7.4 MAGNETIC FORCE ON A CURRENT

7.4.1 Force on Current in a Straight Wire

Before we derive a general result for force on current in an arbitrary wire, let us consider a simple example of current $\vec{I}$ in a straight wire of length L placed in a magnetic field $\vec{B}$ as shown in Fig. 7.19. Note that the force we derive here is the force on the conducting electrons of the wire and not the force on the wire. This force on the moving electrons in the wire is transformed into an effective force on the body of the wire in a complicated process. Despite this subtle difference in the force on electrons of the wire and the force on the wire, it is common to call the magnetic force on the moving electrons of the wire as the "magnetic force" on the wire. See remarks at the end.

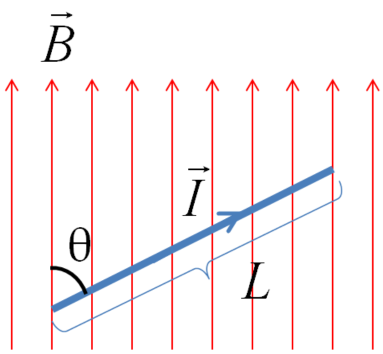

Figure 7.19: A straight wire carrying current in a uniform magnetic field.

To calculate the magnetic force on a current we proceed as follows. Let N be the number of conduction electrons per unit volume, A the cross-sectional area, and $\vec{v}_d$ be their average drift velocity. The total charge of electrons in length L of the wire that are moving is

$$Q = -eNAL.$$

Since the are moving with velocity $\vec{v}_d$, the force on them is

$$\vec{F}_m = Q\vec{v}_d \times B = -eNAL\vec{v}_d \times B$$

Recall that current vector is related to the drifting electrons and geometry as

$$\vec{I} = -eNA\vec{v}_d.$$

Therefore, we can write the force in terms of current also.

$$\vec{F}_m = L\vec{I} \times B.$$

This equation is written in another form by transfering the direction information to the length of the wire. Thus, we define a length vector $\vec{L}$ as having magnitude equal to the length of the wire and the direction of the current.

$$\vec{L} = \{\text{Magnitude: Length of wire; Direction: direction of current.}\}$$

Using the length vector we switch the vector from current to length.

$$\vec{F}_m = I\vec{L} \times B.$$

Therefore, if the current and magnetic field make an angle θ then, the magnitude and direction the force are found to be:

$$\text{Magnitude:} \quad F_m = ILB\sin\theta \tag{7.45}$$

Direction: Use Right-hand Rule for I-to-B cross-product. (7.46)

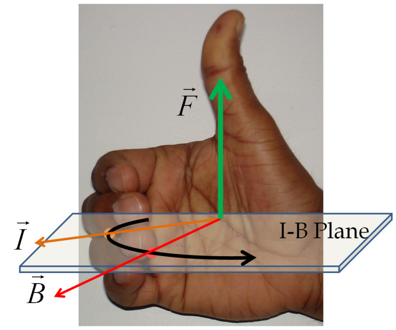

Figure 7.20: Right-hand rule of magnetic force on current. Note: this is the same rule as a force on a positive charge.

Effective force on the wire vs magnetic force on the current

The magnetic force obtained above is the net force on conduction electrons in the wire drifting at an average velocity $\vec{v}_d$. We found that the force is perpendicular to the direction of the current as well as the magnetic field. This force on conduction electrons will bend the paths of the drifting electrons towards the side of the wire. The conductive electrons collide with the atoms in the body of the wire. As a result, the atoms of the wire gain a side-way momentum component. If all the momentum of the conduction electrons are transfered to the atoms, the inertial force on the atoms will have the direction and magnitude equal to the magnetic force on the conduction electrons. In this sense, we may say that magnetic force on conduction electrons causes a change in momentum of the wire itself.

The force responsible for collision between conduction electrons and atoms are electrical in nature and can do work. Therefore, although, magnetic forces on the conduction electrons do not do any work, the inertial force on the atoms can do work. Therefore, if a wire is placed in a magnetic field and a current is passed in the wire, the wire will gain kinetic energy and it would appear that magnetic force has done work on the wire. However, this interpretation is not correct since the magnetic force on moving charges cannot do work as explained above. The work observed in the change in the kinetic energy of the wire is actually done by the electric force on the atoms by the moving conduction electrons. We may drop the subscript m from the force on the current in the wire and write the force on the wire as

Effective force on a current carrying wire : (7.47)

$$\text{Magnitude:} \quad F = ILB\sin\theta$$

Direction: Use Right-hand Rule.

