4.3 ELECTRIC FIELD INSIDE A CONDUCTOR

Vanishing of electric field inside conductors

We saw above that when you bring an electric charge near a metal the conduction electrons in the metal respond to the electric field of the charge and move from one region to another region leading to the creation of positive and negative sides on the metal surface. The material in the volume of the metal, however, remains neutral containing a large number of conduction electrons that balance the charges of the protons in the nuclei in the volume of the material. When the movement of the electrons has stopped the force on the conduction electrons, both at the surface and in the volume, must be balanced otherwise the conduction electrons will continue to move. Therefore, we conclude that the net electric field inside metals must be zero when the static condition has reached.

$$\boxed{\vec{E}_P = 0 \quad \text{(Static; P inside conductor.)}} \tag{4.1}$$

How does the vanishing of the electric field inside the conductor occur? We can express this effect in terms of the electric fields from three sources when a metal is placed in an external electric field of some nearby charges as shown in Fig. 4.6. The polarization of the metal creates additional electric field such that the net electric field is a vector sum of the fields of $+q$, and the surface charge densities $-\sigma_A$ and $+\sigma_B$. That means, the net field will be different from what

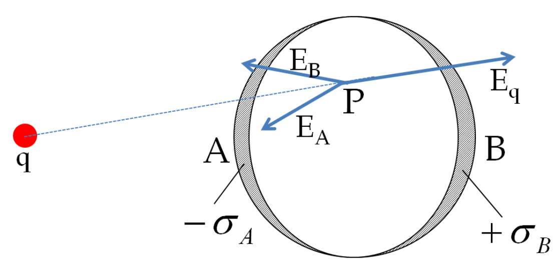

Figure 4.6: In the presence of an external charge q the charges in metal redistribute. The electric field at any point has three contributions, from $+q$, and induced charges $-\sigma_A$ and $+\sigma_B$. The redistribution of charges is such that the sum of three contribution at any point P inside the conductor is zero: $\vec{E}_{\text{P}} = \vec{E}_q + \vec{E}_A + \vec{E}_B = 0$.

it was in the absence of the metal.

$$\vec{E}_{\text{net}} = \vec{E}_{\text{of q}} + \vec{E}_{\text{of }\sigma_A} + \vec{E}_{\text{of }\sigma_B}. \tag{4.2}$$

We have asserted above that in the static condition the electric field on the conduction electrons must be zero. This means that the net electric field inside the metal must be zero.

$$\boxed{\vec{E}_{\text{net}} = 0 \quad \text{(at points inside the conductor)}} \tag{4.3}$$

Vanishing of electric field and charging

The vanishing of electric field inside a conductor also has implications on the charging of a conductor. Suppose you put some extra electrons, say a total charge $-Ne$ anywhere in a conductor. The new charges put in a conductor will disturb the balance that existed before, and the conduction electrons in the metal will rearrange themselves. One can ask the following question: after the static condition has reached again, where will be the charges? The general rule is that the charges will redistribute so that the electric field at space points that are inside the conductor will be zero when the equilibrium has reached again.

This is required for the consistency of our arguments. If there were any net charge in a volume element of the conductor, electric field in its neighborhood will not be zero as can be easily seen by either drawing the electric field lines of the net charge, or by using the Gauss's law with a Gaussian surface enclosing the volume element that has a net charge (Fig. 4.7).

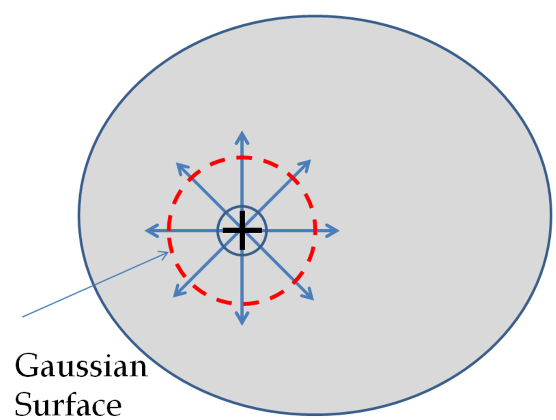

Figure 4.7: The electric field lines of a net positive charge inside a volume point of a conductor will give a net electric flux non-zero. But since, electric field is zero inside, a calculation of electric flux from electric field will be zero. These two values of electric flux contradict each other.

