1.9 HEAT TRANSFER

When two objects at different temperatures come in thermal contact, either directly or indirectly, the two bodies are not in thermal equilibrium with each other. We find that the nature tends to bring objects to the thermal equilibrium by transferring energy from the regions of higher temperature to the regions of lower temperature. This process is called **heat transfer**. There are basically three ways that heat is transferred from one system to another. They are called **conduction**, **convection** and **radiation** depending on the the physical mechanism of the energy transport, which we will describe in detail below.

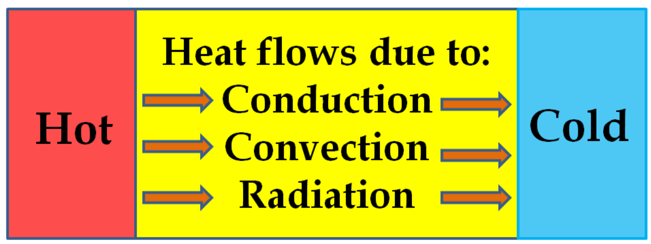

Figure 1.19: Three mechanisms of heat transfer between a body at a higher temperature (Hot) to a body at a lower temperature (Cold).

1.9.1 Conduction

The heat transfer mechanism that requires a physical contact between the bodies is called **conduction**. Materials differ in their abilities to conduct heat from one part to another. Heat flows more readily through a good conductor of heat such as aluminum than through a poorer conductor such as wood. The larger conductivity of aluminum over wood is easily demonstrated by touching an aluminum sheet and a wooden piece, which had been in the same oven for some time so that they have the same temperature. The aluminum sheet feels hotter because it easily transfers heat from other points of the aluminum sheet to the point of touch. As a result, a large amount of energy is given to your skin in a short amount of time as compared to what happens when you touch wood. This quick transfer of heat raises the temperature of your hand giving the sensation that aluminum sheet is hotter than the wood, even though they are at the same temperature.

Just because a particular object is sensed hotter than another does mean the hotter object is at a higher temperature. We sense the rate at which heat energy is deposited at the site of touch to ascertain the sense of hot and cold. We do not necessarily sense the temperature of a substance. Of course, if you touch two identical aluminum sheets at different temperatures, you will be able to correctly tell which sheet is at a higher temperature, since the two sheets have the same conductivity.

To study the conductivity quantitatively, consider a bar of length L and uniform cross-section area A whose ends are kept in contact with two large objects, also called heat baths, of constant temperatures T_1 and T_2 as shown in Fig. 1.20.

The "heat baths" may not even be baths at all; they may be an oven or some other device so that the temperature can be kept steady at T_1 or T_2 on the two sides of the bar under study.

We will assume that the heat baths are so large that their temperatures do not change during the experiment. We also assume that heat passing from one bath to the other through the bar does not leak out from the side surface of the bar. This can be accomplished by insulating the bar on the sides.

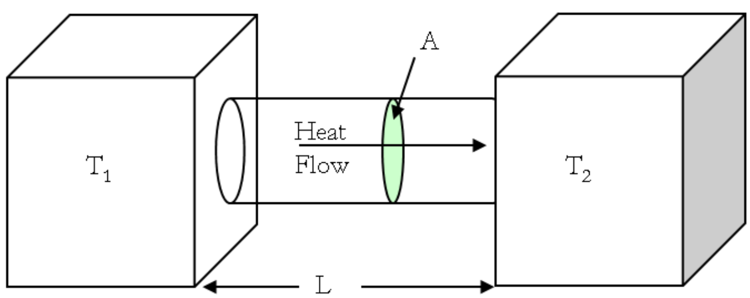

Figure 1.20: Study of conductivity of heat for the material connecting the two blocks at temperature T_1 and T_2 . As $T_1 > T_2$ heat flows in the direction from T_1 towards T_2.

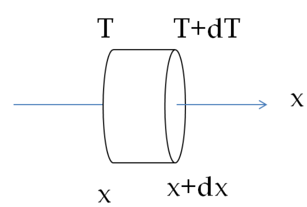

Figure 1.21: Heat transfer through an infinitesimal element.

