3.1 OPTICAL INSTRUMENTS

A large purpose behind studying ray optics is to understand and design better optical instruments. In this section we will only touch upon the basics of some of the most common instruments. The reader is advised to consult a more specialized treatment for further study.

3.1.1 The Human Eye

The human eye is perhaps the most important optical instrument. In many instruments the human eye plays an important role in the observation of the image. In Fig. 3.1, I have drawn a schematic view of a human eye. You can liken the eye to a camera where light enters through a pin-hole (the pupil), and a converging lens system consisting of the cornea and the eye lens that focus light on the retina which acts as a sensitive detector. The pupil is the black center in the colored diaphragm called iris. The pupil is black because no light is reflected from it. The retina consists of light-sensitive cells, called rods and cones, and nerve endings. The retina converts the light energy into electrical signals, which are then carried to the brain for further processing that give rise to the sense of vision. At the center of the retina lies an extremely sensitive area called fovea where best focusing takes place.

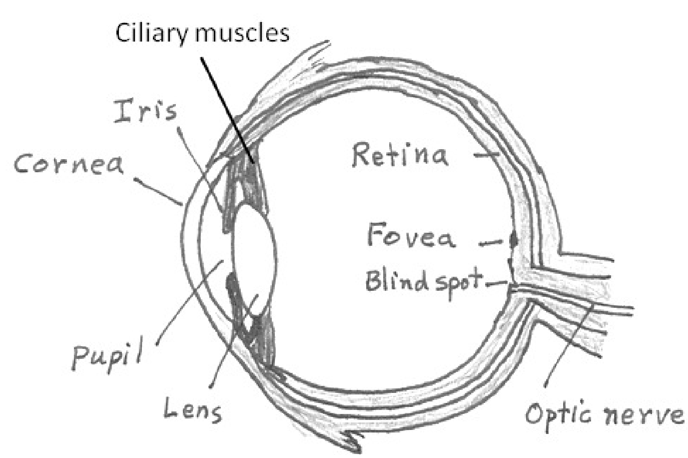

Figure 3.1: Schematic drawing of a human eye.

Accomodation and Near Point

Contrary to the popular belief, most of the bending of light rays in the eye is not done by the eye lens but by the cornea, which is itself a converging lens of focal length approximately 2.3 cm. A finer focusing is done by the eye lens, a converging lens of less power than cornea, having a focal length of approximately 6.4 cm. The ciliary muscles adjust the shape of the eye lens for focusing on the nearby and far objects. By changing the shape of the eye lens the eye can change the focal length of the lens. This mechanism of the eye is called the **accomodation**. When the ciliary muscles are relaxed the lens is thin and has a longer focal length so that the distant objects are properly focused at the retina. For a nearby object, the eye needs a smaller focal length lens, which is achieved by contracting the ciliary muscles and thereby thickening the lens.

The sharpness of the image seen by the eye also depends on the distance from the eye and whether the eye muscles are relaxed or contracted. The nearest point an object can be placed so that eye can form a clear image on the retina by maximum accommodation is called the **near point** of the eye.

To find the sharpest image point for you, hold a sharpened pencil at an arms length and look at its tip with your naked eye. Now, as you bring the pin closer, you should notice that there is point at which the tip of the pencil appears clearest. That would be your near point. For most people the near point is around 25 cm from the eye. For the best view you want whatever you are looking at to be at the near point. Similarly, there is a **far point**, which is the farthest distance an object is clearly visible.

The apparent size of the object perceived by the eye depends on the angle the object subtends on the eye. The same object appears smaller when it is moved farther away. As shown in Fig. 3.2 when the object is at A, it subtends a larger angle at the eye and hence forms a larger image OA′, than when it is moved farther away to B. Since the near point of your eye is the closest distance for clear image you would perceive an object to be the largest and clearest if it is at the near point.

