5.3 INTERFEROMETERS

Interference of light has been used to build instruments called interferometers that allow extremely precise and sensitive measurements of length, wavelength, and refractive index. There are basically two types of interferometers - two-beam and multiple-beam interferometers. In a two-beam interferometers, e.g. Michelson interferometer and Mach-Zehnder interferometer a coherent source of light, such as the light from a laser beam, is split up into two parts by an optical device such as a partially silvered mirror, and the two partial beams are brought together after traveling in different paths. In a multiple-beam interferometers, such as the Faby-Perot interferometer, a coherent source of light undergoes multiple partial reflections at the two sides of a medium and the partially transmitted beams from multiple refractions are made to interfere. Below you will study in detail the workings of the two most common interferometers.

5.3.1 Michelson Interferometer

In a Michelson interferometer shown in Fig. 5.10, light from an extended source first splits into two beams by a half-silvered beam splitter at 45-degrees to the ray. The two resulting waves travel perpendicular to each other, and reflect off mirrors M_1 and M_2. The reflected rays recombine resulting in interference which can be either viewed by naked eye or projected using a converging lens on a screen.

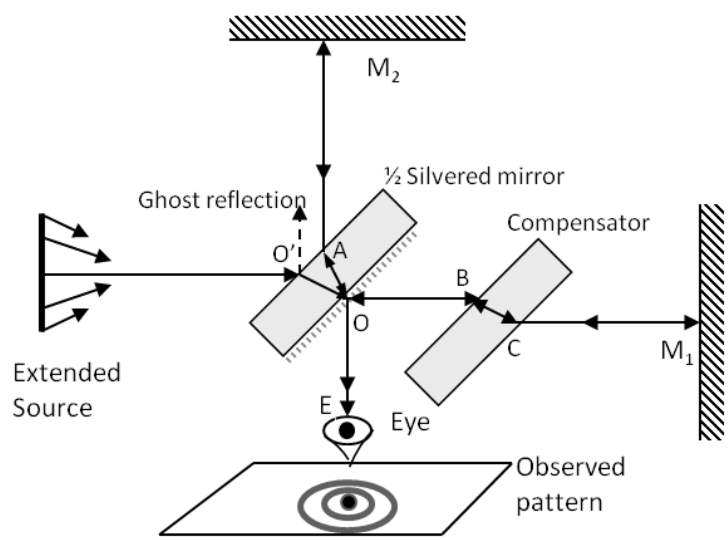

Figure 5.10: Michelson's interferometer.

To compare the phases of the rays in the two arms we start from a point, say O$'$, when the two were together and go to a point, say E, when they come together again. Therefore we will calculate the phase difference between the paths $\mathrm{O'OBCM_1CBOE} \equiv \mathrm{OM_1E}$ and $\mathrm{O'OAM_2AOE} \equiv \mathrm{OM_2E}$.

The optical path lengths O$'$O is common to the two paths. The compensator is inserted in the path of the ray $\mathrm{OM_1O}$ so that the optical path lengths(OPL) for BC+ CB for the path on arm 1 is equal to the OPL for OA+AO for the path on arm 2. Therefore, the net optical path length difference between the two arms comes from the distances to the mirrors and the phase shift when reflecting off of a higher refractive index material.

The ray $\mathrm{O'M_2E}$ has one reflection at $\mathrm{M_2}$ that changes phase by π radians, while ray $\mathrm{O'M_1E}$ has two such reflections, one at $\mathrm{M_1}$ and the other at O. Therefore, the condition of destructive interference will result if the phase difference is 0, 2π, 4π, etc, because the difference in phase flip due to reflections make the phases of the waves in the two arms off by π radians already.

$$2\times|\mathrm{OB} + \mathrm{CM_1} - \mathrm{AM_2}|\times\frac{2\pi}{\lambda_0} = m'\times 2\pi,\ \ m' = 0,\ 1,\ 2,\ \cdots\ \ \text{(Destructive)}. \tag{5.42}$$

where λ_0 is wavelength in air. Simplifying this equation, we find the following.

$$|\mathrm{OB} + \mathrm{CM_1} - \mathrm{AM_2}| = m'\frac{\lambda_0}{2},\ \ \ m' = 0,\ 1,\ 2,\ \cdots\ \ \ \text{(Destructive)}. \tag{5.43}$$

But, this only gives the condition for a ray coming horizontally from the source. How do we find the interference conditions for rays at other angles? To address this question, notice that when you look at the incoming rays from the beam splitter, you are seeing reflections of the source in mirrors $\mathrm{M_1}$ and $\mathrm{M_2}$. We can redraw a conceptual diagram of the Michelson's interferometer, where we display the image of the source in the mirrors, and how light from one point on the source must reflect to reach the back of the eye after converging from the eye lens (Fig. 5.11).

