6.6 DIFFRACTION GRATING

A material consisting of periodic reflectors or periodic transmission slits is called a diffraction grating. The diffraction gratings with periodic reflectors are more specially called **reflection gratings** and those with periodic slits are called **transmission gratings** (See Fig. 6.16).

A transmission grating can be made from a glass plate by machining fine parallel lines on it so that the space between lines serve as slits. In a transmission grating, light is incident on one side of the slits and emerges on the other side having passed through the slits (Fig. 6.16a), so that each slit serves as a new source of wave. The reflection gratings are made by etching parallel grooves on a glass or metal plate from which light can be reflected off (Fig. 6.16b). Music and computer CDs acts as reflection gratings. You will notice that white light bis separated in component colors due to diffraction off the grooves of a CD.

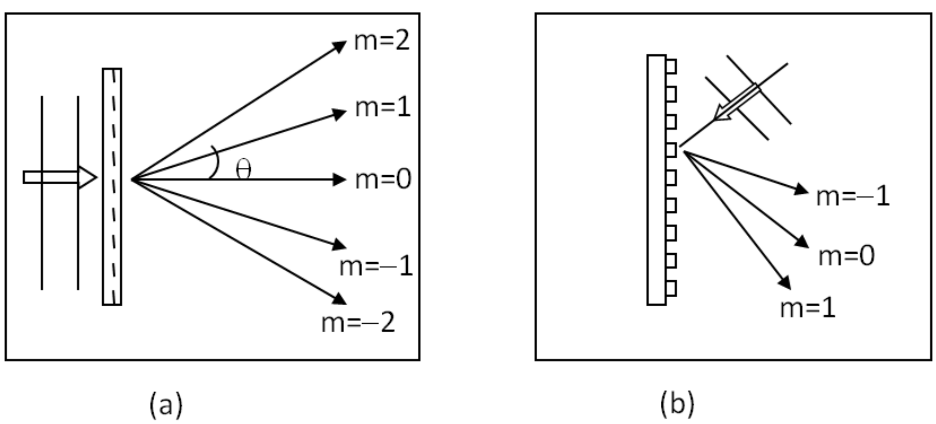

Figure 6.16: (a) Transmission grating and (b) reflection grating.

The periodic structure in the diffraction grating cause periodic phase differences in waves leaving the grating and arriving at a screen where they interfere giving rise to a pattern of bright and dark regions. In both types of diffraction gratings, the slit width is narrow so that the wave spreads out over the screen and we usually see only central maximum of diffraction. The diffraction pattern consists of peaks of the interference of light from slits. Increasing the number of slits makes the interference peaks narrower so that a diffraction grating can be used to resolve lights of nearby wavelengths. This makes diffraction grating a useful tool for spectroscopy.

Diffraction through multiple slits is understood in much the same way we have discussed diffraction through two slits above. If light is incident on N slits, each of width b, and separation distance a between the slits, then you can show that the intensity on the screen at an angle θ will be given by the following expression.

$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2 \left[\frac{\sin(N\alpha)}{N\sin\alpha}\right]^2, \tag{6.38}$$

where $I(0)$ is the intensity at $\theta = 0$, and α and β are defined as before.

$$\alpha \equiv \frac{\pi a \sin\theta}{\lambda} = \frac{ka\sin\theta}{2}$$
$$\beta \equiv \frac{\pi b \sin\theta}{\lambda} = \frac{kb\sin\theta}{2}$$

The parameters α and β control the effect of the interference of waves from adjacent slits and the diffraction from secondary wavelets within the same slit respectively. In the case of narrow slits, i.e. when the slit widths are much smaller than the wavelength, we can assume $\beta << 1$ and make the approximation $\sin\beta \approx \beta$, which gives $\sin\beta/\beta \approx 1$. In these circumstances, one finds only the central maximum for the diffraction peak, the resultant intensity is an interference pattern modulated by the envelop of the central peak of the single-slit diffraction.

The diffraction pattern from multiple slits consists of major and minor peaks. The major peaks, also called **principal peaks**, occur where the waves from all slits interfere constructively while at the sites of **minor peaks** only some of the waves interfere constructively. The positions of the principal peaks are therefore easily obtained by considering the interference of only the adjacent slits, which is same as the two-slit interference condition worked out before.

