5.10 PROBLEMS

Problem 5.10.1. The mean free path is the average distance a molecule travels in-between collisions. In reality a molecule will travel different distances in-between collisions. Suppose a molecule of size 0.3 nm travels the following distances between collisions: 11 nm, 5 nm, 12 nm, 15 nm, 25 nm, 2 nm, 11 nm, 30 nm, 18 nm, 20 nm, 14 nm, 15 nm, 10 nm, 28 nm, 9 nm, 17 nm, 5 nm, 10 nm, 6 nm, 13 nm, 17 nm, 15 nm, 10 nm, 22 nm, 9 nm, 23 nm, 19nm, 4 nm, 15 nm, 12 nm, 10 nm, 16 nm, 11 nm. (a) Find the mean free path from this data. (b) What is the standard deviation of space between collisions from the mean free path? (c) What is the most probable free length? (d) What is the rms free length? (e) Supposing, the temperature of the gas is 200 K and the pressure of the gas is 1 atm, what would be the diameter of molecules of the gas?

Problem 5.10.2. The probability of different free lengths between collisions is given by a probability function $\rho(r)$, similar to what was done for the Maxwell distribution of speed, except that $\rho(r)$ is a different function.

$$\rho(r) = A \exp\left(-r/\xi\right)$$

where A and ξ (read "shee") are constants. The product ρdr gives the probability that a molecule will travel a distance r between collisions. The integral of ρdr from $r = 0$ to $r = \infty$ must equal 1, and the average r is equal to the mean free path λ. Find A and ξ in terms of λ.

Problem 5.10.3. What should be the maximum temperature of a sun-sized star if Helium atoms cannot escape the gravitational pull of the star? Hint: Equate gravitational escape speed with the rms speed.

Problem 5.10.4. Describe in words how the mean free path of molecules of a fixed amount of gas will be affected if (a) the density is doubled by shrinking the volume to half while keeping the temperature same, and (b) the mean molecular speed is quadrupled by doubling the temperature while keeping the volume fixed.

Problem 5.10.5. A container of volume 2 L contains 0.3 moles of Helium gas kept at constant temperature 300 K. There is a small hole of area 0.003 mm^2 though which gas molecules escape. The density of molecules outside is so low that no molecules get back into the container. Find the pressure inside the container after 0.2 seconds. Assume all the atoms arriving at the hole leave the container.

Problem 5.10.6. A container of volume V contains a gas at a constant temperature T. Let P_0 be the pressure of the gas. There is a small hole of area A though which gas molecules escape. The density of molecules outside is so low that no molecules get back into the container. (a) Show that the pressure inside decreases exponentially: $p(t) = p_0 \exp(-t/\tau)$, where the time constant τ depends on V, A and average speed. (b) Using numerical values given in problem 5.10.5, verify your answer. Assume all the molecules arrive at the hole leave the container.

Problem 5.10.7. Humidity in a room is to be maintained at 30%. On a humid day the humidity is found to be 90%. If the room has a volume of 8 cubic meters, how much water vapor must be removed?

Problem 5.10.8. The Maxwell-Boltzmann energy distribution given in Exercise 5.9.6 above can be used to find the pressure at different heights. Assume that energy of air molecules is mostly gravitational potential energy so that the ratio of number of molecules at a height y to the number at ground level, $y = 0$, is given by:

$$\frac{g(y)}{g(0)} = \exp(-mgy/k_B T)$$

(a) Convert this equation to the relation between the pressure at height $y = 0$ and $y = h$ by appropriate arguments. (b) How does the pressure change with height at the same temperature? Show a graph of p vs y. (c) Find the pressure at a height 100 m if the pressure at the ground level is 1 atm. (d) How does the pressure at a particular height change with temperature? Show a graph of p vs T. Does your graph confirm your expectations?