Common Eye Defects

The most common types of eye defects are **near-sightedness** and **far-sightedness** (Fig. 3.3). In a near-sighted eye parallel rays focus in front of the retina. That is, in the near-sighted eye the focal length of the lens is shorter than required to focus on the retina. To increase the net focal length of the combination of the convex lenses (cornea

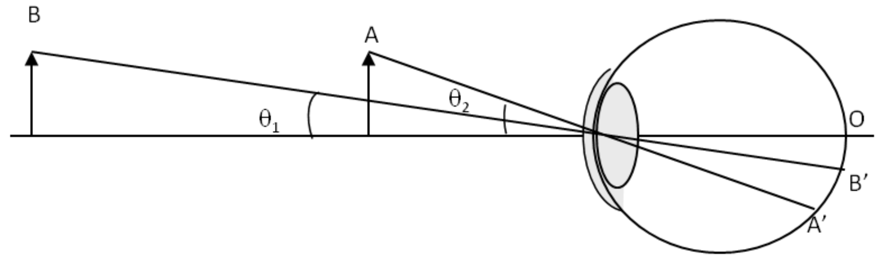

Figure 3.2: Size determined by an eye by angle subtended by it.

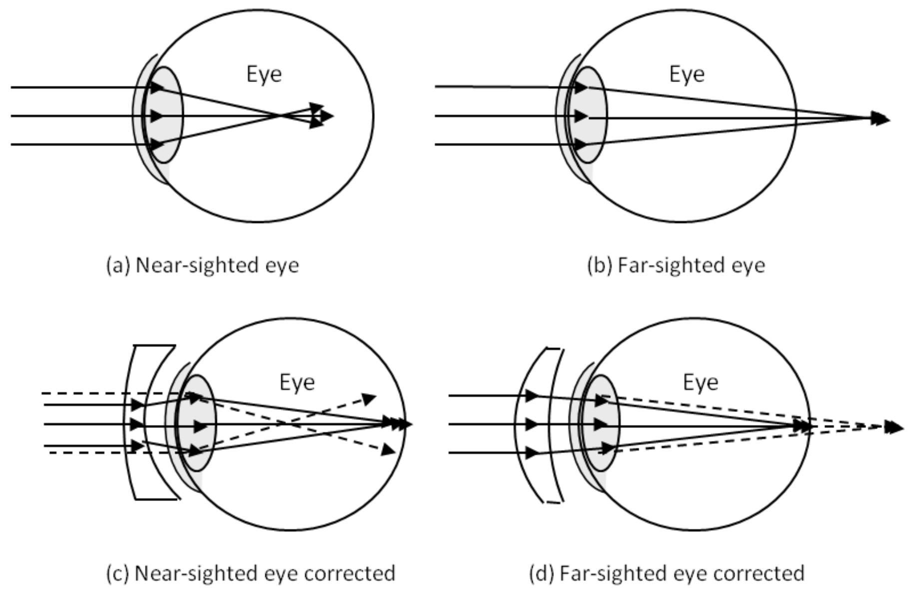

Figure 3.3: Common eye problems related to focusing.

and eye lens) we need to place a concave lens in front of the eye. A concave lens in front of the eye bends the rays outward and helps focuses them on the retina. The far-sighted eye has the opposite problem with parallel rays focusing behind the retina. To correct for this problem one needs a converging lens in front of the eye.

The focal length of corrective lenses is specified by giving its power in diopters, abbreviated as D. The numerical value of the power of a lens in diopters is the inverse of the focal length expressed in meters.

$$P(\text{diopters}) = \frac{1}{f(\text{in meters})}. \tag{3.1}$$

Thus, a prescription of -2.5 D would require a lens of focal length $\frac{1}{2.5}$ m, i.e. 4 cm. The negative sign means that you need a diverging lens.

Example 3.1.1. Effective Focal Length of Eye

The cornea and eye lens have the focal lengths 2.3 cm and 6.4 cm respectively. Find the net focal length and power of the eye.