Let ΔQ amount of heat flows from one bath to the other in time duration Δt. The **power flux** of heat transfer from one bath to the other is defined as the rate of heat transfer per unit cross-sectional area of the bar. Note the use of letter t here: the letter t in this section stands for the time and not the temperature.

$$\boxed{\Phi = \frac{\Delta Q/\Delta t}{A}} \quad \text{(Definition).} \tag{1.29}$$

Experiments show that the power flux is directly proportional to the difference in the temperature across the ends and inversely proportional to the length of the rod.

$$\boxed{\Phi = k\,(T_1 - T_2)\,\frac{1}{L}, \qquad (T_1 > T_2)} \tag{1.30}$$

The proportionality constant k is called the **thermal conductivity** which has units of J/s.m.K. Note that Eq. 1.29 says what we mean by the power flux for the energy we are referring as the heat or thermal energy, and Eq. 1.30 says how this flux depends on the temperature gradient in the material. The constant k in Eq. 1.30 distinguishes the conduction rates in different materials. Thermal conductivity of some common materials are given in Table 1.7.

Table 1.7: Thermal conductivities at 25°C

(Handbook of chemistry and physics, 82nd edition, 2001-2002)

Substance	Conductivity k(W/m.K)	Substance	Conductivity k(W/m.K)
Metals		**Fluids**	
Aluminum	237	Air	0.023
Copper	401	Ethanol	0.169
Gold	317	Mercury	8.25
Iron	80	Water	0.6
Silver	429	Water vapor	0.027
Common Solids			
Asbestos	0.7	Porcelain	1
Asphalt	0.06	Pyrex glass	1
Brass	120	Rock	1
Brick, Dry	0.04	Rubber	0.05
Cork	0.04	Sand	0.33
Glass wool	0.04	Steel	52
Mica	0.2-0.7	Styrofoam	0.03
Polyurethane	0.06	Wood	0.04-0.35

A material with larger thermal conductivity is a better conductor of heat. From the table you can see that thermal conductivities of metals are 10^1 to 10^4 times the conductivities of other materials. Therefore, metals are considered good conductors of heat. Materials whose thermal conductivities are small compared to good conductors are called insulators. Wood and air are good example of insulators. Water is not as good a conductor as metals but a much better conductor than air. That is why you feel much colder in the lake water than air of the same temperature.

From the definition of heat flux in Eq. 1.29 and the experimental results of conduction in Eq. 1.30, we have the following relation for the rate of flow of heat through a bar of length L and cross-section area A.

$$\frac{1}{A}\frac{\Delta Q}{\Delta t} = k\,(T_1 - T_2)\,\frac{1}{L} \qquad (T_1 > T_2) \tag{1.31}$$

The relation actually holds for each thin section of the connecting rod. Therefore, it should be more appropriately written in terms of time-derivative of Q. Let the heat flow be in the x-direction as shown in Fig. 1.21, then

$$\boxed{\frac{dQ}{dt} = -kA\frac{dT}{dx}.} \tag{1.32}$$

Here the minus sign ensures that Q flows from a high T to a low T region. This equation has a direct analogy with the electric current I (in place of dQ/dt) through a wire of resistance (R) when a voltage difference ΔV (in place of $T_1 - T_2$) is applied across the wire. In electric circuits, the relation among I, R and V is given by Ohm's law.

$$I = \frac{1}{R}V$$

Therefore, it is a common practice to define a **thermal resistance** R_{th} of a slab of thickness Δx and area of cross-section A, made up of a material of conductivity k, by the following

$$R_{th} = \frac{\Delta x}{kA} \tag{1.33}$$

The insulating properties of building materials, such a fiberglass, are indicated by the thermal resistance of unit area slab, called the ***R*-factor**.

$$R\text{-factor} = \frac{\Delta x}{k} \tag{1.34}$$

A commonly used fiberglass in the US is R-19, which corresponds to $\frac{1}{19}$ BTU per hour per square foot area of the insulation. In calculating the R-value of a multi-layered installation, the R-values of the individual layers are added. Table 1.8 shows the R-factors of several common insulations used in the construction industry.

Example 1.9.1. Heat Transfer Through a Copper Rod. A half-m copper rod of cross-sectional area $3.14 \times 10^{-6}\ m^2$ is insulated so that heat cannot escape from the sides. One end of the rod is free of insulation and placed in a steam container maintained at $150°C$. The other end is also free of insulation and placed in ice water maintained at $0°C$. Find the amount of heat transferred from the steam container to the ice water in 30 minutes.

