

8.5 Einstein's Gravitation

While Slipher and Hubble were making progress in the observation of galaxies, much progress was also taking place in the theoretical physics. In 1916 Einstein published a theory of gravitation that changed our view of the world. He claimed that gravitation was related to the curvature of the spacetime, which depended on the energy and momentum of matter and fields. This gave physicists valuable new tools for studying the large scale structure of the universe since gravitation is the dominant force at large scale. Einstein's equation for gravitation, also called the **General Theory of Relativity**, can be broadly written as

$$\text{Geometry of spacetime} = G_N \times \text{Energy and momentum of matter and fields}, \quad (8.37)$$

where G_N is Newton's gravitational constant. Solution of this equation shows that the presence of mass and energy in a region changes the geometrical structure of the spacetime. Without any mass or energy the spacetime is flat, with mass and energy the spacetime becomes "curved". The notion of gravitational force on a particle is replaced by the following: a particles placed in this region will follow "straight" paths called geodesics in this curved spacetime.

How does a curved space differ from a flat space? For the purpose of illustration, consider two points A and B on a spherical surface, as in Fig 8.10. Suppose no points outside of the surface exists, i.e. no points in the volume of the spherical surface are accessible. Then, the shortest distance between A and B will be along the great circle, which is not the Euclidean straight path. The shortest path on a flat space will have the length given by Euclidean formula, but the shortest path on a curved surface will be given by some other formula. The shortest path between the two points A and B on the surface of a sphere turns out to be the arc distance on a great circle that contains the two points as shown in Fig. 8.10. The great circles on a sphere are examples of "straight paths" on a curved space and are called **geodesics**.

As a physical example, the motion of Earth around the Sun due to the gravity of the Sun is understood by Earth following a geodesic around the Sun in a curved spacetime that the presence of Sun has created as illustrated in Fig. 8.11. In this picture, the Sun is in the tip of the funnel with two types of geodesics, one pointing towards the Sun and the other going around the Sun.

8.5.1 Successes of Einstein's Theory

Einstein's theory had an important early successes. In 1900s there was a long-standing mystery about the discrepancy in the accounting of the **precession of the perihelion of mercury** which could not be understood from Newton's law of gravitation. Accurate measurements on the path of mercury had shown that the

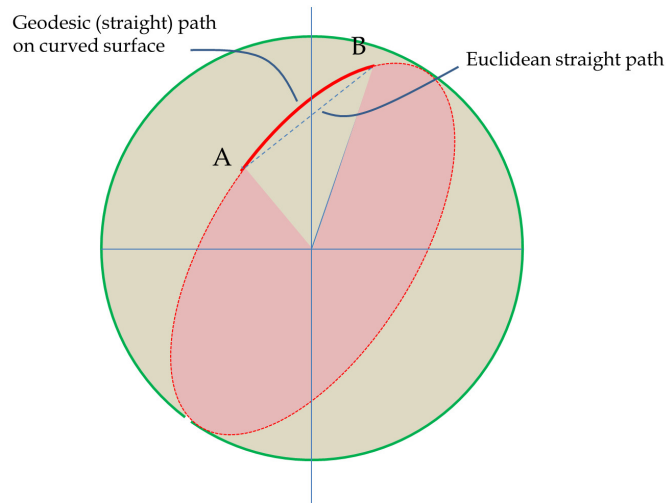


Figure 8.10: The geodesic on a curved space. Note the “straight” path on the curved surface, called the geodesic, is not the Euclidean straight path shown dashed here.

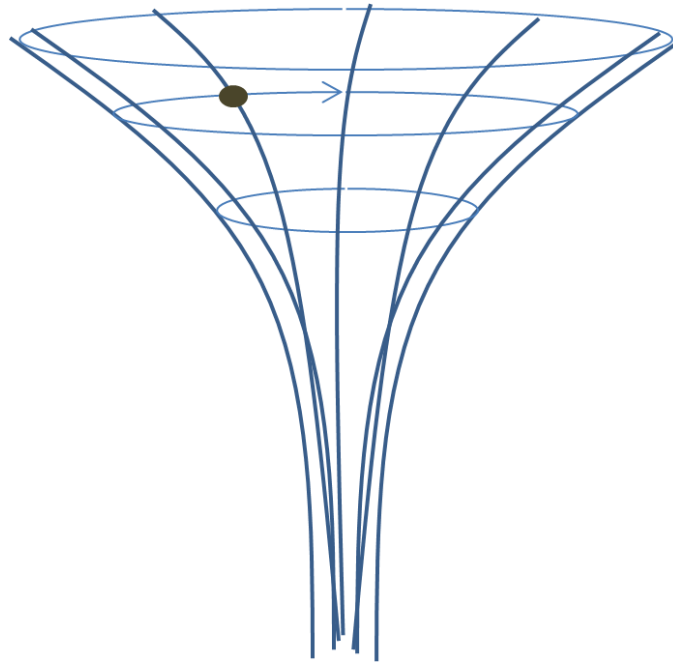


Figure 8.11: The spacetime around the Sun is curved. This figure illustrates geodesics in the curved spacetime of the Sun. The Earth moves on a geodesic of this curved spacetime.

point of the closest approach of Mercury to the Sun, the perihelion, does not occur at the same place each year. Other planets also behave this way, but the effect on mercury is quite pronounced due to its proximity to the Sun. The perihelion of mercury, moves around the Sun each year at a rate of 5,600 arcsec per century. The effects of other planets and non-inertial nature of Earth-based frame accounted for 5,557 arcsec per century. This gave a discrepancy of 43 arcsec per century. When Einstein applied his equations to the problem of the motion of mercury, he was successful in accounting for this discrepancy.

