5.1 ELECTRIC DIPOLES

5.1.1 Induced Dipole Moments

If you place an atom in an electric field, its electrons will be pulled one way and the protons will be pulled the other way which distorts the electron cloud around nucleus as illustrated in Fig. 5.2. The polarized atom has a dipole moment, called the **induced dipole moment**, denoted by $\vec{p}_{\text{ind}}$, which is proportional to the applied electric field.

$$\vec{p}_{\text{ind}} = \alpha \vec{E}_{\text{at atom}} \tag{5.1}$$

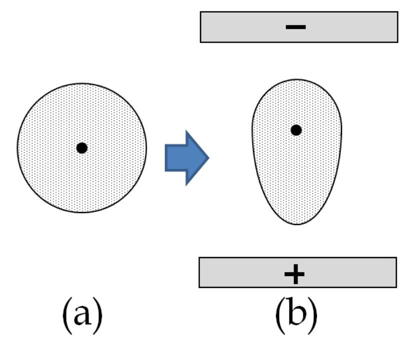

Figure 5.2: Induced polarization of a spherically symmetric atom in an external electric field: (a) no electric field, (b) an external electric field.

The constant of proportionality α is called the **atomic polarizability**. The SI-units of polarizability is obtained from the units of dipole moment and electric field.

$$[\alpha] = \frac{[p]}{[E]} = \frac{\text{C.m}}{\text{N/C}} = \frac{\text{C.m}^2}{\text{V}} = \frac{\text{C}^2\text{s}^2}{\text{kg}}.$$

Experimentally determined atomic polarizabilities of several atoms are given in Table 5.1. Notice that the atomic polarizability decreases across a period of the periodic table as the electrons of the atoms become more tightly bound to the nuclei. However, as you go down in a group to larger atoms, e.g. H, Li, Na, K, ... etc, the valence electrons are further out from the nucleus and less tightly bound, and therefore more easily polarizable as indicated by an increase in the atomic polarizability.

Every physical particle that has both positive and negative charges will behave similar to atoms when placed in an external electric field such that an electric dipole will be induced in the particle regardless of whether or not the particle has a symmetric or asymmetric charge distribution before electric field is applied.

Table 5.1: Atomic polarizability (α) ($\times 10^{-40}$C.m^2/V) (Source: CRC Handbook of Chemistry and Physics 82nd edition, 2001).

Period 2	Li	Be	B	C	N	O	F	Ne
	27	6.2	3.4	2.0	1.2	0.89	0.62	0.44
Group I	H	Li	Na	K	Rb	Cs	Fr	
	0.8	27	27	48	53	66	54	

5.1.2 Permanent Dipole Moments

When dissimilar atoms bond in a molecule, a separation of charge results due to the bonding electrons being attracted more to one atom than to the other due to the difference in the force of attraction for the electron by nuclei of different atoms. The separation of charges within bonds gives rise to **permanent dipole moments**. This dipole moment would be present even in the absence of any applied electric field.

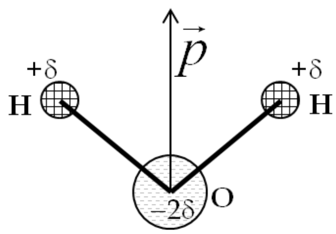

Figure 5.3: Permanent dipole of water molecule. Note the charges on hydrogen and oxygen atoms are much less than a charge of an electron.

For instance, in a water molecule, electrons are attracted more to the oxygen atom than to the hydrogen atoms making the oxygen atom negatively charged and the hydrogen atoms positively charged as shown in Fig. 5.3. The separation of charges between oxygen and hydrogen atoms results in a permanent dipole moment along each O-H bond. Since the angle between the two O−H bonds in water is 104.5°, the vector addition of the dipole moment vectors along the O−H bonds gives a net dipole moment vector of magnitude 6.1×10^{-30} C.m pointed towards the middle of the H−O−H angle. This is a very large dipole moment compared to the dipole moments of other molecules and is responsible for many of the useful properties of water.

