5.2 LINEAR DIELECTRIC

5.2.1 Polarization of a Dielectric Material

What happens to a dielectric material when it is placed in an electric field? A dielectric material consists of numerous microscopic dipoles, whose dipole moments will give rise to a net dipole moment of the material. To take into account the possibility that the dipole moment in the material may vary from place to place we define a quantity called polarization which is the density of dipole moment at a particular place.

$$\vec{P} = \frac{\sum \vec{p}}{\text{Volume}} = \text{dipole moment per unit volume} \tag{5.11}$$

where small letter $\vec{p}$ is the dipole moment of one microscopic dipole and capital letter $\vec{P}$ the polarization of the material.

You can see that if the microscopic dipoles are randomly oriented as would be the case at high temperature, then the polarization will be zero everywhere in the sample. We call the material unpolarized. At room temperature, microscopic dipoles in most materials are randomly pointed and therefore, most materials do not have a net polarization. Consequently microscopic dipoles $\vec{p}$ sum up to zero polarization for the sample.

On the other hand, if the microscopic dipoles are aligned, we would have a net polarization of the material. When an external electric field is applied, the torques on the dipoles lead to the alignment of microscopic dipoles with the external field. This makes the material polarized.

5.2.2 Electric Field Inside a Dielectric

The electric field of the aligned dipoles can be significant even at macroscopic distances. As each dipole produces its own electric field, the net electric field at any point is equal to the superposition of the applied electric field and the electric field of all the dipoles. The net effect of aligned dipole moments is to reduce the electric field inside a dielectric. This effect is called electric field **screening**.

To illustrate the effect of screening consider two parallel metal plates of large surface area A on each side of one plate facing the other plate. Let the plates be separated by a distance d (see Fig. 5.9). Let one of the plates be charged with total charge Q and the other with charge $-Q$. The charges will be only on the side of each plate facing the other plate so that surface charge density on the plates would be $\pm Q/A$ respectively. We will compare the electric properties of the two systems - one with nothing between the plates and the other with a dielectric between the plates.

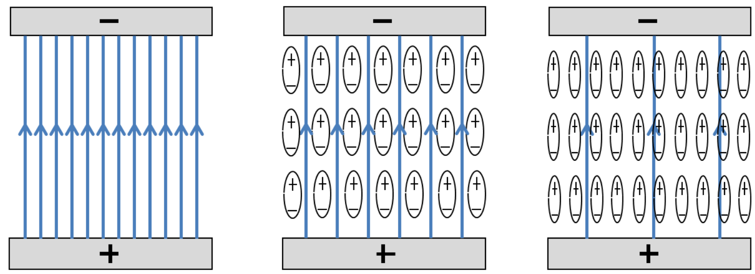

Figure 5.9: Electric field is screened inside a dielectric. Electric field lines between plates charged to same charge on plates $\pm Q$. (a) Vacuum between parallel plates, (b) Dielectric between parallel plates of dielectric constant $\epsilon_r = 2$, and (c) Dielectric between parallel plates of dielectric constant $\epsilon_r = 3$.

Let $\vec{E}_P$ denote the electric field at some point P between the plates when there is nothing between the plates and $\vec{E}_P'$ be the electric field at that same point when there is a dielectric present. As we have argued above that $\vec{E}_P'$ is the net electric field which is the sum of the original electric field $\vec{E}_P$ and the electric field from the dipoles of the material. Now, the electric field produced by the dipoles of the material will depend on the degree to which the material is polarized, i.e. on the polarization of the dielectric. The polarization, in turn depends on the net electric field since the torque on each microscopic dipole comes from the net field at the location of the dipole.

If the applied field is not too great, we expect that the magnitude of the polarization $\vec{P}_P$ at point P would increase linearly with the magnitude of the net electric field at P. We write the proportionality for any point in the material as

$$\boxed{\vec{P}_P = \epsilon_0 \chi_e \vec{E}_P{}',} \tag{5.12}$$

where χ_e is a property of the material called **electric susceptibility**. A material that obeys this linear relation between the polarization and the net electric field is called a **linear dielectric**. The net electric field in a linear dielectric can also be written in terms of the applied electric field at P, $\vec{E}_P$, as

$$\boxed{\vec{E}_P{}' = \frac{\vec{E}_P}{\epsilon_r},} \tag{5.13}$$

where the proportionality constant ϵ_r is called the **dielectric constant** or **relative permittivity** of the material is related to the electric susceptibility χ_E as

$$\boxed{\epsilon_r = 1 + \chi_e.} \tag{5.14}$$

The dielectric constant is a dimensionless quantity. In Table 5.2, I have listed the dielectric constants of some commonly used materials. Note in the table the large dielectric constant of water, which is directly attributable to the high polarity of the water molecule. The quantity $\epsilon_r\epsilon_0$ is called the **permittivity** of the dielectric, just as ϵ_0 is called the permittivity of vacuum or free space. Permittivity is denoted by the same symbol ϵ without the subscript zero.

We see from above that a linear dielectric medium can be characterized by three inter-related properties: dielectric constant ϵ_r, electric susceptibility χ_e and permittivity ϵ. The following equation summarizes the relations among the three quantities.