Solution. This problem is a direct application of Eq. 1.31 relating heat flux to the difference in temperature.

$$\frac{1}{A}\frac{\Delta Q}{\Delta t} = k\frac{T_1 - T_2}{L} \quad (T_1 > T_2)$$

Table 1.8: R-factor of Common Materials

Material	R-factor
Roof (no insulation)	3.3
3″ fiberglass	10
6″ fiberglass	20
6″ cellulose	23
Concrete block (8″)	2.0
Brick (4″)	4
Wooden frame	5
Wooden wall with 3 1/2″ fiberglass	12
Concrete slab with 1″ foam perim	45
Floor with 6″ insulation	25
Single pane windows	1
Double pane windows	2
1/2″ air space between panes	2

Here, the following quantities are given:

$$A = 3.14 \times 10^{-6}\ \mathrm{m}^2$$
$$\Delta t = 30\ \mathrm{min} = 1800\ \mathrm{sec}$$
$$k = 401\ \mathrm{J/s.m.K}$$
$$T_1 - T_2 = 150\ \mathrm{K}$$
$$L = 0.5\ \mathrm{m}$$

Therefore,

$$\Delta Q = kA\Delta t\frac{(T_1 - T_2)}{L} = 680\ \mathrm{J}.$$

Example 1.9.2. Conduction by Two Welded Metals

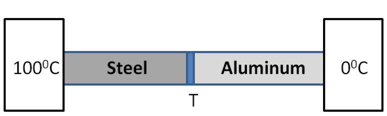

Figure 1.22: Example 1.9.2.

An aluminum and a steel cylindrical rod, each of diameter 1 cm and length 25 cm, are welded together. The other end of the aluminum rod is placed in a large tank of water at 20°C, while the other end of the steel rod is in a boiling water at 100°C. The rods are insulated so that no heat escapes from the surface. What is the temperature at the joint when a steady state has reached so that the rate of flow of heat is same throughout?

Solution. Let T be the temperature at the joint. The steady state condition requires that rate at which heat is transferred to the joint from the hotter end of the steel must be same as the heat transferred by aluminum from the joint to the colder end.

$$\text{Hot end to middle: } \frac{\Delta Q}{\Delta t} = A_{\text{steel}} k_{\text{steel}} \frac{(T_{\text{hot}} - T)}{L_{\text{steel}}}$$
$$\text{middle to cold end: } \frac{\Delta Q}{\Delta t} = A_{\text{Al}} k_{\text{Al}} \frac{(T - T_{\text{cold}})}{L_{\text{Al}}}$$

Equating the two rates gives

$$A_{\text{steel}} k_{\text{steel}} \frac{(T_{\text{hot}} - T)}{L_{\text{steel}}} = A_{\text{Al}} k_{\text{Al}} \frac{(T - T_{\text{cold}})}{L_{\text{Al}}}.$$

The areas of cross-section and lengths of the rods cancel out from the two sides, and we find the following for the temperature at the joint.

$$T = \frac{k_{\text{steel}} T_{\text{hot}} + k_{\text{Al}} T_{\text{cold}}}{k_{\text{steel}} + k_{\text{Al}}} = \frac{80 \times 373.15 + 237 \times 273.15}{40 + 237}\ \text{K} = 298.4\ \text{K} = 25.2°\text{C}$$

Note the temperature in the middle of the two metals is not the average of the temperatures at the two ends since the metals have different conductivities.

Example 1.9.3. Conduction in Radial Direction. A 5 m long copper pipe of diameter 2.0 cm carries steam at 150°C. The pipe is insulated by Styrofoam of thickness 1 cm, and the outside space is at room temperature of 20°C. Find the rate of heat loss from the pipe. Use $k = 0.03$ W/m.K for Styrofoam.

Figure 1.23: Example 1.9.3.

Solution. In the present case the heat is not transferred in one direction as was the case with the example of heat along the length of a bar. Here, the heat exits radially outward in all directions from an axis. Therefore, we use the cylindrical coordinate system (Fig. 1.24). At the radial distance r, the area A through which heat passes is the area of a cylindrical shell of length L and width $2\pi r$. Eq. 1.32 for rate of flow through the shell at radial distance r would be

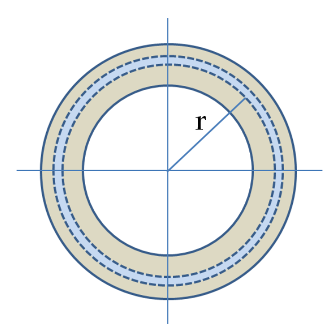

Figure 1.24: Example 1.9.3.

$$\frac{dQ}{dt} = -k(2\pi r L)\frac{dT}{dr}$$

Since, there is no source or sink of heat in the insulation, the rate of flow of heat must be independent of the distance from the center. Rearranging the equation, and integrating from $T = T_1$ when $r = R_1$ (the inside radius of the insulation) to $T = T_2$ when $r = R_2$ (the outer radius of the insulation) will give the following.

