1.5 ORDER OF MAGNITUDE ESTIMATES

It is a common misconception that physics is an exact science. One of the most important skills in developing an understanding of a physical phenomenon is the ability to understand it qualitatively, and figure out roughly how things might work. The process of determining a reliable rough estimate usually involves identification of correct principles and a good guess about the relevant variables. Estimating based on physical principles is very useful in developing physics intuitions. Note that estimating does not mean randomly guessing a number or a formula, but it means employing reasonable physics to relate variable which ought to be related based on sound physical reasoning. Enrico Fermi of the University of Chicago was famous for making rough estimates in the so-called "back of the envelop" calculations, and therefore these types of estimations are also called Fermi Problems.

Often, it is possible to estimate the value of an unknown quantity within a factor of 10 of the exact answer. A factor of 10 is also called one order of magnitude. A factor of 100 corresponds to two orders of magnitude and so forth.

To make some progress in estimating, you need to have some definite ideas about how variables may be related. The following strategies can help you in practising the "art of guessing".

- When dealing with an area or a volume of a complex object, introduce a simple model of the object such as a sphere or a cube.

- Guess the linear dimension, such as the radius of the sphere first, and then use your guess to obtain the volume or area.

- There is no need to go beyond one significant figure.

- Finally, check to see if your answer is reasonable. If you get some wacky answer, check to see if your units are right.

Example 1.5.1. Counting marbles in a jar. Estimate the number of marbles in a jar that is 10 cm in diameter and 20 cm tall as shown.

Solution. Counting the marbles in the Fig. 1.7 across we find that there are approximately 10 marbles in the diameter of the jar. Now, since the jar is 10 cm in diameter, we approximate the diameter of one marble to be 1 cm.

Gauss proved that when spheres are packed as well as it can be, they occupy 74% of the space [You need this information from geometry to make a better guess. But if you didn't know this, you could still make a reasonable guess about the percentage of space occupied by the marbles, and that would be OK.] Now we know how to estimate the number of marbles in the jar: divide 74% of the volume of the jar by the volume of one marble. The actual number of marbles is probably less than this number since you would not get the optimal packing assumed here. I leave the final number for you to work out.

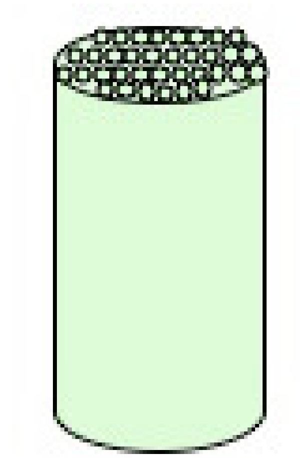

Figure 1.7: Marble Jar

Example 1.5.2. The spherical cow. Estimate the area of cowhide on the body of one thousand cows.

Solution. The area of the surface of a cow is very difficult to figure out exactly. But it is possible to approximate a cow by a sphere of diameter about 1 meter for the purposes of surface area as indicated in Fig. 1.8. We could also assume a cow as a cube of 1-meter side with more or less the same result. Multiplying the surface area of one cow by 1000 we get an estimate of the area of the cowhide of 1000 cows.

$$A = \frac{4\pi\,(1\text{ m})^2}{\text{cow}} \times 1000 \text{ cows} \;\longrightarrow\; 10^4 \text{ m}^2.$$

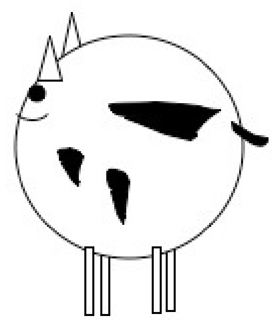

Figure 1.8: Provervial spherical cow.