10.4 FARADAY'S LAW

10.4.1 Induced EMF

The EMF in a circuit is the net work done by the driving force per unit charge in the circuit. The induced electromotive force would be the cumulative work done by the driving force per unit charge throughout the circuit.

As you well-know that the work on a charge in an electromagnetic field is done by the electric force. Therefore the EMF in a circuit at some instant t, whether produced as a result of $\vec{v} \times \vec{B}$ on the moving wires or as due to changing magnetic field $d\vec{B}/dt$, shows up as the electric field in the circuit, which does work on the charges and gives them a drift velocity which constitutes the induced current.

Since electric field is equal to the force per unit charge, the induced EMF will be equal to the line integral of the electric field around the circuit. The line integral calculated in the direction of the electric field gives a positive value for the EMF. The closed line integral is indicated by placing a circle on the symbol of the integral as before.

$$\boxed{\mathcal{E}_{\text{ind}} = \oint \vec{E}_{\text{ind}} \cdot d\vec{l}.} \tag{10.12}$$

Again, note that the line integral of electric field in a loop gives a value of zero for the static electric field. However, here the same integral is non-zero. You might say, "*what is true for static is false for the dynamic!*"

10.4.2 Relating Electric and Magnetic Fields

Now, we use the formula in Eq. 10.12 for the EMF in a loop into the Flux Rule given in Eq. 10.3 to obtain the following equation for a circuit and a surface attached to the circuit at the instant t,

$$\boxed{\oint_{\text{Loop(t)}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_{\text{Surface(t)}} \vec{B} \cdot d\vec{A},} \tag{10.13}$$

where I have used Eq. 10.4 to write the magnetic flux explicitly in terms of the magnetic field. This equation is called the **Faraday's law**. Sometimes this equation is also called the **integral form** of the Faraday's law.

Note that the time derivative on the right side of Eq. 10.13 is *outside the integral* whose limits may be functions of time if the surface

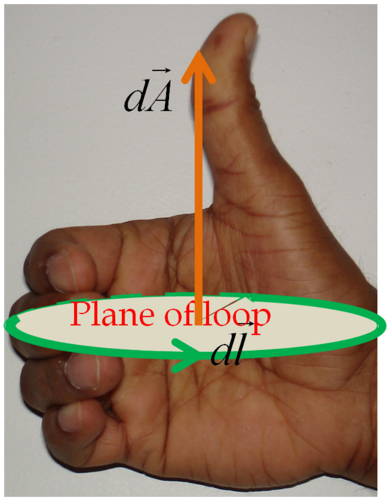

Figure 10.11: Right-hand rule illustrating relation between area vector for magnetic flux calculation and loop direction vector for EMF calculation in Faraday's law in Eq. 10.13.

attached to the loop changes with time, as would be the case when the loop moves in time. The derivative of the parameters that define the attached surface gives the contribution of the magnetic force on the conductor at the boundary.

The minus sign in Eq. 10.13 is needed to give the correct direction of the induced EMF in the circuit when the integral is conducted in the direction of the electric field and the right-hand illustrated in Fig. 10.11 used for the direction of the area vector. . The relation between the electric and magnetic fields given by the Faraday's law applies to any loop in space whether or not the points on the loop are occupied by a physical wire.

The Faraday's law says that magnetic field and electric field are intimately related with each other - a changing magnetic field gives rise to an electric field. You might wonder if a changing electric field will similarly give rise to a magnetic field. So far we have seen that only magnets and electric currents create magnetic field. Maxwell showed that a changing electric field also gives rise to a magnetic field as we will see in a later chapter.

Analogy between Faraday's law and Ampere's law

The problem of magnetic field of steady currents and the problem of electric field from changing magnetic fluxhere are completely analogous mathematically if there are no charges around as seen from the following equations.

Magnetic field of steady current:

$$\oint\!\!\int \vec{B}\cdot d\vec{A} = 0 \quad \text{(since no magnetic charges)}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \tag{10.14}$$

Electric field with no charges:

$$\oint\!\!\int \vec{E}\cdot d\vec{A} = 0 \quad \text{(since no electric charges)}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

This suggests that the problem-solving techniques of Ampere's law can be fruitfully employed here by recognizing that $-d\Phi_B/dt$ of a Faraday's law problem corresponds to $\mu_0 I_{enc}$ in the corresponding Ampere's law problem. I will present an example below that makes the use of this mathematical analogy to deduce the electric field from a changing magnetic field.