$$\frac{dQ}{dt} \int_{R_1}^{R_2} \frac{dr}{r} = -2\pi k L \int_{T_1}^{T_2} dT.$$

This gives the following relation for the rate of flow of heat from the inner surface at radius R_1 to the outer surface at radius R_2.

$$\frac{dQ}{dt} = \frac{2\pi k L(t_1 - T_2)}{\ln{(R_2/R_1)}}.$$

Note that we do not need to convert the temperature into Kelvin since the change in Kelvin is the same in degree Celsius. Putting the numerical values for our case, we get

$$\frac{dQ}{dt} = \frac{2\pi \times 0.03(\mathrm{W/m.K}) \times 5\mathrm{m} \times 130\ \mathrm{K}}{\ln(2\ \mathrm{cm}/1\ \mathrm{cm})} = 177\ \mathrm{W}.$$

1.9.2 Convection

Convection occurs in fluids when the higher energy molecules from one part of a fluid move to another part resulting in a bulk motion of the fluid. If the movement is caused by forcing the hot fluid with a mechanical stirrer or a blower, the convection is called **forced convection**, otherwise it is called the **free or natural convection**. For example, when you heat water on a stove, higher energy molecules from the bottom rise and mix with lower energy molecules above due to natural or free convection (Fig. 1.25).

Since the molecules in the hot region have more kinetic energy, they tend to exclude more space from other molecules. As a result, the volume per molecule in the hotter region is more and so the density in the hotter region is less than the colder region. This the reason why hot water rises in a container. Similarly, hot air from the surface of the Earth rises in the atmosphere. The flow of the fluid creates a convection current that leads to a heat transfer from the hot region to the cold region.

Convection plays a major role in the atmosphere as hot air with water vapor from near the surface of the Earth rises and cools at a lower temperature above the Earth where they condense and form clouds. Convection is also thought to play an important role in bringing heat from the center of the Sun to the surface of the Sun. The **convection cells** photographed at the solar surface shown in Fig. 1.25b are evidence of the role of convection in the solar dynamics.

1.9.3 Radiation

The third form of heat transfer does not require any medium. Radiation carries energy from one system to another by light and other energetic particles, which are collectively called the radiation. Unlike the conduction or the convection, the transfer of energy by way of radiation can occur even across a vacuum between two bodies as evidenced from radiation from far away stars reaching the Earth.

The radiation mode of heat transfer is based on the fact that all

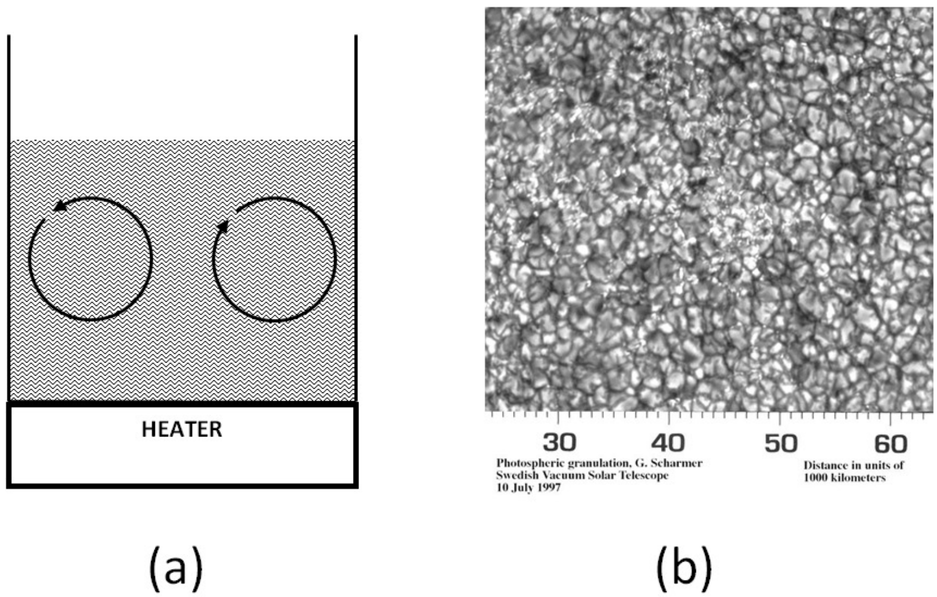

Figure 1.25: (a) Convection currents in water on a heater carry energy from the larger temperature bottom to the cooler top. (b) Granular convection cells at the surface of sun that carry energy from the interior of the sun to the surface. Credits: http://solarscience.msfc.nasa.gov/images/granules.jpg

materials above absolute zero temperature (-273.15°C or zero degree Kelvin) emit electromagnetic waves. The intensity of the electromagnetic waves emitted when the object is at a particular temperature varies with the frequency of the radiation and the chemical composition of the material. The distribution of intensity with frequency for a blackbody, an ideal object that is both a perfect absorber and a perfect emitter of radiation, is given by the **Planck radiation law**. According to the Planck radiation law the power per unit area $d\Phi$ in the radiation of frequencies between f and $f + df$ is given by the following formula.

