Ongoing work on multiple body no-slip collisions

February 24, 2017

An outline of planned numerical experiments along with preliminary results. All bodies are disks, of non-negligible, equal (within a single trial) radii, and unless otherwise stated all collisions between body and wall or between two bodies are no-slip collisions. A dotted boundary, when present, gives the boundary of the center of mass and therefore indirectly gives the radius. Colored segments represent the trajectories, with equally spaced hues for multiple bodies. In some cases the radius may be inferred by the (light) rendering of the body boundaries at collisions, using the trajectory colors.

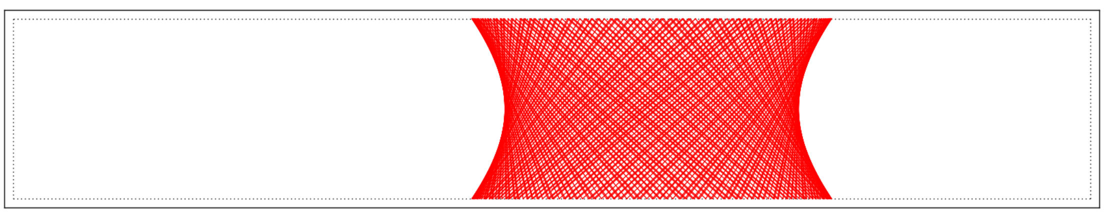

Figure 1: Testing the program with a single body. Boundary collisions occur when the enter is distance r from the edge, indicated by the dotted line. Here r=0.1, with the strip bounded above at 1 and below at -1.

1 Systems which are regular for one body: circles and equilateral triangle

For small radii in billiards demonstrating one body regularity, two body systems also demonstrate regular behavior. The behavior becomes increasingly chaotic when the radius increases and bodies collide, as in the equilateral triangle in Figure 2, for which all orbits are 4-periodic or 6-periodic in the single body case. In Figure 3, for small radii the circle billiard has two caustic circles for each body, but becomes irregular as the radius of the bodies increases. However, for radii approaching (half) the radius of the billiard, regular behavior reappears.

