3.7 Exercises

Matter Waves

Ex 3.1. What is the wavelength of a 50-kg person running at 10 m/s?

Ex 3.2. What is the wavelength of an electron ($m_e = 9.1 \times 10^{-31}$ kg) moving at (a) 10 m/s, (b) 10^4 m/s, (c) 10^8 m/s?

Ex 3.3. What is the wavelength of a proton ($m = 1.67 \times 10^{-27}$ kg) moving at (a) 10 m/s, (b) 10^4 m/s, (c) 10^8 m/s?

Ex 3.4. (a) What is the wavelength of a 100 keV photon? (b) What is the wavelength of a 100 keV electron?

Ex 3.5. What is the wavelength of a 1 TeV proton in an accelerator?

Ex 3.6. (a) At what energy would an electron have the wavelength 0.5 μm, which is the wavelength of a light in the visible spectrum? (b) How does this energy compare with the energy of a photon of wavelength 0.5 μm?

Ex 3.7. (a) The momentum of a nonrelativistic particle is doubled, what becomes of its wavelength? (b) What will be the case if the particle was relativistic?

Ex 3.8. (a) The kinetic energy of a nonrelativistic particle is doubled, what becomes of its wavelength? (b) What will be the case if the particle was relativistic?

Evidence for Matter Waves

Ex 3.9. In a Davisson-Germer experiment electrons were accelerated by 72 V before striking a nickel crystal. The electrons reflect off from crystal planes with spacing of 215 pm. At what angles will there be the first two diffraction orders?

Ex 3.10. In a G. P. Thomson experiment electrons were accelerated by 50 V before striking a copper powder. The first-order diffraction occurs at angle 14° with respect to the central direction. What is the spacing of the lattice planes from which the electron waves reflect off in this direction?

Heisenberg's Uncertainty Principle

Ex 3.11. A proton ($m = 1.67 \times 10^{-27}$ kg) is confined in a nucleus of diameter 5×10^{-15} m. (a) What is the uncertainty in its momentum? (b) What is the uncertainty in its speed? (c) If the momentum of the proton is of the order of the uncertainty in the momentum, what will be the kinetic energy of the proton in MeV unit?

Ex 3.12. An electron's position in the hydrogen atom is known with an uncertainty of 10^{-10} m. (a) What is the uncertainty in its momentum? (b) What is the uncertainty in its speed? (c) If the momentum of the electron is of the order of the uncertainty in the momentum, what will be the kinetic energy of the electron in eV unit? (d) If the binding energy of the electron in the hydrogen atom is 13 eV, can the electron with the kinetic energy calculated in (c) stay bound?

Ex 3.13. To get a feel for the weirdness of the microscopic world, imagine a world where the "Planck constant" is large, say, 1.0 J.s, instead of 6.62×10^{-34} J.s. In the imagined world what will be uncertainty in the position of a ball of mass 0.2 kg if the speed of the ball is known to be between 15 m/s and 16 m/s?

Particle in a Box

Ex 3.14. An electron is in the lowest energy state of a confined one-dimensional space of width a with infinite potential walls on the two side. The walls are moved in slowly to a new width of $\frac{1}{2}a$. What would be the energy of the electron in the lowest state compared to its energy at the start?

Ex 3.15. An electron (mass, $m = 9.1 \times 10^{-31}$ kg) is placed in a confined one-dimensional space of width $a = 0.5$ nm with infinite potential walls on the two sides. (a) What are the energies corresponding to the four lowest states? (b) The electron is in a state that has one node in the middle of the potential well. What is the (i) energy of the electron and (ii) the time-independent wave function of the electron when in this state? (c) How much energy will be needed to excite this electron to the next higher level? (d) What is the wavelength of the photon that will be released when the electron in (b) makes a transition to the lowest energy state?

Ex 3.16. We wish to prepare an electron with energy 10 eV in $n = 2$ state of a one-dimensional confined infinite potential well. What should be the width of the potential well?

Ex 3.17. An electron is confined in a one-dimensional infinite potential well of width $a = 100$ pm. (a) What is the uncertainty in its position? (b) What is the uncertainty in its momentum? (c) If momentum is equal to the uncertainty in momentum, what is the lowest kinetic energy of the electron? (d) What energy level of the well does this energy correspond to?

Ex 3.18. An electron is in a one-dimensional potential well of width a with infinite potential walls on the two sides. The electron is in the lowest energy state. What is the probability that the electron will be in (a) the right half of the well, (b) the right one-quarter of the well, (c) the right one-fifth of the well?

