UAS-Based LiDAR Mapping

Video B

LiDAR Point Positioning

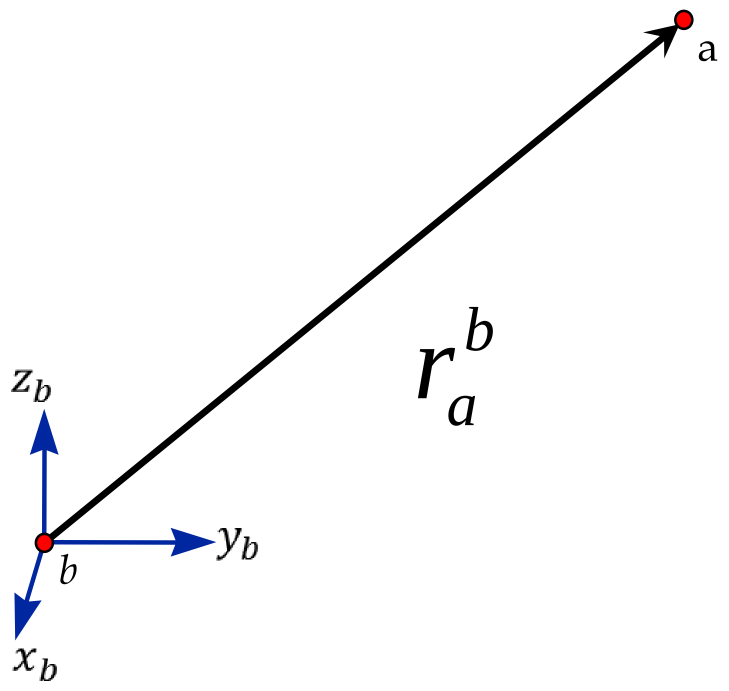
LiDAR Point Positioning Equation

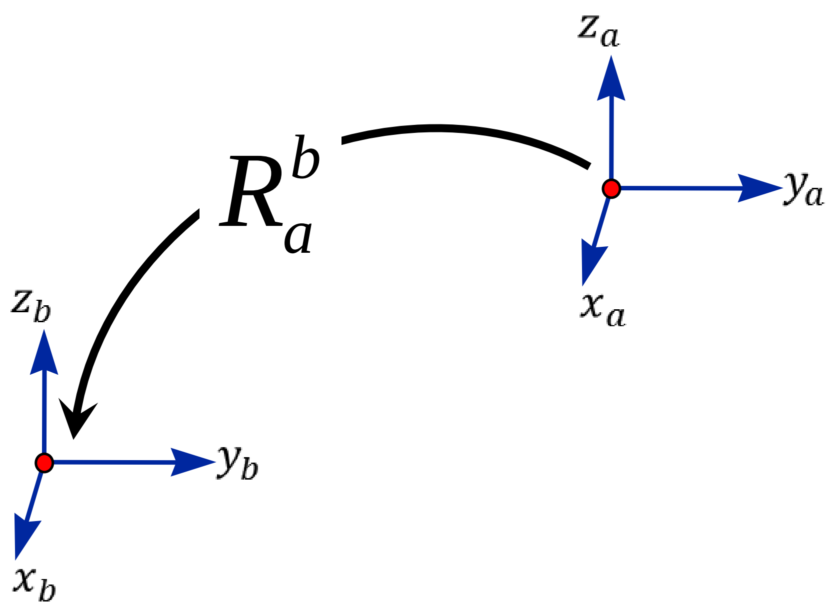
- **Objective:**
  - How are the LiDAR measurements used to generate the ground coordinates of the laser footprints?
- **We will be focusing on Mobile LiDAR Systems since they are the most general ones.**
  - The model for static LiDAR can be derived as a special case.
- **Procedure:**
  - Involved coordinate systems
  - Relationship between these coordinate systems (mounting parameters)
  - LiDAR equation
- **LiDAR & Photogrammetric Mapping**

Notation

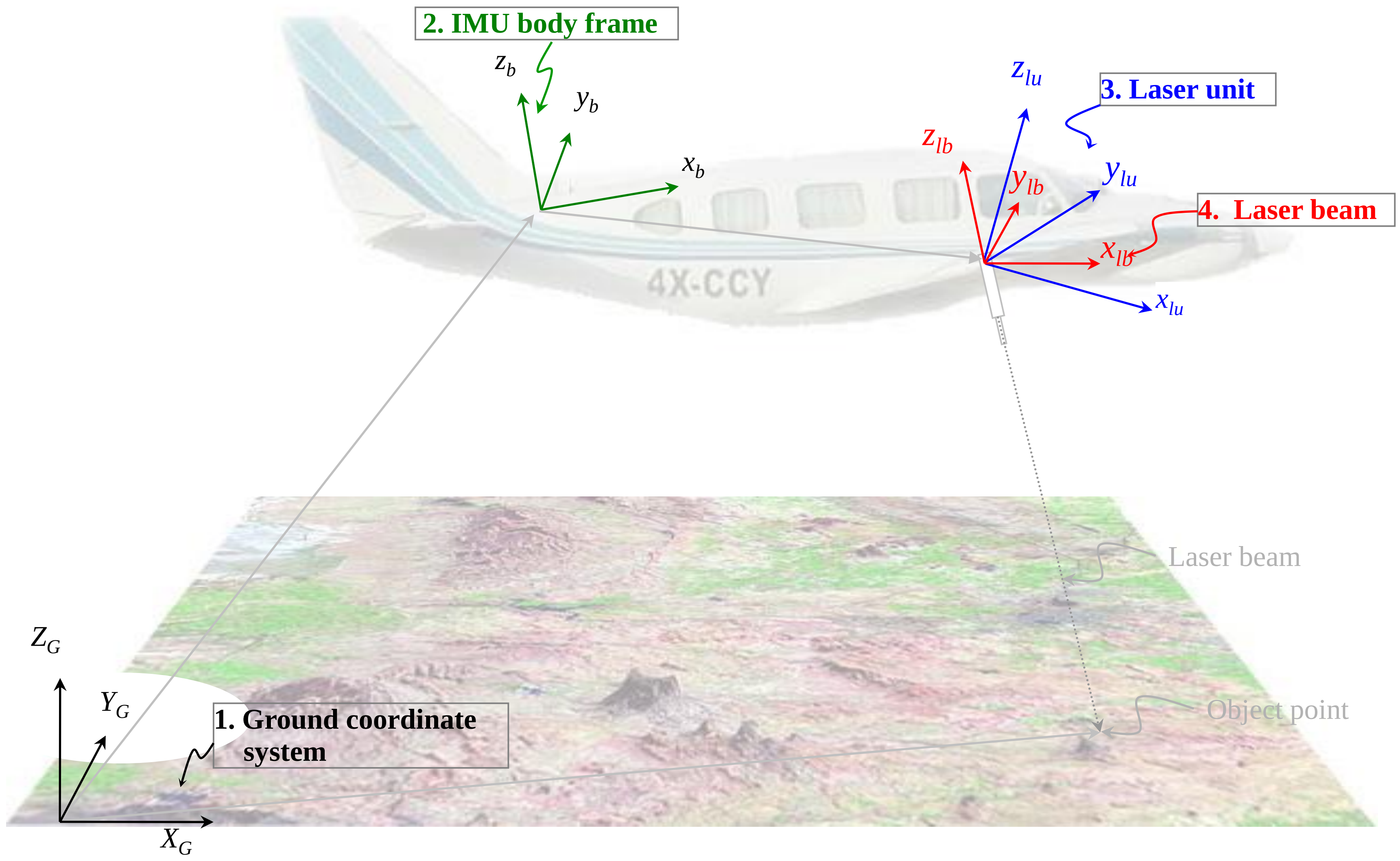
- r_a^b Stands for the coordinates of point ***a*** relative to point ***b*** – this vector is defined relative to the coordinate system associated with point ***b***.

- R_a^b Stands for the rotation matrix that transforms a vector defined relative to the coordinate system denoted by ***a*** into a vector defined relative to the coordinate system denoted by ***b***.



