

10.7 MOTIONAL EMF FROM ROTATION

10.7.1 Alternating Current Generator

The changing magnetic flux as a result of the changing orientation of a loop is exploited to construct a source of current called the **alternating current (AC) generator**. The orientation of a loop with respect to a magnetic field can be changed by rotating the loop. To rotate the loop one usually mounts the loop on a structure with a handle that can rotate the loop about an axis perpendicular to the direction of the magnetic field and to the area vector of the loop as shown in Fig. 10.25. We will address later the question of how we connect the loop to the outside world to harness the energy in the induced current. For now, we focus our attention on how a current can be induced in a rotating loop. When the conducting loop

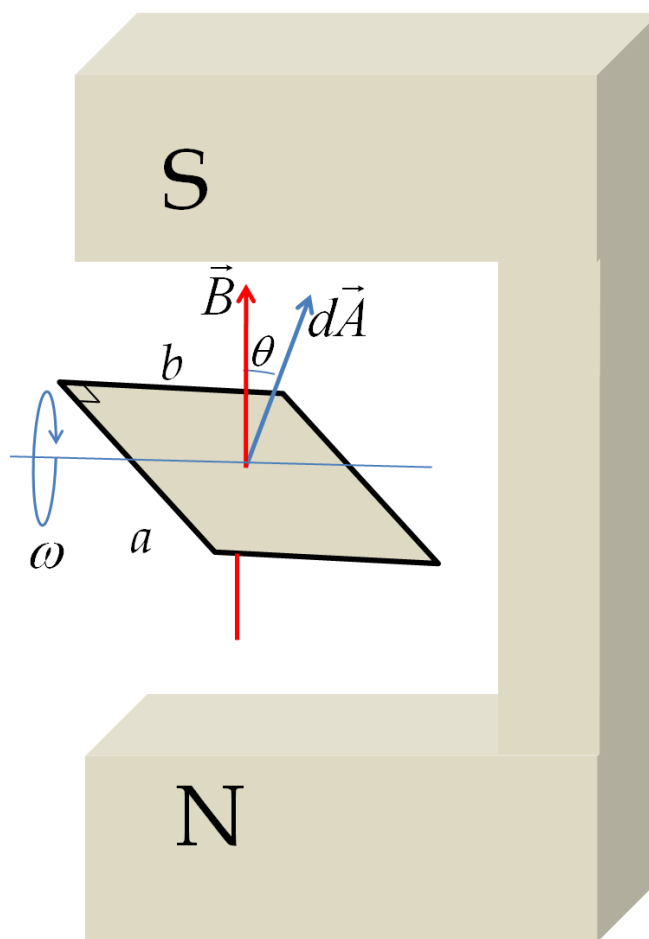


Figure 10.25: To induce an EMF in a loop, the loop is rotated about an axis that is perpendicular to the magnetic field and area vector.

is rotated at angular frequency ω (rad/s), about an axis perpendicular to the the direction of magnetic field, the angle θ between the area vector and the magnetic field will vary in time as

$$\theta(t) = \theta_0 + \omega t, \quad (10.24)$$

where θ_0 is the starting angle at time $t = 0$. To simplify the formulas we assume that the zero of time is the instant when the angle is zero. This gives

$$\theta(t) = \omega t. \quad (10.25)$$

The area of the loop and magnetic field are kept constant so that magnetic flux through the area of the loop at time t is given by

$$\Phi_B = AB \cos(\omega t). \quad (10.26)$$

If the closed circuit of the loop has N turns, then the open surface will have N such contributions to the magnetic flux. By the flux rule, the induced EMF \mathcal{E} would equal the negative of the rate of change of the magnetic flux,

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \omega NAB \sin(\omega t). \quad (10.27)$$

This induced EMF will induce current in the wires. In order for the current to come out of the rotating loop, the loop cannot be a closed wire but an open wire with which contacts have to be made from outside wires thus completing the loop for the current flow as shown in the diagram in Fig. 10.26. The open loops are mounted on a drum, called the **armature**, which rotates about an axle. Slip rings are mounted on the axle to which outside wires make contact with the rotating open loops.