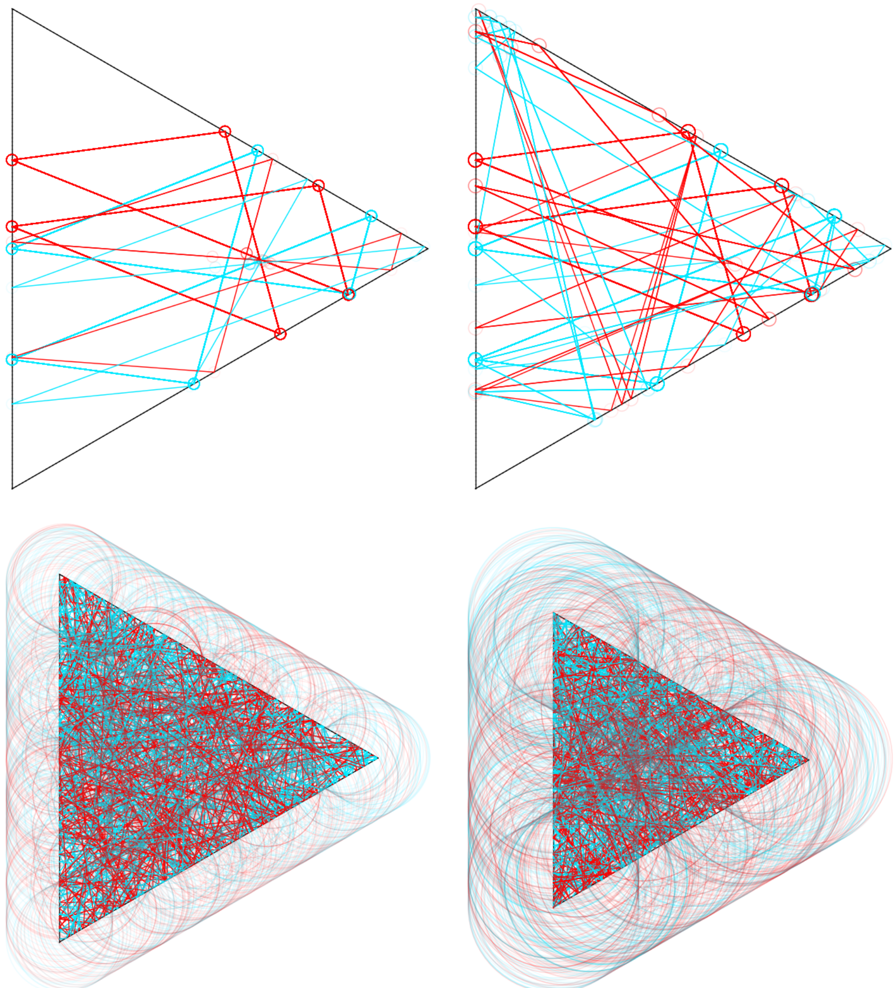

Figure 2: Two body no-slip equilateral triangle billiards with radius 0.02, 0.03, 0.25, and 0.5. All four examples were continued for 2000 collisions, however the top two examples display periodic repetition between occasional collisions.