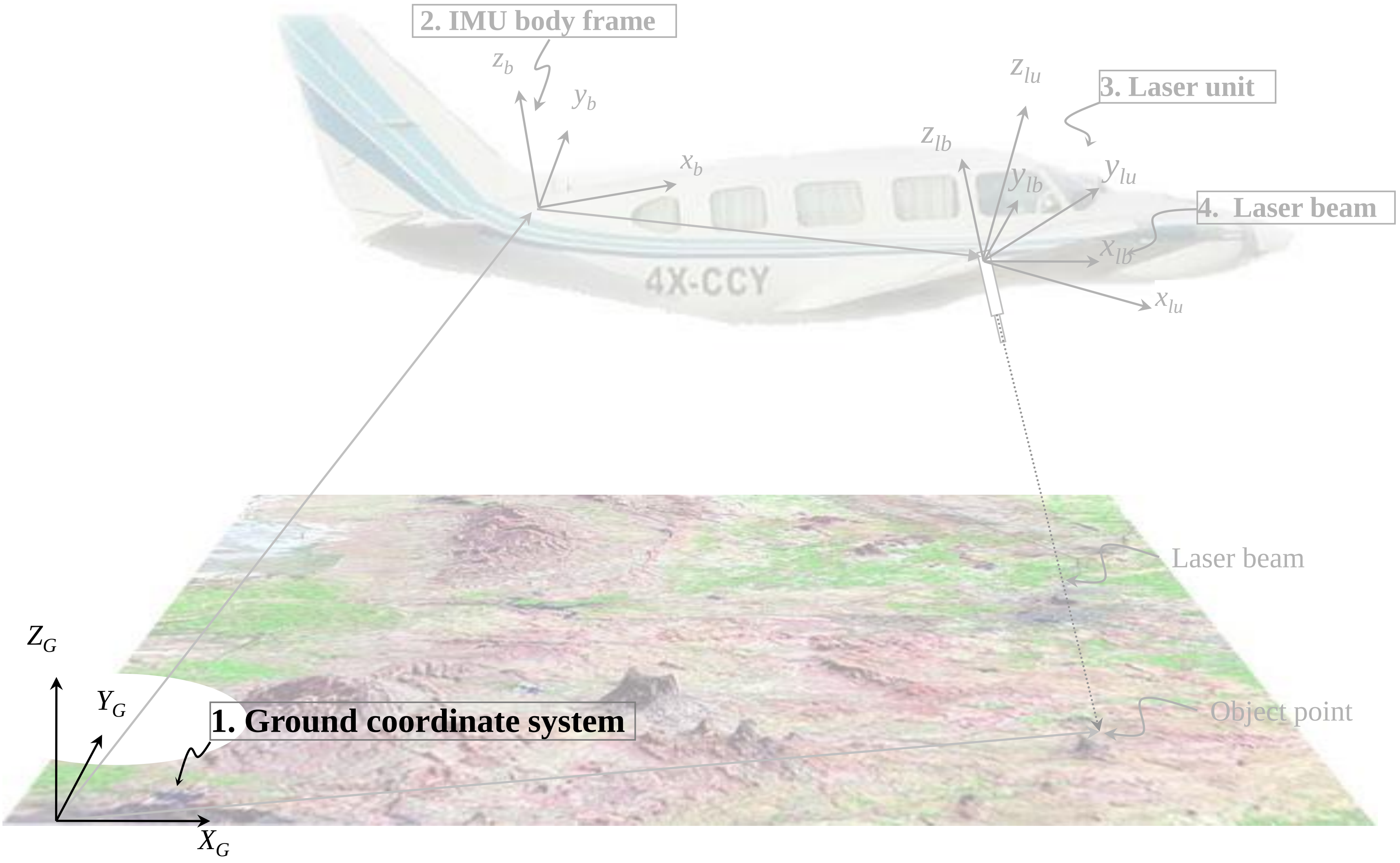


LiDAR Equation: Coordinate Systems



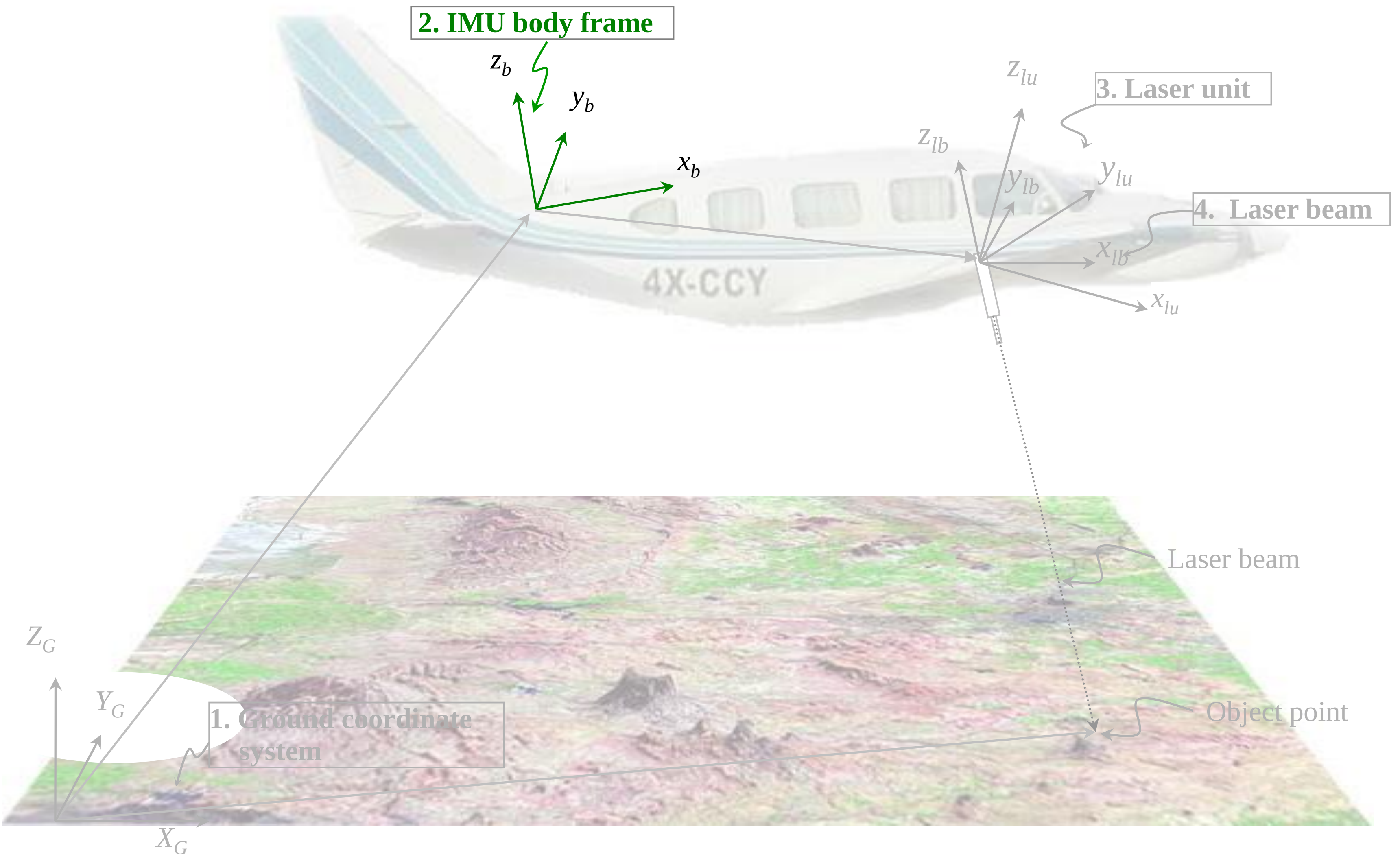
Four coordinate systems involved

LiDAR Equation: Coordinate Systems



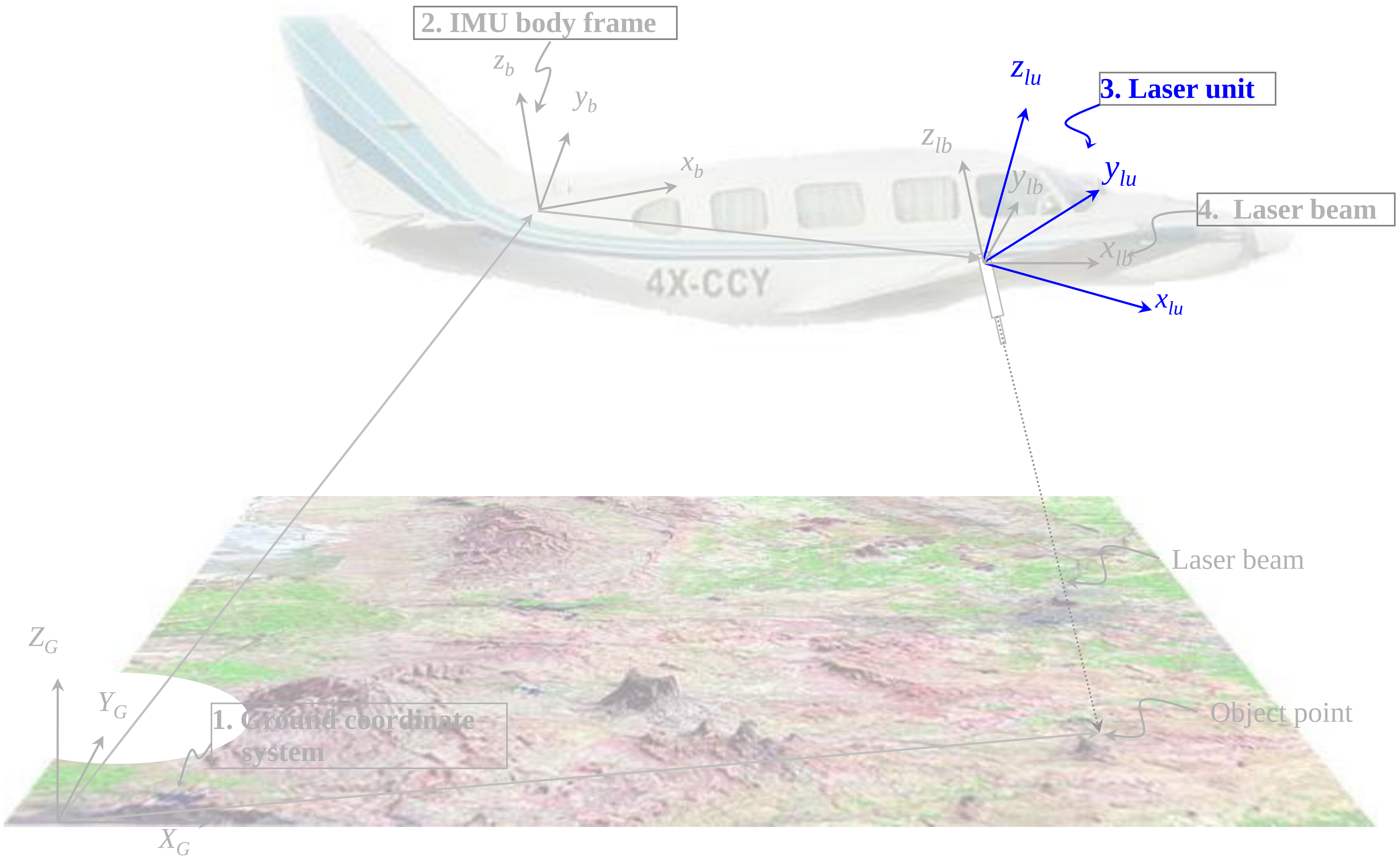
Ground/mapping coordinate system (X_G, Y_G, Z_G)

