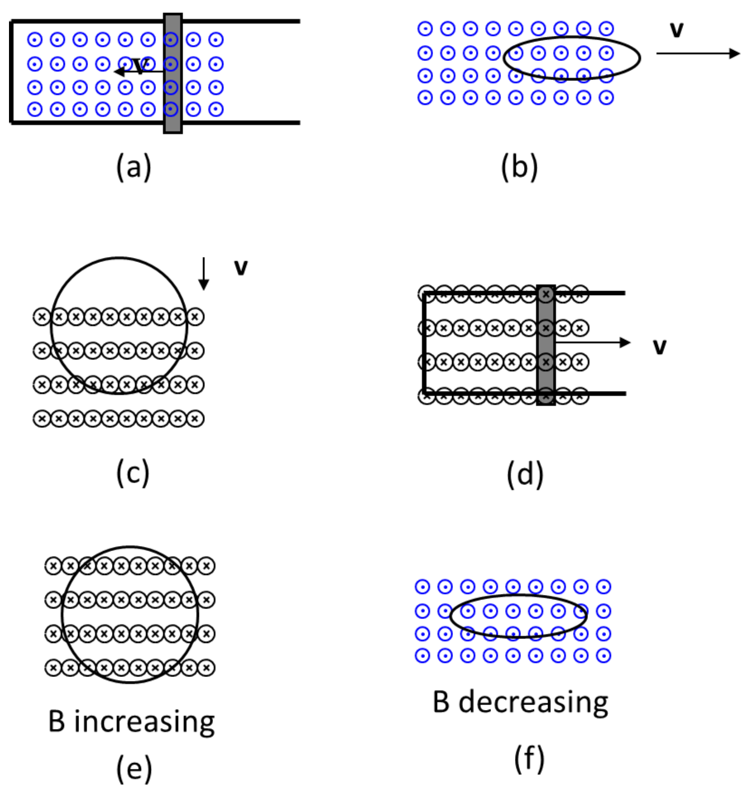
10.11 EXERCISES

Induced EMF and lenz's law

Ex 10.11.1. Use Lenz's law to determine the direction of induced current in each case. Here a circle with a dot, $\odot$ represents magnetic field pointed out-of-page and a circle with an x, $\otimes$ for a magnetic field pointed in-the-page, and lines are conductors.



Faraday's law problems

Ex 10.11.2. A metal bar of mass 500 g slides outward at a constant speed of 1.5 cm/s over two parallel rails separated by a distance of 30 cm which are part of a U-shaped conductor. There is a uniform magnetic field of magnitude 2 T pointing out-of page over the entire area. The railings and the metal bar have an equivalent resistance of 150 Ω. (a) Determine the induced current, both magnitude and direction. (b) Find the direction of the induced current if the magnetic field is pointed into the page. (c) Find the direction of the induced current if the magnetic field is pointed into the page and the bar moves inwards. Ans: (a) 6×10^{-5} A.

Ex 10.11.3. A current is induced in a circular loop of radius 1.5 cm between two poles of a horseshoe electromagnet when the current in the electromagnet is varied. The magnetic field in the area of the loop is perpendicular to the area and has a uniform magnitude. If the rate of change of magnetic field is 10 T/s, find the magnitude and direction of the induced current if resistance of the loop is 25 Ω. Ans: 2.83×10^{-4} A.

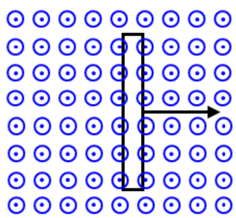


Figure 10.52: Exercise 10.11.4.

Ex 10.11.4. A metal bar of length 25 cm is placed perpendicular to a uniform magnetic field of strength 3 T. (a) Determine the induced EMF between the ends of the rod when it is not moving. (b) Determine the EMF when the rod if moving perpendicular to its length and magnetic field with a speed of 50 cm/s. Ans: (b) 0.375 V.

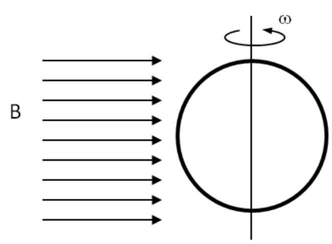


Figure 10.53: Exercise 10.11.5.

Ex 10.11.5. A circular loop of wire of radius 10 cm is mounted on a vertical shaft and rotated at a frequency of 5 cycles per second in a region of uniform magnetic of 2 G perpendicular the axis of rotation. (a) Find an expression for the time-dependent flux through the ring. (b) Determine the time-dependent current through the ring if it has a resistance of 10 Ω. Ans: (b) $1.97 \times 10^{-5} \sin(10\pi t)$ A

Ex 10.11.6. The current in a long solenoid of radius 3 cm is varied with time at a rate of 2 A/s. A circular loop of wire of radius 5 cm and resistance 2 Ω surrounds the solenoid. Find the electric current induced in the loop. Ans: 7.1 μA.

Ex 10.11.7. The current in a long solenoid of radius 3 cm and 20 turns/cm is varied with time at a rate of 2 A/s. Find the electric field at a distance 4 cm from the center of the solenoid. Ans: 3.33×10^5 N/C.

