5.6 STATIC EQUILIBRIUM

5.6.1 Static Equilibrium and Newton's First Law of Motion

The basis for the first law of motion was discovered by Galileo from his observations on the motion of bodies along inclined planes. Newton stated his first law of motion as,

Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

You can see this law at play in many ordinary situations. For instance, your body in a moving car lurches forward when you suddenly stop the car, and similarly you tend to slide back when a car is suddenly started from rest with high acceleration. In both instances, the body is tending to resist the change in its state of motion, whether originally moving or at rest.

According to the first law of motion, a body at rest will remain at rest until a force acts on it. This principle underlies the stability of many physical structures. Questions like, how trusses can support large weights above them, how far above a ladder is it safe to climb, how much weight can be placed before a beam will give way, and many more, can be understood based on the balance of forces on and within physical structures.

To maintain a stable structure, no part of a body may have a left-over force or imbalance of forces. We say that a system is in static equilibrium if the net forces on each particle of the system is zero since if any particle of the system has a left over force, then, according to Newton's first law, a state of rest for that particle cannot be maintained.

Consider a system consisting of N parts. Let there be a net force $\vec{F}_1$ on part 1, $\vec{F}_2$ on part 2, etc. Each part may have more than one force on it, and the forces, $\vec{F}_1$, $\vec{F}_2$, etc, represent the vector sums of all forces on each part. Then, for the whole system to be in a static equilibrium we require the following condition.

$$\boxed{\left.\begin{array}{c} \vec{F}_1 = 0 \\ \vec{F}_2 = 0 \\ \vdots \\ \vec{F}_N = 0 \end{array}\right\}} \tag{5.18}$$

External and internal forces

The forces on each part of the system can be separated into two categories, internal and external. For instance, the net force $\vec{F}_1$ on part 1 is a vector sum of forces on part 1 by other parts, $2, 3, \cdots, N$ of the N-part system, which we call **internal forces**, and forces on part 1 from object that are outside the system which we call **external forces**. Suppose we sum all the internal forces on part 1 and call it $\vec{F}_1^{\text{int}}$ and sum all the external forces on part 1, call it $\vec{F}_1^{\text{ext}}$, then we can write $\vec{F}_1$ as

$$\vec{F}_1 = \vec{F}_1^{int} + \vec{F}_1^{ext}.$$

Similarly for forces on other parts of the system. The static equilibrium condition in Eq. 5.18 can now be expressed in terms of the internal and external forces on each part.

$$\left.\begin{array}{c} \vec{F}_1^{\text{int}} + \vec{F}_1^{\text{ext}} = 0 \\ \vec{F}_2^{\text{int}} + \vec{F}_2^{\text{ext}} = 0 \\ \vdots \\ \vec{F}_N^{\text{int}} + \vec{F}_N^{\text{ext}} = 0 \end{array}\right\} \tag{5.19}$$

From the third law of motion, we know that every force comes in pairs. Therefore, if there is force on 1 by 2, then there is also an oppositely directed force of the same magnitude on 2 by 1. This tells us that if we add all the equations in Eq. 5.19, the internal forces will all cancel each other, and we will obtain an equation that has only external forces on various parts.

$$\boxed{\vec{F}_1^{\text{ext}} + \vec{F}_2^{\text{ext}} + \cdots + \vec{F}_N^{\text{ext}} = 0} \tag{5.20}$$

We will prove below that if internal forces between parts are directed along the lines joining them, i.e. if force between parts 1 and 2 has the direction that is either parallel or anti-parallel to the displacement vector from part 1 to part 2, and the same for other internal forces, then the following identity for torques of external forces on different parts of the system can be proven from Eq. 5.19. Let $\vec{r}_1$, $\vec{r}_2$, $\cdots$, $\vec{r}_N$ be displacement vectors from an arbitrary point P to parts $1, 2, \cdots, N$ respectively. Then

$$\vec{r}_1 \times F_1^{\text{ext}} + \vec{r}_2 \times \vec{F}_2^{\text{ext}} + \cdots + \vec{r}_N \times \vec{F}_N^{\text{ext}} = 0, \tag{5.21}$$

which is the same as the sum of torques of external forces about an arbitrary point P.

$$\boxed{\vec{\tau}_1^{\text{ext}} + \vec{\tau}_2^{\text{ext}} + \cdots + \vec{\tau}_N^{\text{ext}} = 0} \tag{5.22}$$

Equations 5.20 and 5.22 give us an effective way to determine if a body consisting of N parts will be in static equilibrium.

