4.10 Exercises

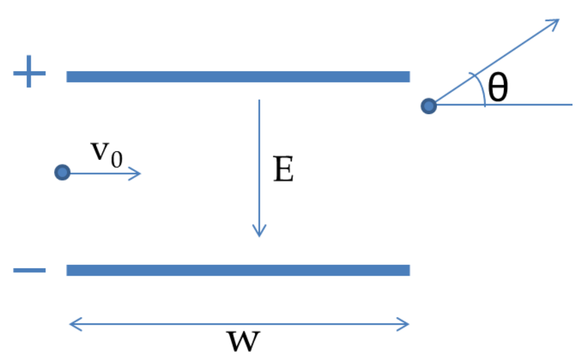
The Discovery of Electron

Ex 4.1. An electron released at a cathode is accelerated by placing a voltage of 3,000 V across the anode and cathode. (a) What will be the speed of the electron when it strikes the anode? (b) Does it matter how far apart are the cathode and anode? Why or why not?

Ex 4.2. An electron is shot in a region of space with speed 1.5×10^5 m/s where electric and magnetic field are uniform and perpendicular to each other. On the other side of the space the electron comes out with the same velocity as it enters the region. (a) If the electric field has the magnitude 3.0×10^4 N/C, what will be the magnitude of the magnetic field? (b) Suppose the region of space is near Earth, the direction of the electric field is towards the East, and the electron enters the region from above. What is the direction of the magnetic field?

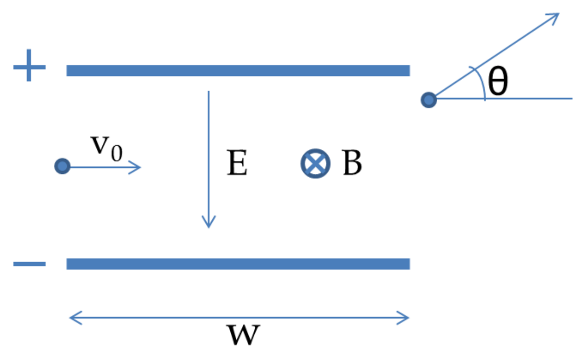
Ex 4.3. In a particular run of the Thomson's e/m experiment, an electron is accelerated by an acceleration voltage of 1,500 V. The electron is undeviated when it enters the region where there is a crossed electric and magnetic field which are perpendicular to each other and to the velocity of the particle. The electric field in the crossed field region is produced by placing 100 V across two plates that are separated by 1.5 cm distance. What must be the magnetic field in the crossed field region?

Ex 4.4. An electron with speed v_0 enters a space between two charges plates in which there is a uniform electric field of magnitude E perpendicular to the velocity of the particle. Let the width of the plates be w and the separation between plates be sufficient that the electron emerges on the other side of where it enters. At what angle to the original direction would the electron come out? [Note: no magnetic field here.]



Ex 4.5. An electron with speed v_0 enters a space between two charges plates in which there is a uniform electric field of magnitude E perpendicular to the velocity of the particle. There is also a uniform magnetic field of magnitude B perpendicular

to the electric field. Let the width of the plates be w and the separation between plates be sufficient that the electron emerges on the other side of where it enters. At what angle to the original direction would the electron come out?



The Charge of Electron

Ex 4.6. Atomic weight of an element is mass in grams of the Avogadro number of atoms of that element. (a) How many atoms are there in 11.5 grams of sodium? (b) An electric current of 2 A is passed through a sodium chloride (Na^+Cl^-) solution. How long will it take to deposit 11.5 grams of sodium on the cathode? (c) How many grams of chlorine will bubble off at the anode? Atomic weights: Na 23, Cl 35.5.

Ex 4.7. An electric current of 2 A is passed through a copper sulfate ($Cu^{+2}SO_4^{-2}$) solution. How long will it take to deposit 2.0 grams of copper on the cathode? Atomic weight: Cu 63.5.

Ex 4.8. In a Millikan oil drop experiment, one of the drops is falling steadily. The drop is found to fall 1.0 cm in 12 sec when there is no electric field. If the density of the oil is 0.956 g/cm^3 and viscosity of air is 1.81×10^{-5} kg/m.s, what is the radius of the drop?

Ex 4.9. In a Millikan oil drop experiment, one of the drops is falling steadily. The drop falls 1.0 cm in 12 sec when there is no electric field. A voltage of 5,000 V is now applied across the plates separated by 1.5 cm and the time for the drop to rise 1.0 cm is observed which is found to be 30.0 sec. If the density of the oil is 0.956 g/cm^3 and viscosity of air is 1.81×10^{-5} kg/m.s, what is the e/m of the drop?

Ex 4.10. In the original paper in Aug, 1913 issue of Physical Reviews, R. A. Millikan, reported the readings on several drops. The data on one of the drops, Drop No. 6, is given below.

