## 1.11 PROBLEMS

**Problem 1.11.1.** A lake freezes from the top at different rates depending upon the thickness of the ice layer already formed on the lake. If the temperature of the air above the ice is $-15°$C and the water below is $0°$C, at what rate the thickness of the ice would increase at the time when the thickness of ice is 2 cm. Assume the density of ice to be 0.9 g/cc, and the heat of fusion of ice 334 J/g.

**Problem 1.11.2.** A 16-liter gas cylinder has the Oxygen gas at the room temperature of $20°$C. The gauge pressure shows a pressure of 20,000 kPa. The amount of gas in the cylinder is some definite amount, but the calculated value will depend upon the model you use. (a) What is the calculated value of number of moles if you assume the ideal gas behavior? (b) What is the calculated value of number of moles if van der Waals gas behavior is assumed?

**Problem 1.11.3.** A metallic container of fixed volume of $3.5 \times 10^{-3}$ m$^3$ immersed in a large tank of temperature $27°C$ contains two compartments separated by a freely movable wall. There are 1.2 moles of nitrogen on one side and 1.5 moles of oxygen on the other side of the wall. Find the temperature, pressure and volume of the two sides when equilibrium has reached. Assume ideal gas behavior.

**Problem 1.11.4.** A bimetallic strip of length 8 cm at $25°$C made of brass and steel is to be used for measuring the temperature. When the bimetal strip is placed in $200°$C superheated steam, the strip bends such that the strip makes an arc of angle theta as of a circle of radius $r$ shown in the figure. Find this angle.

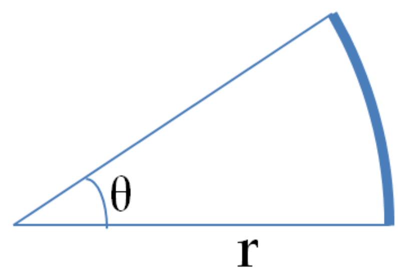


Figure 1.32: Problem 1.11.4.

**Problem 1.11.5.** By knowing the surface temperature of a star and the amount of radiation emitted by it, you can determine the size of the star by assuming the emissivity to be 1. Find the radii of the Sun and the star Rigel, which is the bright blue star in the Constellation Orion by this method. Assume the amount of radiation emitted and the surface temperatures of the two stars are: $3.9 \times 10^{26}$ W and 5780 K for sun, and $2.7 \times 10^{32}$ W and 11000 K for Rigel.

**Problem 1.11.6.** It is extremely difficult to keep liquid helium from evaporating off. Special arrangements must be made to minimize the loss of liquid helium in low temperature physics experiments. Consider a cylindrical metallic container 20 cm tall and 8 cm diameter for storing liquid helium at 4 K. Let the metal container be separated from the outside by a vacuum between the metal and the surrounding metallic wall which is maintained at liquid nitrogen temperature of 77.3 K by pouring liquid nitrogen in a space outside. In this arrangement, heat can enter the liquid helium container by radiation only.

From the surface at 77.3 K to the surface at 4 K. If the emissivity of the metal is 0.25 what is the rate at which helium will evaporate off? Assume the heat of evaporation of helium to be $2.1 \times 10^4$ J/kg.

**Problem 1.11.7.** The frequency of a pendulum in a grandfather clock made of brass is related to its length $L$ and acceleration due to gravity $g$ of earth by the following equation.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

The clock is calibrated to give the accurate time when the temperature is 20°C. How much will the clock be off in an 8 hour period when the temperature is 30°C?

**Problem 1.11.8.** One of the major problems with a standard of length based on a physical material such as a platinum rod is the expansion of the rod with the temperature. We are supposed to compare other rods with the standard platinum-iridium rod which is assumed to be of a "fixed" length of 1 m. If the standard rod is 1 m at 20°C, what will be the error in assuming the the rod is still 1 m at 27°C? Note: $\alpha_{Pt/Ir} = 8.7 \times 10^{-6}$ per deg C.

**Problem 1.11.9.** Newton's law of cooling states that rate of heat loss by a body is proportional to the difference in temperatures between the body ($T$) and the surroundings ($T_0$).

$$\frac{dQ}{dt} = -\beta\,(T - T_0)$$

This accounts for the net heat lost by all three mechanisms. Consider the cooling of a cup of hot coffee with the initial temperature $T_1$ in an ambient temperature $T_0$. We can replace $dQ$ by $mc_P dT$ and obtain the following.

$$\frac{dT}{dt} = -\alpha\,(T - T_0)$$

where $\alpha = \beta/mc_P$. (a) Solve this equation and provide a sketch of $T$ vs $t$ for $\alpha = 1$ and $\alpha = 2$. (b) What is the physical significance of the constant $\alpha$.

**Problem 1.11.10.** Find the virial expansion corresponding to the van der Waals equation of state.

**Problem 1.11.11.** For a tighter fit, rivets are usually made slightly larger than the rivet holes, and, to make the rivets fit into the holes, they are cooled before being driven in the holes. Consider a rivet hole of diameter 4 mm at 20°C. What would be the diameter of an aluminum rivet at 20°C which just fits the rivet hole when it is cooled by dry ice (solid $CO_2$) to a temperature of $-78$°C?

**Problem 1.11.12.** Specific heat often depends on the temperature. Consider an experiment on a metal piece between 100 K and 200 K. Suppose the curve fitting of the experimental data produces the following formula for the molar specific heat at constant pressure

$$C_P = -\left(0.5\ \frac{\mathrm{J}}{\mathrm{mol.K}}\right) + \left(0.4\ \frac{\mathrm{J}}{\mathrm{mol.K}^2}\right) T - \left(0.001\ \frac{\mathrm{J}}{\mathrm{mol.K}^3}\right) T^2$$

Find the amount of heat required to raise the temperature of two moles of the metal from 120 K to 180 K.

**Problem 1.11.13.** At very low temperatures the molar specific heats of metals vary as $T^3$.

$$\frac{C_P}{3R} = 77.9 \left(\frac{T}{\Theta}\right)^3,$$

where $\Theta$ is called the Debye temperature. Here $R$ is the universal gas constant and $C_P$ is the molar specific heat for constant pressure. The Debye temperature of aluminum is 420 K. How much heat will be involved when the temperature of a 3-kg aluminum block is raised from 4 K to 20 K?

**Problem 1.11.14.** A long cylindrical brass pipe of inner radius $R_1$ and outer radius $R_2$ carries hot water at temperature $T_1$ inside the pipe, while the temperature in the space outside the pipe is $T_2$. The length of the pipe is $L$. Show that the rate of the loss of heat per unit length of the pipe is given by the following formula.

$$\frac{1}{L}\frac{dQ}{dt} = 2\pi k \frac{(T_1 - T_2)}{\ln\left(R_2/R_1\right)}$$