$$d\Phi = \frac{2\pi h}{c^2} \frac{f^3}{\exp\left(\frac{hf}{k_B T}\right) - 1} df \quad (0 \le f < \infty) \tag{1.35}$$

where $c(3 \times 10^8$ m/s) is the speed of light, $h(6.63 \times 10^{-34}$J.s) the **Planck constant** and $k_B(1.38 \times 10^{-23}$J/K) the Boltzmann constant. The total radiation emitted by a black body at a given temperature is given by the **Stefan-Boltzmann law** in terms of the power radiated per unit surface area.

$$\boxed{\left(\frac{\text{Power}}{\text{Area}}\right)_{\text{emitted}} = \sigma T^4.} \tag{1.36}$$

where σ is a universal constant called the **Stefan-Boltzmann constant** with magnitude 5.7×10^{-8}W/m^{2}K^4. You can obtain the Stephan

Boltzmann law from the Planck distribution, given in Eq. 1.35, by integrating the later from $f = 0$ to ∞. Stephan-Boltzmann law gives an expression for the total radiation emitted by a blackbody. To obtain the heat flux emitted by a real body we multiply the black-body formula by a surface property of the material called the **emissivity** ϵ.

$$\left(\frac{\text{Power}}{\text{Area}}\right)_{\text{emitted}} = \epsilon\sigma T^4 \quad \text{(All objects)} \tag{1.37}$$

The emissivity has a range of $0 \leq \epsilon \leq 1$. The emissivity of a body is related to its ability to absorb radiation. The larger the emissivity, the more the radiation emitted.

The ability of a body to absorb radiation is given by absorptivity. We define **absorptivity** α as the ratio of radiation it absorbed to the radiation incident on it.

$$\text{Absorptivity,} \quad \alpha = \frac{\text{Radiation absorbed}}{\text{Radiation incident}} \tag{1.38}$$

To simplify calculations we often assume that the absorptivity of a material is equal to the emissivity.

$$\alpha = \epsilon.$$

This relation is strictly true only for an ideal black body, where both quantities have the value equal to 1, but for most materials, this relation can be taken as a starting point of analysis. The absorptivity and emissivity are experimental quantities which are normally found by empirical means for each material.

Example 1.9.4. Surface Temperature of the Sun. The Sun radiates 4×10^{26}J of energy per second. Assuming that the radiation from the Sun is very close to that expected from a blackbody, and the radius of the sun to be 7.0×10^8m, find the surface temperature of the sun.

Solution. The amount of energy radiated per unit time per unit area is given by the Stefan-Boltzmann law.

$$\left(\frac{\text{Power,}\ P}{\text{Area,}\ A}\right)_{\text{emitted}} = \sigma T^4.$$

Here, the area is equal to the surface area of a sphere, which is $4\pi R^2$. Solving for T, we find

$$T = \left(\frac{P}{4\pi R^2 \sigma}\right)^{1/4}.$$

Now, we put values of p, R and σ to obtain the following temperature at the surface of Sun.

$$T = \left(\frac{4.0 \times 10^{26}\text{W}}{4\pi(7.0 \times 10^8\text{m})^2(5.7 \times 10^{-8}\text{W/m}^2\text{K}^4)}\right)^{1/4} = 5,800\ \text{K}.$$

1.9.4 Approach to Equilibrium

By Conduction and Convection

It is a common experience that a hot cup of water placed in a room temperature environment cools down, and eventually the temperature of the cup reaches the room temperature. The heat loss in this process is primarily due to the conduction and convection. Newton's law of cooling asserts that for cooling due to convection the rate of change of temperature of a hot body is proportional to the difference between its temperature and the ambient temperature. Let $T(t)$ be the instantaneous temperature of a hot body at time t that has been placed in fixed-temperature environment of temperature T_0. Then we can write **Newton's law of cooling** as follows.

