1.5 PROBLEMS

Problem 1.5.1. A rocket of mass M is moving at constant velocity $\vec{v}$ in zero gravity environment. It ejects fuel from the back at a steady rate of $(dm/dt = \alpha)$ at speed u with respect to the rocket. What is the increase in speed of the rocket when a mass m_f of fuel has been ejected? Do this problem in the frame of the rocket.

Ans: $v_x(T) - v_0 = u \ln\left(\frac{M}{M-m_f}\right)$.

Problem 1.5.2. A stone is tied to a string and rotated in a vertical circle at constant speed v inside an accelerating train. The circle of rotation is perpendicular to the train's velocity as observed from a frame outside the train. The train has a constant acceleration of magnitude A and direction East with respect to an observer outside the train. The train was at rest at $t = 0$. Find the tension in the string at four instances in the vertical circle of the motion of the stone: (a) when the stone is at the top, (b) when the stone is at the bottom, (c) when the stone is horizontal left, and (d) when the stone is horizontal right.

Ans: $T_x = mA$, $T_y = mg - m\frac{v^2}{R}$.

Problem 1.5.3. Calculate the percentage mistake made in evaluating the acceleration due to gravity when ignoring the rotation of earth?

Ans: 0.35%.

Problem 1.5.4. Calculate the percentage mistake made in evaluating acceleration due to gravity when ignoring the revolution of earth around the sun?

Ans: 0.06% of g.

Problem 1.5.5. You are inside a large enclosed container and everything in the entire container, including you, is rotating at a constant rate about some fixed axis, but you do not know the rate of rotation. To find the rate of rotation you place a penny at different places on a frictionless floor, and discover that the penny accelerates everywhere you place the penny. The directions of the acceleration of the penny at various places on the flat floor meet at a point X. (a) If the penny accelerates at 30 m/s^2 at a distance of 10 meters from point X, what is the rate of rotation of the entire container? (b) What is the acceleration of the penny when it is at a point 20 meters from point X?

Ans: (a) $\Omega = \sqrt{3}$ rad/s. (b) 60 m/s^2.

Problem 1.5.6. A pendulum of length L and mass m in an accelerating frame is in equilibrium at an angle θ_0. (a) What is the acceleration of the accelerating frame with respect to an inertial frame? (b) When the pendulum is disturbed from the equilibrium angle by a small angle θ about θ_0, the pendulum oscillates. Determine the frequency of small oscillations in the accelerating frame.

Ans: (a) $A = g \tan\theta_0$. (b) $\omega = \sqrt{\dfrac{A^2 + g^2}{l}}$.

Problem 1.5.7. A stone is dropped from rest from a height of 200 meters in a place at a latitude of 60 degrees and longitude 30 degrees. Where will it land on the surface of the Earth?

Ans: Local directions: x vertically up, y towards East, z towards North. $x = 0$, $y = -\frac{1}{3}\Omega_z g T^3$, $z = \frac{1}{6}\Omega_x \Omega_z g T^4$. Here $\Omega_x = \Omega \sin\lambda$ and $\Omega_z = \Omega\cos\lambda$, Ω is the rotation speed of Earth, λ is the latitude at the point on the surface of the Earth, $T = \sqrt{2h/g}$, h is the height of fall.

Problem 1.5.8. A stone is dropped from rest from a height of 200 meters in a place at a latitude of 30 degrees and longitude 60 degrees. Where will it land on the surface of the Earth?