4.8 EXERCISES

Fundamentals of electromagnetic waves

Ex 4.8.1. The electric field of an electromagnetic wave in vacuum is given by the following vector function of space and time, where y is in meters and t is in seconds.

$$\vec{E} = \hat{u}_x 3 \times 10^5 (\text{N/C}) \ \cos\left(2\pi \times 10^{15}\ t + ky + \frac{\pi}{2}\right)$$

(a) Evaluate the frequency, wave number and wavelength of the wave.

(b) What is the direction of the travel of the wave?

(c) What are the directions of the electric and magnetic fields of the wave at the origin at $t = 0$?

(d) What are the directions of the electric and magnetic fields of the wave at the origin at $t = 0.25 \times 10^{-15}$ seconds?

(e) What is the amplitude of the magnetic field wave at following places and instants?

  (i) $x = z = 0,\ y = 0.03$ m, $t = 0$.

  (ii) $x = z = 0,\ y = 0.045$ m, $t = 0.05$ ns.

(f) Plot the electric field versus y-coordinate of space at following instants on the same graph.

  (i) $t = 0$.

  (ii) $t = 0.25 \times 10^{-15}$ sec.

  (iii) $t = 0.5 \times 10^{-15}$ sec.

  (iv) $t = 0.75 \times 10^{-15}$ sec.

Ex 4.8.2. Prove the following integrals.

(a) $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} \sin^2\theta d\theta = \frac{1}{2}$; (b) $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} \cos^2\theta d\theta = \frac{1}{2}$;

(c) $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} \sin^2(\theta+2) d\theta = \frac{1}{2}$; (d) $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} \cos^2(\theta+3) d\theta = \frac{1}{2}$;

(e) $\dfrac{1}{\pi}\displaystyle\int_0^{\pi} \sin^2(2\theta) d\theta = \frac{1}{2}$; (f) $\dfrac{1}{\pi/5}\displaystyle\int_0^{\pi/5} \cos^2(10\ \theta) d\theta = \frac{1}{2}$;

(g) $\dfrac{1}{1/f}\displaystyle\int_0^{\pi} \sin^2(2\pi f\theta) d\theta = \frac{1}{2}$; (h) $\dfrac{1}{1/f}\displaystyle\int_0^{\pi/5} \cos^2(2\pi f\theta + g) d\theta = \frac{1}{2}$;

Ex 4.8.3. A spherical wave from a point source at the origin has an electric field that drops with the radial distance r from the origin as follows.

$$\vec{E} = \hat{u}_\phi \frac{E_0}{r} \cos(kr - 2\pi f t)$$

where $E_0 = 5 \times 10^4$ N/C, $f = 2 \times 10^{14}$ Hz, and k is the wave number. Assume the speed of light to be equal to the speed in vacuum.

(a) Find the wavelength (λ) of the electromagnetic wave.

(b) Which way is the electric field pointed at $r = 1$ cm at $t = 0$?

(c) What is the intensity of the source at the origin?

(d) What is the intensity of light at $r = 1$ cm?

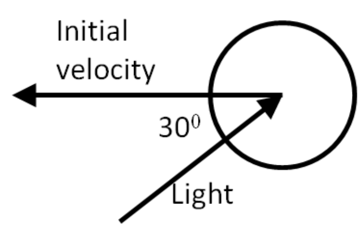
(e) What is the intensity at $r = 2$ cm?

(f) Plot the wave function at $t = 0$ from $r = \lambda$ to $r = 5\lambda$.

Ex 4.8.4. A monochromatic light source radiate 10 W of light isotropically. (a) What is the intensity of light 1 m from it? (b) What is the intensity of light at 2 m? Ans: (a) 0.8 W/m^2; (b) 0.2 W/m^2.

Ex 4.8.5. A microscopic spherical dust particle of radius 2 μm and mass 10 μg is moving in the outer space at a constant speed of 30 cm/sec. A wave of light strikes the particle from the opposite direction of its motion, and gets absorbed. Assuming the particle decelerates uniformly to zero speed in one second, what is the average electric field amplitude in the light? Ans: 7.35×10^6 N/C.

Ex 4.8.6. In the problem above, suppose light strikes the particle at 30 degrees to the initial velocity as shown in the figure. What will be the particle's velocity (direction and magnitude) after 2 s? Assume complete absorption.



Ans: With initial velocity towards the negative x-axis, we get $v_x = -0.135$ m/s and $v_y = 0.0954$ m/s.

