4.7 EXAMPLE WAVE FUNCTIONS

In this section, we will present two examples of wave functions that are very useful for analytic purposes.

4.7.1 Plane Wave

A plane wave is a simple three-dimensional wave where the wave travels along an axis and the wave function has the same value throughout any plane normal to the direction of wave. If the wave is traveling along x-axis, then on each plane parallel to the yz plane, the wave displacement has the same value (see Fig. 4.13). Therefore, the wave function for a plane wave traveling towards the positive x-axis would not depend on the y or z-coordinates of the point. The wave-fronts of such waves are planar as shown in Fig. 4.13.

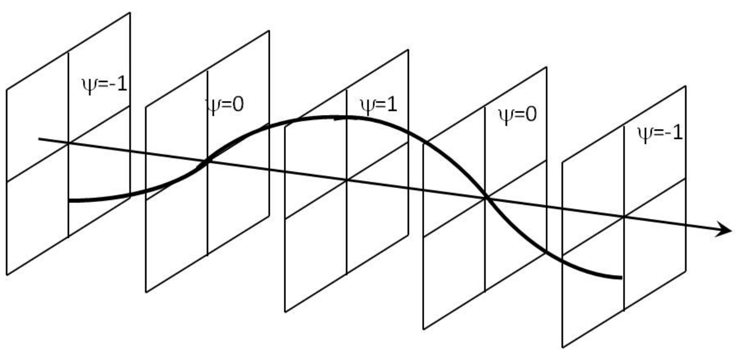

Figure 4.13: A plane wave has same wave amplitude in the entire plane. It is a volume wave. Although it looks similar to the wave on a chord it is three-dimensional rather than one-dimensional.

The analytic expression of the wave function ψ for a plane wave of amplitude A, wavenumber k, angular frequency ω, traveling towards the positive x-axis, passing through the origin at $t = 0$ with the displacement equal to A is identical to the sinusoidal wave on a string since the wave is essentially one-dimensional.

$$\psi(x,t) = A\ cos(kx - \omega t). \tag{4.31}$$

Similar to the wave function for waves on a string, other possibilities at the origin at time $t = 0$ are taken into account by introducing a phase constant ϕ in the argument of the cosine and sine functions.

$$\psi(x,t) = A\ cos(kx - \omega t + \phi). \tag{4.32}$$

The following more general function describes the displacement for a plane wave traveling in the direction of the wave vector $\vec{k}$.

$$\psi(x,y,z,t) = A\ cos(\vec{k} \cdot \vec{r} - \omega t + \phi), \tag{4.33}$$

where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$. The magnitude of the wavevector is the wavenumber of the wave, and the wavelength λ of the wave is related to the magnitude of the wavevector as, $\lambda = 2\pi/|\vec{k}|$.

4.7.2 Spherical Wave

If the energy from a source spreads out radially, such as the optical energy from the sun, it is more appropriate to use spherical coordinates to describe these waves as shown in Fig. 4.14.

$$\psi(r,t) = \frac{A}{r}\, cos(kr - \omega t + \phi), \tag{4.34}$$

where r is the radial distance from the origin.

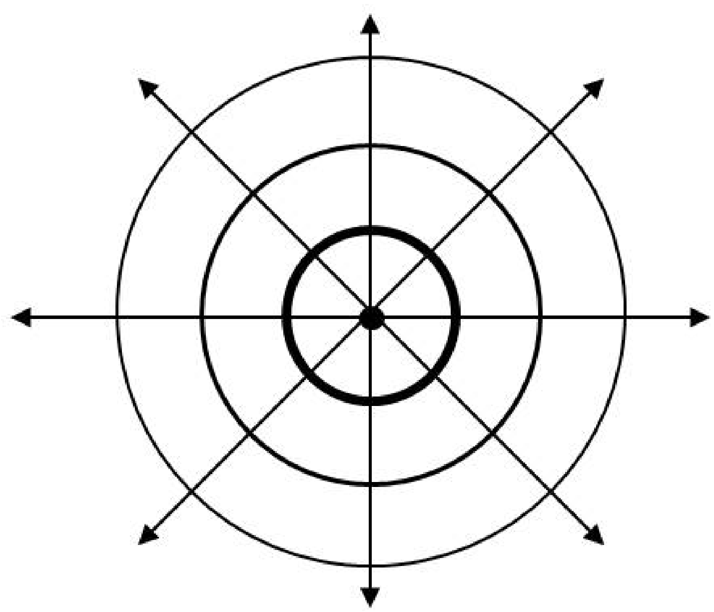

Figure 4.14: Spherical wave fronts emanating from a source at the center. Same amplitudes at all points that are equal distance from the source at the center. The amplitude drops with the radial distance from the source, shown with drop in thickness of lines, as the distance from the source increases.

4.7.3 Wave Pulses

So far we have discussed waves of infinite extent such as the sinusoidal and plane waves. Sinusoidal waves are given by a sine or a cosine function of space and time. For these types of waves, the displacement oscillates in time according to a combination of sine and cosine functions of time having a definite frequency. To be periodic in space and time, the sinusoidal functions must have a domain that cover the entire space and entire time, viz. $-\infty < x < \infty$ and $-\infty < t < \infty$ for a wave traveling along x-axis. These waves are idealization of the situations encountered in real life.

Real waves, such as water waves, a pulse traveling on a string, or sound of a drum are not sinusoidal waves. They can be of any shape. For instance, suppose a very long taut string is given a snap at one end once and then the end is held fixed in one place as shown in Fig. 4.15. This results in a bump in the string, which moves towards the other end. Similarly, suppose you turn a Laser on for a very brief period, you would get a pulse of light wave. Or, if you hit a piano key, there again you will get a pulse of sound wave. The wave functions for these waves cannot be described by a single wavelength or a single frequency.

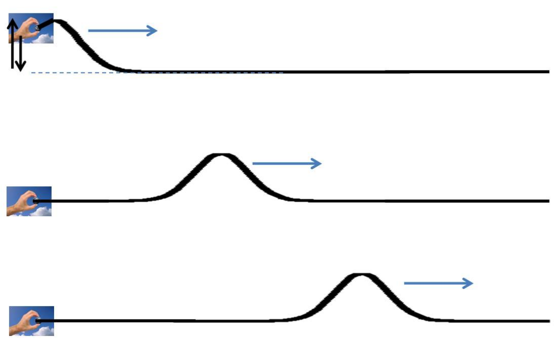

Figure 4.15: Generation and movement of a wave pulse on a string.