10.6 MOTIONAL EMF FROM AREA CHANGE

Faraday's flux rule states that a changing magnetic flux produces an induced current in a conducting loop. In the last section we have studied the effects of a changing magnetic field. In this section and the next, we will look at other ways of changing the magnetic flux, notably by changing the area where the magnetic field passes through the area A of the loop and by changing the angle θ between the magnetic field and the area vector. The EMF generated as a result of the changing A and θ are called **motional EMF**.

10.6.1 EMF by Changing Area

The changing magnetic flux due to a changing area of the loop in a magnetic field can be illustrated by constructing a circuit in which a part of the circuit moves with respect to other parts. Such a circuit can be easily constructed by placing a metal cross bar on a U-shaped metal as shown in Fig. 10.20. The metal cross bar and the U-shaped metal form a closed loop. Now we place the structure in a magnetic field. When the cross bar slides on the U-shaped metal, then the area of the loop will change with time, which would result in a changing magnetic flux through theloop. According to the Faraday's flux rule, the magnitude and direction of the induced current in the circuit would be related to the rate at which magnetic flux is changing through the loop of the circuit.

Note that, as the rod *cd* in Fig. 10.20 slides, the area of the conducting circuit *abcda* changes with time. The normal to the flat surface attached to the loop can be either in the direction of out-of-page or into-the-page. Suppose we take the normal direction as coming out-of-page, the magnetic flux through the area of the circuit would also increase with time. The change in magnetic flux $\Delta\Phi_B$ in time Δt equals the product of the uniform magnetic field and the change in area, which is equal to $Lv\Delta t$ shown shaded in Fig. 10.20.

$$\Delta\Phi_B = BLv\Delta t. \tag{10.15}$$

If the normal direction to the area was chosen to be into-the-page, then we we would have the following for the change in the magnetic flux.

$$\Delta\Phi_B = -BLv\Delta t. \tag{10.16}$$

Therefore, the magnitude of the rate of change of magnetic flux,

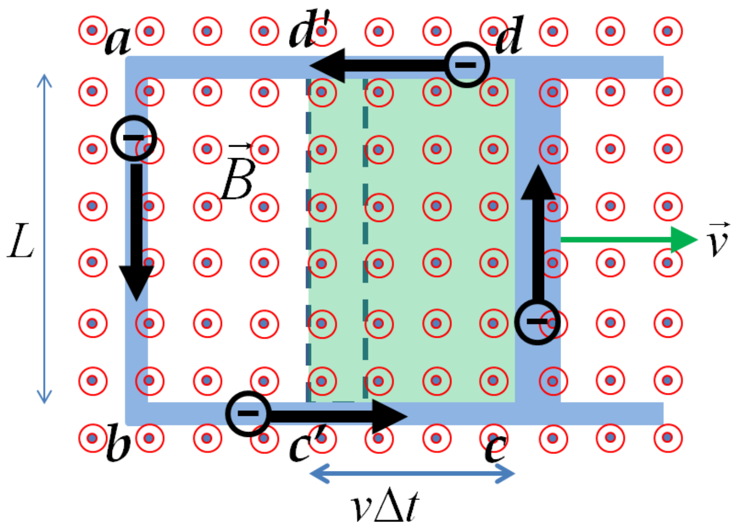


Figure 10.20: An electric current is induced in a circuit as a result of the movement of a part of the circuit in a magnetic field. Magnetic flux through the area of the circuit changes at the rate BvL, which is equal to the EMF induced in the circuit.

which is independent of the choice of the normal, is

$$\left|\frac{\Delta\Phi_B}{\Delta t}\right| = BLv. \tag{10.17}$$

If the net resistance of all conductors in the circuit at the instant t is R, then the induced current at time t would have the magnitude given by

$$I = \frac{1}{R}\left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{BLv}{R} \tag{10.18}$$

We find it easier to work out the direction of the induced current by using the Lenz's law. When the bar moves to the right, the magnetic flux of out-of-page field increases due to the increasing size of the loop with time, which includes the additional field lines with time (Fig. 10.21). Lenz's law tells us that the magnetic field of the induced current will tend to oppose the change in the flux. Therefore, we need the induced current to produce magnetic field pointed into-the-page direction inside the loop area. Using the right-hand rule of the Biot-Savart's Law, we deduce that for a current to produce the magnetic field into-the-page direction inside the loop, the induced current has to be in the downward direction in the moving bar.

Energy for changing area of the loop

We have found that when the area of a conducting loop changes, an induced current flows in the loop. Since the loop has some resistance

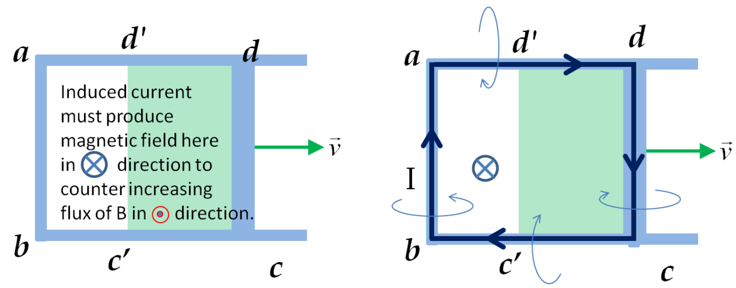


Figure 10.21: Illustrating the use of Lenz's law. Lenz's law says that the magnetic field of the induced current will oppose the change in magnetic flux that causes the induced current.

