12.8 EXERCISES

Elementary AC circuits

Ex 12.8.1. Find reactances of the following capacitors and inductors in AC circuits with the given frequencies in each case.

(a) 2-mH inductor with a frequency 60-Hz of the AC circuit. [Ans 0.754 Ω.]

(b) 2-mH inductor with a frequency 600-Hz of the AC circuit.

(c) 20-mH inductor with a frequency 6-Hz of the AC circuit.

(d) 20-mH inductor with a frequency 60-Hz of the AC circuit.

(e) 2-mF capacitor with a frequency 60-Hz of the AC circuit.

(f) 2-mF capacitor with a frequency 600-Hz of the AC circuit. [Ans 1.33 Ω.]

(g) 20-mF capacitor with a frequency 6-Hz of the AC circuit.

(h) 20-mF capacitor with a frequency 60-Hz of the AC circuit.

Ex 12.8.2. A 50-mH inductor with internal resistance 0.2 Ω is connected across the terminals of a 5-V AC source (i.e. Vrms = 5 V) of frequency 60 Hz. Find

(a) the reactance of the inductor,

(b) the impedance of the circuit, [Ans: approx. 18.9 Ω.]

(c) the current amplitude,

(d) the phase constant for current,[Ans: approx. -89°.]

(e) the real power, [Ans: 1.33 VA.]

(f) the power factor,

(g) the apparent power. [Ans: 0.0133 W.]

Ex 12.8.3. A 200-μF capacitor is connected across the terminals of 15-V AC source (i.e. Vrms = 15 V) of frequency 66 Hz. Assume negligible resistance in the circuit. Find

(a) the reactance of the capacitor

(b) the impedance of the circuit,

(c) the current amplitude,

(d) the phase constant for current,

(e) the real power,

(f) the power factor, and

(g) the apparent power.

Ex 12.8.4. A 20-μF capacitor is connected across a 10-V AC source of variable frequency f. Find the current amplitude in the circuit for the following values of f. Assume negligible resistance.

(a) 1 Hz [Ans: 7960 Ω, 1 mA.]
(b) 10 HZ
(c) 100 Hz
(d) 1 kHz
(e) 10 kHz
(f) 100 kHz
(g) 1 MHz, and
(h) 1 GHz

Ex 12.8.5. A 150-μF capacitor is connected in series to a 2-kΩ resistor and a 3-V AC source (i.e. $V\text{rms} = 3$ V) of variable frequency ω rad/sec.

(a) Plot the RMS-voltage across the capacitor versus ω.

(b) Plot the RMS-voltage across the resistor versus ω.

(c) Plot power delivered to the resistor versus ω.

Ex 12.8.6. A 5-H inductor of negligible internal resistance is connected across a 2-kΩ resistor and a 5-V AC source (i.e. $V\text{rms} = 5$ V) of variable frequency ω rad/sec.

(a) Plot voltage across the inductor versus ω.

(b) Plot voltage across the resistor versus ω.

(c) Plot power delivered to the resistor versus ω.

Ex 12.8.7. A starter for a fluorescent light bulb is a capacitor. When an AC voltage 170 V $\cos(2\pi\times 60\times t)$ is applied across the capacitor, a peak current of 1.2 A flows in the capacitor. What is the capacitance? Ans: 18.7 μF.

Ex 12.8.8. Suppose the voltage source for a series RL-circuit were given as $V_0 \sin(\omega t)$ instead of cosine worked out above. Find the expression for current amplitude and phase constant.

Ex 12.8.9. A 300-Ω and 0.2-H inductors connected in series to an AC source. The voltage across the resistor is found to be (30 V) $\cos(200t)$ where t is time in seconds.

(a) Find the amplitude and phase of the current in the circuit.

(b) Find the amplitude and phase of the voltage across the inductor.

(c) Find the amplitude and phase of the voltage of the source.

Ans: (a) 100 mA, 0°, (b) 4 V, 90°, (c) 30.3 V, 7.6°.

Ex 12.8.10. A 4-H inductor is connected to a 25-mF inductor in series with a 150-V source whose frequency is twice the resonance frequency. Assume negligible resistance in the circuit.

(a) Find the impedance of the circuit.

(b) What are phase relations among the voltages across the inductor and capacitor, and the source voltage.

Ans: (a) 19 Ω, (b) 180° more than the phase of the voltage drop across the capacitor.

Ex 12.8.11. A 20-Ω resistor, 50-μF, and 30-mH inductor are connected in series with an AC source of amplitude 10 V and frequency 125 Hz.

(a) What is the impedance of the circuit?

(b) What is the amplitude of the current in the circuit?

(c) What is the phase constant of the current? Is it leading or lagging the source voltage?

