6.5 Nuclear Fission

Refresh your memory about the binding energy per nucleon curve you studied earlier in Fig. 6.2. You will notice that the binding energy of large nuclei is lower than mid-size nuclei. Consequently, if a large nucleus could be split into medium-size nuclei, energy will be released since the number of nucleon will be same in the product nuclei as the original nucleus and the binding energy per nucleon is higher in the product than in the original nucleus. The reaction of splitting of a large nucleus into smaller nuclei is called **fission**. The fission was discovered by Otto Hahn and Fritz Strassman in Germany in 1938.

Since the difference in binding energy per nucleon is about 1 MeV per nucleon, the energy released for a nucleus having 200 nucleons is about 200 MeV per fission. That is an enormous amount of energy compared to chemical reactions where the energy relased is ~ 1 eV per reaction.

The splitting of large nuclei is induced by making them more unstable than they already are. For instance, a neutron capture of a slow neutron by ^{235}U leads to an excited state of ^{236}U which decays by fission into many products, denoted by Y_1 and Y_1 in the following equation for the fission reaction.

$$^{235}\mathrm{U} + \mathrm{n[slow]} \longrightarrow\ ^{236}\mathrm{U}^* \longrightarrow\ Y_1 + Y_2 + \mathrm{neutrons[fast]}. \tag{6.63}$$

The reaction is called ^{235}U fission even though it is actually $^{236}\mathrm{U}^*$ fission. You do not get the same Y_1 and Y_2 each time a fission of $^{236}\mathrm{U}^*$ occurs but you get a wide range of nuclides of A values with different yields as shown in Fig. 6.9. On average two to three neutrons are released with each fission. These neutrons are high energy neutrons as opposed to the neutron that was captured on the reactant side, which is a slow neutron. To make use of the fast neutrons produced on the product side, they have to be slowed down by a moderator as done in a nuclear reactor.

Each product pair (Y_1, Y_2) must be consistent with the conservation laws we have given above for nuclear reactions - various combinations of A of Y_1 and Y_2 together with how many neutrons are released in the process are allowed by the conservation of nucleon number. For instance, here are two typical product channels.

$$\begin{aligned}
&[\text{Nucleons } = 235 + 1 = 236]\ \ ^{235}\mathrm{U} + \mathrm{n} \longrightarrow\ ^{235}\mathrm{U}^* \\
&\qquad\qquad \longrightarrow\ ^{144}\mathrm{Ba} +\ ^{89}\mathrm{Kr} + 3\mathrm{n}\ \ [\text{Nucleons } = 144 + 89 + 3 = 236] \\
&[\text{Nucleons } = 235 + 1 = 236]\ \ ^{235}\mathrm{U} + \mathrm{n} \longrightarrow^{235} \mathrm{U}^* \\
&\qquad\qquad \longrightarrow\ ^{140}\mathrm{Xe} +\ ^{94}\mathrm{Sr} + 2\mathrm{n}\ \ [\text{Nucleons } = 140 + 94 + 2 = 236]
\end{aligned}$$

The product nuclei are too rich in neutron and have a high value of N/Z ratio. For nuclei of A near 100, the stable nuclei have $N/Z \sim 1.3$ and near $A = 150$, $N/Z \sim 1.4$, while the product nuclei have $N/Z \sim 1.55$. The product nuclei are therefore unstable due to the N/Z imbalance and undergo a series of beta decay which transforms neutrons to protons inside these nuclei, thus bringing the N/Z

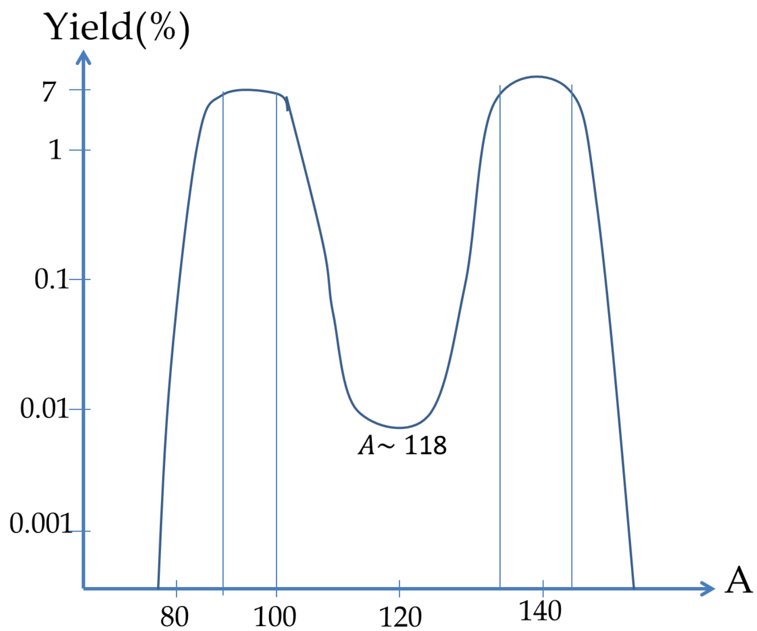

Figure 6.9: The mass distribution of fragments of ^{235}U fission. Most of the fragments fall in the range of $A \sin 100$ and $A \sim 140$. The fragments are not stable and undergo a series of β^- decay because they are neutron rich.

