4.8 ENERGY, POWER AND INTENSITY

All waves transport energy without an actual transport of material. For concreteness, consider a wave of frequency f, wavelength λ, and amplitude A on a taut string with tension T and mass per unit length μ. Then, it is possible to prove that the energy E_λ in the wave over the space of one wavelength would be

$$E_\lambda = 2\pi\mu\lambda f^2 A^2 \tag{4.35}$$

In one period this energy crosses any point of the string. Therefore, the rate of energy flow in the string, i.e. average power in the wave is

$$P_{\text{av}} = \frac{E_\lambda}{1/f} = 2\pi\mu\lambda f^3 A^2 \tag{4.36}$$

In three-dimensions, wave carry energy through space. For instance, a plane wave $\psi(x, y, z, t) = A\cos(kx - \omega t)$ carries energy towards the positive x-axis. For the waves in the three-dimensional space, we define intensity as the rate at energy passes through planes perpendicular to the direction of the wave per unit area of the planes.

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}} \tag{4.37}$$

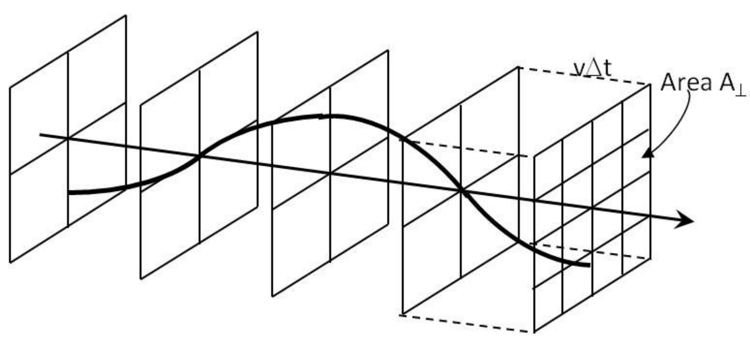


Figure 4.16: A wave passing through a cross-sectional area; intensity of the wave is defined as average power passing per unit are perpendicular to the wave direction. The wave shown here has speed v. The energy contained in the box of area $A_\perp$ and height $v\Delta t$ pass through $A_\perp$ in time Δt.

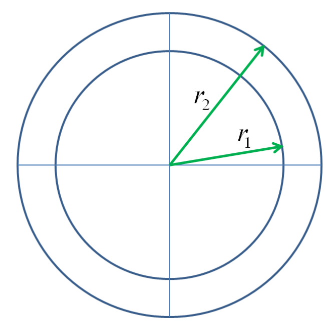
Example 4.8.1. Power of a wave. The amplitude of a wave on a long string is increased from 10 cm to 20 cm while keeping the same frequency. How is the intensity of the waves changed?

Solution. Solution Taking the ratio of intensity for the two cases, we find that common factor cancels out, and we obtain the important result that intensity goes as square of the amplitude.

$$\frac{I_2}{I_1} = \frac{A_2^2}{A_1^2} = \frac{(20 \text{ cm})^2}{(10 \text{ cm})^2} = 4.$$

The intensity quadruples when the amplitude is doubled.

4.8.1 Intensity of a Spherical Wave



Imagine a point vibrating source that emits a wave isotropically in all directions. As the wave spreads out in three-dimensional space isotropically, i.e., without a preferred direction, the wave will be a spherical wave. The energy passing through the surface at radius r_1 from the source point must equal the energy through the surface at radius r_2.

Therefore, the intensities I_1 and I_2 at two distances r_1 and r_2 respectively from a spherical wave source will follow the following inverse square law.

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} \quad \text{(spherical wave)} \tag{4.38}$$

Beware that inverse square law for intensity does not hold true if there is a directional preference for the wave or if the wave amplitude does not drop as $1/r$. For instance the amplitude of a plane wave does not drop with distance; hence, the intensity of a plane wave is constant rather than dropping as $1/r^2$ from the source.

$$\frac{I_1}{I_2} = 1 \quad \text{(plane wave)}$$