5.4 EXERCISES

Interference

Ex 5.4.1. Consider two sources A and B of waves of wavelength 4 cm. The waves travel towards a screen on the positive x-axis at x = 100 cm. While the source A is fixed at x = 0, source B can be moved between x = -8 cm and x=-2 cm. Determine whether the waves are in-phase or out-of-phase when they arrive at the screen at the following positions of source B, and find the value of the phase difference assuming the waves start synchronized from their source points A and B.

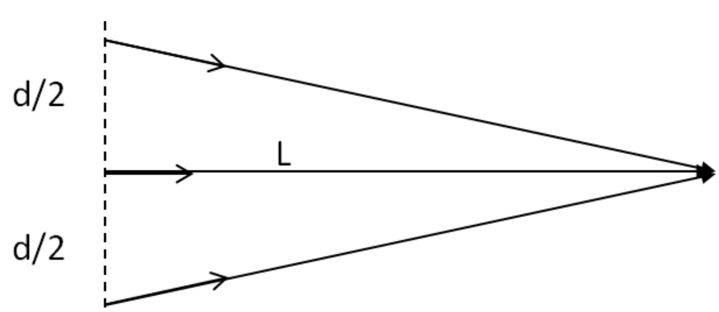
(a) x = -8 cm, (b) x = -6 cm, (c) x = -5 cm, (d) x = -4 cm, (e) x = -3 cm, (f) x = -2 cm.

Ans: (a) in-phase, (b) out-of-phase, (c) phase difference $\dfrac{\pi}{2}$ rad, (d) in-phase, (e) phase difference $\dfrac{\pi}{2}$ rad, (f) out-of-phase.

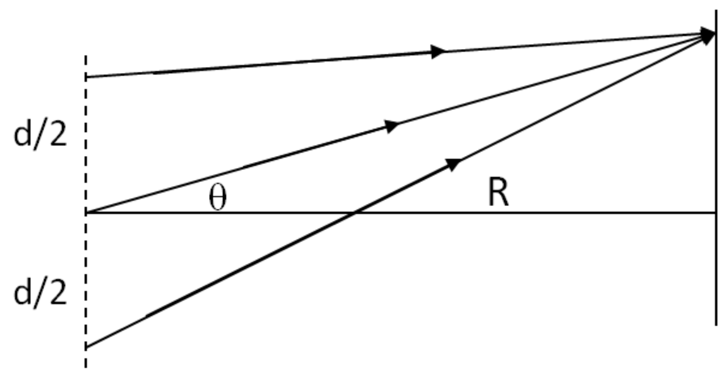
Ex 5.4.2. A laser light of wavelength 632.8 nm passes through two narrow slits separated by 10 μm and form an interference pattern on a screen 1.5 m away. (a) Find the location of three bright and two dark spots. (b) What is the maximum order of interference seen on the screen? Ans: (a) Bright at 0, $\pm 3.6°$; Dark: $\pm 1.8°$; (b) 15.

Ex 5.4.3. A laser light of wavelength 589 nm passes through two narrow slits separated by 10 μm and form an interference pattern on a screen 1.5 m away. (a) Find the location of three bright and two dark spots. (b) What is the maximum order of interference seen on the screen? Ans: (a) Bright at 0, $\pm 3.4°$; Dark: $\pm 1.7°$; (b) 16.

Ex 5.4.4. Three waves of identical wave amplitude E_0, and identical wavelength λ start in phase at the plane shown in the figure with a dashed line. The waves interfere at a screen and we wish to find a formula for the intensity on the screen in terms of the intensity in a single wave. What would be the net intensity at the symmetric point shown in the figure? Assume $L >> d$.



Ex 5.4.5. Three waves of identical wave amplitude E_0, and identical wavelength λ are in phase at the plane shown in the figure with a dashed line. Find a formula for the intensity of light at the arbitrary point in terms of angle θ, assuming $\theta << 1$ when expressed in radians. This exercise is an extension of the previous exercise.

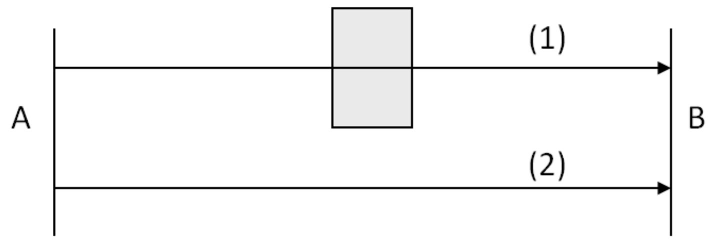


Ex 5.4.6. The sunlight is incident on an opaque board with two narrow slits separated by 250 μm. A spectrum of bands is observed on a white board screen one meter away from the slits. (a) What is the separation between the red (assume 650 nm) and blue (assume 400 nm) bands in the first order? (b) What is the separation between the red (assume 650 nm) and blue (assume 400 nm) bands in the second order? Ans: (a) $y_{\text{red}} = 2.6$ mm, $y_{\text{blue}} = 1.6$ mm; (b) $y_{\text{red}} = 5.2$ mm, $y_{\text{blue}} = 3.1$ mm.

