2.4 EXERCISES

Flux calculations

Ex 2.4.1. Find electric flux through a rectangular area 3 cm $\times$ 2 cm between two parallel plates where there is a constant electric field of 30 N/C for the following orientations of the area: (a) parallel to the plates, (b) perpendicular to the plates, and (c) the normal to the area making 30° angle with the direction of the electric field. Note that this angle can also be given as $180^\circ + 30^\circ$. Ans: (c) 0.016 N.m^2/C

Ex 2.4.2. The electric flux through a square shaped area of side 5 cm near a large charged sheet is found to be 3×10^{-5} N.m^2/C when the area is parallel to the plate. Find the charge density on the sheet. Ans: 4.43×10^{-14} C/m^2.

Ex 2.4.3. Two large rectangular aluminum plates of area 150 cm^2 face each other with a separation of 3 mm between them. The plates are charged with equal amount of opposite charges, $\pm 20\mu$C. The charges on the plates face each other. Find the flux through a circle of radius 3 cm between the plates when the normal to the circle makes an angle of 5° with a line perpendicular to the plates. Note that this angle can also be given as $180^\circ + 5^\circ$. Ans: $\pm 4.24 \times 10^5$ N.m^2/C.

Ex 2.4.4. A square surface of area 2 cm^2 is in a space of uniform electric field of magnitude 10^3 N/C. Amount of flux through it depends on how the square is oriented relative to the direction of the electric field. Find the electric flux through the square, when normal to it makes the following angles with electric field: (a) 30°, (b) 90°, and (c) 0°. Note that these angles can also be given as $180^\circ + \theta$. Ans: (a) $\frac{\sqrt{3}}{10}$ N.m^2/C

Ex 2.4.5. The y-component of the velocity in a fluid flowing in the y direction is a function of x given by $v_y(x) = v_0 \frac{x}{L}$, where v_0 and L are constants. (a) Find the volume flow per unit time through a rectangle in the yz-plane between $a < y < b$ and $c < z < d$. (b) Find the flux through a rectangle in the xz-plane between $a < x < b$ and $c < z < d$. Ans: (b)$\pm \frac{v_0}{2L}(d-c)\left(b^2 - a^2\right)$.

Ex 2.4.6. A vector field is pointed along the z-axis, $\vec{v} = \frac{\alpha}{x^2+y^2}\hat{u}_z$. (a) Find the flux of the vector field through a rectangle in the xy-plane between $a < x < b$ and $c < y < d$. (b) Do the same through a rectangle in the yz-plane between $a < z < b$ and $c < y < d$. (Leave your answer as an integral.)

Spherical Symmetry Problems

Ex 2.4.7. A non-conducting sphere of radius 3 cm has a uniform charge density of 3 nC/m^3. (a) Find the total charge contained in the sphere. (b) Find charge contained within 2 cm of its center. (c) Find the flux through a spherical surface of radius 4 cm centered at the center of the charged sphere. (d) Find the flux through a spherical surface of radius 2 cm centered at the center of the charged sphere. (e) Find the flux through a spherical surface of radius 2 cm centered about a point 6 cm from the center of the charged sphere. Ans: (a) 2.26×10^{-13} C, (b) 6.70×10^{-14} C, (c) 2.56×10^{-2} N.m^2/C, (d) 7.57×10^{-3} N.m^2/C, (e) 0.

Ex 2.4.8. Some charge are put on a copper spherical ball of radius 2 cm where the charges settle on the surface of the ball and distribute uniformly. The electric flux through a spherical surface of radius 30 cm concentric with the spherical ball is -3×10^4 N.m^2/C. (a) Find the electric flux through a 5-cm radius spherical surface concentric with the copper ball. (b) What is the electric flux through a spherical surface of radius 1 cm concentric with the copper ball? (c) What is the electric flux through a cube of side 1 cm side which has the center as the center of the ball? (d) What is the electric flux through a cube of side 4 cm that has the same center as the center of the ball. Ans: (a) -3.0×10^4 N.m^2/C.

Ex 2.4.9. When a charge is placed on a metal sphere, it ends up at the outer surface. Use this information to determine the electric field of $+3\ \mu$C charge put on a 5 cm aluminum spherical ball at the following two points in space: (a) a point 1 cm from the center of the ball (an inside point), and (b) a point 10 cm from the center of the ball (an outside point). Ans: (a) 0, (b) 2.7×10^6 N/C.

Ex 2.4.10. A gold spherical shell of radius 3 cm has a cavity of radius 2 cm. There lies a small copper ball with a charge of $+1.5$ nC at the center of the cavity. As a result, the inner surface of the gold shell has a charge -1.5 nC and the outer surface has a charge of $+1.5$ nC. (a) Determine the electric flux through the spherical surfaces of radii (i) 2.5 cm, and (ii) 4 cm concentric with the gold shell. (b) Find the value of electric field at a point on these spherical surfaces. Ans: (a) (ii) 1.69×10^2 N.m^2/C.

