1.12 Exercises

Galilean Relativity

Ex 1.1. A man standing still at a train station watches two boys throwing baseball in a moving train. Suppose the train is moving towards East with a constant speed of 20 m/s and one of the boys throws the ball with a speed of 5 m/s with respect to him towards the other boy who is 5 meter from him towards West. What will be the velocity of the ball as observed by the man on the station. Ans: 15 m/s East.

Ex 1.2. When observed from the Sun at a particular instant the Earth and Mars appear to move in the opposite directions with speeds 108,000 km/h and 86,871 km/h respectively. What is the speed of Mars at this instant when observed from Earth?

Ex 1.3. A man is running on a straight road perpendicular to a train track and away from the track at a speed of 12 m/s. The train is moving with a speed of 30 m/s with respect to the track. What is the speed of the man with respect to a passenger sitting at rest in the train?

Ex 1.4. A man is running on a straight road that makes 30° with the train track. The man is running in the direction on the road that is away from the track at a speed of 12 m/s. The train is moving with a speed of 30 m/s with respect to the track. What is the speed of the man with respect to a passenger sitting at rest in the train?

Ex 1.5. In a frame at rest with respect to the billiard table, a billiard ball of mass m moving with speed v strikes another billiard ball of mass m at rest. The first ball comes to rest after the collision while the second ball takes off with speed v in the original direction of the motion of the first ball. This shows that momentum is conserved in this frame. (a) Now, describe the same collision from the perspective of a frame that is moving with speed v in the direction of the motion of the first ball. (b) Is the momentum conserved in this frame?

Ex 1.6. In a frame at rest with respect to the billiard table, two billiard balls of same mass m are moving towards each other with the same speed v. After the collision, the two balls come to rest. (a) Show that momentum is conserved in this frame. (b) Now, describe the same collision from the perspective of a frame that is moving with speed v in the direction of the motion of the first ball. (c) Is the momentum conserved in this frame?

Lorentz transformations

Ex 1.7. Describe the following physical occurrences as events: (a) A postman rings a doorbell of a house precisely at noon, (b) At the same time as the doorbell is rung a slice of bread pops out of a toaster which is located 10 m from the door in the East direction from the door, (c) 10 sec later an airplane arrives at the airport, which is 10 km from the door in the East direction and 2 km to the South.

Ex 1.8. A train arrives at a location $(x = x_0, y = 0, z = 0)$ at time $t = 0$ in a frame S at rest with respect to the train station. A spaceship is passing by with velocity $(V, 0, 0)$ with respect to S. Suppose the pilot in the spaceship was at $(0, 0, 0)$ at $t = 0$ and uses a coordinate system in which he is at the origin and sets the zero of his clock at that instant. What will be the coordinates and time the pilot will assign to the event of the train arrival?

Ex 1.9. In a frame S two events are observed: event 1 - a pion is created at rest at the origin and event 2- the pion disintegrates after time τ. Another observer in a frame S' is moving towards the positive x-axis with a constant speed V observes the same two events in his frame. The origins and times in the two frames are such that origin and zero time in one frame is equal to the origin and zero time in the other frame. (a) Find the positions and timings of these two events in the frame S' and (b) describe how the observer in S' will describe these two events.

Some Consequences of Lorentz Transformations

Ex 1.10. A clock in a spaceship runs one-tenth the rate at which an identical clock on Earth runs. What is the speed of the spaceship?

Ex 1.11. An astronaut has a heartbeat rate of 66 beats per minute as measured during his physical exam on Earth. The heartbeat rate of the astronaut is measured when he is in a spaceship traveling at $0.5c$ with respect to Earth by an observer (A) in the ship and by an observer (B) on Earth. (a) Describe an experimental method by which observer B on Earth will be able to determine the heartbeat rate of the astronaut when the astronaut is in the spaceship. (b) What will be the heartbeat rate(s) of the astronaut reported by observers A and B?

Ex 1.12. A spaceship (A) is moving at speed $\dfrac{c}{2}$ with respect to another spaceship (B). Observers in A and B use Cartesian coordinates which have the same directions of the axes in space. With respect to these Cartesian axes, the motion of A is towards the positive x-axis. That is, from the perspective of B, the coordinates of A move towards the positive x-axis. Observers in A and B set zero of their times such that when the origins of of the two coordinates coincide the times there are also zero. That is the event given by (x, y, z, t) is $(0, 0, 0, 0)$ in A and also $(0, 0, 0, 0)$ in B. Observer at the origin of B turns on a laser at $t = 0$ and $t = \tau$ in his time. What will the time duration between on and off as observed by an observer in A?

Ex 1.13. Same two observers as in Exercise 1.12. But, now we look at two events occurring in spaceship A. A photon arrives at the origin of A at its time $t = 0$ and another photon arrives at $(x = 1\text{ m}, 0, 0)$ at $t = 0$ in the frame of ship A. (a) Find the coordinates and times of the two events as observed by frame B. (b) In which frame, the two events are simultaneous and in which frame they are not simultaneous?

Ex 1.14. Same two observers as in Exercise 1.12. A rod of length 1 m is laid out on the x-axis in the frame of B from origin to $(x = 1\,\text{m}, 0, 0)$. What will be the length of the rod observed by an observer in frame of spaceship A?

