4.8 EXERCISES

Constant Speed and Constant Velocity

Ex 4.8.1. A ball is rolling in a straight groove with a constant speed of 1.5 m/s. (a) How much distance does the ball go in 20 sec? (b) There is a stop at the end of the track 100 m from the place it was let go at $t = 0$, what is the velocity of the ball at times (i) $t = 20$ sec, (ii) $t = 70$ sec?

Ex 4.8.2. A car is moving on a straight road in a fixed direction at a constant speed of 50 km/h with respect to the road. You wish to state the kinematic vectors of the motion of the car by using a Cartesian coordinate system whose positive x-axis is pointed in the direction of the motion of the car and the origin is fixed at some point on the road. (a) What is the expression for the velocity of the car? (b) What is the position vector at time $t = 0.01$ hr?

Ex 4.8.3. The rate of change of the x-coordinate of a moving object is found to be constant 10 m/s. If the x-coordinate at $t = 2$ sec was 100 m, what is the x-coordinate at $t = 10$ sec?

Ex 4.8.4. A function $f(t)$ of time t has a constant rate of change u given by

$$\frac{df}{dt} = u.$$

Find the change in the value of f in the interval between $t = t_1$ and $t = t_2$.

Ex 4.8.5. A function $h(t)$ of time t changes linearly with time, i.e. as one power of t, as given by

$$h(t) = b\ t + c,$$

where b and c are constants. Find the rate of change of h.

Ex 4.8.6. A function $q(t)$ of time t changes linearly with time, i.e. as one power of t. It has a value q_1 when $t = t_1$ and a value q_2 when $t = t_2$. Find an analytic expression for the function $q(t)$.

Ex 4.8.7. A function $s(t)$ of time t has a constant rate of change u. At $t = 0$ the value of s is 1. Find an analytic expression for the function $s(t)$.

Ex 4.8.8. The y-coordinate of a particle varies at a constant rate of 4 m/s. At $t = 0$, the y-coordinate was found to be 3 m. Find an analytic expression for the function $y(t)$.

Constant Acceleration - 1 Dimension

Ex 4.8.9. The x-coordinate of an object varies with time according to $x(t) = 3+5t+9t^2$ where t is in seconds and x is in meters. (a) Find the x-component of the average velocity between $t = 0$ and $t = 1$ sec. (b) Find the x-component of acceleration at $t = 0.5$ sec and $t = 1$ sec.

Ex 4.8.10. A car moves on a straight road such that its position from a reference point is given as $\{x(t) = 3+5t+2t^2+0.4t^3, y = 0, z = 0\}$, where t is in seconds and x in meters. (a) Is the acceleration of the car constant? (b) Find the acceleration at $t = 0.5$ sec and $t = 1$ sec. (c) Find the average velocity between $t = 0$ and $t = 1$ sec. (d) Find the average acceleration between $t = 0$ and $t = 1$ sec.

Ex 4.8.11. Starting from rest a box slides down an incline at a constant acceleration of 2 m/s^2 pointed down the incline (not vertically down). (a) Find the speed of the box at $t = 1.5$ sec. (b) Find the average velocity of the box between $t = 0$ and $t = 1.5$ sec (beware of direction!). (c) Determine the distance traveled on the incline in 1.5 sec. (d) Verify that the displacement during the 1.5 sec interval from $t = 0$ to $t = 1.5$ sec is equal to the product of the average velocity for the interval and the time interval.

Ex 4.8.12. A hockey puck is hit up an incline. The puck slows down at a constant acceleration of magnitude 5 m/s^2 and stops after 1.2 sec. Let x-axis be pointed up the incline. (a) What are the directions of velocity and acceleration at $t = 0.5$ sec? (b) Find the initial velocity, i.e. the velocity of the puck immediately after leaving the stick. (c) How far up the incline does the puck go before coming to rest? (d) Find the position of the puck at $t = 1$ sec.

Ex 4.8.13. A bucket of water is hung with a rope which goes around a pulley. The other side of the rope is wound on a wheel that is rotated with a handle so that the bucket goes up with a constant acceleration of 0.5 m/s^2 starting from rest. (a) How much time will it take to raise the bucket by 1.5 m? (b) What will the bucket's velocity be when it reaches the 1.5 meter mark from the starting place?