LiDAR Equation: Coordinate Systems



IMU body frame coordinate system (x_b, y_b, z_b)

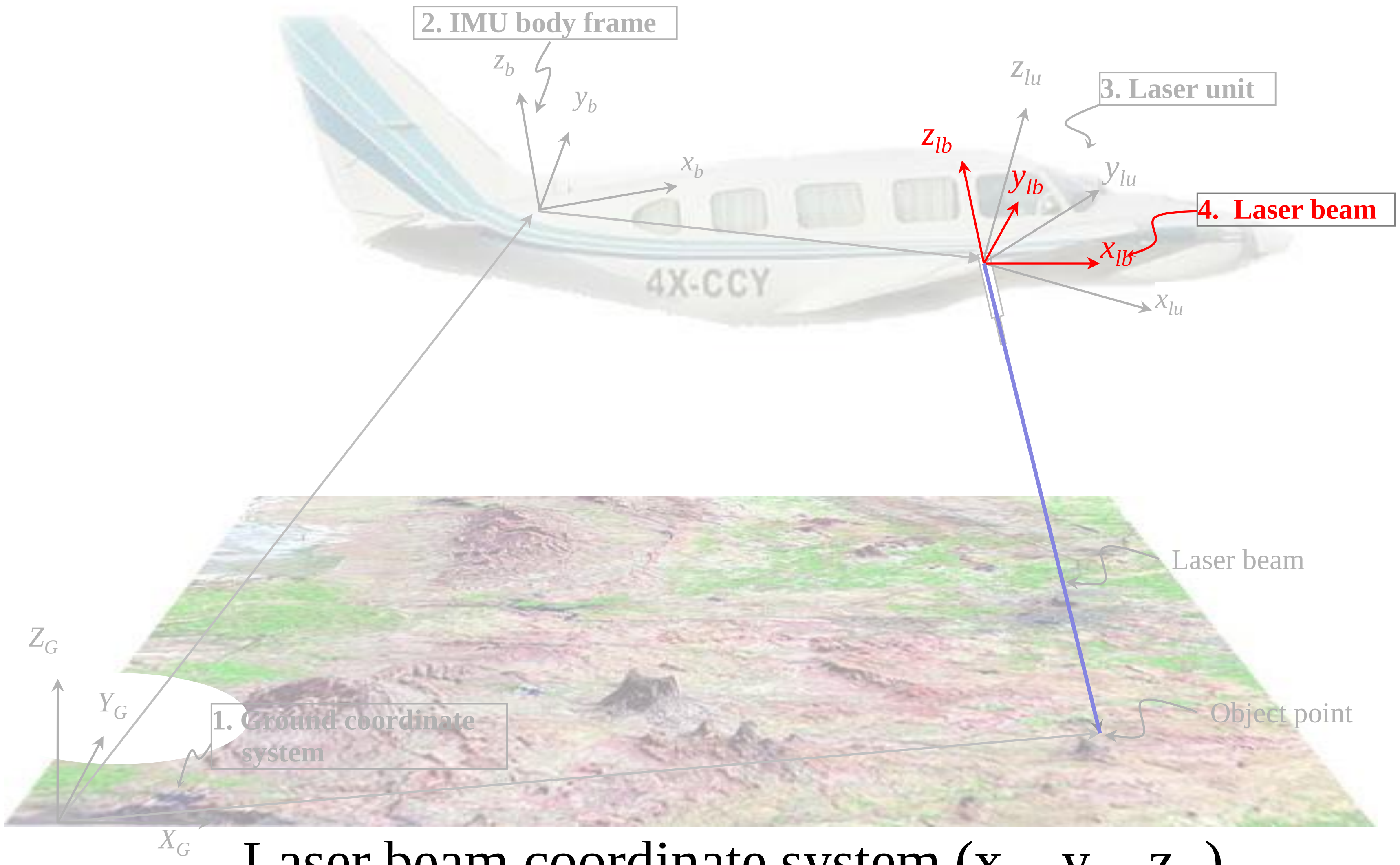
LiDAR Equation: Coordinate Systems



Laser unit coordinate system (x_{lu}, y_{lu}, z_{lu})

- Origin at the laser beam firing point

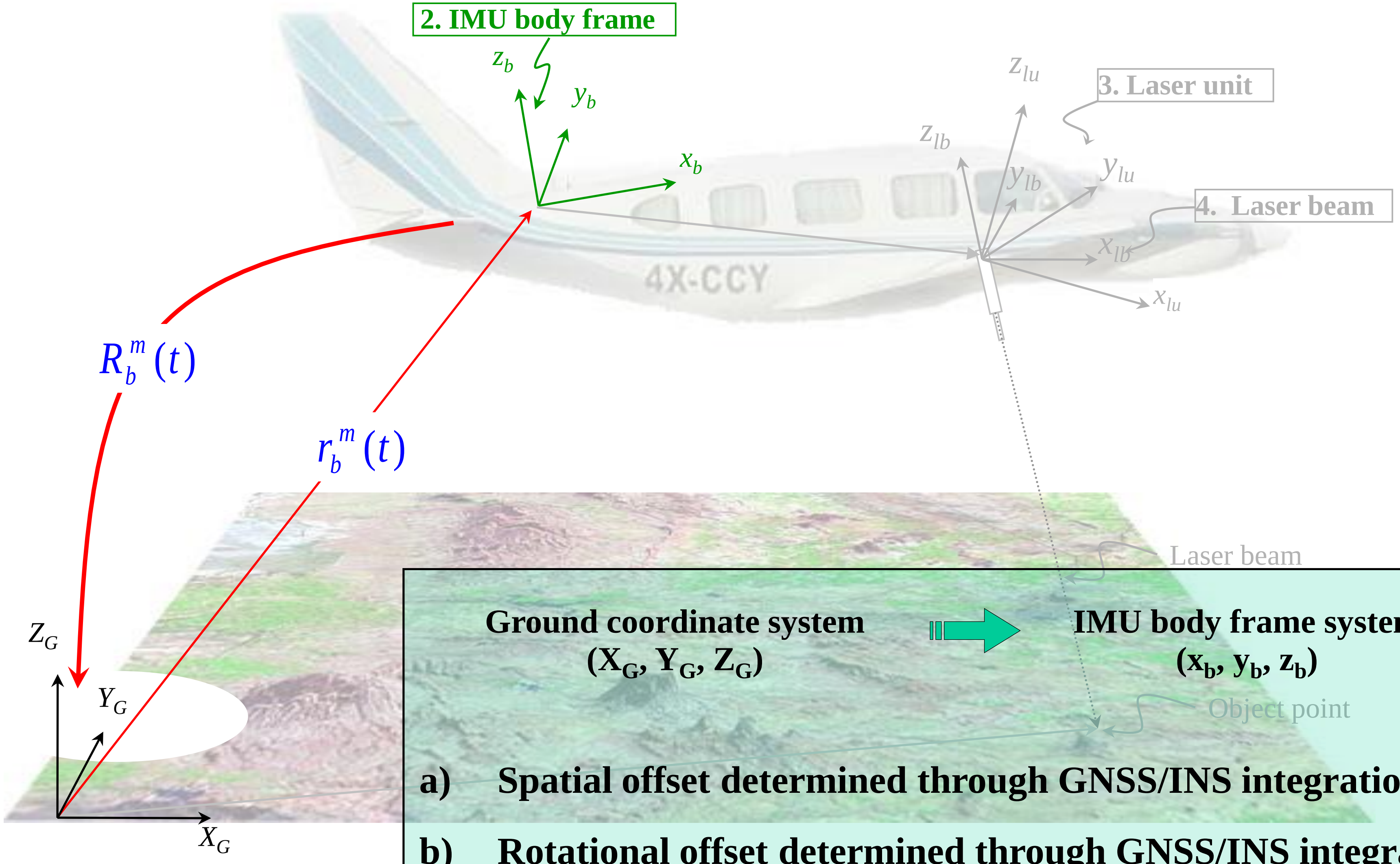
LiDAR Equation: Coordinate Systems



Laser beam coordinate system (x_{lb}, y_{lb}, z_{lb})