If the Gaussian surface encloses any net charge, the flux through it will be non-zero.

$$\oint\!\!\!\!\int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \quad \Longrightarrow \vec{E} \neq 0. \tag{4.4}$$

A non-zero electric field inside the conductor will cause the acceleration of free charges in the conductor, violating the premise that the charges are not moving inside the conductor. Hence, we conclude that any excess charge put inside an isolated conductor will end up at the boundary surface when the static condition has reached.

Electric field shielding

One can take advantage of the vanishing of electric field inside a conductor and construct a metal cage, called the Faraday cage (Fig. 4.8), to screen out electric field from entering an enclosed space. One does not even need to cover all the space; usually a metal screen is sufficient to shield from the external electric field.

Figure 4.8: A Faraday cage made of metal wire for screening out electric field.

Is the electric field inside a metal always zero?

The answer is no. The argument for the vanishing of the electric field inside a metal required static conditions on the conduction electrons. When a metal wire carries current the conduction electrons are not static. Therefore, the electric field will not be zero inside a metal in which conduction electrons are not static. Therefore, often-quoted simplistic rule that, "the electric field inside a conductor is zero," applies only to static situations. If electric field were zero in all situations, then there will be no electric current in a metal wire. We will find that the conduction of electricity requires non-zero electric field inside a conductor.

Conductors as equipotential space

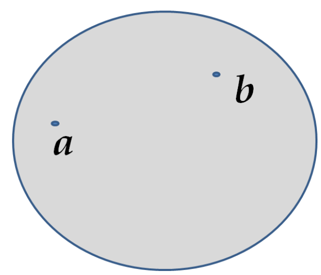

Figure 4.9: Electric potential difference between two points in a conductor is zero.

The vanishing of electric field inside a conductor also means that electric potential inside the conductor is constant as the following calculation shows (Fig. 4.9).

$$V_b - V_a = -\int_a^b \vec{E}_{in} \cdot d\vec{r} = 0 \left(\text{since} \quad \vec{E}_{in} = 0.\right)$$

$$\Longrightarrow V_b = V_a \quad \text{(for any two points in one conductor.)} \tag{4.5}$$

Therefore, a metal has the same potential everywhere when in the metal is an electrostatic equilibrium.

Implications for conductors with cavities

For a conductor with a cavity, if we put a charge +q inside the cavity, then the charge separation takes place in the conductor, with -q amount of charge on the inside surface and a +q amount of charge at the outside surface (Fig. 4.10).

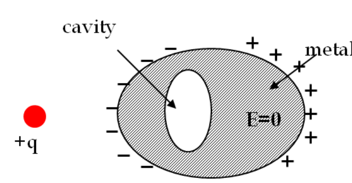

Figure 4.11: A charge outside a conductor with a cavity. The cavity remains free of charge. The polarization of charges on the conductor happens at the outer surface of the conductor.

For the same conductor with a charge +q outside it, there will be no excess charge on the inside surface; both the positive and negative induced charges reside on the outside surface (Fig. 4.11).

If there are two cavities in a conductor, with one of them having a charge +qa inside it and the other a charge -qb, the polarization of

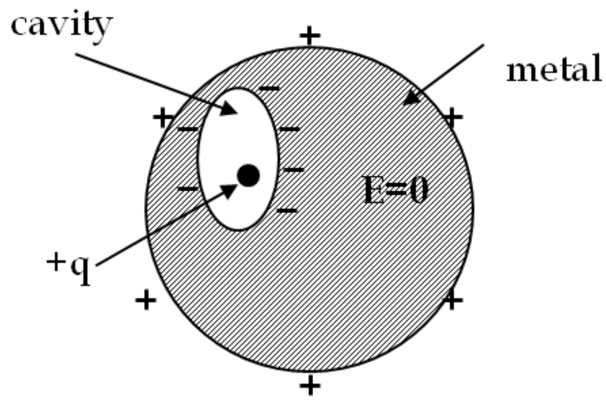

Figure 4.10: A charge inside a cavity of a metal. Charges at the outer surface do not depend on how the charges are distributed at the inner surface since E field inside the body of the metal is zero.

