## 9.7 EXERCISES

### Rotational Kinematics

**Ex 9.7.1.** A CD-ROM disk starts to rotate about its axis from rest at a constant angular acceleration of magnitude 4 rad/sec$^2$. (a) Set up a coordinate system so that one of the Cartesian axis is pointed in the direction of the angular acceleration and use the kinematics of the constant acceleration to find the angle in radians the disk rotates in 15 sec. (b) Find the angular velocity vector at the 15-sec mark.

Ans: (a)450 radians, (b) 60 rad/sec.

**Ex 9.7.2.** Starting from rest a wheel rotates an angle of 200 radians in 20 seconds at a constant angular acceleration. The axis of rotation is pointed towards the North. (a) Find the magnitude and direction of angular acceleration. (b) Find the magnitude and direction of the angular velocity at $t = 20$ sec.

Ans: (a) Magnitude: 1 $rad/s^2$, (b) Magnitude: 20 $rad/s$.

**Ex 9.7.3.** At $t = 0$ a wheel is rotating at 3 rad per sec with the axis pointed up. You give it a steady torque so that the rotation of the wheel picks up angular speed at a steady rate. After some time of constant angular acceleration, you find that the rotational speed of the wheel is 30 rad per sec. During this time the wheel had rotated by a total angle of 20 radians. Find the magnitude and direction of the angular acceleration.

Ans: Magnitude: $7.1 rad/s^2$.

**Ex 9.7.4.** A tire of radius 25 cm rolls on a flat road without slipping such that the axis of rotation is always pointed towards the East and the tire is rolling towards the North. (a) Find the angle rotated when the tire moves a distance of 200 cm in 10 sec. (b) Set up a Cartesian coordinate system so that one axis is towards the direction of motion of the center of mass and another axis is pointed towards the axis of rotation. (c) Find the relation between the components of the velocity of the center of the wheel and the angular velocity of the wheel.

Ans: (a) 8 rad, (c) $\frac{4}{5}$ rad/s.

**Ex 9.7.5.** A circular groove of diameter 50 cm is made in a horizontal table. A penny (diameter 1 cm) is rolled in the groove. Suppose, the penny rolls at right angle without slipping. (a) Find the total angle the penny rotates about an axis through the center as it goes around the groove once. (b) When the penny rotates at a steady rate, and it takes 10 sec to go around the circle. What is its instantaneous rotation velocity of the penny? (c) Going around the circle can be considered as a revolution of the CM of the penny about an axis that goes through the point of the circle that is momentarily in contact with the groove. What is the angular velocity vector of the CM about this axis?

Ans: (a) 100 $\pi$ radians; (b) 10 $\pi$ rad/sec; (c) $\pi/5$ rad/sec.

## Angular Momentum of Particles

**Ex 9.7.6.** A ball of mass 200 grams is moving in a circle of radius 25 cm with a uniform speed of 10 m/s. When observed from above the motion appears counterclockwise. Find the angular momentum about the center of the circle treating the ball as a point particle of mass $m$. Give both the magnitude and the direction of the angular momentum.

Ans: 0.5 $kg.m^2/s$, up.

**Ex 9.7.7.** The magnitude of the angular momentum of a steel ball of mass 400 grams moving in a circle in a horizontal plane of radius 50 cm is 3 kg. m$^2$/sec about the center and the direction of the angular momentum vector is towards the ground. Find the speed of the ball and the clockwise or counterclockwise sense of its motion in the circle as observed from above the circle. Treat the ball as a point particle.

Ans: 15 m/s, clockwise.

**Ex 9.7.8.** Two point particles of masses $m_1$ and $m_2$ are moving uniformly in circles of radii $r_1$ and $r_2$ with the same center in the same direction but on the opposite sides such that the total angular momentum of the two has a constant magnitude $l_0$. The speeds of the two particles are $v_1$ and $v_2$ respectively. Set up a coordinate system so that the particles move in the $xy$-plane and write out the components of the angular momenta of the two masses about the center, and show how the two speeds and radii are related.

Ans: $m_1 v_1 r_1 + m_2 v_2 r_2 = l_0$.

**Ex 9.7.9.** Let the position vector of a particle of mass 50 g be the vector $(x = 2, y = t^2, z = 0)$, where $t$ is in sec and coordinates in m. What is the angular momentum at $t = 30$ sec?

Ans: $6 kg.m^2/s$, along $z$-axis.

