1.3 BENDING OF LIGHT AT PLANE INTERFACE

1.3.1 Critical Angle and Total Internal Reflection

According to the law of refraction, if $n_2 < n_1$, then the angle of refraction (θ_2) ¿ angle of incidence (θ_1). Thus, if you look at various incident rays corresponding to increasing angles of incidence, you will reach a particular angle of incidence, called the **critical angle** θ_c, for which the angle of refraction will be 90° as shown in Figs. 1.15 and 1.16.

$$n_1 \sin\theta_c = n_2 \sin 90^\circ \quad (n_2 < n_1).$$

Hence,

$$\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right) \quad (n_2 < n_1) \tag{1.16}$$

All rays of light that are incident on the interface at greater angle of incidence than the critical angle θ_c are reflected back in the n_1-medium. The phenomenon is called the **total internal reflection**. The concepts of critical angle and total internal reflection find their use in Fiber optics as explained next.

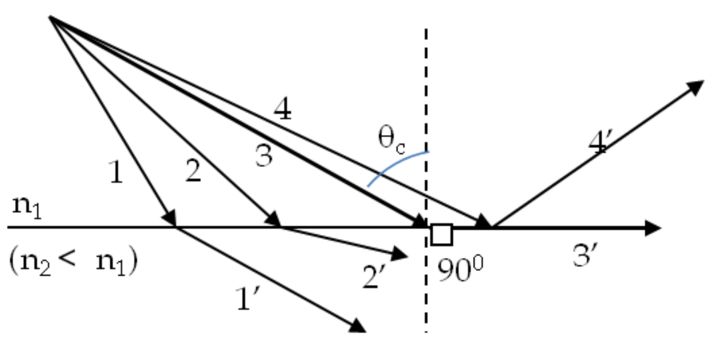


Figure 1.15: Critical angle and total internal reflection. Ray 3 corresponds to the critical angle while ray 4 is totally internally reflected. There is no ray in the medium labeled n_2 correponding to ray 4.

Example 1.3.1. Cricitcal Angle. Find the critical angle for air/water interface.

Solution. The critical angle lies in the medium of higher refractive index, here water, and has the following value.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.00}{1.33}\right) = 48.8^\circ.$$

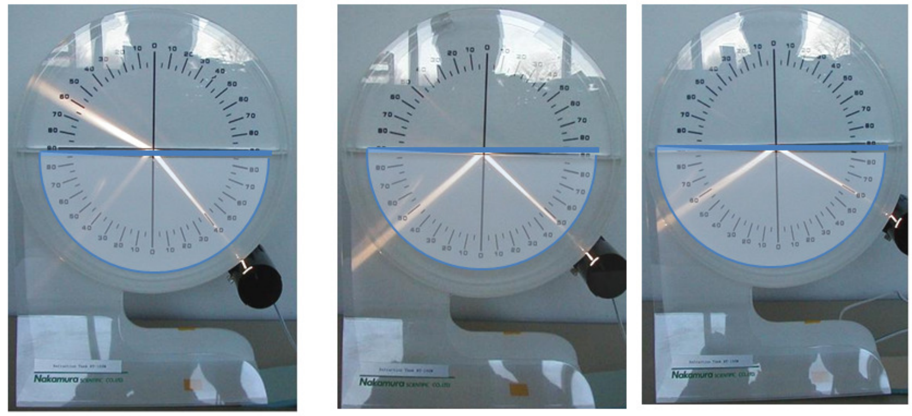


Figure 1.16: Total internal reflection demonstrated (MS). Rays from water incident on the horizontal water/air interface at (a) 40°, (b) 49° (infinitesimally larger than the critical angle) and (c) 60° at water/air interface are shown. For incident rays greater than critical angle, there are no refracted rays in air.

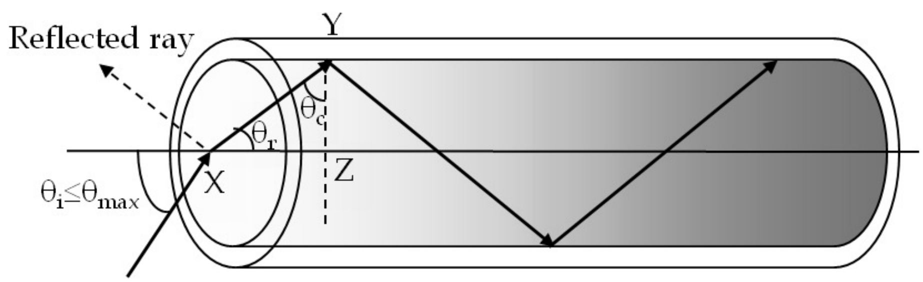


Figure 1.17: Fiber optics. A thin layer of a material of lower refractive index than the refractive index of the material of the fiber surrouonds th fiber. The thin layer is called cladding. If the ray enters the fiber at large enough angle the ray will be totally internally reflected at the fiber/cladding interface and hence travel down the fiber.