Solution. The focal length of a combination of lenses add in inverses. Therefore

$$\frac{1}{f} = \frac{1}{f_{\text{cornea}}} + \frac{1}{f_{\text{lens}}} = \frac{1}{2.3 \text{ cm}} + \frac{1}{6.4 \text{ cm}}$$

Hence, the focal length of eye (cornea and lens together)

$$f = 1.78 \text{ cm}.$$

Power of the eye

$$P = \frac{1}{0.0178 \text{ m}} = 59 \text{ D}.$$

Example 3.1.2. Image of an object placed at the near point

The net focal length of an eye is 1.7 cm. An object is placed at the near point of 25 cm. How far behind the lens a focused image is formed?

Solution. We determine the image distance from the lens equation.

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{1.7 \text{ cm}} - \frac{1}{25 \text{ cm}}$$

Hence,

$$q = 1.8 \text{ cm}.$$

Therefore the image is formed 1.8 cm behind the lens. From the magnification formula, we find $m = -\frac{1.8 \text{ cm}}{25 \text{ cm}} = -0.07$. Since $m < 0$ the image would be inverted in orientation with respect to the object. From the absolute value of m we see that the image is much smaller than the object, it is only 7% of the size of the object.

Example 3.1.3. Near-sighted Eye

A near-sighted eye cannot focus on a far object without a corrective lens. Consider a near-sighted eye that has a far point of 52 cm, i.e. when the eye is relaxed the furthest point the person can see clearly is 52 cm from the eye. Find the power of the corrective lens in diopters needed if the lens is places 2 cm from the eye.

Solution. The eye will look at the image formed by the corrective lens. Hence, we need the image of a far away object to form at 52 cm from the front of the eye, which means that the final virtual image will be at an image distance of 50 cm, since the corrective lens is at 2 cm from the eye. Since, the image is on the same side of the lens, the image distance will be negative, $q < 0$. With the object distance at ∞ we find the focal length to be

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} + \frac{1}{-50\ \text{cm}}$$

Hence, $f = -50$ cm. Since the focal length is negative, we will need a diverging lens. The power of the lens in diopters is obtained from the focal length expressed in meters.

$$P = \frac{1}{f} = \frac{1}{-0.5\ \text{m}} = -2\ \text{D}.$$

Example 3.1.4. Far-sighted Eye

Most people become far-sighted with age and have difficulty reading. Consider a far-sighted person whose near-point is 80 cm. What is the power of the corrective lens needed for reading a magazine placed at a distance of 20 cm? Assume, the corrective lens is placed 2 cm from the eye.

Solution. The lens needed should form a virtual image at a distance of 80 cm from the eye when the print is placed 20 cm from the lens. The required focal length of the lens is obtained by the thin lens formula as follows.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{20\ \text{cm}} + \frac{1}{-80\ \text{cm}} = \frac{3}{80\ \text{cm}}$$

Therefore,

$$f = \frac{80}{3}\ \text{cm}.$$

Since, the focal length is positive, we need a converging lens. The power of the lens in diopter is obtained by expressing the focal length in meters and taking the inverse of that.

$$P = \frac{1}{f} = \frac{3}{0.8\ \text{m}} = 3.75\ \text{D}.$$

3.1.2 The Magnifying Glass and the Angular Magnification

We have seen above that when an object is placed within a focal length of a convex lens its image is virtual, erect and larger than the object. Therefore when you look at the object through the convex lens, you find it magnified if the object is within a focal length of the lens. When a convex lens is used for this purpose, it is also called a **magnifying glass** or a **simple microscope** (Fig. 3.4).

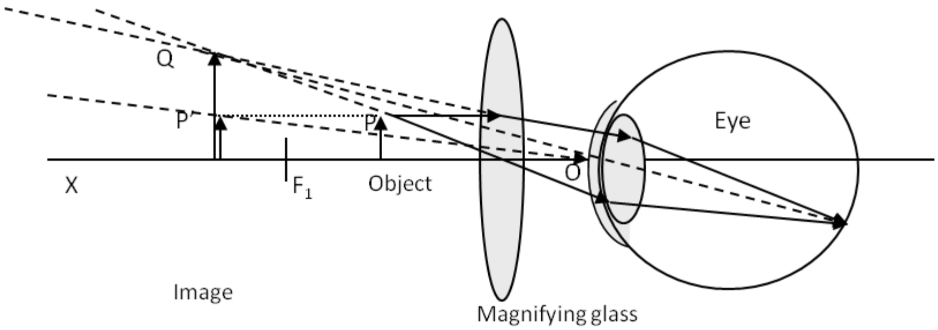

Figure 3.4: The Magnifying glass.

