UAS-Based LiDAR Mapping

Video C

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LiDAR Error Sources & Their Impact

LiDAR Error Sources

- **The quality of the derived point cloud from a LiDAR system depends on:**
  - **Systematic errors** in the system parameters:
    - Biases in the Lever-arm components ($\delta\Delta X$, $\delta\Delta Y$, $\delta\Delta Z$)
    - Biases in the boresight angles ($\delta\Delta\omega$, $\delta\Delta\phi$, $\delta\Delta\kappa$)
    - Biases in the measured ranges ($\delta\Delta\rho$)
    - Scale bias in the mirror angle measurements (δS_α, δS_β)
  - **Random errors** in the system measurements:
    - Position and orientation information from the GNSS/INS unit
    - Ranges between the laser beam firing point and its footprints
    - Mirror angles
- **We would like to investigate the impact of systematic and random errors on the quality of the derived LiDAR surface.**

LiDAR Error Sources: Systematic Errors

- **Objective: Show the effect of systematic errors/biases in the LiDAR parameters on the reconstructed object space**
- **The effects will be derived through mathematical analysis of the LiDAR equation.**
- **These effects will be shown for linear scanners.**

$$\boldsymbol{r_I^m = f(\vec{x})}$$

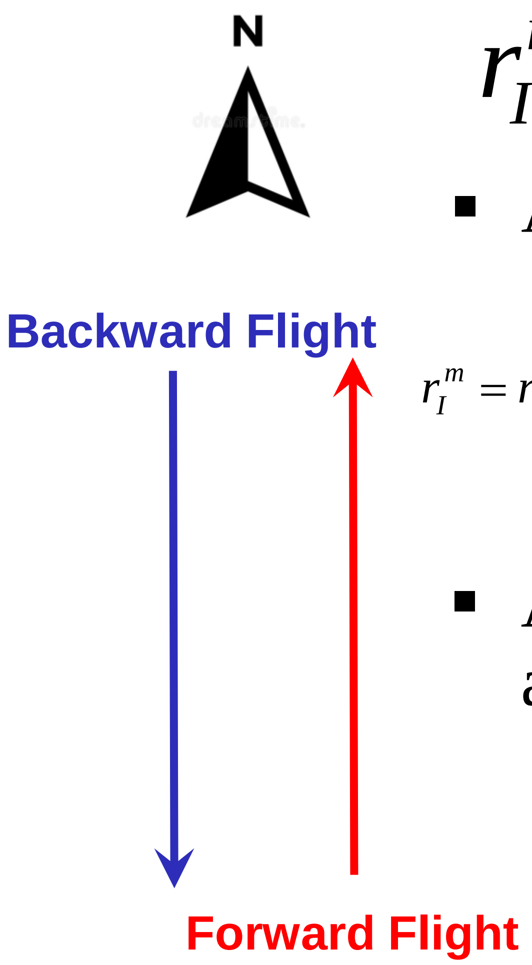
Where: $\boldsymbol{\vec{x} = (\Delta X, \Delta Y, \Delta Z, \Delta\omega, \Delta\varphi, \Delta\kappa, \Delta\rho, S)}$

$$\boldsymbol{\delta r_I^m = \frac{\partial r_I^m}{\partial \vec{x}} \delta \vec{x}}$$ Impact of systematic biases

Where: $\boldsymbol{\delta\vec{x} = (\delta\Delta X, \delta\Delta Y, \delta\Delta Z, \delta\Delta\omega, \delta\Delta\varphi, \delta\Delta\kappa, \delta\Delta\rho, \delta S)}$

Impact of Systematic Errors

- **Mathematical Analysis of the LiDAR Equation**



$$r_I^m = r_b^m(t) + R_b^m(t)\, r_{lu}^b + R_b^m(t)\, R_{lu}^b\, R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

- Assuming small boresight angles and vertical linear scanner:

