5.4 SOME COMMON FORCES

Experiments show that there are four **fundamental forces** in nature that can explain all other forces.

1. Gravitational force
2. Electromagnetic force
3. Weak nuclear force
4. Strong nuclear force

The weak and strong nuclear forces are short-range forces, becoming significant only at nuclear distances. We will not study the nuclear forces in this course.

The electromagnetic and gravitational forces are long-range forces. The electromagnetic force lumps together electric force and magnetic force since both are related to the same property of matter, i.e. the electric charge.

Besides these fundamental forces, we encounter many other forces in everyday life which are fundamentally caused by gravitational and/or electromagnetic force. For instance, the force of friction between two solid surfaces is an average effect of the electric force between the electrons and protons of the two surfaces in contact. Similarly, the force applied by a spring, the force of tension in a string, the force of air resistance, and the force due to surface tension in a fluid, just to name a few of common application, all have their basis in the electric force.

5.4.1 Weight and Gravitational Force

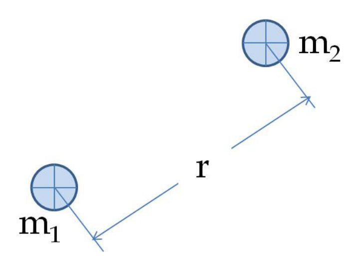


Figure 5.13: Two masses separated by a distance r have an attractive gravitational force between them.

We have already studied the basic aspects of the force called weight. The weight of an object is due to the **gravitation force** between the object and the Earth. In general, gravitational force between any two objects of masses m_1 and m_2 separated by a distance r is given by Newton's law of gravitation, which says that magnitude and direction of the gravitational force are:

Magnitude:

$$\boxed{F = G_N \frac{m_1\, m_2}{r^2}} \tag{5.5}$$

where G_N is a universal constant, called the Newton's gravitational constant, and has the following approximate value.

$$G_N = 6.67 \times 10^{-11}\ \frac{\text{N.m}^2}{\text{kg}^2}. \tag{5.6}$$

Note that G_N is not g, the acceleration due to gravity. I have placed a subscript to G_N to avoid this common confusion. If you are not confused, you may drop the subscript and denote Newton's gravitational constant by G to save time.

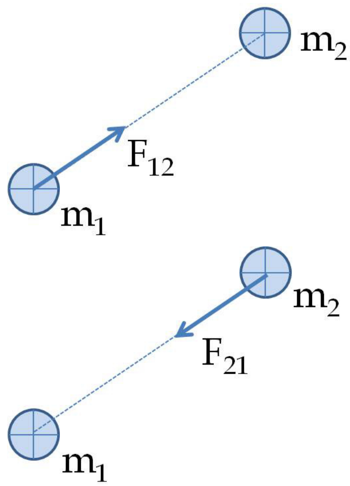


Figure 5.14: The direction of the gravitational force between two masses depends on the body the force acts. The force on 1 by 2, indicated as F_{12} is towards 2. The force on 2 by 1, indicated as F_{21} is towards 1.

For spherical objects, we will show in a later chapter that the distance in Eq. 5.5 would be center-to-center distance. It can be also shown that the second object is inside a spherical shell, then there would be no gravitational force. If the second object is inside a spherical object such as at a point inside the Earth, the mass of the sphere that is within the distance from the center has a net gravitational force; the gravitational force from masses outside cancel out.

Direction:

Force on m_1 is towards the other mass, m_2, and the pair force, i.e. the force on m_2 is towards m_1. The direction says that the force of gravitation between the masses is attractive.

Now, we have learned above that the magnitude of the gravitational force between Earth and an object of mass m, called the weight of the object, is approximately equal to mg, where $g = 9.81\ \text{m/s}^2$. This turns out to be true only when the object is close to the surface of the Earth.

If an object is not near the surface of the Earth, for instance, if the distance of the object from the surface of the Earth is comparable to or exceeds the value of the radius of the Earth, the difference between mg and the actual value of the gravitational force between the object and Earth becomes significant and you cannot say that the weight is mg.

You can address this discrepancy by saying the "value of g is different than 9.81 m/s^2 when you are not near the surface of the Earth" and using a different value of g that would be appropriate for the situation. Although, this viewpoint is acceptable, one rather abandons mg in these situations and uses Eq. 5.5 for the force between Earth and the object.

Demonstration that mg is due to gravitational force

We now demonstrate the gravitational force between Earth and an object at the surface of Earth is equal to weight by applying the formula given in Eq. 5.5 to Earth/Object pair. Let us denote the mass of Earth by M_E and the radius of Earth by R_E. Then, the distance from the center of Earth (since Earth is almost spherical) to an object at the surface is equal to the radius of Earth. Therefore, we set $r = R_E$ in Eq. 5.5. This gives us the following magnitude for the gravitational force between Earth and an object of mass m located at the surface of Earth.

$$F = G_N \frac{M_E\, m}{R_E^2},$$

which can be rearranged so that we write it as m times something that is independent of m so that we can compare with mg formula,

$$W = m\left(G_N \frac{M_E}{R_E^2}\right).$$

The question before us is: Is this equal to mg with $g = 9.81\ \text{m/s}^2$? Let us compare the quantity within the parenthesis to the standard value of g by plugging in the standard values of $M_E = 5.97 \times 10^{24}$ kg and $R_E = 6.37 \times 10^6$ on the right side.

$$G_N \frac{M_E}{R_E^2} = 6.67 \times 10^{-11} \frac{5.97 \times 10^{24}}{(6.37 \times 10^6)^2} = 9.81\ \text{m/s}^2,$$

which is identical to the value of g. This confirms that mg is equal to the magnitude of the gravitational force of Earth on an object at the surface of Earth.

