## 1.4 EXERCISES

### Accelerating Frame

**Ex 1.4.1.** Two friends John and Jane observe the location of a building from their own frames which are accelerating with respect to each other. In Jane's frame John has a constant acceleration $\vec{A}$ whose direction makes an angle $\theta$ with respect to the direction of the building. Let the building be at a distance $R$ from Jane who is fixed to the Earth.

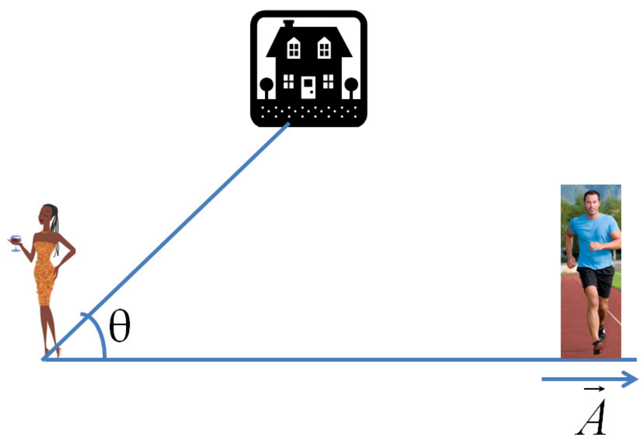


Figure 1.14: Exercise 1.4.1.

Suppose at $t = 0$, John and Jane were at the same location and their relative velocity was zero. (a) Draw a figure in Jane's frame showing a choice of coordinates for both John and Jane supposing that the position vector of the building and acceleration vector $\vec{A}$ are in the $xy$-plane. You can also take the $x$-axis to be the direction of vector $\vec{A}$. (b) Deduce the relation between the coordinates of the building in the two frame at an arbitrary time? (c) Describe the motion of the building in the two frames.

**Ex 1.4.2.** A box sits on a rubber floor in a truck. The driver starts the truck and increases the acceleration steadily from zero to 5 m/s$^2$ in 2 seconds. If the coefficient of static friction between the box and the rubber floor is 0.2, determine if the box will slide, and if so, at what acceleration?

Ans: The box will not slide.

**Ex 1.4.3.** The coordinates of a particle in space from two frames

$Oxyz$ and $O'x'y'z'$ are related as follows.

$$
\begin{aligned}
x' &= x + (5 \text{ m/s})\ t \\
y' &= y + (5 \text{ m/s})\ t + (2 \text{ m/s}^2)\ t^2 \\
z' &= z
\end{aligned}
$$

(a) Describe the relative motion of the two frames with respect to each other. (b) If a particle has a constant velocity with respect to $Oxyz$ with components $(2 \text{ m/s}, 0, 0)$, what are the velocity and acceleration of this particle with respect to $O'x'y'z'$? (c) If a particle has a constant velocity with respect to $O'x'y'z'$ with components $(2 \text{ m/s}, 0, 0)$, what are the velocity and acceleration of this particle with respect to $Oxyz$?

Ans: (c) $v_x = -\ 3$ m/s, $v_y = -\ 3 \text{ m/s} - (4 \text{ m/s}^2)\ t$, $v_z = 0$; $a_x = 0$, $a_y = -\ 4 \text{ m/s}^2$, $a_z = 0$.

**Ex 1.4.4.** A box is sliding on a floor so that its coordinates with respect to a frame $Oxyz$ fixed with respect to the floor are given as

$$
\begin{aligned}
x &= (1 \text{ m}) + (2 \text{ m/s})\ t + (3 \text{ m/s}^2)\ t^2 \\
y &= z = 0
\end{aligned}
$$

What would be the coordinates of this box when observed with respect to another frame $O'x'y'z'$ that has the following acceleration with respect to $Oxyz$?

$$
\begin{aligned}
A_x &= 2 \text{ m/s}^2 \\
A_y &= A_z = 0
\end{aligned}
$$

Assume origins $O$ and $O'$ coincide at $t = 0$ and the axes of the two frames are parallel with each other.

Ans: $x' = 1 \text{ m} + (2 \text{ m/s})\ t + (1 \text{ m/s}^2)\ t^2$, $y' = 0$, $z' = 0$.

