6.8 EXERCISES

Steady Currents

Ex 6.8.1. How many electrons are present in 1 gram samples of (a) Copper, (b) Gold, and (c) Aluminum? Atomic weights: Cu 63.5, Au 197, Al 27; Atomic numbers Cu 29, Au 79, Al 13. Ans: (a) 2.75×10^{23}, (b) 2.41×10^{23}.

Ex 6.8.2. In a thin wire of copper 20 milliamperes of current is flowing. How many electrons are passing through any point every minute? Ans: 7.5×10^{18}.

Ex 6.8.3. A steady current of 2 amps flows through a number 12 wire gauge (0.0808 inch diameter) made of copper. (a) Find the current density. (b) How many conducting electrons are there in one inch of the wire? (c) How many total electrons are there in a one-inch of the wire? (d) Find the drift speed of electrons. Ans: (a) 6.05×10^5 A/m^2, (b) 6.56×10^{21}, (c) $29 \times 6.56 \times 10^{21}$,(d) 4.85×10^{-5} m/s.

Ex 6.8.4. A steady current of 2 amps flows through a number 12 wire gauge (0.0808 inch diameter) made of gold. (a) Find the current density. (b) How many conducting electrons are there in one inch of the wire? (c) How many total electrons are there in a one-inch of the wire? (d) Find the drift speed of the electrons. Ans: (a) 6.05×10^5 A/m^2, (d) 6.42×10^{-5} m/s.

Ex 6.8.5. How much current flows if the current density on the surface of an aluminum plate of width 3 cm is 20 A/m? Ans: 0.6 A.

Ex 6.8.6. A current of 10 amp passes through a cylindrical copper pipe of inner radius 2 cm and outer radius 2.5 cm. Find the current density. Ans: 1.4×10^4 A/m^2.

Ex 6.8.7. A copper pipe of inner radius 1.5 cm and outer radius 1.8 cm surrounds a copper rod of rod of radius 0.5 cm. A 5 amp current flows in the pipe in one direction and the same amount flows in the inner rod in the opposite direction. Write the current density as a function of distance from the center. Ans: $J_{rod} = 6.37 \times 10^4$ A/m^2, $J_{shell} = 1.61 \times 10^4$ A/m^2.

Ex 6.8.8. A steel cylindrical rod of radius 5 cm carries 30 amperes of current uniformly spread through out its cross section. (a) How much current flows within 1 cm of its center? (b) How much current flows between 2 cm and 3 cm of its center? (c) How much of the current flows between 4 cm and 5 cm of its center? Ans: (a) 1.2 A, (b) 6 A, (c) 10.8 A.

Ex 6.8.9. A 4-gauge copper wire of diameter 0.2038 inch carries 10 amp current with a non-uniform current density. The current density increases linearly with distance from the center, $J = as$, with constant a=5 A/m3 and s is the distance from the axis of the wire in m. Find current flowing within 0.1 mm of the outer surface, and compare it with the current that flows within 0.1 mm of the center. Ans: 9.42 A versus 1.73 A.

Ex 6.8.10. A 10-gauge copper wire of diameter 0.102 inch carries 10 amp current with a non-uniform current density. The current density increases with distance from the center as given by $J = \frac{a}{b^2+s^2}$, with constant a= 10 A, b = 0.1 m, and s is the distance from the axis of the wire in m. Find current flowing within 0.1 mm of the outer surface, and compare it with the current that flows within 0.1 mm of the center.

Ohm's Law

Ex 6.8.11. Find the resistances of 10 meters of (a) 14-gauge copper wire (diameter 0.0642 in), (b) 10-gauge copper wire (diameter 0.102 in), and (c) 14-gauge nichrome wire. Ans: (a) $8.1 \times 10^{-2}\Omega$, (b) $3.2 \times 10^{-2}\Omega$, (c) 4.8Ω.

Ex 6.8.12. A 2-meter wire of radius 1 mm of an unknown material with 10 volts across its ends carries a current of 0.5 amp. (a) Find the conductivity of the material. (b) Evaluate the resistivity of the material. Ans: (a) $3.18 \times 10^4\ \Omega^{-1}\text{m}^{-1}$, (b) $3.18 \times 10^{-5}\ \Omega$.m.

Ex 6.8.13. A newly discovered material is made into a slab of width 3 mm, length 2 cm and thickness 1 mm. It is found that 2 mV across its length produces 1 μA current. (a) What is the conductivity of the material? (b) Evaluate the resisitivity of the material. Ans: (a) $1.06\ \Omega^{-1}\text{m}^{-1}$, (b) $9.43 \times 10^{-1}\ \Omega$.m.

