2.5 PROBLEMS

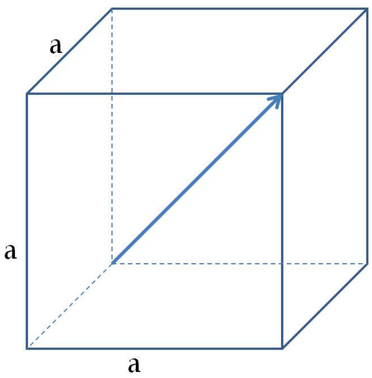

Figure 2.38: Problem 2.5.1.

Problem 2.5.1. Find a unit vector that is perpendicular to the body diagonal of a cube shown in the figure.

Problem 2.5.2. Find the angle between the lines from the center of one face to the corners on the opposite face of a cube.

Problem 2.5.3. Two body diagonals of a cube cross at the center of the cube. Find the angle between them.

Problem 2.5.4. Three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are placed on the adjacent edges of a parallelepiped with their tails at the common vertex. Prove that $\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)$ is equal to the volume of the parallelepiped.

Problem 2.5.5. Prove the following identity for arbitrary vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$:

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\vec{B}\cdot\left(\vec{C}\times\vec{A}\right)=\vec{C}\cdot\left(\vec{A}\times\vec{B}\right).$$

Problem 2.5.6. Prove that, if the magnitude of the two vectors $\vec{A}$ and $\vec{B}$ are equal, then $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are perpendicular to each other.

Problem 2.5.7. Prove that, if $|\vec{A}-\vec{B}|=|\vec{A}+\vec{B}|$, then vector $\vec{A}$ is perpendicular to vector $\vec{B}$.

Problem 2.5.8. Suppose $\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{C}$. Is $\vec{B}=\vec{C}$? Why or why not? Give a graphical interpretation of the given statement also.

Problem 2.5.9. Suppose $\vec{A}\times\vec{B}=\vec{A}\times\vec{C}$. Is $\vec{B}=\vec{C}$? Why or why not? What is the most we can say about $\vec{B}$ and $\vec{C}$? Give a graphical interpretation of the given statement also.

Problem 2.5.10. Suppose $\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{C}$ and $\vec{A}\times\vec{B}=\vec{A}\times\vec{C}$. Is $\vec{B}=\vec{C}$? Why or why not?

Problem 2.5.11. Prove the following identity for vector cosines, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$, where α, β, and γ are angles a vector makes with the positive x, y and z-axes respectively.

Problem 2.5.12. We proved the law of cosines in the text using the scalar product of vectors placed on the sides of a triangle. Prove the law of sines by using the vector product between vectors on the sides of a triangle. The law of sine says that if the sides of a triangle have lengths A, B, and C and the angles opposite to the sides are $\angle A$, $\angle B$, and $\angle C$ respectively, then

$$\frac{\sin\angle A}{A}=\frac{\sin\angle B}{B}=\frac{\sin\angle C}{C}$$

Problem 2.5.13. Prove that an arbitrary vector $\vec{A}$ can always be written as a sum of a vector parallel to a vector $\vec{B}$ and a vector perpendicular to $\vec{B}$. That is, show that

$$\vec{A} = a\vec{B} + b\vec{B}_{\perp},$$

where $\vec{B} \cdot \vec{B}_{\perp} = 0$, and a and b are some scalars. Hint: Use the projection of $\vec{A}$ on $\vec{B}$ to construct the vector you need for the vector parallel to $\vec{B}$. Let $\vec{A}_1$ be the vector parallel to $\vec{B}$ that you need. Then, show that $\vec{A} - \vec{A}_1$ is a vector that is perpendicular to vector $\vec{B}$.