In the figure, plane $\mathrm{Q_1}$ is the image of plane P in mirror $\mathrm{M_1}$ and plane $\mathrm{Q_2}$ is the image in mirror $\mathrm{M_2}$. Points $\mathrm{Q_1}$ and $\mathrm{Q_2}$ are image of point P. To the eye, rays appear to come from the points $\mathrm{Q_1}$ and $\mathrm{Q_2}$. The interference of rays from $\mathrm{Q_1}$ and $\mathrm{Q_2}$ is observed by the eye. Rays from all points P on the circle about the symmetry axis shown in the

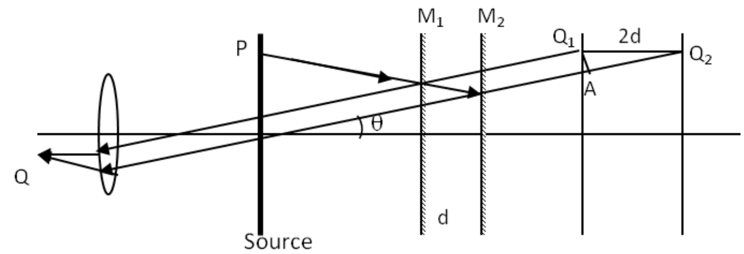

Figure 5.11: Effective geometry of interference of waves in a Michelson's interferometer.

figure will meet on a circle that has point Q. Rays inclined at the same angle will either interfere constructively or destructively depending upon the difference in path which is AQ_2. We have already discussed above the effects of reflection in the two rays. If the distance AQ_2 is an integral multiple of $\lambda_0/2$, then we would have a destructive interference.

$$|AQ_2| = m'\frac{\lambda_0}{2}, \quad m' = 0,\ 1,\ 2,\ \cdots \quad \text{(Destructive)}. \tag{5.44}$$

If the distance between the mirrors is d, and the angle the rays make is θ, then the condition for destructive interference is as follows.

$$2d\cos\theta = m'\frac{\lambda_0}{2}, \quad m' = 0,\ 1,\ 2,\ \cdots \quad \text{(Destructive)}. \tag{5.45}$$

This equation gives us the condition for destructive interference at points of a circle on the retina or screen that contains point Q. Since, the view of an observer is limited, one can only see circles made on the retina by rays that are inclined up to a maximum angle. Various m values in Eq. 5.45 refer to the corresponding angles of inclination for dark circles. Thus $m = 0$ interference is at the center. This is followed by a bright circle. The $m = 1$ destructive interference happens at the following angle of inclination.

$$m = 1: \quad \cos\theta = \frac{\lambda_0}{4d}. \tag{5.46}$$

Suppose you start with the distance to the mirror M_2 greater than to M_1 by some distance d, and move mirror M_2 so that d decreases. That will cause $cos\theta$ to increase, which will imply that the condition for darkness is closer to the horizontal direction than before. This makes $m = 1$ circle on the screen smaller. As d gets smaller the circle for $m = 1$ also gets smaller, and when you have moved by a distance equal to $\lambda_0/2$, the $m = 1$ circle disappears. All other fringes move correspondingly. The circle in space which used to be occupied by

$m = 1$, is now occupied by $m = 2$, and the circle originally occupied by $m = 2$ will now be occupied by $m = 3$, and so on. Hence, if you focus on a particular place on the screen, a change in d can be found by observing the number of fringes that pass, and multiplying that number by half the wavelength.

$$\Delta d = N\frac{\lambda_0}{2} \tag{5.47}$$

This equation is often used for making precise measurements of sub-micrometer distances from a known source of light and number of fringes that pass the detector.

