1.3 UNCERTAINTY IN MEASUREMENTS

1.3.1 Uncertainty and Precision

Recall that the time measured by the NIST-F1 atomic clock is highly precise, the uncertainly being only 3×10^{-16} sec in 1 sec. Even though the NIST atomic clock is highly precise, it is not 100% precise. It is a fact of every measuring device that no measurement is 100% precise. The uncertainties in measurements results from either the difficulties in characterizing the sample perfectly or in the precision of the measuring instruments, or both.

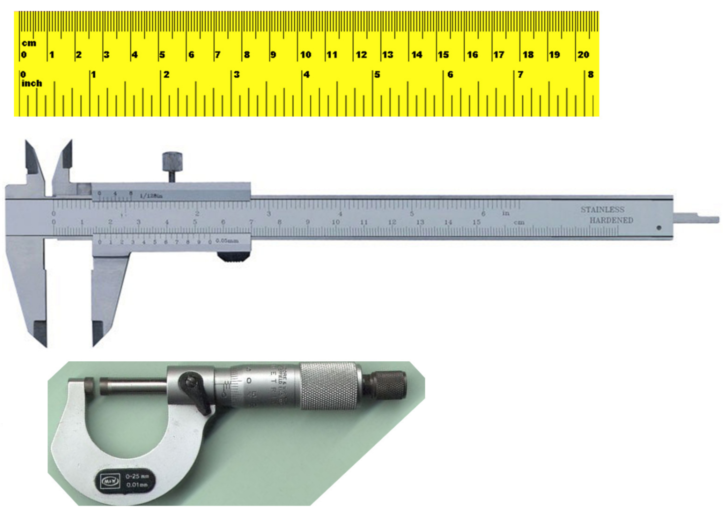

Figure 1.6: The ruler, vernier caliper and micrometer measure lengths to increasing precisions.

The uncertainties are reported in one of the two equivalent ways: **absolute uncertainty** and **relative uncertainty**. The absolute uncertainty is also called the **absolute error** since the absolute uncertainty is indicated by the absolute value of the error in measurements. Suppose we measure the time of swing of a pendulum and find that most values fall between 9 sec and 11 sec with the average value of 10 sec and a spread of 0.4 seconds symmetrically around the average value. The spread here, for example, may come from the standard deviation of the values or some other way you are able to estimate how the observed values are distributed. We write our observation as:

$$10 \text{ sec} \pm 0.4 \text{ sec}.$$

The spread written this way is called absolute uncertainty(error). The problem with the absolute uncertainty(error) is that we do not know how bad the uncertainty is relative to the main value. *The rel-*

ative uncertainty(error) expresses the absolute uncertainty as a percentage of the main value, which is also called **the percentage relative error**. Thus, in our example, the relative uncertainty(error) would be

$$0.4/10, \text{ or, } 4\%$$

. Therefore, the reading of the period can be expressed in two alternate ways:

Absolute uncertainty(error) notation: 10 sec $\pm$ 0.4 sec *or* (10 ± 0.4) sec.

Same reading in relative uncertainty(error) notation: 10 sec, percent relative error 4%.

Although there is an uncertainty in every measurement, I will not be indicating any uncertainty in values given in problems or tables in this book for the sake of brevity. When an uncertainty is not specified, the general practice is to regard the last digit to be uncertain by one or two units. For example, if the length of a rod is given to be 55.7 cm then it is usually assumed that the length is between 55.6 cm and 55.8 cm using one unit of the last digit as uncertainty. I will follow this practice when truncating the digits in numerical problems.

Uncertainty not specified: 55.7 cm will mean 55.5 cm $\pm$ 0.1 cm.

1.3.2 Uncertainty and Significant Figures

The uncertainty in the value of a physical quantity refers to a lack of complete knowledge about that quantity. For instance, a reading of 2.5 cm $\pm$ 0.1 cm for the length of an object says that we are uncertain about the value 2.5 cm by an amount 0.1 cm. Clearly, a claim better than 0.1 cm in this case is not meaningful. We cannot have a reading of the main value more precise than the uncertainty will allow. For instance, a reading of 2.51 cm $\pm$ 0.1 cm is meaningless since it claims that the average value expected is known up to 0.01 cm while it is uncertain by 0.1 cm - now, think about it, how can a quantity be known to a precision of 100^{th} place and uncertain in the 10^{th} place? A normal practice is to round off the uncertainty to one non-zero digit and use that absolute uncertainty figure as a guide to round off the main number noting that the main number can be no more precise than what is allowed by the uncertainty number. **Significant figures** or significant digits refers to the number of digits allowed in the main number. Thus, 2.5 cm $\pm$ 0.1 cm has two significant digits and 2.54 cm $\pm$ 0.02 cm has three.

In numerical calculations where uncertainty is not explicitly stated it will be understood that the last digit displayed is uncertain. We will pay particular attention in our calculations so that the final result we report as our answer does not have too many insignificant digits or that we are not rounding off at too few significant figures. It is also advisable that you keep one or two extra digits in the intermediate steps of a calculation since they tend to effect the final outcome sometimes, and only in the end, round off the final number to the appropriate significant figures. The calculation of the uncertainty in a quantity that depends on other quantities is a complicated matter and you will find simple examples later in the chapter that illustrate how uncertainties propagate from measured to derived quantities. Here, we make do with a simple rule that captures the spirit of significant figures.

