ON THE THEORY OF THERMAL FORCES ACTING ON AEROSOL PARTICLES

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ABSTRACT

Disagreement between experimental results and aerosol thermal force theories for regimes from continuum toward free-molecule is explained through application of first-order slip-flow boundary conditions. The influence of the convective flow on the thermal force is explored through a perturbation technique.

INTRODUCTION

The force arising from a temperature gradient, acting on suspended particles in a fluid, is termed the thermal force. The phenomenon of the thermal force has received extensive experimental and theoretical investigation owing both to the practical interest in utilizing the phenomenon in particle collection and measurement and to the theoretical interest associated with a quantitative description of the phenomenon.

It is the purpose of this note to explain recently observed discrepancies (1, 2) between experiment and theory for the thermal force in one of the dynamic regimes characterizing this force, the so-called slip-flow regime. Before presenting this explanation, it is useful to give brief mention of these various dynamic regimes and their relation to the thermal force.

In a discussion of the thermal force it is convenient to recognize three regimes characterized by the magnitude of the Knudsen number, (l/a), where l is the molecular mean free path[1] of the fluid surrounding a suspended particle and a is the radius of the particle taken as spherical.

The first of these regimes, termed free-molecule, is indicated by a value of $(l/a) \gg 1$. For this regime the particle is so small or the molecular mean free path so large that the particle exercises virtually no influence on the distribution of the molecular velocities of the fluid surrounding it.

The theory for the thermal force for $(l/a) \gg 1$ received early development, owing to interest in the radiometer phenomena, and has long been known to be in agreement with experiment (3-7).

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[1] The molecular mean free path, l, is defined by the formula, $\mu = 0.499\,\rho\,\bar{v}l$. Here $\bar{v}$ is the average molecular velocity, μ is the coefficient of shear viscosity, and ρ is the mass density.

The regime for $(l/a) \sim 1$ may be called transition. This regime presents considerable complexities in problems of rarefied gas dynamics which remain to be resolved by kinetic theory.

The theory for the thermal force in this regime has not yet been formulated adequately. Here one has in the description of the thermal force neither the simplification inherent in the free-molecule regime noted above nor the advantages of a modified continuum analysis used in the slip-flow regime, $(l/a) < 1$. It should be pointed out that the indicated limit between the slip-flow and transition regimes is only tentative.

This note is concerned primarily with the theory of the thermal force for the slip-flow regime. For this regime it is shown here that the continuum equations of mass, momentum, and energy, the Navier-Stokes equations, may be applied with boundary conditions appropriate for $(l/a) > 0$ to the quantitative description of the thermal force.

Various theories have been suggested for the thermal force in the slip-flow regime (8, 9). The most frequently tested of these theories has been that of Epstein (8), who found by a continuum analysis for the thermal force acting on spherical aerosol particles:

$$\vec{K} = -9\pi a \frac{\mu^2}{\rho T} \left(\frac{k_f}{2k_f + k_s} \right) \nabla T_\infty ,$$ [1]

where:

$\vec{K}$ = thermal force;

a = radius of aerosol particle;

k_f = thermal conductivity of gas surrounding particle;

k_s = thermal conductivity of particle;

∇T_∞ = uniform temperature gradient, large distance from particle;

T = absolute temperature;

ρ = mass density; and

μ = coefficient of shear viscosity.

The agreement between experiment and the Epstein theory has been thought to be satisfactory (10, 11), although the thermal force calculated from Eq. [1] has usually been found to be lower than the experimental result. Recently Schadt and Cadle (1, 2) have pointed out serious disagreement between their experimental results and Eq. [1]; in particular they have found that Eq. [1] predicts values of thermal force from thirty to fifty times too low for aerosol particles of large thermal conductivity. Obviously there exists a serious error in the Epstein theory.

Two principal objections may be made to the Epstein theory:

1. Boundary conditions appropriate for the slip-flow regime have not been used.

2. The continuum energy equation has been solved neglecting the convective terms.

Both objections may be met by re-solving the steady-state Navier-Stokes equations for the case of a spherical particle suspended in a gas in the presence of a uniform temperature gradient and in the absence of external forces. Correct boundary conditions will be used and the effect of the convective flow will be noted by a perturbation method.

