6.1 ELECTRIC CURRENT

The flow of charges produces an electric current just as the flow of water produces a water current. A current may be due to the movement of a single free charge, a charge attached to an atom, a molecule or some other structure, or from a collection of charges for instances electrons in a wire. To be specific, we will discuss electric currents mainly in metals caused by drifting conduction electrons. But, our results will be general, and can be applied to other sources of electric current, such as ions in an electrolytic solution, cosmic particles, etc.

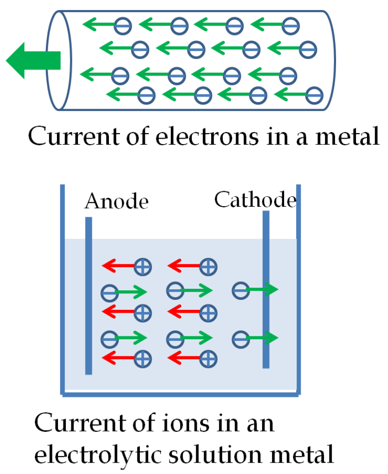

Figure 6.1: Examples of electric current. Electric current in a metal wire is carried by conduction electrons. Electric current in an electrolytic solution is carried by both positive and negatively charged ions.

Electric current is defined as the net charge flowing through the cross-section of the flow per unit time.

$$\boxed{I = \left|\frac{dq}{dt}\right| \quad \text{(through cross-section)}} \tag{6.1}$$

We ignore the sign of the charge of the conducting particles when defining the magnitude or quantity of current. The direction of current is included in the vector current defined below.

Note that symbol q in the definition of current does not refer to all charges or excess charges as was the case in previous chapters, but instead, the quantity q here refers to charges that flow in the wire, which would be the conduction electrons in metals, conducting ions in an electrolyte, conducting holes in the p-type semiconductors etc. In the case of a current carrying metal wire, the wire as a whole is electrically neutral since it has as many electrons as protons. Only a small fraction of the electrons in metals are the conduction electrons. It is important for you to note that electric current has to do with only the movement of charges and not with any excess charges which may or may not be present in a material that has a non-zero electric current.

In the SI system of units, the unit for current is Coulomb/second which is also called Ampere (A) after Andrè-Marie Ampeŕe (1775-1836) who conducted experiments with electric current and magnetism.

$$1 \text{ Ampere (A)} = 1 \frac{\text{Coulomb (C)}}{\text{second (s)}}.$$

One ampere of current in a wire refers to one Coulomb of charge per second flowing in the wire. In order to express electric current more explicitly in terms of the moving charges in the medium, it is important to examine the following three geometries separately -

current in an infinitely thin wire, current on a surface and current through a volume.

6.1.1 Current in a Thin Wire

When electrons flow in an infinitely thin wire, the cross-section is a geometrical point. Therefore, current will be charges flowing a point on the wire as shown in Fig. 6.2.

$$I = \left|\frac{dq}{dt}\right| \quad \text{(past any point)} \tag{6.2}$$

We can express the current in a wire in terms of the number density of the conduction electrons in the wire, and their overall drift speed v_d, which is the average speed with which conduction electrons flow in the wire. Suppose there are N_1 conduction electrons per unit length of the wire and let v_d be their drift speed. With the charge on one electron written as $-e$, we find that in one second a charge equal to $-N_1 e$ will pass through any point in the wire. Therefore, the current I in the wire will be

$$I = \left|\frac{dq}{dt}\right| = N_1 e v_d. \tag{6.3}$$

The vector current includes the information regarding the direction of current also. The vector current is defined by the product of the actual charge density of the flowing particles, which would be $-N_1 e$ for the electrons in the metal wire, and their drift velocity vector $\vec{v}_d$.

$$\vec{I} = -N_1 e \vec{v}_d, \tag{6.4}$$

Note that current I is in opposite direction to the flow of negatively charged electrons in the wire. The minus sign on the right side means that the current vector is in the opposite direction to the direction of the drift velocity of the electrons. This appears crazy at first, but you get used to this difference in the direction of te current vector and the direction of the flow of negatively charged particles that are responsible for the current. In p-type semiconductors, the carriers of current are positively charged pseudo-particles called holes. There the current and the drift velocity would be in the same direction.