The eye helps determine the magnification of the image. Since, the size perceived by eye is dependent on the angle the image or the object subtends at the eye, we will need to compare the subtended angles of the image and the object placed at the same distance from the eye to get a proper account of the magnification due to the instrument itself. Therefore, the magnification of an image when observed by an eye will be given by its **angular magnification** M which is defined by the ratio of the angle ∠QOX subtended at the eye by the image to the angle ∠P′OX subtended by the object when placed at the same distance as the image.

$$M = \frac{\angle \mathrm{QOX}}{\angle \mathrm{P'OX}}. \tag{3.2}$$

Let us find the magnification by a lens of focal length f when the magnifying glass is very close to the eye. The closest distance the view is clearest when we place whatever we are observing at the near point, which we will assume to be 25 cm from the eye - the standard near point distance. The eye muscles are contracted in this state. In the figure the eye looks at the image formed by the magnifying lens. Therefore, the object will be placed at some distance p so that image will form at 25 cm from the eye. We can now use the lens formula to determine p in terms of f. Note that the image distance will be negative since the image is formed on the same side as the object.

$$\frac{1}{p} + \frac{1}{-25\ \mathrm{cm}} = \frac{1}{f} \quad \Longrightarrow \quad p = \frac{f \times 25\ \mathrm{cm}}{f + 25\ \mathrm{cm}} \tag{3.3}$$

Let h_o and h_i denote the height of the object and the image respectively. The **linear magnification** m is the ratio of these heights.

$$m = \frac{h_i}{h_o} \tag{3.4}$$

We need the angles $\theta = \angle \mathrm{P'OX}$ and $\theta' = \angle \mathrm{QOX}$ subtended by the object and image at the eye when they are placed at the distance of 25 cm from the eye. From the geometry, we find the subtended angles to be:

$$\theta \approx \tan\theta = \frac{h_o}{25 \text{ cm}}$$
$$\theta' \approx \tan\theta' = \frac{h_i}{25 \text{ cm}}$$

Therefore the angular magnification for a magnifying glass is equal to the linear magnification.

$$m = \frac{h_i}{h_o} = \frac{\theta'}{\theta} = M. \tag{3.5}$$

Since the linear magnification is related to the object distance p and the image distance q we can write the angular magnification M of the eyepiece/eye system as

$$M = \frac{\theta'}{\theta} = \frac{h_i}{h_o} = -\frac{q}{p} = 25 \text{ cm} \times \frac{f + 25 \text{ cm}}{f \times 25 \text{ cm}} = 1 + \frac{25 \text{ cm}}{f}. \tag{3.6}$$

3.1.3 Compound Microscope

We saw above that the simple convex lens can give a magnified image. It is hard to get large magnification this way. A magnification more than 5x is difficult without introducing aberration in the image. To get increased magnification the simple magnifying glass can be combined with one or more lenses.

The simplest such combination contains two convex lenses, **objective** and **eyepiece** or **ocular** (Fig. 3.5). The objective is a convex lens of short focal length (i.e. high power) while the eyepiece is a convex lens of longer focal length.

The task of the objective is to form a magnified real image inside the focal length of the eyepiece. Therefore, one places the specimen a little bit outside of the focal point of the objective. The lenses are arranged so that a magnified real image by the objective lens is formed inside the focal length of the eyepiece. The image by the eyepiece is therefore a magnified virtual image.

The eye looks at this virtual image of the eyepiece. The object for the lens in the eye is the virtual image formed by the eyepiece. The virtual image formed by the eyepiece is well outside of the focal length of the eye, and hence the eye forms a real image at the retina.