Let the resistance of the external device called load in Fig. 10.26 be R_L before completing the loop and R_{coil} be the resistance of the rotating coil between the magnet poles. Then, the net resistance in the circuit will be

$$R_{\text{net}} = R_{\text{coil}} + R_L. \quad (10.28)$$

In the completed circuit the net resistance is usually dominated by the resistance of the load. Therefore, the induced current in the circuit will be

$$I_{\text{ind}} = \frac{\mathcal{E}}{R_{\text{net}}} \approx N \times \frac{\omega AB}{R_L} \sin(\omega t) \quad (\text{with } R_L \gg R_{\text{coil}}). \quad (10.29)$$

The time dependence of induced current in this circuit is sinusoidal, and since the sine function reverses sign every half cycle, the sine of the current will change every half cycle. Therefore, the induced

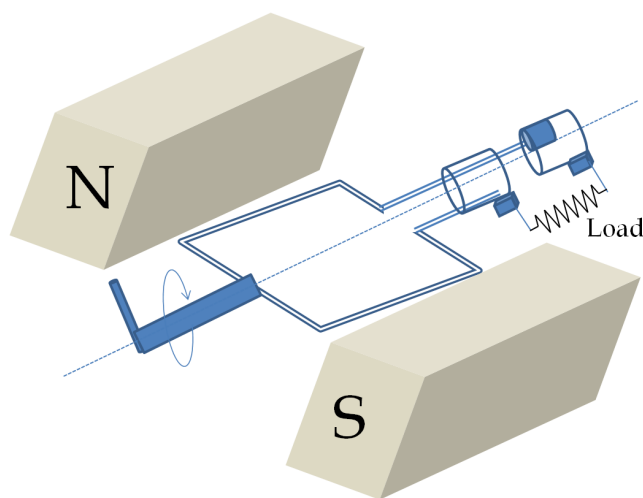


Figure 10.26: The main parts of an AC generator.

current will change direction every half cycle. This type of current is called alternating current (AC). The induced current has a peak value of $N\omega AB/R$ in either direction.

$$I_{\text{peak}} = N\omega AB/R_L. \quad (10.30)$$

The peak current in the coil is directly proportional to the number of turns in the coil, area of each loop of the coil, the strength of the magnetic field and the angular speed of rotation, and inversely proportional to the load resistance in the circuit. Thus, if you increase the number of turns by two, you would get twice as much induced current, assuming other quantities remain unchanged. If you rotate the loops twice as fast you would generate twice as big a current as illustrated in Fig. 10.27.

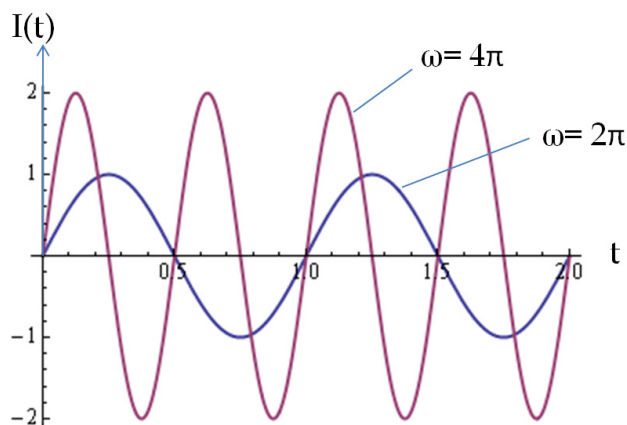


Figure 10.27: Figure illustrates that as period is halved, i.e. as angular frequency is doubled, the peak induced current doubles.

The frequency of rotation of coils sets the frequency of the current

in the circuit. In the late nineteenth century and early twentieth century, several different frequencies were in use. For instance, in 1918, there were ten different frequencies in use in London, England. The frequency of 50 Hz in Europe became more common due to the success of the German electric company AEG. In USA the enormous success of Westinghouse Electric led to the standard use of their 60 Hz by other suppliers.

Example 10.7.1. Induced EMF calculations. A circuit containing 100 turns of circular shape of radius 1 m is rotated in a uniform magnetic field of magnitude 0.5 T with a period of rotation of 2 sec . The axis of rotation is perpendicular to the magnetic field and the area vector of the surface of the loops. If the net resistance in the circuit is 10 Ω , what are the induced EMF and induced current?

