9.2 MICROSCOPIC VIEW OF MAGNETIC MATERIALS

The magnetism of a material is due to the magnetic properties of the constituent particles, electrons, protons and neutrons of their atoms. Electrons, protons and neutrons in an atom each have their magnetic dipole moments, and their vector sum gives the net magnetic dipole moment of the atom. However, due to the low mass of electrons compared to protons and neutrons the electronic contribution to the net magnetic dipole moment is dominant unless electronic magnetic dipole moment happens to be zero. Therefore, in the following, we will ignore the contributions of protons and neutrons.

The magnetic dipole moment of an electron comes from two sources:

1. Motion about the nucleus, also called the orbital magnetic dipole moment. We will denote the corresponding magnetic dipole moment by $\vec{\mu}_l$.
2. Intrinsic magnetic dipole moment. The source of intrinsic dipole moment is the quantum mechanical property called spin. We will denote the corresponding magnetic dipole moment by $\vec{\mu}_s$.

The net angular momentum of an atom will be a vector sum of the two angular momenta of each electron.

$$\vec{\mu} = \vec{\mu}_l + \vec{\mu}_s.$$

9.2.1 Angular Momentum and Magnetic Dipole Moment

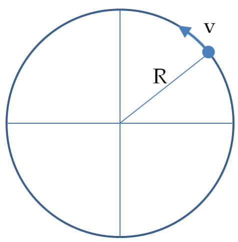

Figure 9.7: An electron bound to an atom moving in a circular orbit about a nucleus.

To develop a sense of the magnetic moment due to motion of charges we will consider a simple system. Consider an electron in a circular of radius R with uniform speed v as shown in Fig. 9.7. Assuming the electron to be moving so rapidly that current can be considered to form a steady current I around the nucleus. Therefore, the magnitude of current given by charge over time will be

$$I = \left|\frac{dq}{dt}\right| = \frac{e}{2\pi R/v}. \tag{9.23}$$

Note here e is the electronic charge without the sign. Therefore, the magnetic dipole moment μ_l of the electron due to its orbital motion will be

$$\mu_l = I\pi R^2 = \frac{1}{2}evR. \tag{9.24}$$

Since, the angular momentum of an electron moving in the circle,

$$|\vec{L}| = m_e v R, \tag{9.25}$$

where m_e is the mass of the electron, we can rewrite the magnetic dipole moment in terms of the magnitude of the orbital angular momentum $\vec{L}$ as,

$$\boxed{\mu_l = \frac{e}{2m} L.} \tag{9.26}$$

The proportionality constant $e/2m_e$ is called the **gyromagnetic ratio** g_e of electron. The gyromagnetic ratio is the ratio of the magnitude of magnetic dipole moment and angular momentum. [Note: Sometimes the gyromagnetic ratio is also called magnetogyric ratio.]

$$\boxed{g_e = \frac{e}{2m}.} \tag{9.27}$$

With the introduction of gyromagnetic ratio, the orbital magnetic moment is compactly written as

$$\boxed{\mu_l = g_e L.} \tag{9.28}$$

Due to the negative charge on the electron, the magnetic dipole moment vector $\vec{\mu}_l$ and the angular momentum vector $\vec{L}$ are in the opposite directions. Therefore, the vector form of this equation will be

$$\boxed{\vec{\mu}_l = -g_e \vec{L}.} \tag{9.29}$$

9.2.2 Angular Momentum in Quantum Mechanics

So far in this book we have discussed classical physics. However, the fundamental basis of magnetism is atomic where the mechanics is different. We will not be doing any quantum mechanics, but we will just quote some of the weird results of quantum mechanics we need here. In particular, we will quote the angular momentum values of atoms seen in experiment since we have seen above that magnetic dipole moment is proportional to the angular momentum.

