7.5 TORQUE ON CURRENT IN LOOP

Torque on a Flat Current Loop

The net magnetic force on a current in a closed loop placed in a uniform magnetic field is zero since the magnetic forces on currents on elments on the opposite sides of the loop cancel each other. As explained above, the magnetic force on the current in the wire leads to effective inertial force on the atoms of the wire. Since magnetic forces that cancel each other act at different parts of the loop, the corresponding effective forces on the atoms of the wire form a couple. Thereofre, if the wire is pivoted properly, the resulting torque can cause an angular acceleration. To study the rotation by the effective forces will give rise to a net non-zero torque causing the physical body of the loop to gain rotatational acceleration.

To calculate the torque of the effective forces on the wire, let us consider a loop of current in the shape of a rectangle placed in a uniform magnetic field. We will examine the torque at an instant when the normal to the loop makes an angle θ with respect to the magnetic field. Let the loop be pivoted about an axis perpendicular to the magnetic field and passing through the middle of two of the sides of the loop as shown in Fig. 7.27.

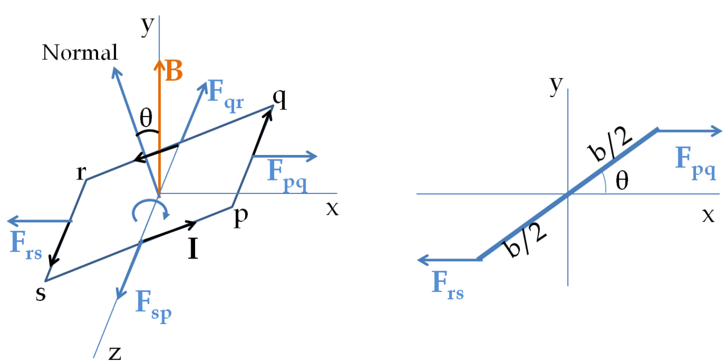

Figure 7.27: Torque on a rectangular current loop. The side pq is of length a while side qr is of length b. Current I flows in the wire and the uniform magnetic field B is pointed along the x-axis. The axis of rotation is pointed along the z-axis. The magnetic force acts on the conduction electrons. The figure on right side shows forces in the xy-plane.

The effective forces on qr and ps, which are equal to the magnetic forces on the conduction electrons making the steady current I in the wire along the lengths of the wire, are parallel to the z-axis and will not have any torque about the z-axis. We only need torques by effective forces $\vec{F}_{pq}$ and $\vec{F}_{rs}$ on pq and rs respectively, which are equal in magnitude but opposite in direction. It is sufficient to figure out torque by $\vec{F}_{pq}$ only. Using the magnetic force on the current in the wire, this effective force on the wire is given by

$$\vec{F}_{pq} = -Ia\hat{u}_z \times B\hat{u}_y = IaB\hat{u}_x$$

where a is the length of the pq segment. To get a better feel of the direction of the forces, Fig. 7.27 shows the forces in the xy-plane. Torque by $\vec{F}_{pq}$ about center of the loop has only the z-component.

$$\tau_z^{pq} = -IaB\frac{b}{2}\sin\theta. \tag{7.48}$$

The torque by $\vec{F}_{rs}$ about center of the loop has the same value and direction as $\vec{\tau}_{pq}$. Therefore, the z-component of the net torque on the loop is

$$\tau_z^{net} = -IabB\sin\theta. \tag{7.49}$$

If you wind the wire in many loops each carrying the current in the same direction then the torque will be enhanced. For an N-loop structure the torque will be N times as much.

$$\tau_z^{net} = -NIabB\sin\theta. \tag{7.50}$$

This torque can be used to understand the working of an electric motor.