Fiber Optics

Fiber optics makes use of the phenomenon of total internal reflection to guide light in a dielectric optical fiber (Figs. 1.17 and 1.18). A thin layer of a transparent material of lower index of refraction, called **cladding**, is deposited on the surface of the fiber to prevent the leakage of light from one fiber to another in a bundle of fibers.

When a ray of light traveling in the fiber strikes the fiber/cladding interface at an incidence angle greater than the critical angle θ_c, it will be totally reflected back in the fiber. Therefore, there is a maximum angle θ_{max} at which a ray must enter the fiber (point X) if it is to travel by total internal reflection. If the ray enters at a greater angle then

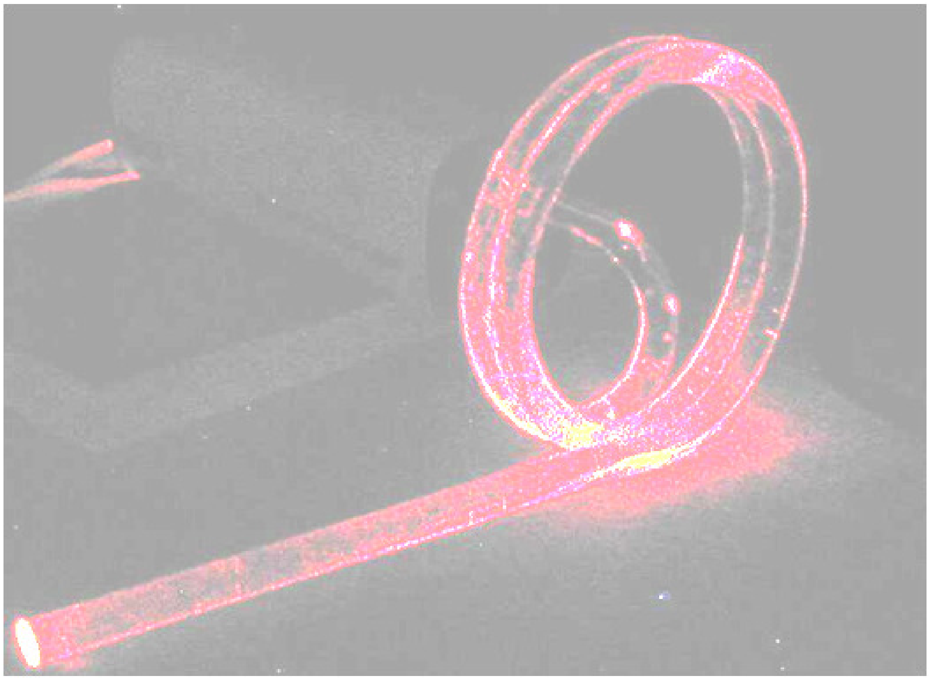


Figure 1.18: Picture of Laser light through an optical fiber.

the refracted ray XY inside the fiber will strike the fiber/cladding interface at less than the critical angle. This would lead to the leakage of light through the fiber/cladding interface. We can find $\theta_{\max}$ as follows. Let n_f be the refractive index of fiber and n_c that of the cladding. From Snell's law at the entry point X we have the following relation.

$$n_f \ \sin\theta_r = n_{\text{air}} \ \sin\theta_{\max} \tag{1.17}$$

From triangle XYZ we have

$$\theta_r = \frac{\pi}{2} - \theta_c. \tag{1.18}$$

The critical angle for fiber/clad interface is given by

$$\sin\theta_c = \frac{n_c}{n_f}. \tag{1.19}$$

From the last three equations, we find the following for $\sin\theta_{\max}$.

$$n_{\text{air}} \sin\theta_{\max} = \sqrt{n_f^2 - n_c^2} \tag{1.20}$$

The product $n_{\text{air}} \sin\theta_{\max}$ is called the **numerical aperture** or **NA**, which is a measure of the light gathering power of the fiber. Optical fibers are used in a number of applications - communication, e.g. transatlantic fiber cable, and medical imaging, e.g. various endoscopes to probe inside organs.

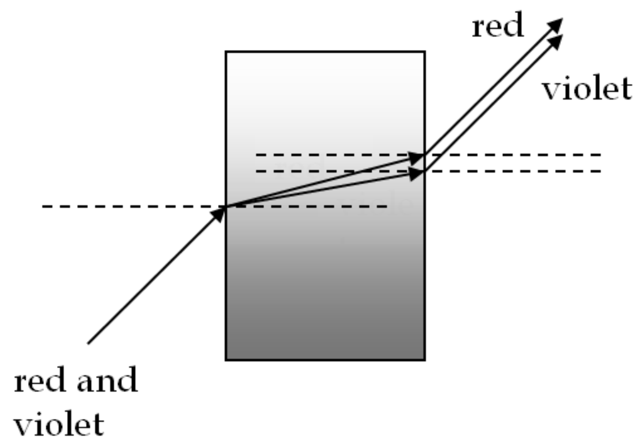


Figure 1.19: Dispersion of light through a glass plate.

