10.12 PROBLEMS

Problem 10.12.1. A long solenoid of radius a with n turns per unit length is carrying a time-dependent current $I(t) = I_0 sin(\omega t)$, where I_0 and ω are constants. The solenoid is surrounded by a wire of resistance R that has two circular loops of radius b with $b > a$. Find the magnitude and direction of current induced in the outer loops at time $t = 0$. Ans: $2\mu_0 \pi a^2 I_0 n\omega/R$.

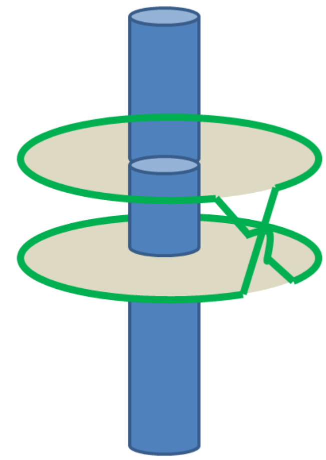
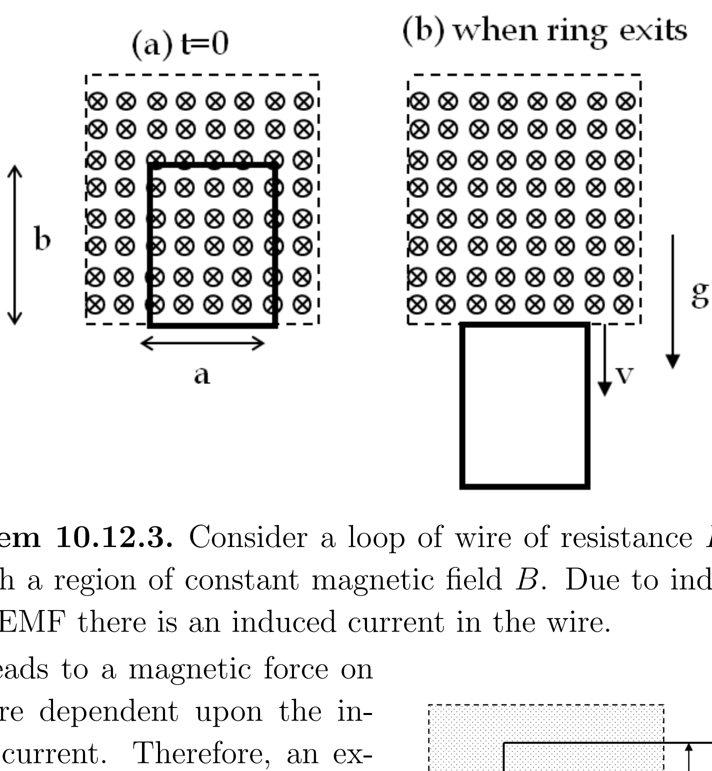


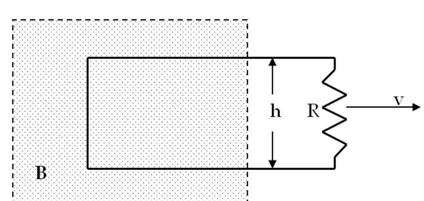
Figure 10.57: Problem 10.12.1.

Problem 10.12.2. A rectangular copper ring, of mass 100 g and resistance 0.2 Ω, is in a region of uniform magnetic field that is perpendicular to the area enclosed by the ring and horizontal to earth surface. The ring is let go from rest when it is at the edge of the nonzero magnetic field region. (a) Find its speed when the ring just exits the region of uniform magnetic field. (b) If it was let go at $t = 0$, what is the time when it exits the region of magnetic field for the following values $a = 25$ cm, $b = 50$ cm, $B = 3$ T, $g = 9.8$ m/s^2? Assume that the magnetic field of the induced current is negligible compared to 3 T. Ans: (a) $v = \dfrac{g}{\omega}\,(1 - e^{-\omega t})$, (b) 2.9 sec.



Problem 10.12.3. Consider a loop of wire of resistance R moving through a region of constant magnetic field B. Due to induced motional EMF there is an induced current in the wire.

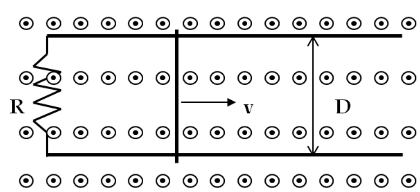
This leads to a magnetic force on the wire dependent upon the induced current. Therefore, an external force is needed for the loop to move to the right at constant velocity. Find the direction and magnitude of the force.



Ans: Magnitude: $F = \dfrac{1}{R} v h^2 B^2$.