5.1.3 Torque on an Electric Dipole

Consider an electric dipole placed in a region of constant electric field. There will be electric forces of equal magnitude but in the opposite directions will act on the two charges of the electric dipole as shown in Fig. 5.4.

Since the external electric field at the site of the dipole is uniform, the forces on $+q$ and $-q$ would have equal magnitude but point in the opposite directions. Therefore, the net force on the dipole will be zero.

However, if the dipole is oriented at an angle to the electric field, then there will be a non-zero torque on the dipole since the forces on the two charges form a couple. Since the net force on the dipole is zero, the torque will be independent of the pivot point for the calculation of the torque with following result,

$$\vec{\tau} = \vec{d} \times q\vec{E} = q\vec{d} \times \vec{E}. \tag{5.2}$$

Replacing $q\vec{d}$ as the dipole moment vector $\vec{p}$, we see that the torque on the dipole is equal to the vector product of the dipole moment

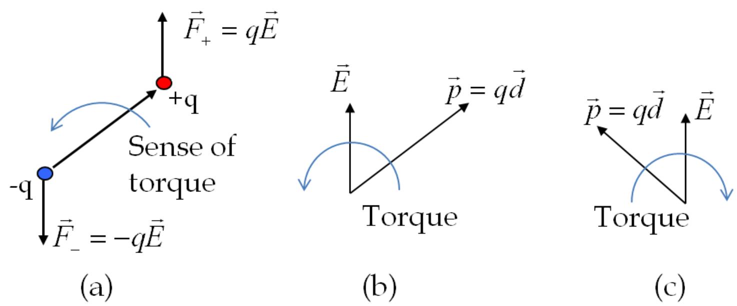

Figure 5.4: An electric dipole in a uniform electric field. (a) The forces on the two opposite charges of the dipole are equal in magnitude and opposite in direction. This results in the vanishing of the net force on the dipole. (b) and (c): The torque of the forces on the two charges have the same sense and therefore add to give a non-zero torque. The direction of the torque depends on the relative orientations of the dipole moment and the external electric field.

vector and the electric field vector $\vec{E}$.

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}.} \tag{5.3}$$

Thus, when a polar molecule such as the water molecule is placed in an external electric field, the molecule experiences a torque that will cause the molecule to acquire angular velocity about an axis that is pointed perpendicular to the plane of $\vec{p}$ and $\vec{E}$. The direction of the dipole moment will oscillate about the direction of the electric field vector $\vec{E}$. The damping of this motion leads to the dipole momet vector $\vec{p}$ settling in the same direction as the electric field $\vec{E}$.

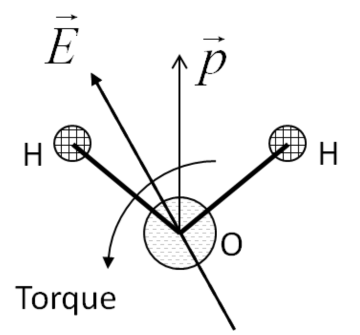

Figure 5.5: Torque on the water molecule tends to align $\vec{p}$ with $\vec{E}$.

5.1.4 Dipole in a Non-Uniform Electric Field

Electric force on a dipole is zero if the electric field is uniform since the oppositely directed forces on $+q$ and $-q$ of equal magnitude cancel each other exactly. However, if electric field is non-uniform, such as the electric field of a point charge, the two charges of the dipole will be in different electric fields, and hence the net force on the dipole will not be zero (Fig. 5.6).

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q\left(\vec{E}_+ - \vec{E}_-\right) = qd\left(\frac{\vec{E}_+ - \vec{E}_-}{d}\right) = p\frac{\Delta\vec{E}}{d} \tag{5.4}$$

where $\Delta\vec{E}$ is the difference in the electric field at the two ends, and d is the distance between charges of the dipole. The force on a dipole is seen to be towards the direction of increasing electric field. This means that a dipole is pulled in the direction of increasing electric field.

