## 9.3 MAGNETIC MATERIALS

### 9.3.1 Magnetization

The density of magnetic dipole moment in a material is called its magnetization. The magnetization gives the strength of a magnet: a magnet with larger magnetization is a stronger magnet. Let $\vec{\mu}$ be the the average magnetic dipole moment per atom, and N the number of atoms per unit volume, then dipole moment in a small volume $\Delta V$ if all the dipoles are aligned in the volume will be

$$\text{Average dipole moment} = N\vec{\mu}\Delta V. \tag{9.46}$$

Magnetization $\vec{M}$ is defined as

$$\text{Magnetization, } \vec{M} = \frac{\text{Average dipole moment}}{\Delta V} = N\vec{\mu}. \tag{9.47}$$

If dipoles are not aligned in the volume, then $\vec{M}$ will be less than $N\vec{\mu}$. The circulating current picture, also called bound currents, of magnetic dipole moment is often helpful. As shown in Fig. 9.8, if magnetization is uniform, then the circulating currents in neighboring cells cancel each other, leaving circulating currents on the surface only since the surface elements do not have neighboring cells outside the body. Therefore, we say that a uniformly magnetized body is

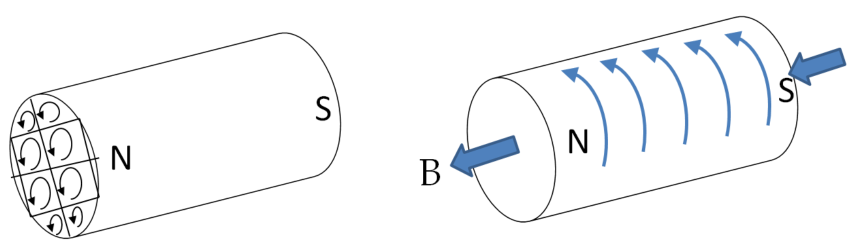


Figure 9.8: Replacing the magnetic dipoles by equivalent current loops, called "Bound Currents", for uniformly magnetetized sample shows that the magnet is equivalent to a surface current. The magnetic field of the bar magneti comes out of the North pole and enters the South pole. The magnetic field lines are bunched inside the magnet from the South pole to the North pole.

equivalent to a body with a bound surface current. If a body is not uniformly magnetized, then the cancellation in the body will not be perfect and there will be bound currents inside the body as well.

### 9.3.2 Diamagnets, Paramagnets, and Ferromagnets

Depending on the electronic and nuclear configurations of an atom, it may or may not have a net permanent magnetic dipole moment. Even when atoms of a material have permanent magnetic dipole moments, the bulk material may not be magnetic at room temperature if the atomic dipole are randomly oriented due to thermal agitation. Consequently, at a macroscopic level they cancel out resulting in a zero net magnetic dipole moment in most materials. However, when an external magnetic field is applied, the permanent magnetic dipoles of each atom tends to align with the external field. In addition, magnetic dipoles are induced in all materials in the direction opposite to the magnetic field. The combined effect of the permanent and induced dipoles is a net alignment of the atomic dipoles in the material. We say that the sample is magnetized. Due to the permanent magnetic dipoles aligning with the external field and the induced dipoles aligning against the external field, the net magnetization may be either parallel or anti-parallel to the applied field.

When the net magnetization is parallel to the applied magnetic field we call the material a **paramagnet**, and when it is anti-parallel we call it a **diamagnet**. Clearly, paramagnetism is possible only in the materials whose atoms have a permanent dipole moments. The magnetic polarization of a paramagnetic material is such that its north pole faces the south pole of the external magnet and its south pole faces the north pole of the external magnet - this makes paramagnets attracted towards stronger magnetic field. Diamagnets, on the other hand, having the opposite magnetic polarization, are repelled from the stronger magnetic field regions. A superconductor is an ideal diamagnet since in the superconducting state it repels external magnets.

Both paramagnetic and diamagnetic materials lose their magnetism when the external field is removed. But there are some materials that remain magnetic at macroscopic level even when the external magnetic field is removed. In these materials, called **ferromagnets**, magnetic dipoles exert strong influence on each other overcoming the randomization due to thermal effects as long as the temperature is not greater than a characteristic temperature called the Curie point. The magnetic strength of ferromagnets depends not only on the temperature and the applied field, but also upon its magnetic history. Iron, Cobalt and Nickel are some of the most common ferromagnets.