Transformations of velocities

Ex 1.15. A proton beam is shot in the forward direction from a moving spaceship. The protons move with respect to the spaceship at speed $0.5c$. The spaceship itself moves at a speed $0.5c$ with respect to Earth. What is the speed of the proton with respect to Earth?

Ex 1.16. A spaceship is moving away from Earth at a speed $0.5c$. In side the spaceship there is an electron accelerator. The electron accelerator speeds the electrons to $0.8c$ with respect to the spaceship but the velocities of the electrons are in the direction of Earth. What will be the speed of an electron with respect to Earth?

Ex 1.17. A pion moving at a speed of $0.9c$ with respect to the laboratory splits into two photons, one in the forward direction and the other in the backward direction, each traveling at the speed of light. What are the speeds of the forward and backward emitted photons with respect to the laboratory?

Ex 1.18. Two spaceships are on a collision course moving perpendicularly to the same point as observed by an observer at rest with respect to the meeting point. Suppose the common point is the origin, spaceship A is on the negative x-axis and moving towards the positive x-axis direction with speed $0.9c$, and spaceship B is on the negative y-axis and moving towards the positive y-axis direction with speed $0.9c$. (a) What will be the velocity of spaceship A with respect to spaceship B? (b) What will be the velocity of spaceship A with respect to spaceship B? (c) If the spaceships are a distance D from the origin at time $t = 0$, when will they collide in the rest frame of the meeting point, in the frame of spaceship A, and in the frame of the spaceship B?

The Doppler Effect

Ex 1.19. A prominent emission of light from excited hydrogen atom occurs in the Lyman series line L_α. Light corresponding to the L_α line has a wavelength of 121.6 nm when the emitting atom has a low speed and considered to be at rest. The L_α line in the light from some star is observed to have wavelength of 486.4 nm. Suppose the star is directly moving away from Earth, what is the speed of the star with respect to Earth?

Ex 1.20. (a) Find an approximate Doppler effect formula when $v << c$ keeping terms upto linear in v/c. (b) A police parked at the side of the road sends out electromagnetic wave of frequency 5 GHz. What will be the frequency of the reflected wave from a car that is approaching the police car with a speed of 100 mph? Assume the wave is almost parallel to the velocity of the car.

Ex 1.21. If you move towards red light you would detect light shifted upward in frequency. With sufficient speed red light might turn into a green light for you. (a) With what speed would you be moving towards a red traffic light of wavelength 675 nm so that the light appears green of wavelength 510 nm to you? (b) With your speed what will be observe for the yellow light of wavelength 570 nm?

Ex 1.22. A galaxy is moving at speed of $0.9c$ with respect to Earth. (a) What will be the wavelength observed on Earth corresponding to a radio wave emitted at wavelength 21 cm from the galaxy? (b) The redshift z of a galaxy is defined by $z = \Delta\lambda/\lambda_0$, where $\Delta\lambda$ is the change in the wavelength and λ_0 is the emitted wavelength. Find the redshift of the galaxy.

Ex 1.23. The redshift z of a galaxy is defined by $z = \Delta\lambda/\lambda_0$, where $\Delta\lambda$ is the change in the wavelength and λ_0 is the emitted wavelength. A quasar is found to have a redshift of 20. Supposing the quasar is moving away from Earth, what is the speed of the quasar with respect to Earth?

Relativistic Momentum

Ex 1.24. A proton is moving with speed $0.95c$ with respect to lab. (a) What is its momentum in the lab frame? (b) What is its momentum in its rest frame?

Ex 1.25. A proton is moving with speed $0.5c$ with respect to lab. Find the percentage error you will make in calculating the momentum in the lab frame if you treat this proton by Newtonian mechanics.

Ex 1.26. A proton is moving with speed $0.01c$ with respect to lab. Find the percentage error you will make in calculating the momentum in the lab frame if you treat this proton by Newtonian mechanics.

Ex 1.27. A proton and an antiproton (same mass as proton) in a high-energy accelerator are moving towards each other. Both have speed of $0.90c$ with respect to lab. What is the momentum of proton with respect to the antiproton?

Relativistic Energy

Ex 1.28. An elementary particle called muon has a mass of 1.88×10^{-28} kg. (a) What is its rest energy in Joules? (b) What is its rest energy in MeV?

Ex 1.29. What is the kinetic energy of a proton that is moving at the speed of $0.90c$?

Ex 1.30. What is the kinetic energy of an electron that is moving at the speed of $0.90c$?

Ex 1.31. In an accelerator, a proton and an antiproton move in the opposite direction at speeds $0.9c$ with respect to the lab. Upon collision various particles are released. What is the total energy released upon collision?

Ex 1.32. A neutron moving with speed of $0.5c$ collides with a Beryllium nucleus which has four protons and five neutrons. The Be nuceus can be considered to be at rest. Upon collision, the neutron lodges itself in the nucleus. (a) What is the speed of the new particle with four protons and six neutrons? (b) Is the kinetic energy before the collision equal to the kinetic energy after the collision? (c) What is the rest mass of the particle created in the collision process?

Ex 1.33. A negative pion at rest decays into a muon and an antineutrino. You can assume that the rest mass of antineutrino is zero and that muon is 106 MeV/c^2. Upon decay the muon comes out with kinetic energy 4.5 MeV. Use conservation of energy and momentum to determine the mass of the pion.