2.6 EXERCISES

Gravitational Force

Ex 2.6.1. Evaluate the magnitude of gravitational force between (a) two 5-kg spherical steel balls separated by a center-to-center distance of 15 cm, (b) the Sun and the planet Neptune which have an average separation of 30.15 AU. $M_{Sun} \approx 2.0 \times 10^30\ kg$ and $M_{Neptune} \approx 1.0 \times 10^26\ kg$, 1 AU = 150,000,000 km.

Ans: (a) 7.4×10^{-8} N, (b) 6.5×10^{20} N.

Ex 2.6.2. Find the net gravitational force on a 2-kg steel spherical ball by two other spherical balls with masses 20-kg and 30-kg in the configuration shown in Fig. 2.17. Give both the magnitude and the direction.

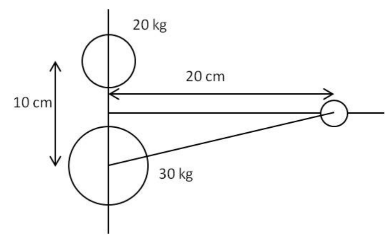

Figure 2.17: Problem 2.6.2.

Ans: 1.45×10^{-7} N, 2.87° below horizontal in third quadrant.

Ex 2.6.3. Find the gravitational force of the Moon on a bucket of water of mass 1 kg placed at two sides of the Earth, one facing the Moon and the other exactly on the opposite side. (a) Find the force of the Moon on the two buckets. (b) Think of a way to use your calculations to explain why there are two tides per day.

Ans Key: $F_1 - F_2 = 2.21\ \mu$N.

Ex 2.6.4. There is a small sphere of mass m a distance D from the edge of a uniform rod of length L and mass M as shown in the Fig. 2.18. (a) Find the gravitational force on the mass m. [Hint: You will need to integrate.] (b) Compare the magnitude of this force to the force between two point masses M and m separated by a distance $D + \frac{L}{2}$. (c) Find the limit of the ratio of the magnitudes for $L >> D$. (d) From these calculations, decide of you can ever replace a rod by a point mass at the center of the rod for the purposes of gravitational force by the rod?

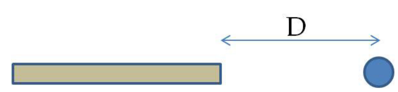

Figure 2.18: Problem 2.6.4.

Ans: (a) Magnitude: $G_N \frac{mM}{L}\left(\frac{1}{D} - \frac{1}{D+L}\right)$.

Ex 2.6.5. There is a small sphere of mass m a distance D from the edge of a uniform rod of length L and mass M placed perpendicularly to the rod as shown in the Fig. 2.19. Find the gravitational force on the mass m.

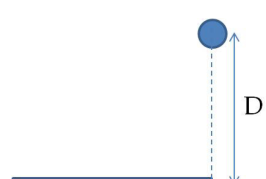

Figure 2.19: Problem 2.6.5.

Ans: With x-axis to the right and the y-axis up the components of the force are $F_x = -G_N \frac{mM}{L}\left[\frac{1}{D} - \frac{1}{\sqrt{L^2+D^2}}\right]$, $F_y = -G_N \frac{mM}{D\sqrt{L^2+D^2}}$.

Ex 2.6.6. There is a small sphere of mass m at a height H from the center of a uniform ring of radius R and mass M as shown in the Fig. 2.20. (a) Find the gravitational force on the mass m. (b) Compare

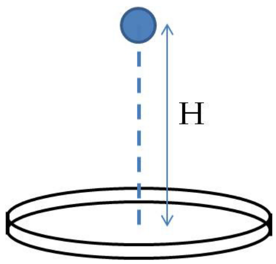

Figure 2.20: Problem 2.6.6.

this force to a force between two point masses m and M separated by a distance H.

Ans: (a) Magnitude: $G_N \frac{mMH}{(R^2+H^2)^{3/2}}$. (b) Hint: different from (a).

Ex 2.6.7. Evaluate the gravitational potential energy between (a) two 5-kg spherical steel balls separated by a center-to-center distance of 15 cm, (b) the Sun and the planet Neptune which have an average separation of 30.15 AU. $M_{Sun} \approx 2.0 \times 10^30$ kg and $M_{Neptune} \approx 1.0 \times 10^26$ kg.

Ans: (a) -1.11×10^{-8} J. (b) -2.95×10^{33} J.