Example 5.3.1. Precise distance measurements by Michelson interferometer

A red laser light of wavelength 630 nm is used in a Michelson interferometer. While keeping the mirror M_1 fixed, as mirror M_2 is moved. The fringes are found to move past a fixed hair cross in the viewer. Find the distance the mirror M_2 is moved for a single fringe to move past the reference line.

Solution. We use the result of the Michelson interferometer interference condition. For a 630 nm red laser light, then for each fringe crossing ($N = 1$), the distance traveled by M_2 if you keep M_1 fixed will be:

$$\Delta d = 1 \times \frac{630 \text{ nm}}{2} = 315 \text{ nm} = 0.315\ \mu\text{m}.$$

5.3.2 Fabry-Perot Interferometer

In discussing the double-beam reflection off a planar dielectric plate, we had ignored the effects of multiple reflections. In 1899 Marie Fabry and Jean Perot in France built an interferometer which used multiple beams by coating the two sides partially with a highly reflecting material. They found that multiple reflections resulted in much narrower interference fringes than other interferometers. Due to the narrow fringes Fabry-Perot interferometer has a much higher resolving power. It is widely used for precision spectroscopy, and as a gas laser cavity, among other things.

The Fabry-Perot interferometer shown in Fig. 5.12 essentially consists of two parallel plates of glass whose inner surfaces are polished to a very high degree of flatness, and then coated with a highly reflecting thin film of silver or aluminum or some similarly good reflector. The outer surface is ground to make a wedge shape so that light escaping the cavity between the plates does not reflect back in. The distance d between plates can vary from a few millimeters to a

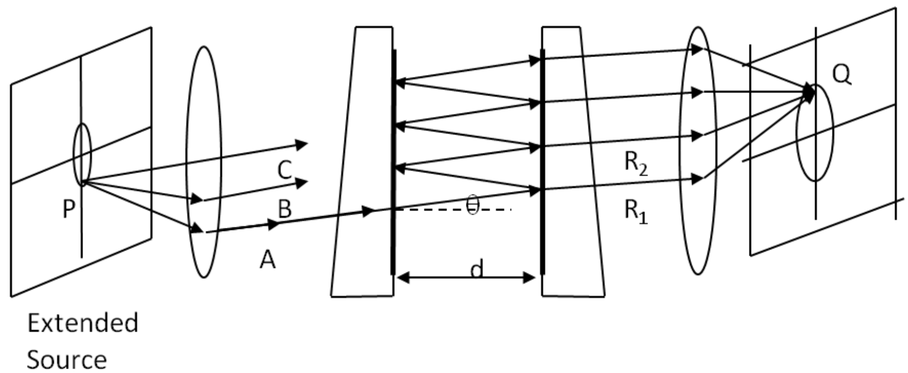

Figure 5.12: Fabry Perot interferometer.

few centimeters. Rays from a broad source enter the space between the reflecting plates, and after multiple reflections emerge on both sides. The rays emerging on the opposite end to the side of entrance are focused on a screen.

An extended light source, such as a mercury lamp is placed at the focal plane of a converging lens. Rays coming from a single point emerge parallel on the other side of the lens at a particular angle of inclination. The transmitted rays from rays of same angle of inclination enter the space between the two partial reflectors. focus on the same point. For instance, the three rays A, B and C from the source point P shown in the figure converge at the same point Q on the screen. The rays from points on the light source in a circle that contains P meet in a circle on the screen. If the condition at Q is met for a constructive interference, we will see a bright ring, while if the condition there corresponds to destructive interference, there will be a dark ring there. The interference pattern seen have alternating bright and dark rings. The reflectivity r of the coating can be adjusted to sharpen the rings as illustrated in Fig. 5.13.

Let λ denote the wavelength of light in the medium between the reflecting plates, and n the refractive index of the medium. Each time a ray between the plates is incident on the reflecting surface, it is partially reflected and partially transmitted. Let r denote reflection coefficient of the reflecting surface. Reflection coefficient tells us the factor by which the reflected amplitude of the electric field decreases when reflecting off a surface. Thus, if $r = 0.9$, the reflected electric