R, the flowing current in the conductor will result in the heating of the conductor.

Where does this energy come from? To answer this question, we need to look at the work done by the forces on the conductor. As we have seen above, the current flows downward in the sliding rod when it is moving out to the right. The force on the moving rod from the magnetic field will be pointed to the left and will have magnitude equal to ILB as shown in Fig. 10.22. For the bar to move at constant

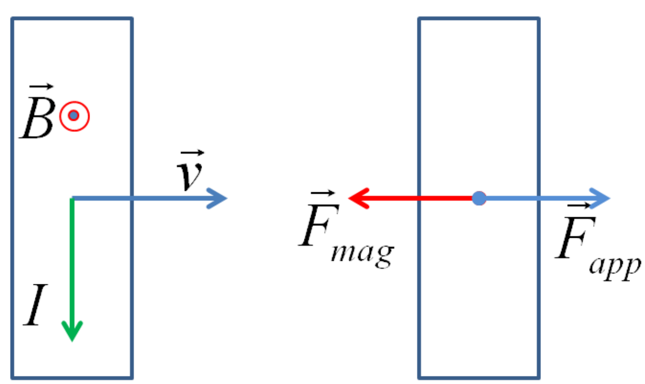


Figure 10.22: The applied force on the moving rod balances the effective magnetic force on the moving rod. The magnetic force on the charges has magnitude equal to ILB where I is the current due to moving charges, L the length of the rod perpendicular to the motion, and B the magnitude of the magnetic field.

velocity, there must be an applied force F_{app} acting on the rod in the opposite direction to balance this force.

$$F_{app} = ILB \quad \text{(Applied force balances magnetic force.)} \tag{10.19}$$

The applied force and the displacment of the rod are in the same direction. The work done by the applied force in a time interval Δt will be

$$W_{app} = \vec{F}_{app} \cdot \vec{v}\Delta t = ILBv\Delta t. \tag{10.20}$$

This must be equal to the energy dissipated in the loop during this time interval. Let us check this directly. We know that the power dissipated in a resistor is $P(t) = I^2R$. Therefore, the energy dissipated in the loop in time Δt should be:

$$\begin{aligned}\text{Energy dissipated } &= P\Delta t = I^2R\Delta t\\ &= I\left(\frac{BLv}{R}\right)R\Delta t \quad \text{(using Eq. 10.18)}\\ &= IBLv\Delta t, \end{aligned} \tag{10.21}$$

which is equal to the work done by the applied force given in Eq. 10.20 as expected!

Example 10.6.1. A Moving Wire. A rectangular wire of length L, width w, and resistance R is placed in a region of uniform magnetic field of magnitude B_0 that is constant in time as shown in Fig. 10.23. The magnetic field is zero outside the region. Initially the wire has all of its enclosed area in the region of non-zero magnetic field.

The wire is then pulled during $t = 0$ to $t = t_0$ interval. At the end of the interval the speed of the wire becomes v. At $t = t_0$ the wire is still within the region of the non-zero field. The wire continues to move at constant velocity till it comes out completely in the region where the magnetic field is zero. Assume an external force is applied on the wire to maintain the constant velocity when necessary. Assume the front of the wire starts to exit the region of the non-zero magnetic field at time $t = t_1$.

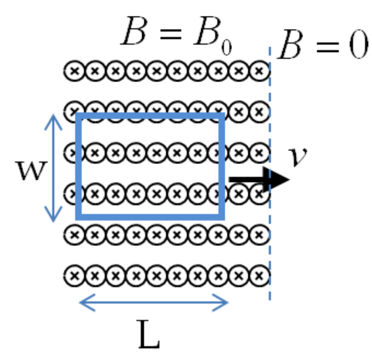


Figure 10.23: Example 10.6.1.

Find the induced current during the following intervals: (a) $0 \le t \le t_0$, (b) $t_0 \le t \le t_1$, (c) $t_1 \le t \le t_2$, (d) $t > t_2$, where t_2 is the time when the wire is completely out of the region of non-zero field.

Solution. Since the induced current in the loop occurs only when the magnetic flux through the area of the loop is changing, we can immediately conclude that there is no induced current in (a), (b) and (d) intervals. The induced current will be non-zero only during (c).

(c) $t_1 \le t \le t_2$: To find the magnitude of the induced current we need to find the induced EMF, which is equal to the rate at which magnetic flux is changing. Since the loop is exiting at constant speed, the rate of change of the flux is constant.