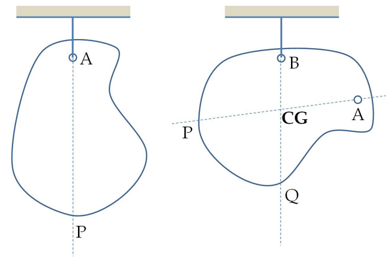


Figure 5.15: The force of gravity of Earth acts at all particles of a body. For the purposes of torque on the body, the force of gravity can be considered to act at a special point called **center of gravity (CG)**. The CG of a body can be located by hanging the body from different points (A and B in the figure) and locating the point where the vertical lines cross.

Example 5.4.1. What is the gravitational force between Earth and a satellite of mass m at a distance $2R_E$ from the center of Earth?

Solution. At $r = R_E$, we found that the gravitational force was equal to mg. The gravitational force given in Eq. 5.5 decreases as distance squared. The distance here is 2 times R_E. Therefore, the force will be $\frac{1}{4}$ of the force at $r = R_E$. Hence, the gravitational force on the satellite will be equal to $\frac{1}{4}mg$, where $g = 9.81\ \text{m/s}^2$.

5.4.2 Tension Force and Hooke's Law

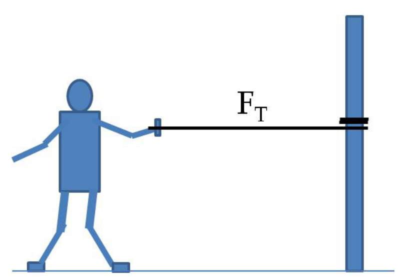


Figure 5.16: A tension force acts between the post and the person.

A wire or string can be used to create forces between two objects. Suppose you tie one end of a wire to a post and pull at the other end. When the wire is loose, there is no force between the post and the person. But, when the wire is taut, the two bodies get linked by the tension force in the wire. The tension force acts on the post in the direction away from the post and towards the person, and it acts on the person in the opposite direction.

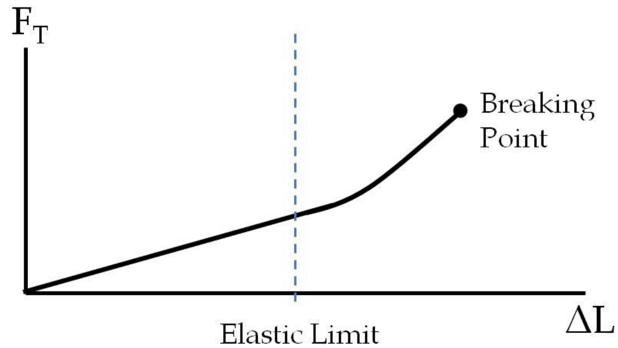


Figure 5.18: Tension in the wire as a function of stretching ΔL.

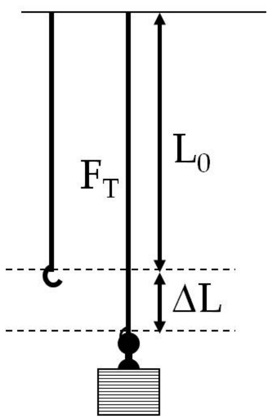


Figure 5.17: Stretching a wire by hanging masses.

The magnitude of the tension force is proportional to the amount of stretch. One way to find the relation between stretching and the corresponding tension is to hang various masses at the bottom of the wire while fixing the top end of the wire to a ceiling as illustrated in Fig. 5.17. In Fig. 5.18 we show a schematic view of the way tension F_T varies with the stretching of wires.

If the stretching is less than the elastic limit of the wire, the wire goes back to its original length once you remove the tension. Beyond the elastic limit, the wire gets permanently changed. Within the elastic limit, the change in length and the tension force are directly proportional and we can write the tension force as

$$\boxed{F_T = k\Delta L,} \tag{5.7}$$

where the proportionality constant k depends on the chemical makeup of the material, and the length and area of cross-section of the wire. This observation was first made by Robert Hooke (1635-1703), a contemporary of Sir Isaac Newton, and is often called **Hooke's law**, which states that "Ut tensio sic uis," in Latin, meaning "As the extension, so the force." Every elastic object obeys Hooke's law within its elastic limit.

The main point of Hooke's law is that if a system, whether a wire, a spring, or any other material body, is deformed from some stable structure, a restoring force develops which tends to bring the system back to the original stable point. In the next section, we will apply Hooke's law to spring force.

