11.5 PROBLEMS

Problem 11.5.1. A 500-pF capacitor is charged fully by a 12-V battery, and then connected to a 125-mH inductor at $t = 0$. Assume the resistance in the circuit to be negligible. Determine (a) the frequency of oscillation, (b) the current as a function of time, (c) the maximum energy stored in the capacitor, (d) the maximum energy stored in the magnetic field of the inductor, and (e) the charge on the capacitor when the current in the inductor is half the maximum. Ans: (a) $20,132\,\text{Hz}$, (b) $760\,\mu\text{A}\,\sin(126500\,t)$, (c) $36\,\text{nJ}$, (d) same as (c), and (e) $6\,\text{nC}$.

Problem 11.5.2. Recall the equivalence of mechanical variables of a mass attached to spring to the electrical variables of an electric circuit, and deduce the mechanical equivalence corresponding to the circuits given in Fig. 11.18.

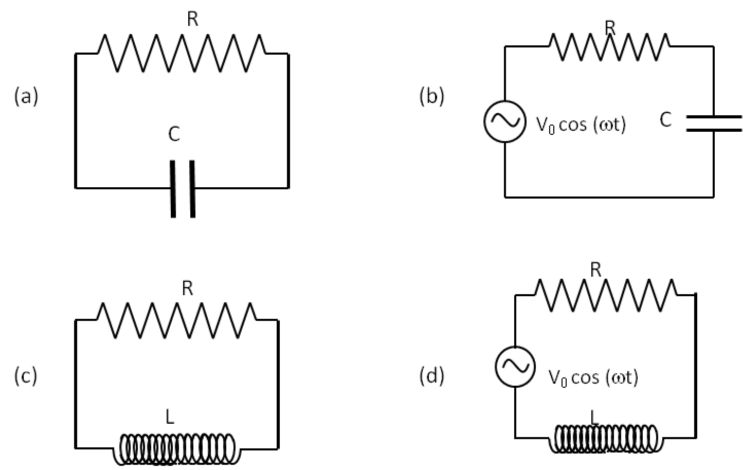

Figure 11.18: Problem 11.5.2.

Problem 11.5.3. Deduce the mechanical equivalence corresponding to the circuits in Fig. 11.19 and find the frequency of oscillation in each case.

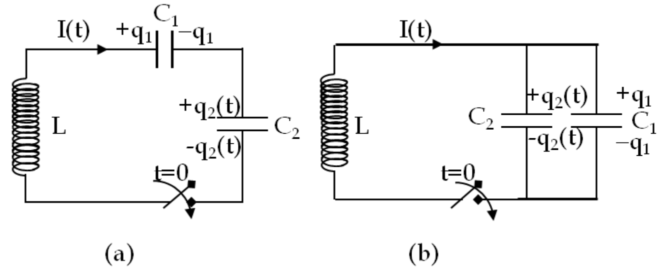

Figure 11.19: Problem 11.5.3.

Problem 11.5.4. A mass m is attached to two springs of spring constants k_1 and k_2 as shown in Figures 11.20(a) and (b). Answer the following questions. (a) Write equations of motion of mass m for each case. (b) Based on the equations of motion, find the corresponding electric circuits that will be governed by similar equations. (c) What is the frequency of oscillation of each circuit?

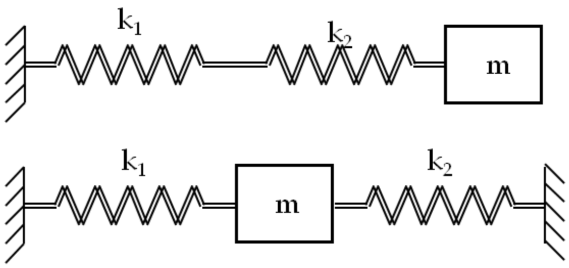

Figure 11.20: Problem 11.5.4.

Problem 11.5.5. A spherical ball of radius 2 mm and mass 0.1 g is attached to a "massless" spring of spring constant 10 N/m in a liquid of viscosity, $\eta = 1.5$ poise (1 poise $= 0.1$ N.s/m^2) such that Stoke's Law ($F_{\text{visc}} = 6\pi\eta R v$) applies. (Density of the fluid = 1.3 g/cc).

(a) Write down the Newton's equation of motion of the particle's center of mass. (b) Draw an equivalent electrical circuit that yields the same equation for a charge on an appropriate capacitor. (c) Is the oscillator, under-damped, critically-damped, or over-damped? Show, how you decided this. (d) Use the analogous electric circuit to find the velocity of the ball at $t = 0.3$ sec if at $t = 0$ the velocity was zero, and the spring was stretched by 0.1 m. (e) What is the value of the Q-factor of this oscillator? (f) If we were to drive the oscillator by a driving force, what will be the driving frequency so that we will see the resonance of the power dissipation?

Ans: (a) Hint: Damped oscillator, (b) Hint: Use electrical-mechanical analogy, (c) underdamped, (d) Hint: Use current in the electrical analogue, (d) 11.2, (e) 50.3 Hz.

Problem 11.5.6. In a damped oscillating circuit the energy is dissipated in the resistor. The Q-factor is a measure of the persistence of the oscillator against the dissipative loss. a. Prove that for a lightly damped circuit the energy E in the circuit decreases according to the following equation.

$$\frac{dE}{dt} = -2\,\beta\,E, \text{ where } \beta = \frac{R}{2L}.$$

(b) Using the definition of the Q-factor as energy divided by the loss over next cycle, prove that Q-factor of a lightly damped oscillator as defined in this problem is

$$Q \equiv \frac{E_{\text{begin}}}{\Delta E_{\text{one cycle}}} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Hint for (b): to obtain Q divide E at the beginning of one cycle by the change ΔE over the next cycle.

Problem 11.5.7. A series RLC-circuit is driven by a sinusoidal EMF. Find the width of the power resonance curve at half the height. Ans: R/L.

Problem 11.5.8. A series RLC circuit with $R = 4$ kΩ, C= 0.3 pF, and $H = 2$ pH is driven at the frequency of the current resonance. How should the resistance in the circuit be adjusted so as to sharpen the width of the current resonance to half the original width? Ans: From 4 kΩ to 2 kΩ.

Problem 11.5.9. A series RLC circuit for tuning into an AM radio station. The circuit has a variable resistor of resistance between 2 Ω and 2000 Ω, a variable capacitor whose capacitance could be varied between 0.1 pF and 100 pF, and a fixed inductance of 4 pH. What settings of the resistor and capacitor would you need so that the circuit would resonate at (i.e. tunes into) a 450 kHz signal? Ans: $C = 31.3\,\text{mF}$. Hint for R: think width of the peak.

Problem 11.5.10. (a) In the series RLC circuit connected to a sinusoidally varying EMF source, find the amplitude of the induced EMF as a function of time. (b) What is the resonance frequency of the induced EMF? Ans: (a) $\dfrac{\omega L V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$, (b) $\omega_R = \dfrac{\sqrt{2}}{\sqrt{2LC - R^2C^2}}$.

Problem 11.5.11. (a) In the series RLC circuit connected to a sinusoidally varying EMF source, find the amplitude of the voltage V_C across the capacitor as a function of time. (b) What is the resonance frequency of V_C? (c) Using results of the problem 11.5.10, find the frequency at which the magnitude of V_C would equal the magnitude of the induced EMF. What is the significance of this frequency?