**Ex 9.7.10.** A baton of length $b$ has two balls of mass $m$ at the ends. The mass of the connecting rod is negligible compared to $m$. The baton is spinning at a constant angular speed $\omega$ keeping the mid-point of rod fixed while the masses move in a plane. Let the coordinate system $Oxyz$ be chosen so that the origin is at the middle of baton, and the $xy$-plane is the plane in which the balls move, and the balls are on the $x$-axis at time $t = 0$. Write expressions for for an arbitrary time $t$: (a) the position vectors of the two masses, (b) the momentum of the two masses, (c) the total momentum, (d) the angular momentum of the individual masses, and (e) the total angular momentum. [You should find in this example that even when total momentum is zero, the total angular momentum is not zero!].

Ans: (e) $\frac{1}{2}mb^2\omega\hat{u}_z$.

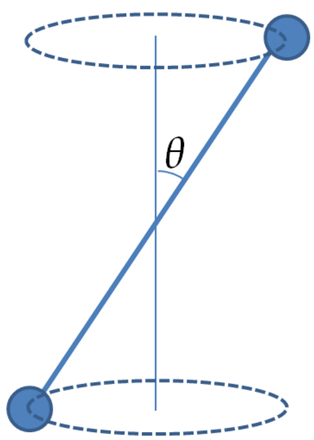



Figure 9.43: Exercise 9.7.11.

**Ex 9.7.11.** Same baton, same mass as above, except this time the baton is spinning so that the plane containing the balls changing with time as shown in Fig. 9.43. Let the connecting rod make an angle $\theta$ with the z-axis. Find the same things.

Ans: (e) Answer key: At $t = 0$, $\vec{L} = mR^2\omega\left(-\frac{b}{R}\cos\theta\hat{u}_x + 2\hat{u}_z\right)$.

## Moment of Inertia and Angular Momentum

**Ex 9.7.12.** A point particle is located at a point P with the coordinates ($x$,$y$,$z$) with respect to a Cartesian coordinate system. Find the moment of inertia of the particle about the $x$, $y$ and $z$-axes, i.e. find the formulas for $I_{xx}$, $I_{yy}$, and $I_{zz}$.

Ans: $I_{xx} = m\,(y^2 + z^2)$, $I_{yy} = m\,(x^2 + z^2)$, $I_{zz} = m\,(x^2 + y^2)$.

**Ex 9.7.13.** Two particles of masses $m_1$ and $m_2$ are located at the ends of a light rod of length $D$. When the rod is placed on a Cartesian coordinate system, the coordinates of the two masses are $(-\dfrac{D}{2}, 0, 0)$ and $(\dfrac{D}{2}, 0, 0)$ respectively. Find the moment of inertia of the system containing both the particles about the $x$, $y$ and $z$-axes, i.e. find the formulas for $I_{xx}$, $I_{yy}$, and $I_{zz}$. Neglect the mass of the rod compared to $m_1$ and $m_2$.

Ans: $I_{xx} = 0$, $I_{yy} = (m_1 + m_2)D^2/4 = I_{zz}$.

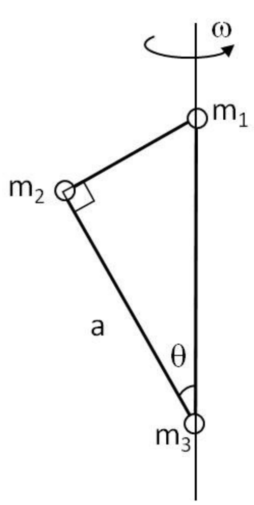



Figure 9.44: Exercise 9.7.14.

**Ex 9.7.14.** Three particles of masses $m_1$, $m_2$ and $m_3$ are rotating with angular speed $\omega$ about the given axis in Fig. 9.44. Find the magnitude and direction of the total angular momentum in two different ways: (a) from a sum of the angular momentum of each particle and (b) first finding the moment of inertia of the three as a system about the axis of rotation, which you can take to be the $z$-axis, and then multiplying the moment of inertial about the axis with the angular velocity about that axis.

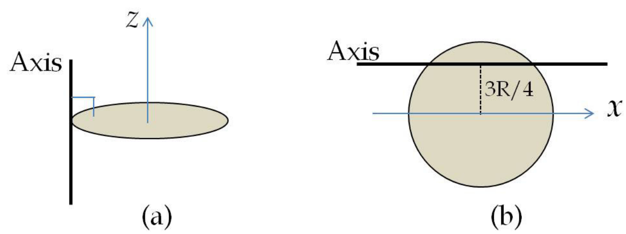



Figure 9.46: Exercise 9.7.17.