**Ex 1.4.5.** A box is sliding on a floor so that its coordinates with respect to a frame $Oxyz$ fixed with respect to the floor are given as

$$
\begin{aligned}
x &= (1 \text{ m}) + (2 \text{ m/s})\ t + (3 \text{ m/s}^2)\ t^2 \\
y &= 0 \\
z &= 0
\end{aligned}
$$

What would be the coordinates of this box when observed with respect to another frame $O'x'y'z'$ that has the following acceleration with respect to $Oxyz$?

$$
\begin{aligned}
A_x &= 0 \\
A_y &= 2 \text{ m/s}^2 \\
A_z &= 0
\end{aligned}
$$

Assume the origins $O$ and $O'$ coincide at $t = 0$ and the axes of the two frames are parallel with each other.

Ans: $x' = (1 \text{ m}) + (2 \text{ m/s})t + (3 \text{ m/s}^2)t^2$, $y' = -(1 \text{ m/s}^2)t^2$, $z' = 0$.

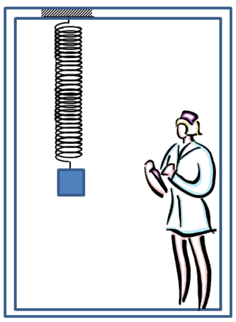

Figure 1.15: Exercise 1.4.6.

**Ex 1.4.6.** A block of mass $m$ is hung from the ceiling of an elevator using a spring of spring constant $k$. Find the change in the length of the spring under the following situations. (a) Elevator going up with constant velocity, $\vec{v}_1$. (b) Elevator going down with constant velocity, $\vec{v}_2$. (c) Elevator going up with acceleration $\vec{a}_1$ point up. (d) elevator going up with acceleration $\vec{a}_2$ pointed down. (e) elevator going down with acceleration $\vec{a}_3$ pointed up. (f) elevator going down with acceleration $\vec{a}_4$ pointed down.

Ans: (a) $\Delta l = mg/k$. (b) $\Delta l = mg/k$. (c) $\Delta l = \frac{m}{k}(g + a_1)$. (d) $\Delta l = \frac{g}{k}(g - a_2)$. (e) $\Delta l = \frac{g}{k}(g + a_3)$. (f) $\Delta l = \frac{g}{k}(g - a_4)$.

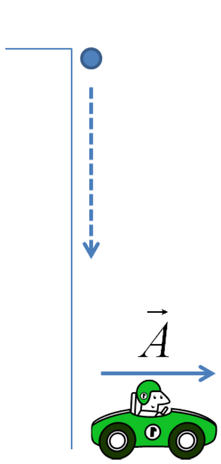


Figure 1.16: Exercise 1.4.7.

**Ex 1.4.7.** A ball is dropped from a 50-m building. In a frame fixed to Earth, the ball drops vertically with a constant acceleration $g$ pointed down. The same ball is observed from a car that is accelerating with respect to the fixed frame. The acceleration of the car is pointed horizontally and away from the building.

Suppose the fixed frame has a coordinate system $Oxyz$ whose $y$-axis is pointed up and the $x$-axis is pointed in the direction of the acceleration of the car. Let $O'x'y'z'$ be the coordinate system whose origin is fixed to the car. The coordinate axes of the two frame are parallel to each other. The time $t = 0$ is chosen when the two origins coincide and the velocity of the car is zero with respect to the ground. (a) Write the coordinates of the ball in $Oxyz$ frame during the flight, i.e., before the ball hits the ground. (b) Write the coordinates of the ball in $O'x'y'z'$ frame during the flight, i.e., before the ball hits the ground. (c) What are the trajectories of the ball in the two frames? Do the trajectory equations make sense, i.e. does the ball fall vertically in both frames?

Ans Key: (c) straight down in one frame, slanted in another frame.


Figure 1.17: Exercise 1.4.8.

**Ex 1.4.8.** A freely falling sky diver looks at another diver directly above him that has his parachute on and finds his position in a coordinate system that has the $y$-axis vertically up as follows.

$$y = 4 + 5t + 3t^2.$$

At $t = 0$, the freely falling person was 100 meters from the ground and had zero speed. Find the position of the person with his parachute on as a function of time when observed from the ground?