A second more popular success was the confirmation of the **bending of light by gravitation**. During the 1919 solar eclipse Arthur Eddington and Frank Dyson observed the bending of light from stars in the line of sight of the sun during the eclipse. Although the observation was of dubious quality but arguments for its interpretation based on Einstein's theory were sufficient to convince many physicists at the time. The effect is called gravitational lensing, which has now been observed in several systems. A famous one is the four image of a distant quasar, called the Einstein cross, due to **gravitational lensing** by a nearby galaxy as shown in Fig. 8.12. The gravitational lensing happens due to the bending of light from the gravitation of a star or galaxy. The light from a star behind a star can be seen due to the bending of the light from the star behind the star in the fore ground which explains multiple images of the same object.

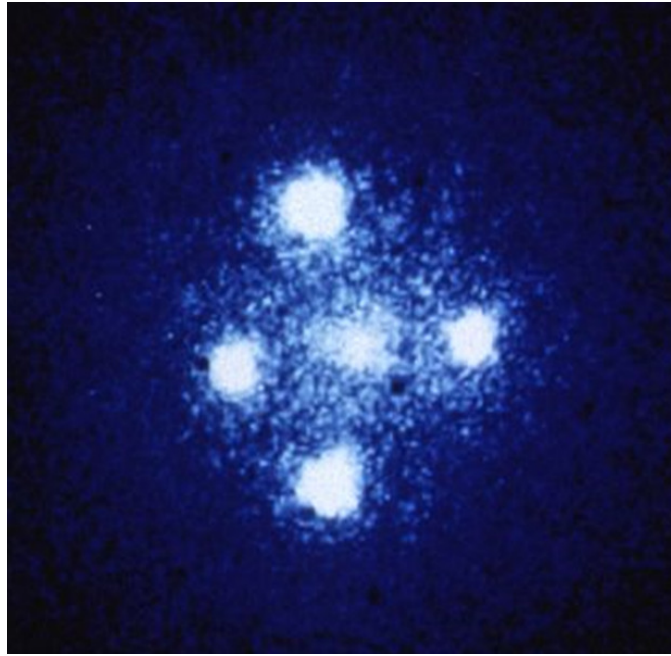


Figure 8.12: Einstein's cross: four images of the same distant quasar formed due to gravitational lensing by a nearby galaxy G2237 + 0305. Credits: European Space Agency.

8.5.2 Some Solutions of Einstein's Equations

The solutions of Einstein's equations are usually written in terms of the rule for the infinitesimal interval ds between two events, say (t, x, y, z) and $(t + dr, x + dx, y + dy, z + dz)$, where t is the time, and x, y , and z are the Cartesian coordinates.

1. No Gravity

If there is no gravity, we have a **flat spacetime**. In this case, the square of the interval ds is

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \quad (\text{flat spacetime}) \quad (8.38)$$

where c is the speed of light. You can write this also in a spherical coordinates r, θ, ϕ . We get

$$ds^2 = c^2 dt^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (\text{flat spacetime}) \quad (8.39)$$

2. Gravity of a single mass

The space outside a nonrotating mass M was found by Karl Schwarzschild, and is called the **Schwarzschild solution** or **Schwarzschild metric**. It is usually written in the spherical coordinates with the mass M at the origin.

$$ds^2 = \left(1 - \frac{r}{r_s}\right) c^2 dt^2 - \left(1 - \frac{r}{r_s}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (\text{mass at origin.}) \quad (8.40)$$

where r_s is called **Schwarzschild radius** and has the following definition.

$$r_s = \frac{2G_N M}{c^2}. \quad (8.41)$$

The Schwarzschild radius is usually very small. For instance, if $M = M_E$, the mass of the Earth, which is about 6×10^{24} kg, we get

$$r_s[\text{Earth}] = \frac{2G_N M_E}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2} = 0.009 \text{ m}.$$

The Schwarzschild metric given in Eq. 8.40 appears to go bad when $r = 0$ and $r = r_s$, called the **singularities** of the metric. Several physicists later found that the singularity at $r = r_s$ was the artefact of the choice of coordinate system and was not physical. The singularity at $r = 0$ was real and could not be removed by changing coordinates.

Example 8.8. Determine the Schwarzschild radius of the Sun and compare it to the radius of the Sun. Data: mass of the Sun = 2×10^{30} kg, radius of Sun = 7×10^8 m.

Solution.

From Eq. 8.41 we obtain

$$r_s[\text{Sun}] = \frac{2G_N M_\odot}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{(3 \times 10^8)^2} = 3,000 \text{ m}.$$

This is very small compared to the radius R of the Sun.

$$\frac{r_s}{R} = \frac{3,000 \text{ m}}{7 \times 10^8 \text{ m}} = \frac{1}{233,333}.$$