This is a commonly encountered behavior of most materials. The basic mechanism of the development of a tension force may be understood in terms of bonding of molecules. When the wire is not pulled, the molecules are positioned at certain distance from each other that balances the attractive and repulsive forces. As the wire is pulled, the distances between molecules increase, increasing both the attractive and repulsive forces making a new equilibrium between the forces on molecules. As long as the external pull is present, the new equilibrium is maintained. When the external pull is removed, the attractive force between the molecules is no longer balanced by the external pull and brings the wire to the original length.

In most problems concerning the tension force, we usually do not know k or ΔL. Therefore, we leave the tension force as F_T or T rather than express it in the corresponding $k\Delta L$ form.

Tension Force:
Magnitude: T, varies
Direction: Along the string, away from the body
Where: At the point(s) string tied or pressed to the body

The tension in a continuous wire is same throughout the wire if the wire is only pulling on the two objects connected at the ends and does not pressing on any other object. For instance, when a mass m is hung from the ceiling by using a wire, the tension in the entire wire is same. However, if the wire goes over a pulley, a ring, or a hook, then the tension now connects two sets of bodies on the two sides of the body in the middle. For instance, in Fig. 5.19, there are tension forces between the ring and the block m and the ring and the support S. The tensions in the two sides of the ring may be different: $T_1 \neq T_2$. Furthermore, while there is one tension force acting on mass m and another one acting on the support S, both tension forces act on the ring R. The net tension force on the ring is the vector sum of the two tension forces acting on it as shown in Fig. 5.19.

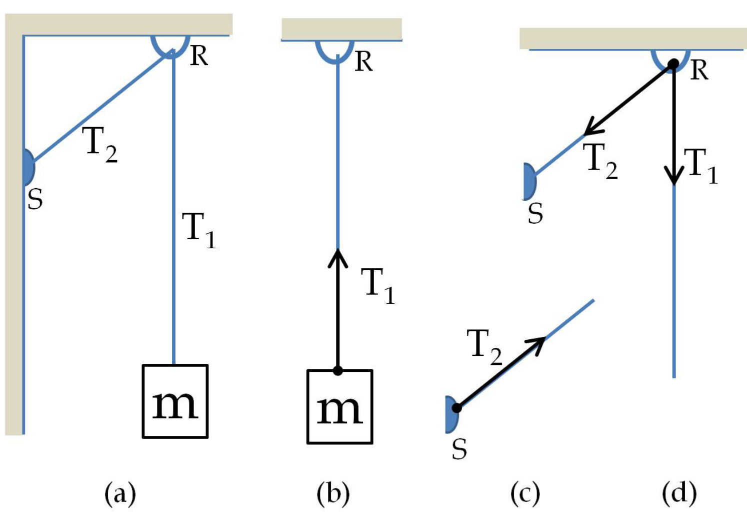


Figure 5.19: The tension in the string acts on three objects, the mass m, the ring R and the support S. The tension force between mass m and ring R may be different from the tension force between the ring and the support R although the same wire connects all three objects. In (b) the tension force on mass m is shown, in (c) the tension force on the support S is shown, and in (d) the tension forces on ring R is shown. The net tension force on ring R is the vector sum of the two tensions.

5.4.3 Spring force and Hooke's Law

When a force is applied on both ends of a spring the distance between coils can change causing a restoring force in the spring that acts to restore the original length of the spring. This force is called **spring force** which acts on the two objects linked by the spring. If the spring is extended, then spring forces on the two objects are such that the two objects are attracted to each other (Fig. 5.20). On the other hand, if spring is compressed, spring forces on the two objects are such that the two objects are repelled from each other.

Spring Force:
Magnitude: $k\Delta L$
Direction: Towards the equilibrium point
Where: At the point spring attached to the body

Spring force is similar to the tension force we have studied in the sense that the force is proportional to the change in length from a reference length as long as the extension or compression is not so large that the spring is permanently deformed. That is, Spring force obeys Hooke's law:

If a spring is either extended or compressed by an amount ΔL, the magnitude of the **spring force** on either object connected to the spring is

$$|\vec{F}_{spring}| = k\Delta L, \tag{5.8}$$

where the proportionality constant k is called the spring constant. Note that when the spring is neither extended or compressed, the spring force is zero.

The directions of the spring forces on the two objects on the two sides of the spring depends on whether the spring is extended or compressed as explained above. When the spring is extended compared to its natural length, the block and the support attract each other, i.e. the force on each is pointed towards the other as shown in Fig. 5.20. When the spring is compressed compared to the natural length, then the block and the support repel each other, i.e. the forces on them are pointed away from each other.

5.4.4 Normal Forces at a Contact Surface

When you press a solid body against another solid body, the bonds among molecules at the interface of the two bodies are compressed. These bonds act as small springs. The compressed "molecular springs" in the two bodies generate a repulsive force on each other. This repulsive force is called the normal force.