Ex 2.6.8. Find the escape speed of a projectile from the following planets, (a) Earth, (b) Mars, (c) Saturn, and (d) Jupiter.

Ans: (a) 11,200 m/s; (b) 5,000 m/s; (c) 36,200 m/s; (d) 60,300 m/s.

Ex 2.6.9. (a) From the average distance of the Earth from the Sun, and the orbital period of the Earth, find the centripetal acceleration of the Earth for its motion about the Sun. (b) Compare the value found in (a) to the gravitational force of the Sun on the Earth per unit mass of the Earth.

Ans: (a) $a_c = 0.006$ m/s^2. (b) $F/M_E = 0.006$ m/s^2.

Ex 2.6.10. (a) A satellite is in a circular orbit around the Earth at a distance R from the center of the Earth. Find a formula for its period in terms of the distance R, the mass M of the Earth and the radius R_E of Earth. (b) Find an expression for the mechanical energy of the satellite.

Ans: (a) $T = 2\pi\sqrt{\frac{R^3}{G_N M}}$. (b) $E = -\frac{1}{2} G_N \frac{Mm}{R}$.

Ex 2.6.11. (a) Find the reduced mass of the Earth/Sun system consisting of the Earth and the Sun. (b) Where is the center of mass of the Earth/Sun system located. (c) Find the angular momentum of the Earth about the center of mass of the Earth/Sun system. (d) Where in the orbit does the Earth move fastest and where does it move slowest? Why? (e) Is the kinetic energy of the Earth with respect to the CM of the Earth/Sun system constant during its flight around the Sun? (f) Is the angular momentum of the Earth with respect to the CM of the Earth/Sun system constant during its flight around the Sun?

Ans: (b) 4.5×10^5 m from the center of the Sun.

Ex 2.6.12. Derive $\vec{r}_1{}'$ and $\vec{r}_2{}'$ in terms of the relative coordinate $\vec{r}$ in the CM frame.

Ex 2.6.13. Derive $\vec{L} = \vec{r} \times \mu \frac{d\vec{r}}{dt}$ from $\vec{L} = \vec{r}_1{}' \times m_1 \vec{v}_1{}' + \vec{r}_2{}' \times m_2 \vec{v}_2{}'$.

Kepler's Laws and Orbit Equation

Ex 2.6.14. The closest and farthest distance of the Earth from the Sun are 0.98 AU and 1.02 AU, where, the Astronomical Unit, 1 AU = 149,598,000 km. Find (a) the eccentricity and (b) the semi-major axis of Earth's orbit.

Ans: (a) 0.02, (b) 1.00 AU.

Ex 2.6.15. The closest and farthest distances of Mars from the Sun are 1.38 AU and 1.67 AU. Find (a) the eccentricity, (b) the semi-major axis, and (c) the period of Mars's orbit.

Ans: (a) 0.095, (b) 1.52 AU, and (c) 1.87 yr.

Ex 2.6.16. The closest and farthest distance of Mercury from the Sun are 0.31 AU and 0.47 AU. Find (a) the eccentricity, (b) the semi-major axis, and (c) the period of Mercury's orbit.

Ans: (a) 0.206, (b) 0.39 AU, (c) 2.92 mo.

Ex 2.6.17. The orbit of Halley's comet is approximately elliptical with $e = 0.967$. Halley's comet comes around every 76 years. Find (a) the distance of the closest approach to the Sun and (b) the farthest distance the comet goes from the Sun.

Ans: (a) 0.604 AU. (b) 36.0 AU.

Ex 2.6.18. A satellite of mass 2000 kg is put in an elliptical orbit of eccentricity 0.5 about Mars, whose mass is approximately 6.4×10^{23} kg and radius 3.4×10^6 m. The distance from the center of the planet to the closest approach of the satellite is equal to $\frac{7}{6}$ times the radius of the planet. (a) Find the distance to the farthest point. (b) Find the energy and angular momentum of the satellite. (c) Find the speed at the closest approach. (d) Using the angular momentum conservation between the farthest and the closest approach, find the speed at the farthest point.

Ans: (a) 1.2×10^7 m. (b) $E = -5.4{\times}10^9$ J, $l = 3.18{\times}10^{13}$ kg.m/s. (c) 4,029 m/s. (d) 1,343 m/s.

Ex 2.6.19. What is the radius of the circular orbit for a satellite of mass 2500 kg about the Earth if it has an energy of -2×10^9 J?

Ans: 2.49×10^8 m.