5.4 INTERNAL ENERGY OF A MONATOMIC IDEAL GAS

The kinetic theory can be used to calculate the internal energy of an ideal gas. The internal energy of a collection of gas molecules will be equal to the sum of the average kinetic and potential energies of all molecules. Since the molecules of a monatomic ideal gas do not interact with each other, they do not possess any potential energy. The molecules of monatomic ideal gas have only kinetic energy.

The average kinetic energy of one molecule is

$$\boxed{K.E. = \frac{1}{2}m < v^2 > . \quad \text{(one molecule)}} \tag{5.20}$$

The quantity $< v^2 >$ is square of the root-mean square speed. We have shown in Example 5.2.2 that $< v^2 >$ is given by the following expression by Maxwell velocity distribution:

$$\boxed{< v^2 >= v_{rms}^2 = \frac{3k_BT}{m}.}$$

Therefore, kinetic energy of one molecule is

$$\boxed{K.E. = \frac{3}{2}k_BT \quad \text{(one molecule)}} \tag{5.21}$$

If the monatomic ideal gas has N molecules, the internal energy U will be

$$U = \frac{3}{2}Nk_BT \quad (N \text{ molecules}) \tag{5.22}$$

This shows that internal energy of an ideal gas only depends on the amount of the gas and the temperature. We can write this equation in term of number of moles n by expressing N as $N = nN_A$ where N_A is **Avogadro's number**. When writing the formula in moles, the product k_BN_A is normally replaced with the gas constant R.

$$\boxed{U = \frac{3}{2}nRT \quad (n \text{ moles})} \tag{5.23}$$

The factor 3 in these equations comes from three translational degrees of motion for each molecule. Each translational degree of freedom contributes $\frac{1}{2}k_BT$ per molecule equally. This result is known as the equipartition of energy.