According to quantum mechanics, the magnitude of angular momentum $|\vec{L}|$ as well as the projection of angular momentum along any direction in space, such as along x, y, or z axis, are quantized, meaning only certain discrete values for L, L_x, L_y, or L_z are allowed as outcomes of measurements. For instance, for the simplest atom, hydrogen atom, you can explicitly show that only the following values are allowed as outcomes in experiments.

$$\text{Values of } |\vec{L}|: \;\; \hbar\sqrt{l(l+1)}, \quad \text{with } l = 0, 1, 2, 3, \cdots, \tag{9.30}$$

$$\text{Values of } L_z: \;\; m\hbar, \quad \text{with } m = 0, \pm 1, \pm 2, \cdots, \pm l, \tag{9.31}$$

where the constant $\hbar$, read as hbar, is equal to the Planck's constant h divided by 2π.

$$\hbar = \frac{h}{2\pi} \tag{9.32}$$

Planck constant h is a universal constant and has the following value in the SI unit.

$$h = 6.627 \times 10^{-34} \text{ J.s.} \tag{9.33}$$

Although I have written the component L_z, the same is true for the x- and y-components, and for that matter projection of the angular momentum in any direction in space. The integers l are called the orbital angular momentum quantum number and the numbers denoted by m are called the magnetic quantum number. The z-component of the magnetic dipole moment is related to the z-component of the angular momentum.

$$\text{Values of } \mu_z \quad m\,\hbar\, g_e, \;\; m = 0, \pm 1, \pm 2, \cdots, \pm l. \tag{9.34}$$

The constant $\hbar g_e$ is called the **Bohr magneton** and is denoted by μ_B. The Bohr magneton has the unit of magnetic dipole moment. Putting the values of the universal constants we find the value of μ_B to be

$$\mu_B = \hbar\, g_e = 9.27 \times 10^{-24} \text{ A.m}^2. \tag{9.35}$$

Equation 9.34 shows that the Bohr magneton is the appropriate unit for the orbital magnetic dipole moments of atoms since the Cartesian components of the $\vec{\mu}_l$ have the values 0, $\pm\mu_B$, $\pm 2\mu_B$, etc.

9.2.3 Spin And Magnetic Dipole Moment

Every electron has an intrinsic angular momentum called **spin** regardless of the state of motion of the electron. Despite the name, spin does not imply that point particle is physically spinning. A better name for this property would be just "the intrinsic angular momentum" but the name "spin" is now ingrained in physics terminology although we do not mean physical spin when we use this term. The spin property accounts for the fact that electrons appear to have additional sources of angular momentum than one can account for based only on the motion of the electron.

Just as the orbital angular momentum creates a magnetic dipole moment for the electron, the spin angular momentum also creates a magnetic dipole moment for the electron. However, the gyromagnetic ratio of the spin angular momentum is approximately twice as much as the gyromagnetic ratio of the orbital angular momentum. Let us denote the spin angular momentum vector by $\vec{S}$ and the corresponding magnetic dipole moment vector by $\vec{\mu}_s$. Then we have

$$\vec{\mu}_s = 2\, g_e\, \vec{S} = \frac{2\mu_B}{\hbar}\, \vec{S}. \tag{9.36}$$

Unlike the magnitude of the orbital angular momentum, a measurement of the magnitude of the spin angular momentum of an electron results in only one value given by

$$\text{Allowed valued of } |\vec{S}|:\ \hbar \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}, \tag{9.37}$$

and a measurement of the spin angular momentum along any direction, say the z-axis, results in only the following two values.

$$\text{Allowed valued of } S_z:\ \pm\frac{1}{2}\hbar. \tag{9.38}$$

These two values of the spin projections are called the spin up and the spin down respectively. The electron is said to be a spin $\frac{1}{2}$ particle. The rules of quantum mechanics often forces the pairing electrons of up and down spins so that atoms that have an even number of electrons have zero net spin angular momentum in any direction. In these cases, the magnetism of a material comes from the orbital angular momenta.