Sometimes, we refer the force on the wire as "magnetic force" on the wire. One should be aware that this use of the term magnetic force leads to inconsistencies with the basic observation that magnetic forces cannot do work on moving charges. The main point here is that although the force on conduction electrons is magnetic, the force on the wire is not magnetic.

7.4.2 General Formula

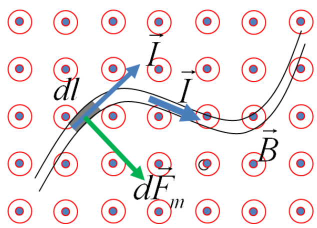

Figure 7.21: Force on an element of an arbitrary shape wire, $d\vec{F}_m = \vec{I} \times \vec{B}\ dl$.

To obtain a more general formula for magnetic force on the current in a wire of arbitrary shape, we consider the force on moving charges in a small element dl of the wire(Fig. 7.21). As above, let $\vec{v}_d$ be the drift velocity of electrons in the wire, N number of electrons per unit volume, A the cross-section area. Then the total charge dq moving in a length dl of the wire will be

$$dq = -eNAdl.$$

Since these electrons are moving with drift velocity $\vec{v}_d$, the magnetic force $d\vec{F}_m$ on the wire element dl will be

$$d\vec{F}_m = dq\ \vec{v}_d \times \vec{B},$$

which can be written in terms of the vector current $\vec{I}$ in the wire as was done above.

$$d\vec{F}_m = \vec{I} \times \vec{B}\ dl,$$

Since we often deal with steady current whose magnitude is constant throughout the wire, it is useful to separate the direction and magnitude of current vector. It is customary to include the direction information with the length of wire by introducing a length vector element $d\vec{l}$ that has the magnitude equal to the length of the elment dl and the direction of the current.

$$d\vec{l} = \{\text{Magnitude: } dl;\ \text{Direction: direction of current.}$$

With this definition, the force on the conduction electrons in the wire element is given as

$$d\vec{F}_m = I d\vec{l} \times \vec{B}.$$

For conduction electrons in a finite wire, we need to add forces on all elments. This can be written as a conceptual integral.

$$\vec{F}_m = \int I\ d\vec{l} \times \vec{B}.$$

If the circuit consists of straight wires, then we can simplify this relation to be a vector sum of the results on each segments. Let there be N_S straight segments in the circuit of vector lengths $\vec{L}_1$, $\vec{L}_2$, $\cdots$, $\vec{L}_{N_S}$ carrying a steady current I and under the magnetic fields $\vec{B}_1$, $\vec{B}_2$, $\cdots$, $\vec{B}_{N_S}$ respectively. Then, this integral simplifies to sum over forces on each segment.

$$\vec{F}_m = \sum_{i=1}^{N_S} F_i = I \sum_{i=1}^{N_S} \vec{L}_i \times \vec{B}_i.$$

More commonly we will encounter situations in which all segments will be under the same uniform magnetic field, $\vec{B}_0$. In this case, magnetic field factors out and we are left with the evaluation of a vector sum of the straight segments of the circuit tat are placed in the non-zero magnetic field.

$$\vec{F}_m = I\left[\sum_{i=1}^{N_S} \vec{L}_i\right] \times \vec{B}_0.$$

If the entire closed circuit is under a uniform magnetic field, then the vector sum of all segments over a closed path will be zero.

$$\sum_{i=1}^{N_S} \vec{L}_i = 0 \quad \text{(over closed loop)}$$

This says that the net magnetic force on current in a closed circuit will be zero if the entire circuit is subject to a uniform magnetic field.

Example 7.4.1. Force on a Straight Wire Segment.

A straight wire carries a steady current of magnitude I is placed in a uniform magnetic field of magnitude B_0 an direction perpendicular to the wire as shown in Fig. 7.22. Find the magnetic force on the wire.

Solution. The angle between the current and the magnetic field is 90°. Therefoire, the magnitude of the magnetic force on the current is

$$\text{Magnitude: } F_m = ILB_0.$$

The direction of the force is obtained by using the right-hand rule of cross-product I-to-B. This gives the direction of the force as coming out of page in the figure.

Example 7.4.2. Force on Current in Two Straight Segments.