Ex 6.8.14. A constant voltage power source across a 12-gauge Nichrome wire (radius 0.0404 in) of length 20 m produces a current of 3 A. (a) What is the voltage across the wire? (b) Find the electric field in the wire. Ans: (a) 18.2 V, (b) 0.91 V/m.

Ex 6.8.15. An aluminum wire of thickness 1 mm and length 50 m is connected across a 3 V battery. Find the current density in the wire. Ans: 2.2 MA/m^2.

Ex 6.8.16. (a) How much is the resistance of a 60-W bulb that operates at a voltage of 220 volts? (b) If in 10 seconds bulb emits 200 J of energy in light, what is its efficiency? Ans: (a) 810Ω, (b) 33.3%.

Ex 6.8.17. A six-inch coil electric heater uses 1500 Watts power at the high setting. If the voltage across the element is 120 V, what is the resistance of the element. Ans: 9.6Ω.

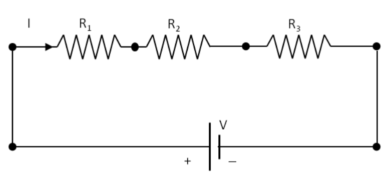
Ex 6.8.18. A copper rod of length 50 cm and diameter 4 mm carries a current of 2 A. If the specific heat of copper is 380 J/kg.K, what is the rate at which its temperature rises? Use density of copper 8.23 g/cc. Ans: 1.36×10^{-4} K/s.

Ex 6.8.19. An aluminum rod of length 25 cm and diameter 2 mm is insulated. When a current is passed through the rod, its temperature rises at the rate of 20 degrees per minute. Find the current. Density of aluminum 2.7 g/cc, specific heat aluminum 900 J/kg.K. Ans: 17.2 A.

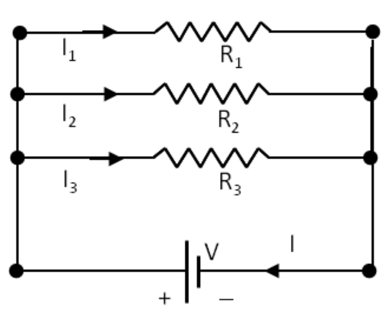
Ex 6.8.20. A 25 cm-copper wire of thickness 0.4 cm is placed inside one liter water at 20°C. The container of mass 100 grams is made up of aluminum which in turn is insulated. A current of 5 A is passed through the copper wire for 15 sec. Find the final temperature of water. Use specific heat of water 4.186 J/g.K.

Simple DC Circuits

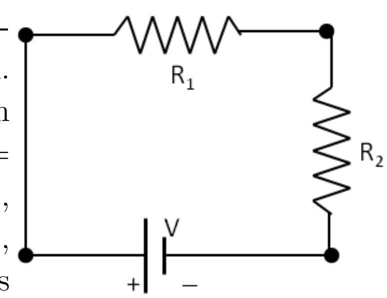
Ex 6.8.21. Find the voltages V_1, V_2 and V_3 across individual resistors and the current I in the circuit for the following values of resistors and voltage V. (a) $R_1 = 2\Omega$, $R_2 = 3\Omega$, $R_3 = 4\Omega$, $V = 12$ V; (b) $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 30\Omega$, $V = 100$ V. Ans: (a) $V_1 = 8/3$ V, $V_2 = 4$ V, $V_3 = 50/3$ V; (b) $V_1 = 50/3$ V, $V_2 = 100/3$ V, $V_3 = 40$ V.



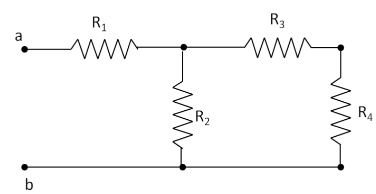
Ex 6.8.22. Find the voltages V_1, V_2 and V_3 across individual resistors, the currents I_1, I_2 and I_3 through each resistor in the circuit for the following values of resistors and voltage. (a) $R_1 = 2\Omega$, $R_2 = 3\Omega$, $R_3 = 4\Omega$, $V = 12$ V; (b) $R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 30\Omega$, $V = 100$ V. Ans: (a) $I_1 = 6$ A, $I_2 = 4$ A, $I_3 = 3$ A; (b) $I_1 = 10$ A, $I_2 = 5$ A, $I_3 = 10/3$ A.



Ex 6.8.23. In the following voltage divider circuits, power P delivered to the resistor R_1 is given. Find the voltage sources needed in each case. (a) $R_1 = 8\Omega$, $R_2 = 800\Omega$, $P = 60$ W; (b) $R_1 = 800\Omega$, $R_2 = 8\Omega$, $P = 60$ W; (c) R_1, R_2, P (find the general formula in this case). Ans: (a) 2213 V, (b) 221 V.