Ans: (a) and (b) $m_2(a \sin\theta)^2\omega$.

**Ex 9.7.15.** Three masses $m_1$, $m_2$ and $m_3$ are rotating about axis with angular speed $\omega$ as shown in Fig. 9.45. Find the magnitude and direction of the total angular momentum.

Ans: $[m_1(a\tan\theta\sin\theta)^2 + m_3(a\cos\theta)^2]\ \omega$.

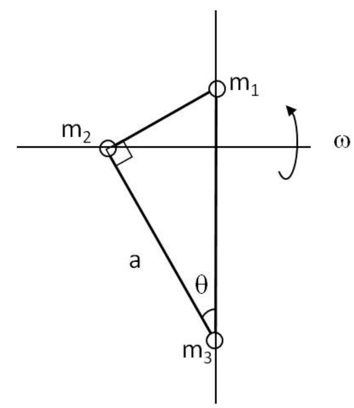
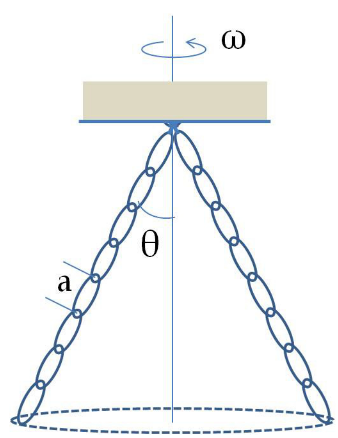



Figure 9.45: Exercise 9.7.15.

**Ex 9.7.16.** 2N rings, each of mass $m$, are linked together into a chain and hung from the middle. The chain is then rotated at an angular speed of $\omega$ as shown in figure on the right. Model the chain as a sequence of masses separated by distance $a$ and find the magnitude and direction of angular momentum.
Ans: $L_z = \frac{ma^2\sin^2\theta}{6}N(1+N)(1+2N)\ \omega$.




**Ex 9.7.17.** Let the $z$-axis of a Cartesian coordinate system be perpendicular to a thin disk shown in Fig. 9.46. Let the $x$-axis be parallel to the axis shown in Fig. 9.46(b) but passing through the center. Find the moments of inertia about the axes shown in the figures.

Ans: $I_{\text{axis, (a)}} = \frac{3}{2}MR^2$, $I_{\text{axis, (b)}} = \frac{13}{16}MR^2$.

**Ex 9.7.18.** A circular hole of radius $a$ is cut out from the center of a disk of mass $M$ and radius $R$. Let the disk with the hole be in the $xy$-plane of a Cartesian coordinate system with the origin of the axes at the center of the disk. Find $I_{xx}$, $I_{yy}$ and $I_{zz}$.

Ans: $I_{zz} = \frac{1}{2}M\left(R^2 + a^2\right)$, $I_{xx} = I_{yy} = \frac{1}{4}M\left(R^2 + a^2\right)$.

## Dynamics of Fixed-Axis Rotation

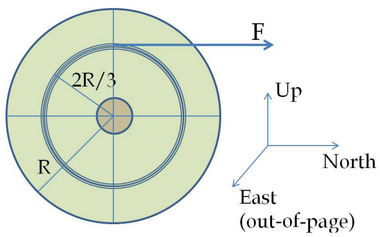



Figure 9.47: Exercise 9.7.19.

**Ex 9.7.19.** A wheel of mass $M$ and radius $R$ is mounted vertically on an axle so that the axle is parallel to a East-West line. A string is wound on the wheel. When the string is pulled, it unwinds at a distance $\frac{2}{3}R$ from the center of the wheel without slipping as shown in Fig. 9.47. (a) What is the angular acceleration of the wheel if the string is pulled steadily at a constant speed? (b) Suppose the tension in the string has a magnitude $T$ at some instant in time, what will be the angular acceleration of the wheel at instant if the effects of friction at the axle can be neglected? (c) Suppose you find that a tension of magnitude $T_0$ is needed to pull the string at a constant speed, what friction must be acting at the axle if friction acts at a distance $r$ from the center of the wheel, which would be the radius of the axle?

Ans: (a) $\alpha = 0$. (b) $\alpha_z = \frac{4}{3}\frac{T}{MR}$. (c) $\frac{2R}{3r}\, T_0$.