Normal forceacts on the two bodies in contact through the surface of contact of the two bodies. The magnitude of the normal force depends in a complicated way on the degree of deformation of

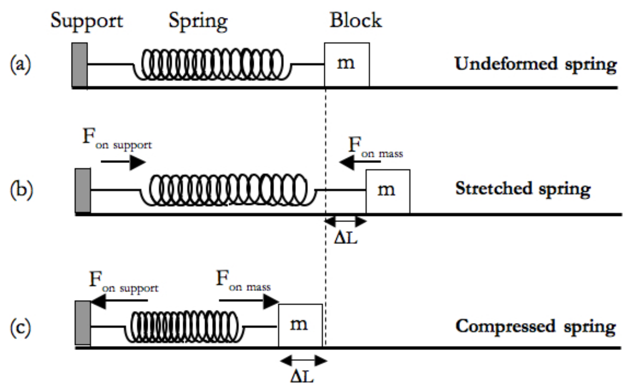


Figure 5.20: Spring force between block and support. The spring force has the magnitude $F = k\Delta L$ and acts on the block and the support. The directions of the forces on the block and support are such that the spring forces tend to restore the original length of the spring.

the bonds near the surface each body and the total area of contact. The force will be greater in the area where the surfaces are pressed together more and less where the surfaces are pressed less. If the surfaces are in contact but not pressing against each other, then the normal force will be zero.

Normal Force:
Magnitude: varies; symbol F_N or N
Direction: Towards the body, perpendicular to the surface of contact
Where: At points of the contact surface

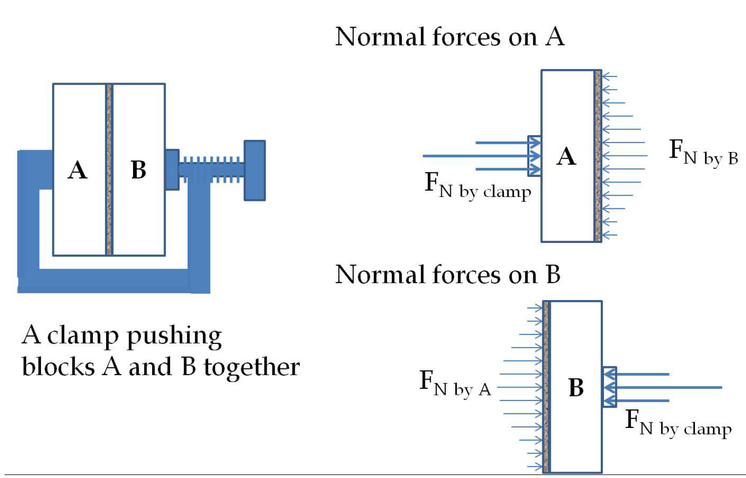


Figure 5.21: The normal forces between blocks A and B is created by pushing the blocks together. The forces by the clamp itself is transmitted through the surfaces between the clamp and the blocks. The normal force is distributed along the surface of contact. Normal force is higher where the surfaces are more pressed together and smaller where they are less pressed together.

Normal force is distributive in nature similar to the force of gravity since it does not act any one point. Despite this fact we will place a net normal force between two bodies to act at one point on the body for the purposes of calculation of torque from the normal force.

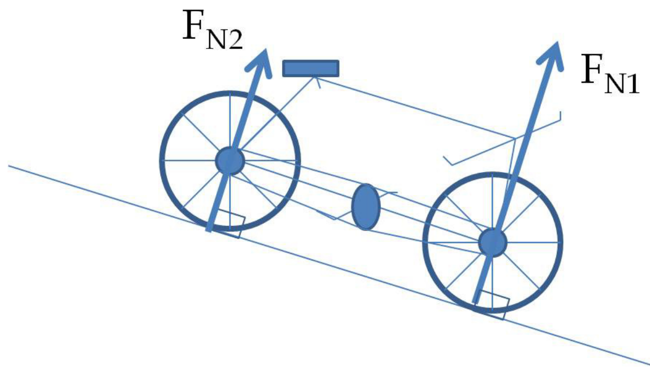


Figure 5.22: The normal forces on the front and back wheels on a bicycle on an incline are not equal.

Just as net torque by gravity of Earth on a body is same as if the net force of gravity acts on the center of gravity, we can envision a net normal force acting at some point of the body giving us the same net torque as would be obtained by calculating the net torque from the distributed actual normal forces between the two bodies. For this purpose, the net normal force may or may not pass through the center of gravity of the body. The net normal force on the body will act on that point of the body that is more pressed than other points. For instance, if a bicycle is on an incline with the front wheel down the incline and back wheel up the incline, then the normal force on the front wheel will be larger than the normal force on the back wheel.

WARNING:
$F_N \neq mg$
$F_N \neq mg\ \cos\theta$
F_N is determined by other forces and acceleration of the body.

Normal force is a reaction force. For a normal force to develop between two bodies, the two bodies need to be subjected to outside forces that are tending to push the bodies into each other.

Unlike weight, which has a formula mg near Earth, and spring force, which has a formula $k\Delta L$, there is no particular formula for a normal force. The magnitude of the normal force is determined by the external forces acting on the body that generate the normal force and the conditions of the motion of the two solid bodies.

Many formulas of normal force are found in elementary textbooks which are specific to the particular problem at hand and should not be memorized. In particular, one often encounters $F_N = mg$ and $F_N = mg\cos\theta$, neither of which are always applicable. The formula for a normal force in a particular situation must be "discovered" by setting up Newton's laws of motion in that situation. In this chapter and the next, we will work out problems where the process of setting up Newton's laws will be explained further.

The direction of the normal force on each body can be deduced from its repulsive nature. Thus, if body 1 is pressed against body 2, the normal force on body 1 will be pointed away from body 2. Similarly, the normal force on body 2 will be pointed away from body 1.

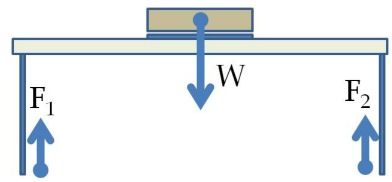


Figure 5.23: The downward weight (W) of the book and upward support forces $\vec{F}_1$ and $\vec{F}_2$ on the legs of the table compress the table and book together.

As an illustration of the normal force consider a book on a table (Fig. 5.23). We know that the weight of the book is supported by the table. But what is really going on at the interface between the book and the table? The book presses on the table with a force equal to its weight. However, if the table did not have a support force upward on its legs, which is itself a reaction due to gravity force on the table and the book, the weight of the book will not succeed in compressing it to the table. You need forces from both directions to compress the book and table together.

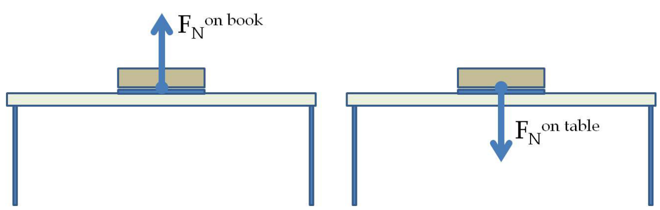


Figure 5.24: Normal force between book and table. When acting on the book, the force is pointed towards the book and away from the table, and when acting on the table it is pointed towards the table and away from the book.

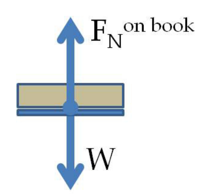
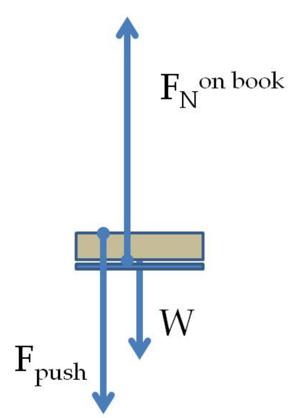


Figure 5.25: Forces on the book.

The weight of the book from one side and the upward support forces on the legs of the table push the book and table into each other, which compresses the "molecular springs" of both the book and the table. Therefore, a normal force develops that acts on the book and the table (Fig. 5.24).

Now, if you look at the forces on the book, you will discover two forces - the force of gravity from Earth pointed down and a normal force from the table pointed up (Fig. 5.25). The two forces may or may not be equal in magnitude depending upon whether or not the book is in equilibrium as we will see later in this chapter.



If the book is in equilibrium, meaning forces on it are balanced, then the magnitude of the normal force will be equal to the weight of the book. Otherwise, if the book is not in equilibrium, the magnitude of the normal force may be greater that or less than weight.

Book in equilibrium (balanced forces): $F_N = mg$

Book not in equilibrium (unbalanced forces):

$$F_N > mg \quad \text{or} \quad F_N < mg.$$

Figure 5.26: Your push on the book acts with the weight to generate a larger normal force. In order for torque to

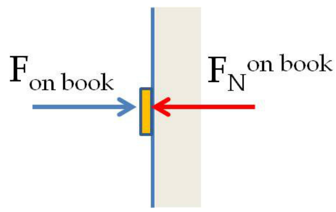


Figure 5.27: Your push on the book against the wall generates a normal force between the book and the wall. Weight and friction on the book are not shown in this figure to keep it simple.

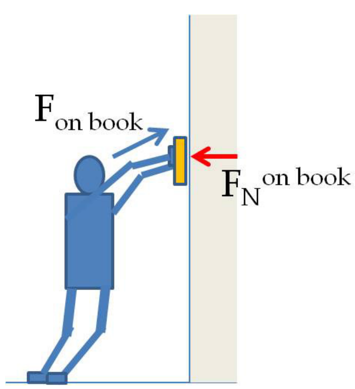


Figure 5.28: Your push on the book at an angle generates normal force and friction between the book and the wall. Weight and friction on the book are not shown in this figure to keep it simple. Note normal force is 90° to the surface of contact.

Now, suppose you apply a force F_{push} on top of the book in the vertical direction (Fig. 5.26). How would the normal force between the book and the table change? The book will press on the table with a force equal to its weight plus F_{push}. The book/table interface will be even more compressed, giving rise to a normal force that would be greater than the normal force when you hadn't applied the force.

As another example, suppose you press the book against a wall at right angle to the wall (Fig. 5.27). What is going on at the interface of the book and the wall? Your force on the book compresses the molecular springs of the book and the wall, which generates a normal force at the interface which acts on the book pushing the book away from the wall and also acts on the wall pushing the wall away from the book.

Let $\vec{F}$ be your push force on the book. Now, the normal force between the book and the wall will not be related to the weight of the book but to the push force.

$$\text{Book in equilibrium (balanced forces):} \quad F_N = F$$
$$\text{Book not in equilibrium (unbalanced forces):}$$
$$F_N > F \quad \text{or} \quad F_N < F.$$

What happens if you press on the book at an angle other than 90° to the wall? The normal force now will be smaller since only a projection of your force is pushing the book's surface into the wall (Fig 5.28). Now, there will also be a force from the book on the wall parallel to the wall. This force is called frictional force which we will study next.

5.4.5 Static Frictional Force

Suppose you try to slide a book resting on a table by applying a horizontal force $\vec{F}$ on the book (Fig. 5.29). What would happen? Recall that due to gravity on the book the book is pressed into the table. As a result, a bonding also develops between the molecules of the two bodies at the interface.

Due to this bonding of the book to the table, when we apply a horizontal force on the book, the book attempts to pull the table along with it. That is, the force applied on the book is transmitted to the interface between the book and the table such that the book now applies a horizontal force on the table that is equal to the applied horizontal force.

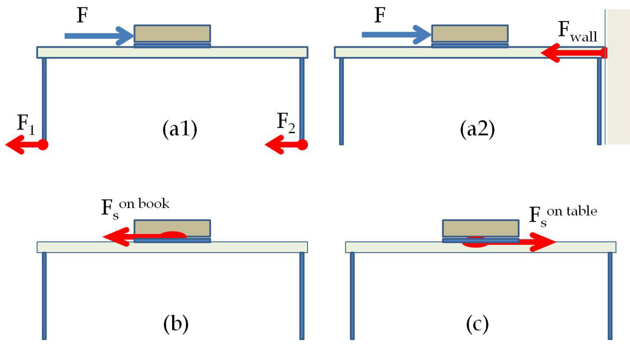


Figure 5.29: Static friction generated as a response to external forces horizontal to the surface of contact. (a1) Force $\vec{F}$ applied on the book is balanced by constraining forces at the floor of the legs of the table. (a2) Force $\vec{F}$ applied on the book is balanced by normal force by the wall. (b) Static friction force acting on the book. (c) Static friction force acting on the table. Other forces on the book not shown here: weight and normal.

If the table is free to move, then the horizontal force from the book at the interface will cause the table to accelerate. However, if the table cannot move, either because it is held against something else, or the legs are bonded to the floor by friction or by some other means, the table will react to the applied force with a generation of a frictional force at the surface between the book and the table. Thus, if an external force is applied parallel to the interface, a static frictional force will develop if there is also a normal force pressing the surface together or the two surfaces are bonded together.

Static frictional force acts on the two bodies in the tangential plane of the contact surface such that the force on frictional force on the body with the applied force on it has the direction of the frictional force opposite to that of the direction of the applied force. The direction of the frictional force on the other body is in the direction of the applied force. In our example above, the direction of the friction force on the table is in the direction of the applied force on the book, and the direction of the frictional force on the book is opposite of the direction of the applied force on it.

The magnitude of the frictional force is determined similarly to the way the magnitude of the normal force is determined. Since friction force is a reaction force, the magnitude depends on the applied force that helps generate frictional force. Therefore, the magnitude of the static friction force depends on the magnitude of the external applied force and whether or not the two bodies are in equilibrium.

As a general rule, the magnitude of static frictional force will increase with the increase in applied force up to a limit when the

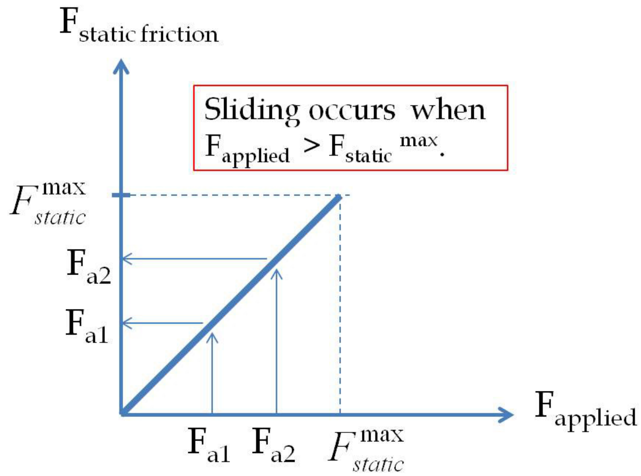


Figure 5.30: The magnitude of static friction depends on the applied force since the frictional force matches the external applied force as long as the later is less than F^{max}_{static}.

Static Friction Force:
Magnitude: varies up to a maximum magnitude; $F_s \le F_s^{\text{max}}$; Inequality law.
Direction: In the tangent plane of the surface of contact to be determined from the dynamical conditions of the body.

static bonds between the two bodies break up and the two bodies start to slide past each other. Thus, static friction force can have any magnitude up to this limit, which is called the **maximum static friction force**, F_s^{max}, as shown schematically in Fig. 5.30. We are, of course, free to apply any amount of force on the bodies, but the static friction force cannot be more than F_s^{max}. When the applied force is less than the maximum static friction force, i.e. when $F_{\text{appl}} < F_s^{\text{max}}$, such as the values F_{a1} and F_{a2} shown in Fig. 5.30, the static friction force matches the applied force and there is no sliding.

The magnitude of the force required to overcome the static bonds depends on the nature of the two surfaces and the degree to which the two surfaces are pushed into each other. This says that F_s^{max} can be stated in terms of magnitude of the normal force, which is a measure of the extent to which the two surfaces are pushed into each other.

$$\boxed{F_s^{\text{max}} = \mu_s\ F_N,} \tag{5.9}$$

where μ_s is called the coefficient of static friction. The constant μ_s depends on the area of contact and the physical nature of the two surfaces in contact. Note that Eq. 5.9 is not a vector relation since friction and normal forces act perpendicularly to each other. Rather, Eq. 5.9 relates only the magnitudes of the maximum static friction and the normal force.

WARNING:
$F_s \neq \mu_s F_N$
$F_s \neq \mu_s mg$

Example 5.4.2. Maximum static friction force - 1

A 1.4-kg book rests on a horizontal wooden table. If the coefficient of static friction force is 0.8, what would be the horizontal force needed to overcome the maximum static friction between the book and the table?

Solution. From the discussion above, we know that we need to overcome static friction force to make the book slide. So, the minimum force we need must be equal to the maximum static friction on the book by the table. According to Eq. 5.9, the magnitude of the maximum static friction force is equal to the product of the coefficient of static friction force and the magnitude of the normal force. The normal force F_N is equal to the force with which the book is pressing on the table vertically. Here, the book is pushing on the table due to its weight. Therefore, the magnitude of the normal force in the present situation must be equal to the weight of the book.

$$F_N = 1.4 \text{ kg } \times \text{ } 9.81 \text{ m/s}^2 = 13.7 \text{ N}.$$

Hence, the magnitude of the minimum horizontal force needed to slide the book

$$F_{\text{min}} = F_{\text{static}}^{\text{maximum}} = \mu_s \; F_N = 0.8 \times 13.7 = 11.0 \text{ N}.$$

Example 5.4.3. Maximum static friction force - 2

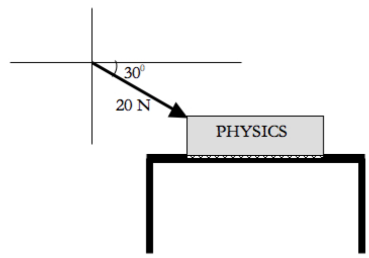
A 1.4-*kg* book rests on a horizontal wooden table. A 2-kg steel block is placed upon the book. If the coefficient of static friction force between the table and the book is 0.8, what would be the minimum horizontal force needed to overcome the static friction between the table and the book?

Solution. Note that the force with which the book is pressing vertically on the table is larger in this example than was the case in the last example. The net force pressing the book into the table here is equal to the total weight of 3.4 kg, which is equal to 33.4 N. Therefore, the magnitude of the minimum horizontal force needed to slide the book now is

$$F_{\text{min}} = 0.8 \times 33.4 = 26.7 \text{ N}.$$

Example 5.4.4. Maximum static friction force - 3

A 1.4-*kg* book rests on a horizontal wooden table. You push on it with a force of 20-N in the direction 30° below the horizontal direction. If the coefficient of static friction force is 0.8, will the book slide?



Solution. In the present example, some of the force with which the book is pressing comes from the vertical component of the applied force, and the rest from the weight of the book, which acts vertically. Therefore, the magnitude of the normal force F_N by the table on the book will be

$$F_N = W + F\ sin\theta = 13.7 + 20\ \text{ sin }\ 30^\circ = 23.7 \text{ N}.$$

The minimum horizontal force required to slide the book must equal the maximum static friction force:

$$\text{Minimum horizontal force needed } = 0.8 \times 23.7 = 18.96 \text{ N}.$$

Now, the question is: Does the horizontal component of the applied force exceed this value?

$$\text{Horizontal component of force} = F\ \cos\theta = 20 \cos\ 30^\circ = 17.32 \text{ N},$$

which is less than 18.96 N required for overcoming the maximum static frictional force. Therefore, the book will not slide.

5.4.6 Kinetic or Sliding Frictional Force

What happens when you apply a horizontal force larger than the maximum static friction force? Once you overcome the maximum static friction force, the two bodies in contact will start to slide relative to each other, and you will find that it takes a smaller force than the maximum static friction force to maintain a steady constant velocity motion. There is still frictional forces on the sliding objects that oppose the sliding motion, called **kinetic or sliding friction force**. The magnitude F of the force needed to slide an object in a uniform motion of constant velocity is usually proportional to the magnitude F_N of the normal force.

$$\boxed{F_k = \mu_k\ F_N, \quad \text{(directed opposite to the sliding motion)}}, \qquad (5.10)$$

where the proportionality constant μ_k is called the **coefficient of kinetic friction**. The constant μ_k depends on the area of contact between the surfaces and the physical nature of the surface of the two bodies such the roughness of the surface.