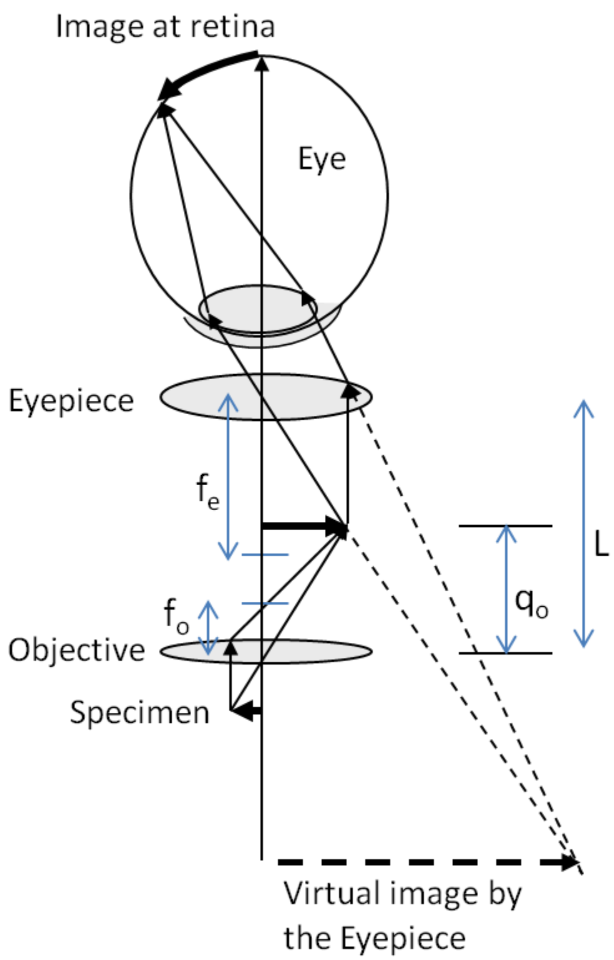

Figure 3.5: The Compound Microscope.

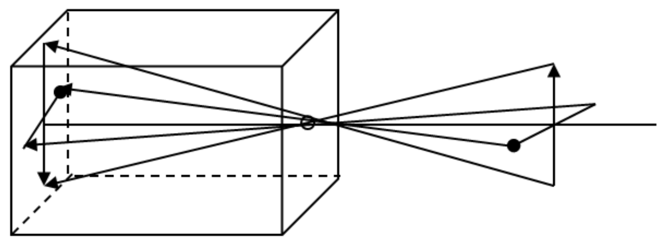

Figure 3.6: A pin-hole camera.

The magnification of the microscope comes from the product of magnification by the objective m_o and the angular magnification M by the eyepiece/eye system.

$$m = -\frac{q_o}{p_o} \approx -\frac{q_o}{f_o} \quad \text{(linear magnification by objective)} \tag{3.7}$$

$$M = 1 + \frac{25\ \text{cm}}{f_e} \quad \text{(angular magnification by eyepiece/eye)} \tag{3.8}$$

where f_o and f_e are the focal lengths of the objective and the eyepiece respectively. Note that the angular magnification from the eyepiece/eye system is same as obtained above for the simple magnifying glass. This is not a surprise since the eyepiece/eye system is identical to the viewing through a magnifying glass, and the same physics would apply here.

The **net magnification** M of the compound microscope will be the product of the linear magnification of the objective and the angular magnification of the eyepiece/eye:

$$M_{net} = -\frac{q_o\,(f_e + 25\ \text{cm})}{f_o f_e}. \tag{3.9}$$

3.1.4 Pin Hole Camera

A **pin-hole camera** is an optical device that projects inverted image of an object on a screen without using any lens or mirror. It can be easily made from a cardboard box and a wax paper. Replace one side of the cardboard box by a wax paper to serve as the screen. Make a small hole in the middle of the opposite side. Rays of light from an object pass the hole and meet on the screen and make an inverted image as shown in Fig. 3.6.