Problem 10.12.4. Find the equivalent inductance of two inductors L_1 and L_2 (a) connected in series and (b) connected in parallel. Ans: (a) $L_1 + L_2$ and (b) $\dfrac{L_1 L_2}{L_1 + L_2}$.

Problem 10.12.5. A metal bar of mass m slides without friction over two rails a distance D apart in a region that has a uniform magnetic field of magnitude B_0 and direction perpendicular to the rails. The two rails are connected at one end to a resistor whose resistance is much larger than the resistance of the rails and the bar.



The bar is given an initially speed of v_0. It is found to slow down. How far does the bar go before coming to rest? Assume that the magnetic field of the induced current is negligible compared to B_0.

Ans: $\dfrac{mRv_0}{B^2 D^2}$.

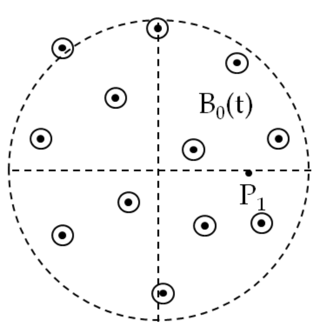
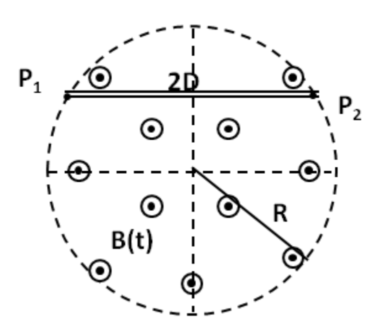


Figure 10.58: Problem 10.12.6.

Problem 10.12.6. A time-dependent but otherwise uniform magnetic field of magnitude $B(t)$ is confined in a cylindrical region of radius 5 cm. Initially magnetic field is in the region has a magnitude of 3 T but it is decreasing at the rate of 400 G/s. An electric field is induced in this space due to changing magnetic field which causes acceleration of charges at rest. Find the direction and magnitude of acceleration of a proton placed at P_1 which is 3 cm from the center. Ans: 1.15×10^5 m/s^2.

Problem 10.12.7. A time-dependent uniform magnetic field of magnitude $B(t)$ is confined in a cylindrical region of radius R. A conducting rod of length $2D$ is placed in the region as shown in the figure. Show that the EMF between the ends of the rod is give by $(dB/dt)D\sqrt{R^2 - D^2}$. Hint: To find the EMF between the ends we need to integrate th electric field from one end to the other. To find the electric field use Faraday's law as "Ampere's law for E".



Problem 10.12.8. A single loop of wire of radius a is inside and coaxial with a cylindrical solenoid of length h, radius b and n turns per unit length, where $b << h$. Call the solenoid circuit 1 and the

loop circuit 2. Find the mutual inductance. Indicate where the assumption $b << h$ is used in your derivation.

Problem 10.12.9. A coaxial cable has an inner conductor of radius a and an outer thin cylindrical shell of radius b. A current I flows in the inner conductor and returns in the outer conductor. The self-inductance of the structure will depend on how the current in the inner cylinder tends to be distributed. Investigate the following two extreme cases.

(a) Let current in the inner conductor be distributed only on the surface and find the self-inductance.

(b) Let current in the inner cylinder be distributed uniformly over its cross-section and find the self-inductance. Compare with your result in (a). Ans: (a) $L = \frac{\mu_0}{2}(b^2 - a^2)$, (b) $L = \frac{\mu_0}{2}(b^2 - a^2) + \frac{\mu_0 a}{3}$.

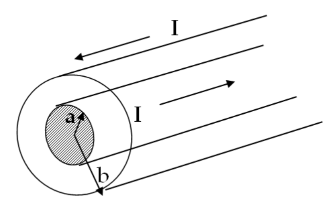


Figure 10.59: Problem 10.12.9.

Problem 10.12.10. A square loop of side 2 cm is placed 1 cm from a long wire carrying a current that varies with time at a constant rate of 3 A/s as shown.

(a) Use Ampere's law and find magnetic field as a function of time from the current in the wire.

(b) Determine the magnetic flux through the loop.

(c) If the loop has a resistance of 3 Ohm, how much induced current flows in the loop and in which direction?

Ans: (b) $\Phi = \frac{\mu_0\, a\, I}{2\pi} \ln\left(\frac{a+b}{b}\right)$, (c) 4.4 nA.

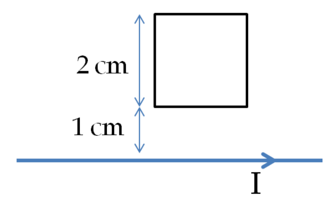


Figure 10.60: Problem **??**.