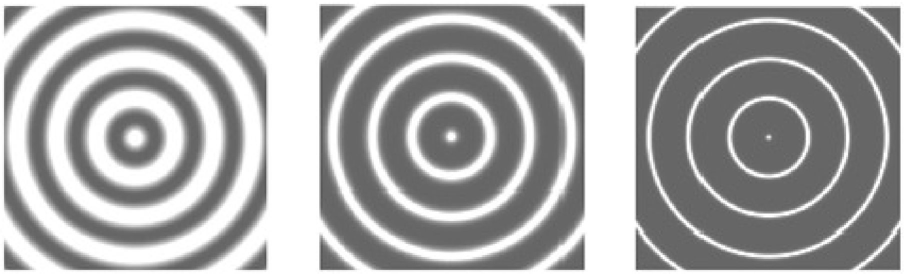

Figure 5.13: Calculated fringes for different reflectivities in a Fabry Perot interferometer: (a) $r = 0.3$, (b) $r = 0.85$, (c) $r = 0.98$.

field is 0.9 times the incident electric field. We will also find use for the reflectance R defined as $R = r^2$, which is the ratio of the intensity of the reflected wave to the intensity of the incident wave after a single reflection. A calculation (usually worked out in a more advanced textbook on optics) of intensity of light at an arbitrary point Q on the screen yields the following result.

$$\frac{I_t}{I_i} = \frac{1}{1 + F\sin^2(\Delta_{12}/2)}, \tag{5.48}$$

where I_i is the intensity of the source, F the **finesse coefficient** or **quality factor** which is related to the reflectance R by

$$F = \frac{4R}{(1-R)^2} \tag{5.49}$$

and Δ_{12} is the phase difference between the optical paths of two successive transmitted waves shown as rays R_1 and R_2 in Fig. 5.12. We redraw the figure with only two successive rays in Fig. 5.14. The phase difference in terms of angle θ, the distance d between the plates, and the wavelength of light λ between the plates is found to be

$$\Delta_{12} = \frac{2\pi}{\lambda}\, 2nd\cos\theta. \tag{5.50}$$

From Eq. 5.48 we find that transmitted intensity is a maximum when $\sin^2(\Delta_{12}/2) = 0$. That means constructive interference will takes place if $\Delta_{12} = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \cdots$.

$$2nd\cos\theta = m\lambda \quad \text{(Constructive)}. \tag{5.51}$$

where m is called the interference order and takes on values $= 0, \pm 1$ $, \pm 1, \pm 1, \cdots$. The function I_t/I_i of quantity Δ_{12} is **Airy's formula**.

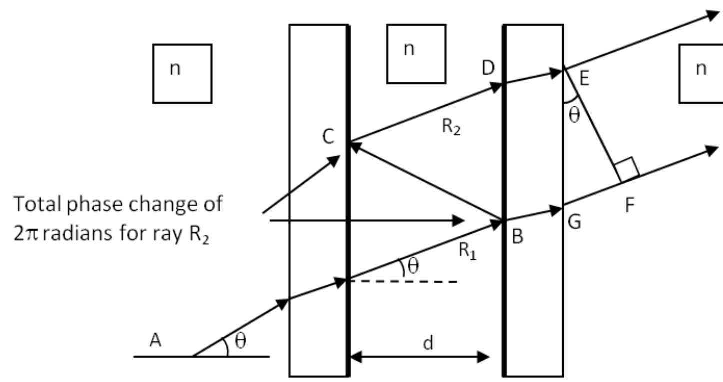

Figure 5.14: Geometry of Fabry-Perot for phase difference calculation between the successively transmitted wave. The phase difference comes from path difference of BCD and GF.

