4.2 Charge of Electron

The knowledge of the charge of an electron can be traced to Faraday's experiments in electrolysis in 1830s. Faraday showed that a mole of singly charged ions carried a charge of 96,500 C. Therefore, the charge on individual singly charged ions would be obtained by dividing this charge by Avogadro number, $N_A = 6.022 \times 10^{23}$, which is the number of ions in one mole.

$$q = \frac{96,500\ \mathrm{C}}{N_A} \approx 1.6 \times 10^{-19}\ \mathrm{C}. \tag{4.7}$$

J.J. Thomson conducted experiments on charged mist to deduce the electronic charge directly but only got values that were in the ballpark of this value.

4.2.1 Millikan's oil drop experiment

In 1908 the American physicist, Robert Millikan, who conducted careful experiments for directly determining the charge on droplets that had been charged to some unknown amounts. The experiment relies on careful measurements on falling charged oil droplets through air to deduce the charge on each droplet. Then, a common factor that multiplied with an integer was sought that would match the charges on various droplets. The common factor was identified to be the charge on one electron. This experiment is called Millikan's oil drop experiment. The schematics of the Millikan oil drop experiment is shown in Fig. 4.4.

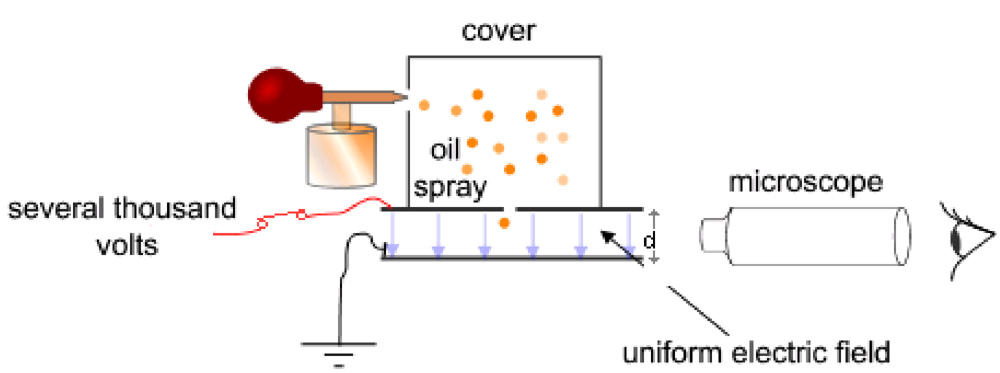

Figure 4.4: Schematic view of the Millikan oil drop experiment. Credits: Wikicommons.

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I will describe here only the most basic aspects of the Millikan oil drop experiment. The actual experiment suffers from several additional complications that must be taken into account. In this experiment we conduct two experiments on each drop.

Experiment # 1: Terminal velocity in the absence of electric field

In Millikan's oil drop experiment an atomizer releases tiny charged oil droplets that fall under gravity in air. Due to the viscosity of air the droplets reach a terminal velocity. Let m be the mass, R the radius of the spherical oil drop, η the viscosity coefficient of air, and v the terminal velocity. Balancing forces we obtain

$$mg = 6\pi\eta R v. \tag{4.8}$$

Let ρ be the density of oil. Then we can write

$$m = \frac{4}{3} 4\pi R^3 \rho. \tag{4.9}$$

Putting this in Eq. 4.8 and solving for R we get

$$R = \sqrt{\frac{9\eta v}{2\rho g}}. \tag{4.10}$$

Therefore, a measurement of terminal velocity can be used to obtain the mass of the droplet.

$$m = \frac{4}{3} 4\pi\rho \left(\frac{9\eta v}{2\rho g}\right)^{3/2}. \tag{4.11}$$