- Origin at the laser beam firing point
  - z-axis (z_{lb}) along the laser beam

LiDAR Equation: Coordinate Systems

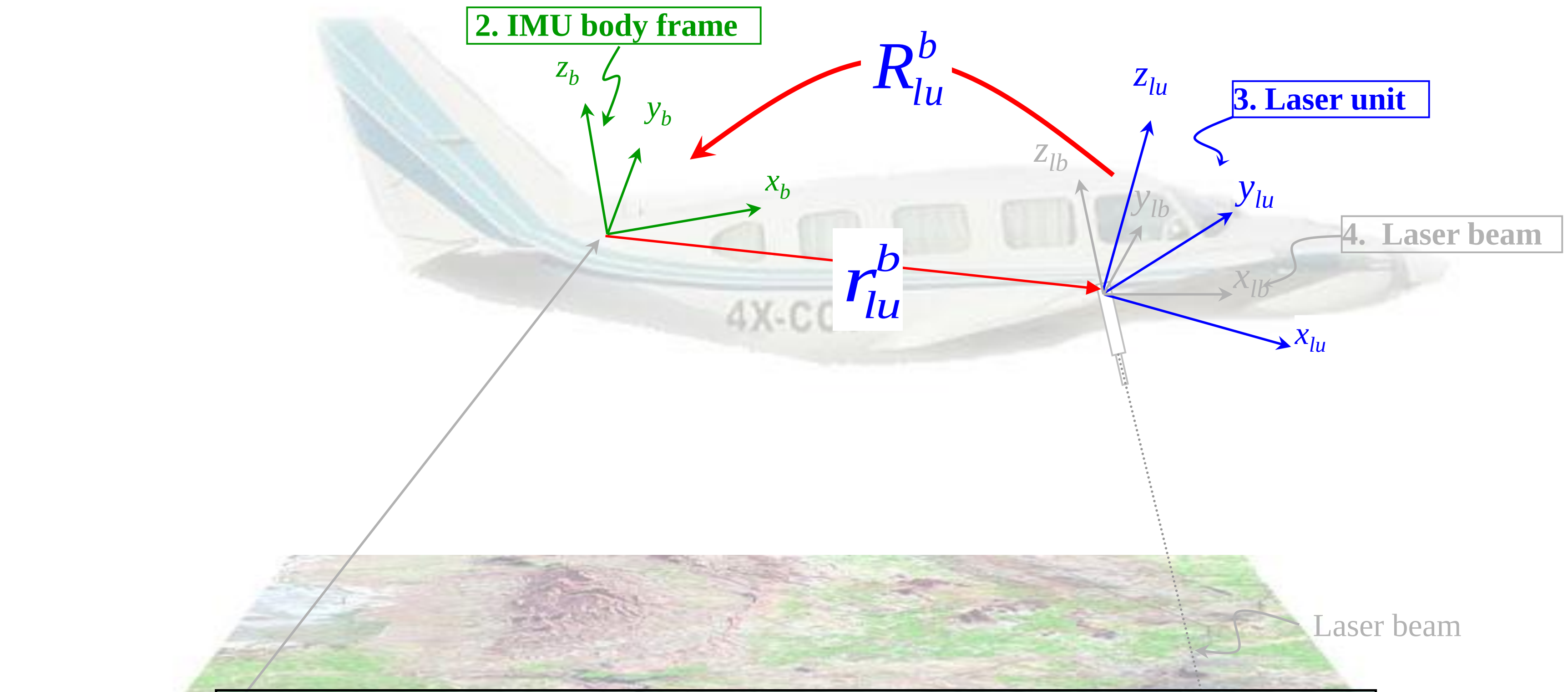


Ground coordinate system (X_G, Y_G, Z_G) → **IMU body frame system (x_b, y_b, z_b)**

a) **Spatial offset determined through GNSS/INS integration**

b) **Rotational offset determined through GNSS/INS integration**

LiDAR Equation: Coordinate Systems

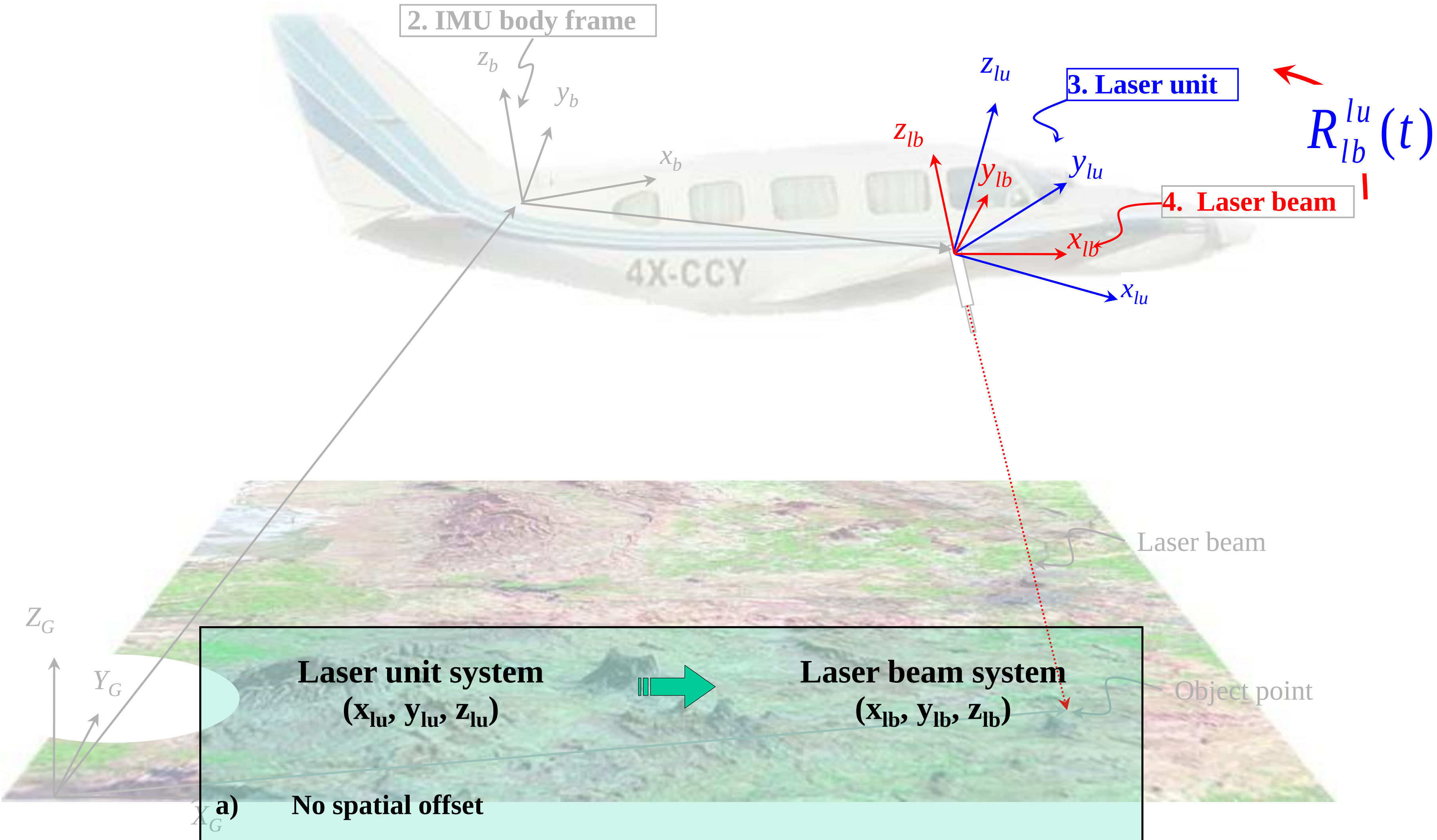


IMU body frame system (x_b, y_b, z_b) → **Laser unit system** (x_{lu}, y_{lu}, z_{lu})

a) **Lever-arm offset ($\Delta x, \Delta y, \Delta z$) determined through system calibration**

b) **Boresight angles ($\Delta\omega, \Delta\phi, \Delta\kappa$) determined through system calibration**