Proof of Eq. 5.21

The force on i by j, denoted by $\vec{F}_{ij}$, and the force on j by i, denoted by $\vec{F}_{ji}$ are related by Newton's third law of motion.

$$\vec{F}_{ij} = -\vec{F}_{ji}.$$

Now, the sum of the torques of force $\vec{F}_{ij}$ on particle i and of force $\vec{F}_{ji}$ on particle j becomes

$$\vec{r}_i \times \vec{F}_{ij} + \vec{r}_j \times \vec{F}_{ji} = (\vec{r}_i - \vec{r}_j) \times \vec{F}_{ij},$$

where the vector $(\vec{r}_i - \vec{r}_j)$ is the displacement vector from particle i to particle j. If force between the particles is pointed along the line joining the two particles, then the internal torque of the forces $\vec{F}_{ij}$ and $\vec{F}_{ji}$ will be zero, leaving only net external torque.

Now, let us take cross product of the first equation in Eq. 5.19 by $\vec{r}_1$, the position of particle 1, of the second equation by $\vec{r}_2$, the position of particle 2, etc., and sum all resulting equations. The sum leads to vanishing of internal torque, leaving the result given in Eq. 5.21.

Deformable objects and static equilibrium

For an arbitrary structure to be in a static equilibrium, the vanishing of the net external force as given in Eq. 5.20 does not ensure static equilibrium. For instance, if you push on a piece of foam from two opposite sides, the net force and net torque on the foam are zero, but the foam gets deformed because of the imbalance between external and internal forces at the points of application of the forces. Therefore, you need to apply the complete condition for the static equilibrium given in Eq. 5.18, viz., the net force at each point of the system must vanish independently.

5.6.2 Problems of Static Equilibrium

We now consider some examples that will help us understand the use of static equilibrium in undeformable objects. The method for solving static problems follow a few basic steps outlined below. Make note of many decisions you will need to make when solving problems involving forces in equilibrium.

1. Pick the system. It can be either the entire structure, a part of the structure, or some small mass element inside the structure. The choice depends on the physical question that need addressing. Once, you have decided on the objects that belong

into the system, other objects that the system interacts with supply the external forces. This process helps us identify the external forces on the system.

2. Identify forces on the system by the external agents, making particular note of their directions and where they act. These usually fall into two categories: (a) long-distance, usually gravity in our case which can be taken to act at the center of gravity, and (b) contact forces at every surface of contact and every point of contact.

   Often, it is helpful to ask if any part of your system is pushing or pushing against something outside of the system. Then, you can use the third law of motion to "discover" the force that the other body would be applying on the system.

   Make a list of forces that you have identified. Remove any force that is by your system since we will be using only the forces on the system.

3. Calculations are usually mistake-free if done in analytic approach of vectors, although sometimes, it may mean more work than is necessary to solve a problem. The analytic approach begins with a judicious choice of Cartesian coordinates.

4. First, we work out components of the force vector. Often it is helpful to draw a separate diagram where all forces come out of the origin of the coordinate system. This diagram is called a free-body diagram and it is just a tool for calculations. Beware that forces may be acting at different points on the body, but they are redrawn in the tail-to-tail fashion in this diagram keeping their directions in space.

5. For the torque equation, we also need to choose a point P about which we will calculate torques. In a static situation, the choice of point P is arbitrary. Therefore, we pick point P such that torques are easy to calculate. A good location for P is where forces cross each other if you extend their vector lines in either or both directions. This choice will make the torques of the forces whose lines pass through P zero, and therefore, we will need to calculate the torques of other forces only.

6. Once point P has been chosen, write out all the displacement vectors $\vec{r}_i$ for $i = 1, 2, \cdots, N$. At this point you should have components of all forces and all displacement vectors if you are planning on working with the analytic approach. If you are

working with the geometric approach, you would have these vectors with arrows drawn out now.

7. Next order of business is to calculate all the non-zero torque vectors, which you can do by any of the methods illustrated in the section on torques above. Most problems in this book involve vectors in one plane and the lever arm method for torques turns out to be convenient.