Solution. We apply the formula for peak current given in Eq. 10.30. Recall that the angular speed of rotation, $\omega = 2\pi/\text{Period}$. Therefore,

$$\omega = \frac{2\pi}{2 \text{ sec}} = \pi \text{ rad/sec.}$$

This gives the induced EMF to be

$$\mathcal{E} = N\omega AB \sin(\omega t) = (490 \text{ V}) \sin(\pi t).$$

Dividing this by the resistance in the circuit gives the induced current to be

$$I = \frac{490 \text{ V}}{10 \Omega} \sin(\pi t) = (49 \text{ A}) \sin(\pi t).$$

Energy required for rotation

Suppose, the net resistance of everything in the circuit is given by R , then the rate at which energy is dissipated in the circuit is given by the power dissipation formula for a resistor.

$$P(t) = I_{\text{ind}}(t)^2 R. \quad (10.31)$$

Where does this energy come from? Note that the induced current in the rotating loop will have a torque $\vec{\tau}_B$ from the magnetic field given by

$$\vec{\tau}_B = I_{\text{ind}} \vec{A} \times \vec{B} \quad (1\text{-turn}) \quad (10.32)$$

for a 1-turn loop. But, if the loop is rotated at a constant angular velocity, this torque must be balanced by a torque $\vec{\tau}_{\text{app}}$ applied by an external agent as shown in Fig. 10.28.

$$\vec{\tau}_{\text{appl}} = -\vec{\tau}_B = -I_{\text{ind}} \vec{A} \times \vec{B}. \quad (10.33)$$

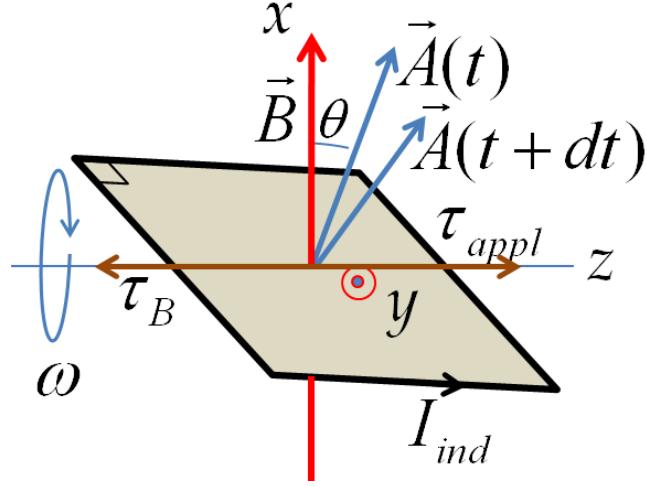


Figure 10.28: Rotational work is done by the external torque that balances the torque on the wires by magnetic field. During the interval t to $t + dt$ the loop rotates by an angle $d\theta$ which is not shown in the figure.

Byt the energy balance, we require that the rotational work done by the external agent must be equal to the total energy dissipated in the circuit. Using the current from Eq. 10.29 in Eq. 10.33 and doing the calculation for work for rotation from $\theta = \omega t$ to $\theta + d\theta = \omega(t + dt)$ we find

$$dW_{\text{rot}} = \frac{\omega A^2 B^2}{R} \sin^2 \theta d\theta \quad (10.34)$$

Replacing $d\theta$ by ωdt we get

$$dW_{\text{rot}} = \frac{\omega^2 A^2 B^2}{R} \sin^2(\omega t) dt. \quad (10.35)$$

Integrating over one cycle we find the work done by the applied torque.

$$\begin{aligned} W_{\text{one cycle}} &= \frac{\omega^2 A^2 B^2}{R} \int_0^{2\pi/\omega} \sin^2(\omega t) dt \\ &= \frac{1}{2} \frac{\omega^2 A^2 B^2}{R} T, \end{aligned} \quad (10.36)$$

where T is period, $T = 2\pi/\omega$. This result can be written in terms of the peak current as

$$W = \frac{1}{2} I_{\text{peak}}^2 R T. \quad (10.37)$$

We define a new measure of current, called the root-mean square (RMS) current, I_{rms} by

$$I_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}}. \quad (10.38)$$

With the rms-current, we can write the energy dissipated in each cycle by a formula that looks similar to the formula in a Direct Current

(DC) circuit.

$$W = I_{\text{rms}}^2 RT. \quad (10.39)$$

Dividing out the period we see that average power P_{ave} dissipated, averaged over full cycles, is given by

$$P_{\text{ave}} = I_{\text{rms}}^2 R. \quad (10.40)$$

A good exercise for the student at this point would be work out the formulas for an N -turn rotating coil.