3.3 THE CARNOT ENGINE

A Carnot engine uses an ideal gas as the working substance. In a Carnot engine a fixed amount of ideal gas is heated and cooled in a particular cyclic process called the **Carnot cycle**, which is shown in Fig. 3.2. The cycle consists of four quasi-static steps conducted such

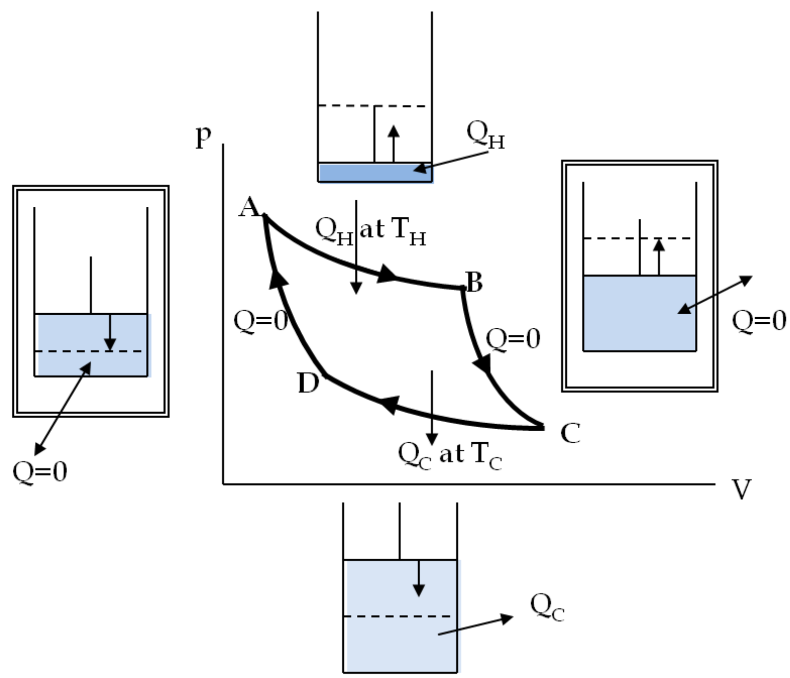

Figure 3.2: The processes of the Carnot engine. Process AB is an isothermal expansion at temperature T_H, BC an adiabatic expansion, CD an isothermal compression at temperature T_C, and DC an adiabatic compression.

that at the end of the last step the gas returns to the initial state thus completing the cyclic process of the engine.

1. An isothermal expansion at high temperature bath T_H from state A to state B with gas absorbing a heat Q_H.

2. An adiabatic expansion from B to C cooling the gas from a temperature T_H to temperature T_C.

3. An isothermal compression from C to D while immersed in the cold bath at temperature T_H with the gas losing heat Q_C to the bath.

4. An adiabatic compression from D to A which raises the temperature of the gas from T_C to the initial temperature T_H.

Since the system is an ideal gas and the processes are all quasi-static it is possible to work out heat and work in each step as we will show below. These calculations show that the ratio of Q_C to Q_H is equal to the ratio of the temperatures of the baths in absolute scale.

$$\boxed{\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \quad \text{(temperatures in absolute/Kelvin scale)}} \tag{3.4}$$

This relation can also used to define the absolute or Kelvin scale of temperature from an operational viewpoint. Using Eq. 3.4 we can write the efficiency of a Carnot engine as

$$\boxed{\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \quad \text{(temperatures in absolute/Kelvin scale)}} \tag{3.5}$$

This formula for the efficiency of a Carnot engine says that a Carnot engine will be 100% efficient only if the temperature of the cold bath T_C were zero degrees Kelvin. When this statement is combined with the Kelvin-Planck statement of the second law, it can be concluded that a temperature of zero degree Kelvin is not reachable since that would imply a possibility of constructing a thermal engine that has 100% efficiency.

Example 3.3.1. The Efficiency of a Carnot Engine.

A Carnot engine operates between $400°C$ and $27°C$. (a) What is the efficiency of the Carnot engine? (b) If the Carnot engine produces 5 kJ of work in each cycle, how much heat must it absorb from the $400°C$ bath in each cycle.

Solution. (a) The efficiency of a Carnot engine can be also written in terms of temperatures of the hot and cold baths as we saw above. In order to make use of the formula for efficiency in terms of temperatures, we must express the temperatures in absolute scale.

$$T_H = 400 + 273.15 = 673.15 \text{ K}$$
$$T_C = 27 + 273.15 = 300.15 \text{ K}$$

Therefore, the efficiency of the Carnot engine is

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{300.15 \text{ K}}{673.15 \text{ K}} = 0.554.$$

The efficiency in this formula is written as fraction. Normally, we write the number in percentage by multiplying this number by 100%. This gives the efficiency of this engine to be 55.4%.