Ex 4.8.14. A drum is rolled down an incline. It is found that the speed of the center of the drum increases steadily with time (i.e. with a constant acceleration) from 2 m/s to 4 m/s over a distance of 1.5 m on the incline. (a) Find the acceleration of the center of the drum. (b) Find the time interval for the given data. (c) How much time will it take to cover the next 1.5 m on the incline?

Ex 4.8.15. A basketball is rolled on a rough horizontal road. The velocity of the center of the ball is found to decrease steadily with time. In a particular roll, the ball stops 20 m from the starting place in 10 sec. (a) What is the speed of the ball at the initial time? (b) What is the acceleration (i.e. deceleration) of the ball? (c) How much distance does the ball cover in the first 5 sec? (d) How much distance does the ball cover in the last 5 sec?

Ex 4.8.16. A skater after an initial push glides with a constant deceleration. He comes to a stop after reaching 120 m in 25 sec. (a) Find his initial speed. (b) Find the deceleration. (c) How much distance did he cover in the first 12.5 sec? (d) How much distance did he cover in the last 12.5 sec?

Ex 4.8.17. A train starts from rest and accelerates on a straight track at a constant acceleration of 3 m/s^2 for 10 sec. It then coasts at a constant speed for 20 sec. (a) Find the speed with which the train is coasting in the last 20 sec. (b) Find the total distance traveled in 30 sec from the start. (c) Find the distance traveled in the first 5 sec. (d) Find the distance traveled in the next 5 sec.

Ex 4.8.18. A car starts from rest and accelerates on a straight road at a constant acceleration of 2 m/s^2 for 20 sec. It then coasts at a constant speed v for another 20 sec. Finally it decelerates and comes to a stop in 10 sec. (a) Find the speed v with which the car is coasting in the second 2 sec constant speed stage. (b) Find the total distance traveled in first 40 sec. (c) Find the deceleration in the last 10 sec. (d) Find the distance traveled in the last 10 sec. (e) Find the total distance traveled over the 50 sec interval.

Free Fall - 1 Dimension

Ex 4.8.19. A steel ball is dropped from rest from a 100-m tower. Assume the effect of the air resistance to be negligible on its motion. (a) Find the time the ball will take to fall to the ground. (b) Find the speed with which the ball will strike the ground. (c) Find the time the ball will take to reach the 50-m mark. (d) Find the time the ball will take to cover the last 50 meters. (e) Find the speed of the ball when it is at the 50-m mark.

Ex 4.8.20. A rocket it shot straight up. It reaches the top of its flight in 3 sec. Assume the effect of the air resistance to be negligible on its motion. (a) Find the velocity of the rocket immediately after it leaves the launch pad. (b) How high does the rocket go?

Ex 4.8.21. A brass ball is launched vertically up with an initial speed of 30 m/s. Assume the effect of the air resistance to be negligible on its motion. (a) Find the location of the ball after 2 sec. (b) How fast is the ball moving at that time? (c) How high does the ball rise before coming to rest?

Ex 4.8.22. A boy throws a rock from the roof of a 10-story building straight down with an initial speed of 10 m/s. Assume 3 meter per story for the building. Assume the effect of the air resistance to be negligible on its motion. (a) How long will it take the rock to strike the ground? (b) With what speed will the rock strike the ground?

Ex 4.8.23. Two balls are launched straight up at different times from a tennis ball launching machine. The second ball is launched at the time the first ball is at the top of its flight. If the launching speed of the balls are 20 m/s, where would the two balls hit each other? Assume the effect of the air resistance to be negligible on the motion.

Ex 4.8.24. Two rocks are thrown vertically down from a 100 m tall tower at different times. While the first rock is let go from rest, the second rock, one second later, is thrown at some speed v_0 such that the second rock strikes the first rock 2.5 seconds later, i.e. at $t = 3.5$ sec from when the first rock was thrown. (a) Find the initial speed of the second rock. (b) Find the speeds with which the two rocks are moving when they strike.

Projectile Motion

Ex 4.8.25. A ball is launched from the end of a table horizontally with a speed of 10 m/s and lands on the floor 2 m below. (a) Where does the ball land on the floor? (b) How much time was the ball in air? (c) What is the velocity of the ball immediately before it strikes the floor? Ignore the air resistance.

