9.8 PROBLEMS

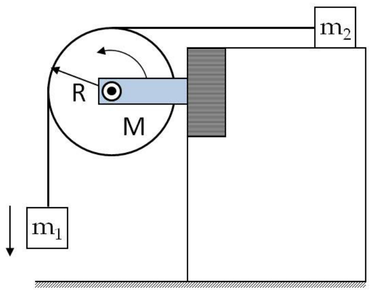


Figure 9.55: Problem 9.8.1.

Problem 9.8.1. Consider two blocks of masses m_1 and m_2 attached through a massless string which goes over a pulley of mass M and radius R as shown in Fig. 9.55. Block m_1 slides on a frictionless table while block m_2 moves in the vertical direction. Assume the pulley to be a disk of uniform mass density with no friction at the axle. Note that the pulley here is not an "ideal" pulley. Find the acceleration of the masses and the tension in the string at various points. You can ignore the mass of the connecting strings compared to all other masses in the problem.

Ans check: If $m_1 = m_2 = M$ then $a = 2g/5$, $T_1 = 3mg/5$, $T_2 = 2mg/5$.

Problem 9.8.2. In the problem above, suppose an average frictional torque from the axle on the pulley has the magnitude τ_f and the radius of the axle is r. How would your answer for the acceleration and tensions be modified? Assume, block m_2 still slides frictionlessly.

Ans: $a = \frac{m_1 g - \frac{r}{R}F_k}{m_1+m_2+\frac{1}{2}M}$. Work out T_1 and T_2 as well.

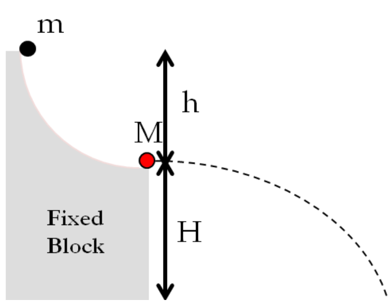


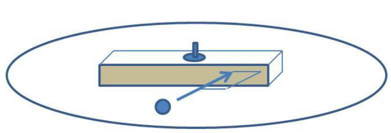
Figure 9.56: Problem 9.8.3.

Problem 9.8.3. A marble of mass m rolls down a circular track without slipping starting from rest at a height h and hits another marble of mass M horizontally. As a result of the collision, the marble m comes to rest while the marble M leaves the track with a horizontal velocity, which you need to determine as an intermediate step towards answering the following question. Where does the marble with mass M land on the floor a height H below?

Ans: $D = \frac{m}{M}\sqrt{\frac{20}{7}hH}$.

Problem 9.8.4. A car rounds a circular bend at a constant speed of 10 m/s. The radius of the bend from the CM of the car is 40 meters. The road can be assumed to be flat. There is enough friction so that car does not slide away. The distance between the left and right wheels is 1.5 meters and between the front and back wheels is 2.5 meters. The CM of the car is in the middle at a height of 30 cm from the ground. Find the value of the magnitudes of the normal forces on the four wheels of the car.

Ans: $F_{\text{left wheels}} = 16{,}000$ N, $F_{\text{right wheels}} = 13{,}000$ N.



Problem 9.8.5. A putty of mass m is tossed at a rod of mass M and length l that is free to rotate frictionlessly about an axle through the center. The putty strikes the rod horizontally with a speed v_0 at $\frac{1}{4}l$ from the center. (a) Find the angular speed of the rod if the putty sticks to the rod. (b) Find the angular speed of the rod if the putty

bounces straight back. You can assume the putty is small enough that we can treat it as a point mass.

Ans: (a) $\omega = \frac{12mv_0}{3ml+4Ml}$, (b) $\omega' = \frac{6mv_0}{Ml}$.

Problem 9.8.6. A spherical ball of mass M and radius R is attached to a thick rod of mass m and length l which is pivoted at the other end. Treating the system as a physical pendulum find the frequency of small oscillation.

Ans Key: If $m = M$ and $l = R$ the $f = \dfrac{1}{2\pi}\sqrt{\dfrac{25g}{142l}}$.

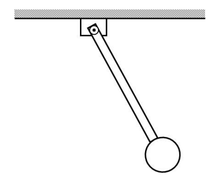


Figure 9.58: Problem 9.8.6.

Problem 9.8.7. When a string is twisted, it applies a torque at the ends. The magnitude of the torque depends on the twist angle similar to the spring force. For small angles of twist, Hooke's law can be applied to express the torque as a linear function of the angular displacement, i.e the twist angle, θ.

$$\tau = \kappa\theta,$$

where κ is a constant. Now, consider tying a string to the center of a disk of mass M and radius R, and hanging the mass from a fixed support as shown in Fig. 9.59. The string is then given a twist by a small amount and released. The disk rotates back and forth with a definite frequency. Find the frequency of these oscillations. Hint: Write the equation of motion for the rotation of the disk.

Figure 9.59: Problem 9.8.7.

Ans: $f = \frac{1}{2\pi}\sqrt{\frac{2\kappa}{MR^2}}$.

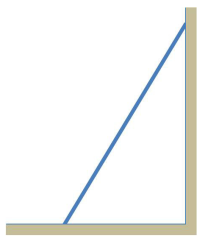


Figure 9.60: Problem 9.8.8.

Problem 9.8.8. A meter stick of length l and mass m is held against a frictionless wall and a frictionless floor. When the stick is let go from rest, it starts to slide down the wall keeping in contact with the wall till it gains enough speed and then the stick leaves the wall at a particular point. Find the point at which the meter stick leaves the wall.

Ans: $h = \frac{2}{3}h_0$ where h_0 is the initial height.

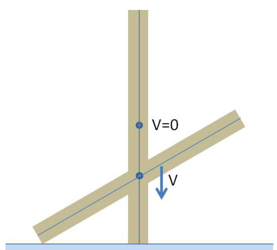


Figure 9.61: Problem 9.8.9.

Problem 9.8.9. A meter-stick is set almost vertically on a frictionless table. When the stick is let go at rest from the nearly vertical orientation it falls to the table. Since there is no horizontal force on the stick and it starts out at rest, the center of mass of the stick falls straight down. Find the speed of the CM as a function of the position of the CM with respect to the table. Hint: Let the y-axis be the vertical direction and the z-axis into-the-page, and write the y-component of equation of motion for the translation of CM and the z-component of the rotational motion of the stick about the CM.

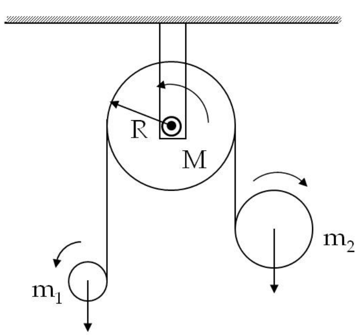


Figure 9.62: Problem 9.8.10.

Problem 9.8.10. A "massless" tape is wound on one disk of mass m_1 and radius R_1. The tape then goes over a pulley of mass M and radius R and the other end of the tape is wound on another disk of mass m_2 and radius R_2. The disks are then released from rest with tapes taut on each side. Assume the tape unwinds smoothly on each side. Determine the accelerations of each mass and the angular acceleration of the pulley.