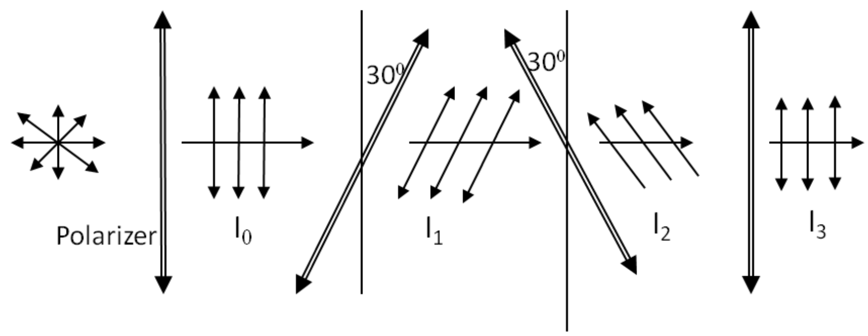
Ex 4.8.7. Radiation from the Sun has the intensity of 1.38 kW/m^2 at the surface of the Earth. (a)What is the intensity light at the surface of the Sun? (b) Compare the force by the radiation pressure on the Earth with the gravitational attraction of the Sun, assuming complete absorption of light by the Earth? Ans: (a) $64,000\,\text{kW/m}^2$.

Ex 4.8.8. A Styrofoam spherical ball of radius 2 mm and mass 20 μg is to be suspended by radiation pressure in a vacuum tube in a earth-based lab. How much intensity will be needed if completely reflected by the ball? Ans: 2.34×10^6 W/m^2 .

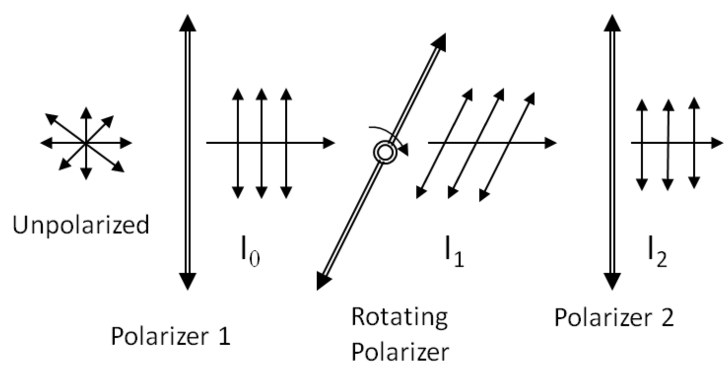
Polarization of light

Ex 4.8.9. Find the Brewster's angle of a ray incident from water on a water/glass interface. Use $\frac{4}{3}$ and $\frac{3}{2}$ as refractive indices for water and glass respectively. Ans: 48.4°.

Ex 4.8.10. Use Malus's law to find the intensities I_1, I_2, and I_3 in terms of I_0. Ans: $I_1 = 0.75\, I_0$, $I_2 = 0.19\, I_0$, $I_3 = 0.14\, I_0$.



Ex 4.8.11. A polarizer is rotated at an angular speed between two identical stationary crossed polarizers. Find the intensity of the emerging wave in terms of the intensity I_0 of the light after the first fixed polarizer. Ans: $I_2 = I_0\ \cos^4(\omega\, t)$.



Ex 4.8.12. Explain why is the numerator of the coefficient of the reflection for a wave polarized perpendicular to the plane of incidence is not zero?

Ex 4.8.13. (a) Express the Fresnel equations for the coefficients of transmission in the two cases in terms of the incidence angle. (b) Plot for air/water interface. (c) Is there an angle at which either of the two coefficients of transmission disappears? Hint: Express $\theta_2 = \sin^{-1}(n_1 \sin\, \theta_1/n_2)$.

Ex 4.8.14. Plot the following time-varying vectors in the xy-plane at $t = 0$, 0.2, 0.4, 0.6 and 0.8 in units of the period of the cosine or sine functions given. State the direction of rotation of the electric field vectors when observed from the positive z-axis.

(a) $\hat{u}_x \cos(2\pi t)$
(b) $\hat{u}_y \sin(2\pi t)$
(c) $\hat{u}_x \cos(2\pi t) + \hat{u}_y \sin(2\pi t)$
(d) $\hat{u}_x \cos(2\pi t) - \hat{u}_y \sin(2\pi t)$

Ex 4.8.15. A circularly polarized light has the following electric field.

$$\vec{E} = E_0 \left[\hat{u}_x \cos\left(2\pi f t - \frac{2\pi}{\lambda} z\right) + \hat{u}_y \sin\left(2\pi f t - \frac{2\pi}{\lambda} z\right) \right]$$

The wave is passed through a linear polarizer whose polarizing axis is pointed 45° counterclockwise to the x-axis. Find the intensity of the emergent wave in terms of the intensity of the incident wave.