5.2 DOUBLE-BEAM INTERFERENCE FROM DIELECTRIC FILMS

5.2.1 Reflected Waves from Plane Interface

Bright fringes from oil spills and soap films are common examples of phenomenon of interference due to the reflections from the sides of dielectric films (Fig. 5.6). In this section we will work out the conditions necessary for the constructive and destructive interferences for reflections from a rectangular thin film. You can use the results to deduce the thickness of oil slick or soap bubbles from the interference patterns observed in them.

Figure 5.6: Interference from a thin film. The color of the fringes are observed in the colored version at the following web-site : http://en.wikipedia.org/wiki/Soap_film. (Picture courtesy of Wikicommons.

For the sake of concreteness, we consider a light ray incident on a

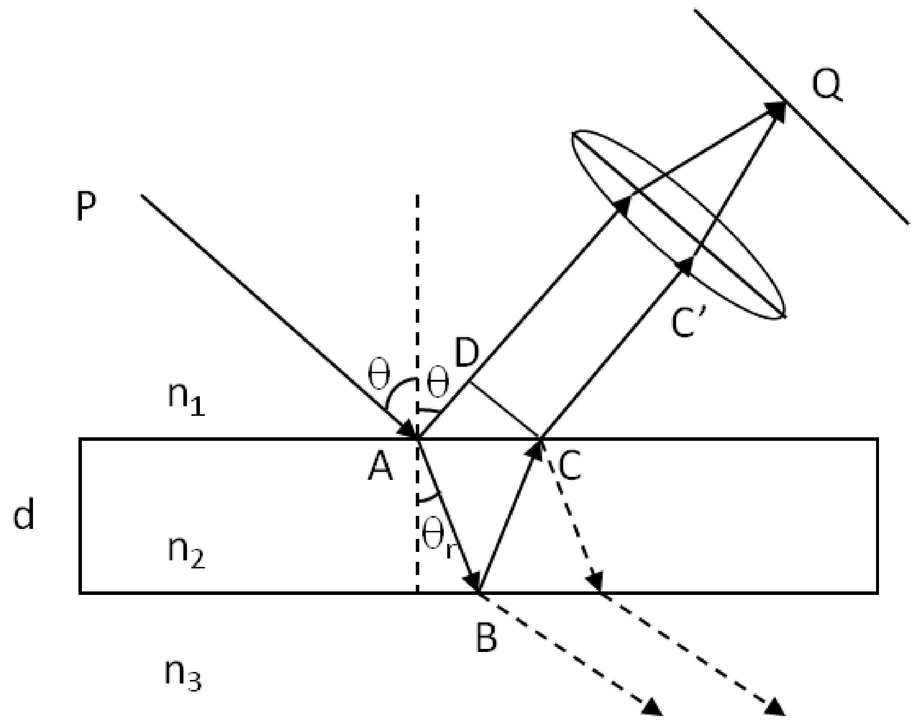


Figure 5.7: Interference from reflections off two sides of a planar film.

dielectric film of thickness d at an angle θ as shown in Fig. 5.7. The amplitude of the original ray PA is split into a reflected part in AD and a transmitted part in AB. The refracted ray AB travels in the medium with refractive index n_2 until it encounters the n_2/n_3 interface at B. At that interface the amplitude splits again in a reflected part in BC and a transmitted part which we will ignore in the present discussion.

The reflected ray BC travels back to the n_1/n_2 interface and refracts into the first medium in the direction CC′) which travels parallel to the ray AD. The parallel rays are brought to the point Q of the focal plane of a converging lens where they undergo constructive or destructive interference depending upon the difference in their phases.

Unlike the Young's double-slit experiment, the phase difference between two waves PADQ and PABCQ here arises due to two factors: (1) geometrical path difference and (2) any phase change due to reflections at A and B. Often, the phase difference is expressed as optical path length difference. As we know that the phase of a wave changes by 2π radians when the wave travels one entire wavelength, the relation between the phase difference and optical path

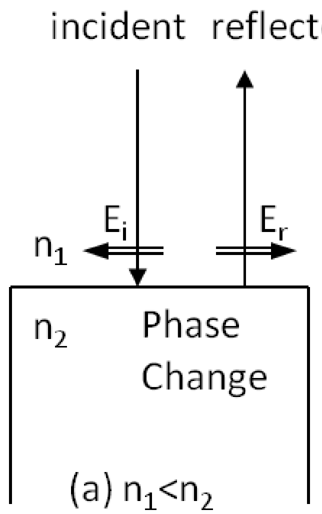


Figure 5.8: Reflection at an interface for $n_1 < n_2$ causes phase of the wave to change by π radians.