### 9.3.3 Auxiliary Field $\vec{H}$ and Magnetic Field Inside Magnets

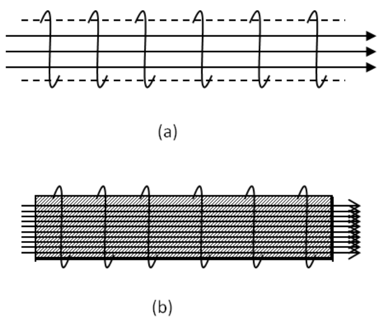


Figure 9.9: Solenoid without any core (a) and with iron core (b). Magnetic field inside the solenoid, $\vec{B}_{in}$, is stronger with the iron core.

Consider a solenoid obtained by winding a thin insulated copper wire over an iron core. When a current passes through the copper wire, magnetic field lines inside the solenoid passes through the iron core and align the atomic magnetic dipoles of iron, which produce additional magnetic field in the same direction. Thus, magnetic field inside the solenoid is stronger with the iron core than without it (Fig. 9.9).

Clearly Ampere's law will not give the correct answer if $I_{enc}$ has contributions only from the current in the wire. If $I_{enc}$ is current in the copper wire only, we will predict $B_{in}$ to be:

$$B_{in} = \mu_0 \, n \, I_{wire} \quad \text{(Wrong!)} \tag{9.48}$$

where $n$ is the number of windings per unit length. The actual $B_{in}$ within the iron core is much larger than the magnetic field of the current in the wire. The mistake is in the omission of the "bound currents" to the eneclosed current on the right-side of Eq. 9.49. The trouble is that we usually do not know the magnetization of the iron core, and hence we do not have a way of including bound current in $I_{enc}$.

A crude way to include the effect of the magnetic material in changing the magnetic field is to replace the constant the magnetic permeability of the vacuum $\mu_0$ by a material-dependent magnetic permeability $\mu$. In the case of the magnetic material inside the solenoid the modified magnetic field will be given by the following expression.

$$B_{in}^{\text{material}} = \mu \, n \, I_{wire} \quad \text{(use } \mu \text{ of the material)} \tag{9.49}$$

Therefore, in general, we write the magnetic field of a material in terms of the magnetic field in the absence of the material as

$$\boxed{\vec{B} = \frac{\mu}{\mu_0} \, \vec{B}_0.} \tag{9.50}$$

A more sophisticated description based on a new field $\vec{H}$ in place of the field $\vec{B}$ by isolating the effect of magnetization $\vec{M}$ from the magnetic effect of the physical current.

$$\boxed{\vec{B} = \frac{1}{\mu_0} \, \left(\vec{H} + \vec{M}\right).} \tag{9.51}$$

The new field is sometimes called auxiliary field and some other times magnetic field when $\vec{B}$ is called magnetic induction. The auxiliary field obeys a law similar to the Ampere's law for $\vec{B}$ but with the current being only the real current, and not on the magnetic dipoles of the magnetic material.

$$\oint \vec{H} \cdot d\vec{l} = I^f_{enc} \tag{9.52}$$

where $I^f$ denotes the current in the wire, also called free current. Note the absence of $\mu_0$ on the right side of the Ampere's law for $\vec{H}$ field. The unit of $H$ is that of $B/\mu_0$ which is the same unit as magnetization or magnetic dipole per unit volume.

$$[H] = \frac{[B]}{[\mu_0]} = \frac{A}{m} = [M]. \tag{9.53}$$

It can be shown that the difference of magnetic field $\vec{B}/\mu_0$ and $\vec{H}$ is equal to the magnetization $\vec{M}$.

$$\frac{1}{\mu_0}\vec{B} - \vec{H} = \vec{M}. \tag{9.54}$$

In iron and other ferromagnets, $B/\mu_0 >> H$ so that the magnetic field $\vec{B}$ is dominated by $\vec{M}$. Since $M = 0$ outside the magnet, magnetic field $\vec{B}$ at those points are from $\vec{H}$ alone.

$$\frac{1}{\mu_0}\vec{B} = \vec{H} \quad \text{(outside magnets)} \tag{9.55}$$

By introducing the auxiliary field $\vec{H}$ we can determine the magnetic field $\vec{B}$ outside magnets even when we do not know the magnetization. Since $\vec{H}$ obeys a law similar to the Ampere's law, one can try to borrow the techniques of Ampere's law to find $\vec{H}$ for a given current if symmetry allows the calculations.