The least precise value in an expression tends to control the maximum number of significant figures left at the end of a calculation.

Example 1.3.1. Illustration of simple rule of significant figures The length, width and height of a rectangular parallelepiped are given to be 1.5 cm, 0.600 cm, and 0.105 cm. What is the volume of the parallelepiped up to the correct number of significant figures using the simple rule of significant figures?

Solution. To apply the simple rule of significant figures in a calculation, we need to identify the least precise number in the input numbers. Here the length of 1.5 cm has two significant figures, the width of 0.600 cm and height of 0.105 cm each have three. Thus, the least precise number has two significant digits. Therefore, the volume will have no more than two significant digits.

$$\begin{aligned} \text{Volume} &= 1.5\ \text{cm} \times 0.600\ \text{cm} \times 0.105\ \text{cm} \\ &= 0.0945\ \text{cm}^3\ \text{(from calculator; incorrect number of digits)} \end{aligned}$$

Now we need to round off the number obtained above to the correct number of significant digits, which is two here. The final number 0.0945 has one digit to the left of the decimal and four digits to the right of the decimal. The decimal is readily moved to get rid of the leading zeros if we write the number using a power of ten as a multiplier. Therefore, the only digits which matter are the contiguous non-zero digits, i.e., 9, 4 and 5 here.

We seek a two-digit approximation of number 945: the digit 4 is called the least significant digit. Now, we need to decide about rounding off the digits to the right of the left significant digit: will 945 be rounded off to 940 or 950? Since we have a 5 after the least significant digit 4 here, we can round up the number to 950.

$$\text{Volume} = 0.095\ \text{cm}^3 \text{ or } 9.5 \times 10^{-2}\ \text{cm}^3$$

1.3.3 The Scientific Notation For Numbers

In the scientific notation, we use powers of 10 to write a decimal number. This helps us avoid any ambiguity in the reporting of the precision in the data. For example, 36000 is written as 3.6×10^4 and 36000. as 3.6000×10^4, which clearly shows that 36000 has two significant digits and 36000. has five.

In the scientific notation, it is a normal practice to force as many powers of ten (positive or negative) as necessary to make the multiplier of ten contain only one non-zero digit from 1 and 9 in the non-decimal part, and place the required trailing zeros on the right side of decimal to display appropriate number of significant figures.

A major advantage of the scientific notation is that it permits us to easily read off the significant figures in the decimal number. Thus, 1.5×10^{-3} m has two significant digits, 9.05×10^6 s has three, and 9.050×10^3 kg has four. Note that in the scientific notation, the decimal number multiplying a power of 10 is greater or equal to 1 and less than 10. The following examples illustrate the standard scientific notation and the same numbers not in standard scientific notation.

Scientific	1.5×10^{-3} m	9.05×10^6 s	9.050×10^3 kg
Non-scientific	0.0015 m	90.5×10^5 s	9050. kg

1.3.4 Rounding Off

In numerical calculations, we must often round off numbers to display the result up to the correct number of significant digits. This requires making decision about the least significant digit. We use the following rules for whether or not to increment the least significant digit when eliminating the insignificant part of a number.

- If the fraction to the right of the least significant digit is greater than $\frac{1}{2}$, then we increment the least significant digit by one.
- If the fraction to the right of the least significant digit is less than $\frac{1}{2}$, then we do not increment the least significant digit.
- If the digit to the right of the least significant digit is equal to 5, then we increment the least significant digit only when it is odd. This leads to incrementing of least significant digit in a group of numbers only half the time, and therefore reduces the chance of the introduction of a systematic error by either incrementing all the time or not incrementing all the time. Thus, 3.5 may

be rounded up to 4 and 4.5 may rounded down to 4. As long as you practice this consistently, then over a large group of numbers you would not introduce any systematic error which might creep in by rounded up a borderline case all the time.

1.3.5 Uncertainty and Accuracy

The **accuracy** of a measurement does not refer to the precision of the measurement, but instead, to the closeness of the measured value to the "true value", also called the accepted value. The accuracy is often expressed in a similar way to the percentage error, except that our objective now is to compare the best experimental value to the accepted value. The deviation of the experimental value from the accepted value is often written as a percentage, which we may call the **percentage deviation from the accepted value**.

Percent deviation from the accepted value.

$$\text{Percent deviation from the accepted value} = \frac{|\text{Accepted Value - Experimental Va}}{|\text{Accepted Value}|}$$

Thus, if you measured the width of a metal plate to be 5.7 cm while it was manufactured to the "exact" specification of 6.0 cm. Then, your percent error will be 5% as obtained from the following calculation.

$$\text{Percent deviation } = \frac{|6.0 - 5.7|}{|6.0|} \times 100\% = 5\%.$$

The causes of percent deviation may be a defect in the ruler or a faulty procedure of measurement.