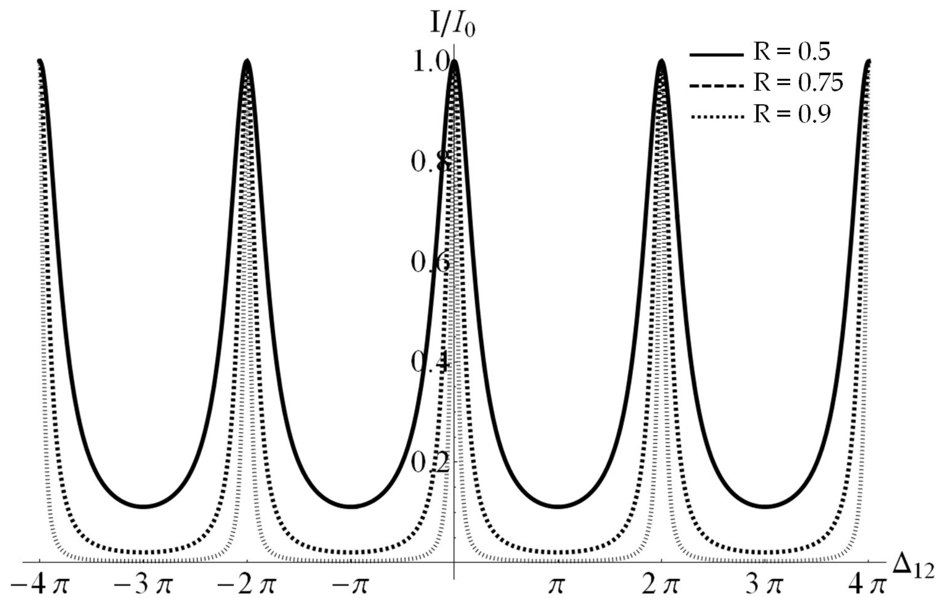

Figure 5.15: Sharpness of peaks increases with reflectance R.

To give you an idea of the sensitivity of the interference pattern with reflectance R and hence finesse coefficient F, we plot I_t/I_i versus Δ_{12} for a number of R values in Fig. 5.15. You can see that fringes become sharper as the reflectivity R, or equivalently finesse coefficient F of the plates increase.

The width of a fringe at half-height provides a standard way of describing the sharpness of the fringes. If the peak width is denoted by symbol ϵ, then, since phase Δ_{12} takes values $2m\pi$ at the peak maxima, the intensity will have half the maximum at following values.

$$\Delta_{12} = 2m\pi \pm \frac{\epsilon}{2} \tag{5.52}$$

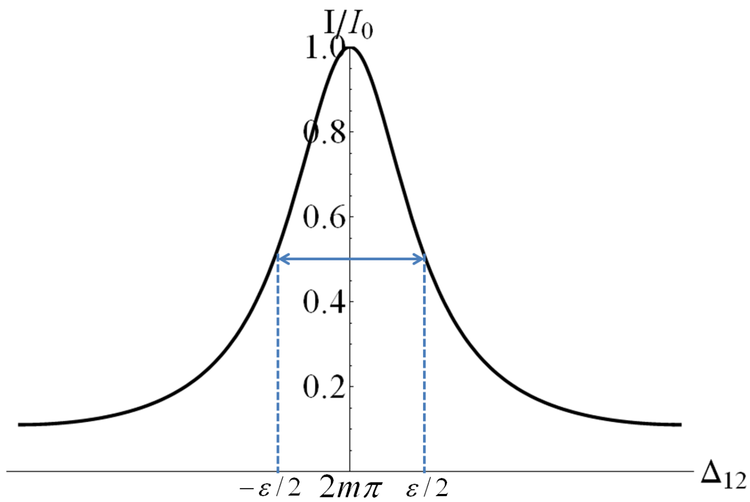

Figure 5.16: Defining the half-width of a peak.

Therefore at half-height we obtain the following relation by setting the intensity to half the value at the maximum and Δ_{12} as given above.

$$\frac{1}{2} = \frac{1}{1 + F\sin^2\left(m\pi \pm \frac{\epsilon}{4}\right)}, \tag{5.53}$$

One can solve this equation to obtain the width ϵ at half height. The sharp peaks will have very small ϵ. In that case you can show by a simple algebra that

$$\epsilon = \frac{4}{\sqrt{F}}. \tag{5.54}$$

It is customary to define a quantity called finesse [not to be confused with the finesse coefficient F defined above] which is useful for discussing resolution of peaks.

$$\text{finesse} = \frac{\text{Separation of adjacent peaks}}{\text{Width at half height}} \tag{5.55}$$

Here the separation of adjacent peaks is 2π radians. Therefore finesse is found to be related to be related to the finesse coefficient F as follows.

$$\text{finesse} = \frac{2\pi}{\epsilon} = \frac{\pi\sqrt{F}}{2}. \tag{5.56}$$

The interference condition in the Fabry-Perot interferometer depends upon three physically controllable parameters - the refractive index n of the medium between the plates, the separation d of the plates, and the wavelength λ of light. Depending upon our particular purpose for using a Fabry-Perot interferometer, we may vary one of these parameters and study the change on the screen. When plates are

