4.5 The Bohr Model of Atom

4.5.1 Bohr's Assumptions

1. **Bohr orbits**. The electron in a Hydrogen atom orbits the nucleus in stable circular orbits. This assumption is now known to be false since one cannot localize an electron in any orbit!

2. **Angular momentum quantization**. The angular momentum L of the electron is restricted to integer multiples of $\hbar$.

$$L = n\,\hbar, \qquad n = 1, 2, 3, \cdots , \tag{4.23}$$

where $\hbar = h/2\pi$. This assumption is now known to be false since it does not take into account the spin of the electron!

3. **Energy conservation**. Light is emitted when an electron in an atom in a higher energy state makes a transition to a lower energy state. The frequency f of the emitted light is related to the energy states as follows.

$$hf = E_{\text{initial}} - E_{\text{final}}. \tag{4.24}$$

Bohr's treatment of the Hydrogen atom based on these assumptions has only historical significance since some of his assumptions have turned out to be false and since there is much better explanation of Hydrogen atom based on Quantum Mechanics which was developed almost ten years after Bohr proposed his model. Even so, the quantum language that Bohr introduced is still in use as we will find below.

4.5.2 Bohr's Orbits

Bohr's assumptions can be shown to lead to formulas for radii for allowed Bohr orbits if we apply Newton's second law of motion to an elctron in circular orbit. Let us assume the speed of the electron to be constant when in the circular orbit. This will give the following for $F = ma$ with F being the electrostatic force on the electron and a the centripetal acceleration.

$$F = ma \quad \Longrightarrow \quad \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r}. \tag{4.25}$$

The angular momentum quantization condition in Eq. 4.23 gives

$$L = n\hbar \quad \Longrightarrow \quad mvr = n\hbar, \qquad n = 1, 2, 3, \cdots . \tag{4.26}$$

Substituting v from this equation into Eq. 4.25 gives the folowing for the radius r of the orbit.

$$r = \left[\frac{4\hbar^2\epsilon_0}{me^2}\right] n^2, \qquad n = 1, 2, 3, \cdots . \tag{4.27}$$

The quantity within brackets [ ] is called the **Bohr radius** and is usually denoted by letter a_0.

$$a_0 \equiv \frac{4\hbar^2\epsilon_0}{me^2} = 5.292 \times 10^{-11}\ \text{m}. \tag{4.28}$$

We rewrite r in Eq. 4.26 as r_n and replace the the quantities within brackets [ ] by a_0 to obtain a simpler formula for the radii of the allowed Bohr oribits.

$$r_n = n^2\, a_0, \qquad n = 1, 2, 3, \cdots . \tag{4.29}$$

4.5.3 Allowed Energies for a Hydrogen atom

Bohr then worked out the energy of electron when in different orbits. Thus, when an electron is in the orbit of radius r_n it has the following energy

$$E_n = KE_n + PE_n = \frac{1}{2}mv_n^2 + \left(-\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n}\right), \tag{4.30}$$

where v_n is the speed when in the orbit labeled n, which from Eq. 4.26 will be

$$v_n = \frac{n\hbar}{mr_n}.$$

Simplifying Eq. 4.30 leads to

$$E_n = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r_n} = -\left[\frac{me^4}{24\pi^2\epsilon_0^2\hbar^2}\right]\frac{1}{n^2}. \tag{4.31}$$

Let us write the constant within bracket as E_1 and rewrite the energy formula more compactly as

$$E_n = -\frac{E_1}{n^2}, \tag{4.32}$$

with

$$E_1 = \frac{me^4}{24\pi^2\epsilon_0^2\hbar^2} = 2.178 \times 10^{-18}\ \text{J} = 13.61\ \text{eV}.$$

4.5.4 Bohr's derivation of Rydberg formula

Using Eq. 4.32 into Eq. 4.24 we can find the frequency f of the emitted light when the electron in the hydrogen atom goes from energy state $n = n_1$ to $n = n_2$ such that $E_{n_1} > E_{n_2}$.

$$hf = E_{n_1} - E_{n_2} = E_1\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]. \tag{4.33}$$

Let us write this equation in terms of inverse of the wavelength $\lambda = c/f$ of the light as in the Rydberg formula.

$$\boxed{\frac{1}{\lambda} = \frac{cE_1}{h}\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right].} \tag{4.34}$$

This is exactly the Rydberg formula for the Balmer series given in Eq. 4.21, when $n = 3, 4, \cdots$ if we can show that cE_1/h is equal to R_H. A numerical calculation of cE_1/h shows that this is indeed the case.

$$\frac{cE_1}{h} = \frac{3 \times 10^8 \text{ m/s} \times 2.178 \times 10^{-18} \text{ J}}{6.63 \times 10^{-34} \text{ J.s}} = 1.095 \times 10^7 \text{ m}^{-1},$$

which agrees with the Rydberg constant given in Eq. 4.22. Thus, Bohr was able to account for the discreteness in the spectrum of the hydrogen atom and explain the origin of the Rydberg formula, although he had to make some rather ad-hoc and [now we know] wrong physics assumptions.

Example 4.3. Electronic transition. Suppose a hydrogen atom has been somehow excited to $n = 3$ state from which the atom makes a transition to the $n = 1$ state accompanied by a release of a photon. What will be the energy, wavelength, and frequency of the light emitted?

Solution.

We can use Eq. 4.33 with $n_1 = 1$ and $n_2 = 3$ to find the energy of the photon.

$$E = E_1 \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = \frac{8}{9}\, E_1,$$

where $E_1 = 13.61$ eV.

$$E = \frac{8}{9} \times 13.61 \text{ eV} = 12.1 \text{ eV}.$$

The frequency of the photon will be

$$f = \frac{E}{h} = \frac{12.1 \text{ eV}}{4.14 \times 10^{-15} \text{ eV s}} = 2.92 \times 10^{15} \text{ Hz}.$$

The wavelength will be

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{2.92 \times 10^{15} \text{ Hz}} = 1.027 \times 10^{-7} \text{ m}.$$