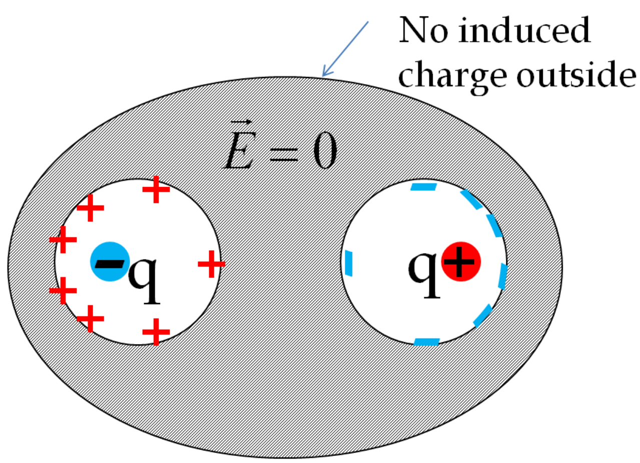

Figure 4.12: The induced charges by two equal and opposite charges in two separate cavities of a conductor. If the net charge on the cavity were non-zero, the external surface will be charged to amount of the net charge.

the conductor results in -qa on the inside surface of the cavity a, +qb on the inside surface of the cavity b, and qa-qb on the outside surface (Fig. 4.12). The charges on the surfaces may not be uniformly spread out - their spread depends upon the geometry; the only rule obeyed is that when the equilibrium has reached, the charge distribution in a conductor will be such that the electric field by the charge distribution in the conductor cancels the electric field of the external charges at all space points inside body of the conductor.

Summary: conductors in electrostatics

We now summarize special aspects of electrostatics in metals.

- A metal has a large number of conduction electrons.

- No extra charges at a point in the volume. All extra charges on the surface, either inner or external surface, depending upon whether there are charges inside cavities of the material.
- Electric field inside conductor is zero.
- A conductor has the same potential everywhere in its body. The surface of the conductor is an equipotential surface.

Example 4.3.1. Electric Field Of An Isolated Charged Metallic Sphere.

As our first example we find the electric field of an isolated metallic sphere of radius R that has a total excess charge $+q$ placed on it. The entire charge $+q$ on the metallic sphere will end up at the surface. Furthermore, as there are no other charges or bodies around, the charge distribution on the spherical surface will be spherically symmetric. Note that if there were other objects around then the charges on the surface of the sphere will not necessarily be spherically symmetric - there will be more in certain direction than in other directions. An application of Gauss's law immediately shows that the electric field at a space point P depending upon the location of P as follows.

$$\vec{E}_P = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{u}_r, & \text{if } r > R \\ 0, & \text{if } r \le R \end{cases} \tag{4.6}$$

The electric field outside the charged spherical conductor is indistinguishable from that of a point charge $+q$ placed at the center without the conductor present (Fig. 4.13).

The difference between the charged metal and a point charge occurs only at the space points inside the conductor: for a point charge placed at the center of the sphere, the electric field will not be zero at points of space occupied by the sphere, but for a conductor with the same amount of charge has a zero electric field at those points. However, there is no distinction at the outside points in space where $r > R$, and we can replace the isolated charged spherical conductor by a point charge at its center with impunity.

Example 4.3.2. Electric Potential of a Charged Metallic Sphere.

The electric potential of a charged metal sphere can be calculated from the electric field of the charged metal. To find the electric potential we will integrate the electric field from the reference point at $r = \infty$ to the field point $P(r, \theta, \phi)$ and multiply the result with minus 1. We will get two different expressions for points inside the

