9.5 LINEAR AND NONLINEAR MAGNETIC MEDIA

9.5.1 Linear Media - Paramagnetic And Diamagnetic Material

In paramagnetic and diamagnetic materials magnetization $\vec{M}$ is proportional to the field if the field is not too great. When this happens we call the medium a linear medium. Customarily, the proportionality is expressed between $\vec{M}$ and $\vec{H}$ rather than between $\vec{M}$ and $\vec{B}$.

$$\vec{M} = \chi_m \vec{H}, \tag{9.72}$$

where the constant of proportionality χ_m (letter chi, pronounced as kaai) is called the volume **magnetic susceptibility**. Magnetic susceptibility is a dimensionless quantity and measures how strong a magnet a material will be when it is placed in a given magnetic field. Substances that have positive susceptibility will magnetization in the same direction as the applied magnetic field and are called **paramagnets**. Substances that have negative susceptibility are magnetized with magnetization in the opposite direction to the magnetic field and are called **diamagnets** (see Table 9.1).

Standard tables often have a related parameter called molar magnetic susceptibility χ_M which is defined by multiplying the volume susceptibility with the molar volume V_m, i.e. volume per unit mole. There is additional complication when using standard table, such as one finds in the CRC handbook on chemistry and physics. The values of molar susceptibilities are listed in the CGS units. Since electromagnetic formulas are quite different in different units, there appears a factor of 4π in the conversion of volume susceptibility from CGS to SI, even though it is a dimensionless number.

$$\chi_m = 4\pi \chi_M V_m \tag{9.73}$$

In Table 9-1 both molar and volume susceptibilities are given for easy reference. Putting Eq. 9.72 into Eq. 9.54 of last section, we can obtain the relation between $\vec{B}$ and $\vec{H}$ for a linear medium.

$$\vec{B} = \mu_0 \left(\vec{M} + \vec{H} \right) = \mu_0 \left(1 + \chi_m\right) \vec{H}. \tag{9.74}$$

The constant of proportionality between $\vec{B}$ and $\vec{H}$ is called the permeability μ of the linear medium.

$$\mu = \mu_0 \left(1 + \chi_m\right) \quad \text{(Beware: This } \mu \text{ is not dipole moment!!)} \tag{9.75}$$

The magnetic field in a linear magnetic material is simply given by replacing μ_0 by μ of the medium in the formulas. Thus, magnetic field $\vec{B}$ inside a solenoid with n turns per unit length, current I in the wires and the space filled with a linear magnetic material of magnetic permeability μ will be

$$|\vec{B}| = \mu n I \quad \text{(inside)}. \tag{9.76}$$

Table 9.1: Magnetic susceptibilities (at $20°C$ and atmospheric pressure) Source: CRC Handbook of Chemistry and Physics, 82nd Edition, 2001-2002 (CRC Press, Boca Raton). Note that table in CRC is given in CGS units, while we need susceptibility in SI units for our formulas in the present book obtained from the former by multiplying with 4π.

Material	Molar susceptibility χ_M $(\times 10^{-6} cm^3/mole)$	Molar mass M $(g/mole)$	Density ρ (g/cm^3)	Volume Susceptbility χ_m $(\times 10^{-5})$
Diamagnets				
Bismuth	-280.1	209	9.79	-16.49
Carbon (graphite)	-6	12	2.2	-1.38
Carbon (Diamond)	-5.9	12	3.513	-2.17
Copper	-5.46	63.5	8.96	-0.97
Gold	-28	197	19.3	-3.45
Lead	-23	207	11.3	-1.58
Mercury	-33.5	201	13.5	-2.83
Silver	-19.5	108	10.4	-2.36
Paramagnets				
Aluminum	16.5	27	2.7	2.07
Gadolinium	185000	157	7.9	11,697.93
Magnesium	13.1	24	1.74	1.19
Manganese	511	55	7.3	85.23
Neodymium	5930	144	7.01	362.76
Oxygen (gas)	3449	32	29.3	3,968.45
Platinum	193	195	21.5	26.74
Sodium	16	23	0.97	0.85
Titanium	151	48	4.506	17.81
Tungsten	53	184	19.3	6.99

9.5.2 Non-Linear Media - Ferromagnetic Materials; Hysteresis

Iron, nickel, cobalt and some of the rare earths (e.g. gadolinium, dysprosium) exhibit a unique magnetic behavior - they retain magnetism even when the external magnetic field is no longer acting as long as the temperature is below a transition temperature called the Curie point. Hence, magnetization M is not proportional to H in ferromagnets.

$$\vec{M} \neq \chi_m \vec{H} \quad \text{(Ferromagnets)} \tag{9.77}$$

If magnetization $\vec{M}$ were proportional to $\vec{H}$, it cannot be non-zero when $\vec{H}$ is zero. These materials are called ferromagnets and they constitute non-linear media. The full relation between $\vec{B}$, $\vec{M}$ and $\vec{H}$ is still valid and so is the Ampere's law of $\vec{H}$. The only trouble now is that $\vec{M}$ has to be determined from experiment, which depends on the history of the sample as we will see below.