length difference is simply:

$$\text{Optical Path Length Difference} = \left[\frac{\text{Phase Difference}}{2\pi}\right] \lambda \quad (5.21)$$

But since the wavelength depends on the refractive index of the medium, we will do our calculations using the phase difference and not bother with converting it into optical path length until the end where we will write the interference condition in terms of the wavelength λ_2 in the dielectric film.

We have mentioned above that the reflection of waves is also one of the sources of phase difference. It turns out that when a wave is reflected off from a medium of higher refractive index, the phase changes by 180° or π radians as illustrated in Fig. 5.8. There is no phase change when reflecting from a medium of lower refractive index.

The two rays resulting in a reflection from a dielectric film, actually reflect off at two different interfaces. Let us examine the most common case where the front and the back media are the same and their refractive indices are less than that of the film. This will be the case, for instance, when light is incident on a plastic film in air, or of a soap bubble which has air inside the bubble and outside the bubble.

Case: $n_1 = n_3 < n_2$ (example: air/plastic/air)

The phase difference between the two interfering waves is easily found by following the two rays in Fig. 5.7 starting from a point, such as point P, before the first reflection/refraction at A.

$$\begin{aligned}&\text{Phase difference } (\Delta_{12}) = \\ &\quad (\text{reflection at A} + \text{phase change over AD}) \\ &\quad\quad - (\text{phase change over AB} + \text{reflection at B} \\ &\quad\quad\quad + \text{phase change over BC})\end{aligned}$$

In the present case, the reflection at A causes a phase change of π radians, but the reflection at B does not have any phase change. Phase change for traveling a distance AD in the first medium can be found by noting that phase of a wave changes by 2π radians when it travels one wavelength in the medium. We must be careful here and use the correct wavelength for each path since waves are traveling in different media. Let λ_1 and λ_2 denote the wavelength of light in the two media. Note that although the frequency of light is same in the two media, the wavelength will be different in the two media because the speed of the wave depends on the refractive index.

$$\lambda_1 = \frac{v_1}{f} = \frac{c}{n_1 f} \tag{5.22}$$

$$\lambda_2 = \frac{v_2}{f} = \frac{c}{n_2 f} \tag{5.23}$$

Hence,

$$n_2\lambda_2 = n_1\lambda_1. \tag{5.24}$$

Therefore phase changes over the two paths are as follows.

$$\text{phase change over AD} = \frac{\text{AD}}{\lambda_1} \times 2\pi. \tag{5.25}$$

The phase changes over AB and BC paths are obtained by using the wave length λ_2 in the second medium.

$$\text{phase change over AB} = \frac{\text{AB}}{\lambda_2} \times 2\pi \tag{5.26}$$

$$\text{phase change over BC} = \frac{\text{BC}}{\lambda_2} \times 2\pi \tag{5.27}$$

Therefore phase difference between the two waves is found to be

$$\Delta_{12} = \pi + \frac{2\pi}{\lambda_1}\left[\text{AD} - \frac{n_2}{n_1}(\text{AB} + \text{BC})\right]. \tag{5.28}$$

We would get a constructive interference if the phase difference Δ_{12} is a multiple of 2π, and a destructive interference if it is an odd multiple of π.

$$\Delta_{12} = \begin{cases} m \times 2\pi & m = 0, \pm1, \pm2, \cdots \quad \text{Constructive} \\ m' \times \pi & m' = \pm1, \pm3, \cdots \quad \text{Destructive} \end{cases} \tag{5.29}$$

These interference conditions are more useful when written in terms of the angle of refraction in the film and the thickness of film. To accomplish that we use the geometry given in Fig. 5.7 and the Snell's law at point A to rewrite the distances in terms of d and the angle of refraction θ_r. Notice the following relations among the sides of various triangles in the figure.

$$\mathrm{AB} = \mathrm{BC} = \frac{d}{\cos\theta_r} \tag{5.30}$$

$$\mathrm{AD} = \mathrm{AC}\sin\theta = 2d\tan\theta_r\sin\theta = 2d\frac{n_2}{n_1}\,\frac{\sin^2\theta_r}{\cos\theta_r}, \tag{5.31}$$

where I have made use of the Snell's law for the refraction at A.

$$n_1\sin\theta = n_2\sin\theta_r. \tag{5.32}$$

Substituting for AB, BC and AD in the expression for Δ_{12} gives

$$\Delta_{12} = \pi + \frac{2\pi}{\lambda_2}\times 2d\cos\theta_r. \tag{5.33}$$

Hence, the interference conditions given in terms of the angle of refraction is as follows.