Torque on Magnetic Dipole

We can write the torque on a current loop by introducing a quantity called the magnetic dipole moment of the current loop so that the formula for the magnetic torque has the same form as the electric torque on the electric dipole. For this purpose we define the magnetic dipole moment μ of a flat current loop as equal to the product of area of the loop and the current in the loop, and the direction of the magnetic dipole moment vector is normal to the loop in the direction indicated by "Normal" in Fig. 7.27. Let us denote this direction by a unit vector $\hat{u}_n$ in that direction. Then magnetic dipole moment vector will be

$$\vec{\mu} = Iab\hat{u}_n. \quad \text{(One Loop)} \tag{7.51}$$

For N loops the magnetic dipole moment will be N times this quantity. From the expression for the z-component of torque on the current loops, given in Eqs. 7.49 and 7.50, we deduce that the z-component of torque on the magnetic dipole will be

$$\tau_z = -\mu B \sin\theta = \left(\vec{\mu} \times \vec{B}\right)_z. \tag{7.52}$$

In general, we the torque on a magnetic dipole is given in the vector form as

$$\vec{\tau} = \vec{\mu} \times \vec{B}. \tag{7.53}$$

Torque on a Three-dimensional Current Loop

What happens when the loop of current is not planar? The torque may be obtained by calculating torque by completing loops in each plane by adding "fictitious" branches of wires where current runs in both direction. Consider a loop in two planes as shown in Fig. 7.28.

By introducing branches *c*-*f* and *f*-*c*, we are able to generate two independent loops with same current: *a*-*b*-*c*-*f*-*a* and *c*-*d*-*e*-*f*-*c*. The magnetic dipole moment of the original non-planar current loop is vector sum of the magnetic moments of the two constituent loops so obtained. In Fig. 7.28, the net magnetic moment is:

$$\vec{\mu}_{net} = \vec{\mu}_1 + \vec{\mu}_2. \tag{7.54}$$

Example 7.5.1. Magnetic Moment of a Two-dimensional Loop

Consider a non-planar loop as shown. Find the magnetic dipole moment.

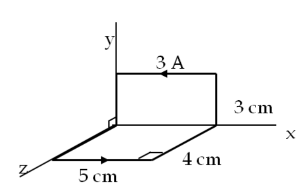

Figure 7.29: Example 7.5.1.

Solution. First we add a horizontal branch at the bend with current in both directions, one of which is used for one plane and the other

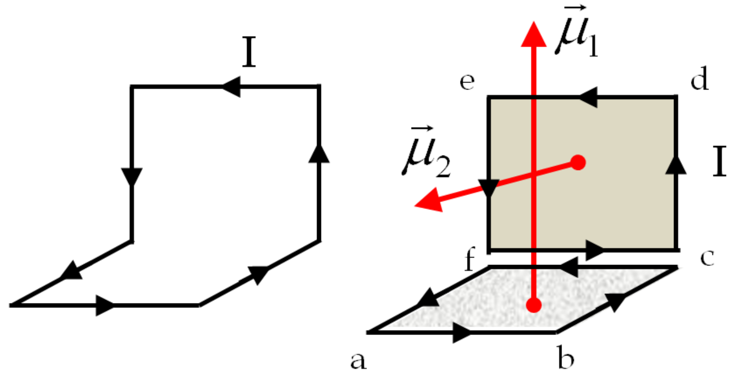

Figure 7.28: A current loop spread over two non-coplanar areas in three-dimensions.

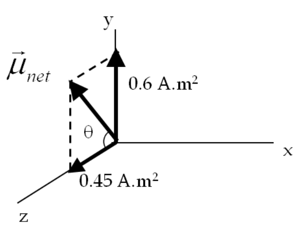

Figure 7.30: Example 7.5.1.

for the other plane. The magnetic dipole moment of the loop in the xz-plane is 0.6 A.m^2 pointed in the positive y-axis direction. The magnetic dipole moment of the loop in the xy-plane is 0.45 A.m^2 pointed along the positive z-axis.

Adding the two parts vectorially, we obtain the magnetic dipole moment to be 0.75 A.m^2, pointed 53° from the z-axis towards the y-axis in the yz-plane.