down towards stable nuclei. For instance, the product ^{140}Xe decays by the following series of β^- decays.

$$^{140}_{54}\text{Xe} \longrightarrow {}^{140}_{55}\text{Cs} \longrightarrow {}^{140}_{56}\text{Ba} \longrightarrow {}^{140}_{57}\text{La} \longrightarrow {}^{140}_{58}\text{Ce} \quad \text{(stable)}. \tag{6.64}$$

Example 6.13. (a) Find disintegration energy of the following channel of disintegration of ^{235}U.

$$^{235}\text{U} + \text{n} \longrightarrow {}^{144}\text{Ba} + {}^{89}\text{Kr} + 3\text{n}.$$

The data needed: $M_{U235} = 235.043924$ u, $m_n = 1.008664$ u, $M_{Ba144} = 143.922953$ u, $M_{Kr89} = 88.91763$ u. (b) Compute the disintegration energy per nucleon.

Solution.

(a) Using the rest energies we get

$$Q = (M_{U235} - M_{Ba144} - M_{Kr89} - 2m_n)\, c^2 = (0.186013\,\text{u})c^2 = 173\,\text{MeV}.$$

(b) There are 236 nucleons involved in the reaction. Therefore, per nucleon, the disintegration energy is 0.733 MeV.

6.5.1 Chain Reaction

We have seen that in the nuclear fission of ^{235}U two to three neutrons are released for every neutron captured. If the new neutrons produced, the second generation neutrons, can be slowed down by moderators, then, they may be captured by two other ^{235}U nuclides, which would, in turn release four to six neutrons. These third generation neutrons can cause fissions of even more ^{235}U nuclides. The process leads to ever more fission, called the **chain reaction** as illustrated in Fig. 6.10.

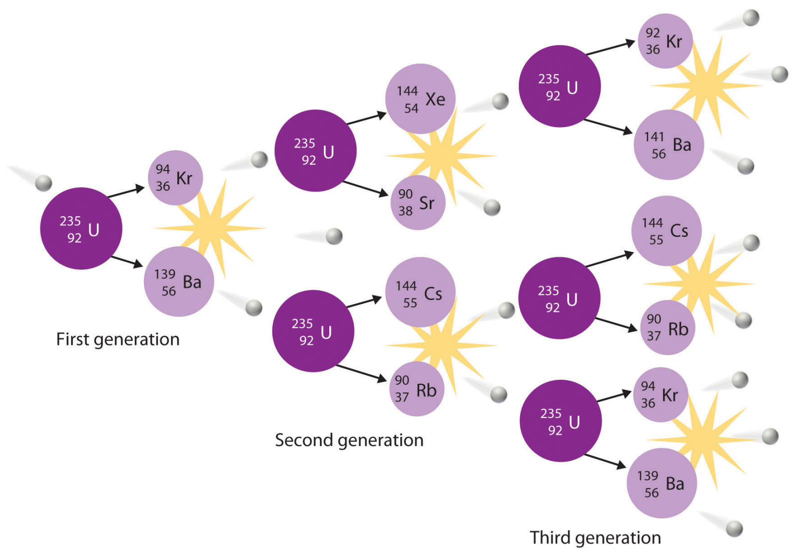

Figure 6.10: Chain reaction in nuclear fission of ^{235}U. When one neutron is captured two to three neutrons are emitted along with fission products. The new neutrons then start fission of additional ^{235}U. Source: catalog.flatworldknowledge.com. **************To the publisher, please have our own artists generate a similar diagram.

6.5.2 Harnessing energy from fission

The nuclear chain reaction outlined above is an uncontrolled process. If unchecked, the chain reaction will release enormous amount of energy. Fission of each atom will release around 212 MeV of energy as you can see from a quick calculation. The binding energy per nucleon in ^{235}U is approximately 7.6 MeV and in the product nuclei of the fission, the binding energy is about 8.5 MeV. Therefore, the energy released will be

$$Q \approx (8.5\,\text{MeV/nuc} - 7.9\,\text{MeV/nuc}) \times 235\,\text{nuc} = 212\,\text{MeV}.$$

In one kg of ^{235}U there are approximately,

$$N = \frac{1.0\text{ kg}}{0.235\text{ kg/mole}} \times 6.022 \times 10^{23}\,\frac{\text{atoms}}{\text{mole}} = 2.56 \times 10^{24}\text{ atoms}$$

Therefore, if all energy in 1 kg of ^{235}U were to be released, we will get

$$E = NQ = 5.43 \times 10^26\,\text{MeV} = 8.69 \times 10^{13}\text{ J}.$$

As a comparison a trinitrotoluene (TNT) i.e. dynamite explosion releases about 4.184×10^9 J of energy. Therefore, the energy released by fission of 1 kg of ^{235}U will release energy equivalent of about 20,000 TNT explosions. Atomic bombs use this fact about uncontrolled nuclear chain reactions.