Cylindrical Symmetry

Ex 2.4.11. Determine if approximate cylindrical symmetry holds for the following situations. State why or why not. (a) A 300-cm long copper rod of radius 1 cm is charged with +500 nC of charge and we seek electric field at a point 5 cm from the center of the rod. (b) A 10-cm long copper rod of radius 1 cm is charged with +500 nC of charge and we seek electric field at a point 5 cm from the center of the rod. (c) A 150-cm wooden rod is glued to a 150-cm plastic rod to make a 300-cm long rod, which is then painted with a charged paint so that one obtains a uniform charge density. The radius of each rod is 1 cm, and we seek an electric field at a point that is 4 cm from the center of the rod. (d) Same rod as (c), but we seek electric field at a point that is 500-cm from the center of the rod.

Ex 2.4.12. A long silver rod of radius 3 cm has a charge of $-5\ \mu$C/cm on its surface. (a) Find the electric field at a point 5 cm from the center of the rod (an outside point). (b) Find electric field at a point 2 cm from the center of the rod (an inside point). Ans: (a) 1.8×10^8 N/C.

Ex 2.4.13. Electric field at 2 cm from the center of long copper rod of radius 1 cm has a magnitude 3 N/C and directed outward from the axis of the rod. (a) How much charge per unit length exists on the copper rod? (b) What would be the electric flux through a cube of side 5 cm situated such that the rod passes through opposite sides of the cube perpendicularly? Ans: 3.33×10^{-12} C/m.

Ex 2.4.14. At a point 3 cm from the center of a long aluminum rod of diameter 3.5 cm there is an electric field of magnitude 20 N/C pointed towards the rod. Assume that charges on the aluminum rod are at the surface only. (a) Find the electric flux through a cylindrical surface of radius 2 cm radius and height 5 cm that has the same axis as the aluminum rod. (b) Find the electric flux through a cylindrical surface of radius 4 cm radius and height 5 cm that has the same axis as the aluminum rod.

Ex 2.4.15. A long copper cylindrical shell of inner radius 2 cm and outer radius 3 cm surrounds concentrically a charged long aluminum rod of radius 1 cm with a charge density of 4 pC/m. All charges on the aluminum rod reside at its surface. The inner surface of the copper shell has exactly opposite charge to that of the aluminum rod while the outer surface of the copper shell has the same charge as the aluminum rod. Find the magnitude and direction of the electric field at points that are at the following distances from the center of the aluminum rod: (a) 0.5 cm, (b) 1.5 cm, (c) 2.5 cm, (d) 3.5 cm, (e) 7 cm.

Planar Symmetry

Ex 2.4.16. A 10 cm $\times$ 10 cm aluminum foil of 0.1 mm thickness has a charge of 20 μC that spreads on both wide side surfaces evenly. You may ignore the charges on the thin sides of the edges. (a) Find the charge density. (b) Find electric field 1 cm from the center assuming approximate planar symmetry. Ans: (a) 1.0×10^{-3} C/m^2, (b) 1.1×10^8 N/C.

Ex 2.4.17. Two 10 cm $\times$ 10 cm aluminum foil pieces of thickness 0.1 mm face each other with a separation of 5 mm. One of the foils has a charge of $+30$ μC and the other has -30 μC. (a) Find the charge density at all surfaces, i.e., on those facing each other and those facing away. (b) Find electric field between the plates near the center assuming planar symmetry.

Ex 2.4.18. A thin copper plate of radius 15 cm is charged with an unknown charge. An electric field of magnitude 10 N/C direction towards West is found at all points near the center of the plate. Determine the amount of charge on the plate and the orientation(s) of the plate consistent with the given data. Make reasonable assumptions and state those assumptions as part of your answer.

Ex 2.4.19. Two large copper plates facing each other have charge densities ± 4.0 C/m^2 on the surface facing the other plate, and zero elsewhere. Find the electric flux through a 3 cm $\times$ 4 cm rectangular area between the plates for the following orientations of the area. (a) If the area is parallel to the plates, and (b) If the area is tilted $\theta = 30^\circ$ from the parallel direction. Note, this angle can also be $\theta = 180^\circ + 30^\circ$. Ans: (a) 5.42×10^8N.m^2/C, (b) 4.69×10^8N.m^2/C.

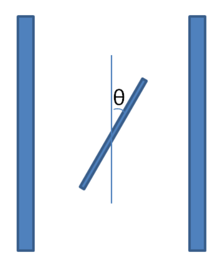

Figure 2.30: Exercise 2.4.19