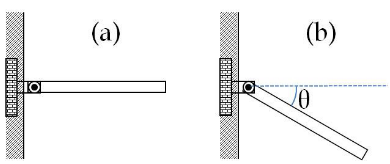



Figure 9.48: Exercise 9.7.20.

**Ex 9.7.20.** A plank of mass $M$ and length $L$ is pivoted at one end. Initially, the plank is supported at the other end so that the plank is horizontal. When the plank is released, the imbalance of the torque about the pivoted end leads to an angular acceleration of the plank. Find the angular acceleration of the plank at two instants (a) immediately after the plank is released such that the plank is horizontal but not supported and (b) when the plank makes an angle $\theta$ with the horizontal. Note: For the purpose of computing the torque, the entire weight may be assumed to act at the center of mass of the body. This does not mean that gravity acts at the center of mass. The force of gravity acts at individual particles of the body at different places.

Ans: With the positive $z$ axis out of page, (a) $\alpha_z = -\frac{3}{2}\,\frac{g}{L}$. (b) $\alpha_z = -\frac{3}{2}\,\frac{g\cos\theta}{L}$.

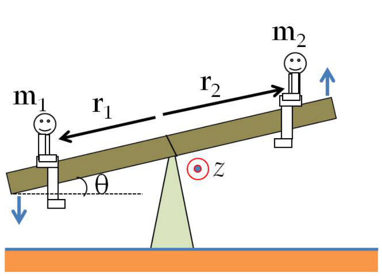



Figure 9.49: Exercise 9.7.21.

**Ex 9.7.21.** This is a long problem and covers a number of important concepts. Two children of mass $m_1$ and $m_2$ are sitting at $r_1$ and $r_2$ from the center on the two sides of a plank of mass $M$ and length $L$ that is pivoted at the center as shown in Fig. 9.49. Consider a particular instant when the plank makes an angle $\theta$ with the horizontal direction and is rotating counterclockwise. Let the origin of Cartesian coordinate system be at the pivot point, which is the point where the plank touches the support, the $z$-axis be perpendicular to the drawing shown in Fig. 9.49, and the positive $z$ direction corresponds to the "coming out-of-page" direction. Assume there is enough static friction between the children and the plank so that the children do not slide on the plank.

(a) Draw free-body diagrams of the forces on each child, choose a coordinate system, and write Newton's second law equations for the two children in the component form.

(b) Draw a diagram showing all forces on the plank and where they act, choose a coordinate system with origin at the pivot point, and find the components of the net torque on the plank about the pivot point.

(c) Draw a free-body diagram of the forces on the plank, and determine the components of the net force on the plank.

(d) Write rotational equation of motion for the plank in the component form. Note that the force by children on the plank will not be $mg$ of the children but the normal and frictional forces between the children and the plank. Note: the force $mg$ on a child acts on that child, not on the plank. The force between the plank and the child consists of normal and frictional forces between the two.

(e) Write any relations between components of the acceleration and the angular acceleration of the plank.

(f) Solve the equations you have generated for (i) acceleration of the children at the instant under consideration, (ii) the angular acceleration of the plank, (iii) the forces between the children and the plank, and (iv) the force of the support on the plank at the pivot point.

Ans: (f) $\alpha = \frac{m_1 g r_1 \cos\theta - m_2 g r_2 \cos\theta}{\frac{1}{12}ML^2 + m_1 r_1^2 + m_2 r_2^2}$.

**Ex 9.7.22.** A cylinder of mass $M$, radius $R$ and length $2R$ is attached to massless rods of length $D$ in two way to form two different physical pendula as shown in Fig. 9.50. Here $D$ is not necessarily large compared to $R$. Consider an arbitrary instant when these pendula make an angle $\theta$ with respect to the vertical line pointed down from the point of suspension.

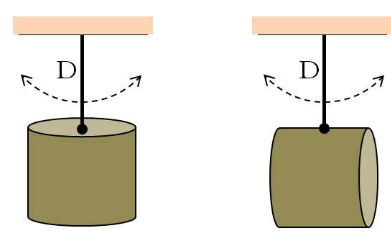


Figure 9.50: Exercise 9.7.22.

(a) Find the moment of inertia of the cylinder about an axis through the point of suspension and perpendicular to the drawing, i.e. coming out-of-page.

(b) Draw diagrams of forces, with information regarding where they act. Then, find the net torques on the bodies about the same axis, i.e. if the axis is the $z$-axis, then find $\tau_z$.