Radius of drop = 2.76 μm; Distance of fall or ascent = 10.21 mm
Electric potential across plates = 5,085 V; Plate separation = 16 mm
Density of oil = 0.9199 g/cm^3; Viscosity of air = 1.824×10^{-5} kg/m.s
Averrage fall time in sec when E-field was off = 11.88 sec
Successive ascent times in sec when E-field was on:
80.708, 22.375, 140.564, 79.600, 34.762, 137.308,
34.638, 22.268, 19.704, 77.630, 42.302.

From this data deduce the charge of the elementary charge by using the assumption used by Millikan: the difference in the rise times for the same drop when electric field is on is due to the charge difference among them, which would be due to the capture of loss of integral number of electrons.

The Nuclear Atom Model

Ex 4.11. The Rutherford scattering formula says that the scattering varies as fourth power of the angle with respect to the forward direction. In a particular experiment with alpha particles of uniform speed incident on a gold foil, 1000 alpha particles arrive every minute at an angle of $\theta = 30°$. How many alpha particles per minute will be observed at (a) $\theta = 45°$, (b) $\theta = 60°$, (c) $\theta = 90°$?

Ex 4.12. The Rutherford scattering formula says that the scattering varies as square of positive charge on the nucleus. Suppose same alpha particle source is incident on aluminum, copper and gold plates and the detector is set at 30° to the forward direction. With aluminum foil 100 alpha particles per minute are observed in the detector. How many will be observed when the target is (a) copper, (b) gold?

Ex 4.13. The Rutherford scattering formula says that the scattering varies as inverse of the square of kinetic energy of alpha particles. In a particular experiment, alpha particles of speed 5×10^6 m/s are incident on a thin gold foil and the detector is set at 10° to the forward direction. The detector detects 1000 alpha particles per minute. How many alpha particles will be detected if the speed of the alpha particles are doubled?

Ex 4.14. The Rutherford scattering formula says that the scattering varies as inverse of the square of kinetic energy of alpha particles. In a particular experiment, alpha particles of speed 5×10^6 m/s are incident on a thin gold foil and the detector is set at 10° to the forward direction. The detector detects 1000 alpha particles per minute. How many alpha particles will be detected if the speed of the alpha particles are doubled?

Ex 4.15. The nucleus of copper has 29 protons and has radius 4.8×10^{-15} m. (a) What must be the speed of the alpha particles if it is to penetrate the nucleus? (b)

If, instead of alpha particle, you shoot energetic protons, what must be speed of the protons at which point the proton will penetrate the nucleus?

Ex 4.16. (a) Polonium is a source of alpha particles of energy around 5 .5 MeV. What is the speed of the alpha particles? (b) Beryllium nucleus has a loosely bound neutron that gives it an unusually large size with radius around 7 fm. Determine if 5.5 MeV alpha particles can penetrate the beryllium nucleus.

Spectroscopy of Hydrogen Atom and the Bohr Model

Ex 4.17. Find the wavelength of light emitted in the Balmer series of hydrogen spectrum when (a) $n = 3$, (b) $n = 4$.

Ex 4.18. There a series, called Lyman series, in the hydrogen spectrum that is associated with transition from $n > 1$ to $n = 1$ state. Find the wavelength of light emitted in the Lyman series of hydrogen spectrum when (a) $n = 1$, (b) $n = 2$, (c) $n = \infty$.

Ex 4.19. If Avogadro number of atoms were arranged in a straight line such that they were separated by 10 Bohr radii, how long will that line will be?

Ex 4.20. In a hydrogen atom spectrum which transitions correspond to the following wavelengths, (a) 486 nm, (b) 433nm, (c) 121 nm, (d) 93.7 nm?

Ex 4.21. Removing one electron from a helium atom creates the He^{+1} ion which is like a hydrogen atom with two protons and two neutrons in the nucleus. (a) Show that the Bohr atom applied to the electron in He^{+1} ion gives the following energy levels of this ion.

$$E_n = -\frac{E_1}{n^2},$$

with

$$E_1 = \frac{mZ^2e^4}{24\pi^2\epsilon_0^2\hbar^2} \quad (Z = 2).$$

(b) What is the ionization energy of the He^{+1} ion? (no need to calculate, you can appropriately modify the ionization energy of H-atom to get the answer. (c) What is the radius of the smallest orbit of the electron in the He^{+1} ion?

Ex 4.22. (a) What is the ionization energy of the Li^{+2} ion? (no need to calculate, you can appropriately modify the ionization energy of H-atom to get the answer. (b) What is the radius of the smallest orbit of the electron in the Li^{+2} ion?

Ex 4.23. A muon is a particle which is similar to an electron, just 207 times heavier. Suppose you replace the electron in a hydrogen atom by a muon and treated the muon/proton atom by the Bohr model. (a) What will be the radius of the smallest orbit? (b) What will be the four lowest energy levels of this atom?