1.3.2 Dispersion of light

Since refractive index depends on the frequency (i.e. color) of light, the angle of refraction will be different for light of different colors. For instance, the refractive index of light flint glass varies from approximately 1.67 for violet light to 1.62 for red light. This phenomenon is called the **dispersion of light**. If the refractive index is higher for higher frequency, the dispersion is called **normal dispersion**, otherwise it is called **anomalous dispersion**. If a mixture of red and violet light is incident on a light flint glass slab at an angle 45°, the angle of refraction of violet and red lights will be 25° and 26° respectively. As a result the colors will emerge separated as shown in Fig. 1.19.

One can magnify the effect of separation of colors by orienting the second refracting face at an angle to the first refracting face as done in a triangular prism. The amount of separation of colors can then be stated by the difference in the angles of deviation of each ray with respect to the original direction (Fig. 1.20).

1.3.3 Bending of Light Though a Prism

You may be familiar with a triangular piece of glass as prism, but a prism may have any shape that provides more than one plane interface for a light ray. In a triangular prism, a light ray incident on one face may emerge from any of the three faces after two or more refractions and reflections, and consequently the direction of light may be changed drastically. In the following example, we illustrate the calculation of the angle of deviation of a light ray passing through two faces of a prism.

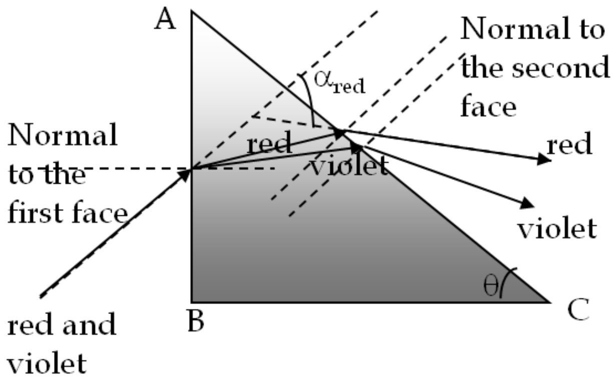
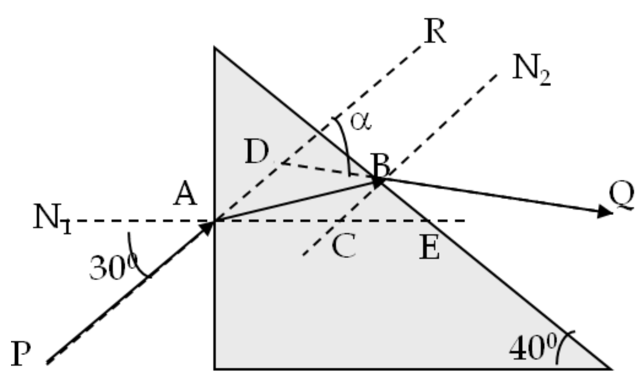


Figure 1.20: Dispersion of light in a glass prism.

Example 1.3.2. Deviation of a Ray by a Prism. Consider a ray of light incident on a right-angled triangular prism with a 40° angle at the base as shown in the figure. Let the angle of incidence be 30°. Find the angle of deviation assuming the index of refraction of glass and air for the given light to be 1.5 and 1.0 respectively.



Solution. Using Snell's law at A we obtain angle ∠BAC to be:

$$\angle\mathrm{BAC} = \sin^{-1}\left(\frac{\sin 30^\circ}{1.5}\right) = 19.5^\circ.$$

Since △BEC in the right angled triangle ∠BEC is 40° because the side CE is parallel to the base, angle ∠ BCA = 90° + 40° = 130°. In triangle △ACB, angle ∠ABC = 180° - 130° - 19.5° = 30.5°. This is the angle of incidence at the second interface. Now applying Snell's law at B, we obtain the angle $\angle\mathrm{N_2BQ}$ to be:

$$\angle\mathrm{N_2BQ} = \sin^{-1}(1.5 \times \sin 30.5^\circ) = 49.6^\circ$$

In rectangle ADBC, we know the following angles.

$$\angle\mathrm{DAC} = 30^\circ,\ \angle\mathrm{BCA} = 130^\circ,\ \angle\mathrm{CBD} = 49.6^\circ.$$

Therefore, we find the angle ∠ADB to be

$$\angle\mathrm{ADB} = 360^\circ - 30^\circ - 130^\circ - 49.6^\circ = 150.4^\circ.$$

Hence the angle of deviation ∠RDQ is equal to $180^\circ - 150.4^\circ = 29.6^\circ$.