(c) Write the rotational equations of motion in the component form for the two bodies. Make sure you have the signs of various components correctly identified, which follows from your choice of the coordinate system.

(d) Make the small angle approximation and then read off the periods or frequencies of the two physical pendula from their equations of motion. What is the ratio of the frequencies of small oscillations of the two pendula?

Ans: (a) $I_{\text{axis}} = \frac{5}{4}MR^2 + 2MDR$. (d) $f = \frac{1}{2}\sqrt{\frac{Mg\left(D+\frac{H}{2}\right)}{I_{\text{axis}}}}$.

## Conservation of Angular Momentum

**Ex 9.7.23.** A disk of mass $M$ and radius $R$ is rotating at an angular speed $\omega$ about an axle through its center and vertical to its plane. Another disk of mass $M'$ and radius $R'$ falls on the axle and gets stuck. The two rotate at a lower speed $\omega'$ about the same axis passing through their common center. Show your choice of a coordinate system and implement the conservation of angular momentum in the component form to find $\omega'$ in terms of the other quantities given.

Ans: $\omega' = \left[\frac{MR^2}{MR^2+M'R'^{\,2}}\right]\,\omega$.

**Ex 9.7.24.** A projectile of mass $1,000,000$ kg is fired from Earth horizontally towards the East at the equator with speed $10^6$ $m/s$. This blast will actually change the angular speed of the rotation of the Earth. Find the new time for one full rotation of the Earth (i.e. the duration for new one-day) afterwards. Assume Earth to be a sphere of mass $6.0 \times 10^{24}$ kg and radius $6.4 \times 10^6$ m and the axis of rotation to be perpendicular to the cross-section of earth at the equator. Note: you must use an inertial coordinate system fixed at the center of the Earth for the law of conservation of angular momentum to be applicable.

Ans: $7.8 \times 10^{-11}$ sec.

**Ex 9.7.25.** A boy of mass 50 kg is standing 7 m from the center of a large circular disk of mass 300 kg and radius 10 m rotating at an angular speed of 0.6 rad/sec. When the boy walks on the platform, the rotation speed changes. Show your choice of a coordinate system and implement the conservation of angular momentum in the component form to find the rotation speed when the boy is 3 m from the center.

Ans: 0.68 rad/sec.

**Ex 9.7.26.** A wheel of mass $M = 10$ kg and radius $R = 0.5$ m is at rest. A putty of mass $m = 0.5$ kg flies at the rim of the wheel striking it at a speed of $v = 15$ m/s. The putty gets stuck after striking the wheel and as a result the wheel and putty start to rotate together. Find the magnitude and direction of angular velocity of the combination in terms of $M$, $R$, $m$ and $v$ before putting in the numbers to get a numerical value for the magnitude. Ignore any resistance at the axle of the wheel and assume all mass in the wheel to be concentrated at the rim. Hint: The flying putty has non-zero angular momentum about the center of the disk.

Ans: 1.4 rad/sec.

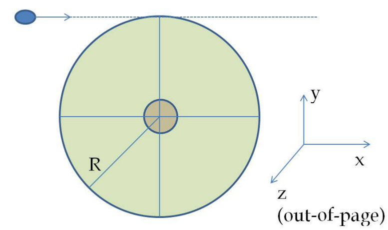



Figure 9.51: Exercise 9.7.26. A coordinate system is shown for convenience.

## Rotational Work and Kinetic Energy

**Ex 9.7.27.** (a) A 12-in wrench is used to tighten a bolt by applying a steady force of 5 N at the end of the handle which is approximately 10 inches from the center of the bolt. How much work is done for each quarter turn?

(b) Suppose, you use a 6-in wrench, and apply the same force, except that the force now would act at a distance of about 5 inches from the center of the bolt. How much work would now be done for each quarter turn?

Ans: (a) 2.0 N.m. (b) 1.0 N.m.

**Ex 9.7.28.** A uniform density disk of mass 50 kg and radius 12 cm is set on an axle through its center and rotated from rest. (a) How much work is required to bring the angular speed from zero to 10 revolutions per second? (b) How much work is required to change the angular speed from 10 rev/sec to 20 rev/sec? (c) If an average force of 20 N acts at the rim in the direction of the tangent to the rim of the disk continuously, how many rotations would it take to reach 10 rev/sec from rest? (d) If you want to get to 20 rev/sec from 10 rev/sec in half as many rotations that it took you to get from rest to 10 rev/sec using the average 20 N force, what average force would you need to act along the tangent of the rim?