A detector such as a photographic plate may be placed in place of the wax paper at the back side to record the image. Pin-hole camera is useful for viewing a solar eclipse where you can look at the image of the sun instead of looking at the sun directly.

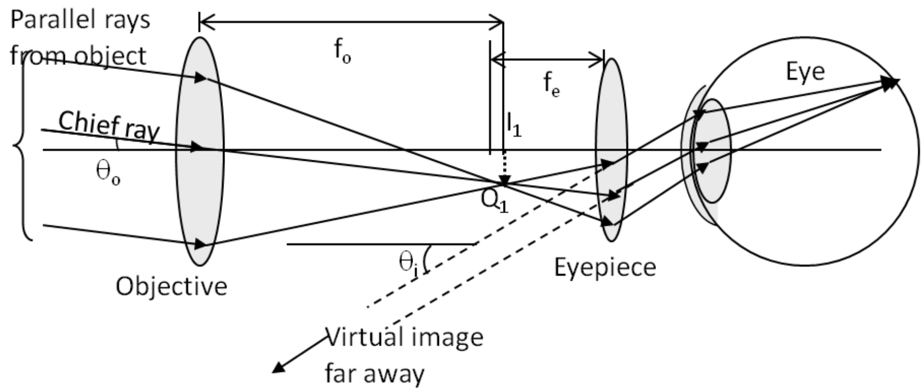

Figure 3.7: A refracting telescope.

3.1.5 Refracting Telescope

Historically, refracting telescope was the first telescope to be used for observing astronomical objects. Although, the accounts vary as to who invented the first refracting telescope, but records suggest that Galileo built his own refracting telescope and was the first one to make observations in astronomy using a telescope. Refracting telescope uses two lenses to view an image of distant objects (Fig. 3.7). The first lens, called the **objective**, forms a real image I_1Q_1 within the focal length of the second lens, which is called the **eyepiece**. The image of the first lens acts as the object for the eyepiece which forms a magnified virtual image which is observed by the eye.

The arrangement of lenses in a refracting telescope looks similar to that in a microscope. However, there are important differences between the two. In a telescope the real object is far away and the intermediate image is smaller than the object while in the microscope the real object is very near and the intermediate image is magnified with respect to the object. In both telescope and the compound microscope the eyepiece magnifies the intermediate image but in the telescope that is the only magnification.

The **magnification of a telescope** is defined by the ratio of the angle θ_i subtended at the eye by the virtual image of the eyepiece to the angle θ_o subtended by the object.

$$M = \frac{\theta_i}{\theta_o}. \tag{3.10}$$

By using the geometry of the chief ray shown in Fig. 3.8 you can show that magnification is also equal to the ratio of the focal length

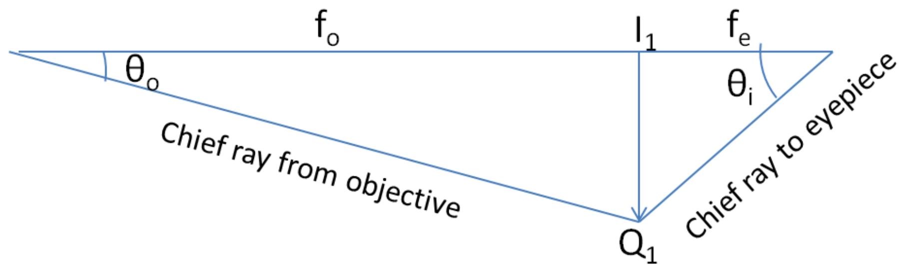

Figure 3.8: Geometry of the chief rays with the assumption that the distances between the image I_1Q_1 and the lenses are each approximately equal to the respective focal lengths.

of the objective to the focal length of the eyepiece.

$$M = \frac{\theta_i}{\theta_o} = -\frac{f_o}{f_i}. \tag{3.11}$$

Typical eyepieces have focal lengths of 2.5 cm, and 1.25 cm. If the objective of the telescope has a focal length of 1 meter, then these eyepieces would result in magnifications of 40-times, and 80-times respectively. The angular magnifications would make the image appear 40-times, or 80-times closer.

