4.4 WAVE DESCRIPTION OF REFLECTION AND REFRACTION

Consider a monochromatic plane wave in air incident on a plane surface of glass of refractive index n at an angle θ_1. A reflected ray comes back in air at angle θ_1' to the normal and a refracted ray is transmitted into glass at angle θ_2. How would we draw plane-wave picture here? First we note that frequencies of incident, reflected and refracted light will be equal to each other as set by the frequency of oscillation of charges generating the wave. But since the refractive indices of air and glass are different, the speed of light in the two media would be different. Hence, the wavelength of the same light will be different in the two media - larger in the air and smaller in the glass. Therefore we draw wavefronts further apart in air than the wavefronts in the glass as shown in the Fig. 4.4.

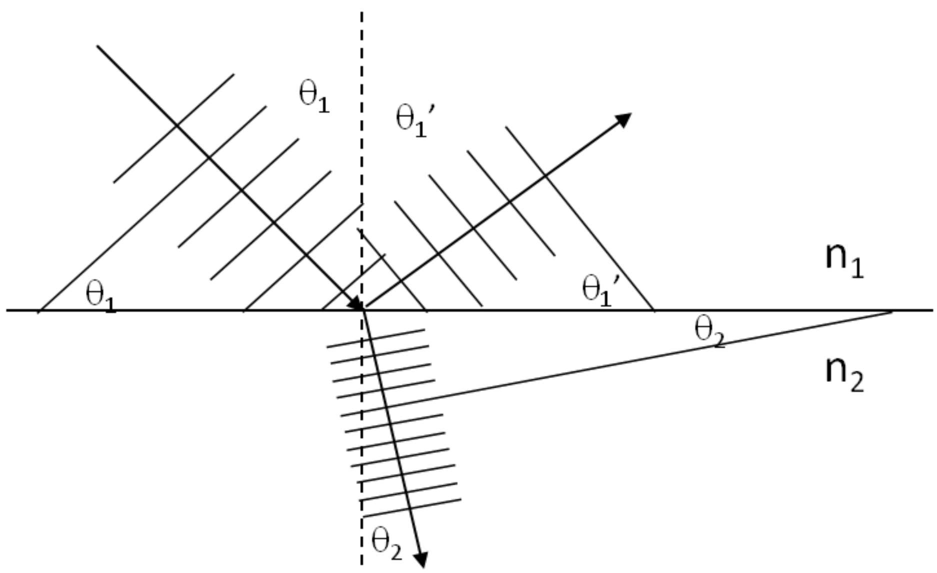

Figure 4.4: Reflection and refraction of plane waves.

Maxwell's equations require that the electric and magnetic fields on the two sides of the interface must be related. These conditions on the interface are called boundary conditions on the fields. Maxwell showed that the laws of reflection and refraction of light emerge as natural consequences of these boundary conditions. You will study boundary conditions on electric and magnetic fields in an advanced course on electricity and magnetism.

We only quote here some of the major results from the wave analysis of Maxwell's equations. The results are tabulated for two cases depending on the direction of the electric field with respect to the plane containing the incident, reflected and refracted rays, called the **plane of incidence**. The resulting relations between the amplitudes are called **Fresnel equations**. In the following we quote results of calculation in two particular cases of interest and discuss some of their implications.

Case 1: Electric field perpendicular to the plane of incidence - Transvere Electric (TE)

The Fresnel equations give us the dependence of the electric field in reflected and refracted waves on the incident wave, the incident angle and the refractive indices. These ratios are called the **coefficients of reflection and transmission**, denoted here by letters ρ and τ respectively. Let E_{0i}, E_{0r}, and E_{0t} be the electric field amplitudes in the incident, reflected and refracted waves respectively.

$$\rho_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{\sin(\theta_2 - \theta_1)}{\sin(\theta_2 + \theta_1)} \tag{4.27}$$

$$\tau_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_2 + \theta_1)} \tag{4.28}$$

Case 2: Electric field parallel to the plane of incidence - Transverse Magnetic (TM)

The Fresnel equations for this case are as follows.

$$\rho_\| = \left(\frac{E_{0r}}{E_{0i}}\right)_\| = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)} \tag{4.29}$$

$$\tau_\| = \left(\frac{E_{0t}}{E_{0i}}\right)_\| = \frac{2\sin\theta_2\cos\theta_1}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)} \tag{4.30}$$

It is interesting to explore how the coefficients of reflection and transmission vary with incidence angle θ_1. In order to do that we will use Snell's law to write the angle of refraction in terms of the angle of incidence and the ratio of the indices of refraction.

