4.3 RADIATION PRESSURE

Imagine an electromagnetic wave incident on a material. The electrons of the surface molecules are accelerated by the electric field in the wave. In addition they also experience a force from the magnetic field. Consequently, there would be a net force on the surface normal to the direction of the wave. This would result in a pressure on the material. We call the pressure by electromagnetic wave the **radiation pressure**, P. Maxwell showed that the radiation pressure P is equal to the energy density of the wave.

$$P = u. \tag{4.24}$$

Do not confuse P with power. Let's see if the units are right.

$$[u] = \text{J/m}^3 = \text{N.m/m}^3 = \text{N/m}^2 = \text{unit of pressure!}$$

Equation 4.24 gives the instantaneous pressure exerted on a perfectly absorbing surface. But, since the energy density oscillates rapidly, we are usually interested in time-averaged quantity, which can be written in terms of the intensity.

$$P_{\text{ave}} = u_{\text{ave}} = \frac{I}{c} \quad \text{(perfectly absorbing)} \tag{4.25}$$

If a surface is perfectly reflecting, then the force on the surface will be twice as great since the direction of the momentum of the incoming wave would be reversed upon reflection. Therefore the average pressure will be twice as much.

$$P_{\text{ave}} = 2\frac{I}{c} \quad \text{(perfectly reflecting)} \tag{4.26}$$

Example 4.3.1. Radiation pressure A laser light of power 3 W is spread evenly across the cross-section of the beam of diameter 2 mm. When the laser light is incident on a perfectly reflecting surface of a spherical particle of diameter 1.5 mm in vacuum, the particle is observed to be suspended in space. Find the density of the particle.

Solution. Here, the force due to the radiation pressure is able to balance the force of gravity. Let ρ be the desired density of the spherical particle and R its radius. Let P be the radiation pressure at the site of the particle. The balancing of forces yields the following relation.

$$PA_{\text{cross-section}} = (\rho V_{\text{particle}})g.$$

Here, the radiation pressure acts on the cross-section area of the particle, which is simply a circle of radius R, and the volume of the particle is just the volume of a sphere.

$$P\pi R^2 = \left(\rho \frac{4}{3}\pi R^3\right) g.$$

Hence the required density is

$$\rho = \frac{3P}{4gR}.$$

Now, we put in the numerical values given here. The values of R, g, and p are

$$\begin{aligned}
&R = 0.75 \text{ mm} = 7.5 \times 10^{-4} \text{ m} \\
&g = 9.81 \text{ m/s}^2 \\
&P = 2\frac{I}{c} = 2 \times \frac{3 \text{ W}}{\pi(1 \times 10^{-3} \text{ m})^2} \times \frac{1}{3 \times 10^8 \text{ m/s}} = 2 \times 10^{-3} \text{ Pa.}
\end{aligned}$$

Hence, $\rho = 0.2$ kg/m^3. It is interesting to compare the density found to the density of air at standard temperature and pressure, which is approximately 1.2 kg/m^3. Clearly, one will need a very powerful laser to suspend an ordinary material.