(d) Write voltage drops across resistor, inductor and capacitor, and the source voltage as a function of time.

(e) What is the power factor of the circuit?

(f) How much energy is used up by the resistor in 2.5 s?

Ans: (a) 20 Ω, (b) 0.5 A, (c) 5.4°, (d) With $V_{\text{Source}} = 10.0\ \text{V}\cos(250\pi t)$, $V_R = 9.96\ \text{V}\cos(250\pi t + 5.4^\circ)$, $V_C = 12.7\ \text{V}\cos(250\pi t + 5.4^\circ - 90^\circ)$, and $V_L = 11.8\ \text{V}\cos(250\pi t + 5.4^\circ + 90^\circ)$, (e) 0.995, (f) 6.25 J.

Ex 12.8.12. A 200-Ω resistor, 150-μF, and 2.5-H inductor are connected in series with an AC source of amplitude 10 V and variable angular frequency ω.

(a) What is the value of the resonance frequency, ω_R?

(b) What is the amplitude of the current if $\omega = \omega_R$?

(c) What is the phase constant of the current when $\omega = \omega_R$? Is it leading or lagging the source voltage?

(d) Write voltage drops across resistor, inductor and capacitor, and the source voltage as a function of time when $\omega = \omega_R$.

(e) What is the power factor of the circuit when $\omega = \omega_R$?

(f) How much energy is used up by the resistor in 2.5 s when $\omega = \omega_R$?

Ans:

(a) $\omega_R = 51.6\,\text{sec}^{-1}$.

(b) 50 mA.

(c) $\phi = 0$.

(d)

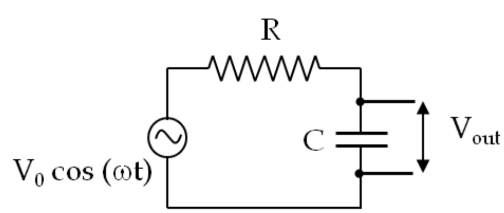
$$\begin{aligned}
V_R(t) &= (10\text{ V})\ \cos(51.6\,t)\\
V_L(t) &= (6.45\text{ V})\ \cos\left(51.6\,t + \frac{\pi}{2}\right)\\
V_C(t) &= (6.45\text{ V})\ \cos\left(51.6\,t - \frac{\pi}{2}\right)\\
V(t) &= (10\text{ V})\ \cos\left(51.6\,t\right)
\end{aligned}$$

(e) $\cos\phi = 1$.

(f) 0.625 J.

Ex 12.8.13. A **low-pass filter** is a circuit that lets out low-frequency electrical signals while suppressing high-frequency signals.

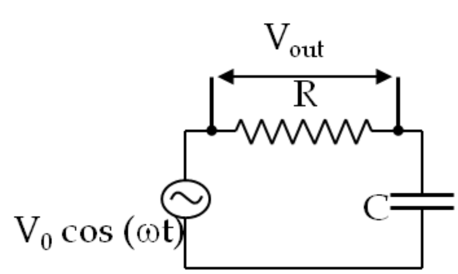
A series RC-circuit with an AC signal source can serve as a low pass filter if the output voltage is taken across the capacitor, the input voltage being the voltage of the source.



Consider a circuit containing a 200-Ω resistor and a 50-μF capacitor in series with a 18-V AC source of variable angular frequency ω. Determine the RMS-voltage across the capacitor (V_C) as a function of frequency, and plot log V_C versus log ω.

Ex 12.8.14. A **high-pass filter** is a circuit that lets out low-frequency electrical signals while suppressing high-frequency signals.

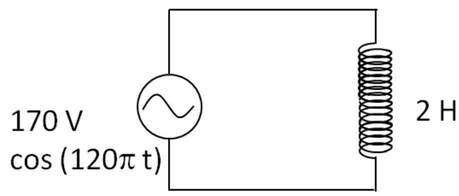
A series RC-circuit with an AC signal source can serve as a low pass filter if the output voltage is taken across the resistor, the input voltage being the voltage of the source.



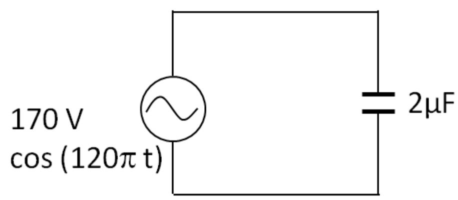
Consider a circuit containing 200-Ω resistor and 50-μF capacitor in series with a 18-V AC source of variable angular frequency ω. Determine the voltage across the capacitor (V_R) as a function of frequency, and plot log V_R versus log ω.