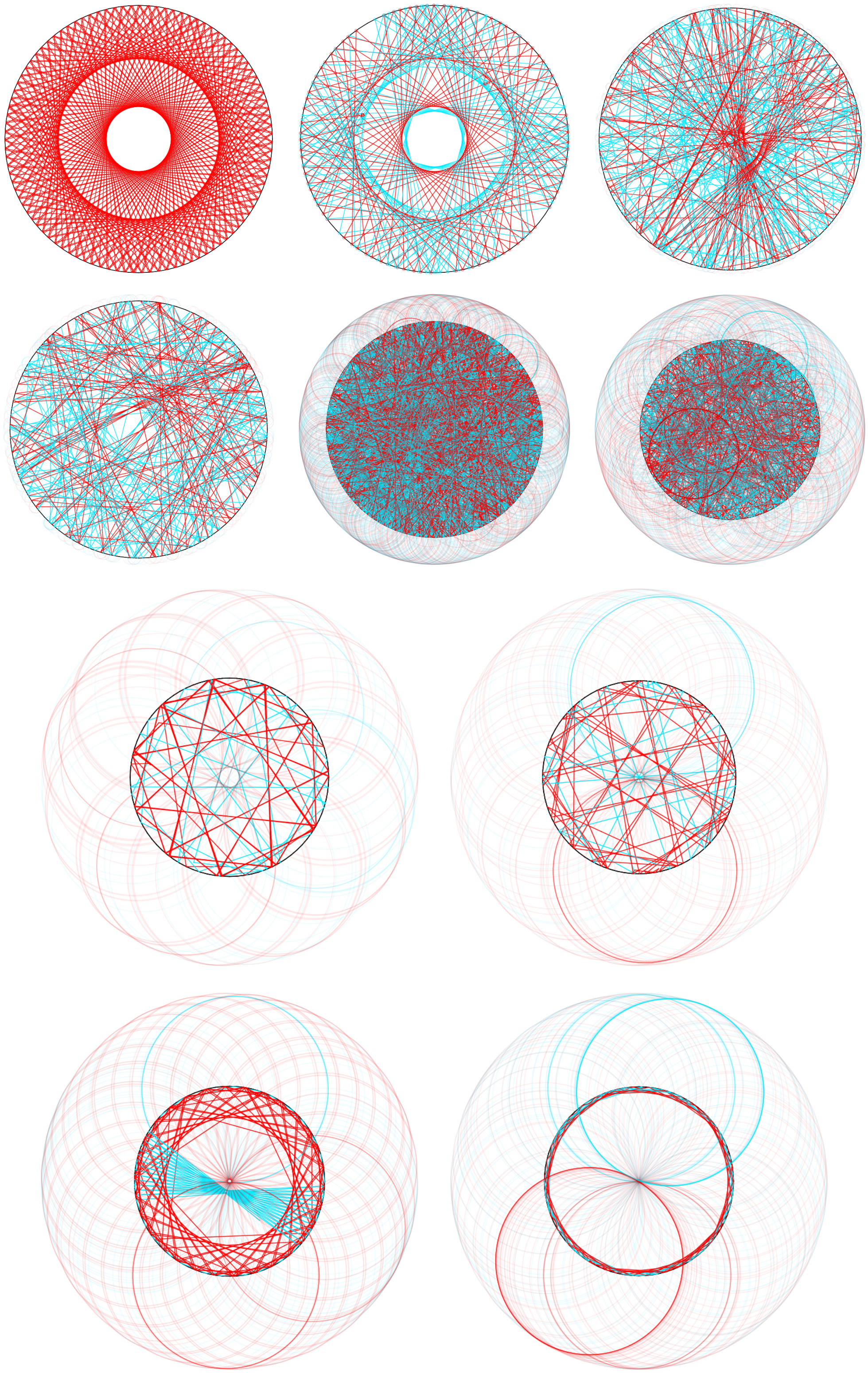

Figure 3: A single body circle (generated with the multibody program) top left, and two bodies with r=0.01, 0,03, 0.05, 0.25, 0.5, 0.0, 0.95, 0.98 , 0.99.

2 Stability

Periodic points involving two bodies colliding appear to be stable in many cases. One specific question that needs to be addressed more systematically is to what extent the stability persists (that is, to what extent the orbits are still bounded) as each rotational velocity, position, and radius is varied. In most of the following examples the (spacial) velocity was varied, except for the last example where varying the radius is considered.

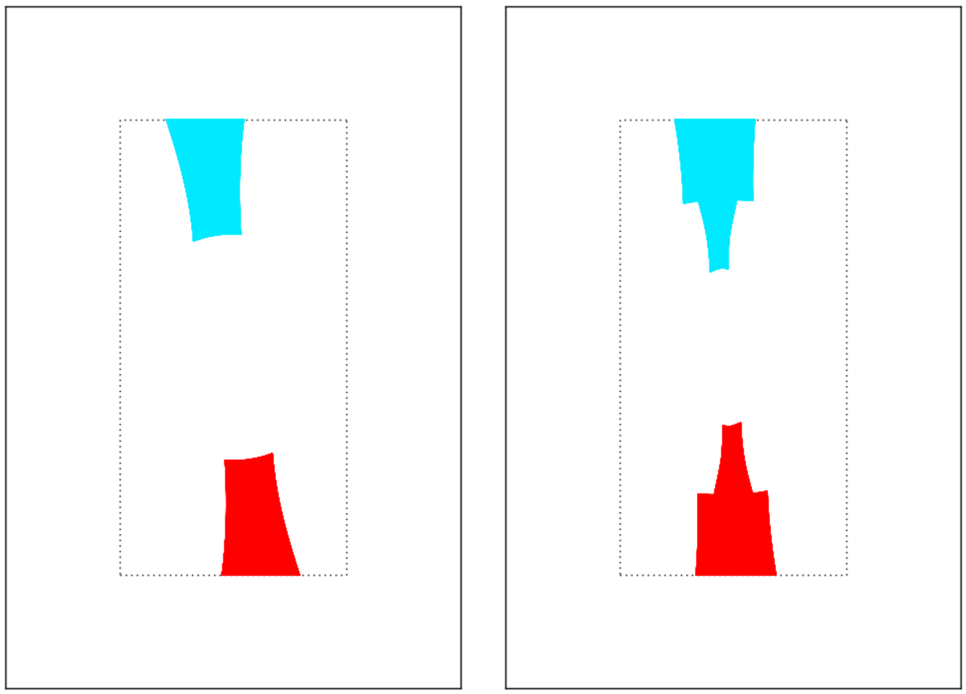

Figure 4: The trajectories of a stable orbit (left) near the up and down periodic orbit, with r=0.5 and boundaries distance 2, initially spaced symmetrically. The stability persists if the vertical initial positions are asymmetric (right).