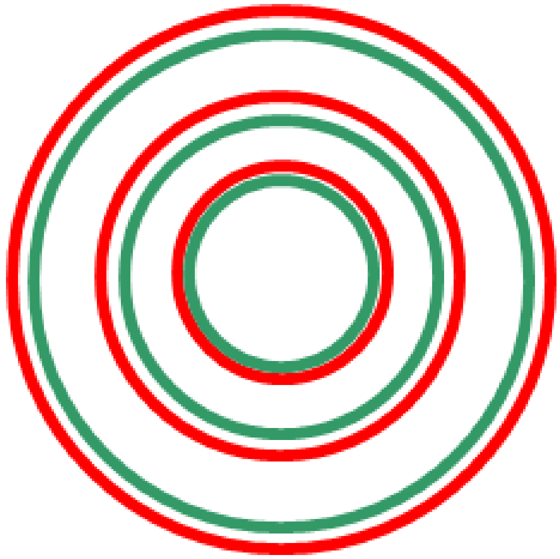

Figure 5.17: The inner ring in each order corresponds to the smaller wavelength.

Visit the following website for the picture:

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/Na.html#Na

Figure 5.18: Separation of sodium yellow doublet by a Fabry-Perot interferometer.

fixed to a definite separation d, the arrangement is also called an **etalon**. For fixed n and d, the interference will depend upon the wavelength. So, if light source consists of two wavelengths, say a red light of 650 nm and a green light of 530 nm, two separate rings for each interference order m are seen (Fig. 5.17).

Since the interference lines in a Fabry-Perot interferometer are very thin, they are easier to resolve than is the case with other interferometers. A celebrated case is the separation of the sodium yellow doublet of wavelengths 588.9950 nm and 589.5924 nm which are shown separated in Fig. 5.18.

5.3.3 Resolving Power

The ability of an interferometer to produce an interference pattern where lights of wavelengths present in the incoming beam are well-separated is important to an experimenter interested in spectroscopy. If the peaks corresponding to two different wavelengths overlap, then they may not be individually discernible depending upon the amount of overlap and that would limit the use of the interferometer. A measure of the resolving power of a spectroscope, called the chromatic resolving power, is defined using the smallest difference in the wavelength $\Delta\lambda_{\text{min}}$ that can be resolved about a wavelength λ.

$$\text{Resolving Power} = \frac{\lambda}{\Delta\lambda_{\text{min}}} \tag{5.57}$$

You may wonder, what is the smallest difference between the two wavelengths so that you can still tell that there are two peaks. The answer is provided by an empirical rule, called the **Raleigh criterion**, which states that the fringes are just resolvable if the peak (max) of one wave is at the same place as the trough (min) of the other. The Raleigh criterion works quite well as a rule of thumb. Using the Raleigh criterion it is possible to show that the resolving power of Fabry-Perot interferometer is related to the finesse coefficient F and is approximately given by the following formula.

$$\text{Resolving Power} = \frac{\lambda}{\Delta\lambda_{\text{min}}} = \pi\sqrt{F}\frac{nd}{\lambda} \tag{5.58}$$

In the next chapter you will study another device called the diffraction gratings which are also used for the separation of light of different wavelengths. It turns out that it is much harder to construct a diffraction grating with as high a resolving power as a Fabry-Perot interferometer. For instance, let $n = 1$, $d = 1$ cm, $\lambda = 650$ nm, and reflectance $R = 0.98$, then the resolving power would be 4.8×10^6. To get this kind of resolving power from a diffraction grating (which will be discussed in the next chapter) that has slit separation of 1000 nm, you will need diffraction from 1,560,000 slits, or 1.56 meter wide diffraction grating!

Example 5.3.2. Separating Sodium yellow D-lines

Sodium yellow D-lines have wavelengths 589.0 nm and 589.6 nm. A spectrometer successfully separates the two wavelengths. What can you say about the resolving power of the spectrometer?

Solution. We use the resolving power definition to figure out the minimum power.

$$\text{Resolving Power} = \frac{\lambda}{\Delta\lambda_{\text{min}}} = \frac{589.3\text{ nm}}{0.6\text{ nm}} = 982, \tag{5.59}$$

where we used the average for the numerator.