Ex 5.4.7. When a laser light of unknown wavelength is passed through two narrow slits separated by 150 μm, an interference pattern is observed on a screen 1.5 m away. The distance between the central spot and first order constructive interference is found to be 5 mm. (a) Find the wavelength of light. (b) Where would the second order constructive interference form on the screen, if at all. Ans: (a) 500 nm, (b) 0.38°.

Double-beam interference

Ex 5.4.8. Two waves start from a coherent source. They travel the same physical distance of $\frac{1}{2}$ meter. But there is a 10 cm thick glass of refractive index 1.5 in the path of one of the waves. What is the phase difference between the two waves if the wavelength is 530 nm in the air. Use refractive index of air = 1.00? Ans: 224°. [Check this ans.]



Ex 5.4.9. A 530-nm light is incident on a double-slit of separation 5.0 μm. One of the slits is covered with a 2.0 μm thick glass plate of refractive index 1.5. Find the location of $m = 1$ constructive interference on a screen one meter away. Ans: 9.4 cm.

Ex 5.4.10. Green light of wavelength 520 nm has a constructive interference when the normally reflected light from an oil spill (refractive index 1.25) on a plastic sheet (refractive index 1.5) is observed. Find the least thickness of the oil spill. Ans: 208 nm. [check this ans.]

Ex 5.4.11. What color will a 150 nm thick soap bubble appear if illuminated by a white light at near normal? Assume a refractive index of 1.4. Ans: It will appear black.

Ex 5.4.12. A transparent plastic film has a thickness of 3 μm and a refractive index of 1.5. Find the first two angles at which a green light of wavelength 530 nm will form bright fringes.

Ex 5.4.13. A laser light of wavelength in air of 530 nm is incident on a wedge made of air between glass plates of refractive index 1.55. Find the distance along the wedge you need to move to go from one bright fringe to the next if the wedge angle is 2°. Ans: 7.6 μm.

Ex 5.4.14. What will be the radii of the tenth and twentieth bright rings formed by a laser light of wavelength 632.8 nm on a hemispherical glass piece of radius 2 cm which is placed over a flat reflecting surface with the curved part facing the flat surface? Use the air gap between the glass piece and the reflecting surface as the dielectric material for the double-beam interference. Ans: 0.347 mm, 0.365 mm.

Ex 5.4.15. A plano-convex lens made of glass of refractive index 1.55 is set on a flat surface with the curved part of the lens facing the flat surface. A light of wavelength 632.8 nm is shone on the lens from the above. One can see 60 bright rings from the center of the lens right up to the edge. What is the radius of curvature of the lens? Ans: 37.7 μm.

Interferometers

Ex 5.4.16. A Michelson's interferometer has two equal arms. A mercury light of wavelength 546 nm is used for the interferometer and stable fringes are found. One of the arms is moved by 1.5 μm. How many fringes will cross the observing field? Ans: $N = 5$.

Ex 5.4.17. In a Michelson interferometer, light of wavelength 632.8 nm from He-Ne laser is used. When one of the mirrors is moved by a distance D, 8 fringes move past the field of view. What is the value of the distance D? Ans: $D = 2.53\,\mu$m.

Ex 5.4.18. Show that the path difference between two successively transmitted waves in the Fabry-Perot interferometer is given by $2d\cos\theta$, where d is the distance between the mirrors.

Ex 5.4.19. Show that the width at half the height of I_t/I_i in the Fabry-Perot interferometer can be written as

$$f(x) = \frac{1}{1 + a\cos x}.$$

Provide explicit expressions for a and x in this formula.

Ex 5.4.20. What should be the reflectivity of the Fabry-Perot etalon so that the yellow-orange doublet of mercury, which correspond to wavelengths 576.959 nm and 579.065 nm, are just resolvable in $m = 5$ order? Ans: $R = 0.944$.

Ex 5.4.21. Sodium yellow D-lines have wavelengths 589.0 nm and 589.6 nm. What would you need in a Fabry-Perot equipment so that these two wavelengths are barely resolvable, i.e., resolved satisfying the Raleigh criterion in $m = 1$? Ans: $R = 0.9686$.

Ex 5.4.22. Hydrogen atom has two red lights in its spectrum of wavelengths 656.272 nm and 656.2852 nm. Figure out the parameters of a Fabry-Perot interferometer that you may use to resolve the two wavelengths successfully.