13.7 EXERCISES

Displacement current

Ex 13.7.1. The voltage across a parallel plate capacitor with area A =800 cm^2 and separation $d = 2$ mm varies sinusoidally as $V = (15\text{ mV})\cos(150\,t)$, where t is in sec. (a) Find the displacement current density between the plates. (b) Find the displacement current between the plates. Ans: (a) Magnitude: 9.95×10^{-9} (A/m^2) $\cos(150\,t+\pi/2)$, (b) 7.97×10^{-10}(A) $\cos(150\,t+\pi/2)$

Ex 13.7.2. The voltage across a parallel plate capacitor with area A and separation d varies sinusoidally with time t as $V = a\,t^2$, where a is a constant. (a) Find the displacement current density between the plates. (b) Find the displacement current between the plates. Ans: (a) Magnitude: $2\epsilon_0 at/d$, (b) $\dfrac{2\epsilon_0\omega aA}{d}\,t$.

Ex 13.7.3. In a region of space the electric field is pointed along the same line, but its magnitude changes sinusoidally. Analytically it is given as

$$\vec{E} = \hat{u}_z(10\text{ N/C})\cos(20\,x - 500\,t).$$

where t is in nano-sec and x in cm.

(a) Find the displacement current density though the yz-plane.

(b) Find the displacement current density through $x = 5$ cm plane.

(c) Find the displacement current through a circle of radius 3 cm in $x = 5$ cm plane.

Ans: (a) $J_{dz}(x=0,t) = -0.443$ (A/m^2) $\sin(500\,t)$, (d) $I_{d0} = 1.19$ mA.

Grad, Div and Curl

Ex 13.7.4. Calculate the gradient of the following scalar fields.

(a) $f = x + y + z$

(b) $f = xyz$

(c) $f = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

(d) $f = \ln\left(\sqrt{x^2+y^2}\right)$

Ans:

(a) $\hat{u}_x + \hat{u}_y + \hat{u}_z$.

(b) $yz\hat{u}_x + zx\hat{u}_y + xy\hat{u}_z$

(c) $\frac{x}{r^3}\,\hat{u}_x + \frac{y}{r^3}\,\hat{u}_y + \frac{z}{r^3}\,\hat{u}_z$, with $r = \sqrt{x^2+y^2+z^2}$.

(d) $\frac{x}{r^2}\,\hat{u}_x + \frac{y}{r^2}\,\hat{u}_y$, with $r = \sqrt{x^2+y^2}$.

Ex 13.7.5. Calculate the divergence and curl of the following vector fields.

(a) $\vec{F}_1 = x\hat{u}_x + y\hat{u}_y + z\hat{u}_z$

(b) $\vec{F}_2 = \dfrac{x\hat{u}_x + y\hat{u}_y}{\left(x^2+y^2\right)^{3/2}}$

(c) $\vec{F}_3 = \dfrac{x\hat{u}_x + y\hat{u}_y + z\hat{u}_z}{\left(x^2+y^2+z^2\right)^{3/2}}$

(d) $\vec{F}_4 = -y\hat{u}_x + x\hat{u}_y$

Ans: (a) 3, (d) 0.

Ex 13.7.6. Find the electric field from the electric potential of a fixed point charge q at the origin by using the following relation of the electric field to the static electric potential.

$$\vec{E} = -\vec{\nabla}V \quad \text{(static)}$$

Ex 13.7.7. A straight wire carries a current I_0. Find the magnetic field at an arbitrary point and calculate the curl of the magnetic field there. Make any observations about the domain of validity of your answer. Ans: $\vec{\nabla} \times \vec{B} = 0$ (x,y,z not on wire).

Ex 13.7.8. Find the divergence of electric field of a point charge at the origin. Make any observations about the domain of validity of your answer. Ans: $\vec{\nabla} \cdot \vec{E} = 0$ (x,y,z not on charge, excluding the origin).

Electromagnetic Wave

Ex 13.7.9. An electromagnetic wave has a frequency of 12 MHz. What is its wavelength in vacuum? Ans: 25 m.

Ex 13.7.10. The electric field of an electromagnetic wave traveling in vacuum is described by the following wave function.

$$\vec{E} = \hat{u}_y (5\ \mathrm{N/C}) \cos\left(k\,x - 6\times 10^9\,t + 0.4\right)$$

where k is the wavenumber in rad/m, x in m, t in sec..

Find the following quantities.

(a) Amplitude.

(b) Frequency.

(c) Wavelength.

(d) Phase constant.

(e) The direction of the travel.

(f) The associated magnetic field wave.

(g) The Poynting vector of the electromagnetic wave.

(h) The intensity of the electromagnetic wave.