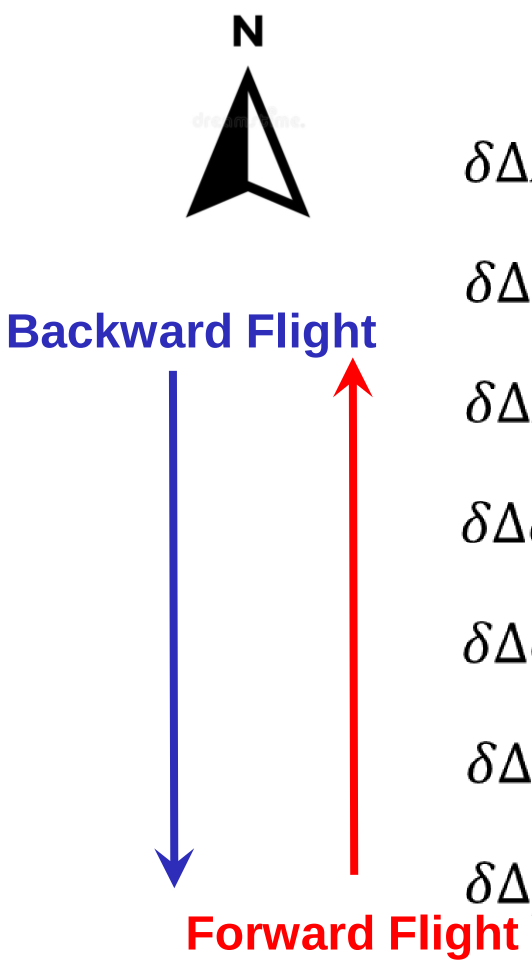
$$r_I^m = r_b^m(t) + \begin{bmatrix} \cos k & -\sin k & 0 \\ \sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} \cos k & -\sin k & 0 \\ \sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\Delta\kappa & \Delta\varphi \\ \Delta\kappa & 1 & -\Delta\omega \\ -\Delta\varphi & \Delta\omega & 1 \end{bmatrix} \begin{bmatrix} -(\rho+\Delta\rho)\sin(S\beta) \\ 0 \\ -(\rho+\Delta\rho)\cos(S\beta) \end{bmatrix}$$

- Assuming heading (κ) angles of 0° and 180° for the forward and backward flight lines, respectively:

$$r_I^m = r_b^m(t) + \begin{bmatrix} \pm\Delta X \\ \pm\Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} \pm 1 & \mp\Delta\kappa & \pm\Delta\varphi \\ \pm\Delta\kappa & \pm 1 & \mp\Delta\omega \\ -\Delta\varphi & \Delta\omega & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \\ z \end{bmatrix} \qquad \boxed{r_I^{lu}(t)}$$

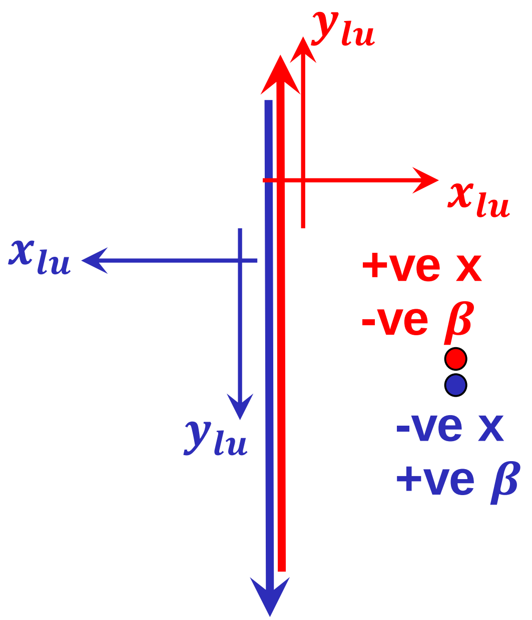
Top sign refers to the forward flight and the bottom sign refers to the backward flight.