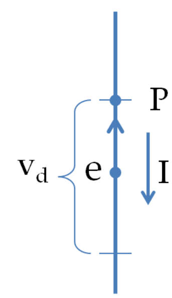

Figure 6.2: The conducting electrons within a distance v_d of point P will pass through P in one second. If there are N conduction electrons per unit length in the wire, then, current in the wire will be: $I = Nev_d$. Note that current I is in opposite direction to the flow of negatively charged electrons in the wire. The vector current includes this information naturally: $\vec{I} = -Ne\vec{v}_d$. This says that electron drift direction given by $\vec{v}_d$ is in opposite direction to the current $\vec{I}$.

Example 6.1.1. Current in a wire. A thin metal wire carries 10 A current. (a) How many electrons flow in the wire through any cross-section of the wire in 30 seconds? (b) Find the drift speed of electrons if there are 10^{22} electrons per meter mobile in the wire? (c) In a section of the wire, electrons are drifting in the direction towards the West. Which way is the current pointed?

Solution. (a) From the definition of current, the total charge flowing through any cross-section of the wire is $\Delta q = I\Delta t = (10 \text{ A}) \times (30 \text{ sec}) = 300$ C. This calculation gives charge regarless of the type. Here we have an electron flow, therefore, the charge flowing in 30 sec would be negative, that is, -300 C. We can find the number of electrons by dividing charge by the charge on one electron. This gives number of elctron flowing any cross-section in one second to be $-300 \text{ C}/\left(-1.67 \times 10^{-19} \text{ C}\right) = 1.9 \times 10^{21}$.

(b) From $|-N_1 e v_d| = |I|$ and given $N = 10^{22} \text{ m}^{-1}$, we immediately find that $v_d = 6.3 \times 10^{-3}$m/s.

(c) Since the electrons are negatively charged, the current vector would be pointed in the opposite direction to the direction of flow. Therefore, the direction of the current will be towards the East here.

6.1.2 Surface Current

Sometimes we are interested in situations where charges flow on a two-dimensional surface. This is the case, for instance, when surface electrons of a metal are driven by electromagnetic waves. The current in this case is called surface current. To define surface current, we note that charges now flow past infinitely many points across any line $L_\perp$ perpendicular to flow as shown in Fig. 6.3. The total current now will be equal to the sum of all charges that cross the line $L_\perp$ per unit time.

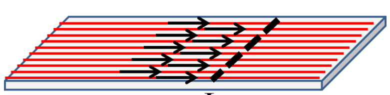

Figure 6.3: The conducting electrons within a distance v_d of line $L_\perp$ will pass through $L_\perp$ in one second. If there are N conduction electrons per unit area on the surface, then, surface current will be: $I = -eNv_dL_\perp$.

$$\boxed{I = \left|\frac{dq}{dt}\right| \quad (\text{past } L_\perp)} \tag{6.5}$$

We can also write this equation in terms of conduction electrons on the surface of the metal. Let N_2 be the number of conduction electrons per unit area and v_d their average drift speed. Then, we see that all the electrons that are in the area $v_d L_\perp$ will cross the line the imaginary line $L_\perp$ in unit time. That means $N_2 v_d L_\perp$ electrons will cross the imaginary line in unit time. With each electron carrying a charge $-e$, the net charge crossing the imaginary line in unit time, or the current is

$$\boxed{I = \left|\frac{dq}{dt}\right| = eN_2 v_d L_\perp.} \tag{6.6}$$

The current flowing on the surface of a conductor depends on the length of the imaginary $L_\perp$ here. Although the direction of the imaginary line is perpendicular to the flow, the length is arbitrary. Therefore, to capture a more intrinsic property of the flow of charges on the surface of a conductor, we define a quantity called the surface current density k by dividing out the length $L_\perp$ of the imaginary line.

$$k = \frac{I}{L_\perp} = eNv_d. \tag{6.7}$$

The vector surface current density is defined by including the information about the direction by using the sign of the charge and drift velocity in the expression above as we have done above for the current vector in an infinitely thin wire.

$$\boxed{\vec{k} = -eN_2\vec{v}_d.} \tag{6.8}$$

The net current vector $\vec{I}$ is then

$$\vec{I} = \vec{k}L_\perp. \tag{6.9}$$

6.1.3 Current Through a Volume

Finally, if charges flow through a three-dimensional volume, e.g. in a regular wire rather than an infinitely thin wire, the cross-section to the flow would be a two-dimensional surface with area $A_\perp$ as shown in Fig. 6.4. Let N_3 be the number of conduction electrons per unit