Ex 4.8.26. A stone is thrown horizontally from a 30-meter cliff with an initial speed of 20 m/s. (a) How far does it travel before hitting the ground? (b) At what time does it strike the ground, if it left the cliff at $2:01:00\ PM$? (c) At what time does it reach a spot that is 5 m from the ground? (d) What is the horizontal distance of the point where the stone is at 5 m height from the ground? Ignore air resistance.

Ex 4.8.27. A basketball is thrown from a height of 1.6 m from the ground. Immediately after leaving the hand the ball had an initial velocity of 5 m/s and in the direction of 40° above the horizontal direction. (a) Where on the floor does the ball land? (b) With what velocity does the ball strike the floor? Ignore air resistance.

Ex 4.8.28. A stone is thrown from a 30-m cliff with initial speed of 20 m/s and at an angle of 60° above the horizontal direction. The stone lands on a plateau that is 10-m above the bottom of the cliff. How far away from the cliff, horizontally, is the place where the stone will land?

Ex 4.8.29. A cannonball must travel 300 m horizontally and 20 m vertically above from the launch site to land on the enemy position. With what speed should the ball be launched at 40° from the horizontal to make the hit? Assume no air resistance.

Ex 4.8.30. A cannonball must travel 400 m horizontally and 50 m vertically below from the launch site to land on the enemy position. With what speed should the ball be launched at 30° from the horizontal to make the hit? Assume no air resistance.

Ex 4.8.31. A rocket launcher launches missiles at a speed of 50 m/s. At what angle with the horizon should a missile be fired so that it strikes a target at 100 m horizontal distance and 30 m vertical distance from the launch site?

Ex 4.8.32. A rocket launcher launches missiles at a speed of 40 m/s. At what angle with the horizon should the missile be fired so that the missile strikes a target at 80 m horizontal distance and 25 m vertical distance from the launch site?

Ex 4.8.33. A football leaves the kicker's foot at 50° from the horizontal direction. The football is required to clear a horizontal bar at a height of 10 m above the ground at a horizontal distance of 40 m. Find the speed with which the ball must leave the kicker's foot so that it would barely go over the bar.

Variable Acceleration

Ex 4.8.34. A particle starts out at rest at $t = 0$ and accelerates in a straight line with an acceleration of 2 m/s^2 from $t = 0$ to $t = 3$ sec, and then with an acceleration of 4 m/s^2 pointed in the same direction from $t = 3$ sec to $t = 5$ sec. (a) Find the position and the velocity of the particle at $t = 3$ sec. (b) Find the position and the velocity of the particle at $t = 5$ sec.

Ex 4.8.35. At $t = 0$ a particle has a velocity of 20 m/s in the direction of the positive x-axis, and accelerates on the x-axis as shown in the Fig 4.19. The y and z-components of the acceleration are zero. (a) Find the position and the velocity of the particle at $t = 2$ sec. (b) Find the position and the velocity of the particle at $t = 5$ sec.

Ex 4.8.36. A particle starts out with a velocity of 20 m/s in the direction of the negative x-axis, and accelerates on the x-axis as shown in the Fig 4.20. (a) Find the position and the velocity of the particle

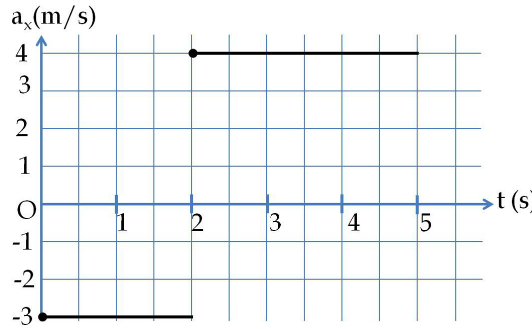

Figure 4.19: Exercise 4.8.35.

at $t = 2$ sec. (b) Find the position and the velocity of the particle at $t = 3$ sec. (c) Find the position and the velocity of the particle at $t = 5$ sec.