Experiment # 2: Terminal velocity in the presence of electric field

Now, we turn on the electric field between the plates. With the plate separation d and voltage V between the plates, the magnitude of the electric field between the plates will be

$$E = \frac{V}{d}. \tag{4.12}$$

With electric field on, the forces on the charged droplet will not be balanced since the droplet would be currently falling at the terminal velocity v when there was no electric force. With electric field on, the droplet comes to a new terminal velocity. Let the new terminal velocity be denoted by v_E. Let the charge on the droplet under observation be $q < 0$ and the electric field be pointed down. That would give an electric force of qE on the droplet pointed up. Let us examine the forces on a droplet that is rising with a constant speed when the electric field is on. Balancing force now gives the following equation.

$$qE - mg - 6\pi\eta R v_E = 0. \tag{4.13}$$

Solving for q we get

$$q = \frac{1}{E}\,[mg + 6\pi\eta R v_E] \tag{4.14}$$

Using Eqs. 4.10, 4.11, and 4.12 we obtain

$$q = \frac{d}{V}\,[v + v_E]\, 6\pi\eta \sqrt{\frac{9\eta v}{2\rho g}}. \tag{4.15}$$

The atomization process does not produce droplets of same size or same charge. The experimentally obtained charges q on a number of droplets are examined and analyzed to deduce the elementary charge.

Often, while a particular drop is being observed with the field on, the droplet will suddenly accelerate as if it has picked up some more charge or lost some of its charge. This discontinuous jump in charge content of the drop gives a new terminal velocity. Suppose that when a charge q_1 was on the droplet of mass m the terminal velocity with field on was $v' = v_1$ and when the charge content on the ball changed suddenly it settled on another terminal velocity $v' = v_2$. Assuming negligible change in mass of the droplet we can obtain the ratio of the two charges

$$\frac{q_1}{q_2} = \frac{v + v_1}{v + v_2}. \tag{4.16}$$

If the ratio q_1/q_2 turn out to be expressible in ratio of integers, it will prove that the charges are themselves an integral multiple of an elementary charge. Millikan's oil drop experiment established the following two fact about the elementary charge.

Result 1: Electric charge is quantized. All electric charges are integral multiples of an elementary charge e.

Result 2: The elementary charge has the value $e = 1.60 \times 10^{-19}$ C.

From the e/m value of J.J. Thomson and e value of Millikan we can calculate the mass m of an electron.

$$m = \frac{e}{e/m} = \frac{1.60 \times 10^{-19}\ \text{C}}{1.76 \times 10^{11}\text{C.kg}^{-1}} = 9.1 \times 10^{-31}\ \text{kg}. \tag{4.17}$$

Example 4.2. When Robert Millikan studied one of the oil drops he measured the falling time (t_0) with no electric field and rising time (t_E) with electric field on for a distance of 10.21 mm. The successive measurements for the falling and rising times showed a consistent value for the fall time without the electric field but a wide variation of the time with the electric field on.

$$\begin{aligned}(t_0, t_E) =& [(18.689, 17.756), (18.73, 17.778), (18.686, 45.978), (18.726, 45.87),\\ &(18.772, 45.716), (18.74, 45.758), (18.724, 694.0), (18.72, 27.95),\\ &(18.816, 118.388), (18.816, 45.03), (18.716, 34.564), (18.804, 44.826),\\ &(18.746, 117.198), (18.746, 44.784)]\end{aligned}$$

The variation in the times when the electric field was on was attributed to different charges on the same droplet. By assuming that the difference in the charges of the same droplet must be an integral multiple of a fundamental unit of charge he was able to deduce a value of the electronic charge. To apply the electric field a voltage of 5100 V across the plates separated by 16 mm was applied. The density of the oil was 0.9199 g/cm^3 and the viscosity of air was 1.824×10^{-5} kg.m/s. Use this data to deduce the electronic charge.

Solution.