- **Mathematical Analysis of the LiDAR Equation**



	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	0
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	0
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	0
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	0
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	0	$-x\ \beta\ \delta S$

- Y- axis is along the flight direction.



Top sign refers to the forward flight and the bottom sign refers to the backward flight.

Impact of Systematic Errors

- **Mathematical Analysis of the LiDAR Equation**

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Backward Flight

Forward Flight

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	**0**	$-x\ \beta\ \delta S$

- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

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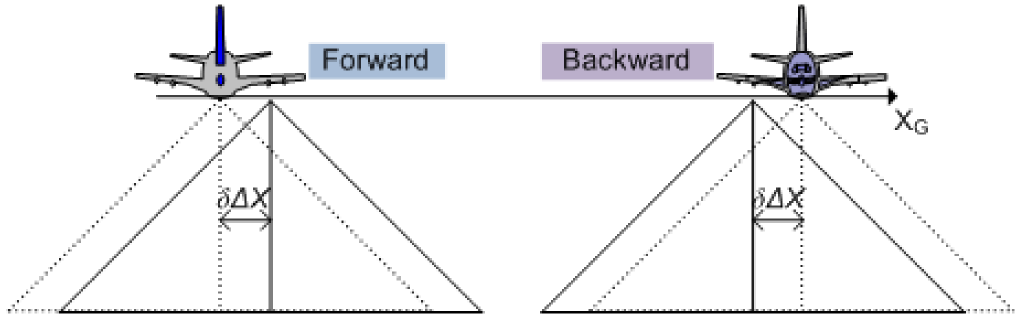
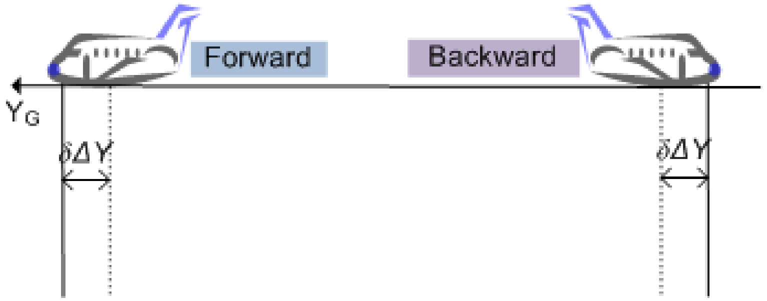
Lever-Arm Systematic Errors (δΔX, δΔY, δΔZ)

- **Linear & Elliptical Scanner**

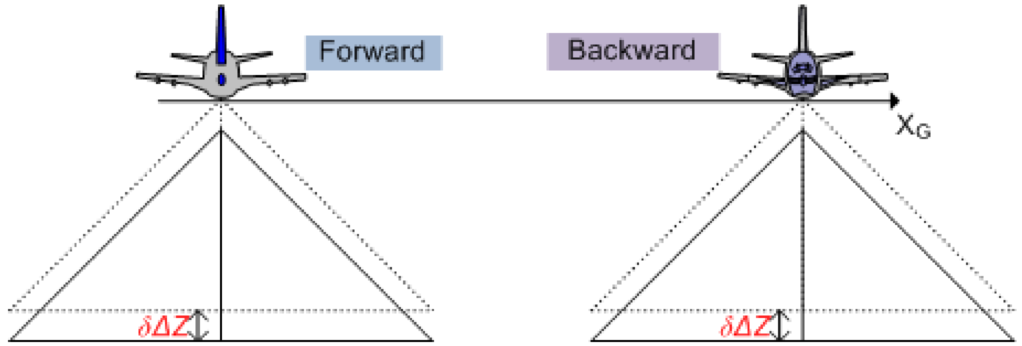
Lever-Arm Offset Bias (δΔX)



