8.6 PROBLEMS

Problem 8.6.1. Helmholtz coils. Two coils of wire of same radius R and N turns each are separated by a distance R equal to the radius of the coils. Current I runs in the coils in the same direction as shown. This arrangement is called the Helmholtz coil and is very useful for generating fairly uniform magnetic field. Let the axis of the coils be the z-axis with origin at the mid-way point between the coils. (a) Find the magnetic field on the axis as a function of coordinate z. (b) Show that the magnetic field at the mid-way point is very uniform with both its first and second derivatives zero.

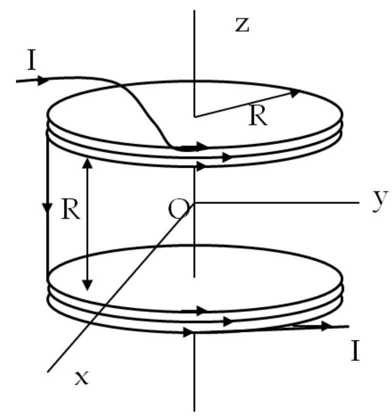

Figure 8.34: Problem 8.6.1

Problem 8.6.2. A non-conducting hard rubber circular disk of radius R is painted with a uniform surface charge density σ. It is rotated about its axis with angular speed ω. (a) Find the magnetic field produced at a point on the axis a distance h meter from the center of the disk. (b) Find the numerical value of magnitude of magnetic field when $\sigma = 1$ C/m^2, $R = 20$ cm, $h = 2$ cm, and $\omega = 400$ rad/sec, and compare it with magnitude of magnetic field of earth which is about $\frac{1}{2}$ Gauss. Ans: (b) $8.2 \times 10^{-7} \times B_{\text{Earth}}$.

Problem 8.6.3. Use the magnetic field for a ring of current I at the axis. (a) Integrate the result from $z = -\infty$ to $z = +\infty$ to show that the integral is equal to $\mu_0 I$. (b) Give reasons why this result is expected based on Ampere's law.

Problem 8.6.4. A circuit with current I has a two long parallel wire sections that carry current in the opposite directions. Find magnetic field at a point P near these wires that is a distance a from one wire and b from the other wire as shown in the figure.

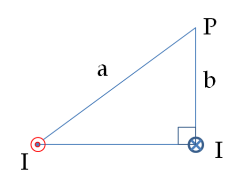

Figure 8.35: Problem 8.6.4

Problem 8.6.5. A very long thick cylindrical wire of radius R carries a current density J that varies across its cross-section. Magnitude of the current density at a point a distance r from the center of the wire is given by the following.

$$J = J_0 \frac{r}{R},$$

where J_0 is a constant. Find magnetic field (a) at a point outside the wire and (b) at a point inside the wire. Write your answer in terms of the net current I through the wire. Ans:

Magnitude:

$$B_{\text{out}} = \frac{\mu_0 J_0 R}{3}\left(\frac{R}{r}\right) = \frac{\mu_0 I}{2\pi r}$$
$$B_{\text{in}} = \frac{\mu_0 J_0 R}{3}\left(\frac{r}{R}\right)^2 = \frac{\mu_0 I}{2\pi R}\left(\frac{r}{R}\right)^2$$

Direction:

If current towards the z-axis, then $\vec{B}$ in $\hat{u}_\phi$ direction.

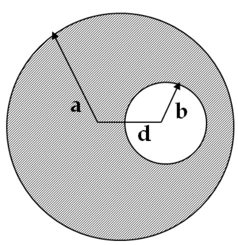

Figure 8.36: Problem 8.6.6

Problem 8.6.6. A very long cylindrical wire of radius a has a circular hole of radius b in it at a distance d from the center. The wire carries a uniform current of magnitude I through it. The direction of the current in the figure is out of the paper. Find magnetic field (a) at a point at the edge of the hole closest to the center of the thick wire, (b) at an arbitrary point inside the hole and (c) at an arbitrary point outside the wire. Hint: Think of the hole as a sum of two wires carrying current in the opposite directions.

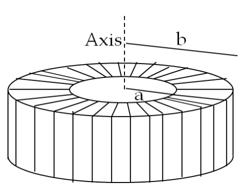

Figure 8.37: Problem 8.6.7

Problem 8.6.7. Magnetic field inside a torus. Consider a torus of rectangular cross-section with inner radius a and outer radius b. N turns of an insulated thin wire is wound evenly on the torus tightly all around the torus and connected to a battery producing a steady current I in the wire. Assume that the current on the top and bottom surfaces in the figure are radial and that the current on the inner and outer radii surfaces is vertical. Find magnetic field inside the torus as a function of radial distance r from the axis. Ans: Magnitude $\frac{\mu_0}{2\pi}\frac{NI}{r}$.

Problem 8.6.8. Two long coaxial copper tubes, each of length L, are connected to a battery of voltage V. The inner tube has inner radius a and outer radius b, and the outer tube has inner radius c and outer radius d. The tubes are then disconnected and rotated in the same direction at angular speed of ω radians per second about their common axis. Find magnetic field (a) at a point inside the space enclosed by the inner tube $r < a$, (b) at a point between the tubes $b < r < c$, and (c) at a point outside the tubes $r > d$. Hint: Think of copper tubes as a capacitor and find the charge density on them based on the voltage applied, $Q = CV$, $C = \dfrac{2\pi\epsilon_0 L}{\ln(c/b)}$. The magnetic field in the region $b < r < c$ is $\dfrac{\mu_0\epsilon_0\omega\, V}{\ln(c/b)}$.