These arguments suggest that the effect of the spin magnetic moment will be present when spins of electrons cannot be paired up. This will definitely happen if a particular atom has either an odd number of electrons or where spins of different electrons are not paired up. For a single unpaired electron, the spin magnetic moment along any axis is either $+1\ \mu_B$ or $-1\ \mu_B$.

If atoms with magnetic dipole moment due to unpaired spins or non-zero orbital angular momentum are placed in a magnetic field, the alignment of atomic dipole moments give rise to macroscopically observable magnetic phenomenon. For instance, if the magnetic dipole moments of N atoms, each with a dipole moment of, say $1\ \mu_B$, are line up in one direction, then the net magnetic dipole moment of the sample will be $N\mu_B$. How large can these number be? Suppose N is of the order of the Avogadro number, then the net magnetic dipole moment will be

$$\mu \sim 6\times 10^{23}\times 9\times 10^{-24}\text{A.m}^2 = 5\ \text{A.m}^2.$$

This is equivalent to 5 A current in a 1 m × 1 m loop, which would have a noticeable large magnetic effect at the macroscopic scale.

Example 9.2.1. Magnetic moment of atoms. Determine the magnetic moment of (a) a sodium atom which has an unpaired electron in $m = 0$ and the spin up state, and (b) an electron in the carbon atom that is in spin up state and has $m = 1$. Use the z-axis as the axis for the angular momentum vector.

Solution. (a) Since $m = 0$, the orbital angular momentum along the z-axis is zero. The spin up state corresponds to the z-component of angular momentum

$$\mu_z = \frac{1}{2}\hbar \times \frac{2\mu_B}{\hbar} = \mu_B.$$

(b) Using the total magnetic moment formula, we find the following

$$\mu_z = \hbar \times \frac{\mu_B}{\hbar} + \frac{1}{2}\hbar \times \frac{2\mu_B}{\hbar} = 2\mu_B.$$

Example 9.2.2. Magnetic moment of an excited helium atom. An excited helium atom has two electrons in different states, one with m = 0, and spin up, and the other m = +1, and spin up state. What is the magnetic moment of the atom? Use the z-axis as the axis of projections of the angular momentum vectors.

Solution. Using the total magnetic moment formula, we find the magnetic moment of each electron, and then add them vectorially.

$$\mu_z = \mu_{1z} + \mu_{2z} = \left[(0+1)\mu_B\right]_1 + \left[(1+1)\mu_B\right]_2 = 3\mu_B.$$

9.2.4 Induced Magnetic Dipole Moment

An atom has a permanent magnetic dipole moment if the vector sum of magnetic dipole moments of its electrons is not zero. Additionally, when an atom is placed in an external magnetic field, a magnetic dipole moment is induced in the atom as we will see now. The induced dipole moment occurs in every material regardless of whether the material has a permanent magnetic dipole moment or not.

To see the emergence of an induced magnetic dipole moment in a material, consider one hydrogen atom and model the motion of the electron as a uniform circular motion of radius R and constant speed v. In the absence of any magnetic field, the net force on the electron would be the electric force from the proton in the nucleus and the acceleration will be centripetal.

$$\frac{1}{\pi\epsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R}. \tag{9.39}$$

When the atom is placed in an external magnetic field $\vec{B}$, there will be an additional force on the moving electron given by $\vec{F} = -e\vec{v}\times\vec{B}$. For simplicity, let the circle of the motion of the electron be the xy-plane and the magnetic field towards the positive z-axis. The magnetic force on the electron will be also towards the center of the circle. This will keep the motion circular but now the centripetal acceleration of the electron will be different. If the radius of the orbit under new forces is to remain the same as the old radius, then the speed must change to v' to satisfy the changes equation of the uniform circular motion:

$$\frac{1}{\pi\epsilon_0}\frac{e^2}{R^2} + ev'B = m_e\frac{v'^2}{R}. \quad \text{(radially in)} \tag{9.40}$$

