6.3 NEWTON'S THIRD LAW OF MOTION

The third law of motion addresses the nature of mutual interactions between two bodies as we have studied in the last chapter. Recall the statement of Newton's third law given in the last chapter.

To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and directed to the contrary parts.

Let $\vec{F}_{AB}$ denote the force on body A by body B, and $\vec{F}_{BA}$ the force on B by A. Then, Newton's third law states that:

$$\vec{F}_{AB} = -\vec{F}_{BA}. \tag{6.13}$$

Here, the minus sign on right side of the equation tells us that the two forces are acting in the opposite directions. Since, only one of these forces acts on any body, you do not use both forces in any problem concerning only one of the two bodies.

What happens if we choose a system that includes both bodies? The consideration of this question leads to an important result of mechanics and gives us another view of Newton's third law.

Let us imagine a world where only two bodies A and B exist which may exert a force on each other. We say that the two bodies make up an isolated system. Any two bodies that interact with each other and not with anything else can be an example of such a world.

According to the second law of motion, the total momentum of the combined system cannot change with time since there is no external force on the isolated system. However, if you look at A or B, the momentum of each is changed by the force by the other. Since the total momentum does not change with time, the change in the momentum $(\Delta\vec{p}_A)$ of A must be exactly equal in magnitude to the change in the momentum $(\Delta\vec{p}_B)$ of B and must be pointed in exactly opposite direction.

$$\Delta\vec{p}_A = -\Delta\vec{p}_B \text{ } (A \text{ and } B \text{ isolated from the rest of universe}). \tag{6.14}$$

This relation gives another view of forces: forces are vehicles for exchange of momentum between interacting objects - while one body gains momentum, the other body must lose momentum at the same time. Let us write Eq. 6.14 for an interval from t_1 to t_2. Let the momenta of A and B at t_1 be $\vec{p}_1^A$ and $\vec{p}_1^B$ and at t_2 be $\vec{p}_2^A$ and $\vec{p}_2^B$ respectively. Then Eq. 6.14 will be

$$\vec{p}_2^A - \vec{p}_1^A = -\left(\vec{p}_2^B - \vec{p}_1^B\right), \tag{6.15}$$

which can be rearranged so that the momenta at one instant are on one side of the equation to obtain

$$\vec{p}_1^{\,A} + \vec{p}_1^{\,B} = \vec{p}_2^{\,A} + \vec{p}_2^{\,B}, \tag{6.16}$$

Eq. 6.16 states a very important principle of physics - **the principle of conservation of momentum**:

If a system is isolated such that the net external force is zero then the total momentum is constant in time, even though momentum of its constituents may change with time.

Since the principle of conservation of momentum plays a major role in physics, we will take up the subject in a separate chapter.

Experimental test of the third law

One way of testing the third law experimentally involves two carts placed on a smooth horizontal track as shown in Fig. 6.5.

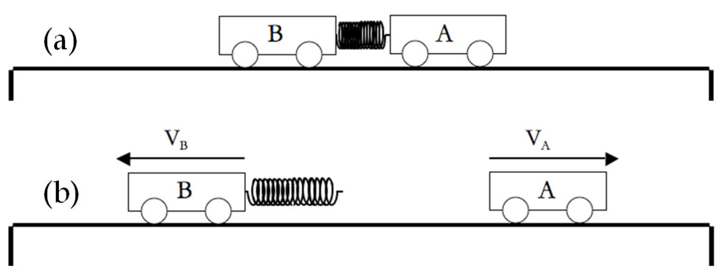

Figure 6.5: (a) Two carts with a compression spring attached to one cart and compressed between them are let go from rest so that their starting momenta are zero. (b) After release, the two carts move in opposite directions such that the magnitude of the change in momenta of the two carts are equal: $|M_A V_A| - 0 = |M_B V_B| - 0$.

We attach a spring to one cart and use the other cart to press the two carts together to hold them still on the track. After we let go of the carts, each cart applies a force on the other through the spring. As a result the momenta of the two carts change which can be determined by the speed of the carts once they are no longer in contact.

The measurement of speeds of the two carts shows that the amount of change in momentum of the two carts are equal. And since the carts are moving in opposite directions, the momentum change vectors of the carts are pointed in the opposite directions. Therefore, the total momentum of the two carts is still zero even when they are moving away from each other. Thus, momentum before release is same as the momentum afterwards, confirming the fact that the forces exerted by the two carts in each other are equal in magnitude and opposite in directions.