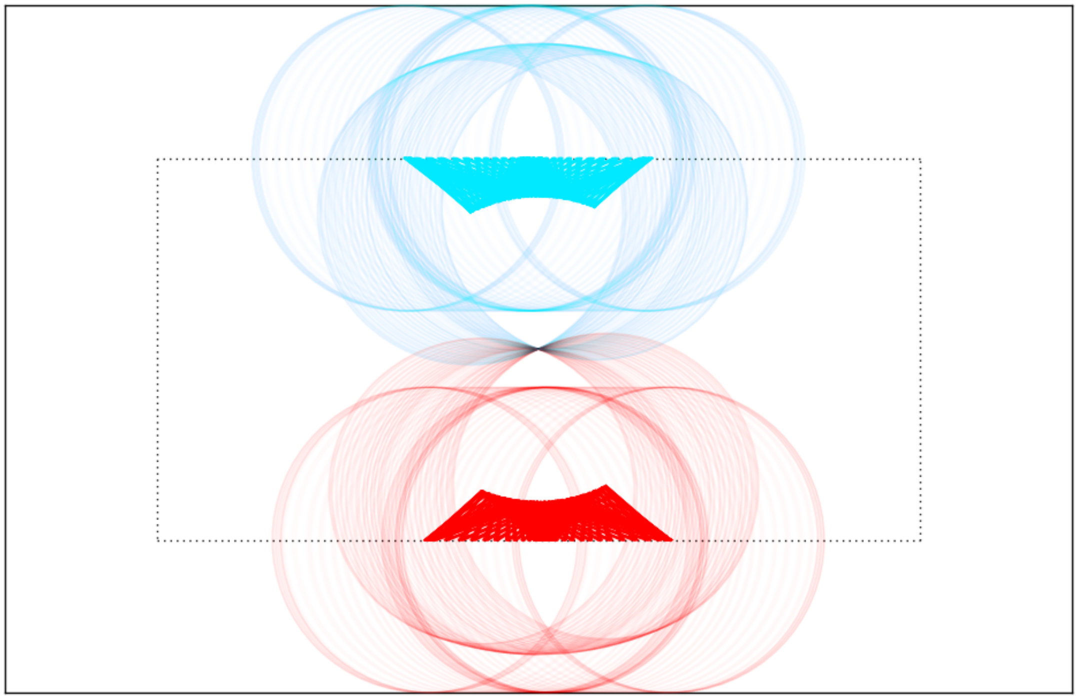

Figure 5: The stability persists even when the horizontal component of the velocity vector is greater than the vertical component along the periodic direction (bottom).

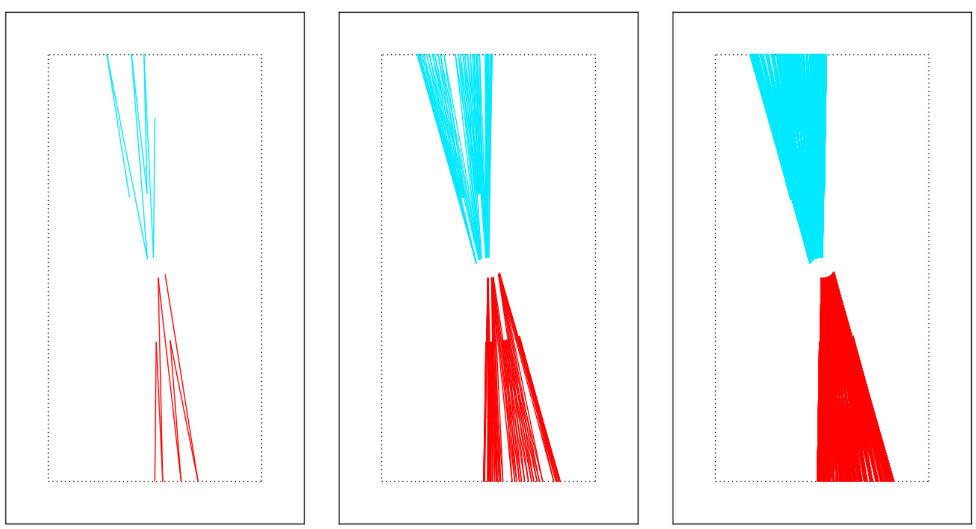

Figure 6: Near the periodic point with two bodies colliding along the vertical starting at different distances from the respective walls. The periodic orbit in this case includes four wall collisions and two two-body collisions at different heights. Strip height 2, radius 0.1, with 10, 100, and 1000 collisions.

