

## 9.6 EXERCISES

### Magnetic dipole in a magnetic field

**Ex 9.6.1.** A magnetic dipole of magnitude  $9.27 \times 10^{-24} \text{ A.m}^2$  is placed in a magnetic field of magnitude 3 T. The dipole makes an angle of  $30^\circ$  with the magnetic field. Determine the torque on the dipole and the energy to flip the direction of the dipole by  $180^\circ$ .

**Ex 9.6.2.** A dipole experiences a torque of magnitude  $5 \times 10^{-20} \text{ N.m}$  when it is pointed  $90^\circ$  to the direction of an external magnetic field of magnitude 4 T. (a) Determine the magnitude of magnetic dipole moment. (b) If the dipole was as a result of a circulating current in a loop of radius 1 mm, what would be the current in the loop? Ans: (a)  $\mu = 1.25 \times 10^{-20} \text{ A.m}^2$ , (b)  $I = 4.0 \times 10^{-15} \text{ A}$ .

**Ex 9.6.3.** Near a bar magnet magnetic field is not uniform. A magnet of magnetic dipole moment magnitude  $3 \times 10^{-22} \text{ A.m}$  is brought near the end of a bar magnet where the magnetic field component along the dipole's direction changes at the rate of 30 T/m. Determine the force on the dipole. Ans:  $9.0 \times 10^{-21} \text{ N}$ .

**Ex 9.6.4.** Determine the energy needed to flip the dipole of moment of magnitude  $10^{-3} \text{ A.m}^2$  in a field of 2 T from being opposite to the field to being fully aligned. Ans:  $6.0 \times 10^{-3} \text{ J}$ .

**Ex 9.6.5.** Determine the energy needed to flip the dipole of moment  $10^{-3} \text{ A.m}^2$  in a field of 2 T from being  $160^\circ$  direction to the field to  $20^\circ$  direction. Ans:  $-3.8 \times 10^{-3} \text{ J}$ .

**Ex 9.6.6.** A dipole of moment  $2 \text{ A.m}^2$  is pointed in along  $x$  axis direction in a region of space where magnetic field is given as follows:

$$\vec{B} = \mu_0 I_0 \left[ -\frac{y}{x^2 + y^2} \hat{u}_x + \frac{x}{x^2 + y^2} \hat{u}_y \right],$$

where  $x$  and  $y$  are in  $m$ . Find the magnetic force on the dipole when  $x = 0.3 \text{ m}$  and  $y = 0.4 \text{ m}$ . Ans: 7.68 N.

**Ex 9.6.7.** A dipole of moment  $3 \times 10^{-10} \text{ A.m}^2$  is placed inside a long solenoid of 2000 turns per meter such that dipole is directed perpendicular to the axis of the solenoid. Determine the force and torque on the dipole when a 8 A current is passed through the solenoid. Ans:  $|\vec{\tau}| = 6 \times 10^{-12} \text{ N.m}$ .

### Microscopic view of magnetic material

**Ex 9.6.8.** Determine the energy needed to flip the magnetic moment of a silver atom in 3.5 T external magnetic field. A silver atom has

one unpaired electron in  $l = 0$  state. Ans:  $\Delta U = 2.78 \times 10^{-23}$  J.

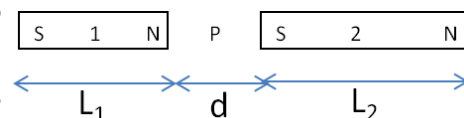
**Ex 9.6.9.** Find the magnetic moment of an atom that has two unpaired electrons. One electron has  $m = +1$  and spin up, and the other electron has  $m=0$  and spin up. Ans:  $\Delta U = 3.71 \times 10^{-23}$  J.