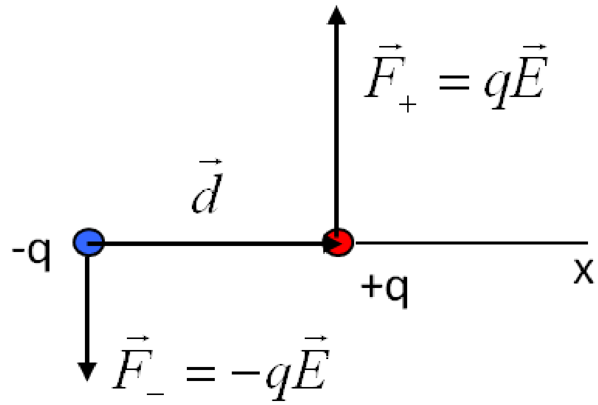

Figure 5.6: A dipole in a non-uniform electric field. Here, the electric field is larger on $+q$ then on $-q$. The dipole will have a non-zero net force and a non-zero net torque.

If the dipole is microscopic, we can take the small d limit. The result depends on the orientation of the dipole moment. For concreteness, let $\vec{p}$ be pointed along the x-axis and write the distance d between the charges in the dipole as Δx. Then $\Delta\vec{E}/d$ will become $\Delta\vec{E}/\Delta x$. As $\Delta x \to 0$, $\Delta\vec{E}/\Delta x$ will be replaced by the x-derivative of the electric field so that the electric force on the dipole will be proportional to the x-derivative of the electric field.

$$p\frac{\Delta\vec{E}}{d} = p_x\frac{\Delta\vec{E}}{d} \to p_x\frac{d\vec{E}}{dx} \quad \text{(dipole oriented along } x \text{ axis)}$$

Similarly, if $\vec{p}$ is pointed along the y-axis, we will get

$$p\frac{\Delta\vec{E}}{d} = p_y\frac{\Delta\vec{E}}{d} \to p_y\frac{d\vec{E}}{dy} \quad \text{(dipole oriented along } y \text{ axis)}$$

and if $\vec{p}$ is pointed along the z-axis, we will get

$$p\frac{\Delta\vec{E}}{d} = p_z\frac{\Delta\vec{E}}{d} \to p_z\frac{d\vec{E}}{dz} \quad \text{(dipole oriented along } z \text{ axis)}$$

For $\vec{p}$ in an arbitrary direction we will need the partial derivatives of $\vec{E}$ with respect to x, y and z to go with x, y, and z components of dipole moment vector $\vec{p}$.

$$\boxed{\vec{F} = p_x\frac{\partial\vec{E}}{\partial x} + p_y\frac{\partial\vec{E}}{\partial y} + p_z\frac{\partial\vec{E}}{\partial z}.} \tag{5.5}$$

Example 5.1.1. Force and Torque on a Dipole in a Non-uniform Field

A dipole has dipole moment of magnitude 5×10^{-12} Cm. The dipole is placed 20 μm from a fixed 3μC charge. Find the force and torque on the dipole (a) when the dipole is pointed away from the fixed charge, (b) when the dipole is pointed 90° from the fixed charge.

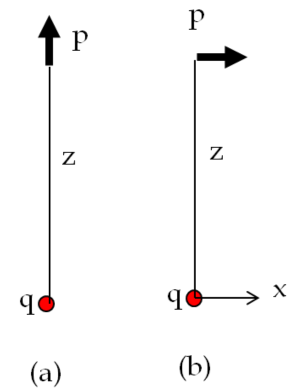

Figure 5.7: Example 5.1.1.