Applications of complex numbers to AC circuits

Ex 12.8.15. In the circuit shown in the figure, find: (a) the current in the circuit, (b) the phase angle between the current in the circuit and the EMF, and (c) average power. Assume $R = 0$ in the circuit. Ans: (a) (225 mA) $\cos(\omega t - \pi/2)$, (b) Phase given in (a), (c) 0.



Ex 12.8.16. In the circuit shown in the figure, find the following. (a) the current in the circuit, (b) the phase angle between the current in the circuit and the EMF, and (c) average power. Assume $R = 0$ in the circuit. Ans: (a) $I_0 \cos\left(\omega t + \frac{\pi}{2}\right)$ with $I_0 = 128$mA, (b) given in (a), (c) 0.



Ex 12.8.17. A 200-Ω resistor and 150-μF capacitor are connected in series across a 120-V (rms) AC power source oscillating at 60-Hz frequency.

(a) Find the amplitude and phase of current in the circuit, assuming phase of source voltage to be zero at $t = 0$.

(b) Find the voltage drops across the resistor and capacitor.

(c) Find the impedance of the circuit.

(d) Find the power dissipated in the resistor.

(e) Find the power dissipated in the capacitor.

(f) Find the power produced by the source.

Ans: Hint: Note the peak voltage $V_0 = \sqrt{2}V_{\text{rms}} = 169$ V. Hint: Use complex method with $\tilde{Z} = \tilde{Z}_R + \tilde{Z}_C$.

(a) $I = (0.845\text{ A})\ \cos(\omega t + 0.088)$.

(b) $V_R = (169\text{ V})\ \cos(\omega t + 0.088)$.

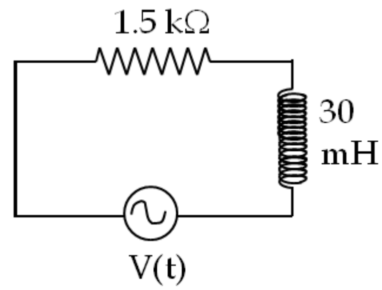
(c) $200.8\,\Omega$.

(d) $P_R = 71.4\,\mathrm{W}$

(e) 0.

(f) $P = 71.4\,\mathrm{W}$.

Ex 12.8.18. A 1.5-kΩ resistor and 30-mH inductor are connected in series across a 120-V (rms) AC power source oscillating at 60-Hz frequency.



(a) Find the current in the circuit.

(b) Find the voltage drops across the resistor and inductor.

(c) Find the impedance of the circuit.

(d) Find the power dissipated in the resistor.

(e) Find the power dissipated in the inductor.

(f) Find the power produced by the source.

Ans:

(a) $0.113\ \mathrm{A}\cos(2\pi \times 60t - 0.0075)$ when $V(t) = (120\sqrt{2}\ \mathrm{V})\cos(2\pi \times 60t)$.

(b) $V_R = (169\mathrm{V})\cos(\omega t - 0.0075)$, $V_L = (1.28\mathrm{V})\cos(\omega t - 0.0075 + \frac{\pi}{2})$.

(c) $1500\,\Omega$.

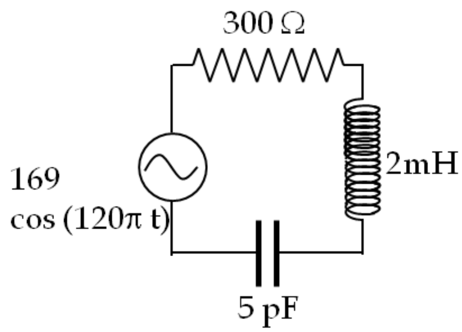
(d) 9.6 W.

(e) 0.

(f) 9.6 W.

Ex 12.8.19. In the circuit shown in the figure, find the following. (a) the current in the circuit, (b) phase angle between the current in the circuit and the voltage drop across the resistor,

(c) phase angle between the current in the circuit and the voltage drop across the inductor, (d) phase angle between the current in the circuit and the voltage drop across the capacitor, (e) the power factor, (f) real impedance of the circuit, and (g) average power dissipated in the circuit.



Ans:

(a) 320 nA $\cos(2\pi \times 60t + \pi/2)$.

(b) 0.

(c) $\angle_V$ - $\angle_I = \pi/2$.

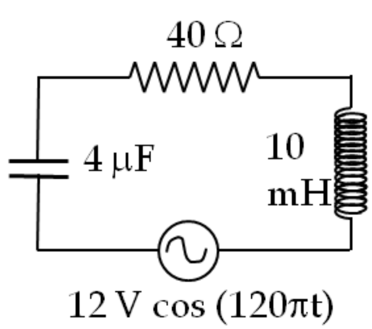
(d) $\angle_V$ - $\angle_I = -\pi/2$.