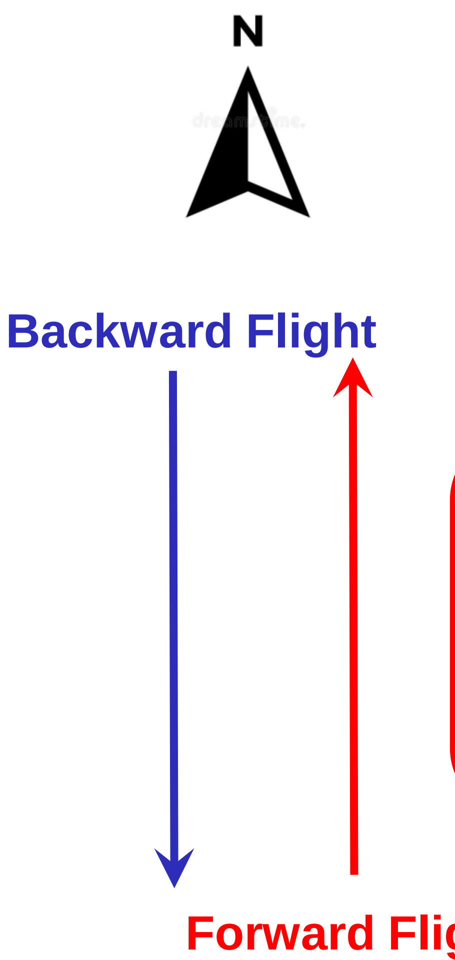
Lever-Arm Offset Bias (δΔY)



Lever-Arm Offset Bias (δΔZ)



- **Mathematical Analysis of the LiDAR Equation**



	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	**0**	$-x\ \beta\ \delta S$

- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

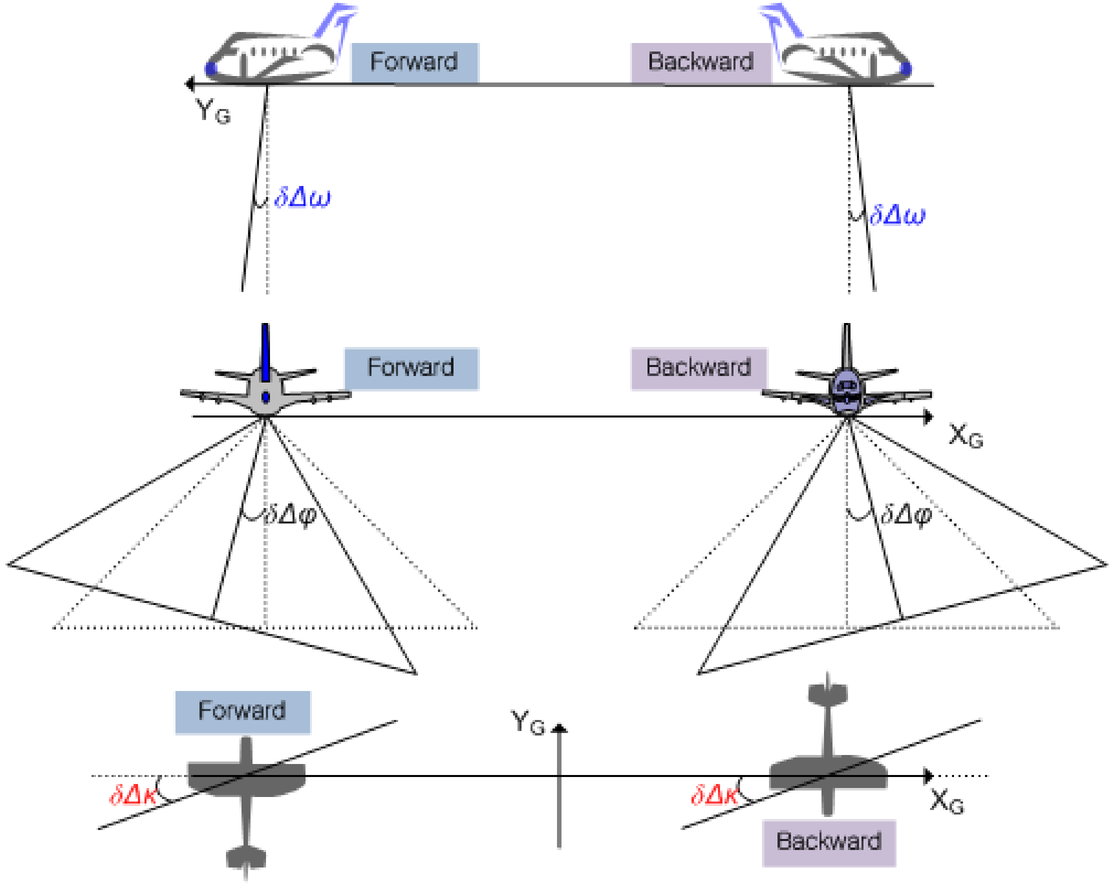
Boresight Systematic Errors (δΔω, δΔϕ, δΔκ)

