9.4 THE MAGNETIC FIELD OF A MAGNETIC MATERIAL

9.4.1 The Magnetic Dipole Field

A magnetic material is made up of microscopic magnets. We find the magnetic field by these microscopic magents by modeling them by tiny current loops and determining the magnetic field due to currents in these loops. To find the magnetic field of a macroscopic magnetic material, it makes sense to review the magnetic field of a single magnetic dipole as obtained in an earlier chapter from the magnetic field on a current I in a circular loop of radius R. The magnetic field at a point on the axis of the loop far away from the loop was found to be

$$\vec{B}_P = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\hat{u}_z \quad \text{(Point P on the axis)} \tag{9.61}$$

where $\mu = I\pi R^2$, the loop was placed in xy plane and $\vec{\mu}$ pointed along z axis. The magnetic field of a magnetic dipole at a point that is not on the axis is more difficult to work out. Here we just cite the magnetic field at a point P that is located at a distance r from the origin and line from origin to P makes an angle θ with respect to z axis for a magnetic dipole placed at the origin in the z axis direction.

$$\vec{B}_P = \frac{\mu_0\mu}{4\pi r^3}\left(2\cos\theta\hat{u}_r + \sin\theta\hat{u}_\theta\right) \quad \text{(P at arbitrary location)} \tag{9.62}$$

This field is called dipole field due to its distance dependence of $1/r^3$. The dipole field is symmetric about the direction of the dipole - you get the same picture when you rotate about the direction of the dipole. Therefore, we can just draw field lines in xz plane as shown in Fig. 9.10. You can rotate the field lines in your mind to figure out what it would look like in three dimensional space.

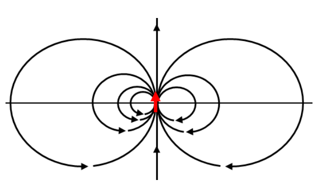

Figure 9.10: Magnetic dipole field lines of a magnetic dipole.

9.4.2 Magnetic Field Of A Thin Bar Magnet

The calculation of magnetic field of a thin bar magnet from the magnetic field of its constituent magnetic dipoles provides an important example. We will calculate the magnetic field of a thin bar magnet using the formula for the magnetic field on a point on the axis of a dipole. We will make use of superposition principle and add contributions of dipoles located along the bar magnet.

Consider a bar magnet that is uniformly magnetized along z axis as shown in Fig. 9.11. We wish to find the magnetic field at a point P on z axis that is far away from the bar magnet.

Let there be N magnetic dipoles per unit length, each with magnetic moment μ. This says that magnetization of the bar magnet is $M = \mu n$ per unit length. Therefore, the z component of the magnetic dipole moment in length dz' of the bar will be

$$d\mu' = M dz'. \tag{9.63}$$

We use prime to denote the dipole moments and locations on the bar to distinguish the location of the field point P. The z component of the magnetic field at point P from the dipoles in the element dz' between z' and $z' + dz'$ of the bar magnet is

$$dB_P = \frac{\mu_0}{2\pi} \frac{M dz'}{(z - z')^3} \tag{9.64}$$

Integrating over the sample, i.e. from $z' = -L/2$ to $z' = L/2$, we find the z component of the magnetic field at point P.

$$B_P = \frac{\mu_0 M}{\pi} \left[\frac{1}{(2z - L)^2} - \frac{1}{(2z + L)^2} \right] \tag{9.65}$$

The direction of the magnetic field is towards positive z axis in the figure. Note, since the point P is far away from the bar ($z >> L$), the pathology of $z = \pm L/2$ does not arise in the application of this formula.

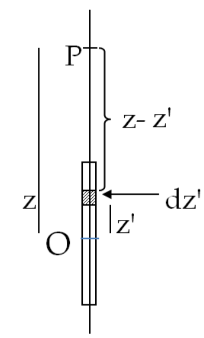

Figure 9.11: Coordinate for calculation of magnetic field of a br magnet.

9.4.3 Magnetic Field Of A Uniformly Magnetized Sphere

Another simple system that lends itself to an easy calculation is a uniformly magnetized sphere where microscopic dipoles are all lined up parallel to each other and distributed with a uniform density. The calculation can find application to an understanding of Earth's magnetic field.

Magnetic field at a point on the axis

We find the magnetic field at a point P on the axis, which can be inside or ouotside the magnetic sphere. Let sphere be magnetized along z axis with magnitude M. We will first calculate the magnetic field at point P from dipoles in a slab of infintesimal thickness dz' perpendicular to z axis located between z' and $z' + dz'$. The calculation is a little different here than the way we dod the calculation for a bar magnet. Here, we make use of the microscopic current picture of dipoles to simplify the calculations.

We first divide up the slab into small cells containing microscopic dipoles and replace these tiny dipoles by loop currents as displayed in Fig. 9.12. The microscopic current picture shows that the slab of magnet is equivalent to a ring of current at the booundary of the slab.

