1.8 THERMAL EXPANSION

Most things expand when heated and contract when cooled although there are some materials that behave anomalously. The response varies among different materials depending on their coefficient of expansion and temperature. It is important to study the thermal expansion and contraction properties of materials in order to design mechanical devices that will provide the desired performance in a given range of temperature. In this section we will study how to characterize the thermal expansion.

1.8.1 Length Expansion

When a solid bar is heated its length expands. In almost all materials, the change in length ΔL is proportional to the change in temperature ΔT as well as the original length of the bar L_0.

$$\boxed{\Delta L = \alpha L_0 \Delta T,} \tag{1.24}$$

where the proportionality constant α is called the **coefficient of linear expansion** and the bar is free to expand or contract. Note that linear dependence of the change in the length on the change in the temperature may fail if the change in temperature is too large. The coefficient of linear expansion usually depends on the temperature and the tension in the bar. The coefficient of linear expansion of some common materials are listed in Table 1.5.

An important application of the thermal expansion is a **bimetallic strip**, which is used to measure the temperature. In a bimetallic strip, two strips made up of different metals such as brass and steel are welded at the ends (Fig. 1.15). When the bimetallic strip is heated the two metals expand to different extent, which causes the bimetal strip to bend since the ends are fixed. The direction of bending can be calibrated to give readings of temperature.

A common approach is to enhance the effect so that a more sensitive equipment for measuring temperature can be constructed is to wind a bimetallic strip as a helix on a support. The winding lets one use a very long bimetallic wire so that the twists along the wire can be magnified. As bimetal changes temperature the helix winds and unwinds which is displayed on an analog dial or converted to a digital output (Fig. 1.16).

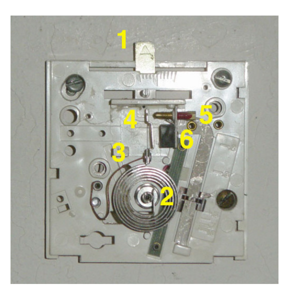

Figure 1.16: A bimetal coil in a thermostat.

Example 1.8.1. Rail Track Joints. Rail tracks are joined together with a gap left for allowing the expansion in hot weather and/or

Table 1.5: Coefficient of linear expansion of common materials at 298 K (CRC Handbook of Chemistry and Physics, 82nd Edition and Kaye and Laby Online, 2006.)

Substance	$\alpha(10^{-6}/^\circ\text{C})$
Metals and alloys	
Aluminum	23.1
Brass	20.3
Copper	16.5
Lead	28.9
Steel	11.7
Common solids	
Cement/Concrete	3-10
Glass (crown)	7-8
Glass (Pyrex)	2.8
Graphite	7.1
Granite	7-14
Invar	0.13
(64Fe/36Ni alloy)	
Quartz	3-6

Figure 1.17: Example 1.8.1.

heating of the rails from the friction. How much gap should be left between two 20 m tracks made of steel if it is laid at 20°C and is to operate up to a temperature of 45°C?

Solution. Each track will expand half of the gap on each side. Hence the total expansion will the equal to the gap. We use the physics of linear expansion to obtain the required gap assuming the heating by friction to be ignorable for this problem.

$$\begin{aligned} \text{Gap} &= \Delta L = \alpha L_0 \Delta T \\ &= 11.7 \times 10^{-6}\text{C}^{-1} \times 20\text{m} \times 25^\circ\text{C} = 0.59 \text{ cm}. \end{aligned}$$

1.8.2 Area Expansion

When a solid plate is heated, both its length and width expand by the mechanism of linear expansion. Consequently, the area change ΔA is proportional to the original area A_0 and the rise in temperature ΔT assuming $\Delta A << A_0$.

$$\boxed{\Delta A = k A_0 \Delta T.} \tag{1.25}$$

The constant of proportionality k can be related to the coefficient of linear expansion α of the material.

$$\Delta A = L_0 W_0 (1 + \alpha \Delta T)^2 - L_0 W_0 \approx 2\alpha A_0 \Delta T.$$

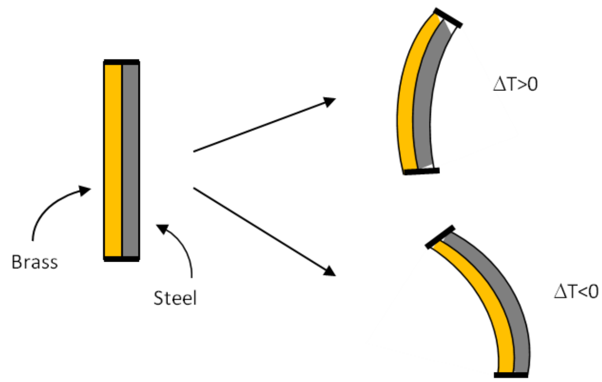

Figure 1.15: A bimetal strip - the two metals expand with different coefficients.

where we have assumed $\alpha\Delta T << 1$ and dropped higher order terms in $\alpha\Delta T$. Hence, the coefficient of area expansion is equal to twice the coefficient of linear expansion.

$$k \approx 2\alpha. \tag{1.26}$$

Note that the coefficient of expansion for the area is not square of the coefficient of the linear expansion, i.e. $k \neq \alpha^2$, but rather $k = 2\alpha$.