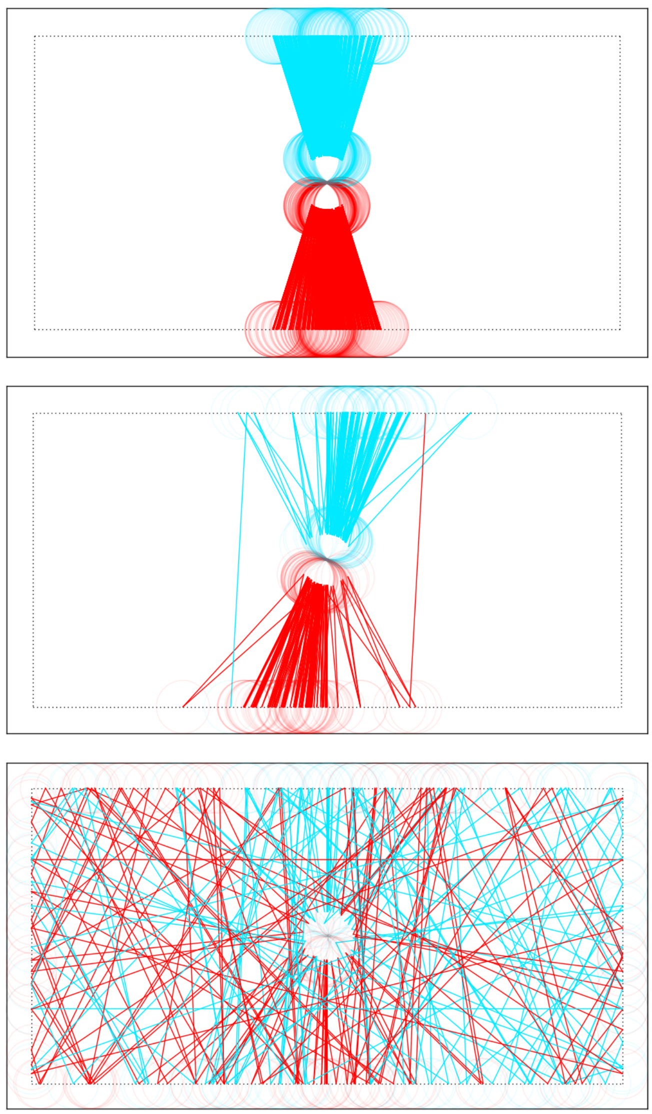

Figure 7: It may be that as in the case of the no-slip Sinai billiard there is some sort of stability threshold as the radius of the disks decreases. For $r = 0.19$ (top) the orbits are very stable, through at least 2000 collisions; for $r = 0.18$ there is an appearance of stability through 150 collisions before the orbit becomes unstable; for $r = 0.17$ the instability appeards immediately.