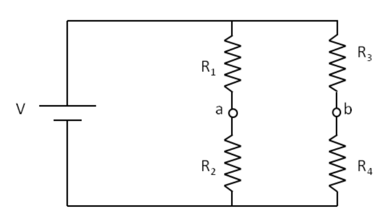
Ex 6.8.24. Find the equivalent resistance between points a and b for the following values of resistors. (a) $R_1 = 100\Omega$, $R_2 = 150\Omega$, $R_3 = 50\Omega$, $R_4 = 75\Omega$; (b) $R_1 = 6\Omega$, $R_2 = 20\Omega$, $R_3 = 15\Omega$, $R_4 = 10\Omega$. Ans: (a) $R = 1850/11\ \Omega$, (b) $R = 154/9\ \Omega$.



Ex 6.8.25. (a) If a 150-V voltage source is connected between the ends a and b, find the power delivered to the resistor R_4 in 6.8.24(a). (b) If a 12-volts voltage source is connected between the ends a and b, find the power delivered to the resistor R_1 in 6.8.24(b). Ans: (a) 17.8 W, (b) 2.95 W.

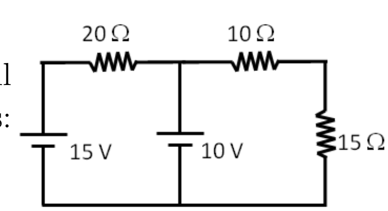
Ex 6.8.26. A battery of EMF 3 volts and internal resistance 2 Ω is connected to a light bulb of resistance 10 Ω. (a) Find the power dissipated in the battery. (b) What is the wattage of the bulb? Ans: (a) 1/8 W, (b) 5/8 W.

Ex 6.8.27. In the given circuit find the voltage between points labeled a and b for the following values of the resistors. (a) $R_1 = 2\Omega$, $R_2 = 3\Omega$, $R_3 = 4\Omega$, $R_4 = 5\Omega$, $V = 12$ V; (b) $R_1 = 5\Omega$, $R_2 = 10\Omega$, $R_3 = 15\Omega$, $R_4 = 20\Omega$, $V = 9$ V. Ans: (a) (8/15) V, (b) (6/7) V.

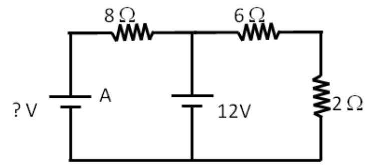


Kirchhoff's Rules

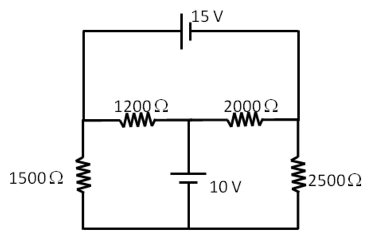
Ex 6.8.28. Find current in all branches in the given circuit. Ans: 0.15 A, 0.25 A, 0.4 A.



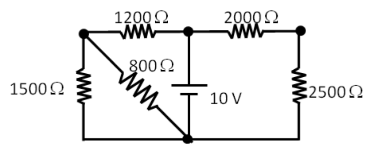
Ex 6.8.29. Ten watts of power is desired in the 8 Ω resistor. What voltage should be applied by source labeled A? Ans: 21 V.



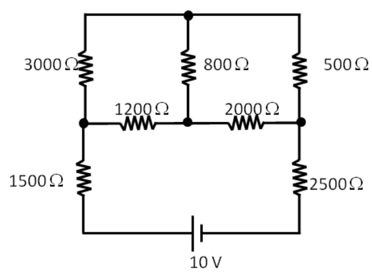
Ex 6.8.30. Find the voltage drop across 2500 Ω resistor. Ans: (175/8) V.



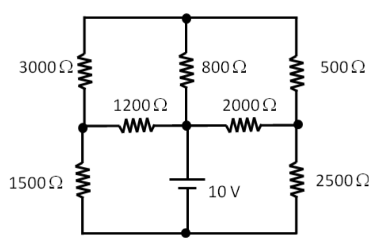
Ex 6.8.31. Find current through the 800 Ω resistor. (Can be done by series/parallel method.) Ans: 3.79 mA.



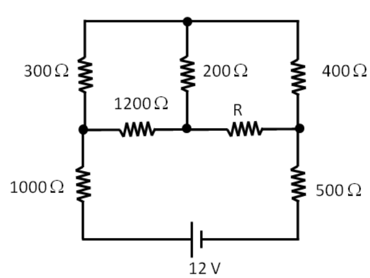
Ex 6.8.32. Find current in each branch.



Ex 6.8.33. Find current in each branch.



Ex 6.8.34. Find the unknown resistance R such that current through 200 Ω resistor is zero. Ans: 1600 Ω.