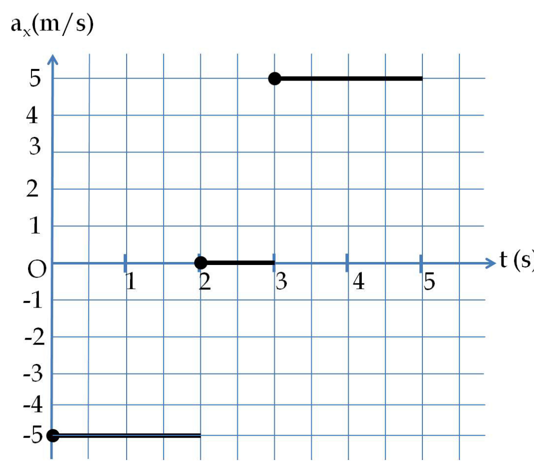

Figure 4.20: Exercise 4.8.36.

Ex 4.8.37. A box slides on a floor in a straight path such that the magnitude of the acceleration is not constant in time. The data is collected with respect to a Cartesian axis system in which only the a_x is non-zero. The x-component of the acceleration is shown in Fig 4.21. As the figure shows, the direction of the acceleration is always pointed towards the positive x-axis, but the magnitude varies with time. (a) Find the velocity of the box at $t = 4$ sec if it starts out at rest at $t = 0$. (b) Find the velocity of the box at $t = 10$ sec.

Ex 4.8.38. A hockey puck is shot on a surface that has different roughness at different places. As a result its acceleration varies from place to place. By placing the x-axis on the line of motion of the puck, we cast the acceleration vector in terms of its x-component which can be plotted with time. Note the acceleration vector cannot be plotted, since they are not ordinary functions; only the components can be plotted. The resulting x-component of the acceleration is shown in

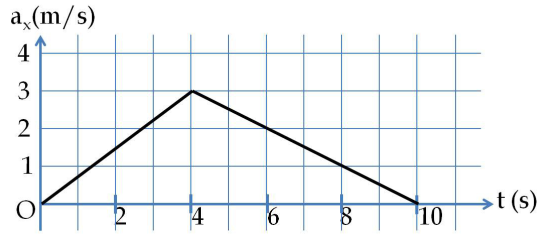

Figure 4.21: Exercise 4.8.37.

Fig. 4.22. The y and z-components of the acceleration are zero. The puck has a velocity of 2 m/s at $t = 5$ sec. Find the initial velocity of the puck at $t = 0$.

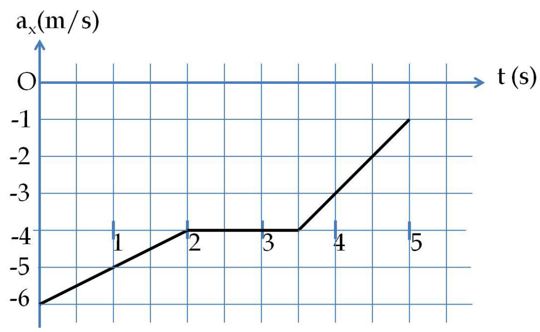

Figure 4.22: Exercise 4.8.38.

Ex 4.8.39. The x-component of the acceleration of a particle changes according to the following analytic expression, $a_x(t) = 4+t^2/2$, where t is in sec and the acceleration in m/s^2. Find the x-components of the velocity and position vectors.

Ex 4.8.40. The x-component of the acceleration of a particle changes according to the following analytic expression, $a_x(t) = 1 + 3t$, where t is in sec and the acceleration in m/s^2. (a) Find the x-component of the velocity at $t = 3$ sec if the particle was at rest at $t = 0$. (b) Find the change in the x-coordinate of the particle in the interval $t = 1$ sec to $t = 3$ sec.

Ex 4.8.41. The x-component of the velocity of a ball is given as $v_x(t) = 20 + 3t^3$, where t is in sec and the velocity is in m/s. (a) Find the change in the x-coordinate between $t = 0$ and $t = 5$ sec. (b) Find the x-component of the acceleration at (i) $t = 0$, and (ii) $t = 5$ sec.