- **Linear Scanner**

Boresighting Pitch Bias (δΔω)

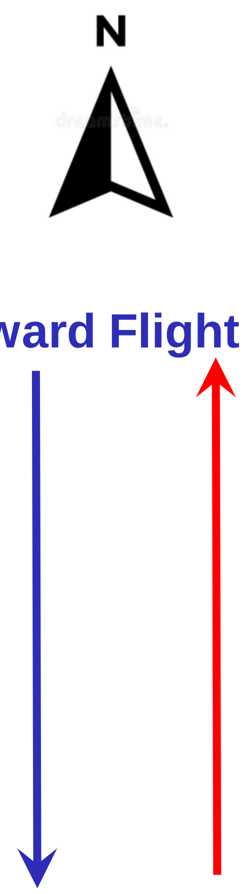
Boresighting Roll Bias (δΔϕ)

Boresighting Heading Bias (δΔκ)



Boresight Pitch Systematic Error (δΔω)

- **Mathematical Analysis of the LiDAR Equation**



	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	$\mathbf{0}$
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	$\mathbf{0}$
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	$\mathbf{0}$
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	$\mathbf{0}$
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	$\mathbf{0}$	$-x\ \beta\ \delta S$

- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

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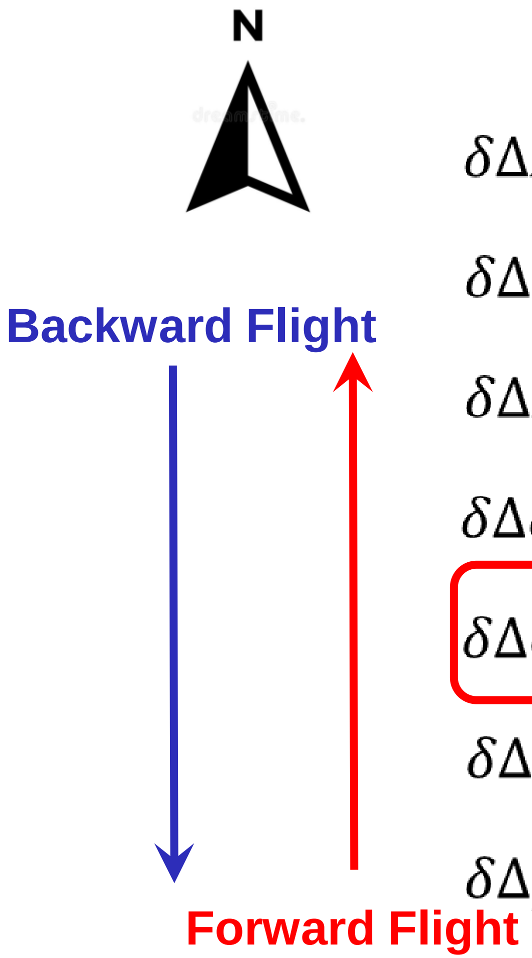
Boresight Roll Systematic Error (δΔϕ)

