3.7 PROBLEMS

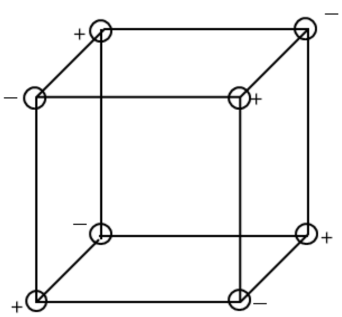

Figure 3.27: Problem 3.7.1

Problem 3.7.1. Sodium chloride forms a cubic crystal with sodium ions Na^+ and chloride ions Cl^- occupying adjacent vertices of the crystal. The distance between the adjacent ions is 2.82×10^{-10} m in NaCl crystal. Each cell of the crystal has eight charges as shown in the figure. Taking zero of potential energy to be when the eight charges in a cell are infinitely apart, find the potential energy stored in one cell. Ans: -4.75×10^{-16} J.

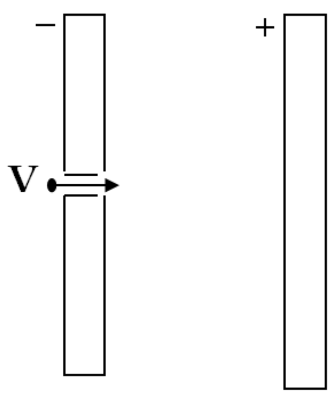

Figure 3.28: Problem 3.7.2

Problem 3.7.2. An electron enters a region between two large parallel plates made of aluminum separated by a distance 2 cm and kept at a potential difference of 200-volts. The electron enters through a small hole in the negative plate and moves towards the positive plate. At the time the electron is near the negative plate, its speed in 4×10^5 m/s. Assume the electric field between the plates to be uniform, and find the speed of electron at (a) 0.1 cm (b) 0.5 cm (c) 1 cm (d) 1.5 cm from the negative plate and (e) immediately before it hits the positive plate. Ans: (a) 1.9×10^6 m/s.

Problem 3.7.3. A non-conducting thin rod of length 2 m has charge density that varies along its length with more charges in the middle part than near the end. If the rod is laid out along the x-axis with its center coinciding with the origin, charge density λ is given by the following expression, $\lambda = (3\text{C/m}^2)\,(|x| - 1)$, where x is in meter. (a) Plot the charge density as a function of the x coordinate. (b) Find the total charge on the rod. (c) Find electric potential at a point P a distance h above the center. Ans: (b) -3 C, (c) $\frac{3}{2\pi\epsilon_0}\left(\sqrt{1+h^2} - |h| - \ln\frac{1+\sqrt{1+h^2}}{|h|}\right)$.

Problem 3.7.4. The surface charge density σ on a thin non-conducting disk of radius R is non-uniform and depends on the distance r from the center as, $\sigma = \sigma_0\ \times (r/R)$, where σ_0 is a constant surface charge density. (a) Find the total charge on the disk. (b) Find the electric potential at a point P located at a height h above the center of the disk. Ans: (b) $\frac{\sigma_0}{4\epsilon_0}\left[R\sqrt{h^2+R^2} + h^2 \ln\frac{|h|}{\left(R+\sqrt{h^2+R^2}\right)}\right]$.

Problem 3.7.5. The volume charge density ρ inside a non-conducting sphere of radius R is non-uniform and depends on the distance r from the center given by the following, $\rho = \rho_0\ (r/R)^2$, where ρ_0 is a constant volume charge density. (a) Find the total charge in the sphere. (b) Find the electric potential at a point located outside the sphere, and (b) Find the electric potential at a point located inside the sphere. Ans: (b) $\frac{\rho_0 R^3}{15\epsilon_0}\frac{1}{r}$, (c) $\frac{\rho_0 R^3}{20\epsilon_0}\left[\frac{7}{3} - \left(\frac{r}{R}\right)^4\right]$.

Problem 3.7.6. The volume charge density ρ inside a non-conducting infinitely long cylinder of radius R is non-uniform and depends on the distance r from the center given by the following, $\rho = \rho_0 \left[1 - (r/R)^2\right]$, where ρ_0 is a constant. (a) Find charge per unit length of in the cylinder. (b) Find the electric potential at a point located outside the cylinder. (c) Find the electric potential at a point located inside the cylinder.

Problem 3.7.7. In a region of space there is a non-uniform electric field given by the following in units of N/C.

$$\vec{E} = \left(5\frac{Nm^2}{C}\right)\frac{x\hat{u}_x + y\hat{u}_y + z\hat{u}_z}{(x^2+y^2+z^2)^{3/2}},$$

where all the coordinates are in meters. Find the potential difference between (a) P_1(1,2,3) and P_2(4,2,3), (b) P_1(1,2,3) and P_3(1,3,3), and (c) P_1(1,2,3) and P_4(2,3,3).

Problem 3.7.8. The electric field at a space point P(x, y, z) due to some unknown charges is given by the following formula.

$$\vec{E} = \begin{cases} s^3\hat{u}_s & 0 < s < 1 \\ \frac{1}{s}\hat{u}_s & 1 < s < \infty \end{cases}$$

where $s = \sqrt{x^2+y^2}$ and $\hat{u}_s$ is a unit vector radially outward from the z-axis. The unit of electric field given is N/C when r is expressed in meters. (a) Find the electric potential at $r = 0.5$ m if the potential at $r = 1$ m is zero. (b) Find the potential difference between a point whose $r = 0.2\ m$ and another point whose $r = 1.2$ m. Ans: (a) 0.25 V, (b) 0.43 V.

