7.7 PROBLEMS

Problem 7.7.1. A particle of charge $+q$ and mass m moves with velocity v_0 pointed in the $+y$-direction as it crosses the x-axis at $x = R$ at a particular time. There is a negative charge $-Q$ fixed at the origin, and there exists a uniform magnetic field B_0 pointed in the $+z$-direction. It is found that the particle describes a circle of radius R about $-Q$. Find B_0 in terms of the given quantities. Ans: Check answer: If $|Q| = 4\pi\epsilon_0 R$, and $q/m = 2v_0 2$, then $B_0 = 1/(2v_0 R)$.

Problem 7.7.2. Consider a region of space where magnetic field is uniform and is given to be $B_x = 0$, $B_y = 0$ and $B_z = 2$ mT. An electron which is first accelerated by a voltage difference of 500 V enters this region with its velocity pointed in the positive x-axis. (a) Find the acceleration of the electron. (b) Find an equation for the trajectory of the electron. Ans: (b) Circular trajectory of radius mv/eB.

Problem 7.7.3. A proton of speed $v0 = 6 \times 10^5$ m/s enters a region of uniform magnetic field of B =0.5 T at an angle $\theta = 30°$ to the magnetic field. In the region of magnetic field proton describes a helical path with radius R and pitch λ (distance between loops). Find R and λ. Ans: $\lambda = 2\pi m/eB)v_0 \cos\theta$.

Problem 7.7.4. A particles path is bent when it passes through a region of non-zero magnetic field although its speed remains unchanged. This is very useful for "beam steering" in particle accelerators. Consider a proton of speed 4×10^6 m/s entering a region of uniform magnetic field 0.2 T over a 5 cm wide region. Magnetic field is perpendicular to the velocity of the particle. By how much angle will the path of the proton be bent? (Hint: The particle comes out tangent to a circle.) Ans: 14°.

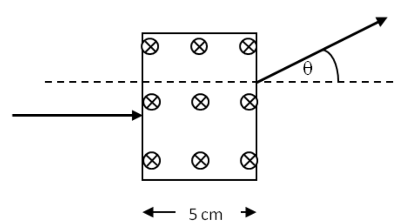

Figure 7.36: Problem 7.7.4.

Problem 7.7.5. Cyclotron. A cyclotron is a device for accelerating charged particles. It was one of the earliest particle accelerators and it is still used today as a first stage in a multistage particle accelerator. Cyclotron uses electric field to boost the speed and magnetic field to bend the trajectory of the particle and bring it in a region where there is electric field to act on it again and again.

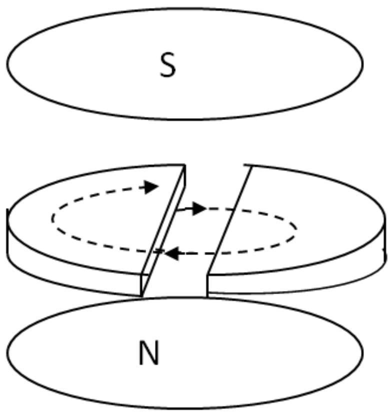

Figure 7.37: Problem 7.7.5. The cyclotron: the poles are shown for a positive particle. The Ds are connected to an AC source not shown

To accomplish this task it used two D-shaped hollow metals with open flat side. They are connected to an oscillating emf source $V = V_0 \cos(2\pi f t)$ which creates an oscillating electric field in the gap between the two D's with the frequency f matching the particle's entering and exiting the gap each half-cycle. A magnetic field is created perpendicular to the plane of D's by magnetic poles below and above the D's.

Particles are injected in the center and accelerated into one of the D's where magnetic field bends it back into the gap, at which time electric field also is in the opposite direction to the last time particle was in the gap. Particle travels with increasing speed in larger and larger radii circles, but the time is same for each half-cycle. The frequency is called the cyclotron frequency. The maximum speed achieved depends on the radius of the largest orbit. Let Q and m be the charge and mass of the particle, and B_0 be the uniform magnetic field. Show that cyclotron frequency f at which voltage must oscillate is given by $f = B_0 Q/2\pi m$.

Problem 7.7.6. (a) Find the speed of the electron in a hydrogen atom if it moves in a circular orbit of radius 0.0527 nm about the proton. (b) If a uniform magnetic field of 3 T perpendicular to the circle of revolution is applied what will be the new speed and orbit size of the electron? Ans: (a) 2.192×10^6 m/s, (b)$R' = 53.5$ pm, $v' = 2.175 \times 10^6$ m/s.

Problem 7.7.7. A thin wire of length L is pivoted at one of its ends and rotated at a constant angular speed ω in a region of uniform magnetic field B_0 pointed in the direction of the axis of rotation. As a result electrons in the wire are pushed to one end and a potential difference results in the wire. Find the potential difference between the ends P_1 and P_2 of the wire. Ans: $\omega B_0 R^2/2$.