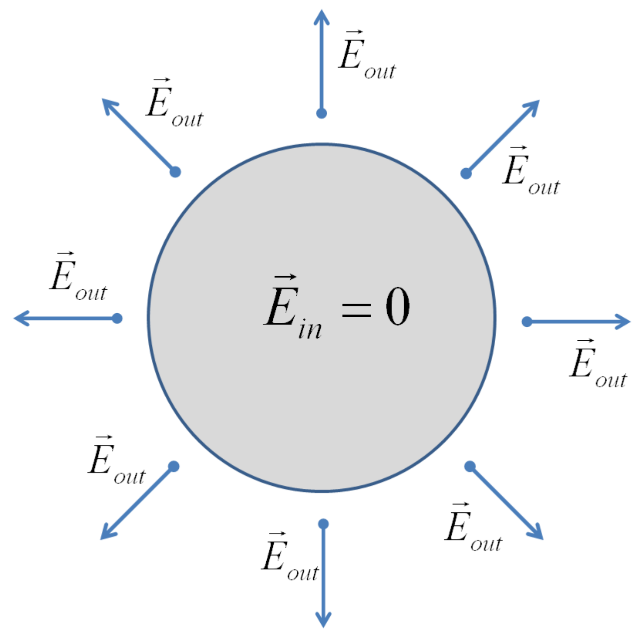

Figure 4.13: Electric field of a positively charged metal sphere. The electric field inside is zero and the electric field outside is same as the electric field of a point charge at the center although the charge on the metal sphere is at the surface.

sphere and the points outside the sphere, which we will label as V_{in} and V_{out} respectively. With $r = \infty$ as the reference it is better to work out the potential at a point outside the sphere first.

$$\begin{aligned} V_{out} &= -\int_{\infty}^{r>R} \vec{E}\cdot d\vec{r} = -\frac{q}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{1}{r^2}dr \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad (r \geq R) \end{aligned}$$

Now, to find the potential at a point inside the sphere, we need to integrate from the reference at $r = \infty$ to a point inside the sphere. The integral will break up into $r = \infty$ to $r = R$ and $r = R$ to $r < R$ since the electric field is different in the two regions. The first integral gives V_{out} evaluated at $r = R$, and we need to perform only the second integral using the electric field for $r < R$. The second integral will be zero since electric field at points inside the metal is zero.

$$V_{in} = -\int_{\infty}^{r<R} \vec{E}\cdot d\vec{r} = -\int_{\infty}^{R}\vec{E}_{out}\cdot d\vec{r} - \int_{R}^{r}\vec{E}_{in}\cdot d\vec{r}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{R}, \quad \text{(constant; independent of } r \text{ for } r \leq R.)$$

The conducting sphere has same potential throughout. A conductor is an equipotential object. We summarize our results for the electric potential function of the charged conducting sphere.

$$\vec{V}_P = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{q}{r}, & \text{if } r \geq R \\ \frac{1}{4\pi\epsilon_0}\frac{q}{R}, & \text{if } r \leq R \end{cases} \tag{4.7}$$

It is instructive to plot the potential of a charged metal sphere to get a visual sense of the potential. The plot is shown in Fig. 4.14. As you can see, the electric potential is constant inside the metal sphere but drops off as $\frac{1}{r}$ outside the metal sphere.

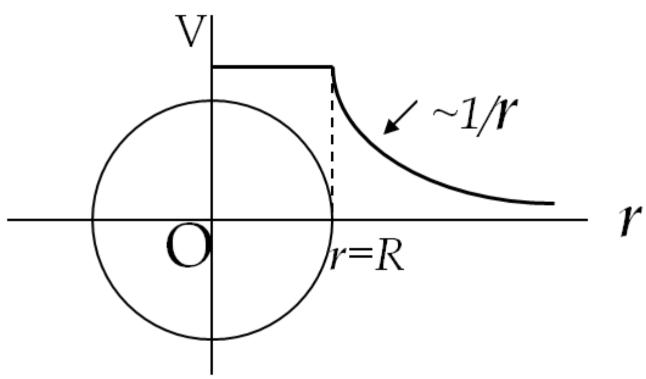

Figure 4.14: Variation of potential with distance for a charged conducting sphere.

Example 4.3.3. Electric Field of Metallic Plate.

In this example we will find the electric field of a large conducting plate containing a net charge q. Let A be area of one side of the plate and h the thickness of the plate. The charge on the metal plate will distribute mostly on the two planar sides and very little on the edges if the plate is thin. This gives two flat surfaces with the surface charge density $\sigma \approx \frac{q}{2A}$ on the two sides of the plate.

$$\sigma = \frac{\text{Charge}}{\text{Surface Area}} \approx \frac{q}{2A}.$$

The electric field is zero inside the volume of the plates. For the electric field at points outside the plates we use Gauss's law since the charges on the planar surface have an approximate planar symmetry. The Gaussian surface we will use would be a box straddling the plate with one end containing the field point P of interest as shown in the Fig. 4.15. The electric flux through the side of the Gaussian box is zero since electric field is perpendicular to the area vectors. The electric field at all points of the two ends have the same magnitude