Subtracting the two equations, we obtain

$$ev'B = \frac{m_e}{R}\left(v'^2 - v^2\right) = \frac{m_e}{R}(v'+v)(v'-v). \tag{9.41}$$

Assuming the change in speed $\Delta v = v' - v$ to be small, we can replace $v' + v$ by $2v$ on the right side since $v' \approx v$ and obtain the following for the change in the speed.

$$\Delta v = \frac{eBR}{2m_e} > 0. \tag{9.42}$$

The change in the speed corresponds to the following change in the z-component of the angular momentum

$$\Delta L_z = m_e R \Delta v = \frac{eBR^2}{2}, \tag{9.43}$$

which has the following change in the z-component of the magnetic dipole moment.

$$\Delta\mu_z = -g_e \Delta L_z = -\frac{eR^2 B_z}{2\hbar}\mu_B. \tag{9.44}$$

Writing this in the vector notation we have

$$\Delta\vec{\mu} = -\frac{eR^2\vec{B}}{2\hbar}\mu_B, \tag{9.45}$$

which shows that the induced magnetic moment is directed opposite to the applied magnetic field. The oppositely directed magnetic dipoles act as tiny magnets that are in the opposite orientation to the external magnetic field and hence reduce the magnetic field inside the material. The phenomenon of reduced magnetic field in a magnetic material is called the **diamagnetic effect**. The induced magnetic dipole moments are usually considerably less than the permanent magnetic dipole moments if the later is present in a material.

Therefore, although induced magnetic dipoles are created in all materials, the diamagnetic effect is most evident in the substances, such as Bismuth, that have no permanent magnetic dipole moments. The interactions between the permanent magnetic dipoles in materials may interact with each other. Depending upon the type of interaction between the magnetic dipoles, the materials with permanent magnetic dipoles are broadly classified as s paramagnets, ferromagnets, and antiferromagnets. We will discuss paramagnets and ferromagnets in detail below.

Example 9.2.3. Induced magnetic moment of a helium atom. The helium has two paired electrons at approximately the Bohr radius (0.053 nm) from the nucleus. Estimate the induced magnetic dipole moment when the atom is placed in a magnetic of 3 T.

Solution. For analytic purpose let us use a Cartesian coordinates with z axis in the direction of the magnetic field. The change in z components of the magnetic dipole moments of the two electrons will be twice the amount for one electron.

$$\Delta\mu_z = -2 \times \frac{eR^2 B_z}{2\hbar}\mu_B,$$

where e is the charge of an electron, R the radius of the orbit, and B_z the z component of the external magnetic field. Putting the numerical values we find

$$\begin{aligned}\Delta\mu_z &= -\frac{1.6 \times 10^{-19} C \times (0.53 \times 10^{-10} m)^2 \times 3T}{1.055 \times 10^{-34} J.s}\mu_B \\ &= -1.3 \times 10^{-5}\mu_B.\end{aligned}$$

Compare this to the z component of the magnetic dipole moment of one unpaired electron due to spin angular momentum along z axis, which $1\mu_B$.

Example 9.2.4. Induced magnetic dipole of bismuth

Bismuth is one of the best diamagnetic materials. Each atom of Bismuth has 83 electrons in different shells around the nucleus. Assume the average radius of orbits to be same as Bohr radius (0.053 nm), and estimate the total induced magnetic dipole moment of a 100-gram Bismuth sample when it is placed in 1.5 T field, assuming the induced dipoles are all in the same direction. Use atomic number of Bismuth 209.

Solution. Here we will have to multiply the result of one electron by the total number of electrons in one atom of bismuth, and then multiply the result by the number of atoms in 100 gram of bismuth.

This gives

$$
\begin{aligned}
\Delta\mu_z &= -\left(100g\frac{6.022\times 10^{23}atoms}{209g}\right)\times 82\frac{elec}{atom}\times\frac{eR^2B_z}{2\hbar}\mu_B/elec \\
&= -1.275\times 10^{24}\mu_B.
\end{aligned}
$$