6.5 FLUIDS AT REST AND PASCAL'S PRINCIPLES

When a fluid is not flowing, we say that the fluid is in a static equilibrium. If the fluid is water the term **hydrostatic equilibrium** is used. In this condition, the net force on any part of a fluid at rest must be zero, otherwise the fluid will start to flow.

In 1653 the French mathematician, and physicist Blaise Pascal (1623-1662) published his Treatise on the Equilibrium of Liquids in which he outlined three principles of static fluids.

6.5.1 Pascal's First Principle

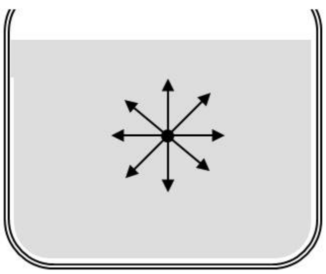

Figure 6.14: Pressure at a point is isotropic, meaning same in all directions from the point.

The force at any point inside a fluid due to the pressure at that point acts with the same magnitude in all directions.

That is, the force due to the pressure in a fluid will be the same on any area at an infinitesimal particle oriented in whichever manner (Fig. 6.14).

6.5.2 Pascal's Second Principle or Simply Pascal's Principle

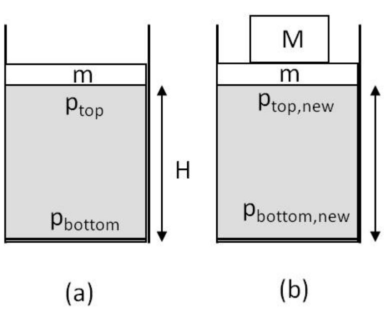

Figure 6.15: Pressure in fluid is changed when the fluid is pressed. The pressure at the top layer of the fluid is different than at the bottom layer. The increase in pressure by putting additional weight is same everywhere, e.g., $p_{\text{top new}} - p_{\text{top}} = p_{\text{bottom new}} - p_{\text{bottom}}$.

When pressure is changed (increased or decreased) at any point in an incompressible fluid, the pressure at all other points in the fluid changes to the same extent.

The key phrase here is "change in pressure". Often, the second principle of Pascal is also referred to as the **Pascal's principle**. Note that this principle does not say that the pressure is same at all points of a fluid, which is certainly not correct since the pressure in a fluid near earth varies with height. This principle rather addresses the change in pressure.

Suppose you have some water in a cylindrical container of height H and cross-sectional area A that has a move-able piston of mass m (Fig. 6.15).

Adding weight Mg at the top slide-able lid would increase the pressure at the top by Mg/A, since the additional weight also acts over area A of the lid.

$$\Delta p_{\text{top}} = \frac{Mg}{A}$$

According to Pascal's second principle, the pressure at all points of water will change by the same amount Mg/A. Thus, the change in pressure at the bottom will also be an increase of Mg/A. This seems surprising since one may expect the pressure at the top to change more, but that is not the case.

$$\Delta p_{\text{bottom}} = \frac{Mg}{A}$$

Since the pressure changes are the same everywhere in the fluid, we do not need to label the change with subscripts top or bottom and will drop the subscripts.

$$\Delta p = \Delta p_{\text{top}} = \Delta p_{\text{bottom}} = \Delta p_{\text{everywhere}}$$

6.5.3 Pascal's Third Principle

In a hydraulic jack, a small force applied over a small area can balance a much larger force on the other side over a larger area.

A **hydraulic jack** is a device that has an incompressible fluid in a U-tube fitted with move-able pistons on the two sides. One side of the U-tube is narrower than the other to make use of Pascal's third principle (Fig. 6.16).

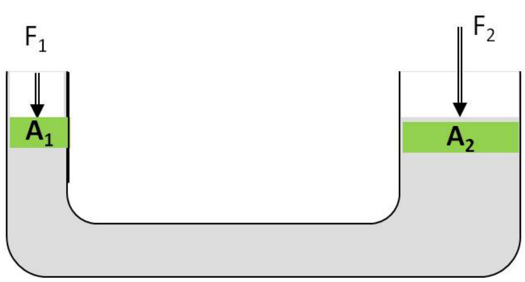

Figure 6.16: A hydraulic jack.

When a force $\vec{F}_1$ is applied on the side with the area of cross-section A_1 it adds pressure P_1 to all points of the fluid according to the second principle stated above. In particular an increase of pressure P_1 in the upward direction takes place on the piston A_2 on the other side. A second force must be applied on the piston A_2 to balance the effect of the increased pressure. Let $\vec{F}_2$ be the force on the second piston with the area of cross-section A_2 to balance the force on the second piston. Then we must have

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}. \tag{6.20}$$

Example 6.5.1. Lifting a car

Find the force needed to balance a 3000-kg car on the wider side of a hydraulic jack with the circular cross-sections of radii 10 cm and 0.5 cm.

Solution. From Pascal's third principle, it is immediately evident that the following force will be needed to balance the weight of the car.

$$F_1 = \frac{A_1}{A_2} F_2 = \left(\frac{\pi r_1^2}{\pi r_2^2} \right) F_2 = 73.5\ N,$$

which is the weight of a 7.5 kg mass.