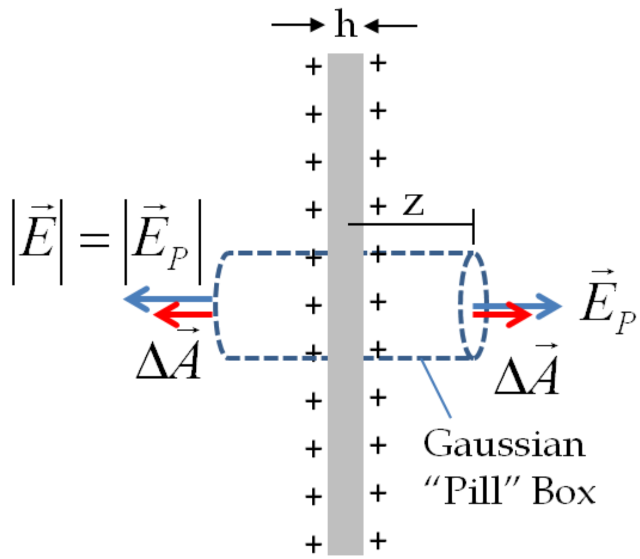

Figure 4.15: Gaussian closed surface for determining electric field at point P.

as at point P. Only the electric flux from the two ends contribute to give

$$E_P \times \Delta A + E_P \times \Delta A = q_{enc}/\epsilon_0 = \left(\frac{q}{2A} \times \Delta A\right)/\epsilon_0.$$

Canceling out the area ΔA of the ends of the Gaussian box from both sides we find that the electric field at point P has the following magnitude

$$E_P = \frac{q}{2\epsilon_0 A}.$$

The direction of the electric field at point P is along the normal to the plate and pointed away from the plate if the plate is positively charged and towards the plate if negatively charged. We can include the information about the direction if we express the electric field in terms of unit normal vector $\hat{u}_n$ from the plate.

$$\vec{E} = \frac{q}{2\epsilon_0 A}\hat{u}_n$$

Note that A is the area of one side of the plate. The total surface of the plate is approximately $2A$ if we ignore the small area at the edges.

You can also obtain this result by using the electric field of uniformly charged large sheets obtained in the last chapter by recognizing that each side of metal plate is like a uniform sheet of charge of density $\sigma = q/2A$. It was shown in the last chapter that a single sheet of charge density σ has electric field equal to $\sigma/2\epsilon_0$. As shown in Fig. 4.16 the electric fields from the two sheets add in regions I and III, and cancel in region II, i.e. inside the conductor. Thus in regions I and III,

$$\vec{E} = 2 \times \frac{(q/2A)}{2\epsilon_0}\hat{u}_n = \frac{q}{2\epsilon_0 A}\hat{u}_n$$

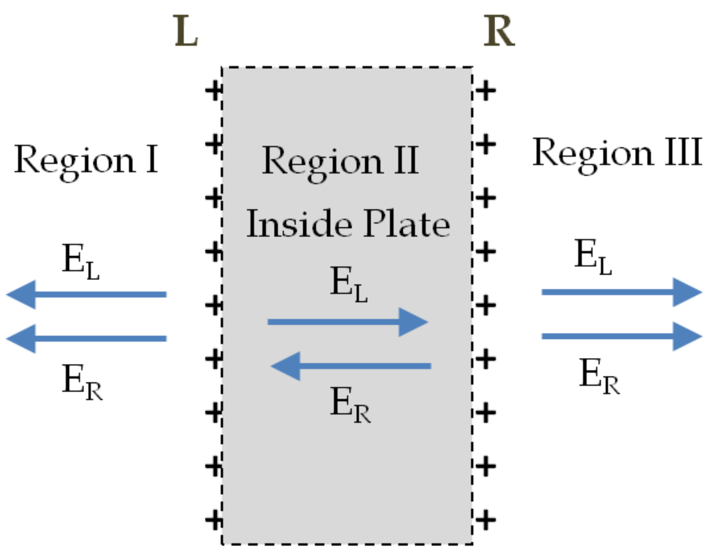

Figure 4.16: Electric field of a charged large thin conducting plate. Here "L" and "R" refer to the left and right side of the plate that is assumed to be infinite in size. Inside the plate, $\vec{E}_L$ cancels $\vec{E}_R$.