THEORETICAL ANALYSIS

The implication in Epstein's derivation of Eq. [1] is that the Navier-Stokes equations can be applied for $(l/a) > 0$, where theory and experiment have shown (12) that the Navier-Stokes equations break down. It has been found that the Navier-Stokes equations may be applied for $(l/a) > 0$ if proper slip-flow boundary conditions are used. The question as to how far toward the free-molecule regime such a slip-flow continuum analysis may be used is unresolved.

In slip-flow aerodynamics it has been observed (12) that the use of higher order stresses and heat flux from the Grad thirteen moment approximation and equations of change corresponding to the second approximation to the distribution function, the Burnett equations, are generally unnecessary. Certainly, for example, a higher order stress contribution such as the thermal stress would usually be found negligible in calculating aerodynamic drag. The situation with the thermal force is not quite so clear. The thermal force is small and exists only for $(l/a) > 0$; for this situation (as is indicated by some preliminary results of the author for photophoresis) it may not always be found that the thermal stress is negligible.

For the purpose of this analysis it will be assumed that the contribution of the thermal stress to the thermal force is small. The correctness of this assumption will be noted briefly in a following discussion. It is therefore appropriate in the analysis simply to apply the Navier-Stokes equations and correct slip-flow boundary conditions.

Accordingly, the steady-state energy equation may be written for the physical system described above in spherical coordinates (origin of coordinates in center of the suspended sphere) for the region external to the suspended sphere:

$$\nabla^2 T_f = \frac{1}{\alpha} \left(\frac{v_\theta}{r} \frac{\partial T_f}{\partial \theta} + v_r \frac{\partial T_f}{\partial r} \right), \qquad [2]$$

where α is the thermal diffusivity and v_θ and v_r are the θ and r components of the fluid velocity. The ϕ component of velocity is neglected as is the dependence of T on ϕ.

For the suspended spherical particle:

$$\nabla^2 T_s = 0. \qquad [3]$$

The boundary conditions are:

$$T_f \to gr \cos \theta \qquad r \to \infty$$

$$T_s \quad \text{finite}$$

which are unexceptional. Here $g \equiv |\nabla T_\infty|$.

Inasmuch as this analysis is to apply to the slip-flow regime, it is necessary to use the temperature jump condition in the matching conditions. The phenomenon of the temperature jump in rarefied gases is well known and confirmed by experiment (12, 13).

For this problem the matching condition including the temperature jump has the form:

$$T_f - T_s = c_t \, l \left(\frac{\partial T_f}{\partial r} \right), \qquad r = a, \tag{4}$$

where c_t is a constant depending on the nature of the gas-aerosol particle interaction. Its calculation and magnitude will be noted in a following discussion.

The other matching condition for the energy equation is:

$$k_f \left(\frac{\partial T_f}{\partial r} \right) = k_s \left(\frac{\partial T_s}{\partial r} \right), \qquad r = a. \tag{5}$$

The mass and momentum equations have their usual form for the Stokes problem for the flow around a sphere:

$$\nabla^2 \vec{v} = \frac{1}{\mu} \nabla p;$$

$$\nabla \cdot \vec{v} = 0. \tag{6}$$

It is assumed that the particle is sufficiently small relative to the extent of fluid that all mass average velocities vanish as $r \to \infty$.

The proper slip-flow boundary condition must be used for the motion of the fluid at the particle surface. At a gas-particle interface, it may be shown (12, 14) that the following conditions apply:

$$v_r = 0, \qquad r = a$$

$$v_\theta = c_m \, l \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]_{r=a} + \frac{3}{4} \frac{\mu}{\rho T} \frac{1}{a} \left(\frac{\partial T_f}{\partial \theta} \right)_{r=a}. \tag{7}$$

It is from the slip-flow boundary conditions, Eq. [7], that one obtains the principal contribution to the thermal force. Epstein in his analysis considered only the second term on the right of the Eq. [7]. In Eq. [7], c_m is a constant whose value depends on the nature of the gas-particle interaction. Available sources (12, 14) give the indicated value for the coefficient of the second term on the right of Eq. [7]. Further study of slip-flow boundary

conditions is necessary to determine any alterations in this value for various gas-surface systems.