Equations for Ferromagnets:

$$\begin{aligned} \vec{B} &= \mu_0 \left(\vec{M} + \vec{H} \right) \\ \oint \vec{H} \cdot d\vec{l} &= I^f_{enc} \end{aligned} \tag{9.78}$$

Ferromagnets lose magnetism if you heat the sample beyond certain characteristic temperature called the Curie point. The Curie points of some common magnets are shown in the Table 9.2. Beyond Curie point ferromagnets are often paramagnetic. While gadolinium is not ferromagnetic at room temperature (298 K), Cobalt and iron retain their ferromagnetic property up to very high temperatures.

Table 9.2: Magnetic properties of common ferromagnets Source: CRC Handbook of Chemistry and Physics, 82nd Edition, 2001-2002.

Material	Saturation magnetization (B per cm^3) $\times 10^{23}$	Number of Bohr magnetons per atom	Curie Point (K)
Cobalt	1.56	1.72	1388
Gadylonium	2.31	7.63	293
Iron	1.88	2.22	1043
Nickel	0.557	0.61	627
Neodymium compound, $Nd_2Fe_{14}B$			583

The name ferromagnetism comes from the most common substance iron which is also called ferric. In recent times we have the

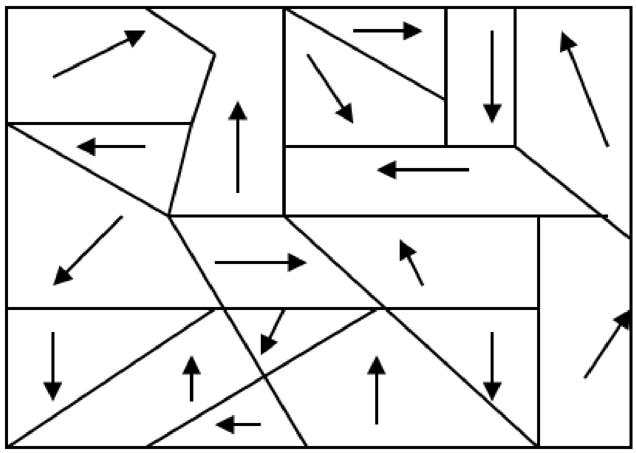

Figure 9.14: Schematic drawing of domains inside an unmagnetized ferromagnetic material.

emergence of very powerful and light weight rare-earth magnets such as neodymium materials $Nd_2Fe_{14}B$ that produce strong magnetic fields and have a Curie point of 583K. Due to the lightweight of the rare earth magnets, they have found uses in microphones, motors and other applications. What is the reason behind ferromagnetism? Due to the inter-atomic forces between magnetic dipoles of unpaired electron spins, they tend to align with each other. As a result domains are formed inside the material where all spins are line up in one direction. The domains are microscopic in size and can be seen under a microscope by special etching methods. In an unmagnetized sample, there are as many domains pointed in one direction as in the opposite direction so that overall the sample is non-magnetic as shown in Fig. 9.14.

When an unmagnetized sample of a ferromagnet is put in an external magnetic field, spins in the domains that are most closely parallel to the external magnetic field line up with the external magnetic field and their boundaries grow while the others shrink resulting in an alignment of all domains and hence a magnetized material. Magnetization of a ferromagnetic material also depends on the history of the sample. For instance consider magnetizing an unmagnetized iron rod. You can do so by wrapping an wire around it and passing a current through the wire that creates an external magnetic field in the iron rod. As we increase the current, thereby creating increased external magnetic field, iron becomes more and more magnetized until a saturation point is reached when all the spins are lined up after which increasing external magnetic field does not increase the magnetization. What will happen if you reduce the current now? As you reduce the current below what is needed for the saturation, iron rod does not trace back to zero magnetization when there is no external magnetic field but retains magnetization even when we turn off the current at point b (Figure 9.15).

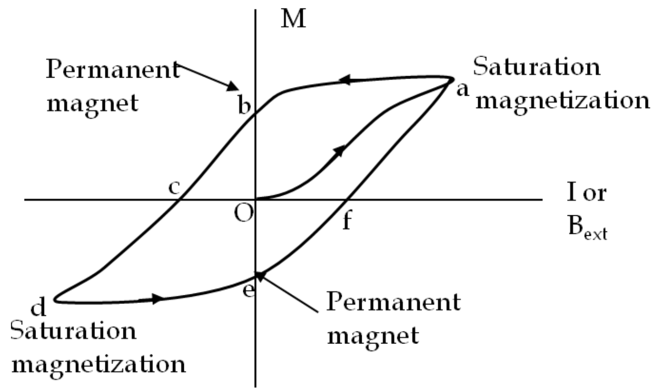

Figure 9.15: The Hysteresis loop in magnetization of a ferromagnet.

If we reverse the current direction the dipoles start to flip in the opposite direction and reach a point where the net magnetization is zero (point c) even when magnetic field on the sample is not zero. Continuing to increase current in this direction, we discover another saturation point (point d). Consider reducing the current back to zero. Once again, magnetization does not retrace its path, but instead when the current is zero, this time (point e) magnetization points in the opposite direction to point b. Now sending the current in the original direction takes magnetization in the rod on path from (e) to (f) to saturation point (a). The loop a-b-c-d-e-f-a is called a **Hysteresis loop**. Clearly, the magnetization of the iron rod depends not only on the magnetic field but also on the history.