Ex 3.19. An electron is in a one-dimensional potential well of width a with infinite potential walls on the two sides. The electron is in the first excited state. What is the probability that the electron will be in (a) the right half of the well, (b) the right one-quarter of the well, (c) the right one-fifth of the well?

Ex 3.20. An electron is in a one-dimensional potential well of width a with infinite potential walls on the two sides. The electron is in the second excited state. What is the probability that the electron will be in (a) the right half of the well, (b) the right one-quarter of the well, (c) the right one-fifth of the well?

Reflection from a finite potential step

Ex 3.21. An electron of energy 10 eV is shot towards a region where there is a repulsive potential barrier of 4 eV. Suppose the barrier starts at $x = 0$ and continues towards the positive x-axis and the electron is shot at the barrier from a far-away place on the negative x-axis. (a) Find the wave number k of the electron when it is in region $x < 0$. (b) Find the wave number k of the electron when it is in region $x > 0$. (c) What is the probability that the electron will be reflected? (c) What is the probability that the electron will be transmitted?

Ex 3.22. A proton of energy 10 MeV is shot towards a region where there is a repulsive potential barrier of 4 MeV. Suppose the barrier starts at $x = 0$ and continues towards the positive x-axis and the proton is shot at the barrier from a far-away place on the negative x-axis. (a) Find the wave number k of the proton when it is in region $x < 0$. (b) Find the wave number k of the proton when it is in region $x > 0$. (c) What is the probability that the proton will be reflected? (c) What is the probability that the proton will be transmitted?

Ex 3.23. Electrons of energy 20 eV are shot towards a region where there is a repulsive potential barrier of energy less than 20 eV. Suppose the barrier starts at $x = 0$ and continues towards the positive x-axis and the electron is shot at the barrier from a far-away place on the negative x-axis. It is found that 25% of the electrons continue on towards positive x-axis and 75% of the electrons are reflected back. What is energy of the barrier?

Ex 3.24. Electrons of unknown energy are shot towards a region where there is a repulsive potential barrier of energy 100 eV. Suppose the barrier starts at $x = 0$ and continues towards the positive x-axis and the electron is shot at the barrier from a far-away place on the negative x-axis. It is found that 25% of the electrons continue on towards positive x-axis and 75% of the electrons are reflected back. What is energy of the electrons?

Ex 3.25. Electrons of unknown energy are shot towards a region where there is a repulsive potential barrier of energy 100 eV. Suppose the barrier starts at $x = 0$ and continues towards the positive x-axis and the electron is shot at the barrier from a far-away place on the negative x-axis. It is found that 25% of the electrons continue on towards positive x-axis and 75% of the electrons are reflected back. We wish to increase the percentage of electrons transmitted in the region of the barrier to 50%. By what factor should the energy of the electrons be increased?

Quantum Tunneling

Ex 3.26. Electrons of energy 10 eV are incident on a finite barrier of energy 20 eV and width 1 nm. What is the percentage of electrons that will tunnel through the barrier?

Ex 3.27. An electron of energy E is incident on a potential barrier of height $U_0 > E$. What will happen to the probability of tunneling if (a) the width of the barrier is doubled, (b) the width of the barrier is halved?

Ex 3.28. The current in the Scanning Tunneling Microscope is proportional to the probability of electrons tunneling between the tip and the surface of the metal. In an experiment the tunneling current increased from I_0 to $2I_0$ when the tip is moved from a distance z to $z - \Delta z$ from the sample. Find Δz in units of $1/\alpha$, where

$$\alpha = \frac{\sqrt{2m\phi}}{\hbar},$$

where ϕ is the work function of the metal.

Ex 3.29. In an unstable nucleus a neutron of energy $E = 1$ MeV is trapped inside a finite potential barrier of height $U_0 = 100$ MeV. Let the width of the nucleus be 3 fm. Estimate the probability that the neutron will tunnel out of the nucleus.

Ex 3.30. The alpha decay of a radioactive nucleus can be modeled as a quantum tunneling effect for alpha particles, which are particles that contain two protons and two neutrons, i.e. the nucleus of the Helium atom. In an unstable nucleus an alpha particle of energy $E = 2$ MeV is trapped by a potential barrier of height $U_0 = 40$ MeV and width 5 fm. Estimate the probability that the alpha particle will tunnel out of the nucleus.