An exact analytical solution of Eqs. [2], [3], and [6] has not been found. A perturbation solution to these equations suggests itself with:

$$T_f = \sum_{n=0} \beta^n T_f^{(n)} \tag{8}$$

$$T_s = \sum_{n=0} \beta^n T_s^{(n)} \tag{9}$$

and:

$$T_f^{(0)} \to gr \cos \theta, \qquad r \to \infty \tag{10}$$

$$T_f^{(n)} \to 0, \quad n > 0, \qquad r \to \infty \tag{11}$$

$$T_f^{(n)} - T_s^{(n)} = c_i l \left(\frac{\partial T_f^{(n)}}{\partial r} \right)_{r=a}, \qquad r = a \tag{12}$$

$$k_f \left(\frac{\partial T_f^{(n)}}{\partial r} \right)_{r=a} = k_s \left(\frac{\partial T_s^{(n)}}{\partial r} \right)_{r=a}, \qquad r = a. \tag{13}$$

Also

$$v_\theta = \sum_{n=0} \beta^{n+1} v_\theta^{(n+1)} \tag{14}$$

$$v_r = \sum_{n=0} \beta^{n+1} v_r^{(n+1)} \tag{15}$$

$$v_\theta^{(n+1)} = c_m l \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta^{(n+1)}}{r} \right) + \frac{1}{r} \frac{\partial v_r^{(n+1)}}{\partial \theta} \right]_{r=a} + \frac{1}{a} \left(\frac{\partial T_f^{(n)}}{\partial \theta} \right)_{r=a} \tag{16}$$

$$v_r^{(n+1)} = 0, \qquad r = a \tag{17}$$

$$v_\theta^{(n+1)} = v_r^{(n+1)} = 0, \qquad r \to \infty \tag{18}$$

Here β is the perturbation parameter. $T_f^{(n)}$, $T_s^{(n)}$, $v_\theta^{(n)}$, $v_r^{(n)}$ are the n-th iterated solutions of Eqs. [2], [3], and [6].

Substituting Eqs. [8] and [9] in Eqs. [2] and [3], respectively, and equating coefficients of equal powers of β, one has for the first several iterations for Eqs. [2] and [3]:

$$\nabla^2 T_f^{(0)} = 0; \tag{19}$$

$$\nabla^2 T_f^{(1)} = \frac{1}{\alpha} \left(\frac{v_\theta^{(1)}}{r} \frac{\partial T_f^{(0)}}{\partial \theta} + v_r^{(1)} \frac{\partial T_f^{(0)}}{\partial r} \right); \tag{20}$$

$$\vdots$$

$$\nabla^2 T_s^{(0)} = 0;$$

$$\nabla^2 T_s^{(1)} = 0.$$

$$\vdots$$

The same type of iterative solution is carried out for Eq. [6].

A difficulty is found in this perturbation solution. It is not possible to meet the condition of Eq. [11] if the solution of Eq. [19] and the corresponding solutions of Eq. [6] are used in Eq. [20]. However, the solutions $T_f^{(n)}$, $n > 0$, are of interest in the calculation of the thermal force only very near $r = a$.

Accordingly, the following approximations are made. Consider the solution of Eq. [20]. Here it is appropriate to use expansions of the functional dependence on r of $T_f^{(0)}$, $v_\theta^{(1)}$, and $v_r^{(1)}$ such that Eq. [11] is satisfied for $T_f^{(1)}$.

For $v_\theta^{(1)}$, $v_r^{(1)}$ near $\rho' = (r/a) = 1$, the following expansions are useful:

$$\frac{1}{\rho'^3} + \frac{1}{\rho'} = \frac{2}{\rho'^2} + \sum_{n>2} a_n \left(\frac{1}{\rho'^n} - \frac{1}{\rho'^{n+1}} \right) \qquad [21]$$

for $v_\theta^{(1)}$ and

$$\frac{1}{\rho'} - \frac{1}{\rho'^3} = \sum_{n>2} b_n \left(\frac{1}{\rho'^n} - \frac{1}{\rho'^{n+1}} \right) \qquad [22]$$

for $v_r^{(1)}$. In the solution $T_f^{(0)}$ the following expansion is used:

$$\rho' = \sum_{n=0} (-1)^n \left(\frac{1}{\rho'} - 1 \right)^n, \qquad 0 < \rho' < 2 \quad [23]$$

satisfactory for ρ' in the neighborhood of 1.

