8.5 POWER

The rate at which a force does work is called the **power**. Thus, if a force $\vec{F}$ does work ΔW is done in time Δt then average power P_{ave} over that period of time is given as

$$\boxed{P_{ave} = \frac{\Delta W}{\Delta t}.} \tag{8.71}$$

If the force varies during the interval, then we will use the average force F_{ave}. Writing ΔW in terms of the average force and the displacement $\Delta\vec{r}$, we find that the average power is given by

$$P_{\text{ave}} = \frac{\vec{F}_{\text{ave}} \cdot \Delta\vec{r}}{\Delta t} = \vec{F}_{\text{ave}} \cdot \frac{\Delta\vec{r}}{\Delta t}. \tag{8.72}$$

Writing the average velocity v_{ave} for $\frac{\Delta\vec{r}}{\Delta t}$.

$$\boxed{P_{\text{ave}} = \vec{F}_{\text{ave}} \cdot \vec{v}_{\text{ave}}.} \tag{8.73}$$

Larger force and rapid displacement would correspond to more power. A person with stronger muscles normally will be able to lift weights more quickly than the same weight lifted by someone with weaker muscles. Consequently, you might say that building muscles gives you more power!

The dimensions of power can be found from the dimensions of work and time in Eq. 8.71.

$$[P] = \frac{[M][L]^2/[T]^2}{[T]} = \frac{[M][L]^2}{[T]^3}. \tag{8.74}$$

Therefore, the SI unit of power would be kg.m^2/s^3. This unit is given the name Watt (W) after James Watts who is given the credit for the invention of the steam engine. One Watt is also 1 Joule/second or J/s.

$$1\ \text{W} = 1\ \frac{\text{kg.m}^2}{\text{s}^3} = 1\ \frac{\text{J}}{\text{s}}. \tag{8.75}$$

A common unit of power in the United States is **Horsepower** (hp). One horsepower equals 746 W. Thus, an engine rated at 300 hp will deliver work at a rate of (300 hp $\times$ 746 W/hp) $= 223,800$ W, or, $223,800$ J/s.

Often, the power is used to denote the rate at which a device uses up energy. For instance, a 60-W electric bulb uses 60 Joules of energy per second, and a microwave oven rated at 1100-W uses 1100 Joules of energy per second.

If we take the limit of infinitesimal time interval in Eq. 8.72, we obtain instantaneous power $P(t)$ at instant t.

$$P(t) = \lim_{\Delta t \to 0} \left[\vec{F}_{ave} \cdot \frac{\Delta \vec{r}}{\Delta t} \right] = \vec{F} \cdot \vec{v}. \tag{8.76}$$

This gives instantaneous power as the dot product of the force $\vec{F}$ and the velocity $\vec{v}$ at instant t.

$$\boxed{P(t) = \vec{F} \cdot \vec{v}.} \tag{8.77}$$

The work done over a finite interval, then, can be obtained by integrating the instantaneous power over the time interval. Therefore, the work W_{12} over an interval from $t = t_1$ to $t = t_2$ would be

$$\boxed{W_{12} = \int_{t_1}^{t_2} P(t) dt.} \tag{8.78}$$

Example 8.5.1. Power calculation An agent applies a constant force of 5 N horizontally on a 2 kg box sliding on a smooth frictionless horizontal surface. (a) What is the instantaneous power of the agent when the velocity of the box is 2 m/s in the same direction as the force? (b) What is the work done by the agent in 15 sec?

Solution. (a) Using the formula for the power of a force given as $P = \vec{F} \cdot \vec{v}$ we find that

$$P = (5\ \text{N})(2\ \text{m/s}) \cos(0^\circ) = 10\ \text{W}.$$

(b) Since the power is constant, the work done will be $W = P\Delta t$. Putting the numerical values we find that the work done is equal to 150 N.m, or 150 J.