(b) We can use the efficiency found to figure out the heat in the engine.

$$\text{Since } \eta = \frac{W}{Q_H}, \quad Q_H = \frac{W}{\eta} = \frac{5 \text{ kJ}}{0.554} = 9.03 \text{ kJ}.$$

Proof of $Q_C/Q_H = T_C/T_H$ in Carnot Cycle

Since the internal energy of an ideal gas is proportional to temperature, the internal energy does not change in the processes AB and CD in Fig. 3.2. Applying first law of thermodynamics to these steps and using the formula for work by a gas in an isothermal process, we conclude

$$Q_H = W_{AB} = nRT_H \ln\left(\frac{V_B}{V_A}\right) \tag{3.6}$$

$$Q_C = W_{CD} = nRT_H \ln\left(\frac{V_D}{V_C}\right) \tag{3.7}$$

From the adiabatic processes BC and DA we have the following relations.

$$\text{BC process: } T_H V_B^{\gamma-1} = T_C V_C^{\gamma-1}$$
$$\text{DA process: } T_H V_A^{\gamma-1} = T_C V_D^{\gamma-1}$$

Taking the ratio we find

$$\frac{V_B}{V_A} = \frac{V_C}{V_D}. \tag{3.8}$$

Now, we take the ratio of Eq. 3.7 to Eq. 3.6 and use Eq. 3.8 to obtain the required result.

$$Q_C/Q_H = T_C/T_H.$$

3.3.1 The Carnot Refrigerator

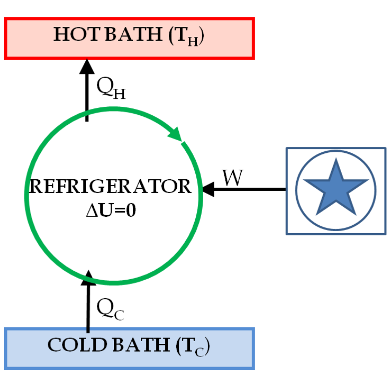

Figure 3.3: Schematics of energy flow in and out of a Carnot refrigerator.

Reversing the direction of each of the four processes of the Carnot engine of Fig. 3.2 leads to the absorption of heat Q_C from the cold reservoir (temperature T_C) and delivery of heat Q_H to the hot reservoir (temperature T_H) with work W done on the system as shown in Fig. 3.3. The net effect of reversing the Carnot cycle will be a transfer of heat from a cold place to a hot place with energy W spent in doing it. The Carnot engine run in reverse is called the Carnot refrigerator.

The efficiency of a refrigerator is defined differently than the efficiency of an engine since we are concerned here with a different task. In a refrigeration cycle we want to take out the maximum heat Q_C possible for a particular amount of energy W expended. Therefore, the ratio of interest will be Q_C to W, which is called the **coefficient**

of performance (COP) of the refrigerator, and is denoted by the Greek letter β.

$$\boxed{\text{Coefficient of performance of a refrigerator: } \beta = \frac{Q_C}{W}.} \tag{3.9}$$

Unlike the efficiency of an engine, the coefficient of performance can be greater than 1. Using the conservation of energy based on the first law of thermodynamics, work W must be the difference of Q_H and Q_C just like the engine. This gives the coefficient of performance to be

$$\beta = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{1}{(Q_H/Q_C) - 1}. \tag{3.10}$$

Since the processes in the Carnot refrigerator are just reverse of the processes in a Carnot engine, here you can show again that, $Q_H/Q_C = T_H/T_C$. Therefore, we can write the coefficient of performance of a Carnot refrigerator in terms of absolute temperatures only.

$$\beta_{\text{Carnot}} = \frac{1}{(T_H/T_C) - 1} = \frac{T_C}{T_H - T_C}. \tag{3.11}$$

Example 3.3.2. Freezing Water in a Refrigerator.

A refrigerator working between $-10°$C and $30°$C is used to freeze 0.5 kg of water at $0°C$ into ice at $0°$C. If the refrigerator is approximately Carnot refrigerator, how much electric energy will it use in the process?

Solution. For a Carnot refrigerator, the coefficient of performance is

$$\beta_{\text{Carnot}} = \frac{Q_C}{W} = \frac{T_C}{T_H - T_C}. \quad \text{(Temperatures in Kelvin.)}$$

Therefore,

$$W = \left(\frac{T_H - T_C}{T_C} \right) Q_C = \left(\frac{40}{263.15} \right) Q_C.$$

We need to remove heat from water for freezing at $0°C$.

$$Q_C = ml = 0.5 \text{ kg} \times 4186 \text{ J/kg} = 2093 \text{ J}.$$

Therefore, electrical energy needed by the refrigerator is

$$W = \left(\frac{40}{263.15} \right) \times 2093 \text{ J} = 318 \text{ J}.$$

3.3.2 Is a Perfect Refrigerator Possible?

A prefect refrigerator will transfer heat $Q = Q_C$ from the cold bath to the hot bath without any work W (Fig 3.4). This will require a

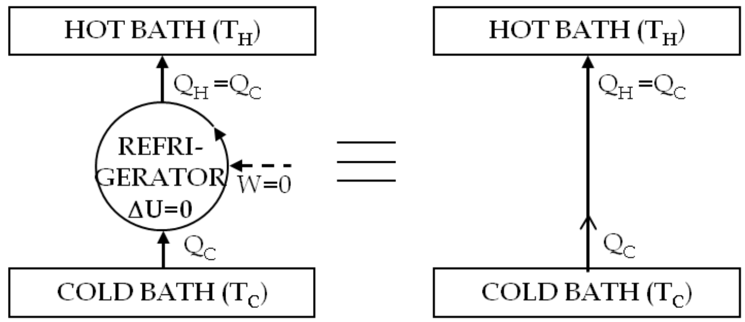

Figure 3.4: A perfect refrigerator where heat is transferred from a lower temperature reservoir to a higher temperature reservoir is not possible.

coefficient of performance, $\beta = \infty$. Since the refrigerator itself goes in a cyclic process, there is no change in the state of the working substance of the refrigerator in one cycle. Therefore, there is a net transfer of heat Q from a lower temperature reservoir to a higher temperature reservoir in each cycle of a perfect refrigerator. This violates the Clausius statement of the second law of thermodynamics. Hence a prefect refrigerator is not possible.