Integration of Equations

Ex 4.8.42. Find function $f(t)$ from the given conditions on the function. (a) $df/dt = 0$ and $f(0) = 1$; (b) $df/dt = 2$ and $f(0) = 5$; (c)

$df/dt = 4t$ and $f(1) = 1$; (d) $df/dt = 5 + 3t$ and $f(0) = 2$; (e) $df/dt = (1/2)t^2$ and $f(0) = 7$; (f) $df/dt = \sin(t)$ and $f(0) = 1/2$; (g) $df/dt = \pi - t$ and $f(1) = 10$; (h) $df/dt = e - 3t$ and $f(0) = 2$.

Ex 4.8.43. A particle moves at a constant speed of 20 m/s along the positive x-axis of a Cartesian coordinate system. (a) Write its velocity? (b) If the particle was at $x = 1$ m at $t = 0$, where will it be at $t = 3$ sec? (c) If the particle was at $x = 1$ m at $t = 1$ sec, where will it be at $t = 3$ sec? (d) What is its acceleration?

Ex 4.8.44. In a particular Cartesian coordinate system, the y and z-components of the velocity are zero and the x-component varies as given by the function, $v_x(t) = 2 + 5t$, where t is in sec and v_x is in m/s. (a) Find the displacement vector in the following time intervals (i) $[0, 1\ \text{sec}]$, (ii) $[0.5\ \text{sec}, 1.5\ \text{sec}]$, and (iii) $[2\ \text{sec}, 3\ \text{sec}]$. (b) Find the average velocity in the same intervals. (c) Find the instantaneous velocity at the following instants in time, (i) 1 sec, (ii) 2 sec, and (iii) 3 sec. (d) Find the instantaneous accelerations at the same instants in time. (e) If the particle was at $x = 0$ at $t = 0$, find the position of the particle at (i) 1 sec, (ii) 2 sec, and (iii) 3 sec.

Ex 4.8.45. In a particular Cartesian coordinate system, the y and z-components of the velocity are zero and the x-component varies as given by the function, $v_x(t) = 2 + 5t - 3t^2$, where t is in sec and v_x is in m/s. (a) Find the displacement in the following time intervals ((i) $[0, 1\ \text{sec}]$, (ii) $[0.5\ \text{sec}, 1.5\ \text{sec}]$, and (iii) $[2\ \text{sec}, 3\ \text{sec}]$. (b) Find the average velocity in the same intervals. (c) Find the instantaneous velocity at the following instants in time, (i) 1 sec, (ii) 2 sec, and (iii) 3 sec. (d) Find the instantaneous accelerations at the same instants in time. (e) If the particle was at $x = 0$ at $t = 0$ and moved on the x-axis, find the position of the particle at (i) 1 sec, (ii) 2 sec, and (iii) 3 sec.

Ex 4.8.46. In a particular Cartesian coordinate system, the y and z-components of the acceleration are zero and the x-component varies as given by the function, $a_x(t) = -20 + 10\ t$, where t is in sec and a_x is in m/s^2. The particle's velocity at $t = 0$ was pointed towards the positive x-axis and had a magnitude of 10 m/s. (a) Find the change in the velocity in the following time intervals (i) $[0, 1\ \text{sec}]$, (ii) $[0.5\ \text{sec}, 1.5\ \text{sec}]$, and (iii) $[2\ \text{sec}, 3\ \text{sec}]$. (b) Find the average acceleration in the same intervals. (c) Find the instantaneous accelerations at the following instants in time, (i) 1 sec, (ii) 2 sec, and (iii) 3 sec. (d) Find the instantaneous velocity at the same instants in time. (e) If the particle was at $x = 0$ at $t = 0$, find the position of the particle at (i) 1 sec, (ii) 2 sec, and (iii) 3 sec.

Ex 4.8.47. In a particular Cartesian coordinate system, the y and z-

components of the acceleration are zero and the x-component varies as given by the function, $a_x(t) = 5t - 3t^2 + 20\exp(-t)$, where t is in sec and a_x is in m/s^2. The particle's velocity at $t = 0$ was pointed towards the positive x-axis and had a magnitude of 10 m/s. (a) Find the change in velocity in the following time intervals (i) $[0, 1 \text{ sec}]$, (ii) $[0.5 \text{ sec}, 1.5 \text{ sec}]$, and (iii) $[2 \text{ sec}, 3 \text{ sec}]$. (b) Find the average acceleration in the same intervals. (c) Find the instantaneous accelerations at the following instants in time, (i) 1 sec, (ii) 2 sec, and (iii) 3 sec. (d) Find the instantaneous velocity at the same instants in time. (e) If the particle was at $x = 0$ at $t = 0$, find the position of the particle at (i) 1 sec, (ii) 2 sec, and (iii) 3 sec.

