4.9 WAVE SPEED AND MEDIUM

How fast a mechanical wave travels in a medium depends on the elastic and inertial properties of the medium. If the medium has a high density then it will be more difficult to set the particles of the medium into motion, and therefore we would expect the wave to have slower speed in a more dense medium. On the other hand, if the particles are more tightly bound to each other they will tend to transfer momentum from one to the next more efficiently. Hence we expect the speed of a wave to be larger in stiffer medium.

Dimensionally we can guess the type of relation we might find among the speed of the wave, the density and the stiffness of the medium. The stiffness of a medium is reflected in the bulk modulus B, which is a measure of the fractional change in volume to an applied pressure. Bulk modulus has dimensions of pressure while the density ρ has dimensions of mass over volume. The dimensions of pressure is [Force]/[Area]. Therefore the ratio of bulk modulus to density has dimensions of the square of speed. Hence, dimensionally speaking, speed of a mechanical wave should be proportional to the square root of B/ρ .

$$\text{Dimensions:} \quad [B] \times \frac{1}{[\rho]} = \frac{[L]^2}{[T]^2}$$

That is, we can say that the speed of the wave would be given by

$$v = \alpha\sqrt{\frac{B}{\rho}}$$

where α is an undetermined dimensionless constant. Figuring out the exact relation will require more work, but the dimension analysis gives us some clues about the type of formula we might find.

In a one-dimensional wave on a taut string, the wave speed is related to the tension of the string F_T and the mass density of the string, which is equal to the mass per unit length μ. By applying Newton's second law of motion to a small segment of string it is possible to show that the wave on a taut string has the speed given as

$$v = \frac{F_T}{\mu}, \tag{4.39}$$

which is a ratio of a restorative tendency (tension) and an inertia property (mass per unit length). Similarly, for a wave equation we would obtain speed by replacing F_T by the bulk modulus B and linear density μ by the volume density ρ to obtain the wave velocity in a bulk of a medium

$$v = \frac{B}{\rho}. \tag{4.40}$$