LiDAR Equation: Coordinate Systems



Laser unit system (x_{lu}, y_{lu}, z_{lu}) ⇒ **Laser beam system (x_{lb}, y_{lb}, z_{lb})**

a) **No spatial offset**

b) **Rotational offset (α, β) defined by the laser steering mirror**

LiDAR Equation: Coordinate Systems

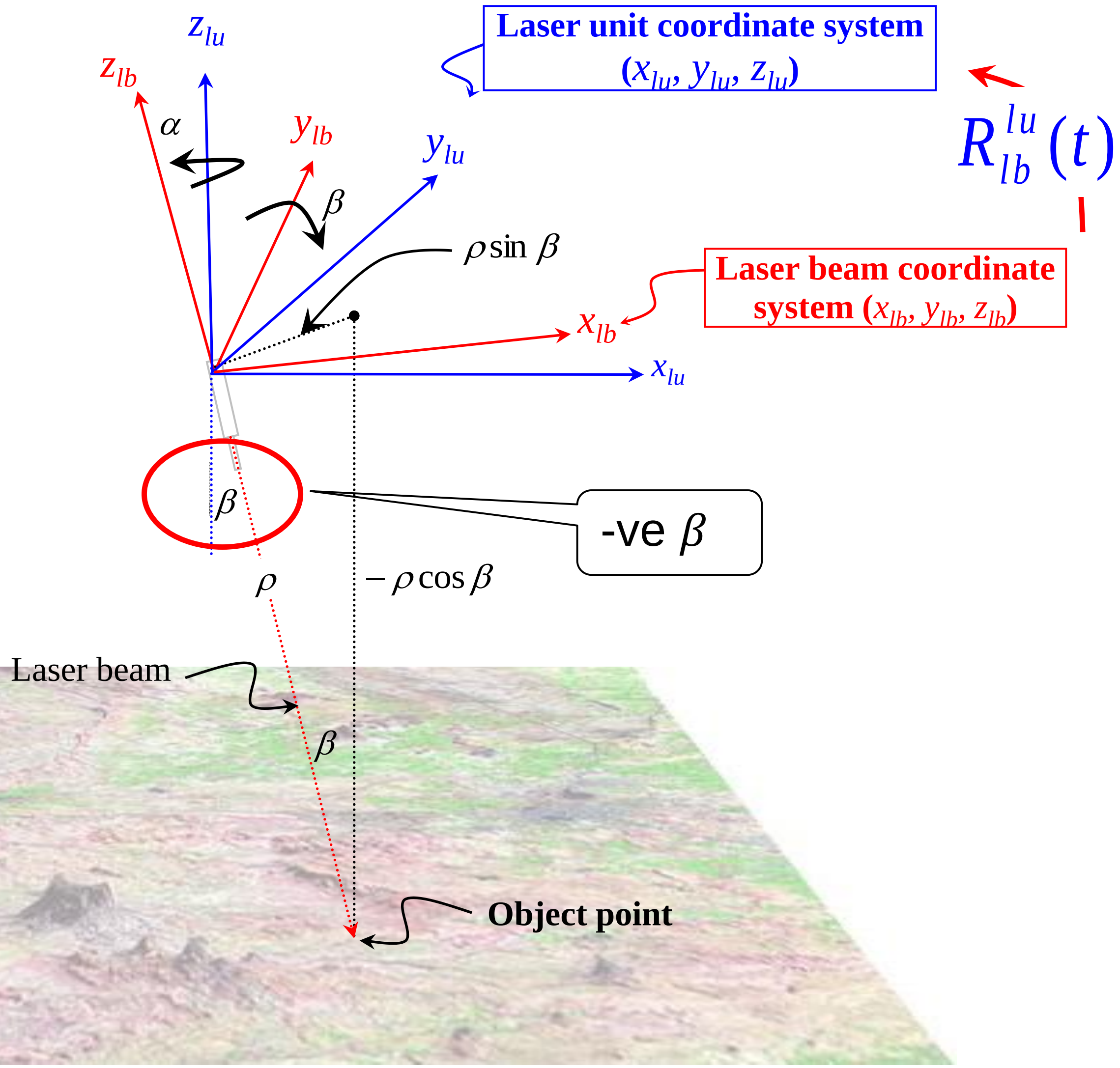
Object point coordinates

w.r.t. laser beam coordinate system

$$r_I^{lb}(t) = \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix}$$

w.r.t. laser unit coordinate system

$$r_I^{lu}(t) = \begin{bmatrix} -\rho\cos\alpha\sin\beta \\ -\rho\sin\alpha\sin\beta \\ -\rho\cos\beta \end{bmatrix}$$



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LiDAR Equation: Coordinate Systems

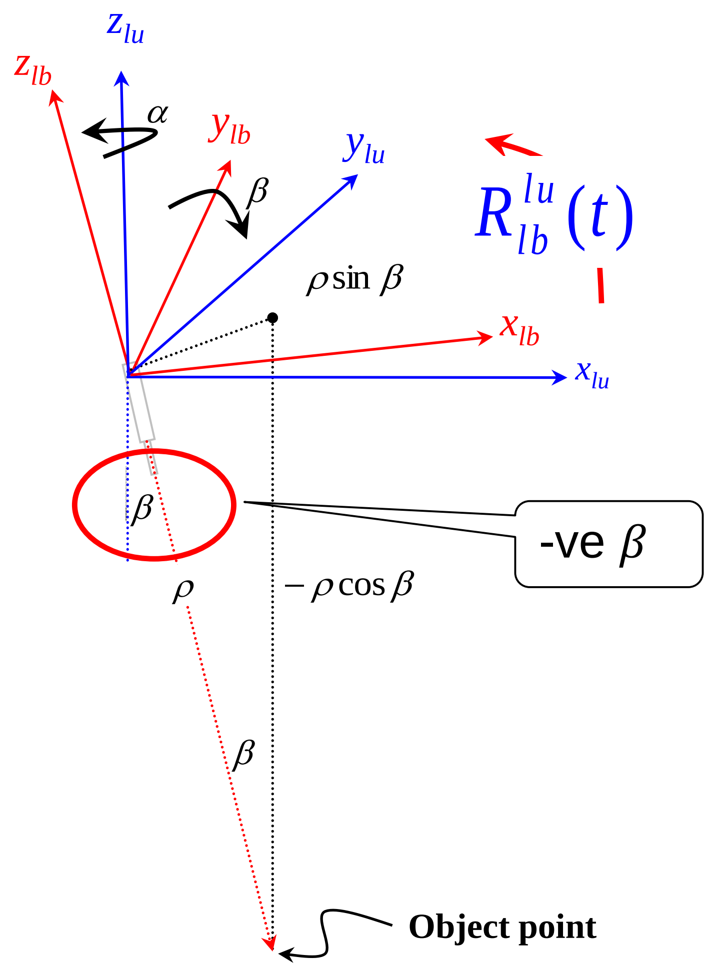
- **Transformation between laser unit and laser beam coordinate systems can be established through the following rotations:**
  - Rotation (α) around z_{lu}
  - Rotation (β) around y_{lu_α}

$$\begin{bmatrix} x_{lu} \\ y_{lu} \\ z_{lu} \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix}$$

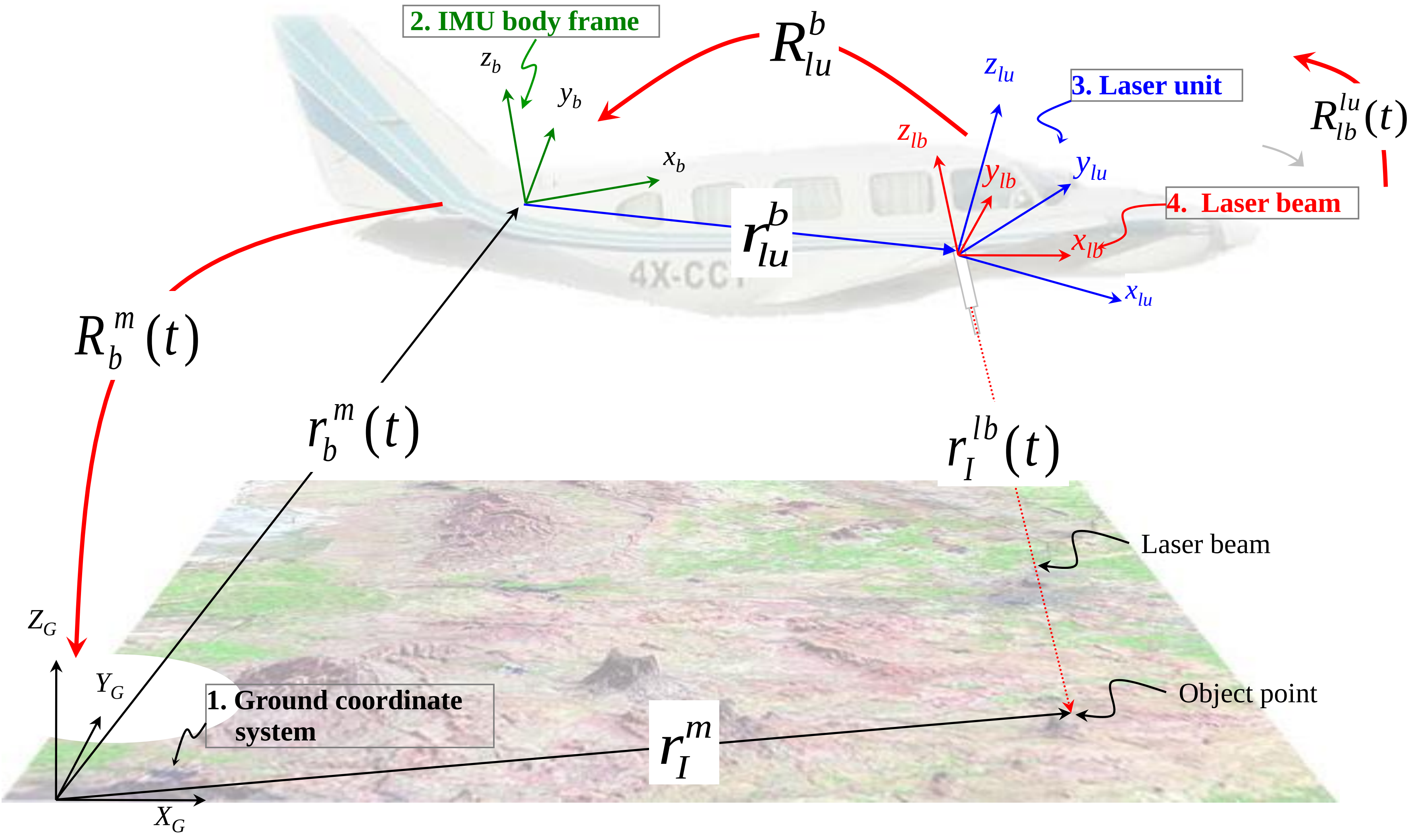
$$\begin{bmatrix} x_{lu} \\ y_{lu} \\ z_{lu} \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & -\sin\alpha & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta & \cos\alpha & \sin\alpha\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -\rho \end{bmatrix}$$

