6.6 EXERCISES

Calculating Momentum

Ex 6.6.1. Provide both the magnitude and direction of the momentum in each question. (a) Find the momentum of a 2 kg ball flying towards the East at a speed of 15 m/s. (b) Find the combined momentum of a 50 kg rider and a 10 kg bicycle moving to the North at a speed of 10 m/s. (c) Find the total momentum of two cars, each of mass 2500 kg, one moving with speed 28 m/s towards the East and the other with speed 29 m/s towards the West. (d) Find the net momentum of a car of mass 3000 kg moving towards the North at a speed of 25 m/s and a truck of mass $10,000$ kg moving towards the East at a speed of 8 m/s. (e) Find the net momentum of three rockets, each of mass 200 kg, one moving horizontally towards the East at a speed of 100 m/s, the second moving towards the North at a speed of 150 m/s and the third is moving straight up at a speed of 75 m/s.

Ex 6.6.2. (a) The Earth (mass $\approx 6.0 \times 10^{24}$ kg) makes a full revolution around Sun in almost a circular path in approximately 365 days. Assuming circular orbit of radius 1.5×10^{11} m, find the magnitude of momentum of the Earth with respect to the Sun. (b) Draw momentum vectors at two points on the "circular" orbit that is separated by six months in time. (c) Is the momentum of Earth constant? Why or why not? (d) What is the momentum of Earth with respect to Earth itself?

Ex 6.6.3. A box has a momentum 500 kg.m/s pointed towards the East. The box crashes into a wall and after some time comes to rest. What is the change in momentum of the box? Provide both magnitude and direction.

Ex 6.6.4. A baseball of mass 0.145 kg moving at a speed of 100 miles per hour (mph) is struck by a bat. After the hit the ball reverses direction and moves in the opposite direction with a speed of 120 mph. (a) Use a Cartesian coordinate and write the components of momenta before the hit and after the hit in the chosen coordinates. (b) What is the magnitude and direction of the change in momentum of the ball?

Ex 6.6.5. A proton ($m = 1.67 \times 10^{-27}$ kg) is moving towards the East with the speed 5×10^{6} m/s. The proton runs into a thin gold foil and emerges on the other side at 30° to the original direction but having the same speed. Determine the change in momentum of the proton. Provide both the magnitude and the direction. Note:

You may find it useful to choose a coordinate system with original direction as the x-axis and the proton emerging in the xy-plane. Use this coordinate system to compute the components of the momenta to find their difference.

Free-Body Diagrams, Axes, Components of $\vec{F} = m\vec{a}$

Ex 6.6.6. Fig. 6.22 shows four different situations of forces on a particle in (a)-(d). Determine the magnitude and direction of the net force in each case. If the mass of the particle is 0.1 kg, what are the accelerations of the particle in the four cases.

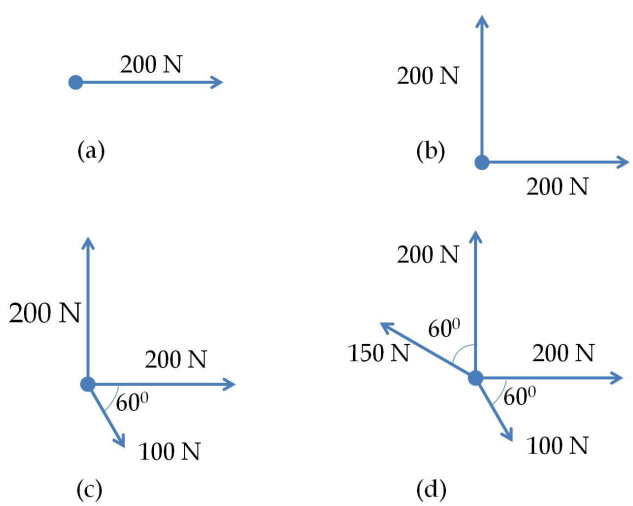

Figure 6.22: Exercise 6.6.6.

Ex 6.6.7. A projectile of mass m is flying freely with only the force of gravity for the Earth acting on the projectile. (a) Draw a free-body diagram for the projectile. (b) Choose a Cartesian coordinate system and set up the Newton's second law expression in the component form. (c) Find the magnitude and direction of acceleration of the projectile.

Ex 6.6.8. A child of mass m in a sleigh sleds down a snowy hill at a constant incline of angle θ that has virtually no friction. Ignore the air resistance also. (a) Draw a free-body diagram for the forces on the child. Use symbols to denote forces. (b) Choose a Cartesian coordinate system and set up Newton's second law in the component form for the constant mass case. (c) Find the magnitude and the direction of acceleration of the projectile. You would get the magnitude of acceleration in terms of g and θ. (d) Find the normal force from the incline on the child in terms of m, g and θ. Do not use a known formula for the magnitude of the normal force, but deduce the magnitude of the normal force from the equations of motion you have written down.