The expansions, Eqs. [21], [22], and [23], will be carried out only to the first order in $1/\rho'^2$. This approximation is equivalent to neglecting $v_r^{(1)}$ in Eq. [20]. When these expansions are used for the functional dependence on r of $v_\theta^{(1)}$ and $T_f^{(0)}$ in Eq. [20] the resulting solution satisfies the condition of Eq. [11]. The same procedure is followed in obtaining the higher iterations.

Following the method outlined above, the perturbation solution of Eq. [2] has the following form for the first several iterations:

$$
\begin{aligned}
T_f = \Bigg\{ & \left\{ r + \frac{\left(\frac{k_f}{k_s} + c_t \frac{l}{a} - 1 \right) a^3}{\left(\frac{2k_f}{k_s} + 2 \frac{c_t l}{a} + 1 \right) r^2} \right\} g \cos \theta \\
& + \beta \Bigg\{ -\frac{2}{3}\epsilon \frac{a \ln(r/a)}{r} - \frac{1}{4}\gamma \frac{a^2}{r^2} + \frac{1}{9}\gamma \frac{a^3}{r^3} - \frac{1}{15}\gamma a^3 \frac{\ln(r/a)}{r^3} \\
& + (-(1/18)\epsilon - (1/3)\gamma) \frac{a}{r} + \left(\frac{1}{6}\epsilon \frac{a}{r} - \frac{1}{4}\epsilon \frac{a^2}{r^2} + \frac{1}{5}\gamma a^3 \frac{\ln(r/a)}{r^3} \right) \cos^2 \theta \\
& + \frac{((1/9)\epsilon + (1/15)\gamma)\left[\frac{k_f}{2k_s} + c_t \frac{l}{a} \right] + (1/36)\epsilon}{\left(1 + \frac{3k_f}{k_s} + 3c_t \frac{l}{a} \right)} (3\cos^2 \theta - 1) \Bigg\} \qquad [24] \\
& + \beta^2 \Bigg\{ f_1 \left(r; \left(\frac{1}{\alpha} \right)^2 \left(\frac{c_t \frac{l}{a} + \frac{k_f}{k_s}}{1 + 2c_t \frac{l}{a} + \frac{k_f}{k_s}} \right)^2 g^3 a^3 \right) \cos \theta \\
& + f_2 \left(r; \left(\frac{1}{\alpha} \right)^2 \left(\frac{c_t \frac{l}{a} + \frac{k_f}{k_s}}{1 + 2c_t \frac{l}{a} + \frac{k_f}{k_s}} \right)^2 g^3 a^3 \right) \cos^3 \theta \Bigg\} + \cdots ,
\end{aligned}
$$

where:

$$\epsilon = \frac{3}{\alpha}\left(\frac{c_t \dfrac{l}{a} + \dfrac{k_f}{k_s}}{1 + 2c_t \dfrac{l}{a} + \dfrac{2k_f}{k_s}}\right)\left(\frac{1}{1 + 3c_m \dfrac{l}{a}}\right)g^2 a^2;$$

$$\gamma = \frac{9}{\alpha}\left(\frac{c_t \dfrac{l}{a} + \dfrac{k_f}{k_s}}{1 + 2c_t \dfrac{l}{a} + \dfrac{2k_f}{k_s}}\right)^2\left(\frac{1}{1 + 3c_m \dfrac{l}{a}}\right)g^2 a^2;$$

$$\beta = \frac{3}{4}\frac{\mu}{\rho T}.$$

The approximate solution of Eq. [6] for the first several iterations has the form:

$$v_\theta = -(3\!/\!2)\beta g \frac{\left(\dfrac{k_f}{k_s} + c_t \dfrac{l}{a}\right)}{\left(1 + 2\dfrac{k_f}{k_s} + 2c_t \dfrac{l}{a}\right)}\left(\frac{1}{1 + 3c_m \dfrac{l}{a}}\right)\left(\frac{a^3}{r^3} + \frac{a}{r}\right)\sin\theta \qquad [25]$$

$$+ \beta^2 h_1(r)\sin\theta\cos\theta + \beta^3 h_2(r)\{\sin\theta + b\cos^2\theta\sin\theta\} + \cdots$$

and similarly for v_r .

DISCUSSION

In the preceding analysis, slip-flow boundary conditions, correct for the physical system, have been applied and approximate expressions for the velocity and temperature distributions in the convective field have been obtained.

It is now possible to derive an expression for the thermal force, valid for the slip-flow regime, with some knowledge of the effect of the convective flow on the thermal force.

