4.12 DOPPLER EFFECT

So far in our discussion, there was no motion of detector or source of wave with respect to the medium. Hence, in all the foregoing studies, we have assumed that the frequency of wave detected is same as that of the source. However, if either the source or the detector moves with respect to the medium, there will be a difference in the frequency detected at the detector and that produced at the source. The effect is called the **Doppler effect**, named after the Austrian physicist Christian Johann Doppler (1803-1853), who in 1842 proposed the effect in his book titled, "Uber das farbige Lict der Dopperlterne," which means "the colored light of double stars".

Doppler effect also provides explanation for the high pitch of an incoming train whistle and low pitch of a train moving away. To be sure, the incoming train whistle also appears louder since the whistle is blown in the direction of the platform, but here we are not concerned with the loudness, but the pitch or the frequency. This is also the reason why the spectrum of light is shifted to the lower frequency, the so-called red-shift, if a star or a galaxy is moving away.

To understand the Doppler effect we will first consider two instances, one in which the source is stationary with respect to the medium while the detector moves at a constant velocity, and the other in which the detector is stationary with respect to the medium and the source moves with a constant velocity. Then we will combine the results to figure out what would happen when both the source and the detector move with respect to the medium. To keep it simple the relative motion will be kept along the line joining the source and the detector, which will be taken to coincide with the x-axis with positive x-axis pointed from detector to source.

Moving Detector and Stationary Source

Let v_D be the speed of the detector while the source is fixed with respect to the medium, and v be the speed of the wave in the still medium (Fig. 4.26). Note that if the detector is moving away from the source with a speed greater than the speed v of the wave in the medium, then the wave will never catch up. Therefore, we will impose the restriction here that if the detector is moving away from the source its speed be less than the wave velocity.

$$v_D < v \ \text{ if detector moving away.}$$

We do not need to put any restriction if the detector is moving towards the source. Let f_0 be the frequency of the wave emitted by the

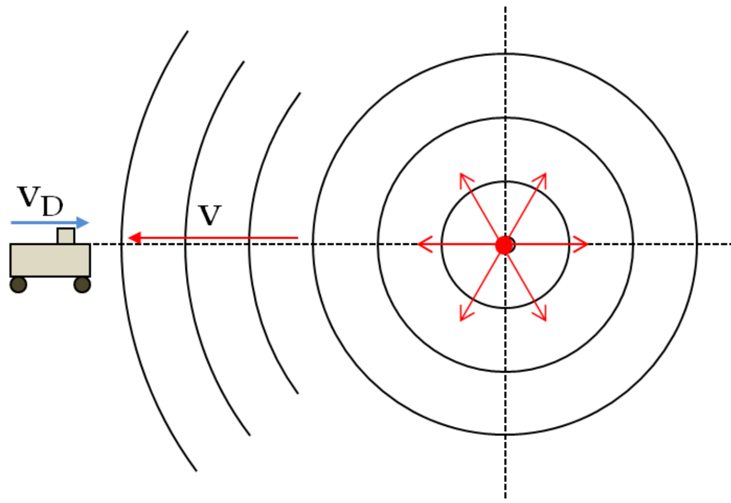

Figure 4.26: Doppler effect when the detector moves while the source remains stationary. When the detector is moving towards the stationary source, the detector encounters the wavefronts more frequency and therefore it will register higher frequency for the wave. If the detector were to move away from the source, the wave fronts will reach to the detector at longer time intervals and hence the detector will register a smaller frequency.

source. That is, there will be a new wave front every $1/f_0$ second. Wavefronts from the source will travel towards the detector at speed v with the distance between the wavefronts equal to $\lambda = v/f_0$, which is the wavelength λ.

For nonrelativistic speeds, e.g. v and $v_D << c$, the speed of light in vacuum, the speed of the detector with respect to the moving wave front will be $v+v_D$ if moving toward the source, and $v-v_D$ if moving away from the source. Hence, the distance λ between the wavefronts will be covered by the detector in different time then $1/f_0$. Let T_D be the time the detector encounters the successive wavefronts. Then T_D will be

$$T_D = \frac{\lambda}{v \pm v_D} \begin{cases} + & \text{when detector moves towards the source} \\ - & \text{when detector moves away from the source} \end{cases} \tag{4.65}$$

The frequency of the wave detected by the detector will be the inverse of this time period. Therefore, the frequency detected by the moving detector will be

$$f = \frac{1}{T_D} = \left(\frac{v \pm v_D}{v}\right) f_0 \begin{cases} + & \text{when detector moves towards the source} \\ - & \text{when detector moves away from the source} \end{cases} \tag{4.66}$$

Thus, when the detector moves towards the stationary source, the frequency at the detector is higher than that produced by the source, and when the detector is moving away from the stationary source, the frequency detected is lower than the one produced. This explains why the pitch appears higher if you run towards a stationary horn and lower if you run away from the stationary horn.