In a refracting telescope the task of the objective lens is to collect light from the source - the more the better. For fainter objects in the sky you need an objective lens of larger diameter. The largest refracting telescope in the world is the 40-inch diameter Yerkes telescope located at Lake Geneva, Wisconsin (Fig. 3.9) and operated by the University of Chicago.

It is very difficult and expensive to build large refracting telescopes. You need large defect free lenses, which in itself is a technically demanding task. Refracting telescope basically looks like a tube with support structure to rotate it in different directions. A refracting telescope suffers from a number of defects. The aberration of lenses causes the image to be blurred. Also as the lenses become thicker for larger lenses more light is absorbed making the faint stars more difficult to observe. Large lenses are also very heavy and deform under their own weight. Some of these problems with refracting telescope is addressed by avoiding refraction for collecting light and instead using a curved mirror in its place as devised by Isaac Newton. These telescopes are called the reflecting telescopes which we discuss next.

Figure 3.9: Yerkes observatory in Wisconsin (USA) built in 1897 has a large objective lens of 40 inches diameter and tube length of 62 feet. (Photo credit: Yerkes Observatory, University of Chicago)

3.1.6 Reflecting Telescope

Isaac Newton designed the first reflecting telescope around 1670 to solve the problem of chromatic aberration that happens in all refracting telescopes. Due to chromatic aberration a rainbow appears around the image and image appears blurred. In the reflecting telescope, light rays from a distant source fall upon the surface of a concave mirror fixed at the bottom end of the tube. The concave mirror focuses the rays on its focal plane. The design problem is how to observe the focused image. Newton used a design in which the focused light from the concave mirror was reflected to one side of the tube into an eyepiece (Fig. 3.10a). This arrangement is common in many amateur telescopes and is called the **Newtonian design**.

Some telescopes reflect the light back towards the middle of the concave mirror by using a convex mirror. In this arrangement the light-gathering concave mirror has a hole in the middle (Fig. 3.10b). The light then is incident on an eyepiece lens. This arrangement of the objective and eyepiece is called the **Cassegrainian design**. Most big telescopes, including Hubble telescope, are of this design (Fig. 3.11). There are other arrangements also possible. Sometimes one puts a light detector right at the spot where light is focused by the curved mirror.