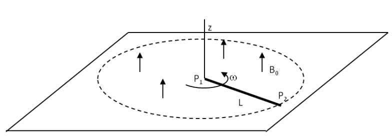

Figure 7.38: Problem 7.7.7.

Problem 7.7.8. A copper rod of mass m and length L is hung from the ceiling using two springs of spring constant k. A uniform magnetic field of magnitude B_0 pointing perpendicular to the rod and spring (coming out of the page in the figure) exists in a region of space covering a length w of the copper rod. The ends of the rod are then connected by flexible copper wire across the terminals of a battery of voltage V. Determine the change in the length of the springs when a current I runs through the copper rod in the direction shown in figure. (Ignore any force by the flexible wire.) Check answer: If $IwB = 1$, and $mg = 2$, then $dy = 3/2k$.

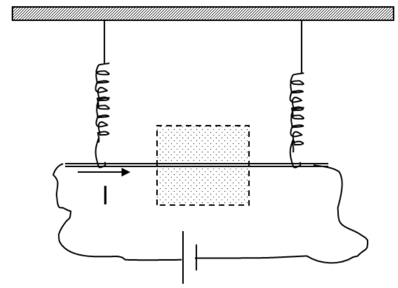

Figure 7.39: Problem 7.7.8.

Problem 7.7.9. An L-shaped conductor sits on a table the ends of which are connected to the terminals of a DC power source resulting in a current I to flow in the conductor (see Figure). There is a uniform magnetic field B_0 pointed perpendicular to the table. Find magnetic force on the L-shaped conductor. Ans: Magnitude: $IB_0\sqrt{a^2+b^2}$

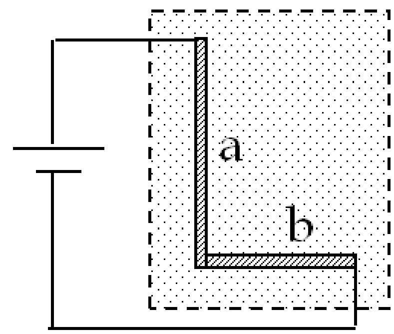

Figure 7.40: Problem 7.7.9.

Problem 7.7.10. In a region a non-uniform magnetic field exists such that $B_x = 0$, $B_y = 0$, and $B_z = ax$, where a is a constant. At some time t, a wire of length L is carrying a current I is located along the x-axis from origin to $x = L$. Find the magnetic force on the wire at this instant in time. Ans: Magnitude $IaL^2/2$.

Problem 7.7.11. In the previous problem find the magnetic force if the magnetic field is $B_x = c$, $B_y = ax$, and $B_z = bx$, where a, b and c are constants. Ans: Magnitude $\frac{IL^2}{2}\sqrt{a^2+b^2}$

Problem 7.7.12. Mass Spectrometer. The accompanied figure shows an arrangement for measuring mass of ions by an instrument called the mass spectrometer. An ion of mass m and charge $+q$ is produced essentially at rest in source S, a chamber in which a gas discharge is taking place. The ion is accelerated by a potential difference V_{acc} and allowed to enter a region of constant magnetic field B_0. In the uniform magnetic field region, the ion moves in a semi-circular path striking a photographic plate at a distance x from the entry point. Derive a formula for mass m in terms of B_0, q, V_{acc} and x. Ans: $m = (qB_0^2/8V_{\text{acc}})\,x^2$

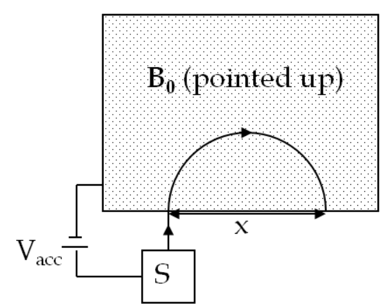

Figure 7.41: Problem 7.7.12.

Problem 7.7.13. A wire is made into a circular shape of radius R and pivoted along a central support. The two ends of the wire are touching a brush which are connected to a DC power source. The structure is between the poles of a magnet such that we can assume there is a uniform magnetic field on the wire. In terms of a coordinate system with origin at the center of the ring, magnetic field is $B_x = B_0, B_y = B_z = 0$, and the ring rotates about the z-axis. Find the torque on the ring when it is not in the xz-plane.

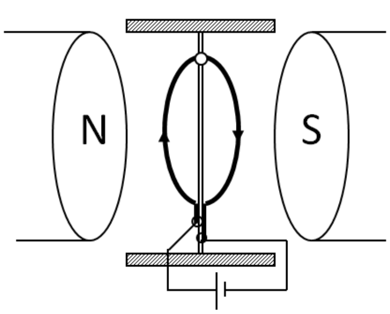

Figure 7.42: Problem 7.7.13.