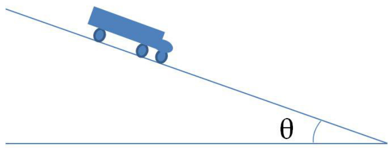

Figure 6.23: A truck sliding down an incline frictionlessly. Exercise 6.6.9

Ex 6.6.9. A truck slides down an icy hill, which slopes at a constant angle θ. (a) Ignoring the friction from the road and the air resistance, determine the instantaneous acceleration of the truck at an arbitrary time during the slide in terms of g and θ. (b)Assuming the forces on the truck remain constant throughout the slide on the incline, predict the speed with which the truck will move after sliding for a distance D on the hill if it was moving down the hill with a speed v_0 at rest at the starting point.

Ex 6.6.10. In a physics experiment designed for studying kinetic friction coefficient a student releases a 500-gram block from rest on an incline. He finds that it takes 1 second to cover a distance of 1 meter on the incline sloped at 15°. Ignoring the effect of air resistance but not that of kinetic friction of the incline, find the coefficient of kinetic friction as follows: first find the magnitude and direction of the acceleration from the given kinematic data assuming the acceleration down the incline to be constant, and then, set up Newton's second law to determine the normal and kinetic friction forces by using the known acceleration. Use the techniques of free-body diagram and choice of axes to work with components.

Ex 6.6.11. A hockey puck of mass 160 grams is shot with a hockey stick on a flat concrete surface. The coefficient of kinetic friction between the bottom of the puck and the concrete surface is 1.5. After the puck is no longer in contact with the hockey stick, the puck moves on the flat surface in a straight line with decreasing speed and comes to rest after traveling a distance of 50 meters. From the given data we wish to find the speed with which the puck leaves the hockey stick and it instantaneous acceleration before coming to rest by utilizing the following steps. (a) Draw a free-body diagram for the forces on the puck at an arbitrary time after the puck has left the hockey stick and before it has come to rest. Use symbols to denote forces. (c) Choose a Cartesian coordinate system and set up Newton's second law in the component form. (d) Find the magnitude and direction of acceleration of the puck. (d) Assuming the acceleration of the puck to be constant, determine the speed of the puck immediately after it leaves the hockey stick.

Ex 6.6.12. A block of mass M is resting on a rotating platform so that it rotates in a circle of radius R. The block starts to slide on the platform if the platform rotates faster than N turns per second. (a) Find the magnitude of the centripetal acceleration of the block when the platform rotates at a rate of one turn per second. (b) Suppose you use a coordinate system whose origin is at the center of the platform and the platform is in the xy-plane. The platform rotates but the axes remain fixed in space. Now, pick an instant when the block is crossing the x-axis. Set up equations of motion in the component form at this instant using symbols for various forces and the expression for the acceleration you found in part (a). (c) Solve the equations of motion to find the coefficient of static friction between the block and the platform.

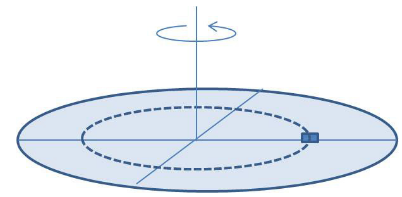

Figure 6.24: Exercise 6.6.12.

Ex 6.6.13. A flat curve on a road has a radius of curvature of 200 m. If the coefficient of static friction between the tires of a car and the road is as low as 0.5, what speed limit in km/h should be posted there so that cars do not slide on the curve. Hint: the static friction force has a maximum value.

Ex 6.6.14. A toy car of mass 250 grams is shot on a horizontal track which then loops in a circle of radius 20 cm. Note that magnitude of centripetal acceleration is still given by the formula v^2/R since the motion is circular, even though v now varies in time, but since the speed of the car in the vertical circle will not be constant, there will be a tangential acceleration in addition to the centripetal acceleration. Suppose that the toy is moving at a speed of 2.4 m/s when it is rounding the track at the top. Find the force with which the car presses the track when it is at the top of the circular path. Hint: Set up $\vec{F} = m\vec{a}$ for the instant when the car is at the top of the circular path.

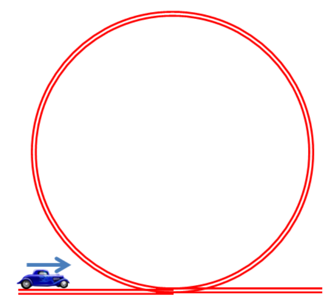

Figure 6.25: Exercise 6.6.14.

Ex 6.6.15. In the previous exercise, find the minimum speed with which the car must travel at the top so that it rounds the circle. Hint: The non-zero normal from the track adds to the net centripetal force.

