3.5 ENERGY IN THE ELECTRIC FIELD

Each electric charge is accompanied by its own electric field. We can show that when it comes to the electrical energy associated with static charges, you have a dual picture - you can think the electrical energy as potential energy of particles interacting via electric force or energy stored in electrical field.

To make this point clear, let us consider a simple situation of two large sheets of area A facing each other with opposite charges of equal amount as shown in Fig. 3.23. The electric force per unit

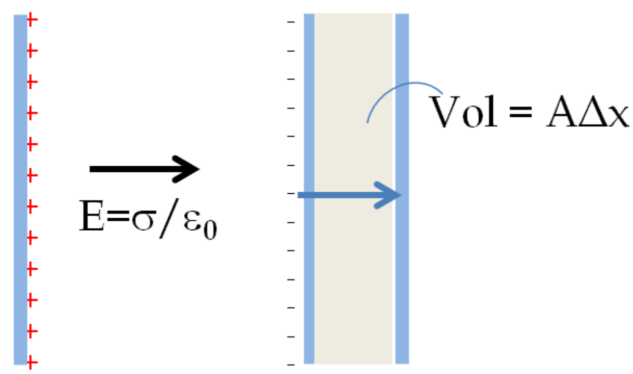

Figure 3.23: Work done in moving the negative plate is stored in electric field energy in the new space between plates.

charge on each sheet is equal to the constant electric field between the plates. Now, suppose while holding one sheet, say the positively charged sheet fixed, I pull the other sheet by a distance Δx.

How much work would I need to do? The applied force will need to balance the force by electric field at the negative plate, which has the magnitude σ/ϵ_0, where $\sigma = Q/A$. Therefore, work required

$$W = \sigma/\epsilon_0 \times Q \times \Delta x.$$

This will increase the potential energy of the charges on the plates by the same amount

$$\Delta U = \sigma/\epsilon_0 \times Q \times \Delta x,$$

which can be written completely in terms of the magnitude of the electric field between the plates and the amount of the new space between the plates.

$$\Delta U = \epsilon_0 E^2 \times A\Delta x. \tag{3.60}$$

The quantity $\epsilon_0 E^2$ has unit of energy per unit volume, or energy density. We notice that by moving the plate by a distance Δx, there is now an electric field in additional volume $A\Delta x$ between the plates. Therefore, if we identify $\epsilon_0 E^2$ as energy density in electric field, then work done by the external agent can be interpreted to go into the energy of the electric field. The energy per unit volume, u_E, in electric field is defined by this expression.

$$u_E = \epsilon_0 E^2 \quad \text{(energy density in electric field} \tag{3.61}$$

The energy in electrostatic E-field therefore gives one a choice of how to account for energy in the system - you may think of electrostatic energy as potential energy of the particle or energy in the electric field. Beware that this situation changes completely when we study the dynamical behavior of moving charges and the corresponding electromagnetic radiation. In an electrodynamics situation the energy and momentum of the dynamic electromagnetic field must be added to those of particles in order to properly account for the conservation of energy and momentum; you do not have the choice of thinking that the energy is either stored in the field or in the particles - you must account for both separately.