Principal maxima:

$$a\sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \cdots. \tag{6.39}$$

The principal maxima are sharp and narrow for a diffraction grating. To understand this, consider the interference of waves near a principal maximum. At the principal maximum waves from all slits interfere constructively. At a nearby point on the screen, the waves from two adjacent slits will be slightly out-of-phase, but waves from slits further away will be more and more out of phase so that for every slit there is some other slit whose waves are completely out of phase thus canceling the waves.

For instance, suppose you look at a point P on the screen where the difference in path between the adjacent slits is 1% of a wavelength.

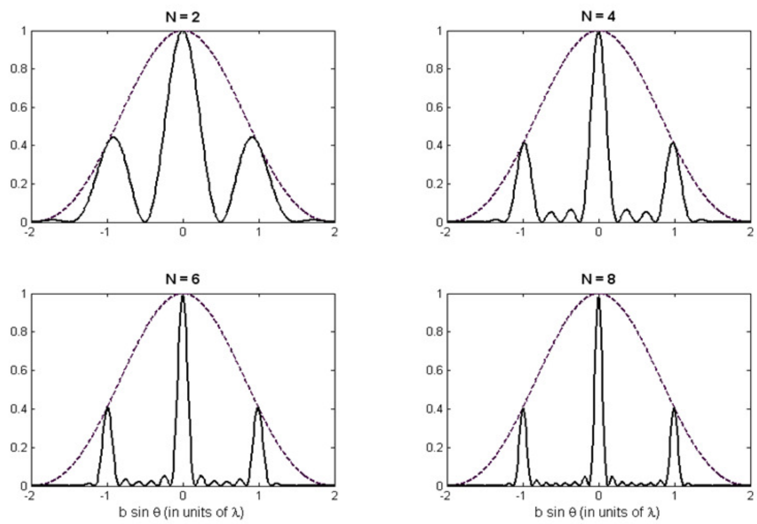

Figure 6.17: Multiple slits diffraction. Here slit width $b = \lambda/2$ and distance between slits $a = 2b$. The number of slits N as shown in each plot. The envelop is the diffraction of a single slit.

Then slit # 1 and slit number #50 will be completely out-of phase there. Thus slit #1 and slit #50 will cancel each other out at P, and similarly slit #2 will cancel slit #51, and so on. Hence, the slightest place away from a principal maximum leads to a very low intensity, and hence the principal maxima peaks are sharper and narrower.

The narrow peaks of diffraction grating occupy little space on the screen, and since the location of principal maxima depend on the wavelength, a grating can be used to separate light of two different wavelengths. If you have more than one wavelength in the incident light, principal peak at $m = 0$ will have all the wavelengths on one central bright fringe, but all other orders ($m \neq 0$) will have different peaks for different wavelengths. Spectroscopes use diffraction gratings to analyze wavelengths of light precisely.

In Fig. 6.17, diffraction patterns of a monochromatic light incident across N-slits of a transmission grating for various values for N are shown. You can easily notice that the peaks become sharper with increasing N.

By visiting the website given in Fig. 6.18, you can see the diffraction pattern for a laser light of wavelength 632.8 nm passed through a diffraction grating that has N slits per mm (not given in the picture in the web site). You can see that the maxima are sharp and occur

Visit the following website for pictures.

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/grating.html#c1

Figure 6.18: Picture of diffraction pattern from a diffraction grating of 632.8-nm Neon laser light projected on a plastic screen.

at specific spots. A diffraction grating with larger number of slits per mm, e.g. $2N$ slits per mm, results in larger separation between maxima on the screen same distance away. Can you figure out why that should be the case?

Example 6.6.1. Diffraction grating for spectroscopy. Find the separation in angles of a light of wavelength 550 nm and 575 nm for the first order ($m = 1$) principal peak of the two waves from a diffraction grating with 800 lines per mm.

Solution. Since there are 800 lines per mm, the distance between adjacent slits is

$$a = \frac{1 \text{ mm}}{800} = 1.25\ \mu\text{m}.$$

Hence, the directions for $m = 1$ principal peaks corresponding to the two wavelengths in the problem are

$$\theta_{550} = \sin^{-1}\left(\frac{0.550\ \mu\text{m}}{1.25\ \mu\text{m}}\right) = 26.1^\circ$$
$$\theta_{575} = \sin^{-1}\left(\frac{0.575\ \mu\text{m}}{1.25\ \mu\text{m}}\right) = 27.4^\circ$$

Therefore,

$$\theta_{575} - \theta_{550} = 1.3^\circ.$$