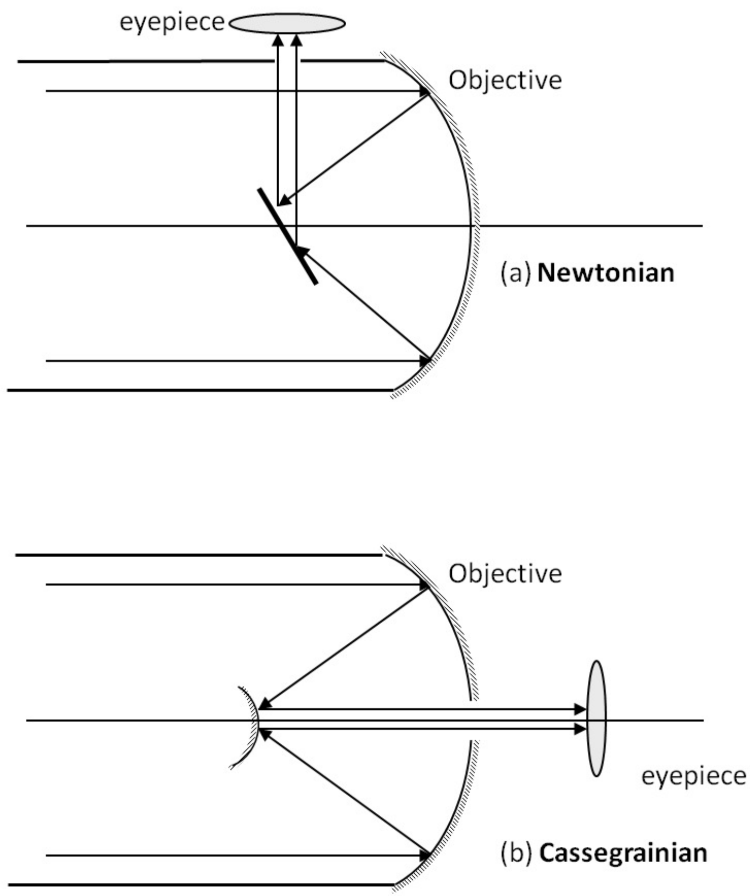

Figure 3.10: Reflecting telescopes: (a) Newtonian design, (b) Cassegrainian design.

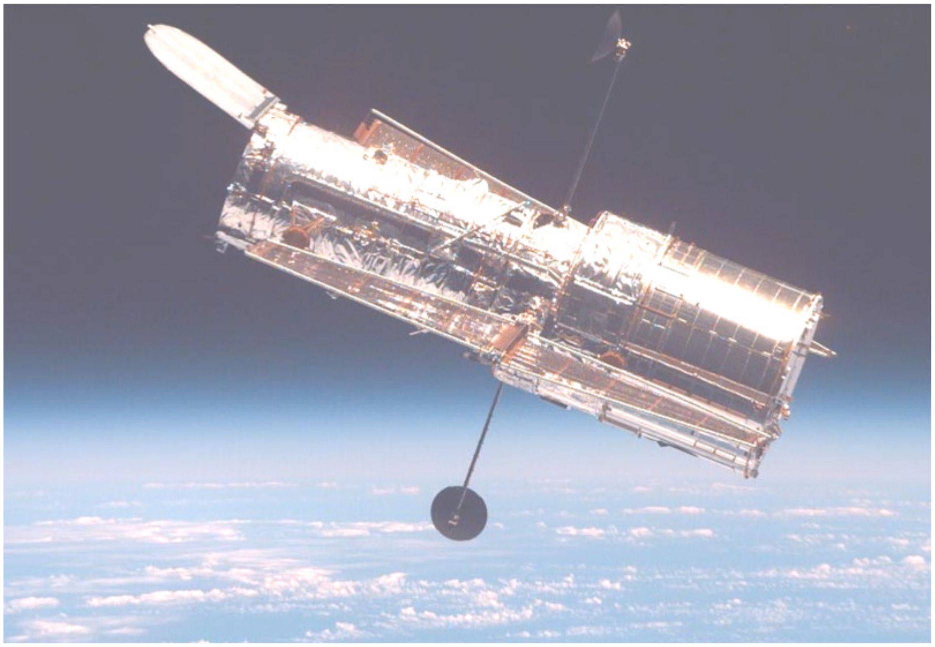

Figure 3.11: Hubble Space Telescope as seen from the Space Shuttle Discovery on mission STS-82. (NASA, Feb 1997, see http://hubble.nasa.gov)

Most of the research telescopes are now of reflecting type. One of the largest telescopes of this kind is the Hale 200-inch or 5-meter built on Mount Palomar in southern California. The measurement 200 inches here refers to the diameter of the mirror. In the year 2012, the largest telescope in the world was the 10 meter Keck telescope at the Keck observatory on the summit of the dormant Mauna Kea volcano, in Hawaii. At the Keck observatory there are two 10 meter telescopes. They are not single mirror, but instead made up of 36 hexagonal mirrors each. Furthermore, the two telescopes on the Keck can work together through an interference which increases their power to an effective 85-meter mirror!

Hubble telescope (Fig. 3.11) is another large reflecting telescope with the primary mirror of diameter 2.4 meter. Hubble was put in the orbit around earth in 1990.

The angular magnification M of a reflecting telescope is also given by a similar formula as that of the refracting telescope.

$$M = -\frac{f_o}{f_e} \tag{3.12}$$

where f_o is half of the radius of curvature of the objective mirror and f_e is the focal length of the eyepiece. Thus, making large objective not only helps collect more light into the telescope it also helps with the magnification of the image.