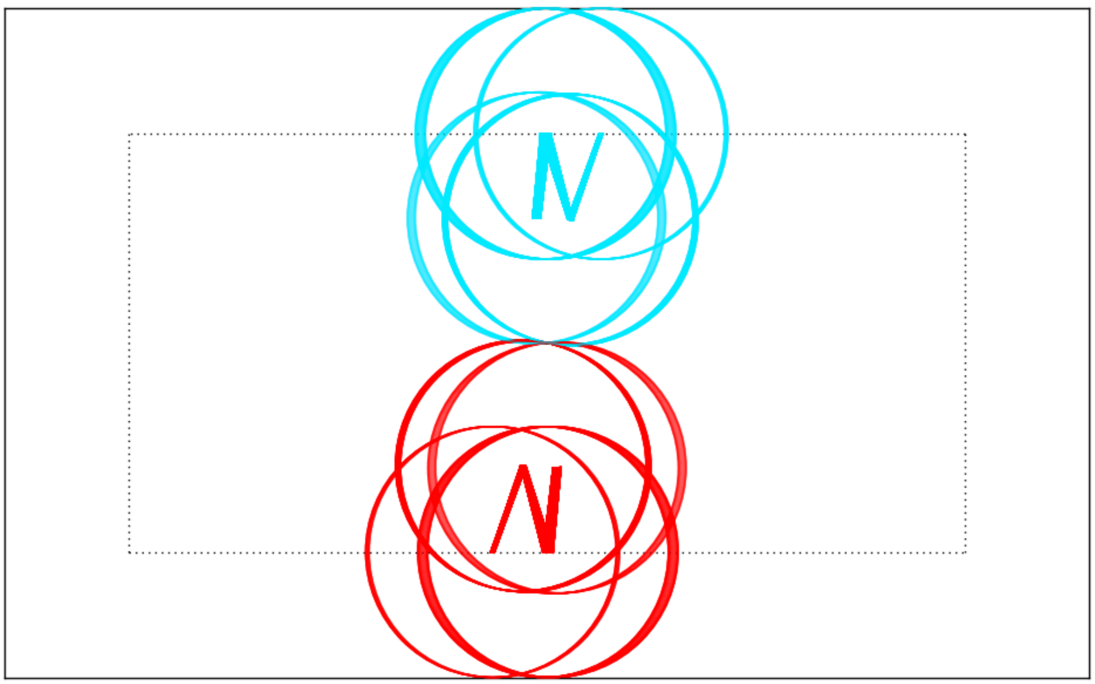

Figure 8: As with one body no-slip billiards, it appears there are examples of resonance in the quasiperiodic orbits yielding higher order periodic orbits.

3 Dispersion on the strip

3.1 Graphical experiments

It appears that if two bodies collide in the infinite no-slip strip their maximum displacement will increase without limit over time.

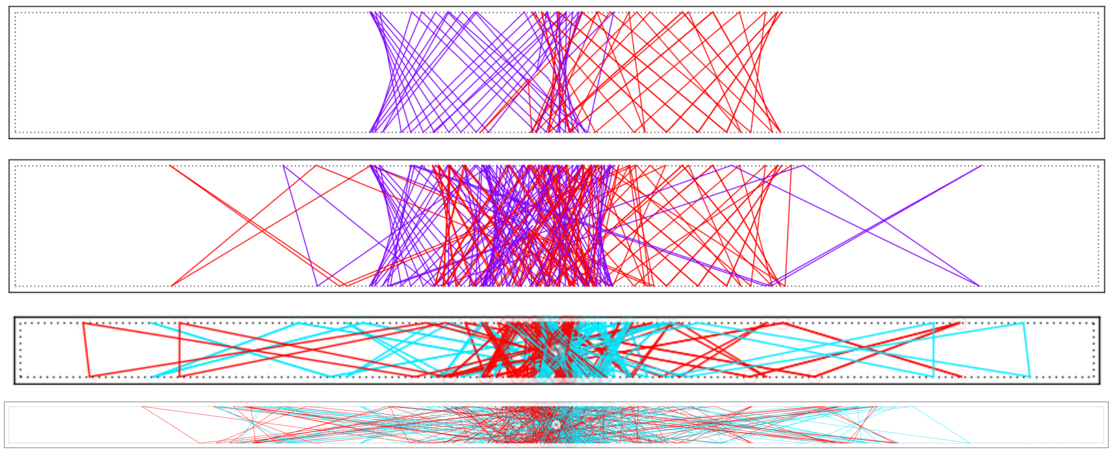

Figure 9: Two bodies colliding on first iteration seem to disperse without boundary. The vertical scale is two units in all cases, with the horizontal scale expended according to the dispersion.

3.2 Statistical experiments

Varying the number of bodies and the radius of the bodies, how does the variance or maximal displacement change over time.

