6.9 PROBLEMS

Problem 6.9.1. A current I flows on the surface of a cylindrical wire of radius R in the longitudinal direction and is distributed uniformly over the circumference. Find the surface current density. Ans: $I/2\pi R$.

Problem 6.9.2. A current I flows through a cylindrical wire of radius R in the longitudinal direction and is distributed uniformly over the area of cross-section. Find the volume current density.

Problem 6.9.3. Charged microscopic particles are sprinkled uniformly over a non-conducting oil which is flowing at speed 10 m/s. The charge density on the surface of the oil is 2.5 μC/cm^2. What is the value of surface current density k? Ans: 0.25 C/m.s.

Problem 6.9.4. Charges are painted over a non-conducting disk such the there is a uniform suface charge density of σ over its surface. The disk is rotated at a uniform angular speed of ω radians per second. (a) Find the surface current density k at a distance r meter from the center. (b) Find the total current I. Ans: (a) $\sigma\omega r$, (b) $\sigma\omega R^2/2$.

Problem 6.9.5. In an electrolytic solution of copper sulfate, the Cu^{+2} ions are flowing to the cathode and SO_4^{-2} ions to the anode. The ions flow between the plates of area A. The number density of positive ions is N_+ per cubic meter each carrying a charge of $+2e$ and the number density of the negative ions is N_- per cubic meter each carrying a charge of $-2e$. the solution is neutral so that you can use same symbol N for the number densities, $N_+N_- = N$. Assume the drift speed of the both ions is v and that the volume current density they generate is uniform. Find the volume current density J and the total current I carried by the ions in the solution. Ans: $I_{\text{tot}} = 4eNvA$.

Problem 6.9.6. In a cylindrical wire of radius R current density J varies with zero current at the center of the cross-section to a maximum value J_0 at the edge. The current density as a function of distance r from the center of wire is given to be $J(r) = J_0\,(r/R)^2$. Find the total current I carried by the wire. Ans: $\pi R^2 J_0/2$.

Problem 6.9.7. A cylindrical wire of radius R carries a steady current. (a) Determine the total current I carried by the wire if the current density is uniform J_0? (b) Determine the total current I carried by the wire if the current density varies with distance s from the center of the wire as $J(r) = J_0\,[\exp(r/R) - 1]\,/(e-1)$? (c) Plot the current density given in part (b) and describe the distribution in words. Ans: (b) $\pi R^2 J_0/(e-1)$.

Problem 6.9.8. One end of Gauge 6 (diameter 0.162 in) copper wire is welded to a Gauge 6 aluminum wire. A current of 10 A passes through the composite wire. (a) Find the current density in each wire. (b) Assuming there is one conduction electron per atom for copper and one conduction electron per atom for aluminum, find the drift velocity of electrons in copper and in aluminum for the current given. (c) Why is the drift velocity in the copper wire greater than that in the aluminum wire? (d) Why do the drift velocities much smaller than common everyday experience of almost instantaneous effect of turning electrical appliances when switch is thrown? Ans: (a) 7.5×10^5 A/m^2, (b) Cu 6.03×10^{-5} m/s, Al 7.78×10^{-5} m/s.

Problem 6.9.9. A cylindrical copper Gauge 6 (diameter 0.162 in) wire carries a current of 30 A. (a) Assuming that current density J is uniform throughout the cross-section, find the electric field inside the copper wire. (b) Why is the electric field not zero inside copper, which is a good conductor? Ans: (a) $E = 3.8 \times 10^{-2}$ V/m.

Problem 6.9.10. A rectangular slab of copper has dimensions as shown in the figure. Find resistance in the three directions. Ans: $1.13 \times 10^{-1}\Omega$, $6.38 \times 10^{-4}\Omega$, $2.83 \times 10^{-4}\Omega$.

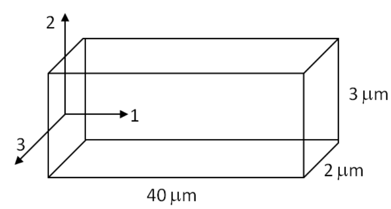


Figure 6.56: Problem 6.9.10.