$$\boxed{\frac{dT}{dt} = -\alpha\,(T - T_0)\,,} \tag{1.39}$$

where α is the constant of proportionality, called the **rate constant** of cooling. The rate of cooling α should not be confused with the same symbol used for the absorptivity - they are different properties. The negative on the right side ensures that the temperature of the hot body will be decreasing with time. Given an initial temperature, $T(0)$ of the body, we can predict the temperature at a later time by solving Eq. 1.39 which result in the following solution.

$$T(t) = [T(0) - T_0]\exp(-\alpha t) + T_0. \tag{1.40}$$

Therefore, we say that the, according to Newton's law of cooling, the temperature of a hot body placed in a lower temperature environment, drops exponentially. The rate constant α depends on the conductivity of the container and other conditions, such as air draft. The solution in Eq. 1.40 is plotted in Fig. 1.26 for two different values of the cooling rate constant to illustrate the role of α in the rate at which the temperature drops.

Example 1.9.5. Time for Cooling a Cup of Coffee. A 100 ml of coffee at temperature 75°C is placed on a table in an environment that has a constant temperature of 25°C. It is found that the temperature of the cup after 3 minutes is 60°C. Find Newton's rate constant of cooling α.

Solution. Since the cooling is primarily due to convection by air in the room, Newton's law of cooling is applicable. Rearranging the equation given above and then taking a natural log we can bring the exponent down.

$$\ln\left[\frac{T(t) - T_0}{T(0) - T_0}\right] = \ln\left[\exp(-\alpha t)\right]$$

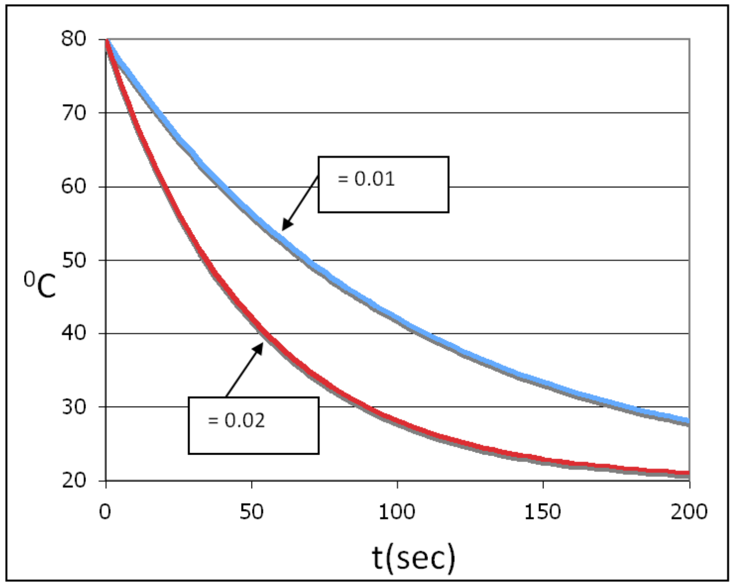

Figure 1.26: Newton's law of cooling illustrated for two different values of α. The upper curve has $\alpha = 0.01$ sec^{-1} and the lower curve has $\alpha = 0.02$ sec^{-1}. The larger k corresponds to faster cooling. Both samples starting at 80°C reach the ambient temperature 20°C asymptotically.

Therefore,

$$\alpha = -\frac{1}{t}\ln\left[\frac{T(t) - T_0}{T(0) - T_0}\right]$$

Now, we put the given values: $t = 3$ min, $T(t) = 60°$C, $T(0) = 75°$C, $T_0 = 25°$C, to obtain the value of the rate of cooling α.

$$\alpha = -\frac{1}{3\text{ min}}\ln\left[\frac{60 - 25}{75 - 25}\right] = 0.12 \text{ per min.}$$

By Radiation

When heat is either gained or lost from a body primarily due to radiation, the rate is not given by Newton's cooling law. Instead we must use Stefan-Boltzmann equation for deducing the law of rate of cooling due to radiation alone.

As an example, consider an object of emissivity ϵ at temperature T in outer space. We assume that the energy received by the body is negligible compared to the energy radiated out. Stefan-Boltzmann says that the body will radiate out energy at the following rate.

$$P = \epsilon\sigma AT^4, \tag{1.41}$$

where A is the surface area and σ the Stefan-Boltzmann constant. The energy loss will result in a decrease in temperature given by the relation of heat to temperature by way of specific heat, c.