We want the multiple readings of $(v+v_E)$ to be in proportion of integers. First, we will average t_0 to get the average time for the fall without electric field. This will give us the best value for v.

$$v = \frac{h}{t_0} = \frac{10.21\ \text{mm}}{18.745\ \text{s}} = 0.545\ \text{mm/s}.$$

Various rise times give different values of v_E corresponding to different amount of charge on the drop. In Eq. 4.16, they are referred to v_1 and v_2 for two such data.

$$\begin{aligned} v_E(\text{mm/s}) =& [0.575, 0.574, 0.222, 0.223, 0.223, 0.223, \\ & 0.0147, 0.365, 0.0862, 0.227, 0.295, 0.228, 0.087, 0.228] \end{aligned}$$

Suppose you sibtract 1 from both sides of Eq. 4.16. you will get the following for the difference of two charges.

$$q_1 - q_2 = \frac{v_1 - v_2}{v + v_2}\, q_2.$$

We require that this be an integer multiple of the electronic charge e.

$$\frac{v_1 - v_2}{v + v_2}\, q_2 = ne.$$

Therefore,

$$v_1 - v_2 = ne_2, \quad e_2 = e/[(v + v_2)q_2].$$

So, if we compare all other velocities to a particular velocity we should get integers times some fixed number e_2. Therefore, let us look at the difference between the velocities with the smallest velocity, which is 0.0147 mm/s. We get the following

$$\begin{aligned} v_1 - v_2(\text{mm/s}) =& [0.56, 0.56, 0.21, 0.21, 0.21, 0.21, \\ & 0, 0.35, 0.07, 0.21, 0.28, 0.21, 0.07, 0.21] \end{aligned}$$

Now, we can try to get smallest integers n for each $v_1 - v_2$. Suppose, we divide each by the smallest one in the list here, which would be 0.07 mm/s.

$$\begin{aligned} (v_1 - v_2(\text{mm/s}))/(0.07\text{mm/s}) = & [7.8, 7.8, 2.9, 2.9, 2.9, 2.9, \\ & 0, 4.9, 1.0, 3.0, 3.9, 3.0, 1.0, 3.0] \end{aligned}$$

Let the charge was q_0 for the reference case and q_1 when the difference in rise time was 0.07 mm/s. The formula for each charge is

$$q = \frac{d}{V}(v + v_E) 6\pi\eta \sqrt{\frac{9\eta v}{2\rho g}}.$$

We have

$$d = 16\ \text{mm}, \quad V = 5100\ \text{V}, \quad \eta = 1.824 \times 10^{-5}\ \text{kg.m/s}, \rho = 0.9199\ \text{g/cm}^3.$$

Therefore,

$$q_0 = \frac{0.016\,\mathrm{m}}{5100\,\mathrm{V}}(0.000545\,\mathrm{m/s} + 0.0000147\,\mathrm{m/s})6\pi \times 1.824 \times 10^{-5}\,\mathrm{kg.m/s}$$
$$\sqrt{\frac{9 \times 1.824 \times 10^{-5}\,\mathrm{kg.m/s} \times 0.000545\,\mathrm{m/s}}{2 \times 919.9\,\mathrm{kg/m^3} \times 9.81\,\mathrm{m/s^2}}} = 1.344 \times 10^-18\,\mathrm{C}.$$

$$q_1 = \frac{0.016\,\mathrm{m}}{5100\,\mathrm{V}}(0.000545\,\mathrm{m/s} + 0.000087\,\mathrm{m/s})6\pi \times 1.824 \times 10^{-5}\,\mathrm{kg.m/s}$$
$$\sqrt{\frac{9 \times 1.824 \times 10^{-5}\,\mathrm{kg.m/s} \times 0.000545\,\mathrm{m/s}}{2 \times 919.9\,\mathrm{kg/m^3} \times 9.81\,\mathrm{m/s^2}}} = 1.518 \times 10^-18\,\mathrm{C}.$$

The difference in their charges is

$$q_1 - q_0 = 1.518 \times 10^-18\,\mathrm{C} - 1.344 \times 10^-18\,\mathrm{C} = 1.74 \times 10^{-19}\,\mathrm{C}.$$

If we assume that the smallest difference in the rise times occur as a result of capture of one electron by the drop, we can deduce that the electronic charge is $e = 1.74 \times 10^{-19}\,\mathrm{C}$, which is not too far from the accepted value of $e = 1.60 \times 10^{-19}\,\mathrm{C}$. Working with other pair of data produced additional results for e.