(e) Approx. 0.

(f) 15 pJ.

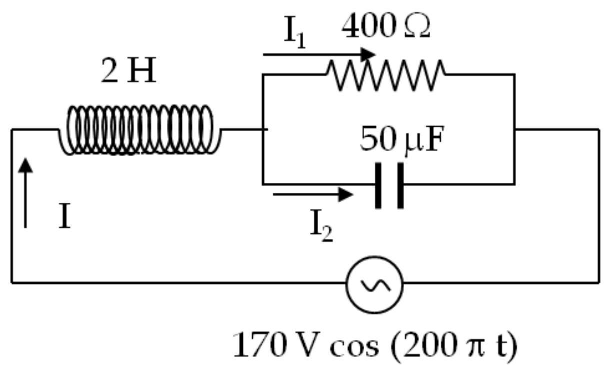
Ex 12.8.20. Consider the circuit given in the figure and find the following.

(a) the current in the circuit. (b) the voltage drops across the resistor, capacitor and inductor. (c) the impedance of the circuit. (d) the power dissipated in the resistor. (e) the power dissipated in the capacitor. (f) the power dissipated in the inductor. (g) the power produced by the source.

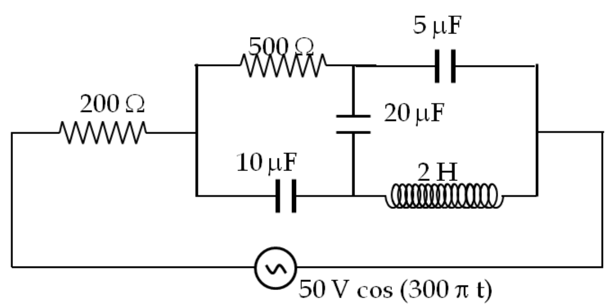


Hint: Replace the resistor, the capacitor and the inductor by their complex impedances.

Ex 12.8.21. Determine current in each branch, power dissipated in the resistor, and power produced by the source. Hint: replace the resistor, the capacitor and the inductor by their complex impedances. Answer check: overall current $I = 0.139\text{ A} \cos(200\pi t - 1.51)$.



Ex 12.8.22. Determine current in each branch, power dissipated in the resistors, and power produced by the source



Hint: This is a bridge circuit. Replace the circuit elements by their complex impedances.

Transformers

Ex 12.8.23. The output voltage and input currents of a 700-W transformer are 9-V (rms) and 2 A (rms). (a) What are the input voltage and output current? (b) What are the ratio or primary turns to secondary turns? (c) Is this a step-up or step-down transformer? Ans: (a) 350 V, (b) 39.

Ex 12.8.24. A hairdryer is rated to work in a 220-V AC line (i.e. $V_{\text{rms}} = 220$ V). It operates at 1000 W power. To connect the hair dryer to 120-V AC line, it has to be connected through a step-up transformer that will convert the 120-V line to yield a 220-V output to which the hair-dryer will be connected.

(a) What must be the turn ratio between the primary and the secondary? Which one must have higher turns? (b) Determine the current that the hair dryer will draw from the 120-V line.

Ex 12.8.25. Many electronic devices work on 12-V DC. To get 12-

V DC from a 120-V AC line (i.e. $V_{\rm rms} = 120$ V), first the voltage is dropped to 12-V AC (i.e. $V_{\rm rms} = 12$ V) by using a transformer, and then the 12-V AC is converted to 12-V DC by using diodes in a rectifier circuit. Consider a speaker of resistance 8 Ω connected to the 12-V DC output of the rectifier.

(a) What must be the ratio of turns in the primary to secondary of the transformer? (b) How much power is delivered to the device? (c) Assuming no power loss anywhere else, what is the rms-current in the secondary?

Ans: (a) 10:1, (b) 18 W, (c) 1.5 A.

Ex 12.8.26. Consider a power plant located 25 km outside a town delivering 50 MW of power to the town. The transmission lines are made of aluminum cables of 7 cm^2 cross-section area. Find the loss of power in the transmission lines if it is transmitted at (a) 200 kV (rms) and (b) 120 V (rms). Ans: (a) 12.6 W and (b) All.

Ex 12.8.27. Neon signs require 12-kV for their operation. A transformer is to be used to change the voltage from 220-V (rms) AC to 12-kV (rms) AC.

(a) What must the ratio of turns in the secondary to the turns in the primary? (b) What is the maximum rms current the neon lamps can draw if the fuse in the primary goes off at 0.5 A? (c) How much power is used by the neon sign when it is drawing the maximum current allowed by the fuse in the primary?

Ans: (a) $600 : 11$, (b) 9.17 mA, (c) 110 W.