4 A few more examples

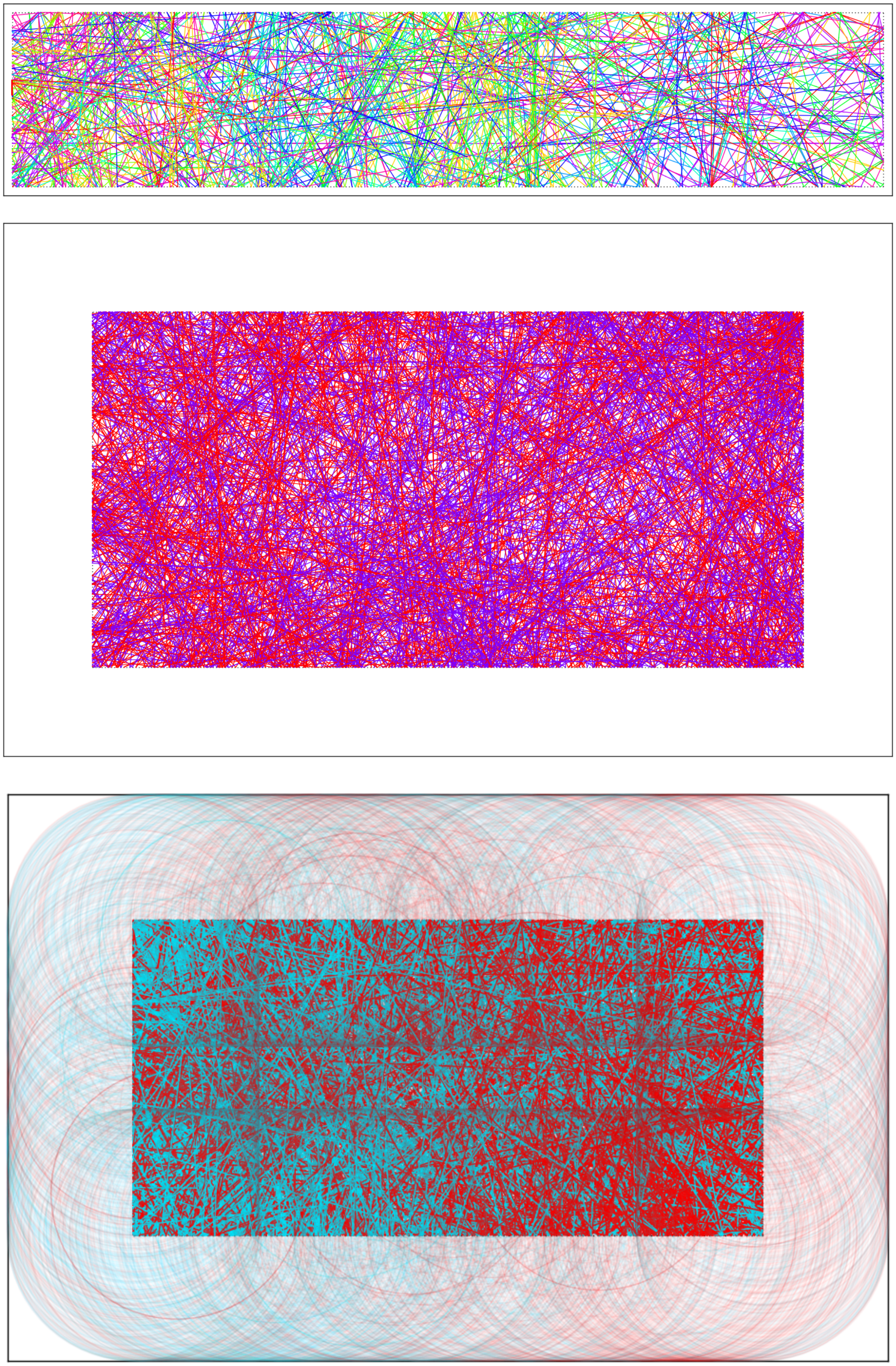

Figure 10: