12.1 INTRODUCTION

12.1.1 AC Source

Alternating current (AC) circuits form the backbone of distribution of electricity to households and industry. The behavior and analysis of AC circuits are very different than DC circuits, so you would have to learn anew how to calculate currents and voltages in an AC circuit. To start with, a current in DC circuit always points in the same direction, but in an AC circuit, the direction of current changes with time (Fig. 12.1).

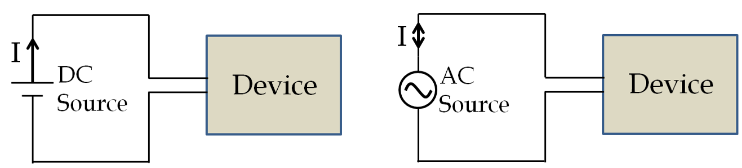

Figure 12.1: In a direct current (DC) the direction of current remains fixed in time. In an alternating circuits (AC) the direction of the current changes with time.

In this chapter we will study AC circuits where the current in a circuit is driven by an oscillating electromotive force V.

$$V(t) = V_0 \cos(2\pi f t + \phi), \tag{12.1}$$

where v_0 is the amplitude of the oscillating voltage, f the frequency of oscillations, and ϕ the phase constant. The amplitude V_0 is also called the **peak voltage** of the source. The argument of cosine, here $(2\pi f t + \phi)$ is called the phase of the source. Sometimes the phase constant ϕ itself is called the phase.

If there is only one source we pick the zero of time such that ϕ can be set to zero. But, if you have two or more sources, their phase constants may be different even when the sources may fluctuate with the same frequency. In that case, the phase constant gives us important information about the relative timings of peaks and troughs of different sources. We keep track of the relative phases of different sources with respect to the zero phase constant of one of the sources.

The voltage across the AC source fluctuates between $-V_0$ and $+V_0$ with the frequency f. That is, one terminal of the source could be positive at one time and negative at another time with respect to the other terminal of the source. This change in voltage leads to current into the source changes direction with time with the same frequency.

In the United States, the frequency of oscillation of the supply line is 60 cycles per second, i.e., 60 Hertz (Hz). In most of the world the frequency of the AC supply line is 50 Hz. In the early days of the AC supply different frequencies were in use by different companies and with the commercial success of Westinghouse in the US 60 Hz became standard in the US and that of AEG in Germany led to the widespread adoption of 50 Hz in Germany. Japan uses both 60 Hz and 50 Hz lines which can be traced to the initial purchase of generators from AEG of Germany and General Electric from US. There does not appear to be any advantage of one frequency over the other. However, instruments and devices built to operate at one frequency may not work properly if connected to a different frequency source.

12.1.2 Phases of Power Lines

The power lines for the industrial machines and equipment as well as the power transmission lines in the US have three phases instead of one because a three-phase line provides a more uniform power than a one-phase line as you will see below. One of the ways three phases are transmitted is through the use of four wires in the transmission line, with one being the ground, and the EMF in the other three with respect to the ground having phase constants 120° or $\frac{2}{3}\pi$ rad apart.

$$
\begin{aligned}
V_1 &= V_0 \cos(2\pi f t) \\
V_2 &= V_0 \cos\left(2\pi f t + \frac{2\pi}{3}\right) \\
V_3 &= V_0 \cos\left(2\pi f t + \frac{4\pi}{3}\right)
\end{aligned}
$$

To understand the difference in power fluctuation with time in a one-phase line compared to a three-phase line consider connecting a resistance to each of the voltage sources. As each of the phases in the three-phase line acts independently, we connect each to a separate load in a Y-to-Y connection (Fig. 12.2). You can look up an advanced book on electric circuits for other ways three-phase lines are connected to circuits. Our objective here is to illustrate power fluctuation in the circuit and a simple Y-to-Y connection will be sufficient. We make a calculation to compare power per load in the two circuits.

Suppose a resistor of resistance R is connected to each phase of

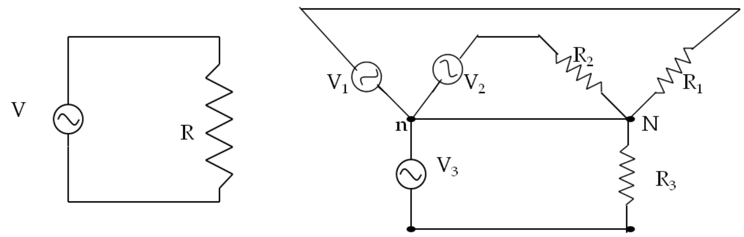

Figure 12.2: (a) One phase connection across a resistor, and (b) a four-wire Y-to-Y connections for a three-phase circuit. There are other configurations for connecting a three-phase circuits.

an AC source. What will be the power delivered in the case of a one-phase versus a three-phase source? We have seen in the last chapter that the instantaneous power delivered to a resistor R in the one-phase line varies with time.

$$P_{\text{one-phase}}(t) = \frac{V_0^2}{R}\cos^2(\omega t). \tag{12.2}$$

The instantaneous power delivered to three identical resistors in a three-phase circuit will be

$$\begin{aligned} P_{\text{three-phase}}(t) &= \frac{V_0^2}{R}\left[\cos^2(\omega t) + \cos^2\left(\omega t + \frac{2\pi}{3}\right) + \cos^2\left(\omega t + \frac{4\pi}{3}\right)\right] \\ &= \frac{3}{2}\frac{V_0^2}{R}. \end{aligned} \tag{12.3}$$

Hence, the total power in a three-phase line is steady as opposed to the pulses in single-phase AC circuits (Fig. 12.3). This is a great advantage, giving three-phase lines greater stability than identical single-phase lines. Nikola Tesla (1856-1943) was the discoverer of polyphase currents who employed two-phase current separated by 90-degree phase. The unit of magnetic field (Tesla) in the SI system of units is named after Nikola Tesla.

12.1.3 Phases of Currents and Voltages

All currents in the circuit will have the same frequency of oscillation as the driving EMF, but their phases and magnitudes would be different. We will therefore denote each current I_a by its magnitude I_{a0} and phase constant ϕ_a.

$$I_a = I_{a0}\,\cos(\omega t + \phi_a) \tag{12.4}$$

The phases of currents and voltages can be compared by their differences from a reference. Usually the phase of one of the source EMF

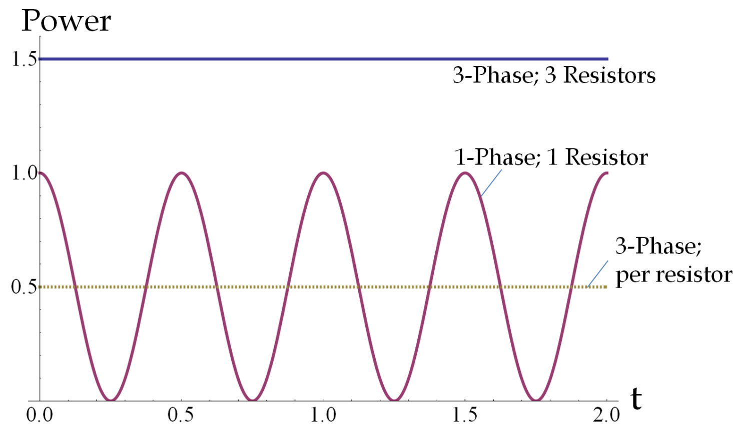

Figure 12.3: Instantaneous power in an AC circuit using one-phase-one-load versus power in a circuit using a three-phase-three-load line. For these graphs, set $V_0 = 1$ V, $R = 1\ \Omega$, and $f = 1$ Hz. The successive peaks in the one-phase circuit corresponds to the positive and negative cycles of the AC current respectively. The dashed line shows the average power delivered to one resistor, which is the same in the two cases.

is taken to be the zero-phase reference. Sometimes we compare two oscillating quantities against one another, and speak of one either leading the other or lagging behind the other. When a current I_1 in the circuit leads another current I_2 we find that $\phi_1 - \phi_2$ is positive. In Fig. 12.4, two sinusoidally oscillating functions whose phase difference or phase shift is $\pi/2$ rad or $90°$ is illustrated.

12.1.4 The Ideal AC Generator

The AC generator is an active element of all AC circuits. The AC generator is the ultimate source of all power in the circuit. In an

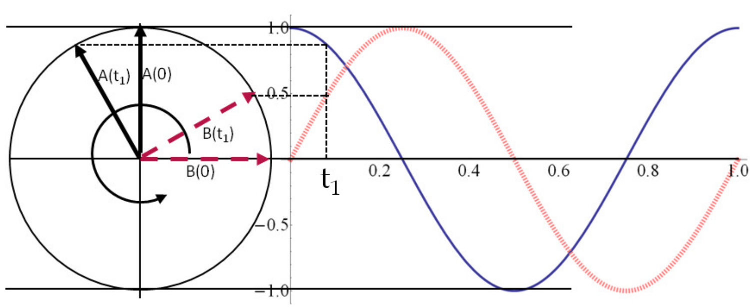

Figure 12.4: Plot of two oscillating quantities that have a difference of $\pi/2$ radians in phase. Here functions represented by the phasors are: $A(t) = \cos(2\pi f t)$ and $A(t) = \cos(2\pi f t - \pi/2)$. The quantity A leads B, or equivalently, B lags A as by $90°$ when the phasors rotate in time. The positions of the phasors at $t = 0$ and $t = t_1$ are indicated in the figure.

