. The traveling spherical waves are given by a wave function whose amplitude drops off inversely with the distance $r$ from the source.

$$\vec{E} = \hat{u}_{\perp r}\frac{E_0}{r}\ \cos(kr - \omega t + \phi), \tag{4.9}$$

where $\hat{u}_{\perp r}$ is a unit vector in the direction in the plane perpendicular to the radial direction. Pictorially, a traveling electromagnetic wave of a single frequency can be represented by drawing wavefronts that are a single wavelength apart. Rather than draw a three-dimensional plot of waves it is customary to draw a cross-section. Thus, for a plane wave we draw parallel lines, each line representing a plane wavefront, while for a spherical wave we draw arcs or circles, each representing a spherical wavefront. (Fig. 4.2). Note that the wave fronts are perpendicular to the direction of wave.