5.9 EXERCISES

Kinetic Theory

Ex 5.9.1. A stream of particles of mass m strikes a wall of area A perpendicularly with speed v. Suppose that upon rebound particles emerge with speed v'. If there are n particles per unit volume, what is the pressure exerted on the wall? (Note the collision with the wall is not elastic.)

Ex 5.9.2. A stream of particles of mass m strikes a wall of area A perpendicularly with speed v, sticks to it and comes to rest. If there are n particles per unit volume, what is the pressure exerted on the wall? (Note the collision with the wall is not elastic.)

Ex 5.9.3. At moderately high temperatures where only rotation and translation modes of a molecule are excited, what is number of excited modes in water molecule?

Ex 5.9.4. What are the rms speeds of monatomic molecules of helium gas at (a) 3 K, (b) 30 K, and (c) 300 K?

Maxwell's Velocity Distribution

Ex 5.9.5. Plot Maxwell distributions for hydrogen gas (H_2) and helium gas (He) at 300 K. Label the peak, average and rms speeds on the plot.

Ex 5.9.6. The molecules for Maxwell distribution have only translational kinetic energy. Therefore, we can express their speed v in terms of energy E of a molecule.

$$\frac{1}{2}mv^2 = E \quad \text{(since PE = 0.)}$$

(a) Change the variable from v to E in the Maxwell distribution and obtain the distribution function for energy from the distribution function of speed. Note that distribution function for energy, lets say $g(E)$ is a function of energy E such that $g(E)dE$ gives the number of particles with energy between E and $E + dE$. The distribution of energy is called the Maxwell-Boltzmann energy distribution. Ans:

$$g(E)dE = \frac{2\pi N}{(\pi k_B T)^{3/2}}\sqrt{E}\exp\left(-E/k_B T\right)dE.$$

(b) Show that integration of $g(E)$ from $E = 0$ to ∞ yields N, the total number of molecules. (c) Plot $g(E)$ vs E for helium gas at $T = 100$ K, $T = 200$ K, and $T = 300$ K. (d) Calculate the average energy and show it is equal to $\frac{3}{2}k_B T$.

Ex 5.9.7. Since k_BT sets up an energy scale, $\sqrt{2k_BT/m}$ has units of speed and sets a reference scale for speeds of molecules. Let us express speed v in units of $\sqrt{2k_BT/m}$ and call the scaled speed u.

$$u = \frac{v}{\sqrt{2k_BT/m}}$$

Speed u will be dimensionless quantity with no units. Show that the Maxwell distribution for molecules with speed between u and $u+du$ is given by the following.

$$h(u)du = \frac{4N}{\sqrt{\pi}}u^2 \exp\left(-u^2\right)du.$$

Ex 5.9.8. If we divide the distribution function $f(v)$ by the total number of molecules in the system, we will get a normalized distribution function $f_N(v)$.

$$f_N(v) = \frac{f(v)}{N}$$

The product $f_N(v)dv$ tells us the fraction of all molecules that have speed between v and $v+dv$. (a) Show that the integration of $f_N(v)$ over all possible speeds $[0,\infty)$ is equal to 1. Therefore, f_N can be interpreted as a probability density with f_Ndv as the probability that a molecule will have speed between v and $v+dv$. (b) Find the most probable speed, average speed, and standard deviation from the average speed. (c) Plot the standard deviation of speed as a function of temperature and state what happens to the standard deviation as temperature is raised. The standard deviation of speed tells us the degree to which the molecular speeds are spread out from the average.

Mean Free Path

Ex 5.9.9. Find the mean free path, mean free time and inter-molecular distances in a container of volume 5 L that contains 2 grams of Helium gas at 25°C and 1 atmosphere, assuming the diameter of each Helium molecule to be 1 Å. Mean free time is the average time a molecule spends in-between collisions. (1 Å= 0.1 nm).

Ex 5.9.10. Repeat for the diatomic Nitrogen gas assumed to be of "sphere" of diameter 3 Å.

Ex 5.9.11. (a) What would happen to the mean free path in Exercise 5.9.9 if you double the pressure while keeping the other quantities same? (b) What happens to the mean free path in Exercise 5.9.9 if you double the temperature while keeping the other quantities same?

(c) What happens to the mean free path in Exercise 5.9.9 if you double the volume while keeping the other quantities same? (d) What happens to the mean free path in Exercise 5.9.9 if you double the amount of gas, i.e. mass, while keeping the other quantities same? (e) What happens to the mean free path in Exercise 5.9.9 if you double the mass and volume while keeping the other quantities same?

Humidity

Ex 5.9.12. A meteorologist reports that the humidity is 60% on a day when temperature is 77°F. What is the partial pressure of water in the air that day?

Ex 5.9.13. (a) Plot the saturated vapor pressure of water versus the temperature. (b) Plot the saturated vapor density of water versus temperature. (c) On your plots, show what happens when you start with 50% humidity at 77°F and lower the temperature to 50°F, and describe in words what you expect.