7.10 PROBLEMS

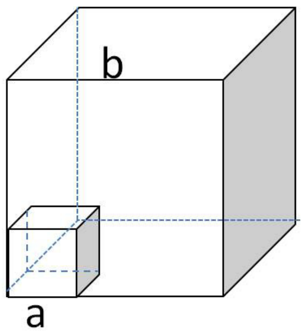

Figure 7.35: Problem 7.10.1.

Problem 7.10.1. A cube of side a is cut out of another cube of side b as shown in Fig. 7.35. Find the location of the CM of the structure. Hint: Think of the missing part as a negative mass overlapping a positive mass.

Problem 7.10.2. Find the CM of cone of uniform density that has a radius R at the base, height h and mass M.

Ans: $Z_{cm} = \frac{1}{4}h$.

Problem 7.10.3. Find the CM of a thin wire of mass m and length L bent in a semicircular shape.

Ans: $Y_{cm} = \frac{2}{\pi^2}L$.

Problem 7.10.4. Find the CM of a uniform thin semicircular plate.

Ans: $Y_{cm} = \frac{4}{3\pi}R$.

Problem 7.10.5. A gun of mass 1.5 kg fires bullets of mass 100 grams each. The bullets come out at a speed of 400 m/s. If each firing lasts 200 msec, what is the average recoil force on the gun?

Ans: 200 N, towards gun. The **recoil force** is the force applied by the exiting bullet on the gun.

Problem 7.10.6. A tennis ball machine of mass M initially at rest on a frictionless surface shoots tennis balls of mass m. When a ball comes out of the muzzle the ball has a velocity of magnitude v_0 with respect to the ball machine and directed at an angle θ with respect to the horizontal direction as shown in Fig. 7.36. Find the velocity of the ball machine immediately after the ball leaves. (Be careful in this problem since the velocity of the ball is given in relation to an accelerating object.)

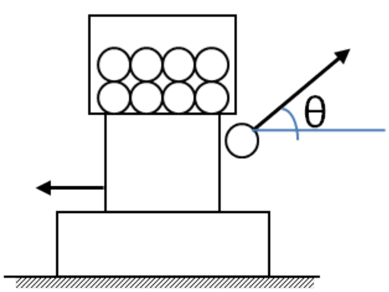

Figure 7.36: Problem 7.10.6.

Ans: Speed = $\left(\frac{M}{M-m}\right) v_0 \cos\theta$.

Problem 7.10.7. There are N cannons on a railway car. The total mass of the car and the cannons is M. The cannons can fire cannon balls of mass m at a speed u with respect to the car regardless of the state of motion of the car. Initially the car is at rest. If cannons are fired horizontally in the direction of the back of the car, the car moves forward. (a) What is the speed of the car when all cannons are fired at the same time? (b) What is the final speed of the car if cannons are fired one after another?

Problem 7.10.8. A 65-kg person jumps from first floor window of a burning building and lands almost vertically on the ground with the horizontal component of the velocity of 3 m/s and the vertical component of velocity of -9 m/s with the axes pointed horizontally in the plane of the motion and vertically up. Upon impact with the ground he is brought to rest in a short time. The force experienced by his feet are different depending on whether he keeps his knees stiff or bends them. Find the forces on his feet in the two cases. (a) First find the impulse on the person from the impact on the ground. Calculate both its magnitude and direction. (b) Find the average force on the feet if the person keeps his leg stiff and straight and his CM drops by only 1 cm vertically and 1 cm horizontally during the impact. You will need to find the time the impact lasts by making reasonable assumptions about the deceleration. Although the force is not constant during the impact, working with constant average force for this problem is acceptable. (c) Find the average force on the feet if the person bends his legs throughout the impact so that his CM drops by 50 cm vertically and 5 cm horizontally during the impact. (d) Compare the results of part (b) and (c), and draw conclusions about which way is better.

Ans: (a) 703 N.s, 106°. (b) $|F_x| =$ 29,100 N, $|F_y| =$ 307,000 N. (c) $|F_x| =$ 5,850 N, $|F_y| =$ 6,080 N.

Problem 7.10.9. A spherical rain drop of radius R and mass M falls vertically through a cloud layer. The drop enters the cloud layer at a height H from the ground and exits the cloud at a height h. While inside the cloud the drop accumulates water molecules so that its mass and size grow with time. The mass of the rain drop at time t when inside the cloud is given as $m(t) = M + \alpha t$, with $t = 0$ at the time the drop enters the cloud. Find the speed of the rain drop as a function of time when it is inside the cloud if it has a speed of v_0 immediately before entering the cloud.

Problem 7.10.10. A chain of mass M and length L is suspended vertically with its bottom touching a pan of mass m on a scale. The chain is then let go from rest. As chain falls, the scale reads the force on the pan needed to balance the force of the chain that is on the pan and the chain link that is impacting the pan at that instant. Assume each chain link is infinitesimally small and comes to rest instantaneously upon impact so that at each instant the entire momentum of an infinitesimal part of the chain link falling on the pan is transferred to the pan. (a) Find an expression of the reading of the scale when a length y has fallen. (b) What is the reading on the scale at the instant the whole chain has fallen to the fan.

Ans: (a) $mg + 3\frac{xM}{L}g$, (b) $mg + 3Mg$.

Problem 7.10.11. Two blocks of mass m each are attached to the ends of a spring of spring constant k. They are placed on a frictionless

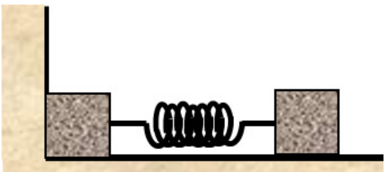

Figure 7.37: Problem 7.10.11.

surface and pushed against a wall with a horizontal force of magnitude F till the spring is compressed by a distance D. The masses are then released (Fig. 7.37). (a) What is the magnitude of the force F applied to hold the masses in place before they are released? (b) What is the amount of normal force from the wall immediately before the masses are released? (c) What are the forces acting on the masses immediately after the masses are released? (d) When does the normal force from the wall stop acting on the mass? (e) Find the position of the CM of the two blocks as a function of time taking the zero of time at the instant they are released.

Ans: (a) kD. (b) kD. (c) Spring, Normal from wall, Weight, Normal from floor. (d) At the instant the spring is not compressed or extended. (e) Hint: After the block leaves the wall the CM moves at a constant velocity.