For $(n_1 = n_3 < n_2)$

$$2d\cos\theta_r = \begin{cases} \frac{|m|}{2}\,\lambda_2 & \text{constructive} \quad m = \pm1, \pm3, \pm5, \cdots \\ |m'|\lambda_2 & \text{destructive} \quad m' = 0, \pm1, \pm3, \pm5, \cdots \end{cases} \tag{5.34}$$

We can rearrange this equation and write in terms of the angle of incidence θ, but that would be a much more complicated expression.

For a nearly normal reflection, the angle of refraction will be zero. This would give an interference condition that would depend on the thickness of the film and the wave length, and hence depending upon the thickness, the light of a particular wavelength will interfere constructively or destructively.

Normal incidence :

Case: $(n_1 = n_3 < n_2)$

$$2d = \begin{cases} \frac{m}{2}\,\lambda_2 & \text{constructive} \quad m = \pm1, \pm3, \pm5, \cdots \\ m'\lambda_2 & \text{destructive} \quad m' = 0, \pm1, \pm3, \pm5, \cdots \end{cases} \tag{5.35}$$

Example 5.2.1. Colors from soap films A soap film reflects blue light of wavelength 475 nm when viewed normally. Find the thickness of the film if the soap water has a refractive index of 1.33?

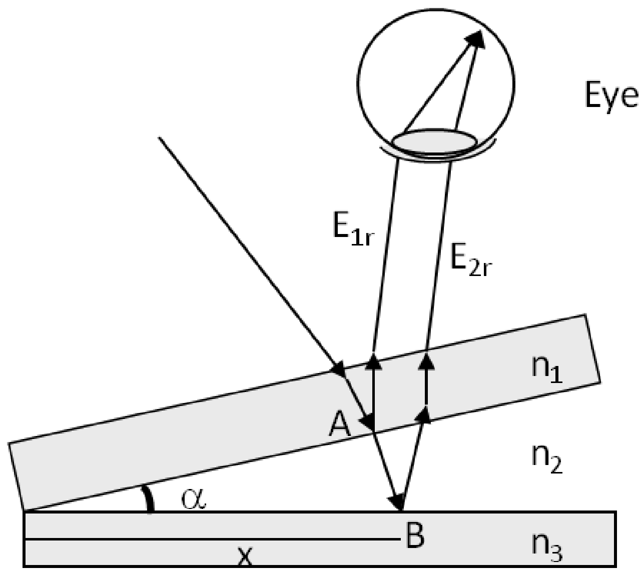


Figure 5.9: Interference from reflection from two sides of a wedge.

Solution. To use the interference formula, first we need to evaluate the wavelength λ_2 inside the soap film, which is different from 475 nm.

$$\lambda_2 = \frac{n_1}{n_2}\lambda_1 = \frac{1}{1.33} \times 475 \text{ nm} = 375 \text{ nm}.$$

We find the thickness of the soap film by using $m = 1$ in the constructive interference condition.

$$d = \frac{1}{2}\frac{\lambda_2}{2} = \frac{1}{2}\frac{375 \text{ nm}}{2} = 89.3 \text{ nm}.$$

5.2.2 Reflected Waves from a Wedge-shaped Film

A wedge is a simple system of linearly varying thickness. The phase difference between the rays reflected from the top and the bottom of a wedge-shaped dielectric film will depend upon where on the wedge this happens. A set-up that makes use of the reflections from a wedge-shaped film is shown in Fig. 5.9. An extended source is used to strike at different parts of the wedge, and observations are made on the reflections of nearly normally incident rays.