For illustrative purposes in Fig. 4.5, I have plotted the coefficients of reflection and transmission for an air/glass (n_1=1, n_2=1.55) interface for the case of electric field parallel to the plane of incidence and magnetic field perpendicular to the plane of incidence. You should notice that there is an angle of incidence for which the coefficient of reflection for this case vanishes completely. The particular value of the incident angle when this happens is called the **Brewster's angle** or **polarization angle**, θ_B.

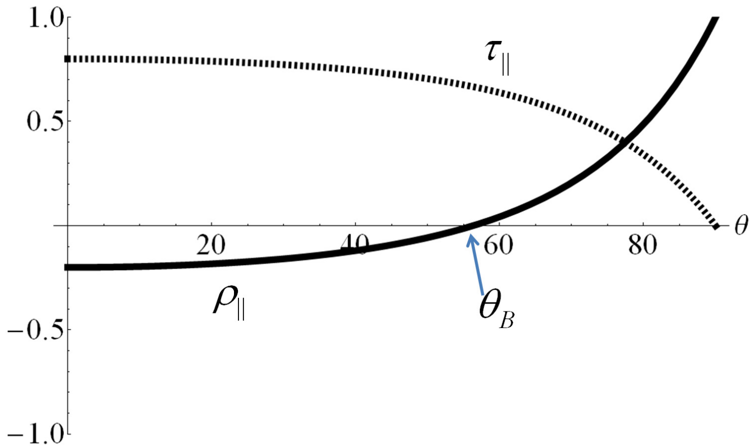

Figure 4.5: Plot of reflection and transmission coefficients for air/glass interface for the case of the electric field parallel to the plane of incidence. Note that the vanishing of the reflected electric field at approximately 57°, which is the Brewster's angle for this interface.

From Eq. 4.29 we find that since $\theta_1 \neq \theta_2$, the reflection coefficient $\rho_{\|}$ will vanish if

$$(\theta_1 + \theta_2)_{\theta_1=\theta_B} = 90^\circ. \tag{4.31}$$

Transfer θ_1 to the right-side, take sine of both sides to obtain

$$\sin_{\theta_B} = \cos\theta_2 = \sqrt{1 - \sin^2\theta_2} = \sqrt{1 - \frac{n_1^2}{n_2^2}\sin^2\theta_B}.$$

This equation can be solved to yield

$$\tan\theta_B = \frac{n_2}{n_1}. \tag{4.32}$$

A wave that has its electric field in the plane of incidence will have no reflection if the wave is incident at the interface at the Brewster's angle. Therefore, if you shine a beam of light on a planar interface at Brewster's angle as shown in the Fig. 4.6, the reflected ray will have its electric field pointed only perpendicular to the plane of incidence, which would be parallel to the plane of the interface. This is one way of obtaining a polarized light light wave with known direction of electric field.

Example 4.4.1. Brewster's angle for air/water interface

Find the angle of incidence of sunlight so that the reflected light is fully polarized horizonal to the water surface.

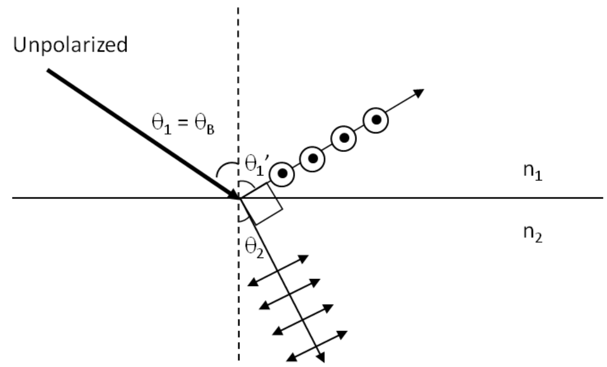

Figure 4.6: Reflection at Brewster's angle gives rise to polarized light.

Solution. The plane of incidence is the plane of the incident, reflected, and refracted rays. The polarization resulting from the refraction at Brewster's angle will be perpendicular to this plane, and hence parallel to the water surface. Therefore, we calculate the Brewster's angle using the following formula.

$$\tan\theta_B = \frac{n_2}{n_1}.$$

Putting $n_1 = 1$ and $n_2 = 4/3$ we find that the Brewster's angle is

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53°.$$

Therefore, the sunlight will be polarized upon reflection from air/water interface if the reflected angle is 53°.