1.8.3 Volume Expansion

The most general type of expansion will the expansion of the volume. Similar to the linear and area expansions, the change in volume ΔV of a material of initial volume V_0 subject to a change of temperature ΔT at a constant pressure is given as a linear relation as long as $\Delta V << V_0$.

$$\boxed{\Delta V = \beta V_0 \Delta T \quad \text{(Constant pressure.)}} \tag{1.27}$$

The proportionality constant β is called the coefficient of volume expansion. Similar to the case of area expansion, it can be shown that the coefficient of volume expansion of a solid is three times the coefficient of a linear expansion as long as $\alpha\Delta T << 1$.

$$\boxed{\beta \approx 3\alpha.} \tag{1.28}$$

If expansion becomes so great that $\Delta V << V_0$ cannot be assumed anymore as is often the case with gases, we will not be able to use Eq. 1.27. For this reason, Eq. 1.27 is mostly useful for liquids and solids. The coefficients of volume expansion of some common fluids are given in Table 1.6.

Table 1.6: Coefficients of volume expansion of some common liquids at 20°C (Handbook of Chemistry and Physics, 82nd Volume, 2001-2002.)

Material	Coefficient of vol exp (β) ($\times 10^{-3}$per°C)
Acetone	1.46
Ethyl alcohol	1.40
Gasoline	0.95
Glycerol	0.520
Mercury	1.811
Water	0.206

1.8.4 Thermal Stress

Materials normally expand when they are heated and contract when they are cooled. Now, if you clamp both ends of a rod with a material that has a much smaller coefficient of expansion and prevent the rod from expanding or contracting, then a tensile or compressive stress, called thermal stress, will develop in the rod as a result of a change of temperature. The resulting stress can be quite large and could cause damage to the material.

An engineer must take this effect into account when designing or choosing a construction material. For instance, the reinforcing rods in concrete are made of steel since the coefficients of linear expansion of steel is nearly the same as that of concrete. If you were to reinforce the concrete with aluminum, which has a coefficient of linear expansion twice that of the concrete, a stress would develop in the concrete when temperature changes.

Another way to reduce stress is to allow the ends to expand or contract freely as done in highways and railroad tracks, where the gaps between blocks are deliberately left to prevent thermal stress from developing.

To calculate the thermal stress in a rod whose both ends are fixed rigidly we can think of the process of the development of stress as occurring in two steps. First let the ends be free to expand (or contract) and find the net expansion (or contraction), then use a force to compress (or expand) the material to the original length and find the resulting stress by using the methods of Statics studied in mechanics. Suppose a bar of length L_0, area of cross-section A, coefficient of linear expansion α, and Young's modulus Y be subject to a change of temperature ΔT. The thermal stress in the bar will be calculated using the following steps.

Step 1: Change in length if expansion were allowed to take place freely.

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$

Step 2: Stress to cause the change in length. This step restores the length, therefore change in length is negative of the change of length in step 1.

$$\text{Stress } = \frac{F}{A} = -Y\frac{\Delta L}{L_0}.$$

Therefore,

$$\text{Thermal Stress } = -Y\frac{\Delta L}{L_0} = -Y\alpha\Delta T.$$

Therefore, if the temperature increases, i.e. if $\Delta T > 0$, thermal stress would attempt to restore the original length. Therefore, the stress would be compressive (Fig. 1.18). If the temperature decreases, i.e. $\Delta T < 0$, then the thermal stress is pointed outward, i.e. tensile.

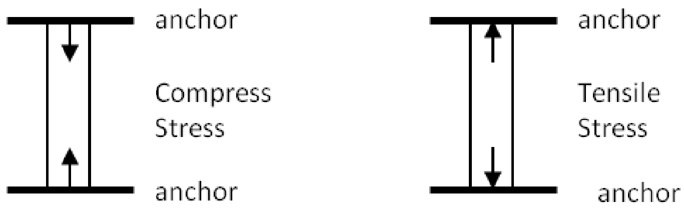

Figure 1.18: Compressive and tensile stresses for anchored beam when $\Delta T > 0$ and $\Delta T < 0$ respectively.

Example 1.8.2. Thermal Stress in Concrete

Concrete blocks are laid out next to each other on a highway without any space between them such that they are prevented from expanding. The construction crew did their work on a winter day when the temperature was 5°C. What will be the stress in the blocks on a hot summer day when the temperature is 38°C ? (b) If the ultimate compressive strength of concrete is 20×10^6 N/m^2, will the blocks fracture? Young's modulus of concrete = 20×10^9 N/m^2, and the coefficient of linear expansion = 12×10^{-6} per degree Celsius.

Solution. (a) Using the formula given above, the thermal stress is found to be:

$$\begin{aligned}\text{Thermal Stress} &= -Y\alpha\Delta T \\ &= (20 \times 10^6 \text{ N/m}^2)(12 \times 10^{-6}\text{C}^{-1})(38 - 5)^\circ\text{C} \\ &= 7.9 \times 10^6 \text{ N/m}^2.\end{aligned}$$

(b) No, the concrete will not fracture by compressive stress as the stress does not exceed the ultimate compressive strength. But, it does exceed ultimate sheer strength of concrete which is only 2×10^6 N/m^2 and it might chip off.