Ans: (a) 5 N/C, (b) 3.77×10^{10} Hz, (c) 7.96 mm. (d) 0.4 rad, (e) towards $+x$-axis, (f) $\vec{B} = \hat{u}_z\,(1.67\times 10^{-8}\ \mathrm{T})\cos(k\,x - 6\times 10^9\,t + 0.4)$, (g) Do: $\frac{1}{\mu_0}\vec{E}\times\vec{B}$, (h) $33.2\ \mathrm{mW/m^2}$.

Ex 13.7.11. The magnetic field of an electromagnetic wave traveling in vacuum is described by the following wave function.

$$\vec{B} = \hat{u}_y (5\times 10^{-7}\ \mathrm{T}) \cos\left(5\times 10^{10} x + \omega\,t\right),$$

where ω is the angular frequency in rad/sec, x in m, t in sec.

Find the following quantities.

(a) Amplitude.

(b) Frequency.

(c) Wavelength.

(d) Phase constant.

(e) The direction of the travel.

(f) The associated electric field wave.

(g) The Poynting vector of the electromagnetic wave.

(h) The intensity of the electromagnetic wave.

Ans: (a) 5×10^{-7}T, (b) 2.39×10^{18} Hz, (c) 126 pm. (d) 0, (e) towards $-x$-axis, (f) $\vec{E} = +\hat{u}_z\,(150\ \text{N/C})\cos(5\times10^{10}\,x + 1.5\times10^{19}\,t)$, (g) Do: $\frac{1}{\mu_0}\vec{E}\times\vec{B}$, (h) 29.8 W/m^2.

Ex 13.7.12. The magnetic field of an electromagnetic wave traveling in vacuum is described by the following wave function.

$$\vec{B} = \hat{u}_x(6\times10^{-8}\ \text{T})\cos\left(-k\,z + 2\pi\times4\times10^{12}\,t\right),$$

where k is the wavenumber in rad/m, x in m, and t in sec. Find the following quantities.

(a) Amplitude.

(b) Frequency.

(c) Wavelength.

(d) Phase constant.

(e) The direction of the travel.

(f) The associated electric field wave.

(g) The Poynting vector of the electromagnetic wave.

(h) The intensity of the electromagnetic wave.

Ex 13.7.13. A plane electromagnetic wave of frequency 20 GHz moves in the positive y-axis direction such that its electric field is pointed either towards the positive z-axis or towards the negative z-axis. The amplitude of the electric field is 10 N/C. The start of time is chosen so that at $t = 0$, the electric field has a value zero at the origin. (a) Write the wave function that will describe the electric field wave. (b) Find the wave function that will describe the associated magnetic field wave. Ans: (a) $\hat{u}_z(10\ \text{N/C})\ \sin(419y - 40\pi\times10^9 t)$, (b) $\hat{u}_z(3.33\times10^{-8}\ \text{T})\ \sin(419y - 40\pi\times10^9 t)$ (y in m and t in s).

Ex 13.7.14. A plane electromagnetic wave of frequency 500 kHz moves in the positive z-axis direction such that its magnetic field is pointed either towards the positive x-axis or towards the negative x-axis. The amplitude of the magnetic field is 10^{-9} T. The start of time is chosen so that at $t = 0$, the magnetic field has an amplitude of 10^{-9} T at the origin. (a) Write the wave function that will describe the magnetic field wave. (b) Find the wave function that will describe the associated electric field wave.

Ans: (a) $\vec{B} = \hat{u}_x\, B_0\, \cos(kz - \omega t + \phi)$, with $B_0 = 10^{-9}$ T, $\omega = 2\pi\times 5\times 10^5 \mathrm{s}^{-1}$, $k = 0.0105\mathrm{m}^{-1}$, $\phi = 0$, (b) $\vec{E} = \hat{u}_y E_0 \cos(kz - \omega t + \phi)$, $E_0 = 0.3\, N/C$, same k, ω, and ϕ as that of $\vec{B}$ wave.

Ex 13.7.15. A plane electromagnetic wave of frequency 300 MHz moves in the positive z-axis direction such that its magnetic field is pointed either towards the positive x-axis or towards the negative x-axis. The amplitude of the magnetic field is 5×10^{-6} T. The start of time is chosen so that at $t = 0$, the magnetic field has an amplitude 2×10^{-6} T at the origin. (a) Write the wave function that will describe the magnetic field wave. (b) Find the wave function that will describe the associated electric field wave.

Ans: (a) $\vec{B} = \hat{u}_x\, B_0\, \cos(kz - \omega t + \phi)$, with $B_0 = 5 \times 10^{-6}$ T, $\omega = 2\pi\times 3\times 10^9 \mathrm{s}^{-1}$, $k = 62.8\mathrm{m}^{-1}$, $\phi = 1.16$rad, (b) $\vec{E} = \hat{u}_y E_0 \cos(kz - \omega t + \phi)$, $E_0 = 1500\, N/C$, same k, ω, and ϕ as that of $\vec{B}$ wave.