Hydrogen Atom According to Schrödinger's Equation

Ex 4.24. The electron in a hydroen atom is in a quantum state with principal quantum number $n = 4$. (a) What are the possible values of the orbital angular momentum l? (b) For each possible values of l, what are the possible values of the magnetic quantum number m_l? (c) How many quantum states of hydrogen atom are possible that have $n = 4$ as their principal quantum number [ignoring the spin for now]?

Ex 4.25. (a)An electron is in s subshell of the K shell. What are its n and l? (b) An electron has $n = 3$ and $l = 2$, what are the spectroscopic letter designations of the shell and subshell for these values?

Ex 4.26. The electron in the hydrogen atom is in the 1s state. (a) What is the probability that the electron is located within $a_0/4$ of the nucleus? (b) What is the probability that the electron is outside a distance $r = 4a_0$ from the nucleus? Here a_0 is the Bohr radius.

Ex 4.27. The electron in the hydrogen atom is in the state $n = 2, l = 1, m_l = 0$. (a) What is the probability that the electron is located within a_0 of the nucleus? (b) What is the probability that the electron is outside a distance $r = 2a_0$ from the nucleus? Here a_0 is the Bohr radius.

Ex 4.28. The electron in the hydrogen atom is in the state $n = 2, l = 1, m_l = 0$. (a) What is the probability that the electron is located in the direction $\theta \leq 45°$ of the z-axis? (b) What is the probability that the electron is located in the direction $\theta \leq 45°$ of the xy-plane?

Ex 4.29. The electron in the hydrogen atom is in the state $n = 2, l = 1, m_l = +1$. (a) What is the probability that the electron is located within a_0 of the nucleus? (b) What is the probability that the electron is outside a distance $r = 2a_0$ from the nucleus? Here a_0 is the Bohr radius.

Ex 4.30. The electron in the hydrogen atom is in the state $n = 2, l = 1, m_l = +1$. (a) What is the probability that the electron is located in the direction $\theta \leq 45°$ of the z-axis? (b) What is the probability that the electron is located in the direction $\theta \leq 45°$ of the xy-plane?

Ex 4.31. An electron makes a transition from 3p state to 2s state. What will be the energy and wavelength of the photon emitted?

Ex 4.32. An electron makes a transition from 6d state to 5p state. What will be the energy and wavelength of the photon emitted?

Angular Momentum and Magnetic Dipole Moments

Ex 4.33. A ring of uniform charge density λ [Coulomb/meter] is rotated at angular speed ω about an axis perpendicular to the ring and passing through the center of the ring. This causes the charges in the ring to go in a circle of radius R. What will be the magnetic dipole of the rotating ring of charges?

Ex 4.34. A disk of uniform charge density σ [Coulomb/meter^2] is rotated at angular speed ω about an axis perpendicular to the ring and passing through the center of the disk. This causes the charges in the disk to go in circles. Let the radius of the disk be R. What will be the magnetic dipole of the rotating disk of charges?

Ex 4.35. The electron in a hydrogen atom is in $n = 2$ shell. (a) What are the possible l for the electron? (b) For each allowed l, list the allowed m_l values. Note $m_l = 0$ will appear twice. (c) For each l, answer: (i) what is the magnitude of the angular momentum for that l, and (ii) what are the values of that the projections of the angular momentum vector can have along the z-axis? (d) Find the directions in space which correspond to the allowed values of the projections of the angular momentum vector and draw the angular momentum vectors in space.

Ex 4.36. The electron in a hydrogen atom is in $n = 4$ shell. (a) What are the possible l for the electron? (b) For each allowed l, list the allowed m_l values. Note $m_l = 0$ will appear four times. (c) For each l, answer: (i) what is the magnitude of the angular momentum for that l, and (ii) what are the values of that the projections of the angular momentum vector can have along the z-axis? (d) Find the directions in space which correspond to the allowed values of the projections of the angular momentum vector and draw the angular momentum vectors in space.

Ex 4.37. (a) The electron in an excited hydrogen atom is in the 2p subshell. Show that there are 6 quantum states that the electron could be in. (b) Find the magnetic dipole moments of each state.

Ex 4.38. (a) List all the quantum states for a hydrogen atom in 3d subshell by listing the quantum numbers for each state. (b) Find the magnetic dipole moments of each state.

Ex 4.39. Suppose the magnetic field between the poles of the magnet in the Stern-Gerlach apparatus is pointed towards the z-axis and varies with z coordinate linearly.

$$B_z(x, y, z) = B_0 + \alpha\, z.$$

A beam of silver atoms in $l = 0$ state and speed v is sent through the space between the poles along the x-axis. Each silver atom will experience a force due to the spin magnetic dipole moment. (a) If the force acts on the beams over a horizontal distance of d before exiting the magnetic field region, what will be the deflection along the z-axis of the atoms with spin $\uparrow$? (b) What will be the deflection along the z-axis of spin $\downarrow$ atoms? (c) If we seek a separation of 5 mm for a beam of speed 120 m/s, what should be the value of α?