$$\begin{bmatrix} x_{lu} \\ y_{lu} \\ z_{lu} \end{bmatrix} = \begin{bmatrix} -\rho\cos\alpha\sin\beta \\ -\rho\sin\alpha\sin\beta \\ -\rho\cos\beta \end{bmatrix}$$

Object point coordinates relative to the laser unit coordinate system



LiDAR Equation & Coordinate Systems



- LiDAR equation is a vector summation procedure.

LiDAR Equation (Mobile Systems)

$$r_I^m = r_b^m(t) + R_b^m(t)\ r_{lu}^b + R_b^m(t)\, R_{lu}^b\, R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

r_I^m	ground coordinates of the object point under consideration
$r_b^m(t)$	ground coordinates of the origin of the IMU coordinate system
$R_b^m(t)$	rotation matrix relating the ground and IMU coordinate systems
r_{lu}^b	offset between the laser unit and IMU coordinate systems (lever arm offset)
R_{lu}^b	rotation matrix relating the IMU and laser unit coordinate systems (boresight matrix)
$R_{lb}^{lu}(t)$	rotation matrix relating the laser unit and laser beam coordinate systems
$r_I^{lb}(t)$	coordinates of the object point relative to the laser beam coordinate system

- Note: There is no redundancy in the surface reconstruction process.

LiDAR Equation (Static Systems)

$$r_I^{lu} = R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

r_I^{lu}	coordinates of the object point under consideration relative to the laser unit coordinate system
$R_{lb}^{lu}(t)$	rotation matrix relating the laser unit and laser beam coordinate systems at a given epoch (t)
$r_I^{lb}(t)$	coordinates of the object point relative to the laser beam coordinate system at a given epoch (t)

- We are only dealing with laser beam and laser unit coordinate systems

- Note: There is no redundancy in the surface reconstruction process.

LiDAR Output

- **232802.510 319978.600 44.300 41.0 9 First**
- **232802.510 319978.600 44.300 41.0 9 Last**
- **232802.360 319979.590 44.460 38.0 9 First**
- **232802.360 319979.590 44.460 38.0 9 Last**
- **232802.250 319980.340 44.550 41.0 9 First**
- **232802.250 319980.340 44.550 41.0 9 Last**
- **232802.100 319981.420 44.470 37.0 9 First**
- **232802.100 319981.420 44.470 37.0 9 Last**
- **…………**
- **……………**

Black Box (non-transparent model)

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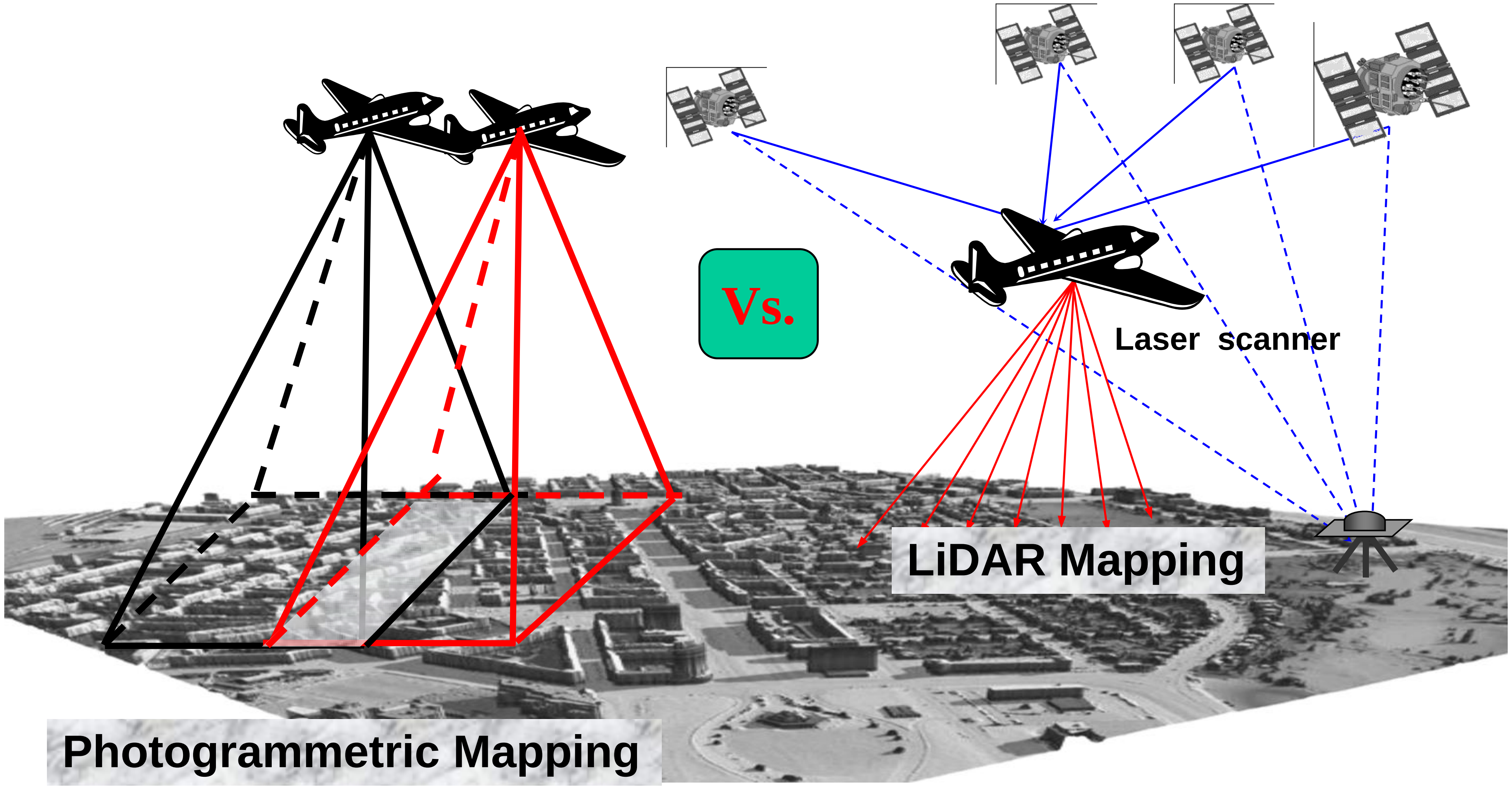
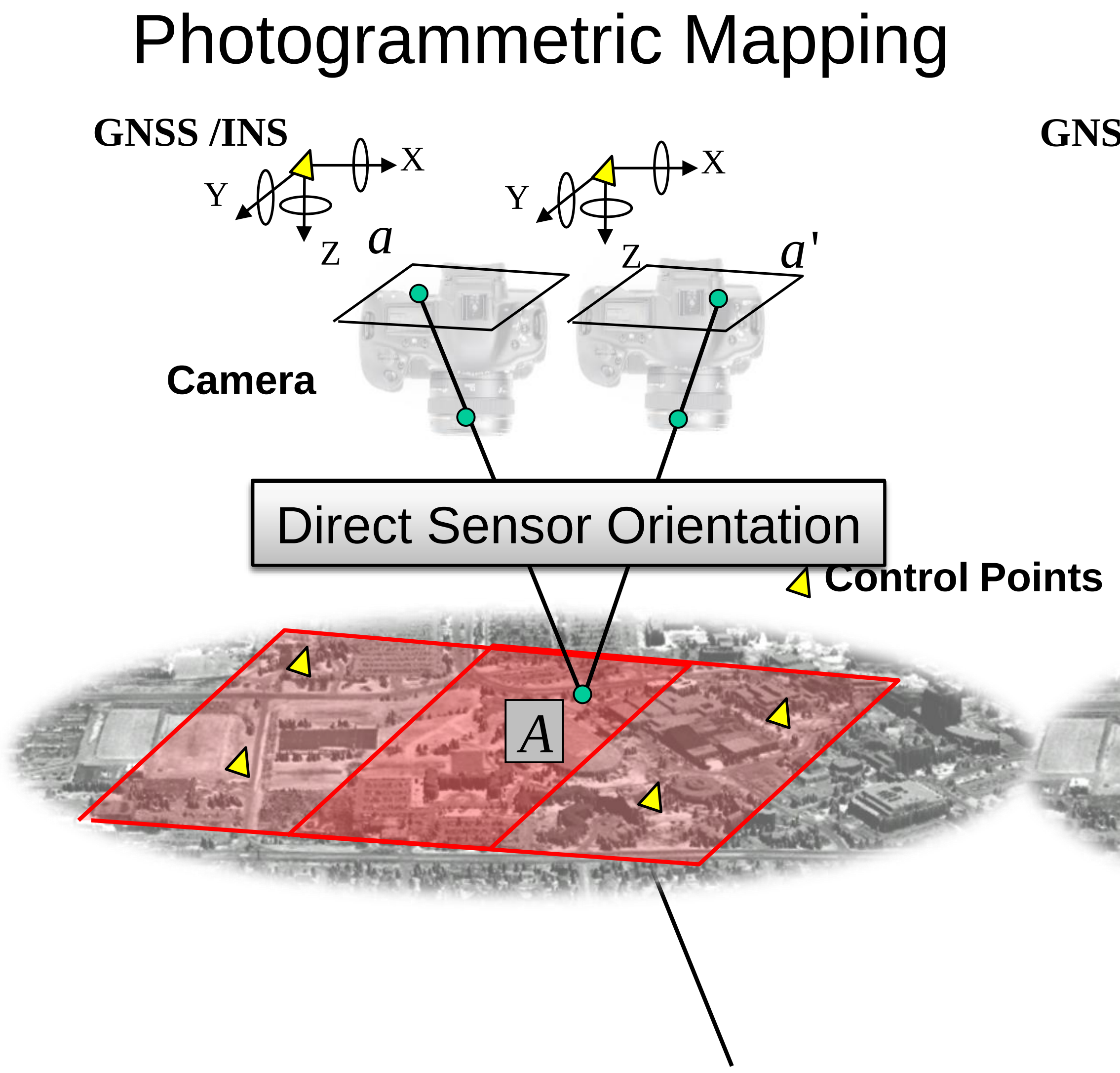
LiDAR Vs. Photogrammetric Mapping