Ex 6.6.16. At an air show a stunt pilot of mass 60 kg flies an air plane in a vertical loop of radius 100 m such that his head always faces the center of the circle. (a) At the top of the loop the pilot feels that he is no longer putting any weight on his seat, i.e. he is weightless. Find his speed when he is at the top of the vertical circle. (b) When he is at the bottom of the circle he is flying at a speed of 400 km/h with what force he must be pushing his seat when he is at the bottom of the circle? Hint: this exercise is similar to the toy car in the vertical circle.

Integrating $\vec{F} = m\vec{a}$

Ex 6.6.17. A particle of mass m is subject to a constant force of magnitude $\vec{F}_0$ and direction towards the West. Choose a Cartesian coordinate system and write the components of $\vec{F} = m\vec{a}$ or $\vec{F} = md\vec{v}/dt$ to answer the following questions. (a) what is the change in velocity of the particle between $t = 0$ and $t = t_1$? (b) What is the displacement of the particle between $t = 0$ and $t = t_1$?

Ex 6.6.18. A particle of mass m is subject to a force that is always pointed towards the East but whose magnitude changes linearly with time. The magnitude of the force is given as $F = bt$. Let x-axis point towards the East. (a) Find the change in the x-component of velocity between $t = 0$ and $t = t_1$. (b) Find the change in x-coordinate of the particle between $t = 0$ and $t = t_1$.

Ex 6.6.19. A particle of mass m is subject to a force that is always pointed towards the North but whose magnitude changes quadratically with time. The magnitude of the force is given as $F = bt^2$. (a) Find the change in the velocity between $t = 0$ and $t = t_1$. (b) Find the displacement of the particle between $t = 0$ and $t = t_1$. Hint: Take y-axis to point towards the North, and work with the y-components.

Ex 6.6.20. A particle of mass m is subject to a force that is always pointed towards East or West but whose magnitude changes sinusoidally with time. With the positive x-axis pointed towards the East, the x-component of the force is given as $F_x = F_0 \cos(\omega t)$, where F_0 and ω are constant. At $t = 0$ the particle is at $x = 0$ and has the x-component of the velocity, $v_x = 0$. (a) Find the x-component of velocity at time t. (b) Find the x-coordinate of the particle at time t.

Ex 6.6.21. A particle of mass m is subject to a force that is always pointed down but whose magnitude changes with time as given by the function, $F = mg\,[1 - \exp(-ct)]$, where c is a constant which takes only positive values. (a) Find the change in velocity between $t = 0$ and $t = t_1$. (b) Find the displacement of the particle between $t = 0$ and $t = t_1$. Hint: Take y-axis to point down, and work with the y-components.

Ex 6.6.22. A particle of mass m is subject to a force that is proportional to the speed and pointed opposite to the direction of velocity: $\vec{F} = -b\vec{v}$, where b is a constant which takes only positive values. Let x-axis be the direction of velocity. (a) Find the change in the velocity with time if the initial velocity has the magnitude u and points towards the East direction. (b) Find the change in the position with time.

Motion of Coupled Systems

Ex 6.6.23. Two large crates of masses $m_1$and m_2 are sitting side by side on the floor. (a) You push horizontally on one crate with a force of magnitude F but the crates do not move. Find the net frictional force on the two crates while you are pushing on the first crate crate. (b) You increase your force to F' and find that crates start to slide while they did not slide when the force was less than F'. What is the value of average coefficient of static friction between the surface of crates and the floor? (c) What is the acceleration of the masses when your force is F'' if the coefficient of kinetic friction between m_1 and m_2 and the table is μ_k (d) With what force does the second crate push back on the first crate?

If you like numbers try these: $m_1 = 1000$ kg, $m_2 = 1200$ kg, $F = 2000$ N, $F' = 3300$ N, $F'' = 3400$ N, and $\mu_k = 0.1$.

Ex 6.6.24. Two masses m_1 and m_2 are connected using a "massless and frictionless" pulleys as shown in Fig. 6.26. When the masses are let go from rest, the mass m_1 moves on a frictionless surface and the mass m_2 moves down towards the floor. Find the time it would take for the mass m_2 to drop a distance h before the mass hits the floor in terms of m_1, m_2, g, and h.

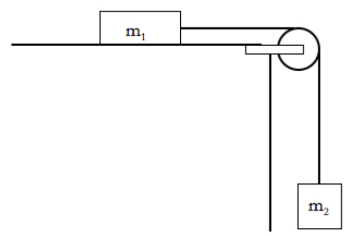

Figure 6.26: Exercise 6.6.24.