The thermal force is obtained by integrating the normal component of the pressure tensor, $\bar{\bar{\pi}}$, over the surface of the particle: $\int \bar{\bar{\pi}} \cdot \vec{n} \, ds$. The form of the pressure tensor for this analysis must now be considered.

In the continuum analysis for this system only the usual first-order pressure tensor was considered. However, it was pointed out that for $(l/a) > 0$ it may be necessary to consider the contribution of higher order stresses, in particular the thermal stress, to the pressure tensor. For this study, it may be shown that the contribution of the thermal stress to the thermal force is negligible.

Unless one considers the thermal stress as small, as here, inclusion of the thermal stress in the pressure tensor results in a coupling of the equations of change. How-

ever, for an assumed small thermal stress, one can uncouple the equations to some approximation and simply superpose the contributions of the thermal and viscous stresses to the thermal force.

The thermal stress tensor has the form (8):

$$\bar{\bar{t}} = -3\delta \frac{\mu^2}{\rho T} (\nabla\nabla T),$$

where δ is a numerical constant which for a Maxwellian molecule is equal to one.

For the temperature solution, Equation [24], the zeroth order term in β, $T_f^{(0)}$ corresponds to the solution:

$$\nabla^2 T_f^{(0)} = 0 ,$$

and it may be seen that for any shape particle the thermal stress term corresponding to this solution can give no contribution to the thermal force, i.e.,

$$\int (\nabla\nabla T_f^{(0)})\cdot \vec{n} \, dS = 0.$$

The effect of the convective flow on the temperature distribution is accounted by terms first order and higher in β in Equation [24]. From Equation [24] it may be seen that the first-order term in β gives no contribution to the thermal force for this physical system:

$$\int (\nabla\nabla T_f^{(1)})\cdot \vec{n} \, dS = 0.$$

Not until the second-order term:

$$\int (\nabla\nabla T_f^{(2)})\cdot \vec{n} \, dS \neq 0$$

does one obtain a contribution to the thermal force from the thermal stress term. This contribution to the thermal force is multiplied by terms including $\beta^2 a^3 g^3$ which, for an imposed temperature gradient of order 1 at from 0.01–1.0 atm., is in the range 10^{-18}–10^{-24} and represents a negligible contribution to the thermal force. This result is borne out by experiment (1, 2, 10, 11) which gives within a fairly large experimental error a linear dependence of the thermal force on the imposed temperature gradient. Larger values of β cannot be considered since the perturbation method becomes questionable.

There remains, therefore, as the only important contribution to the thermal force the viscous stresses arising from the condition of Eq. [7] at the fluid-particle boundary. These viscous stresses may be calculated from the velocity distribution given by Eq. [25] and an analogous expression for v_r .

The effect of accounting for the convective flow in the energy equation on the velocity distribution may be noted by observing that terms in Eq. [25] of second order and higher in β are associated with the perturbations of the temperature field produced by the convective flow.

The first-order term in β in Eq. [25] gives the important contribution to

the thermal force. The second order term gives no contribution. The third and higher order terms give contributions which are negligible for small values of β, as described above for the thermal stress.

Thus, to the first order in the perturbation parameter β for the temperature distribution and to the second order for the velocity distribution, the convective flow has no effect on the thermal force.

To this approximation, then, the following expression is obtained for the thermal force in the slip-flow regime:

$$\vec{K} = -9\pi \frac{\mu^2}{\rho T} a \left(\frac{1}{1 + 3c_m \dfrac{l}{a}} \right) \left(\frac{\dfrac{k_f}{k_s} + c_t \dfrac{l}{a}}{1 + 2\dfrac{k_f}{k_s} + 2c_t \dfrac{l}{a}} \right) \nabla T_\infty . \qquad [26]$$

This equation has been compared with the experimental data of Schadt and Cadle (2). It is found that the equation is in agreement with the experimental results for the slip-flow regime. Before discussing this comparison, brief consideration should be given to the definition and calculation of c_m and c_t of Eq. [26].

The constants c_t associated with the temperature jump and c_m associated with the velocity slip were introduced in the analysis above in Eqs. [4] and [7] as constants whose values depend on the nature of the gas-particle interaction.

