1.13 Problems

Problem 1.1. Consider the relativistic form of $\vec{F} = m\vec{a}$ given in the chapter. Show that if force $\vec{F}$ is parallel to the velocity $\vec{u}$, then this equation becomes

$$F = m\gamma^3 \frac{du}{dt}, \quad \text{where } \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}.$$

Problem 1.2. Fizeau Experiment. In 1851 Hippolyte Fizeau carried out an experiment to measure the speed of light in moving water. When water is stationary the speed is

$$u = \frac{c}{n},$$

where n is the refractive index of water. Fizeau found that if water was flowing with speed v in the same direction as the direction of light his data for the speed of light would fit with the following formula.

$$u = \frac{c}{n} + v - \frac{v}{n^2}.$$

This formula is an approximate formula for an exact formula that you can derive based on velocity transformation law you have studied in this chapter. (a) Show that the exact expression for the speed of light will be

$$u = \left(\frac{c + nv}{nc + v} \right) \frac{c}{n}.$$

(b) Expand the expression in part (a) for $v/c << 1$ keeping only the linear term in v/c and show that you get the approximate formula agrees with Fizeau's experiment.

Problem 1.3. Hafele and Keating Experiment Four cesium atomic clocks were flown around the world, once eastward and once westward, and their times were then compared with a reference clock at the U.S. Naval Observatory. The time difference in the clocks can be understood in terms of the time dilation due to gravity and relative motion. You will calculate here prediction of the time dilation due to motion only. Let R be the radius of Earth, ω the angular rotation speed of Earth, u the speed of flight with respect to the surface of Earth. Calculate the difference in times due to motion if $u = 232$ m/s, trip around Earth takes 48 hours in the Earth-bound clock in the following steps. We will assume that a clock based on the center of Earth is an inertial clock and the clocks at Earth's surface and in the planes move with uniform speed relative the clock at rest at the center of Earth. (a) First show that the time dilation formula simplifies for $v << c$ to be

$$\Delta t_{\text{lab}} = \Delta t_{\text{rest}} \left(1 + \frac{1}{2} \frac{v^2}{c^2} \right).$$

(b) Use the simplified formula for the time dilation to find the times elapsed t_s for the clock at the surface of Earth and that elapsed in the air plane t_a if the time elapsed in the clock at the center of the Earth is t_0. Assume the airplane is going towards East. (c) Deduce the relation between t_s and t_a. (d) Use your relation to find the time difference between the Earth-based clock and the flying clock for a 48-hour trip as observed in the Earth-based clock. Use speed of the flight to be $u = 232$ m/s and ignore the altitude of the plane compared to the radius of Earth.