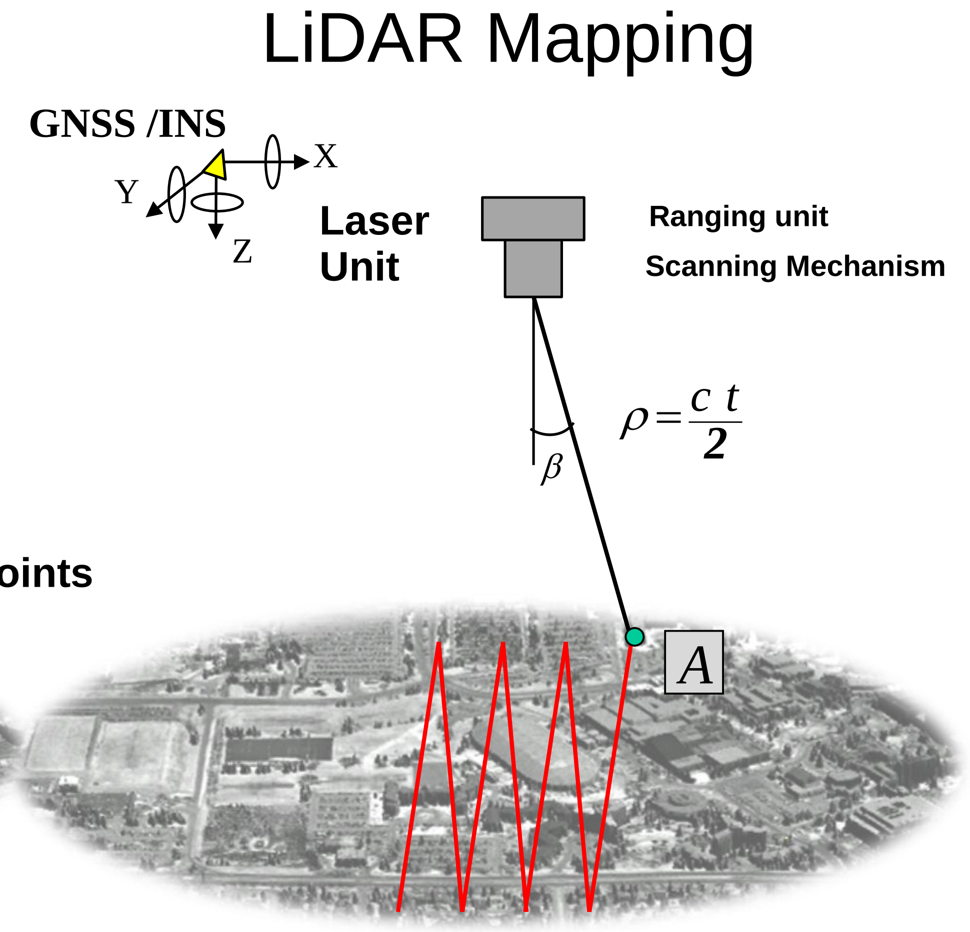
	LiDAR	Photogrammetry
Georeferencing	Direct Only (Mobile Systems)	Indirect/Direct
Reconstruction	Single Pulse	Intersection of Conjugate Light Rays
System Calibration	Mounting Parameters + Laser Ranging/ Scanning Unit	Mounting Parameters + Camera Calibration
Point Positioning Equation	Vector Summation Procedure	