Ex 10.11.8. The magnetic field between the poles of a horseshoe electromagnet is uniform and has a cylindrical symmetry about an axis from the middle of the South pole to the middle of the North pole. The magnitude of the magnetic field changes at a rate of dB/dt due to the changing current through the electromagnet. Determine the electric field at a distance r from the center. Ans: Magnitude $(dB/dt)A/2\pi r$.

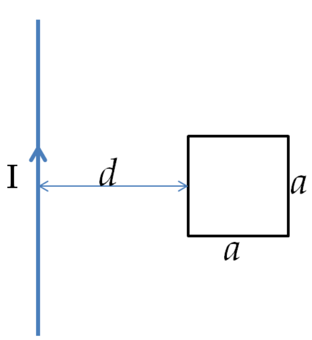


Figure 10.54: Exercise 10.11.9.

Inductance

Ex 10.11.9. (a) Determine the mutual inductance of a long wire and a square loop of side a lying in the same plane with a separation d. (b) Determine the amount of magnetic energy stored in the area of the square loop when a steady current I flows in the straight wire. Ans: (a) $\mathcal{M} = \frac{\mu_0 a}{2\pi} \ln\left(\frac{d+a}{d}\right)$.

Ex 10.11.10. (a) Determine the self-inductance per unit length of a coaxial cable with an inner hollow conductor of radius a and outer conductor of radius b. (b) Find the amount of energy in magnetic field stored per unit length of the coaxial cable if a current I flows in the inner wire and returns in the outer conductor. (Ignore the effect of the magnetism of the material.) Ans: (b) $U/h = \frac{\mu_0 I}{4\pi}\,(1/a - 1/b)$.

Ex 10.11.11. (a) Find the self-inductance of a loop of wire with N turns around a rectangular cross-section toroid of internal radius a, external radius b and height h. (b) Find the amount of energy in the magnetic field if a current I runs in the wire. Ans: (a) $L = (\mu_0 N h/2\pi)\ln(b/a)$.

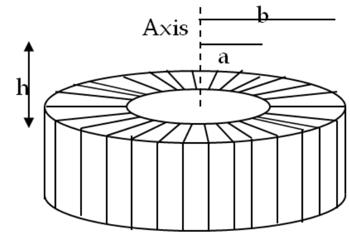


Figure 10.55: Exercise 10.11.11.

Ex 10.11.12. A large coil of inductance 200 H is connected to a varying current source such that current passing through the inductor is given by $I = 120\cos(400t)$ Amps. Find the voltage across the inductor. Ans: $9.6 \times 10^6 \sin(400t)$ V.

Ex 10.11.13. A 200-Ω resistor is connected to a 2-mH inductor and a 10-V source in series. The voltage source is turned on at $t = 0$, and the current starts to build. (a) Determine the current at large values of t. (b) What is the voltage drop across the inductor when maximum current flows through it? (c) What is the voltage drop across the resistor when the steady state has reached? (d) Determine the time it will take to build to 50% of the final current in the circuit. (e) Determine the time it will take to build $(1 - 1/e) \times 100\%$ of the final current. (f) Determine the time it will take to build to 90% of the final current. Ans: (a) 5.0×10^{-2} A, (d) 6.93×10^{-6} sec, (f) 2.3×10^{-5} sec.

Ex 10.11.14. A 100-V source is connected to a 1 kΩ resistor and a 30 H coil and the current is allowed to reach maximum. A switch is then thrown that disconnects the voltage source, but leaves the resistor and the inductor connected in another loop. Let us call this time $t = 0$. (a) Determine the time it will take for the current in the circuit to drop to 10% of its value at $t = 0$. (b) How much energy is dissipated in that time? Ans: (a) 69 ms, (b) 4.9 mJ.

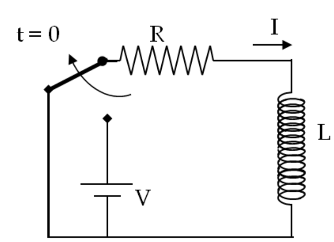


Figure 10.56: Exercise 10.11.14.

Ex 10.11.15. A 100-V source is connected to a 1 kΩ resistor and a 30 H coil and the current is allowed to reach maximum. A switch is then thrown that disconnects the voltage source, but leaves the resistor and the inductor connected in another loop. Let us call this time $t = 0$. After the current has decreased to 50% of its value, the connection to the battery is made by throwing the switch again in the reverse direction. (a) Find the time when the current reaches 75% of the maximum. (Same Figure as the exercise above.) (b) How much energy is supplied in total by the battery, both in the initially bringing the current to maximum and in bringing the current back to the 75% level from 50%.

Ex 10.11.16. A coil of resistance 0.2 Ω and inductance 0.4 mH is connected across a battery of voltage 12 volts and internal resistance 0.01 Ω and a switch S. The coil has radius 5 cm and length 30 cm. The switch is closed at $t = 0$. (a) Find the time it will take the current to reach 99% of its final value. (b) Find the amount of energy transferred to the coils as heat during that time? (c) How much energy is stored in the coils as magnetic field at $t = 4$ msec? (d) At what rate is the current increasing at t = 4 msec? Ans: (a) 9.2×10^{-4} sec, (b) 0.6 J, (c) 0.54 J, (d) 4.1×10^{3} A/s.