AC generator a sinusoidally varying EMF $\mathcal{E}(t)$ is generated either by changing the magnetic field with time or moving wires in a magnetic field or both. Since the electric field generated by the changing magnetic field is a non-conservative field, in general, we cannot treat this EMF as a voltage between the two terminals of the AC generator. However, we can think of an "ideal generator" where with some assumptions it would be possible to introduce the concept of voltage for AC generators also. We will make two assumptions to help us in this regard:

1. The wires in the generator are perfect conductors.
2. The magnetic field outside the generator is zero.

By 'perfect conductors" we meaning that the net force on the conduction electrons is zero. The vanishing of net force on the conduction electrons is necessary for ideal conductors otherwise we would have an infinite acceleration.

The vanishing of the magnetic field outside the generator will imply that the electric field outside the generator will obey

$$\vec{\nabla} \times \vec{E} = 0 \quad \text{(outside)}. \tag{12.5}$$

Therefore, the electric field outside the generator will be conservative, and we can use the electric field at the outside points to define a potential difference or the voltage between points in space, specifically the two terminals of the generator.

Now consider the simple rotating wire generator discussed in the chapter on the Faraday's law. We had found that the EMF induced by the magnetic force on the moving charges in these generators with N turns is given by

$$\mathcal{E}(t) = \omega NAB \sin(\omega t). \tag{12.6}$$

This must be equal to the line integral of the electric field around a closed path, say aba in Fig. 12.5, where the segment ab goes through the generator and the segment ba is outside the generator. The line integral of the electric field around this closed loop separates into two parts - one for the inside and the other for the outside.

$$\oint \vec{E} \cdot d\vec{l} = \int_{a\ \text{[inside]}}^{b} \vec{E}_{\text{in}} \cdot d\vec{l} + \int_{b\ \text{[outside]}}^{a} \vec{E}_{\text{out}} \cdot d\vec{l}$$

Since we have assumed the wire ab inside the generator to be a perfect conductor, the electric field in them will be zero.

$$\int_{a\ \text{[inside]}}^{b} \vec{E}_{\text{in}} \cdot d\vec{l} = 0 \quad \text{(since perfect conductor)}$$

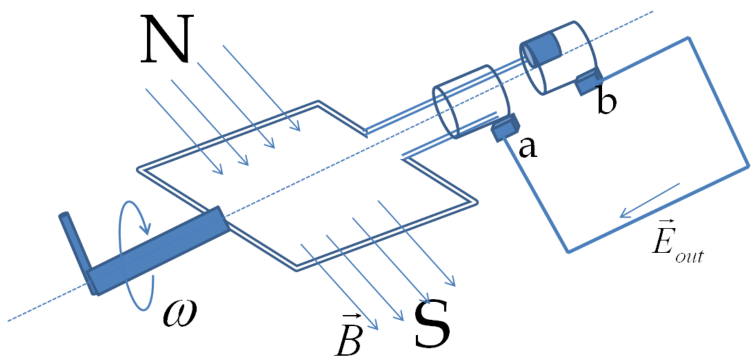

Figure 12.5: A generator consisting of a coil that rotates in a magnetic field. The line integral of the electric field is taken from a to b inside the generator and b to a outside. The coil in the generator is assumed to be a perfect conductor and the magnetic field outside is assumed to be negligible. With this assumptions the line integral of the electric field in the loop is equal to the voltage between the terminals a and b.

Therefore, the line integral will have the contribution only from the segment that is outside the generator. Now, that part has the conservative electric field, therefore, the integral will be independent of the path and will depend only upon the end points a and b. The integral will give the negative of the potential difference $(V_a - V_b)$ between the ends.

$$\int_{b\ \text{[outside]}}^{a} \vec{E}_{\text{out}} \cdot d\vec{l} = -(V_a - V_b) \ \ \text{(since conservative } \vec{E} \text{ field).}$$

This result must equal the EMF given in Eq. 12.6.

$$V_b - V_a = (\omega NAB)\sin(\omega t),$$

which can be written as time-dependent voltage $V(t)$ for the "ideal" generator.

$$V(t) = V_0 \sin(\omega t), \tag{12.7}$$

with $V_0 = \omega NAB$ for the N-turn generator. We will call this voltage the AC voltage. Note that we can write the sine as cosine by subtracting $pi/2$ radian from the phase.

$$V(t) = V_0 \cos\left(\omega t - \frac{\pi}{2}\right), \tag{12.8}$$

Since the phase constant of any one quantity in a circuit can be chosen arbitrarily, we will often set this phase constant to zero instead of $-pi/2$ in the calculations below.