$$P = -\frac{dQ}{dt} = -mc\frac{dT}{dt} \quad (dT/dt < 0), \tag{1.42}$$

where negative sign is included to indicate the energy loss with increasing time. For simplicity in writing we do not distinguish the specific heat into the constant volume, constant pressure, or some other process. If we neglect any effect due to expansion or contraction of the sample, a more appropriate specific heat will be the constant volume specific heat. Equating the two rates, viz. the emitted rate from Eq. 1.41 and the loss of energy rate from Eq. 1.42, we find the required law for temperature change of the body when the body loses energy solely due to the radiation.

$$\frac{dT}{dt} = -\frac{\epsilon\sigma A}{mc}T^4 \quad \text{(solely due to radiation.)}$$

Integrating this equation from $[t = 0, T(0)]$ to $[t, T(t)]$, we find the temperature at arbitrary time is given by the following complicated formula.

$$T(t) = \left[\frac{1}{T(0)^3} + \frac{3\epsilon\sigma A}{mc}t\right]^{-1/3}. \tag{1.43}$$

Example 1.9.6. Cooling by Radiation

A copper spherical ball of radius 5 cm is heated to a temperature 150°C and hung by a non-conducting wire in a vacuum chamber whose walls are kept cold at very low temperature so that the energy received by the copper ball by radiation is negligible compared to the energy emitted by the ball. Find the temperature of the ball after 10 minutes. Use specific heat of copper to be 379 J per Kg per degree K, and density of copper to be 8230 kg/m^3. Assume emissivity 1 for the copper ball.

Solution. We use the following values in Eq. 1.43.

$$\begin{aligned}
&\epsilon = 1\\
&T(0) = 150 + 273.15 = 423.15 \text{ K}\\
&A = 4\pi R^2 = 4\pi(0.05\text{m})^2 = 0.0314 \text{ m}^2\\
&\sigma = 5.7 \times 10^{-8}\text{W/m}^2\text{K}^4\\
&m = \rho V = (8230 \text{ kg/m}^3) \times \frac{4}{3}\pi(0.05\text{m})^3 = 12.93 \text{ kg}\\
&t = 600\text{sec.}
\end{aligned}$$

Therefore, the temperature of the ball after 30 seconds is

$$T(t) = \left[\frac{1}{423.15^3} + \frac{3 \times 1 \times 5.7 \times 10^{-8} \times 0.0314}{12.93 \times 379} \times 600\right]^{-1/3} = 373 \text{ K}.$$

Example 1.9.7. Radiation in a Spherical Cavity

Consider a spherical cavity kept at temperature T_1 that has a spherical ball which is maintained at temperature T_2 with $T_1 > T_2$ (Fig. 1.27). We wish to find the rate at which heat will be transferred from 1 to 2 by radiation mechanism. Note that the temperature of the ball inside the cavity will rise. Assume that inside the ball there is a mechanism for taking the heat out so that its temperature does not rise. Similarly, assume that there is a mechanism for continuously supplying energy to the cavity so that its temperature is also kept at T_1. Let the surface areas of 1 and 2 be A_1 and A_2 respectively. Let ϵ_1 and ϵ_2 be their emissivities. Also, assume emissivity and absorptivity of the ball are equal.

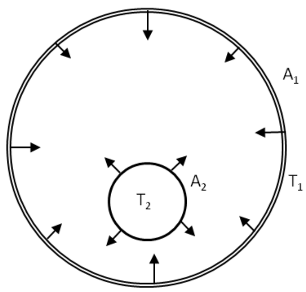

Figure 1.27: Example 1.9.7.

Solution. We can work out the transfer of heat through radiation by looking at the surface that will be absorbing the heat, which in the present case is the ball. In time Δt, the ball will absorb radiation from the cavity and will also emit some radiation, the difference of which will be the amount of heat transferred.

$$\text{Energy absorbed - Energy emitted} = A_2\epsilon_2\epsilon_1\sigma T_1^4\Delta t - A_2\epsilon_2\sigma T_2^4\Delta t$$

Hence the rate of transfer of heat from 1 to 2 through radiation is given as follows.

$$\frac{\Delta Q}{\Delta t} = A_2\epsilon_2\sigma\left(\epsilon_1 T_1^4 - T_2^4\right)$$

1.9.5 Thermal Energy Balance

When a body is in thermal equilibrium with another body, the temperature of the two bodies must be equal, and there would be zero net heat transfer from one body to the other. Two bodies at different temperatures will have a heat transfer from the hotter body to the colder body, causing the temperature of the colder body to rise and that of the hotter body to decrease until their temperatures become equal at the equilibrium.