Problem 3.7.9. Two point charges $q_1 = -5\ \mu$C and q_2=+5 μC are fixed at $z = -2$ cm and +2 cm respectively. (a) Find the electric potential from these charges at an arbitrary point P(x, y, z) in space with the reference of zero potential at infinity. (b) Use the electric potential formula you obtained to find components of electric field at an arbitrary point. (c) Which way is electric field pointed on the x-axis at $x = 1\ m$? Ans: (c) -1.8×10^7 N/C $\hat{u}_z$).

Problem 3.7.10. Two point charges $q_1 = -5\ \mu$C and $q_2 = +5\ \mu$C are fixed at $z = -2$ cm and +2 cm respectively. (a) Find the electric field at an arbitrary point on the x-axis using electric field formula for point charges. (b) Deduce the electric potential at an arbitrary point on the x-axis from the electric field you wrote down for part (a). (c) Compare your answer to part (b) of the present problem with that of part (a) in the previous problem.

Problem 3.7.11. A charge of +5 μC is put on a metallic sphere of radius 2 cm. Static charges on metals reside on the surface. If potential is zero at infinity, what is the potential at an arbitrary point inside the sphere? Ans: 2.25 MV.

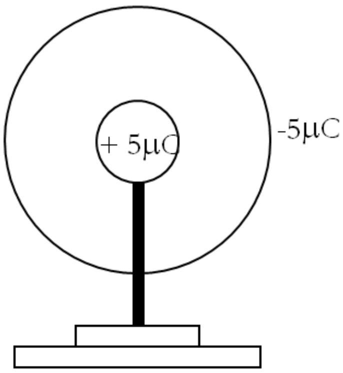

Figure 3.29: Problem 3.7.12

Problem 3.7.12. A metallic sphere of radius 2 cm is charged with +5 μC charge, which spreads on the surface of the sphere uniformly. The metallic sphere stands on an insulated stand and is surrounded by a larger metallic spherical shell, of inner radius 5 cm and outer radius 6 cm. Now, a charge of -5 μC is placed on the inside of the spherical shell which spreads out uniformly on inside surface of the shell. If potential is zero at infinity, what is the potential of (a) the spherical shell and (b) the sphere? Ans: (b) 1.35 MV.

Problem 3.7.13. A very long wire has a uniform charge density of +1 μC/m. (a) Using Gauss's law find electric field at an arbitrary point a distance s from the wire. (b) From the electric field you found, deduce the formula for electric potential at an arbitrary point. (State your reference point.) (c) If an electron starts at rest from a distance 4 cm from the wire, what will be its speed when it is at 0.5 cm from the wire? Ans: (c) 1.35×10^8 m/s.

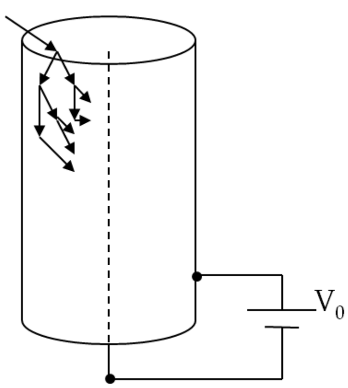

Figure 3.30: Problem 3.7.14

Problem 3.7.14. In a Geiger counter a thin metallic wire at the center of a metallic tube is kept at a high voltage with respect to the metal tube. Ionizing radiation entering the tube knocks electrons off gas molecules or sides of the tube which then accelerate towards the center wire knocking off even more electrons. This process eventually leads to an avalanche which is detectable as a current. A particular Geiger counter has a tube of radius R and the inner wire of radius a is at a potential of V_0 volts with respect to the outer metal tube. Consider a point P at a distance s from the center wire and far away from the ends. (a) Find a formula for the electric field at a point P inside assuming infinite wire approximation, (b) Find a formula for the electric potential at a point P inside. (c) Use V_0= 900 V, a = 3 mm, R = 2 cm, and find the value of the electric field at a point 1 cm from the center. Ans: (c) 4.75×10^4 N/C.

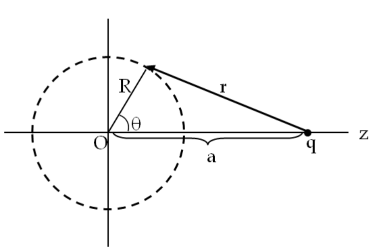

Figure 3.31: Problem 3.7.15

Problem 3.7.15. Consider a charge q at $z = a$. Show that the average of its potential over an imagined spherical surface of radius R with center at the origin is equal to the value fo the potential at the origin. This is a particular case of a general result for electric potential: the average value of V over an imaginary spherical surface is the same as the value of V at the center if there are no charges inside the sphere.

Problem 3.7.16. Electric field in a region is pointed away from the z-axis and the magnitude depends upon the distance s from the axis. The magnitude of $\vec{E}$ is give as $E = \frac{\alpha}{s}$, where α is a constant. Find the potential difference between points P_1 and P_2, explicitly stating the path over which you conduct the integration for $\vec{E} \cdot d\vec{r}$ line integral.

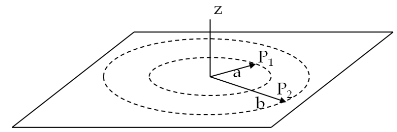

Figure 3.32: Problem 3.7.16