However, it is possible to control the chain reaction by materials that would absorb some of the excess neutrons produced in the fission. This is done in a system called **nuclear reactor**, first invented by Enrico Fermi in the US. Fig. 6.11 shows parts of a nuclear reactor and the power plant based on the nuclear reactor.

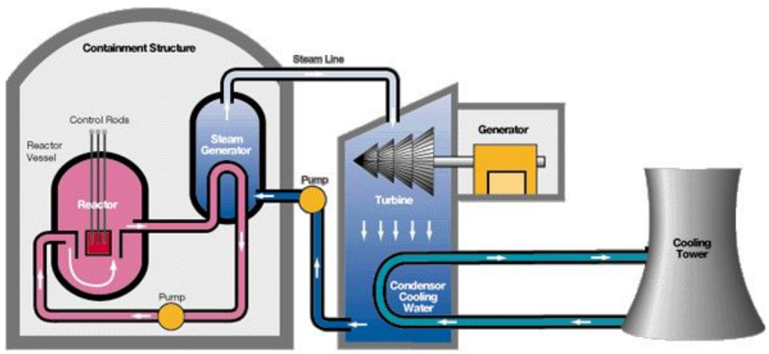

Figure 6.11: Components of a nuclear reactor and a steam power plant that converts the energy released to electrical energy. From: http://www.freeinfosociety.com/site.php?postnum=3115.

Sustainable chain reaction

Recall that in the fission process on average 2.5 neutrons are released for each neutron absorbed. Therefore, in an uncontrolled chain reaction, each fission event will lead to two or three new fission events. Let K denote the number of fission events induced by neutrons released by one fission event.

$$K = \text{Number of fission events induced by neutrons released by one fission event.} \tag{6.65}$$

This factor K is called **reproduction constant**. When $K > 1$, the nuclear reaction is a runaway chain reaction, $K < 1$, there are fewer and fewer fission reactions with time, and when $K = 1$, the reaction is sustained at a particular level of activity, neither increasing nor decreasing. The condition $K = 1$ is called the critical condition for the operation of a nuclear reactor. For sustainable chain reaction we desire to operate first at $K > 1$, reach a desired level of activity, and then maintain with $K = 1$. To operate a nuclear reactor we need mechanisms of controlling the reaction rates.

Moderator

The neutron emitted in the fission process have energy of the order of 2 MeV. They have a very small cross-section of capture by ^{235}U. The cross-section for neutron capture by ^{235}U is highest for slow neutrons. To slow the neutrons, the fuel rods containing the fissile material ^{2}35U are surrounded by moderating materials, such as water or graphite, called **moderators**. Neutrons scatter from the nuclei of the moderators which thermalzes neutrons, making them prime for capture by ^{2}35U. The presence of moderator material increases the probability of neutron capture by the ^{235}U nuclide.

Control Rods

The rate of nuclear reaction in the uranium fuel rods is actively controlled by moving**control rods** between the fuel rods. The control rods are made from elements such as boron or cadmium which have a high cross-section for absorbing slow neutrons. For instance, ^{113}Cd has a cross section of $\sim 10^4$ barns for slow neutrons in the following reaction

$$^{113}\text{Cd} + n \longrightarrow \ ^{114}\text{Cd}^* \longrightarrow \ ^{114}\text{Cd} + \gamma,$$

which is a little more than the reaction cross-section of neutron capture by ^{235}U. The control rods compete with the fuel rods for the slow neutrons and in this way control the number of slow neutrons available for capture for fission.

Enrichment

Finally, the uranium fuel rods are mostly ^{238}U, not ^{235}U. The uranium ore contains 99.3% ^{238}U and only 0.7% ^{235}U. The isotope ^{238}U is not as fissile as the isotope ^{235}U. Furthermore, nuclide ^{238}U has a high probability of capture of neutron of high energy. In this sense, ^{238}U nuclide deplete the fuel rod of high energy neutrons. If the fuel rods are made with the natural abundance levels of each isotope, the neutrons released from the fission of ^{235}U will be quickly absorbed by ^{238}U and no chain reaction will ensue. Therefore, fuel rods are made by enriching uranium to a higher level than 0.7%, usually between 3 and 5%.

Steam engine

In a power plant using a nuclear reactor, the heat produced in the nuclear reactor is used to convert liquid water into steam. The steam drives a turbine which converts the thermal energy in steam into electrical energy. In 2014, around 13% of the world's electricity was produced from nuclear energy based on nuclear reactors.