Solution. (a) Let us pick the z-axis to be the direction in which the dipole is pointed. Then the z-component of force is found to be

$$\vec{F} = p_z \frac{d}{dz}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{u}_z\right) = -\frac{qp}{2\pi\epsilon_0}\frac{1}{z^3}\hat{u}_z = 3.4 \times 10^7 \hat{u}_z \text{ N}.$$

$$\vec{\tau} = \vec{p} \times \vec{E} = p\hat{u}_z \times E\hat{u}_z = 0.$$

(b) Let the x-axis be the direction the dipole is pointing while located on the z-axis.

$$\vec{F} = p_x \frac{d}{dx}\left(\frac{1}{4\pi\epsilon_0}\frac{q}{z^2}\hat{u}_z\right) = 0.$$

$$\begin{aligned}\vec{\tau} &= \vec{p} \times \vec{E} = p\hat{u}_x \times E\hat{u}_z \\ &= -pE\hat{u}_y = -\frac{qp}{4\pi\epsilon_0}\frac{1}{z^2}\hat{u}_y = -335\ \hat{u}_y \text{ N.m.}\end{aligned} \tag{5.6}$$

5.1.5 Potential Energy of a Dipole in an External Electric Field

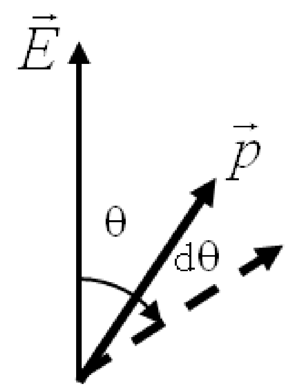

Figure 5.8: Rotating a dipole from θ to $\theta + d\theta$ will cost energy given by work against the torque on the dipole by the electric field.

When an electric dipole $\vec{p}$ is placed in an external electric field $\vec{E}$ the dipole experiences a torque if $\vec{p}$ is not along $\vec{E}$. That means, it would take work to rotate a dipole with respect to the direction of the electric field. To find the work required to rotate a dipole by a finite angle with respect to the field, we first write the work for an infinitesimal rotation from an arbitrary angle θ to $\theta + d\theta$ (Fig. 5.8). The work by the applied torque will be a rotational work opposing the torque due to the electric field. That is the applied torque will be in the opposite direction to the torque on the dipole by the electric force and will be given by

$$\vec{\tau}_{\text{appl}} = -\vec{p} \times \vec{E},$$

and the work done by the applied torque will be

$$dW = \vec{\tau}_{\text{appl}} \cdot d\vec{\theta},$$

where $d\vec{\theta}$ is the angular displacement vector. In Fig. 5.8 while $\vec{p} \times \vec{E}$ is coming out of page the vector $d\vec{\theta}$ is going into the page. Therefore,

$$dW = pE \sin\theta d\theta. \tag{5.7}$$

Integrating from an initial angle θ_i to a final angle θ_f we obtain the change in energy $U_f - U_i$ of the dipole.

$$U_f - U_i = -pE\cos\theta_f + pE\cos\theta_i. \tag{5.8}$$

The formula for the potential energy of the dipole simplifies if the reference of zero potential energy is chosen at the angle, $\theta = \pi/2$ radians. This choice gives the following formula for the potential energy of the dipole when it is pointed at an angle θ with respect to the external electric field.

$$U = -pE\cos\theta, \quad \left(\text{Reference } U = 0 \text{ at } \theta = \frac{\pi}{2}\right) \tag{5.9}$$

which can be written more compactly as follows.

$$\boxed{U = -\vec{p}\cdot\vec{E}.} \tag{5.10}$$

This relation shows that the energy of a dipole is least when the dipole moment and the external electric field are in the same direction and largest when the two are in the opposite direction.

Example 5.1.2. Rotating an electric dipole. A dipole of moment 50×10^{-12} C.m is aligned with an electric field between two parallel plates separated by 5 mm that have a potential difference of 1 kV. How much energy will it take to flip the orientation of the dipole?

Solution. We use the formula for the change in energy.

$$\Delta U = (-pE\cos\pi) - (-pE\cos 0) = 2pE.$$

Putting numbers in now, we get

$$\Delta U = 2 \times 50 \times 10^{-12}\text{C.m} \times \frac{1000\text{ V}}{0.005\text{ m}} = 20\mu\text{J}.$$