Would the temperatures of all bodies in thermal contact become equal? Not necessarily, if the body can expend energy to maintain the difference in temperature. For instance, the temperature of our bodies remain quite constant at around 36°C in environments of different temperatures. The key is that the loss in heat from our bodies is compensated by the generation of energy within our bodies by biochemical reactions. Therefore, although our bodies are not in thermal equilibrium with the environment, they are in a **steady state** of nearly constant temperature. The idea of the supply of energy to compensate for the heat loss by a body, or the removal of energy to compensate for a gain in heat by a body, in order to maintain the temperature of the body is called the **thermal energy balance**.

Example 1.9.8. The Surface Temperature of the Moon.

Although the Moon gains energy from the Sun, the temperature of the Moon remains quite steady. This happens because of thermal energy balance in the absorption and emission of energy by the Moon as we will illustrate in this example by finding the average temperature of the Moon.

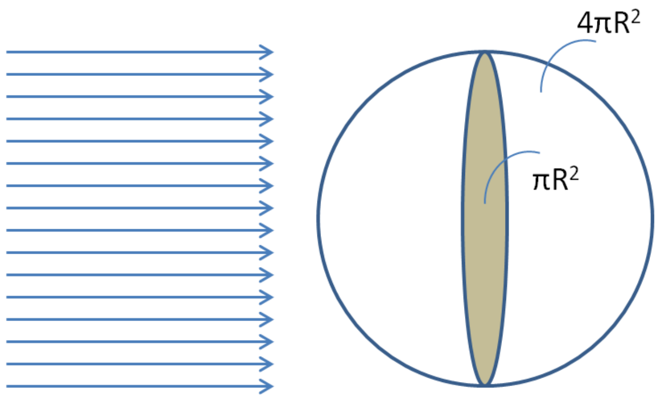

Figure 1.28: Example 1.9.8.

Consider the Moon to be a spherical black body of radius 1.74×10^6 m. The intensity of the sunlight on the Moon is approximately 1.4 kW/m^2, the same as on the earth. Let R be the radius of the moon, and S the power per unit area incident on the moon from the sun. To simplify out calculations, we will assume the Moon to be a perfect black body so that $\epsilon = \alpha = 1$.

To find the balance that would result between the energy gained and energy lost we look at their mechanisms in the case of the Moon. The energy gained will be from the radiation of the Sun incident on the Moon. Since the effective area of the Moon that intersects the almost parallel rays from the Sun is the are of the cross-sectional area of the Moon, we will multiply the flux of the radiation from the Sun by the area (πR^2) as shown in Fig. 1.28. On the other hand, the radiation emitted from the Moon will be emitted from the surface area of the Moon, not the intersected area. Therefore, the energy lost by radiation emitted will be from the surface area ($4\pi R^2$) of the moon. The two energies must be equal for the balance. Setting the two rates equal gives

$$\pi R^2 S = \sigma(4\pi R^2)T^4.$$

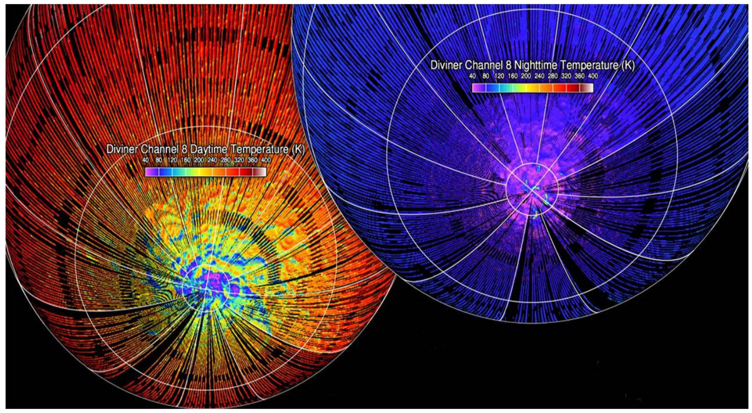

Figure 1.29: NASA's Lunar Reconnaissance Orbiter, an unmanned mission to comprehensively map the entire moon produced this thermal map of the north and south polar regions of the Moon in 2008. The temperature of the Moon ranges from approx. 35 K to 400 K. Photo credit NASA.

Therefore,

$$T = \left(\frac{S}{4\sigma}\right)^{1/4} = \left(\frac{1.4 \times 10^3 \mathrm{W/m^2}}{4 \times 5.7 \times 10^{-8} \mathrm{W/m^2K^4}}\right)^{1/4} = 280 \text{ K}.$$

This is approximately 7°C. The temperature of the Moon actually varies over a wide range as found by NASA's Lunar Reconnaissance Orbiter. See Fig. 1.29.