8. Finally, since Equations 5.20 and 5.22 are vector equations. they give rise to three equations per vector equation.

   Depending on how much is known about the vectors and displacements, and the particular question you are trying to answer, you may need just Eq. 5.20, or just Eq. 5.22, or both. Examples below will illustrate the decision making process that goes into selecting particular set of equations we work with in different situations.

Example 5.6.1. A simple problem. Lets begin with a really simple problem. A 10-kg block is hung from the ceiling using a light string. What is the force between the ceiling and the block?

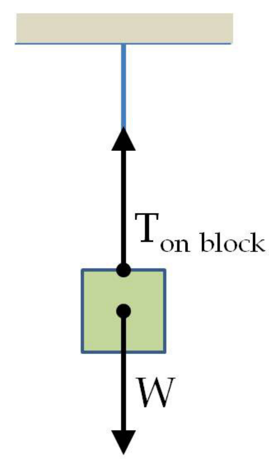

Figure 5.38: Example: 5.6.1.

Solution. Let us first understand the problem and get a sense of what we need for the answer. The force between the ceiling and the block is the tension in the string. The directions of the tension force on the block and ceiling are known here, so we only need to find the magnitude. Let T denote the magnitude of the tension force, although we will sometimes also write $\vec{T}_{\text{on block}}$ and $\vec{T}_{\text{on ceiling}}$ when we want to be specific about the individual force of the force pair acting on a particular body.

What should be our system? Could block alone be enough to solve the problem? Or, do we have to include the ceiling as well? Let us look at this choice from the simplest to more complicated. It would be great if forces on the block alone could solve the problem. In this case, the two forces on the block, namely, the weight of the block and the tension in the string, are external forces (see Fig. 5.38). Although these two forces act at different points on the block, they act along the same line. These two forces must be balanced since we have an equilibrium. The balancing of forces on the block will generate one equation for one unknown T. Therefore, we have sufficient information if we choose the block alone to be the system.

Balancing the forces on the block can be done analytically, by placing one of the axes, say the y-axis along the common line of action of the forces in the free-body diagram as shown in Fig. 5.39.

Only the y-components of forces are non-zero. Therefore, the y-component of the force-balancing equation, namely Eq. 5.20, results in the following relation.

$$\begin{aligned} &\text{y-component:} \quad T - 98.1\ \text{N} = 0. \\ &\text{x-component:} \quad 0 = 0. \\ &\text{z-component:} \quad 0 = 0. \end{aligned}$$

Therefore, $T = 98.1$ N, where we now put the units back. Note that I have include the information about the x and z-components for pedagogical reasons. You do not need to include them in your answer since it is quite clear from the figure itself that the x and z-components are zero.

We write the answer as follows: The force between the ceiling and the block has a magnitude of $mg = 98.1$ N. The tension force on the block is pointed up towards the ceiling, and the tension force on the ceiling is pointed down towards the block. Note that, since the question is about force, we must state both the direction and magnitude of the force in the answer. The force between the ceiling and the block refers to both the force on the ceiling and the force on the block.

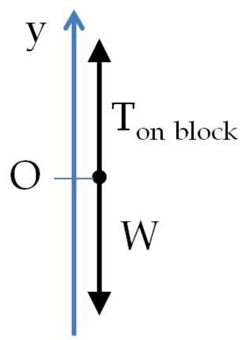

Figure 5.39: Example: 5.6.1. Tail-to-tail force vectors or a free-body diagram and axes for calculating components.

Example 5.6.2. Hanging a picture frame. A picture frame of mass m is hung by using two light strings. The two strings are tied to two different hooks in the ceiling such that the two strings make different angles with the horizontal direction as shown in Fig. 5.40. The picture hangs in equilibrium. What are the tensions in the two strings?

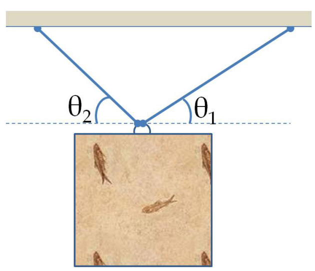

Figure 5.40: Example: 5.6.2. A picture frame hung from ceiling using two strings.

Solution. Once again, we start with first getting a feel for the problem. A rough drawing of the situation helps here. The situation is shown in Fig. 5.41. Since the strings are taut, there will be tension forces in the strings that would act on the frame as well as on the hooks in the ceiling. Now, since we have two strings, and there is no obvious symmetry arguments that can relate the magnitudes of the tensions in the two strings, we will use two different symbols, T_1 and T_2, for the magnitudes of the tensions in the two strings as shown in the figure.

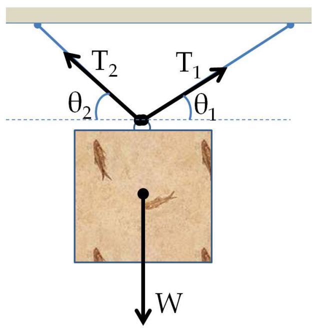

Figure 5.41: Example: 5.6.2. Forces on the picture frame.

Now, we need to decide about the system. Here, we have the choices: picture frame only, picture frame and the hooks, hooks only, etc. We again start with the simplest first. Fig. 5.41 shows three external forces on the picture frame. Here we know the directions of all three forces and the magnitude of one (W). So, we have two unknowns - the magnitudes T_1 and T_2. Since all forces fall in one plane, the force balancing equation will give us two relations. Therefore, the force balancing equation has sufficient information to solve the problem. Before we proceed to the solution, we point out that the torque equation will not give us any useful information since all three forces cross at one point when their lines of action are extended, which will give zero torque for each force with respect to the crossing point, which can be chosen for the arbitrary point P.

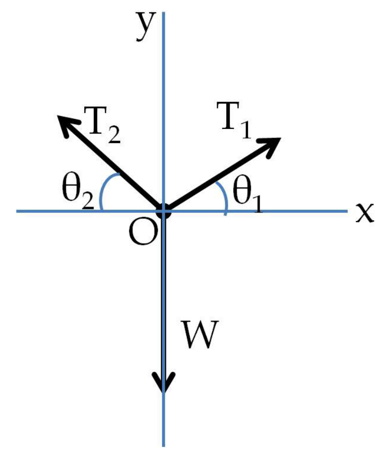

Figure 5.42: Example: 5.6.2. Tail-to-tail forces or free-body diagram and axes.

To implement the force balancing equation, we can work in the analytic picture of vectors as shown in Fig. 5.42. We draw the forces in the tail-to-tail fashion, called a free-body diagram, and then choose axes. With the choice of axes shown in Fig. 5.42, the z-components are all zero and the x and y-components of force balancing equations are:

$$\text{x-component:}\quad T_1\cos\theta_1 - T_2\cos\theta_2 = 0. \tag{5.23}$$

$$\text{y-component:}\quad T_1\sin\theta_1 + T_2\sin\theta_2 - mg = 0. \tag{5.24}$$

$$\text{z-component:}\quad 0 = 0.$$

Note the minus signs in the x-component of T_2 and the y-component of weight. The minus signs arise for the projection of vectors that fall on the corresponding negative axes. I have also listed the zero equal zero for the z-component for pedagogical reasons. When you are solving a problem and if it is obvious that a particular direction will give no information, you do not need to bother with listing that direction. I will also stop doing this pretty soon.

Now, we can solve Eqs. 5.23 and 5.24 for T_1 and T_2. Although, it isn't difficult to solve these equations, the answer looks complicated and we will give it below. As an example of a simpler calculation, let us solve these equations for particular numerical values of mass of the frame and the angles.

Case: $m = 0.5\ kg$, $\theta_1 = 30°$, $\theta_2 = 60°$

Putting these values in Eqs. 5.23 and 5.24 we find the following two equations in two unknowns.

$$\sqrt{3}T_1 - T_2 = 0.$$
$$T_1 + \sqrt{3}T_2 = 9.81.$$

Multiplying the first equation by $\sqrt{3}$ and adding the two equations gets rid of T_2 and we find

$$4T_1 = 9.81 \quad\Longrightarrow\quad T_1 = 2.45\ \text{N},$$

which we put in the first equation to obtain $T_2 = 4.24$ N. A student may prove that for general m, θ_1, and θ_2, the solution of Eqs. 5.23

and 5.24 give the following for T_1 and T_2.

$$T_1 = \frac{mg}{\sin\theta_1 + \cos\theta_1 \tan\theta_2}. \tag{5.25}$$

$$T_2 = \frac{mg}{\sin\theta_2 + \cos\theta_2 \tan\theta_1}. \tag{5.26}$$

Although Eqs. 5.25 and 5.26 appear complicated, they give more insight into the physical content than just numerical values. These general solutions let you answer many other questions. For instance, we can ask, what would happen if the strings were symmetric in their directions? It is a simple matter to set $\theta_1 = \theta_2 = \theta$, where we now write θ for equal angle of the two, to obtain the following conclusion.

$$T_1 = T_2 = \frac{mg}{2\sin\theta} \quad \text{(Symmetric situation.)}$$

Example 5.6.3. A Seesaw A seesaw of mass M has a support in the middle and can be balanced if it is perfectly horizontal. A child of mass m_1 sits tight at a distance d_1 from the middle. At what distance from the middle should another child of mass m_2 sit so that the seesaw is in static equilibrium?

Figure 5.43: Seesaws in a Montreal park.

Solution. Once again, we face the decision about what should be the system that would help answer the question. Should it be the seesaw alone, or the seesaw and the two children? Here we need to find the distance of the second child from the pivot point of the seesaw. This information will be contained in the torque equation.

Let us see what happens if we think of only the seesaw as the system. Then the external forces on the seesaw will come from Earth, the support at the pivot, child 1 and child 2. These forces are: weight of the seesaw as Mg, the force from support as F_N, force by child 1 as F_1 and force by child 2 as F_2.

WARNING!! If the seesaw is not in equilibrium, which is the point of fun in the seesaw, then often-made assumption that the force by child is equal to his/her weight would be false. We will discuss non-equilibrium situations when we introduce Newton's second law of motion in the next chapter. Presently, we leave the forces by children undetermined, and use equilibrium of each child to deduce the magnitude of the force each child applies on the seesaw.

Fig. 5.44 shows all the forces on the seesaw. The base of each force arrow shows the location where the force acts. Figure also shows a choice of coordinate system. The force from the support is the normal force here, and is labeled as $\vec{F}_N$. The line of force for the force from the support goes through the center of the seesaw where the force of gravity also acts.

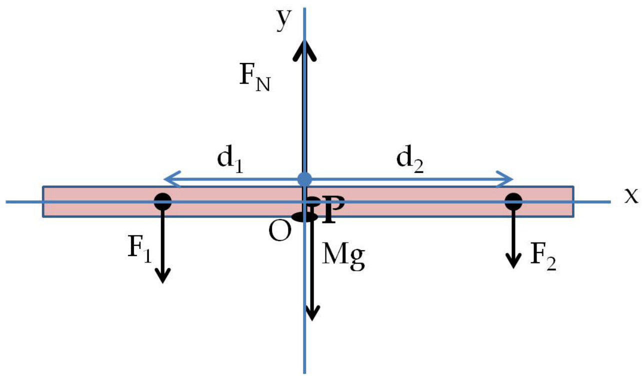

Figure 5.44: Example 5.6.3. External forces and their locations on the system consisting of seesaw only. The base of each force arrow shows the location where the force acts.

Now, let us figure out the force each child applies on the seesaw. To find the magnitude of the force by each child, we will need to set up a problem with that child as the system (Fig. 5.45). For instance, let us look at child 1 as the system. We find that there are two external forces on child 1 - his/her weight and the force by seesaw. The force by seesaw is equal to negative of force $\vec{F}_1$ by third law of motion. Since, the child 1 is in equilibrium, the forces must be balanced. This gives the following condition.

$$\text{Child 1 in equilibrium:} \qquad F_1 - m_1 g = 0 \implies F_1 = m_1 g.$$

Similar arguments for child 2 gives

$$\text{Child 2 in equilibrium:} \qquad F_1 - m_2 g = 0 \implies F_2 = m_2 g.$$

Therefore, we find that if a child is in equilibrium, the child presses on the seesaw by a force equal to his/her weight.

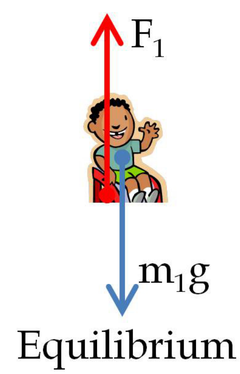

Figure 5.45: In equilibrium, the two forces on the child are balanced.

Now, we know the directions of all forces, and the magnitudes of all but the force from the support. Force balancing equation will clearly give us the value of the magnitude of the force from the support, but we are not after that information here. If the question had asked about the force from support, we would write the force balancing equation at this point and solve for the force. Our question is about the distance of child 2 from the support. That means, we will need to work with the torque equation for sure.

Choosing point P for torque calculations: Recall that torque equation requires specification of a point P, whose choice is up to us, for the calculations of torques about that point. Now, if we place point P at the support, then we don't need to know the support force since it will have zero contribution in the torque equation. This way, the torques equation for the system with only the seesaw will produce an equation in the unknown distance.

Torque calculations: With the choice of point P at the center of the seesaw, the contribution of $\vec{W}$ and $\vec{F}_N$ to the torques equation are zero and we need to calculate only the torques of pushes by each child on the seesaw. A calculation of torque can be done using the geometric method or lever arm or analytic approach of vector calculations using components. For the sake of comparison, I will work out the geometric and analytic methods.

Geometric approach:
In the geometric picture for torque as across product we need to know the directions of each torque so that we can decide if they will add or subtract from each other. Since the force and displacement vectors are in one plane, the torques will be perpendicular to this plane, pointing either above the plane or below the plane.

In Fig. 5.44 we use the right-hand rule to find the direction of the torques. The torque from F_1 is pointed towards the positive z-axis (coming out of page), and the torque from F_2 is towards the negative z-axis (going into the page). Therefore, they are oppositely directed and their vector sum will actually be subtraction of their magnitudes.

The magnitudes of the two torques are obtained from the magnitudes of the forces and displacements and the angle between them. Since the angle between the displacement vectors from point P to the point of action of the forces are both 90°, the magnitudes of the torques are simply multiplications of the magnitudes of the forces and the displacements.

$$\begin{aligned}
&\text{Torque of } \vec{F}_1 \text{ about P:} \quad \vec{\tau}_1 = \{m_1 g d_1,\ \text{pointed out of the page}\}\\
&\text{Torque of } \vec{F}_2 \text{ about P:} \quad \vec{\tau}_2 = \{m_2 g d_2,\ \text{pointed into the page}\}
\end{aligned}$$

Let out-of-page direction be taken as positive, then the sum of the torques becomes

$$m_1 g d_1 - m_2 g d_2 = 0,$$

which can be solved for the unknown d_2 to yield

$$d_2 = \left(\frac{m_1}{m_2}\right) d_1$$

Analytic approach:
The analytic approach for this problem is really not needed. But, for the sake of demonstration of the calculation, I will present the calculation of the torques in this problem using analytic method.

For the analytic method, we start with the components of the forces. The axes are given in Fig. 5.44. It is easy to show that the forces and displacements have the following representations in the given coordinate system.

$$\begin{aligned}\vec{F}_1 &= -m_1 g \hat{u}_y\\ \vec{F}_2 &= -m_2 g \hat{u}_y\\ \vec{d}_1 &= -d_1 \hat{u}_x\\ \vec{d}_2 &= d_2 \hat{u}_x\end{aligned}$$

It is not difficult to work out the cross product here. You do not need to use the determinant. Just do it directly using the cross product results of the base vectors themselves.

$$\begin{aligned}\vec{\tau}_1 &= \vec{d}_1 \times \vec{F}_1 = m_1 g d_1 \hat{u}_z\\ \vec{\tau}_2 &= \vec{d}_2 \times \vec{F}_2 = -m_2 g d_2 \hat{u}_z\end{aligned}$$

Therefore, the sum of the two is

$$\vec{\tau}_{net} = \left(m_1 g d_1 - m_2 g d_2\right) \hat{u}_z.$$

Setting the net torque to zero gives the same result as obtained above.

Example 5.6.4. A cantilever A beam that extends beyond its

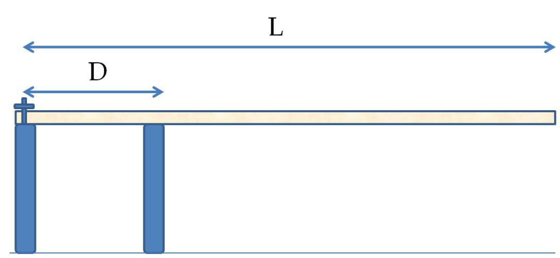

Figure 5.46: Example 5.6.4.

support is called a **cantilever**. Consider a cantilever of length L and mass M resting and bolted to two supports, one at the end and the other is at a distance D from it. (a) Find the conditions on forces so that the cantilever is in static equilibrium. (b) For $L = 30$ m, $M = 1000$ kg, and $D = 12$ m meters, find the magnitude and directions of the forces on the beam from the supports.