5.4.7 Rolling Friction

The static and kinetic frictions are caused by the interlocking of microscopically uneven surfaces of two solid bodies in contact which vary directly with the area of contact between the two bodies. When a circular hoop or a spherical ball rolls on a flat surface, the surface of contact is much reduced (Fig. 5.31). We say that a body is rolling over the other body without sliding if the point of contact of the bodies is momentarily at rest. In a sense, the rolling motion is due to a static friction between the bodies since the point of contact between the two bodies does not move in a rolling motion. Consequently, the

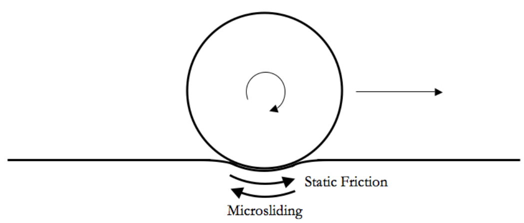


Figure 5.31: Schematic drawing of the two major forces involved in the rolling friction. If the static friction is not enough then the ball will slide instead of rolling. Both the rolling ball and the rolling surface are deformed at the point of contact.

rolling friction is considerably less than the sliding friction.

The contributing factors to rolling resistance are the deformation of the surface of contact due to weight of the wheel, adhesion of the two surfaces and relative micro-sliding between the surfaces. For instance, a surface that is easily deformable under pressure, such as rubber or sand will give a larger rolling resistance than a surface which is more resistant to deformation, such as steel and concrete. Thus, a car rolling on sand will slow down faster than a car on a concrete surface.

Since, a rolling motion is just a reduced static friction, we expect the rolling friction to depend upon the degree to which the bodies are dug into each other at the point of contact. Therefore, we also write the magnitude of the rolling friction F_r in terms of the magnitude of the normal force between the two as was done for the sliding friction.

$$F_r = k_r\ N, \tag{5.11}$$

where k_r is the coefficient of rolling friction. Rolling friction coefficients are typically $\frac{1}{100}$th of the sliding friction between the same two surfaces. Thus, between an iron block and an iron plate, the sliding friction coefficient is approximately 0.02, and the rolling friction of a wheel of same mass rolling on an iron plate is approximately 0.0002.

A wheel will roll as long as the horizontal force is greater than the rolling friction force needed to get the wheel rolling but less than the maximum static friction allowed between the two surfaces. If the force exceeds the maximum static friction force, then the wheel will start to slide. In winter one has to be careful in driving over frozen roads since static friction coefficient between rubber tires and ice is considerably less than that between tires and concrete, and it is easily exceeded if wheels are turning rapidly. The coefficient of static friction between rubber tires and concrete road is approximately $\mu_s = 1.7$ and the rolling friction coefficient is approximately $\mu_r = 0.01$. Therefore, the minimum force needed to get a 3000-kg car rolling on a horizontal concrete road is $\mu_r\ N = \mu_r\ m\ g = 294$ N, and the maximum force that can be exerted on the car before it starts to slide is $\mu_s\ N = \mu_s\ m\ g = 50,031$ N. At the maximum force on the car, it will have an acceleration of $1.7\ g = 16.7\ \text{m/s}^2$. If you try to accelerate the car at more acceleration than $1.7\ g$, the car will slide.

Although rolling of a wheel avoids the sliding friction at the surface, but at the axle, sliding friction is present. Ball bearings between the axle and the wheel reduce friction by making use of the rolling friction of the ball bearings rather than sliding of the wheel against the axle.

5.4.8 Viscous drag

When a solid moves through a fluid as when a baseball moves through the air or car travels in air or a fish swims in water, it encounters a friction due to the viscosity of the fluid. The **fluid resistance** or the **viscous drag**, as it is commonly called, increases with the speed of the moving body. For slowly moving objects viscous drag ($\vec{F}_d$) is proportional to speed and acts in the opposite direction to the velocity.

$$\boxed{\vec{F}_d = -b\ \vec{v}, \quad (b > 0)} \tag{5.12}$$

where b is a constant that depends on the viscosity of the fluid and the area of cross-section of the object perpendicular to the velocity. For objects moving fast the drag force can vary as square of the speed, or even the cube of the speed. Due to the increasing drag force, the cost of incremental speed boost to an object is much higher at higher speeds. It is much more costly to increase the speed of a car from 150 km/h to 160 km/h than from 15 km/h to 25 km/h for the same additional 10 km/h rise. Drag is responsible for the existence of a safe limit of a terminal speed when you sky dive. When the parachute opens it increases the drag force by increasing the cross-section upon which air resistance can act. At some critical speed, called the terminal speed, v_t, the weight of the person plus the parachute is balanced by the drag force by air.

$$b\ v_t = m\ g \quad \Longrightarrow \quad v_t = \frac{mg}{b}. \tag{5.13}$$