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0} = 1 + \chi_e.} \tag{5.15}$$

In addition to the linear dielectrics, there are special materials, called **ferroelectrics**, such as Barium Titanate ($BaTiO_3$), that do not respond to an external electric field in the manner outlined above. Their polarizations are not linear in the electric field. Furthermore, while the polarization in linear dielectrics go to zero due to thermal randomization of the dipoles when the external electric field is removed, the dipoles in the ferroelectrics remain aligned even when the external electric field is removed as long as the temperature is lower than a critical value called **Curie point**.

If the electric field at a point is so great that electrons can be ripped off from the molecules, a **dielectric breakdown** of the medium occurs. The loose electrons accelerate and strike other molecules with great energy and cause more electrons to come off molecules leading to an avalanche of moving electrons and ions. This is responsible for the lightning shown in Fig. 5.10. Usually it takes considerable electric field to reach the dielectric breakdown level. For instance, you need approximately 3 MV/m of electric field to cause the electric breakdown of the dry air.

Figure 5.10: Lightning as an example of dielectric breakdown (Credits: NOAA Photo Library, NOAA Central Library; OAR/ERL/National Severe Storms Laboratory (NSSL), Photographer C. Clark)

A capacitor is rated for the dielectric breakdown because when they are charged electric field between the plates increases and if they are charged too much there may be a dielectric breakdown and charges will flow from one plate to another. The dielectric breakdown of a capacitor will burn out the capacitor and make it useless. The

Table 5.2: Dielectric constants at 20°C (Kaye-Laby Online, 2005)

Material	Dielectric constant K	Material	Dielectric constant K
SOLIDS			
Cellophane	7.0	Pyrex Glass	4.7
Epoxy Resin	3.6	Quartz	4.3
Mica	7.0	Teflon	2.1
Mylar (polyester)	3.25		
Paper	3.7	LIQUIDS	
Polycarbonate	2.8	Castor oil	4.5
Polypropylene	2.2	Water	80.4
Polystyrene	2.6	GASES	
Porcelain	5.5	Air (1 atm.)	1.00054

maximum electric field that a dielectric material can withstand before a dielectric breakdown occurs is called its **dielectric strength**. Common insulating materials have dielectric strengths tens of times more than that of air. For instance, the commonly used insulating material polycarbonate has a dielectric strength of approximately 300 MV/m, which can be compared favorably to 3 MV/m for dry air.

5.2.3 Capacitance with Dielectrics

A common way to increase the capacitance of a capacitor is to fill the space between the plates with a dielectric such as mica. With the reduced electric field between the plates the potential difference between the plates would be less. Therefore, you would need to put a larger amount of charge on the plates to produce the same potential difference. That means the capacitance is larger when there is a dielectric material between the plates. Usually mica or paper is put between the plates of a capacitor to increase the capacitance. We can derive the relation between the capacitance when there is no dielectric and the capacitance when there is dielectric between the plates.

Previously, we have worked out the formula for the electric field in a parallel plate capacitor without a dielectric. We had found that the magnitude of the electric field between plates is uniform with the magnitude given by

$$E_0 = \frac{Q/A}{\epsilon_0}, \quad \text{(No dielectric.)} \tag{5.16}$$

The integrate of this electric field from one plate to the other gives the potential difference V_0 between the plates,

$$V_0 = \frac{Qd}{\epsilon_0 A}. \tag{5.17}$$

The ratio of charge Q to the voltage difference V_0 is the capacitance C_0 of the capacitor.

$$C_0 = \frac{\epsilon_0 A}{d}. \tag{5.18}$$

With a dielectric between the plates the magnitude of the electric field between the plates will be reduced by a factor of the dielectric constant, ϵ_r.

$$E = \frac{E_0}{\epsilon_r}, \quad \text{(with dielectric.)} \tag{5.19}$$

Therefore, the potential difference V with dielectric between the plates will be less.

$$V = \frac{V_0}{\epsilon_r} = \frac{Qd}{\epsilon_r \epsilon_0 A}. \tag{5.20}$$

This gives the capacitance C for the capacitor with the dielectric in terms of the capacitance C_0 as

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon_r \epsilon_0 A}{d} = \epsilon_r C_0.} \tag{5.21}$$

Example 5.2.1. Dielectric Between Plates of a Capacitor. The space between plates of a parallel-plate capacitor of area 400 cm^2 and separation 3 mm is fill with mica. The capacitor is then connected to 12-V battery and charged fully. Determine (a) the capacitance, (b) the charge on the plates, (c) the electric field between the plates and (d) the electrostatic energy stored.

Solution. The dielectric constant of mica from the table, $\epsilon_r = 7$.
(a) To find the capacitance we can use the formula for the capacitance [To keep the expressions simple I will express all quantities in the SI units and suppress them in the calculations.]

$$\begin{aligned} C &= \frac{\epsilon_r \epsilon_0 A}{d} \\ &= \frac{7 \times 8.5 \times 10^{-12} \times 0.04}{0.003} = 7.9 \times 10^{-10} F. \end{aligned} \tag{5.22}$$

(b) To find the charge on the positive plate we just multiply the capacitance with the voltage across the plates. This gives $7.9 \times 10^{-10}\text{F} \times 12$ V, or 9.5×10^{-9} C.
(c) The electric field between the plates is equal to the the voltage divided by the separation of the plates. This gives $12/0.003 = 4,000$ V/m, or 4,000 N/C.
(d) The electrostatic energy will be the energy stored in the capacitor, which is equal to $\frac{1}{2}QV = 5.7 \times 10^{-8}$ J.