7.4 TURBULENCE

When you let water out of a tap, first you get a smooth laminar flow when the speed of flow is not high, but as you turn up the flow it becomes more roiled and turbulent as shown in a numerical simulation in Fig. 7.14. You see the same transition from a laminar flow to a turbulent flow in all fluids when speed is increased. The mathematics needed to understand the turbulence phenomena is quite complicated. Only recently with the advent of computers has it become possible to find the numerical solutions of some cases of interest such as the flow of air around automobiles and airplanes.

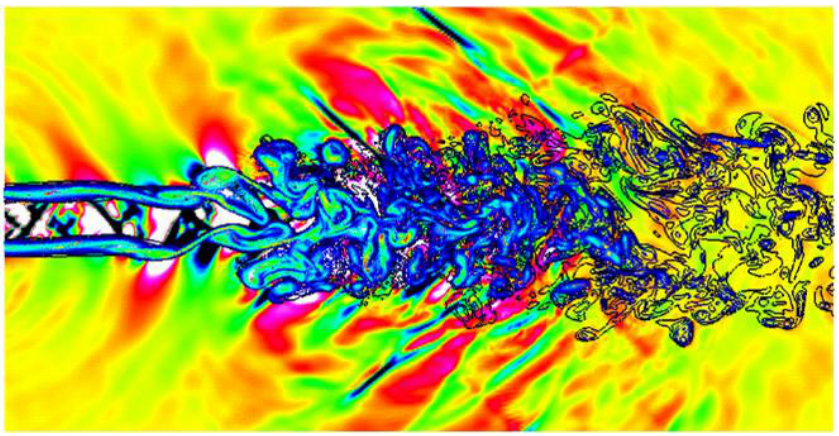

Figure 7.14: Numerical simulations of supersonic turbulent jet. The simulations were performed by Jonathan Freund, Parviz Moin and Sanjiva K. Lele of Center of Turbulence Research, Stanford University.)

The easiest way to decide whether a flow will be turbulent or laminar is based on a dimensionless parameter called Reynolds number, named after the British engineer Osborne Reynolds (1842-1912) who introduced it in a paper in 1883. Basically, **Reynolds number** (Re) is dimensionless combination of the momentum density, an inertial property, the dissipative property as given by viscosity, and a characteristic length scale of the system, such as diameter of the tube in which the flow occurs.

$$Re = \frac{\rho v D}{\eta} \tag{7.36}$$

where ρ is the density, v the flow speed, η the coefficient of viscosity, and D a characteristic length, e.g. diameter of the cross-section of the tube if the flow is in a cylindrical pipe. Let us check the dimensionless aspect of Re. Putting the units for various quantities, we do find that all the units cancel out.

$$[Re] = \frac{[\rho][v][D]}{[\eta]} = \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}}{\text{s}} \times \text{m} \times \frac{\text{m.s}}{\text{kg}} = \text{Dimensionless}$$

The Reynolds number of a flow is higher for higher speed flow. Reynolds number is also higher for a fluid flowing through a thicker pipe. Experiments show that, regardless of the chemical make-up, the flow is laminar if the Reynolds number for the flow is less than about 2,000, turbulent for Reynolds number above 4000, and in-transition for Reynolds number between 2,000 and 4,000.

If a fluid is less viscous then it will become turbulent at lower speed. Viscosity tends to maintain the layer flow. With increase in speed, the liquid molecules migrate more rapidly from one layer into the other destroying any ordering and the flow develops eddies or vortices at multiple scales. Turbulence mixes fluid rather well. In the internal combustion engine, for instance, air and fuel mix well due to the turbulence in the chamber, which is necessary for a more efficient burn. Turbulence is also an important factor that determines the flight path of baseballs and tennis balls. For a "standard air at sea level", the density and viscosity of air are 1.23 kg/m^3 and 1.73×10^{-3} Pa.s respectively. If a ball of diameter 5 cm is moving at 201 km/h or 126 mph, then from the perspective of the ball the Reynolds number of air will be 2000. Spinning of a baseball gives the air an effective speed greater than the nominal speed, and hence a higher Reynolds. Similarly, the dimples in a golf ball help with the development of turbulence around the ball, which make the drives longer.

Example 7.4.1. Reynolds number of water

A 2-cm diameter pipe carries pure water of density 1 g/cc and viscosity 1.002 centipoise. At what speed will the Reynolds number be 2000?

Solution. For Reynolds number to be 2000, the water has to flow at 10.02 cm/s obtained directly from the formula. At that flow speed, 113 liters of water will flow per hour. Compare this to the normal kitchen water fountain flow rate in the US which is more than 1000 liters per hour. Clearly when you turn on the kitchen faucet on full, the flow is turbulent. This is visible as frothy and bubbly flow when the water comes out of the faucet.