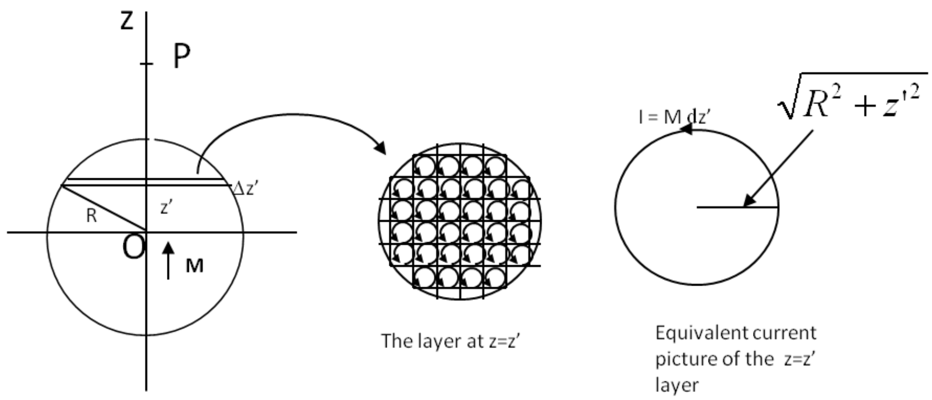

Figure 9.12: Equivalent current picture for uniformly magnetized sphere of radius R. Note the current inside the material add up to zero due to uniform nature of magnetization $\vec{M}$. The equivalent current, also called bound currents, in a uniformly magnetized material are all on the surface of the spere.

Note that each cell has a magnetic moment $\Delta\mu = M\Delta A dz'$, where $\Delta A dz'$ is the volume of a cell. In order for the loop of current to represent the dipole moment in the cell we must have current $I = M dz'$ so that $I\Delta A$ equals $\Delta\mu$.

$$I = M dz' \tag{9.66}$$

Since M is uniform, all the equivalent bound currents have the same magnitude but on the internal sides of the adjacent loops, two equal and opposite currents cancel. Therefore, the only bound current in the slab at z' is one on the outside strip. Thus for the purposes of calculation of magnetic field, the slab of magnetic dipoles can be replaced by a circular wire of radius of the slab at z' and current $M dz'$. We take the origin at the center of the sphere, then at z', the radius of the slab will be $\sqrt{R^2 - z'^2}$. Therefore, the magnetic field at point P from dipoles of the slab between z' and $z' + dz'$ is

$$dB_z = \frac{\mu_0 (M dz')}{2} \frac{R^2 - z'^2}{\left(R^2 - z'^2 + z^2\right)^{3/2}} \tag{9.67}$$

We must integrate this from $z' = -R$ to $z' = R$ to include all the dipoles of the sphere. This gives

$$B_z(0,0,z) = \frac{\mu_0 (M)}{2} \int_{-R}^{R} \frac{R^2 - z'^2}{\left(R^2 - z'^2 + z^2\right)^{3/2}} dz'. \tag{9.68}$$

This is the magnetic field at a space point on the z axis. The result of integration depends upon whether the field point is inside the sphere or outside. A student is strongly encouraged to perform the integration and show the following result.

$$B_z(0,0,z) = \begin{cases} \frac{2}{3}\mu_0 M & (r < R) \\ \frac{\mu_0}{2\pi}\left(\frac{4}{3}\pi R^3 M\right)\frac{1}{r^3} & (r > R) \end{cases} \tag{9.69}$$

Magnetic field at a non-axis point

The calculation for magnetic field at a non-axis point is involved and will be skipped. We will quote the results and let it a challenge to a more ambitious student to prove them. The magnetic field at a distance r from the center of the sphere and at an angle θ from the direction of the dipole is given by the following formula in spherical coordinates.

$$\vec{B}(r,\theta,\phi) = \begin{cases} \frac{2}{3}\mu_0 \vec{M} & (r < R) \\ \frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3}\left(2\cos\theta\hat{u}_r + \sin\theta\hat{u}_\theta\right) & (r > R) \end{cases} \tag{9.70}$$

where $\vec{m}$ is the total magnetic field of the spherical magnet.

$$\vec{m} = \frac{4}{3}\pi R^3 \vec{M} \tag{9.71}$$

The magnetic field outside is that of the total magnetic moment placed at the center of the sphere. Figure 9.13 displays a plot of the magnetic field of a uniformly magnetized sphere.

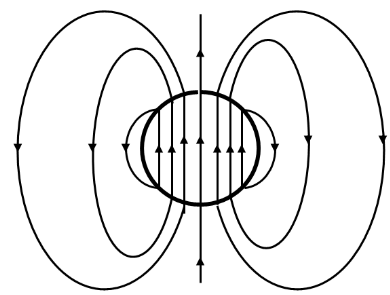

Figure 9.13: Magnetic field lines of a uniformly magnetized sphere.