1.5 HEAT CAPACITY AND SPECIFIC HEAT

The amount of heat that can change the temperature of a body by one degree is called the heat capacity of the body. Therefore, if heat of amount ΔQ raises the temperature by ΔT then the **heat capacity** would be given as

$$\boxed{\text{Heat Capacity } = \frac{\Delta Q}{\Delta T} \to \frac{dQ}{dT}.} \tag{1.8}$$

The heat capacity is a misleading term, since it seems to imply that heat may be contained in the system rather than having only a transitory nature. Although the term "heat capacity" is still in use in certain disciplines such as Chemistry, in physics, we now use a different related concept called the specific heat. The specific heat of a material is defined as the heat capacity of the material per unit mass. The specific heat is usually denoted by the small letter c. Therefore, specific heat c is equal heat capacity divided by mass m:

$$\boxed{\text{Specific Heat, } c = \frac{\text{Heat Capacity}}{m} \to \frac{1}{m}\left(\frac{dQ}{dT}\right).} \tag{1.9}$$

Sometimes specific heat is defined per mole rather than per unit mass. Then, we call the specific heat the molar specific heat and denote the later by the capital letter C.

$$\boxed{\text{Molar Specific Heat, } C = \frac{\text{Heat Capacity}}{n} \to \frac{1}{n}\left(\frac{dQ}{dT}\right).} \tag{1.10}$$

The specific heat and heat capacity of a substance depend on the manner in which the temperature of the substance is changed. For instance, different amount of heat will be required to raise the temperature by 1°C if you keep the pressure fixed during the change than if you keep the volume fixed, or if you kept neither of them fixed. We say that the specific heat is a **process-dependent property**.

Two processes of particular interest are the constant pressure process (the isobaric process) and the constant volume process (the isochoric process). The specific heats at constant pressure and at constant volume are denoted by c_P and c_V respectively.

$$\boxed{\text{Specific Heat at Constant Pressure, } c_P = \frac{1}{m}\left(\frac{dQ}{dT}\right)_P} \tag{1.11}$$

$$\boxed{\text{Specific Heat at Constant Volume, } c_V = \frac{1}{m}\left(\frac{dQ}{dT}\right)_V,} \tag{1.12}$$

where subscript p and V are attached on the right side to emphasize the quantity that must remain constant during any change in the temperature of the system. The dimension of specific heat is Energy/(mass $\times$ temperature) and therefore it has units of J/kg.C or J/kg.K in the SI system. Another commonly used unit is cal per g per deg C. Table 1.2 lists specific heats of some common substances.

For many substances, under normal conditions, specific heat is nearly independent of temperature. Therefore, you can deduce a formula for heat Q needed if m kg of a substance is heated so that its temperature rises from T_1 to T_2.

$$\boxed{Q \approx \begin{cases} m\ c_P\,(T_2 - T_1) & \text{(constant } p \text{ process)} \\ m\ c_V\,(T_2 - T_1) & \text{(constant } V \text{ process)} \end{cases}} \tag{1.13}$$

Often specific heat is a function of temperature, $c(t)$. In that case, to obtain heat needed for a finite change in temperature, you will need to integrate over temperature to obtain heat needed for a finite change in temperature.

$$\boxed{Q = \begin{cases} m \int_{T_1}^{T_2} c_P(T)dT & \text{(constant } p \text{ process)} \\ \\ m \int_{T_1}^{T_2} c_V(T)dT & \text{(constant } V \text{ process)} \end{cases}} \tag{1.14}$$

Example 1.5.1. Heating a Block of copper. Use table of specific heat and compute heat needed to raise the temperature of a 2-kg block of copper from 20°C to 200°C at 1 atmosphere.

Solution. From Table 1.2 we find that the specific heat of copper is 0.379 J/g.K. Since, the specific heat of copper in the temperature range given in this problem is independent of temperature, we can use Eq. 1.13 for the heat calculation.

$$\begin{aligned} Q &= m\ c_P\,(T_2 - T_1)\,, \\ &= 2kg \times \left(0.379 \frac{\text{J}}{\text{g.K}} \times \frac{1000\text{g}}{1\text{kg}}\right) \times 180\text{K} = 136,000 \text{ J}, \end{aligned}$$

where I have rounded off the answer to three figures.

Example 1.5.2. Low Temperature Study of Aluminum. The specific heat of aluminum at constant pressure is found to vary with temperature as $c_P(T) = \left(7.0 \times 10^{-4}\text{J/g.K}^4\right) T^3$, where T is the temperature expressed in degrees Kelvin. How much heat is needed to raise the temperature of 50 gs of aluminum from 5 K to 9 K?

Solution. Note that the specific heat is temperature dependent. Therefore, we would have to integrate the specific heat to find heat required.

$$\begin{aligned}
Q &= m\int_{T_1}^{T_2} c_P(T)dT \\
&= 50\text{g} \times \left(7.0\times 10^{-4}\text{J/g.K}^4\right)\int_{T_1}^{T_2} T^3 dT \\
&= \frac{3.5\times 10^{-2}}{4}(\text{J/K}^4)\left(T_2^4 - T_1^4\right) = 52\ \text{J}.
\end{aligned}$$

Table 1.2: Specific heats (25°C and 1 atm pressure, except as noted). (Source: Kaye and Laby, 2006).

Substance	c_P(J/gK)
Metals and alloys	
Aluminum	0.888
Bismuth	0.122
Brass (alloy)	0.387
Copper	0.379
Gold	0.128
Lead	0.127
Mercury	0.140
Nickel	0.429
Silver	0.235
Steel (alloy)	0.460
Tungsten	0.133
Zinc	0.385
Miscellaneous	
Carbon: Diamond	0.42
Carbon: Graphite	0.644
Ethyl alcohol	2.40
Glass (Pyrex)	0.70
Granite	0.79
Ice	2.1
Polycarbonate	1.1
Porcelain	0.7
Steam	2.01
Teflon	0.97
Water (0°C)	4.217
Water (15°C) (standard)	4.1855
Water (30°C)	4.178
Water (60°C)	4.184
Water (90°C)	4.204