Capacitors

Ex 6.8.35. Determine the time constant of a circuit that has a 30 μF capacitor in series with a 200 Ω resistor. Ans: 6 ms.

Ex 6.8.36. A 200 pF capacitor is connected in series to a 2 kΩ resistor, a 12 V battery and a switch. (a) Determine the time it will take to get the capacitor 50% charged. (b) Determine the maximum charge on the capacitor if allowed to charge for a long time. (c) Determine energy lost during the charging process. Ans: (a) 2.8×10^{-7} s, (b) 2.4×10^{-9} C, (c) 1.44×10^{-8} J.

Ex 6.8.37. A charged 2 μF capacitor with a charge of ± 10 μC is connected in series to a 200 kΩ resistor and a switch. When the switch is closed current starts to run in the circuit which decreases in time. (a) Determine the maximum current in the circuit at $t = 0$. (b) Determine the time for the current to drop to 10% of the maximum value. (c) Determine the time to lose 90% of the energy stored in the capacitor. Ans: (a) 2.5×10^{-5} A, (b) 0.92 sec, (c) 0.46 sec.

Ex 6.8.38. Determine the equivalent capacitance of the following connected capacitors. Use $C_1 = 20$ μF, $C_2 = 30$ μF, $C_3 = 40$ μF. Ans: (a) 50 μF, (b) 90 μF, (c) 120/13 μF, (d) 140/9 μF.

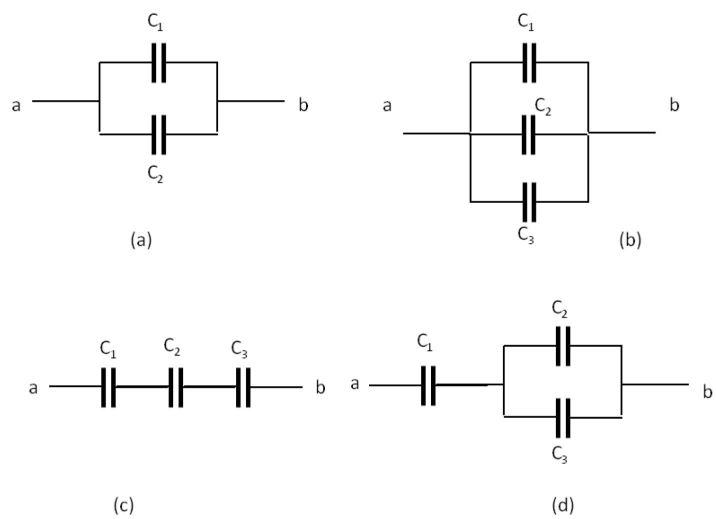


Figure 6.52: Exercise 6.8.38.

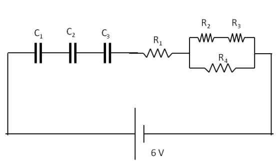


Figure 6.53: Exercise 6.8.39.

Ex 6.8.39. Determine the time constant of following circuit. Use $C_1 = 20$ μF, $C_2 = 30$ μF, $C_3 = 40$ μF, and $R_1 = 2$ kΩ , $R_2 = 5$ kΩ, $R_3 = 10$ kΩ, and $R_4 = 4$ kΩ. Ans: 47.6 ms.

Ex 6.8.40. Find the time constant for the charging of capacitor in the given circuit.

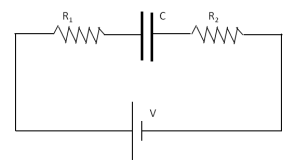


Figure 6.54: Exercise 6.8.40.

Ex 6.8.41. Find the time constant for charging of capacitor in the given circuit. (Hint: its not equal to RC).

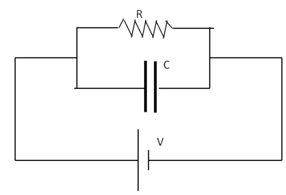


Figure 6.55: Exercise 6.8.41.

Ex 6.8.42. In a certain RC circuit the charge on the capacitor is found to build according to the following rate in charge per unit time. Here t is in msec.

$$\frac{dQ}{dt} = -2Q + 3.$$

The capacitor plates initially contain $\pm 5\mu C$ charges. (a) What is the value of time constant of the circuit? (b) Find charges on the capacitor plates at $t = 1.2\ ms$. (c) Set up a circuit with resistor(s), capacitor(s), and voltage source(s) with appropriate values so that your proposed circuit obeys the equation given above. Ans: (a) $\tau =$ 0.5 ms, (b) 6.36 μC.

Ex 6.8.43. Determine the time constant of the following circuit. Ans: $\left(\frac{R_1 R_2}{R_1+R_2}\right) C$.