Ans: (a) 4,400 J, (b) 13,00 J, (c) 47, (d) 120 N.

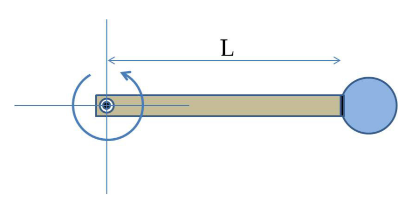



Figure 9.52: Exercise 9.7.29. Ignore the mass to the left of the hole in the rod.

**Ex 9.7.29.** A steel ball of radius $R$ is attached at the end of a cylindrical steel rod of radius $r$ and length $L$ as shown in Fig. 9.52. The other end of the rod is pivoted about a shaft, and the assembly is rotated. (a) How much work is required to bring the angular speed to $\omega$ starting from rest? (b) How much work is required to change its angular speed from $\omega$ to $2\omega$? Use the symbol $\rho$ for the density of steel.

**Ex 9.7.30.** A ceiling fan rotates at a steady speed of $\omega$. Let the magnitude of average torque applied by the motor be $\bar{\tau}$. (a) Find the rotational work done by the motor in $N$ revolutions. (b) If the moment of inertia of the fan about the axis of rotation is $I_0$, what is the change in rotational energy of the fans in each turn when the fans are rotating at a steady speed? (c) What happens to the work done by the motor? Explain.

Ans: (a) $2N\pi\bar{\tau}$, (b) No change, (c) Goes towards the work by drag.

## Rolling Motion

**Ex 9.7.31.** A bicycle moving in a straight line speeds from 5 m/s to 15 m/s over a distance of 200 meters at a constant acceleration without slipping. Each wheel has a radius of 25 cm and a mass of 250 grams with all masses assumed to be at the rim. Find the following quantities: (a) the angular speed of the wheel at the initial and the final instants, (b) the magnitude of angular rotation of each wheel over the time interval, (c) the angular acceleration during the interval, (d) the net torque on each wheel about the center of the wheel, and (e) the net force on the bicycle, assuming the mass of the bike is 1.2 kg.

Ans: (a) 20 rad/s and 60 rad/s, (b) 800 radians, (c) 2 rad/s2, (d) 0.03N.m.

**Ex 9.7.32.** A yoyo of radius $R$ and mass $M$ has a massless thread wound on the inside wheel of radius $r$. The thread is then held and the yoyo is allowed to fall. As the yoyo falls the thread unwinds smoothly. Find the tension in the thread and the speed of fall of the yoyo when it has fallen a height $h$ vertically. For purposes of the moment of inertia, assume the yoyo to be a uniform thin disk of of mass $M$ and radius $R$. You may use $R = 3$ cm, $M = 300$ g, $r = 2$ cm, and $h = 0.8$ m if you prefer numerical problem.

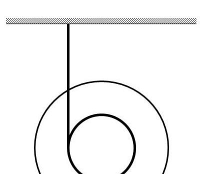


Figure 9.53: Exercise 9.7.32.

Ans: 1.56 N, 2.7 m/s.

**Ex 9.7.33.** A spherical marble of radius $R$ and mass $M$ rolls down in a straight line on an inclined plane with the angle of inclination $\theta$ without slipping. What will be the speed of the marble when it has rolled down a distance D without slipping as measured on the incline?

Ans: $\sqrt{\frac{10}{7} g D \sin\theta}$.

**Ex 9.7.34.** A heavy drum of mass $M$ and radius $R$ is being pushed up an incline of angle of inclination $\theta$ by a man who is applying a force of magnitude $F$ horizontally with the incline at a height of $\frac{3}{2}R$ from the incline. (a) What would be the acceleration of the center of mass of the drum if the coefficient of rolling friction is $\mu_r$? (b) How long will it take to move the drum a distance $L$ on the incline?

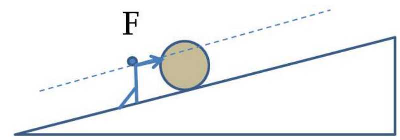



Figure 9.54: Exercise 9.7.34.

Ans: (a) $a = \dfrac{F}{m} - g\left(\sin\theta - \mu_r \cos\theta\right)$, (b) $t = \sqrt{\frac{2\Delta x}{a}}$.