8.4 CONSERVATION OF ENERGY

The principle of conservation of energy has a broader scope than stated above in Eq. 8.38 as we explain in this section. The forces in the work-energy theorem can also be divided into internal forces and external forces as opposed to conservative and non-conservative forces. For instance, suppose you are pulling a cart, then if we can consider you and the cart as one system, then the force of the Earth on you and the cart will be external forces while the force on the cart by you and its pair force, i.e. the force of cart on you, will be the internal forces. The work energy theorem then becomes.

$$W_{\text{int}} + W_{\text{ext}} = K_f - K_i. \tag{8.62}$$

A force can be either conservative or non-conservative. As we have seen above that we can write the work done by a conservative force as causing a change in the potential energy of the object upon which the force acts. Therefore, another way of writing Eq. 8.62 is in terms of conservative and non-conservative forces.

$$W_{\text{int}}^{\text{cons}} + W_{\text{int}}^{\text{non-cons}} + W_{\text{ext}}^{\text{cons}} + W_{\text{ext}}^{\text{noncons}} = K_f - K_i. \tag{8.63}$$

We now combine all the work by non-conservative forces and write the result as W_{NC} and combine all the work by conservative e forces and write the result as change in potential energy $U_i - U_f$ to obtain

$$W_{\text{NC}} + U_i - U_f = K_f - K_i. \tag{8.64}$$

This equation can be rearranged to obtain the change in $(K + U)$ as

$$[K_f + U_f] - [K_i + U_i] = W_{\text{NC}}, \tag{8.65}$$

which is written more simply as

$$\boxed{E_f - E_i = W_{\text{NC}},} \tag{8.66}$$

Therefore, the energy of a system does not change in the absence of non-conservative forces.

Example 8.4.1. Two bodies interacting by a conservative force Show that the mechanical energy of an isolated system consisting of two bodies interacting with a conservative force is conserved.

For discussion, let us label two objects labelled 1 and 2. Since the system is isolated, there is no external force on the system and we have

$$W_{\text{int}} = (K_f - K_i)_1 + (K_f - K_i)_2. \tag{8.67}$$

The work W_{int} is the work by force by 1 on 2 and force by 2 on 1. Since these forces are are assumed to be conservative, their works can be written in terms of changes in the potential energies of objects 1 and 2 respectively.

$$W_{\text{on }1} = (U_i - U_f)_1 \tag{8.68}$$
$$W_{\text{on }2} = (U_i - U_f)_2. \tag{8.69}$$

Using Eqs. 8.67, 8.68 and 8.69 we find

$$[E_f - E_i]_1 = -[E_f - E_i]_2\,, \quad \Leftrightarrow \quad \boxed{\Delta E_1 = -\Delta E_2,} \tag{8.70}$$

where $E = K + U$ for each particle, the subscripts i and f refer to the initial and final states respectively, and ΔE is the change in energy. Therefore, if there are no external forces on a system so that $W_{\text{ext}} = 0$, i.e. when a system is isolated, and all the internal forces are conservative, then the total energy of the system cannot change.

It is interesting to note that for two bodies in this example, the work on 1 by force from 2 and the work on 2 by force from 1 are conduits for the transfer of energy from one to the other. The energy of the parts of a system, such as the energy of object 1 or the energy of object 2 may change with time due to the interaction between them, but the increase in energy of one will be accompanied by an equal decrease in the energy of the other.

Further Remarks

Since all the fundamental forces in nature are conservative forces, at the most basic level all systems are conservative and energy of an isolated system is always conserved.

<u>Principle of conservation of energy: The energy of an isolated system cannot change with time.</u>

The principle of conservation of energy is one of the most fundamental laws of nature that holds true even when Newton's laws of motion fail although we have arrived at it through the use of Newton's laws.

Example 8.4.2. Skier down hill with air resistance. Use the same problem as in Example 8.3.1 above, but this time, suppose that the work done by the air-resistance cannot be ignored. Let the work done by the air resistance when the skier goes from A to B along the given hilly path be -2000 Joules. The work done by the sir resistance is negative since the air resistance acts in the opposite direction to the displacement. Suppose the mass of the skier is 50-kg, what is the speed of the skier at point B?

Solution. Note that mechanical energy is not conserved in this situation since we have non-conservative force on the system. Therefore, we must use the full work-energy theorem given in Eq. 8.66.

$$W_{\rm NC} = (K_f + U_f) - (K_i + U_i)\,.$$

$$W_{\rm NC} = \left(\frac{1}{2}mv_B^2 + 0\right) - (0 + mgh)$$

Solve for v_B and then plugging in the numbers to find v_B.

$$v_B = \sqrt{\frac{2(W_{\rm NC} + mgh)}{m}} = 18 \text{ m/s}.$$

The speed v_B is less when the work by air resistance is taken into account. The mechanical energy of the skier is lost to the air particles and the internal energy of the molecules of the skier. We say that the energy of the system is converted to the thermal energy.