We again consider the case $n_1 = n_3$. If $n_1 < n_2$, then the reflection at A will cause a phase shift of π radians and no phase shift for the reflection at B. If $n_1 = n_3 > n_2$, then the reflection at A will not cause phase shift but the reflection at B will cause phase shift of π

radians. Therefore, as far as the phase shift contributions in the two waves labeled E_{1r} and E_{2r} are concerned, the situation here is similar to the plane plate worked out in the last section.

Since we are looking at near normal directions, the angles of incidence and refraction would be nearly zero. Therefore we will have $\cos\theta_r \approx 1$ in the condition for interference worked out in the last section. Let x be the distance from the wedge and α be the angle at the corner of the wedge, then the thickness d of the dielectric material in the wedge at the place shown in the figure will be

$$d = x \tan\alpha \approx x\alpha, \tag{5.36}$$

where I have approximated α to be much less than 1 radian. Depending on the values of d and x, we will have constructive and destructive interferences on the screen as given by the following conditions.

$$2d \approx 2\alpha x = \begin{cases} \frac{m}{2}\lambda_2 & \text{constructive} \quad m = \pm1, \pm3, \pm5, \cdots \\ m'\lambda_2 & \text{destructive} \quad m' = 0, \pm1, \pm3, \pm5, \cdots \end{cases} \tag{5.37}$$

Since all places on the wedge corresponding to the same thickness give rise to the same type of interference, you observe bands. These bands are also called Fizeau fringes.

Example 5.2.2. Interference from a wedge

A laser light of wavelength in air of 630 nm is incident in the apparatus shown above. The slides are made of glass of refractive index 1.60, and the wedge is water with refractive index 1.33. Find the distance along x you need to move to go from one bright fringe to the next if the wedge angle is 5°.

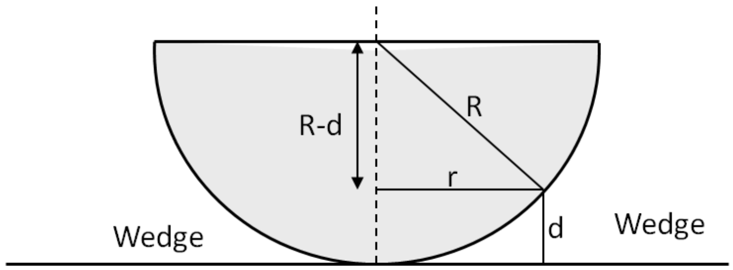
Solution. Let us work out the constructive interference conditions for $m = 0$ and $m = 1$. Note that the angle α of the wedge must be expressed in radians.

$$m = 0: \quad x = \frac{1}{\alpha}\frac{\lambda_{\text{water}}}{4} = \frac{180}{5\times\pi}\,\frac{1}{4}\frac{1.0\times 630 \text{ nm}}{1.33} = 1.36\ \mu\text{m}.$$
$$m = 1: \quad x = \frac{1}{\alpha}\frac{3\lambda_{\text{water}}}{4} = \frac{180}{5\times\pi}\,\frac{3}{4}\frac{1.0\times 630 \text{ nm}}{1.33} = 4.07\ \mu\text{m}.$$

Hence, $\Delta x = 2.71\ \mu$m.

Example 5.2.3. Newton's rings

Alternating bright and dark fringes take on the shape of circles when a hemispherical surface is put on a flat surface as shown in the figure. These rings are called Newton's rings.



Solution. The thickness d at a horizontal distance r from the center is given by the following relation from the Pythagoras theorem.

$$(R-d)^2 + r^2 = R^2. \tag{5.38}$$

Solve for d keeping the root with $d < R$.

$$d = R - \sqrt{R^2 - r^2}. \tag{5.39}$$

The condition for constructive interference in case of near normal incidence is as follows.

$$d = R - \sqrt{R^2 - r^2} = \frac{m}{2}\frac{\lambda_0}{2}. \tag{5.40}$$

where m is an odd number and λ_0 is the wavelength in air, the medium of the wedge. The bright fringes form at following radial values from the center.

$$r = \sqrt{\frac{m}{2}R\lambda_0}. \tag{5.41}$$

where we have assumed $\lambda_0 << R$ and dropped a term. Since bright and dark fringes form in circles, they look like rings. Isaac Newton reported these rings in his book Optiks: or, a Treatise on the Reflexions, Refractions, Inflexions and Colours of Light. We now call these rings Newton's rings. Newton pressed a double convex lens against the curved sides of the planar convex lens to create wedge that produced colored rings.