Ex 6.6.25. Two blocks of masses 0.5 kg and 2.5 kg are tied with a string which goes over a massless and frictionless pulley as shown in Fig. 6.27. The 2.5-kg block moves over an incline of angle of inclination 60°. The coefficient of kinetic friction between the 2.5-kg block and the surface of the incline is 0.08. Find the accelerations of the two blocks.

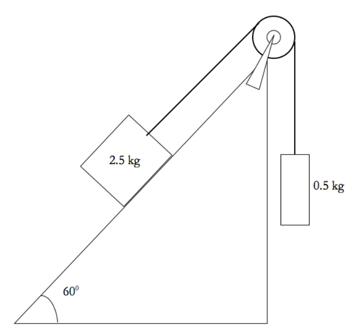

Figure 6.27: Exercise 6.6.25.

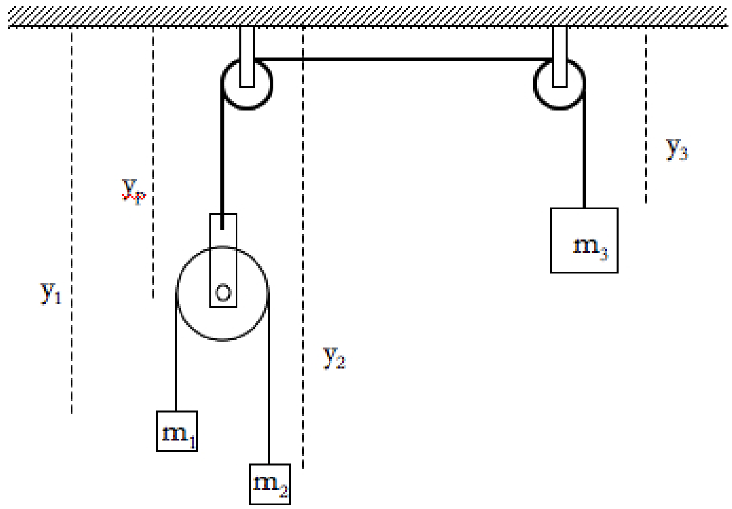

Figure 6.29: Exercise 6.6.27.

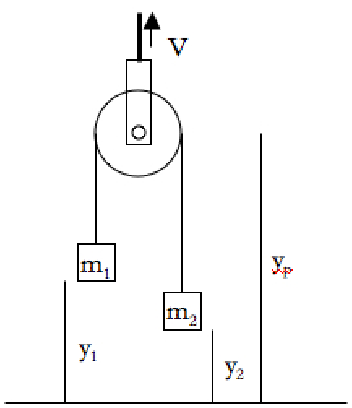

Figure 6.28: Exercise 6.6.26.

Ex 6.6.26. A pulley has masses m_1 and m_2 around it. The support of the pulley is pulled up at a constant velocity V (Fig. 6.28). (a) Find the constraint equation relating Δy_1, Δy_2, and Δy_p. (b) Find the tension in the string and the acceleration of the masses. Assume the pulley to be massless and frictionless.

Ex 6.6.27. In a three-pulley system shown Fig. 6.29, three masses and one of the pulleys are allowed to move. (a) Find the constraint equation(s) among a_1, a_2, and a_3 by first figuring out a relation among y_1, y_2, y_3, and y_p. (b) Find the tension in the strings and the acceleration of each mass. Assume pulley to be "ideal".

Ex 6.6.28. Two masses m_1 and m_2 are connected using two massless and frictionless pulleys as shown in Fig. 6.30. The mass m_1 moves on a frictionless surface and the mass m_2 moves vertically. The masses are let go from rest. (a) Find the values of the magnitudes of accel-

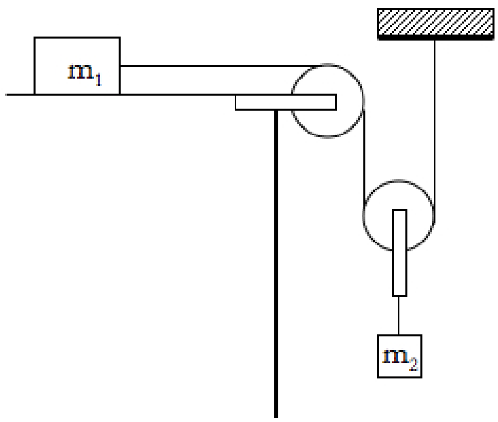

Figure 6.30: Exercise 6.6.28

erations of the masses m_1 and m_2 after a time $t = T$ has elapsed. (b) Find the speeds v_1 and v_2 of the masses m_1 and m_2 when m_2 has dropped a distance h. Assume when $t = T$, the mass m_1 has not hit the pulley at the end of the table of the mass m_2 has not hit the floor. If you like numbers, try these $m_1 = 5$ kg, $m_2 = 0.4$ kg, and $h = 5$ cm.