Simple semimacroscopic derivations (e.g., references 13, 14) of the slip-flow boundary conditions for monatomic molecules impinging on an infinite plane wall give the following values for c_t and c_m :

$$c_t = \frac{15}{8} \left(\frac{2 - a_t}{a_t} \right) ; \qquad [27]$$

$$c_m = \frac{2 - a_m}{a_m} . \qquad [28]$$

Here a_t is termed the thermal accommodation coefficient and a_m the momentum accommodation coefficient or "reflection coefficient." These coefficients are best defined by:

$$a_t = \frac{E_i - E_r}{E_i - E_w} ; \qquad [29]$$

$$a_m = \frac{G_i - G_r}{G_i} . \qquad [30]$$

Here, E_i is the average incident molecular energy flux at a point on a surface, E_r is the average energy flux for molecules leaving the surface at that point, and E_w represents that energy flux which would be emitted if the molecules left in Maxwellian equilibrium with the surface at that point.

Similarly, G_i and G_r represent the average tangential components of the momentum of molecules striking and leaving the surface.

Values of $a_t = a_m = 1$ correspond to the situation where incident molecules achieve complete thermodynamic equilibrium with the surface before leaving; $a_t = a_m = 0$ corresponds to complete specular reflection of the incident molecules. From present evidence, it is suggested (15) that for unprepared or "technical" surfaces, a_m is usually very close to 1.0 and a_t may vary from 0.5 to 1.0. For air on various surfaces values of a_t very near 1.0 have been reported (16).

It should be pointed out that molecular beam experiments with helium on LiF crystal surfaces (15) have given values of a_m very near zero. Apparently quantum effects are involved in the explanation of these results. A test of Eq. [26] would be interesting for this special case inasmuch as the predicted result for $a_m = 0$ is a value of zero for the thermal force.

More rigorous deductions of the slip-flow boundary conditions have been made. These deductions begin with a linearized Boltzmann equation from which a form of the distribution function is found. The temperature jump and velocity slip conditions follow from Maxwell's assumption concerning the reflection of gas molecules from a wall where the accommodation coefficients are introduced. These more elaborate derivations result usually in relatively small changes in the expression for c_t and c_m from the values found by the simpler semi-macroscopic arguments. For example, for $a_t = a_m = 1.0$, values of c_t range from 1.875 to 2.48; values of c_m range from 1.00 to 1.27. A review of these values and deductions may be found in reference 12.

This brief discussion cannot present the complexities inherent in the *a priori* evaluation of c_t and c_m for a given gas-surface system. Further investigation is required before it will be possible to deduce c_t and c_m for a particular system.

The effect of changes in c_t and c_m on the thermal force can be seen easily by inspection of Eq. [26]. It is noted that the magnitude of the thermal force becomes less sensitive to changes in c_t and more sensitive to changes in c_m as (l/a) increases. As the magnitude of (k_f/k_s) increases the thermal force becomes less sensitive to changes in the magnitude of c_t.

In comparing Eq. [26] with thermal force experimental data it seems justified to assume that $a_t = a_m = 1.0$, in view of the discussion given above. Once the values of the accommodation coefficients have been set, thereby accounting for the gas-particle interaction, the magnitudes of c_t and c_m depend on the nature of the gas. The variation in these magnitudes has been noted above. Consequently, within these limits, c_m and c_t must at present be regarded as semi-empirical constants.

Figures 1 and 2 present a comparison between some of the data of Schadt and Cadle (2) and Eq. [26]. The value of c_t was taken to be 2.16, and is the same as calculated by Welander (17) and is also the same as the average of
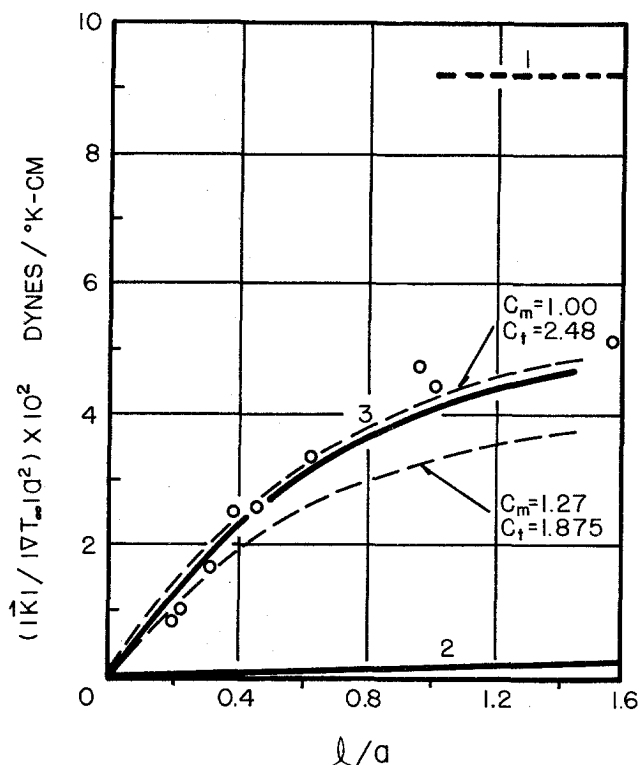
FIG. 1. Sodium chloride particles in air; particle size: 0.22–0.35 μ radius. $T = 307°K\cdot$ Curve 1: Waldmann equation (ref. 6) Curve 2: Epstein equation (ref. 8) Curve 3: Eq. [26] this note $c_m = 1.00$, $c_t = 2.16$. $\bigcirc$: Experimental points of Schadt and Cadle (ref. 2).