Problem 6.9.11. A platinum wire is connected to a precision constant voltage source of 1.5 V and internal resistance 0.1 Ω. Current through the wire is measured by a precision Ammeter. At 20°C a current of 1.010 A flows through the platinum wire. When the wire is dipped in a non-conducting fluid of unknown temperature we find that current decreases to 1.000 A. Find the temperature of the fluid. Ans: 22.7°C.

Problem 6.9.12. Two rectangular cross-section wires of copper and aluminum are welded together seamlessly. A steady current of 50 A is passed though them. The areas of cross-sections of the wires are 4 mm^2 each. (a) What is the value and direction of the electric field in each wire? Assume uniform electric field. (b) What excess charges are responsible for the difference in electric field in the two materials, and where are they located? Density of copper = 8000 kg/m^3; Density of aluminum = 2700 kg/m^3; Atomic weight of copper = 65; Atomic weight of aluminum = 27; Conductivity of copper = $5.9 \times 10^7\Omega^{-1}\text{m}^{-1}$; Conductivity of aluminum = $3.7 \times 10^7\Omega^{-1}\text{m}^{-1}$.

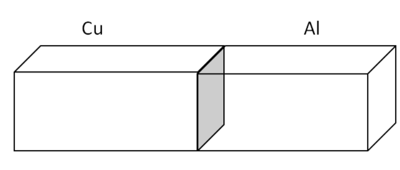


Figure 6.57: Problem 6.9.12.

Ans: (a) Cu 0.21 V/m, Al 0.34 V/m; (b) 1.15×10^{-12} C/m^2.

Problem 6.9.13. A cylindrical shell made of a material of resistivity ρ has inner radius a, outer radius b and length L. Find its resistance for flow of current along its length.

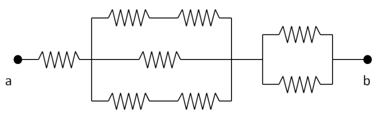


Figure 6.58: Problem 6.9.14.

Problem 6.9.14. Determine the equivalent resistance between a and b for the network of resistors shown in the figure. Each resistor is equal to 2Ω. Ans: 4Ω.

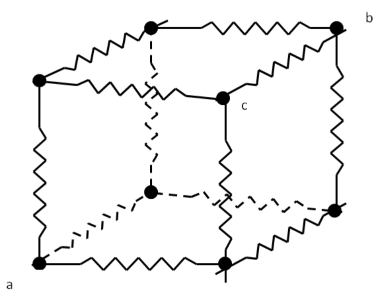


Figure 6.59: Problem 6.9.15.

Problem 6.9.15. (a) Determine the equivalent resistance between points a and b of twelve identical resistors of resistance R each connected along the edge of a cube. (b) Do the same between points a and c. Use symmetry instead of setting up Kirchhoff equations.

Problem 6.9.16. (a) Find current through each resistor. (b) How much power is consumed by each resistor? (c) If the battery can supply at most 200 Joules of energy, how long the current will flow in the circuit?

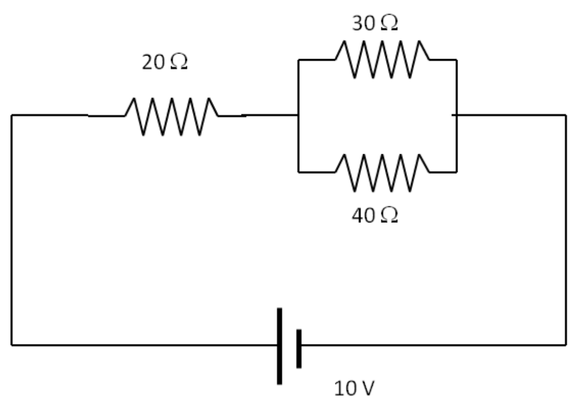


Figure 6.60: Problem 6.9.16.

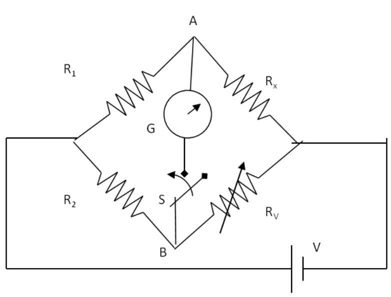


Figure 6.61: Problem 6.9.17.

Problem 6.9.17. A Wheatstone bridge is used to measure resistance by "balancing the circuit". The circuit is balanced if no current flows through the galvanometer G between A and B. In the circuit R_1 and R_2 are fixed precision resistors with known values and R_V is a variable resistor whose value can be read off accurately. The unknown resistor is R_x. The variable resistor is adjusted till no current flows in the branch from A to B as indicated by no deflection in the galvanometer G. Find a formula for R_x in terms of R_1, R_2 and R_V.

Problem 6.9.18. Calculate current in each branch of the circuit. Ans: 1.5 A, 0.73 A, and 0.82 A.

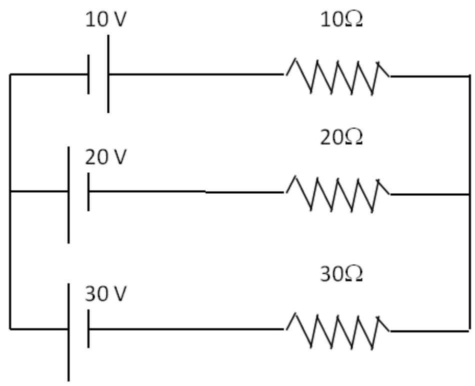


Figure 6.62: Problem 6.9.18.

Problem 6.9.19. A variable resistor R is connected to the terminals of a battery of fixed EMF V and internal resistance R_{in}. Find the value of variable resistor R for which the battery delivers maximum power to the resistor. Ans: $R = R_{in}$.

Problem 6.9.20. An aluminum rod of length L is shaved into a conical shape tapering linearly from radius a at one end to radius b at the other end. Prove that the resistance in terms of the given dimensions and resistivity ρ. [Hint: Try using resistors in series.] Ans: $4\rho L/\pi ab$.

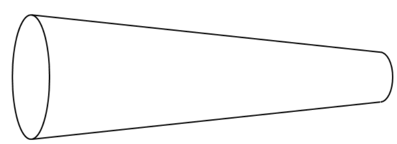
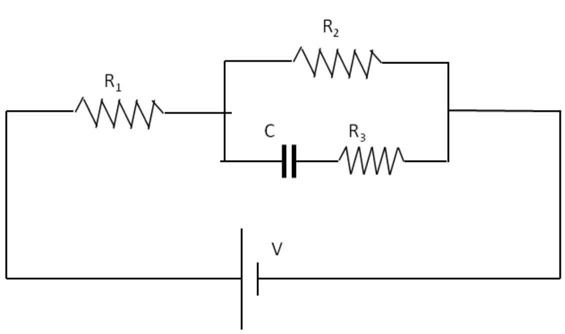


Figure 6.63: Problem 6.9.20.

Problem 6.9.21. Determine the time constant of the given circuit. Ans: $\tau = \left(\frac{R_1 R_2}{R_1+R_2} + R_3\right) C$.



Problem 6.9.22. Find the equivalent resistance between a and b. You may connect a battery of voltage V across a and b and then solve for current through the battery. The R_{eq} will then be V/I.

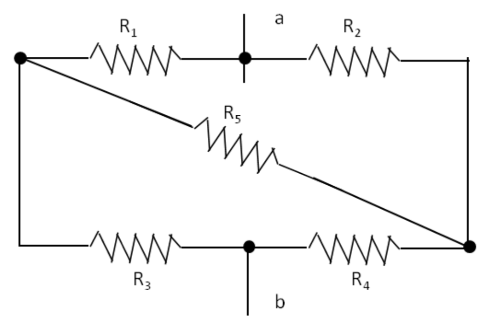


Figure 6.64: Problem 6.9.22.