Let us calculate how much flux changes during the time interval from t_1 to t_2. Taking the direction of the area vector to point into-the-page direction, the flux at $t = t_1$ is $\Phi_B(t_1) = B_0Lw$, but the flux at $t = t_2$ is zero since the loop is entirely in the region of no magnetic field. In time interval $(t_2 - t_1)$ the loop moves a distance L at speed

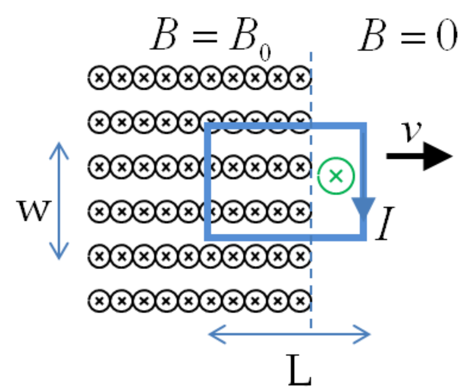
v. Therefore, the time duration, $(t_2 - t_1) = L/v$. This gives the following for the induced EMF in the loop during this time

$$\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{B_0 L w}{L/v} = B_0 v w.$$

Dividing the EMF by the resistance of the loop we obtain the magnitude of the induced current in the loop as

$$I = \frac{B_0 v w}{R}.$$

The direction of the induced current will be determined by Lenz's law. The magnetic field of the induced current will oppose the change in magnetic flux. Since, there is a loss of magnetic flux of the magnetic field into-the-page, the induced current must produce magnetic field in the direction of into-the-page. Therefore, the induced current will circulate in the clockwise direction as shown in the figure.



Example 10.6.2. Motional EMF in a Moving Rod. The circuit for the induced EMF can happen even in situations where there is no obvious circuit, for instance in a metal rod. Consider a metallic rod sliding in a region of uniform magnetic field as shown in Fig. 10.24. As the rod moves in the external magnetic field, there will be a magnetic force on the electrons and protons of the metal. Since conducting electrons in the metal are mobile, their acceleration due to magnetic force will move them towards one end where they will accumulate, leaving the other end positively charged. As a result of the separation of charges in the rod, there would be an electric field induced in the rod.

Because of the induced electric field, electrons in a sliding metallic rod experience both magnetic and electric forces. As shown in Fig. 10.24, the two forces are in opposite directions, and a stationary condition is established once the electric force can balance the magnetic force. Before the stationary condition is reached, there is a current in the moving rod but once the stationary condition is reached the current also stops.

$$\text{Stationary condition: } \vec{E}_{ind} = \vec{v} \times \vec{B}. \tag{10.22}$$

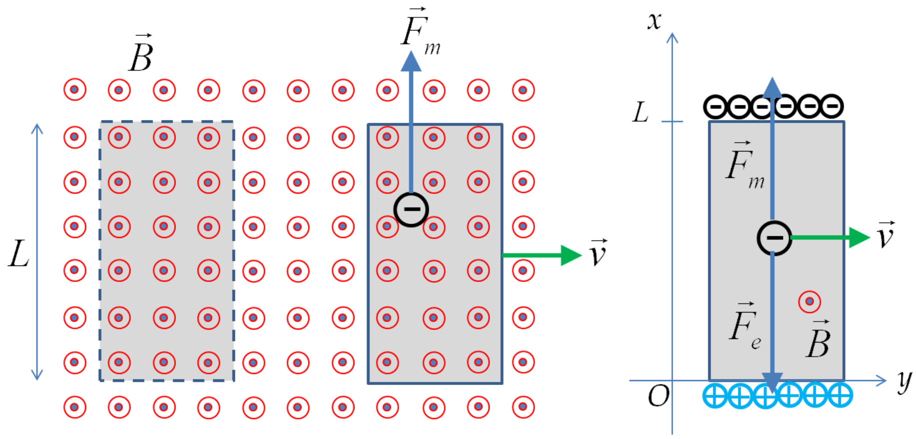


Figure 10.24: Magnetic force on a moving electron in the rod gives the electron acceleration perpendicular to the velocity of the rod causing electrons to accumulate at one end of the rod leaving the other side positively charged. Separation of charges creates an electric field that exerts an electric force on the electron.

The line integral of the electric field from the positive end to the negative end of the bar gives the electromotive force (EMF) induced in the bar. Using the coordinates displayed in Fig. 10.24, we obtain $\vec{E}_{ind} = vB\hat{u}_x$. Therefore, denoting the induced EMF by $\mathcal{E}_{ind}$, we have

$$\mathcal{E}_{ind} = \int_0^L vBdx = vBL. \tag{10.23}$$

Note that this is the same EMF we found earlier when the rod was moving on a U-shaped rail and was part of a closed loop. This is not a coincidence and points to teh fact that moving conductors in a magnetic field induces an electric field locally at each point regardless of the presence or absence of a physical circuit.