Ans: $x' = 0$, $y' = (104) + 4$ t - $\frac{1}{2}(g-3)t^2$, $z' = 0$.

## Rotating Frame

**Ex 1.4.9.** A large platform is rotating uniformly with an angular speed $\Omega$ about the $z$-axis of a fixed frame $Oxyz$ with the origin at the center of the platform. A man is on the platform at a distance $R$ from the center. (a) What is the motion of the man in the fixed frame? (b) What is the motion of the man with respect to a frame $O'x'y'z'$ that has the same origin and $z$-axis as the fixed frame but whose $x$ and $y$-axes rotate with the platform? (c) Let the man be along the $x'$ axis at $t = 0$ when he starts to walk directly towards the center at a constant speed $v'$. That is, he is walking along the $x'$ axis towards origin with speed $v'$ with respect to the origin $O'$. What is the motion of the person as seen from the fixed frame?

**Ex 1.4.10.** A car in Los Angeles, California, USA, is moving with a velocity of 30 m/s towards North with respect to an observer on the ground. Find the magnitude and direction of the Coriolis force on the car. Los Angeles, USA has latitude 34.0522° N, and longitude 118.2428° W.

Ans: $F_c = 7.3$ N towards the local East.

**Ex 1.4.11.** A car of mass 3000 kg in Canberra, Australia, is moving with a velocity of 30 m/s towards North with respect to an observer on the ground. Find the magnitude and direction of the Coriolis force on the car. Canberra, Australia has latitude 35.2828° S, and longitude 149.1314° E.

Ans: Magnitude $F_c = 7.56$ N. Hint: Figure out the direction using the cross product.

**Ex 1.4.12.** In exercise, Ex 1.4.9, write the equation of motion of the CM of the man with respect to (a) the fixed frame and (b) the rotating frame.

Ans: (a) In the fixed frame, Centripetal: $F_s = m\Omega^2 R$; Vertical: $F_N - mg = 0$.

**Ex 1.4.13.** A satellite is in the Geocentric orbit about Earth at a distance $R$ from the center of the Earth. The satellite revolves in a circular motion with the angular speed $\Omega$, which is equal to the angular speed of the Earth about the same axis, as observed from a fixed frame. (a) Describe the motion of the satellite with respect to a frame with the origin at the center of Earth and which rotates with the Earth. (b) Write the equation of motion of the satellite in the two frames.

Ans: (b) Fixed Frame: $G_N \frac{Mm}{R^2} = m\frac{v^2}{R}$

**Ex 1.4.14.** A bead of mass $m$ can slide frictionlessly on a rotating ring of mass $M$ and radius $R$ which rotates at an angular speed of $\omega$ about a vertical axis through its center. When the bead is at a particular location, it does not slide. (a) Find this special position of the bead using calculations in a rotating frame. (b) Repeat the calculation in an inertial frame.

Ans: (a) $\theta = \cos^{-1}\left(\frac{g}{\omega^2 R}\right)$ with $\omega^2 R > g$.

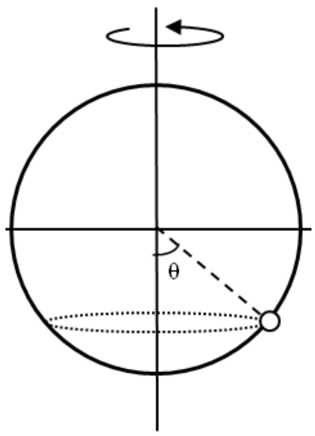

Figure 1.18: Exercise 1.4.14.

**Ex 1.4.15.** A block of mass $M$ is attached to a string of length $l$. The other end of the string is attached to a post at the center of a rotating platform. When the platform is rotating at a steady angular speed $\Omega$ the block moves in a circular motion. Find the radius of the circular motion of the block performing the calculations in a rotating frame.

Ans: $\frac{l}{\omega^2}\sqrt{\omega^4 l^2 - g^2}$.

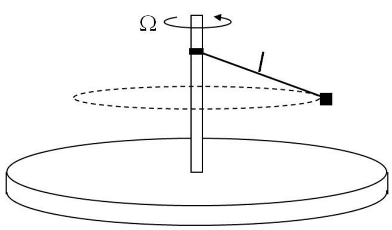

Figure 1.19: Exercise 1.4.15.