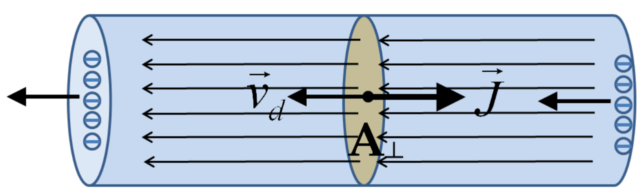

Figure 6.4: The volume current density $\vec{J}$ is defined for a system where the moving charges flow through a volume. The current in a metal wire is carried by electrons drifting throughout the cross-section of the wire. If N_3 is the number of conduction electrons per unit volume that drift with velocity $\vec{v}_d$, the current density is defined as $-eN_3\vec{v}_d$.

volume in the wire and v_d be the average drift speed of the conduction electrons. Then, all conduction electrons within a volume $v_dA_\perp$ would pass through the cross-section per unit time. Therefore, current in the wire will be

$$I = \left|\frac{dq}{dt}\right| = eN_3v_dA_\perp. \tag{6.10}$$

We define a volume current density J by dividing out the area of cross-section from the current.

$$J = \frac{I}{A_\perp} = eN_3v_d. \tag{6.11}$$

The vector volume current density is defined by including the information about direction by using the sign of the charge and drift velocity in the expression above.

$$\boxed{\vec{J} = -eN\vec{v}_d.} \tag{6.12}$$

The net current vector $\vec{I}$ is then

$$\boxed{\vec{I} = \vec{J}A_\perp.} \tag{6.13}$$

Sometimes, the current density is non-uniform through the volume. For instance, current flowing in a metal wire has more current density near the surface than near the center. The current density will then be integrated over the cross-sectional area to obtain the current. Formally, we write the relation as follows.

$$\vec{I} = \int_{A_\perp} \vec{J} dA_\perp. \tag{6.14}$$

NOTATION: In the three geometries, infinitely thin wire, surface, and volume, I have used three different symbols for the number density, viz., N_1, N_2, and N_3, respectively. In practice, however, the same symbol N is used for all three of these densities. From now on, we will use N for number densities, whose meaning in terms of N_1, N_2, and N_3 will be evident from the context. Thus, we wil have three different formulas for the current I in the system,

$$I = eNv_d, \tag{6.15}$$
$$I = eNv_dL_\perp, \tag{6.16}$$
$$I = eNV_dA_\perp, \tag{6.17}$$

and one formula for the magnitude of the surface and volume current densities: $k = eNv_d$ and $J = eNv_d$.

Example 6.1.2. Volume Current in a Thick Wire. A cylindrical copper wire of thickness 4 mm carries 12 A current. (a) Find volume current density assuming the wire to be uniform in thickness everywhere. (b) Find the drift speed of electrons in the wire. (c) What would be the drift speed if the wire carrying 12 A current was 0.04 mm thick, i.e. one-hundredth as thick?

Solution. (a) From definition, $J = I/A_\perp = 12\ \text{A}/\pi\,(o.002\ \text{m})^2 = 9.6 \times 10^5\ \text{A/m}^2$.

(b) We can figure out v_d from $J = eNv_d$ if we know the number of conduction electrons per unit volume in copper wire. Since, there is one conduction electron per atom of copper, an Avogadro number of atoms in 63 grams of copper, and the density of copper being 8.9 g/cm^3, we have the following density of conduction electrons.

$$N = 8.9\ \frac{\text{g}}{\text{cm}^3} \times \frac{6.022 \times 10^{23}}{63\ \text{g}} \times 10^6 \frac{\text{cm}^3}{\text{m}^3} = 5.6 \times 10^{30}\text{m}^{-3}.$$

Therefore, the drift speed is given by

$$v_d = \frac{J}{eN} = \frac{9.6 \times 10^5 \text{A/m}^2}{5.6 \times 10^{30}\text{m}^{-3} \times 1.6 \times 10^{-19}\text{C}} = 7.0 \times 10^{-5}\text{m/s}.$$

(c) For thinner wire, the current density will scale accordingly. For same I, the current density varies inversely with area of cross-section, which means J varies as inverse square of thickness. Therefore, if thickness is one-hundredth, then J will be 100^2 times. Therefore,

$$J = J_{\text{part(a)}} \times 100^2 = 9.6 \times 10^9 \text{A/m}^2.$$

Since the charge density is the same, the drift speed will scale with J. Therefore, the drift speed will also become 100^2 times the old value, giving the following for drift speed now.

$$v_d = 7.0 \times 10^{-5}\text{m/s} \times 100^2 = 0.7\ \text{m/s}.$$