Ex 4.8.48. In a particular Cartesian coordinate system, $y = 0$ and $z = 0$, and the x-coordinate of a particle varies with time as given by $x(t) = 2\sin(3t) + C$, where t is in sec and x in m, and C is a constant to be determined by the data. At $t = 0$ the particle was at $x = 1$ m and the velocity of the particle was pointed towards the positive x-axis and had a magnitude of 10 m/s. (a) Find the value of constant C. (b) Find the displacement in the following intervals, (i) $[0, 1 \text{ sec}]$, (ii) $[0.5 \text{ sec}, 1.5 \text{ sec}]$, and (iii) $[2 \text{ sec}, 3 \text{ sec}]$. (c) Find the instantaneous velocity at the following instants in time, (i) 1 sec, (ii) 2 sec, and (iii) 3 sec. (d) Find the instantaneous accelerations at the following instants in time, (i) 1 sec, (ii) 2 sec, and (iii) 3 sec.

Ex 4.8.49. In a particular Cartesian coordinate system, $v_y = 0$,$v_z = 0$, and v_x varies with time as given by, $v_x(t) = 20\cos(5t) + C$, where t is in sec and v_x is in m/s, and C is a constant to be determined by the data. The particle's acceleration at $t = 0$ was pointed towards the negative x-axis and had a magnitude of 3 m/s^2. (a) Find the value of constant C. (b) Find the displacement in the following intervals, (i) $[0, 1 \text{ sec}]$, (ii) $[0.5 \text{ sec}, 1.5 \text{ sec}]$, and (iii) $[2 \text{ sec}, 3 \text{ sec}]$. (c) Find the average velocity in the same intervals. (d) Find the instantaneous velocity at the following instants in time, (i) 1 sec, (ii) 2 sec, and (iii) 3 sec. (e) Find the instantaneous accelerations at the same instants in time. (f) If the particle was at $x = 0$ at $t = 0$, find the position of the particle at (i) 1 sec, (ii) 2 sec, and (iii) 3 sec.

Relative Motion

Ex 4.8.50. John and Betsy are separated by 20 m on a platform and watch a train approach the station on a straight track at a constant speed of 5 m/s. Find the position, velocity, and acceleration of the train at $t = 10$ s with respect to John and Betsy if the front of the train was 200 m from Betsy and 220 m from John at $t = 0$.

Ex 4.8.51. On a straight East-West road, two cars A and B are moving towards East with respect to a person P on the ground. According to P, the car A is moving with a constant velocity of 30 m/s and the car B with a constant acceleration of 5 m/s^2. At $t = 0$, car B was at rest with respect to P. (a) Find the velocity of the car B with respect to the car A at $t = 20$ s. (b) Find the velocity of the person P with respect to the car A at $t = 20$ s. (c) Find the acceleration of the car B with respect to the car A at an arbitrary time t.

Ex 4.8.52. Two cars A and B start from a junction as observed by a person P on the ground. Car A goes north at a constant acceleration of 5 m/s^2 having started out at rest at the junction. Car B goes East at a constant velocity of 25 m/s. (a) Find the velocity of person P with respect to car B at $t = 10$ s. (b) Find the velocity of car B with respect to car A at $t = 10$ s. (c) Find the acceleration of car A with respect to car B at an arbitrary time.

Ex 4.8.53. On a straight East-West road, two cars A and B are moving towards East with respect to a person P on the ground. According to person P, car A is moving with a constant velocity of 30 m/s and car B with a constant acceleration of 4 m/s^2. At $t = 0$, the car B was at rest with respect to P. (a) Find the velocity of person P with respect to the car B at $t = 30$ s. (b) Find the velocity of the car A with respect to the car B at $t = 30$ s. (c) Find the acceleration of the car A with respect to the car B at an arbitrary time.