values reported by Payne. The value of c_m was taken as 1.0, as given both by Eq. [28] and by the lower limit reported by Payne. It was found that these values gave an approximate fit to the experimental data for sodium chloride aerosols at small values of (l/a). It is seen that the same values of c_t and c_m also adequately represent the experimental data for TCP droplets.

Also shown in Fig. 1 are the limiting curves corresponding to the extremes of the reported ranges of values of c_t (1.875–2.48) and c_m (1.00–1.27) noted above for a value of 1.0 for the accommodation coefficients. For these ranges of values for c_t and c_m it is seen that the maximum variation in the calculated thermal force is from 20% to 30%.

For comparison, the curves representing the Epstein equation are shown in Figs. 1 and 2. The complete failure of Eq. [1] for the sodium chloride aerosols is evident. The other curves shown represent calculated values from the Waldmann equation (6) derived for the free-molecule regime. Equation [26], valid for the slip-flow regime, predicts a value of 7 for
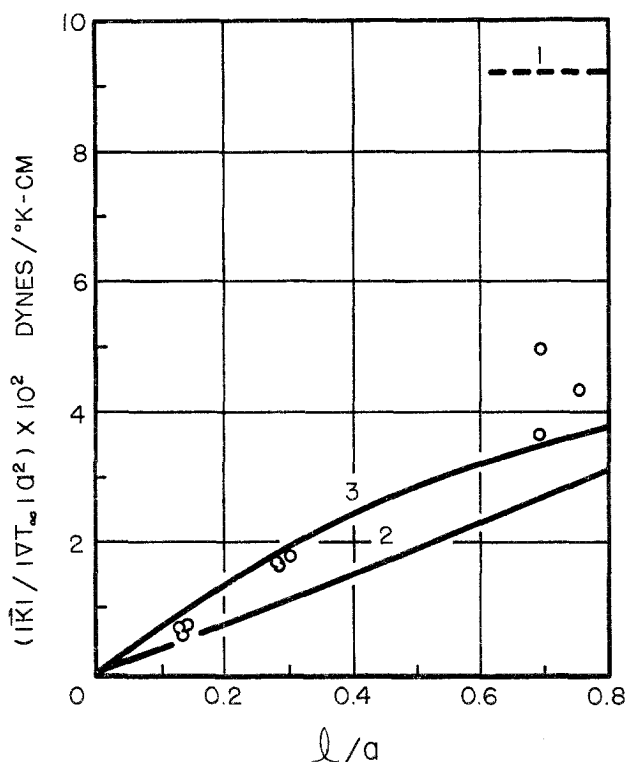
FIG. 2. TCP droplets in air. Droplet size: 0.46–0.49 μ radius. $T = 307°$K. For legend, see Fig. 1.

$\{\mid K \mid /(\mid \nabla T_\infty \mid a^2)\} \times 10^2$ for $(l/a) \to \infty$, whereas the indicated prediction of the Waldmann equation is $\sim$9.

In view of the various assumptions involved in the derivation of Eq. [26] and evaluation of c_t and c_m, the agreement between values of the thermal force predicted by Eq. [26] and the data reported by Schadt and Cadle seems satisfactory. Further work remains to be done to determine the range of validity of the slip-flow continuum analysis presented here.

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