Moving Source and Stationary Detector

Consider a source of wave that moves with a speed v_S with respect to the medium in which the wave travels at the speed v while the detector is fixed in its place with respect to the medium as shown in Fig. 4.27.

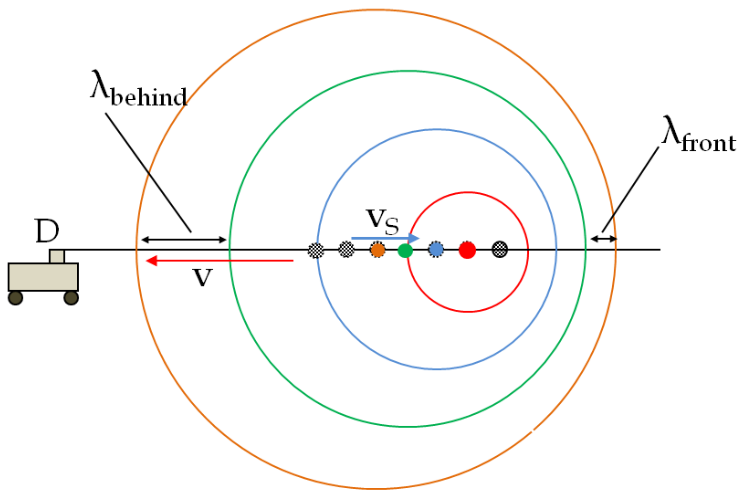

Figure 4.27: Doppler effect when the detector is stationary while the source is moving. The source sends out wavefronts spaced equally in time at the rate of $1/f_0$ per wavefront. But, these waves are centered at different places because they are generated at the instantaneous position of the source which is changing with time. The distance between the wavefronts in the front would be less than the distance in the back of the wavefront. The wavefronts travel at the same speed in the medium regardless of the state of motion of the source. This means that the detector behind the source will register longer time periods or lower frequencies and a detector in the front of the source will show higher frequency with respect to the frequency with which the waves were emitted at the source.

Even though the source is moving, it still emits wave at the same frequency f_0. Waves move out in expanding spheres centered at the the instantaneous location of the source which is changing in time due to the movement of the source. Hence, to an observer stationary with the medium, the wave fronts will be spaced differently on the two sides of the source. In front of the source, they will appear more closely spaced and in the back more widely separated, because as the source moves, it reduces the distance between a previously emitted wavefront and itself before laying the next wavefront's source point.

Therefore, wavelength will be different in front of the source than behind it.

$$\lambda = \begin{cases} (v - v_S)/f_0 & \text{in front of the source, } v_S < v \\ (v + v_S)/f_0 & \text{behind the source} \end{cases} \tag{4.67}$$

Since the speed of the wave is v in the medium, the frequency observed at the detector will be v/λ .

$$f = \left(\frac{v}{v \pm v_S} \right) f_0 \begin{cases} + & \text{behind the source} \\ - & \text{in front of the source, } v_S < v \end{cases} \tag{4.68}$$

Both Source and Detector Moving

It is quite common that both source and detector move with respect to the medium. In that case we can imagine a stationary frame of reference in the medium, and then perform a two-step process. First we find the frequency in the imagined stationary frame using the source-moving transformation, and then find the frequency detected by the detector-moving transformation. Let f_0 be the frequency of the source, and v_S, v_D and v be speeds of the source, the detector and wave with respect to the stationary medium. Let f_M be the frequency in the imagined frame stationary with respect to the medium, and f be the frequency observed in the detector.

The following two-step process will give the frequency observed by the detector in terms of the frequency of produced at the source.

$$f_M = \left(\frac{v}{v \pm v_S} \right) f_0 \begin{cases} + & \text{behind the source} \\ - & \text{in front of the source, } v_S < v \end{cases} \tag{4.69}$$

$$f = \left(\frac{v \pm v_D}{v} \right) f_M \begin{cases} + & \text{when detector moves towards the source} \\ - & \text{when detector moves away from the source} \end{cases} \tag{4.70}$$

We can combine these equations to obtain one equation giving us the frequency detected to the frequency produce, as long as we remember the meaning of different signs.

$$f = \left(\frac{v \pm v_D}{v \pm v_S} \right) f_0 \tag{4.71}$$

Note: Because of the nonrelativistic calculation, we cannot use the formulas derived here for light; you will need to use special theory of relativity for correct relation called the **relativistic Doppler effect**. We quote the relation without derivation.

$$f = \sqrt{\frac{c - v_S}{c + v_S}} f_0 \begin{cases} v_S > 0 & \text{if source moving towards receiver} \\ v_S < 0 & \text{if source moving away from receiver} \end{cases} \tag{4.72}$$