Solution. We will consider beam of the cantilever as the system. The beam has force from the support and bolt at the support at the end, an upward normal force from support in the middle, and the weight of the beam. As shown in Fig. 5.47, we will combine the forces of the bolt and the support at the end into a horizontal force

$\vec{F}_{11}$ and a vertical force $\vec{F}_{12}$, the sum of the two forces at the end give us the net force $\vec{F}_1$ at that point.

$$\vec{F}_1 = \vec{F}_{11} + \vec{F}_{12}.$$

Normally, one writes the force $\vec{F}_{11}$ as $\vec{F}_{1x}$ and $\vec{F}_{12}$ as $\vec{F}_{1y}$. This notation is confusing since symbols F_{1x} and F_{1y} stand for the x and y-components of the vector $\vec{F}_1$ and are not vectors themselves. Therefore, we will use $\vec{F}_{11}$ and $\vec{F}_{12}$ for the the horizontally and vertically pointed vectors whose sum is the force at the end.

From physical grounds we expect that all four forces $\vec{F}_{11}$, F_{12}, $\vec{F}_2$ and the weight $\vec{W}$ will be in one plane, which we take to be the xy-plane of the coordinate system shown.

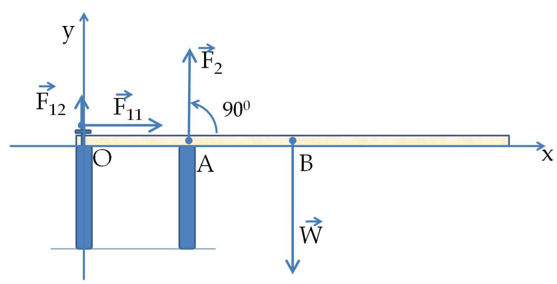

Figure 5.47: Example 5.6.4. Forces on the cantilever beam.

I leave for the student to do this problem using lever arm and geometric methods. Here, I will use the analytic approach. For the torque equation, we need to choose a suitable point P about which we will work out the torques. Since we do not know either $\vec{F}_1$ (or equivalently $\vec{F}_{11}$ and $\vec{F}_{12}$) or $\vec{F}_2$, it does not save us more work whether we place point P at O or A. Let us us pick O because it is nicely on one end so that we can measure displacements of the forces from the end point. We will also name the displacements as $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_w$ for the four forces. Let us organize the information regarding forces and displacements in a table.

Force	Force in components	Displacement	Torque
$\vec{F}_{11}$	$F_{11}\hat{u}_x$	$\vec{r}_1 = 0$	0
$\vec{F}_{12}$	$F_{12}\hat{u}_y$	$\vec{r}_1 = 0$	0
$\vec{F}_2$	$F_2\hat{u}_y$	$\vec{r}_2 = D\hat{u}_x$	$DF_2\hat{u}_z$
$\vec{W}$	$-Mg\hat{u}_y$	$\vec{r}_2 = (L/2)\hat{u}_x$	$-(L/2)Mg\hat{u}_z$

Balancing the forces and torques means setting the sum of the x, y, and z-components zero independently. Therefore, we obtain the following three equations in the present case from the force in component and torque columns in the table.

$$\begin{aligned} F_{11} &= 0 \\ F_{12} + F_2 &= 0 \\ DF_2 &= \frac{1}{2} LMg \end{aligned}$$

These equations can be solved to yield the following for the magnitudes of the forces.

$$F_{11} = 0 \tag{5.27}$$

$$F_{12} = Mg\left(1 - \frac{L}{2D}\right) \tag{5.28}$$

$$F_2 = \left(\frac{L}{2D}\right) Mg \tag{5.29}$$

Equation 5.27 says that there is no horizontal component of force $\vec{F}_1$ at the end, and Eq. 5.28 shows that if the second support is less that $L/2$ from the end, then the force in the vertical direction at the end support is downward, i.e. the force there is dominated by the push back from the bolt at the top of the beam.

(b) Plugging in the numerical values given we find the following.

$$\begin{aligned} F_{11} &= 0, \\ F_{12} &= -2500 \text{ N}, \\ F_2 &= 12000 \text{ N}, \end{aligned}$$

where I have rounded off to two significant digits.