A current of magnitude I flows through two straight segments of length L each. The structure is in a uniform magnetic field of magnitude B_0 and direction perpendicular to the wires as shown in Fig. 7.23. Find the magnetic force on the currents.

Solution. Since the current in the two segments ab and bc are in different directions while magnetic field is in the same direction, the magnetic force on them would be in different directions. The forces on the two segments are shown in Fig. 7.24. We need to add the two forces as vectors to obtain the net force on the two together. We add them analytically using the Cartesian directions as shown in the figure. The magnitude of each force is ILB_0 since the current in each

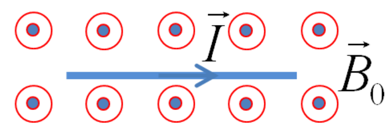

Figure 7.22: One straight wire of a circuit in a uniform magnetic field. The magnetic field is pointed out of page.

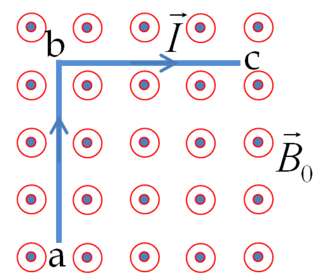

Figure 7.23: Two straight segments of a circuit in a uniform magnetic field. The magnetic field is pointed out of page.

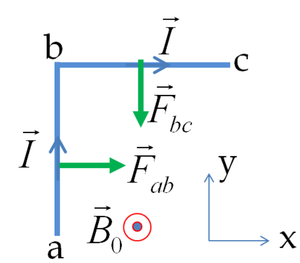

Figure 7.24: Forces on two segments.

wire is perpendicular to the magnetic field. The sum of the two forces give the following net force.

$$\vec{F}_{net} = ILB_0\left(\hat{u}_x - \hat{u}_y\right).$$

Therefore, the magnitude of the net force is $\sqrt{2}ILB_0$ and the direction is 45° clockwise from the positive x-axis.

Example 7.4.3. Force on a Current in a Semi-Circular Wire. A metal wire bent into a shape of a semicircle of radius R and placed in a region of constant magnetic field of magnitude B_0 and direction perpendicular to the plane of the wire as shown in Fig. 7.25. A steady current of magnitude I is passed through the wire. Find the magnetic force on the current in the wire.

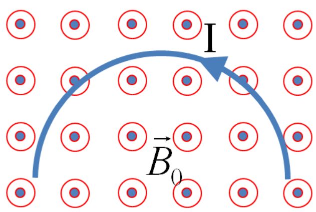

Figure 7.25: A semi-circular wire with current I in uniform magnetic field $\vec{B}_0$.

Solution. Here current changes direction over the wire. Therefore, we will need to work with the general formula. For a steady current I in an infinitesimal element $d\vec{l}$ of the wire and uniform magnetic field $\vec{B}_0$ the magnetic force is given by

$$d\vec{F}_m = Id\vec{l} \times \vec{B}_0.$$

Here, the infinitesimal element is in the shape of arc between θ and $\theta + d\theta$ is shown in Fig. 7.26. Therefore, the length of the infinitesimal element is $dl = Rd\theta$ and the direction of the vector is tangential to the circle at that point. The magnetic force on current in any element of the wire is pointed radially outward as shown in the figure. We use the Cartesian axes given in the figure to calculate the force analytically. The x-component of the magnetic force will cancel out due to symmetry in the situation. Therefore, we need only the y-component.

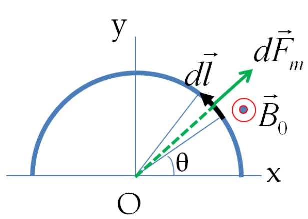

Figure 7.26: Force on any segments is radially outward. The x-component of the forces on right-side of the semi-circular wire is canceled by the x-component on the left side.

$$\left[d\vec{F}_m\right]_y = \left[Id\vec{l} \times \vec{B}_0\right]_y = IRd\theta B_0 \sin\theta,$$

when upon integration from $\theta = 0$ to $\theta = \pi\ rad$ gives the y-component of the net force. Therefore, the net magnetic force on the current in the semicircular wire is

$$\vec{F}_m = 2IRB_0\hat{u}_y.$$

This says that the magnetic force on the current in a semicircular wire has magnitude $2IRB_0$ and points from the center to the middle of the wire direction. Note that the magnetic force on a circular arc is not equal to IBL where L of the wire would be πR here.