### 9.3.4 Nuclear Magnetism

The protons and neutrons in an atom reside in the nucleus having a volume of the order of $10^{-45} m^3$. Contrast this to the volume occupied by an atom, which is of the order of $10^{-30} m^3$. Since the repulsive electrical forces between protons in the nucleus is tremendous, you would expect that protons cannot stay in the tiny space of the nucleus. There is, however, a much stronger force called the strong nuclear force that acts between protons, between protons and neutrons, and between neutrons and neutrons that is responsible for overcoming the repulsive electrical force between protons in the nucleus.

A proton is a spin 1/2 particle similar to an electron. The projection of spin angular momentum along any direction is again quantized giving the values $\pm\hbar/2$ in any measurement of the spin angular momentum projection along the chosen axis. The gyromagnetic ratio for protons $g_p$ is approximately 2000 times smaller than the gyromagnetic ratio of electrons due to different masses of the two kinds of particles.

$$g_p = (m_e/m_p)\, g_e \tag{9.56}$$

where $m_e$ and $m_p$ are masses of electron and proton respectively. The magnetic moments of nuclei are given in terms of nuclear magneton $\mu_N$ instead of Bohr magneton $\mu_B$. The nuclear magneton $\mu_N$ is defined by replacing the electron mass in the Bohr magneton definition by a mass of a proton.

$$\mu_N = g_p\hbar = \frac{e\hbar}{2m_p} = 3.15 \times 10^{-27} A.m^2. \tag{9.57}$$

The magnetic moment of a proton inside hydrogen nucleus is $2.7928\mu_N$. Note that nuclear magnetic moments are of the order of $10^{-27}$ A.m$^2$ per nucleus while electron magnetic moments are of the order of $10^{-24}$ A.m$^2$. Hence, nuclear magnetism can be mostly ignored when studying magnetic properties of materials. Even so, the resonance of nuclear magnetic moments provides a useful analytical tool.

The resonance of a nuclear magnetic moment takes place when its direction is flipped back and forth dynamically with respect to a direction defined by a static magnetic field. The dynamical magnetic field is provided by electromagnetic waves. The frequency of the wave needed to accomplish the flip depends on the chemical environment of the atom. The effect, called **nuclear magnetic resonance or NMR**,index provides an important tool for the identification of chemical structure and the study of chemical environment of an atom. Nuclear magnetic resonance has also found medical applications through the invention of a machine called the **Magnetic Resonance Imaging (MRI)**. In MRI, we align the nuclear magnetic moments of protons, usually of hydrogen atoms in the body, by means of a strong static magnetic field, and then apply a time dependent magnetic field on the sample that flips the direction of the nuclear magnetic moments. In doing so, the sample absorbs energy from the dynamic field. The energy needed to flip a proton's spin magnetic moment from being aligned parallel to static field $\vec{B}$ to being anti-parallel to it is given by

$$\Delta U = \Delta\left(-\vec{\mu}\cdot\vec{B}\right) = 2\mu B. \tag{9.58}$$

The energy taken from the electromagnetic wave of appropriate frequency is accounted for in terms of energy of each photon of the electromagnetic wave,

$$E_{\text{photon}} = hf, \tag{9.59}$$

where $h$ is the Planck constant and $f$ the frequency of the electromagnetic wave.

Hence, the energy conservation for the absorption of electromagnetic wave by the sample yields the required frequency.

$$hf = \Delta U = 2\mu B \implies f = \frac{2\mu B}{h}. \tag{9.60}$$

**Example 9.3.1. Energy for changing the magnetic dipole of proton.** Determine the frequency of electromagnetic wave needed to flip the magnetic moment of a proton in hydrogen atom aligned in a 2 T field. Use 2.7928 $\mu_N$ for the magnetic moment of the proton.

**Solution.** We just use the formula derived above putting in the numerical values.

$$\begin{aligned} f &= \frac{2\mu B}{h} \\ &= \frac{2 \times 2.7928 \times 3.15 \times 10^{-27}\text{A.m}^2 \times 2\ \text{T}}{6.627 \times 10^{-34}\text{J.s}} \\ &= 5.3 \times 10^{7}\ \text{Hz} = 53\ \text{MHz}.. \end{aligned}$$