3.8 Problems

Problem 3.1. A free-particle moves in a potential free space. Consider a free particle of mass m and energy E in a one-dimensional space represented by the x-axis. (a) Write the time-independent Schrödinger equation for this particle. (b) Let k be the wave number defined by $\hbar k = \sqrt{2mE}$. Solve the Schrödinger equation and write your wave function using k rather than E. (c) Write your solution in complex exponential notation. (d) What will be the solution of the time-dependent Schrödinger equation for this particle? (e) What is the probability density of this particle? (e) Find a relation between the probability current and probability density for this particle.

Problem 3.2. Electrons in some organic molecules with alternating single and double bonds behave as if they were confined in a potential well of width equal to the length of the molecule. Suppose the length of a particular molecule is 5 nm. (a) What will be the energy levels of the molecule? (b) What will be the wavelength of a photon released when the molecule makes a transition from the first excited state to the lowest energy state?

Problem 3.3. A particular electron in a one-dimensional potential well of width a is not in one of the stationary states of the well. Instead, the quantum state of the electron has the following combination of two states for x in the well and zero outside.

$$\psi(x,t) = \frac{1}{\sqrt{2}} sin(k_1 x)\ e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}} sin(k_2 x)\ e^{-iE_2 t/\hbar},$$

where $E_1 = \hbar^2 k_1^2/2m$, $E_2 = 4E_1$, $k_1 = \pi/a$, $k_2 = 2k_1$. (a) Find the real probability density, $\psi^*\psi$, as a function of x and t. (b) How does the probability vary with time at the mid point of the well? Does this probability oscillate wit time? If so, then find the oscillation frequency. (c) Describe what you will observe at the mid point of the well.

Problem 3.4. Consider a particle in one-dimensional well. (a) What will be the behavior of the particle classically if has energy E? (b) Can the value of energy E be any real number? Why or why not? (c) We have seen in the chapter that quantum mechanically this particle can have only quantized values of energy, which are given by $E_n = nE_1$ with $n = 1, 2, 3, \cdots$. Show that the difference in energy between the adjacent levels as a percentage goes down as $1/n$ as levels go up. (d) What is the probability of finding the particle somewhere in the well if the situation is considered classical? (e) Compare the probability of finding the particle anywhere in the well when the particle is in a large n quantum state to the probability expected classically. (f) What can you conclude about the large n quantum states compared to the classical states?