LiDAR Vs. Photogrammetric Mapping

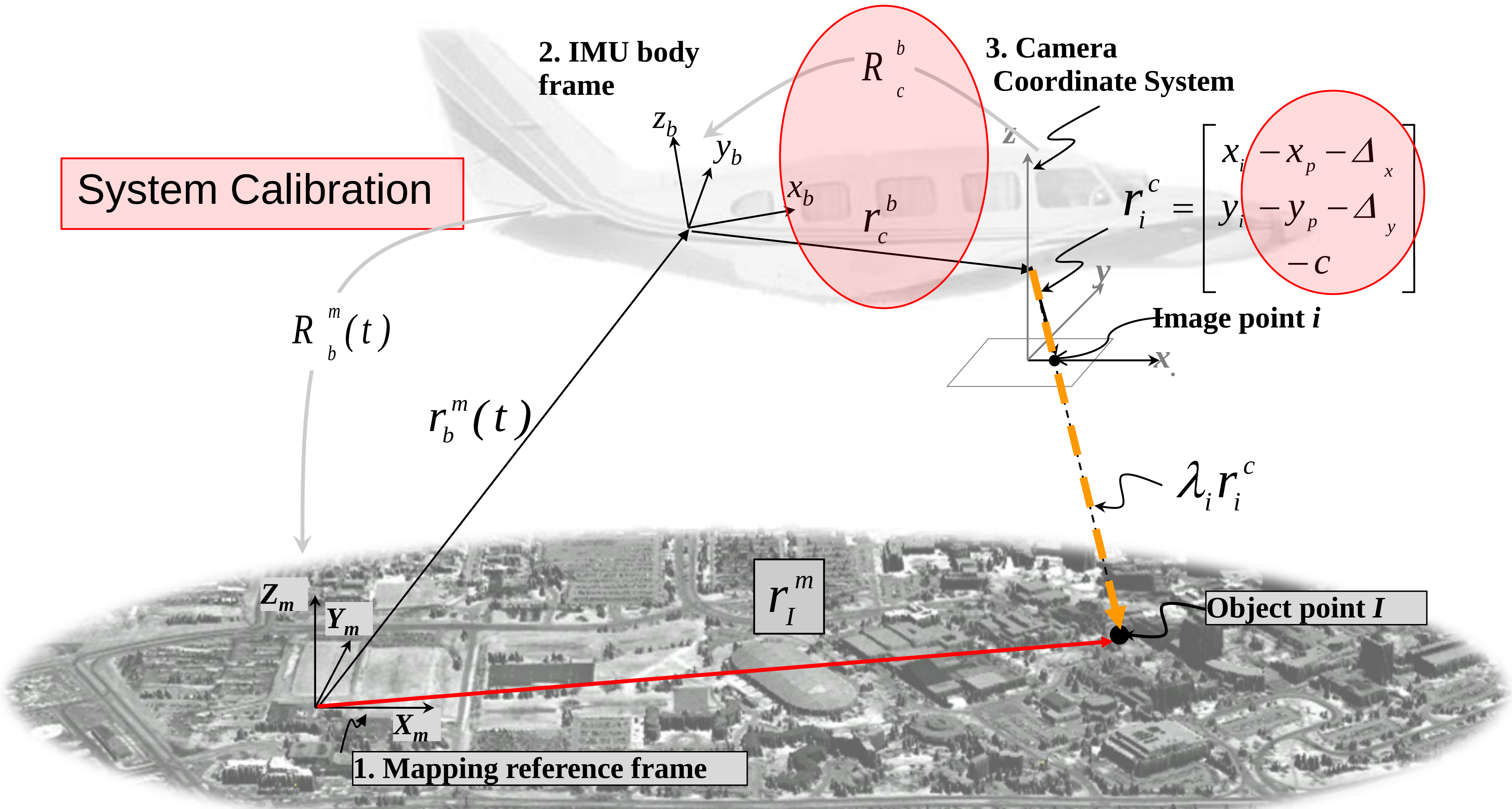


LiDAR Vs. Photogrammetric Mapping



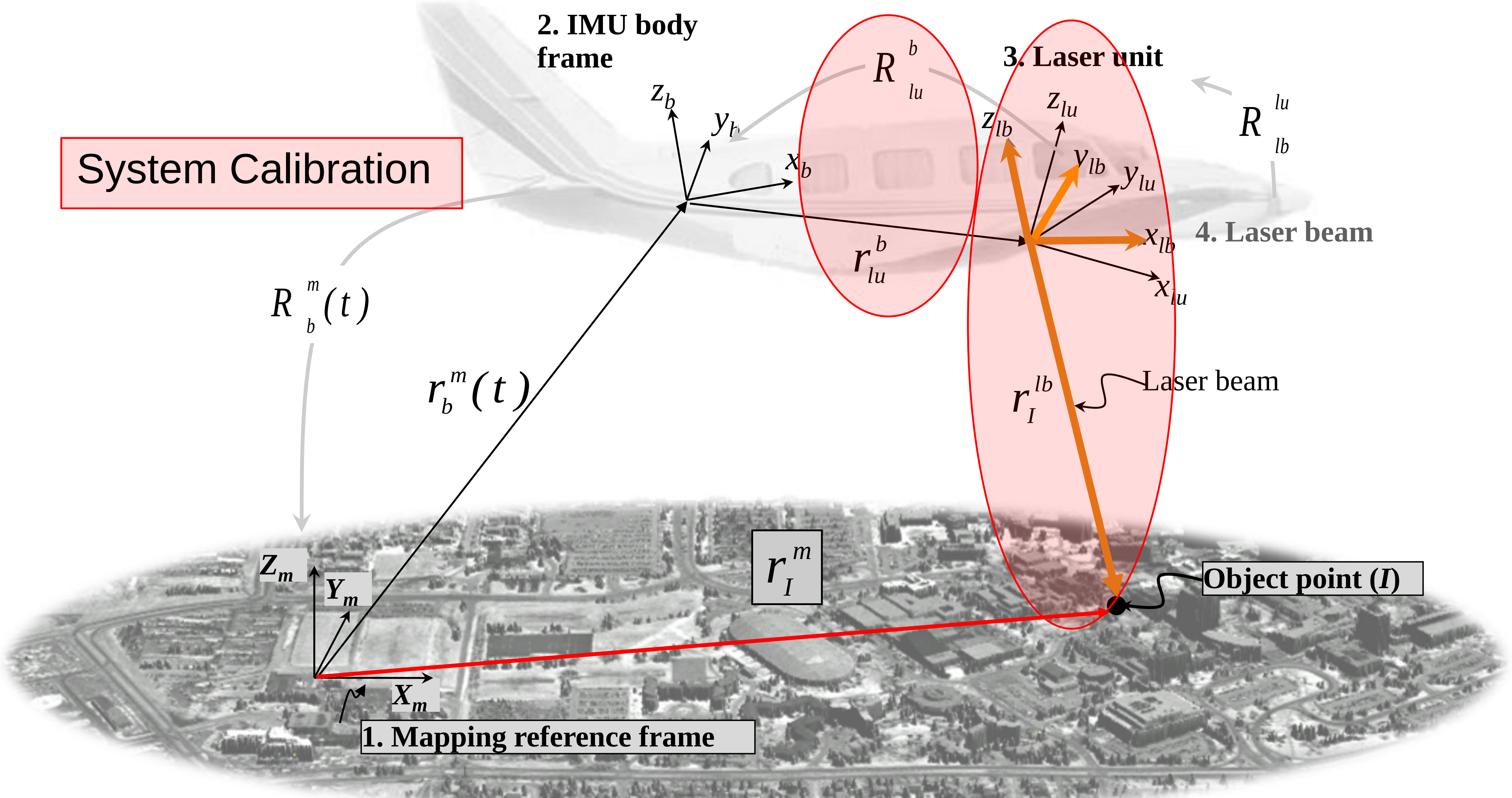


Photogrammetric System Calibration



$$r_I^m = r_b^m(t) + R_b^m(t)\, r_c^b + \lambda_i\, R_b^m(t)\, R_c^b\, r_i^c$$

LiDAR System Calibration

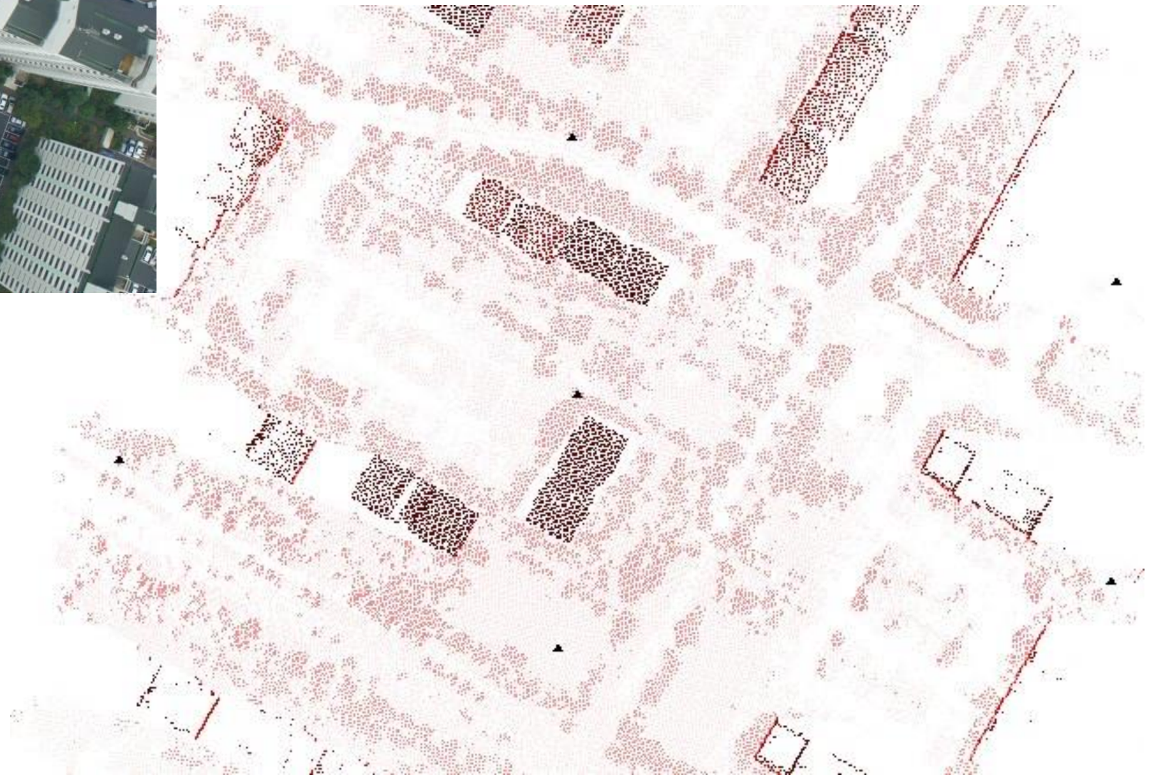


$$\boxed{r_I^m = r_b^m(t) + R_b^m(t)\, r_{lu}^b + R_b^m(t)\, R_{lu}^b R_{lb}^{lu} r_I^{lb}}$$

LiDAR Vs. Photogrammetric Mapping

Optical Imagery

LiDAR Range Image



LiDAR Vs. Photogrammetric Mapping

Optical Imagery

LiDAR Intensity Image

LiDAR Vs. Photogrammetric Mapping

LiDAR (Pros)	Photogrammetry (Cons)
Dense information along homogeneous surfaces	Almost no positional information along homogeneous surfaces
Day or night data collection	Day time data collection
Direct acquisition of 3D coordinates	Complicated and sometimes unreliable matching procedures
Vertical accuracy is better than the planimetric accuracy	Vertical accuracy is worse than the planimetric accuracy

LiDAR Vs. Photogrammetric Mapping

Photogrammetry (Pros)	**LiDAR (Cons)**
High redundancy	No inherent redundancy
Rich with semantic information	Positional; difficult to derive semantic information
Dense positional information along object space breaklines	Almost no information along breaklines
Planimetric accuracy is better than the vertical accuracy	Planimetric accuracy is worse than the vertical accuracy
Transparent Model	***Non-transparent model***