Ex 4.40. (a) The ground state of a sodium atom is an $l = 0$ state with one electron whose spin is not paired up with any other electron. This gives sodium atom a spin $\frac{1}{2}$. What will be energy difference between the spin $\uparrow$ and spin $\downarrow$ states of a sodium atom placed in a magnetic field of strength 2 T? (b) What are the energy and wavelength of the photon you would need to excite the lower of these states to the upper state?

Ex 4.41. A hydrogen atom in $l = 1$ state is placed in a magnetic field of magnitude 1.5 T. (a) Find the magnetic energy of the electron in various m_l states. [Ignore spin.] (b) Find the Larmor frequency of the electron. [Ignore spin.]

Ex 4.42. A hydrogen atom in $l = 1$ state is placed in a magnetic field of magnitude 1.5 T. Now, do not ignore spin. Find the magnetic energy of the electron in various (m_l, m_s) states.

Ex 4.43. A hydrogen atom in $l = 2$ state is placed in a magnetic field of magnitude 1.5 T. (a) Find the magnetic energy of the electron in various m_l states. [Ignore spin.] (b) Find the Larmor frequency of the electron. [Ignore spin.]

Ex 4.44. A hydrogen atom in $l = 2$ state is placed in a magnetic field of magnitude 1.5 T. Now, do not ignore spin. Find the magnetic energy of the electron in various (m_l, m_s) states.

Pauli's Exclusion Principle

Ex 4.45. The quantum states of a particle in a one-dimensional box of size a are designated by the principal quantum number n and the spin quantum number m_s, which can be $+\frac{1}{2}$ or $-\frac{1}{2}$. The energy of a state with a particular value of $n = 1, 2, 3, \cdots$ is given by

$$E_n = n^2 \frac{\hbar^2}{2ma},$$

where m is the mass of the particle. Suppose there are five particles in the box and they do not interact with each other except for the Pauli's exclusion principle. (a) Write the electronic configuration of the particles in the box in the lowest energy state. Your answer will state the n and m_s values of each particle. (b) What is the energy of the ground state? (c) What are the configuration and the energy of the first excited state?

Ex 4.46. The quantum states of a particle in a one-dimensional box of size a are designated by the principal quantum number n and the spin quantum number m_s, which can be $+\frac{1}{2}$ or $-\frac{1}{2}$. The energy of a state with a particular value of $n = 1, 2, 3, \cdots$ is given by

$$E_n = n^2 \frac{\hbar^2}{2ma},$$

where m is the mass of the particle. Suppose there are six particles in the box and they do not interact with each other except for the Pauli's exclusion principle. (a) Write the electronic configuration of the particles in the box in the lowest energy state. Your answer will state the n and m_s values of each particle. (b) What is the energy of the ground state? (c) What are the configuration and the energy of the first excited state?

Ex 4.47. The quantum states of a particle in a two-dimensional box of size $a \times a$ are designated by the principal quantum numbers n_1 and n_2 and the spin quantum number m_s, which can be $+\frac{1}{2}$ or $-\frac{1}{2}$. The energy of a state with particular values of $n_1 = 1, 2, 3, \cdots$ and $n_2 = 1, 2, 3, \cdots$ is given by

$$E_n = \left(n_1^2 + n_2^2\right) \frac{\hbar^2}{2ma},$$

where m is the mass of the particle. Suppose there are nine particles in the box and they do not interact with each other except for the Pauli's exclusion principle. (a) Write the electronic configuration of the particles in the box in the lowest energy state. Your answer will state the n_1, n_2, and m_s values of each particle. (b) What is the energy of the ground state? (c) What are the configuration and the energy of the first excited state?

Ex 4.48. The quantum states of a particle in a two-dimensional box of size $a \times a$ are designated by the principal quantum numbers n_1 and n_2 and the spin quantum number m_s, which can be $+\frac{1}{2}$ or $-\frac{1}{2}$. The energy of a state with particular values of $n_1 = 1, 2, 3, \cdots$ and $n_2 = 1, 2, 3, \cdots$ is given by

$$E_n = \left(n_1^2 + n_2^2\right) \frac{\hbar^2}{2ma},$$

where m is the mass of the particle. Suppose there are ten particles in the box and they do not interact with each other except for the Pauli's exclusion principle. (a) Write the electronic configuration of the particles in the box in the lowest energy state. Your answer will state the n_1, n_2, and m_s values of each particle. (b) What is the energy of the ground state? (c) What are the configuration and the energy of the first excited state?