**Ex 9.6.10.** Find the possible magnetic moments of an electron in an atom that has angular momentum of  $l = 3\hbar$ .

**Ex 9.6.11.** In a hydrogen molecule, the nuclear magnetic moments of two protons may point in the opposite direction or in the same direction. When the nuclear magnetic moments of hydrogen atoms point in the opposite direction, the molecule is called para-hydrogen, and they point in the same direction it is ortho-hydrogen. When hydrogen is placed in a magnetic field, the energy of the two types of hydrogen molecules are different. Find the difference in energy of ortho- and para-hydrogen molecules when the field is 4 T if the dipoles of the ortho-hydrogen are aligned with the external field.

## Magnetic field of a magnet

**Ex 9.6.12.** Find the magnitude and direction of magnetic field midway between the poles of two bar magnets of magnetization  $M_1$  and  $M_2$  per unit length placed as shown. Assume the lengths of the magnets to be  $L_1$  and  $L_2$ , and separation to be  $d$ .



**Ex 9.6.13.** Five hundred magnetic dipoles of moment 3 A.m<sup>2</sup> each are placed uniformly in a circle of radius 2 cm. Find the magnitude and direction of magnetic field at the center (a) if the dipoles are pointed towards the origin, and (b) if the dipoles are aligned perpendicular to the plane of the circle. Ans: (b) Magnitude: 19 T.

**Ex 9.6.14.** A thin ring of radius  $R$  is made of nickel, and magnetized in the direction perpendicular to the ring. Determine the magnitude and direction of magnetic field for a uniform magnetization  $\vec{M}$  (magnetic dipole moment per unit length) of the ring. Ans: Magnitude:  $\mu_0 M / 2R^2$ .

**Ex 9.6.15.** A thin annular plate of inner radius  $R_1$ , and outer radius  $R_2$  is made of iron and magnetized perpendicular to its surface. Determine the magnitude and direction of magnetic field at the center for a magnetization  $\vec{M}$  (dipole moment per unit face area) of the thin plate. Ans: Magnitude:  $\frac{\mu_0 M}{2} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$ .

**Ex 9.6.16.** A small sphere of radius  $a$  is carved out at the center of uniformly magnetized sphere of radius  $R$  that has a magnetization  $\vec{M}$ . Find magnitude and direction of magnetic field (a) at the center, (b) at a point on the axis that is a distance  $2R$  from the center, and (c) at a point on the axis that is between  $a$  and  $R$  from the center.

## Linear and non-linear media

**Ex 9.6.17.** A long copper rod of radius  $R$  is carrying a current  $I$  uniformly distributed over the cross-section. Find the magnitude and direction of the auxiliary field, magnetization and magnetic field inside and outside the wire. Ans: Magnitudes at a point a distance  $r$  from center :  $H_{\text{in}} = Jr/2$ ,  $M = \chi_m Jr/2$ ,  $B_{\text{in}} = \mu_0(1 + \chi_m) Jr/2$ . Here  $J = I/\pi R^2$ .

**Ex 9.6.18.** A solenoid is constructed by wounding wires over a long platinum rod. For a winding of 3000 turns per meter and a current of 14 A through the wires, find the magnetization of platinum. Ans: Magnitude: 11.2 A/m, Direction: same as the magnetic field.

**Ex 9.6.19.** Thin insulated wires run parallel in a water tank. There are 2500 wires per meter, each carrying the same current of 8 A. Find the magnetization of water. Ans: Magnitude: 900 A/m, Direction: Opposite to the magnetic field.

**Ex 9.6.20.** Thin insulated wires are placed with a density of 5000 wires per meter parallel on a plate of copper, and an aluminum plate is placed over the wires. Find the magnetization of copper and aluminum when 25 A current is passes through each wire. Ans: Magnitudes:  $M(\text{Cu}) = 60$  A/m,  $M(\text{Al}) = 129$  A/m, Opposite directions.

**Ex 9.6.21.** A long tungsten cylindrical wire of thickness 2 mm is surrounded by oxygen gas at 1 atm and  $20^\circ\text{C}$  in a glass tube. Determine the magnetization of oxygen when 3 A current is passed though the wires, and the magnetic field at a point in space occupied by oxygen gas. Ans: Magnitude  $\frac{1.91 \times 10^{-2}}{r}$  A/m where  $r$  is in meter.