- **Mathematical Analysis of the LiDAR Equation**



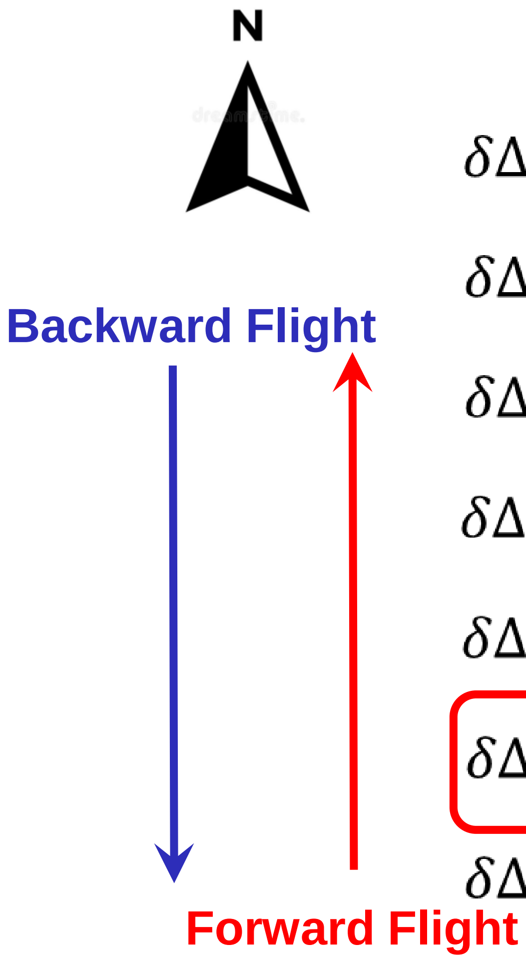
	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	**0**	$-x\ \beta\ \delta S$

- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

Boresight Heading Systematic Error (δΔκ)

- **Mathematical Analysis of the LiDAR Equation**



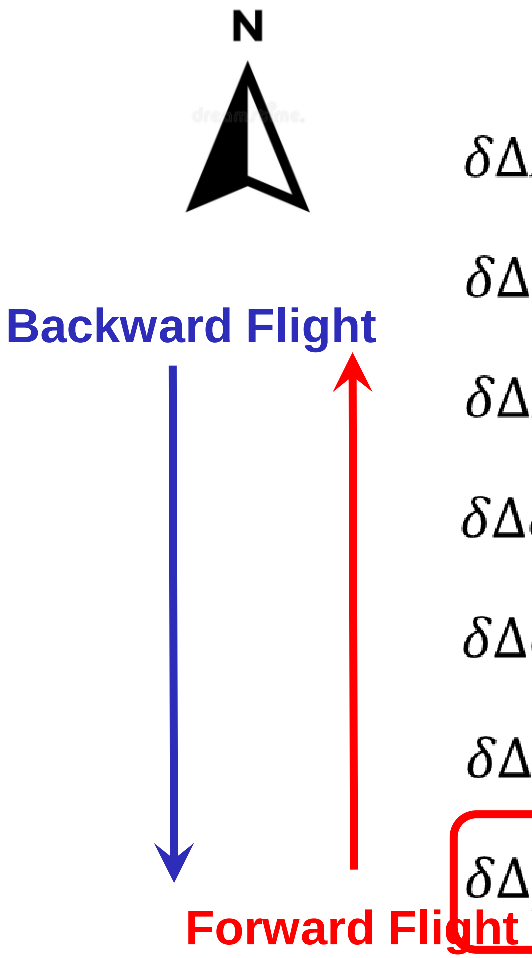
	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	**0**	$-x\ \beta\ \delta S$

- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

Laser Beam Range Systematic Error (δΔρ)

- **Mathematical Analysis of the LiDAR Equation**



	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	**0**	$-x\ \beta\ \delta S$

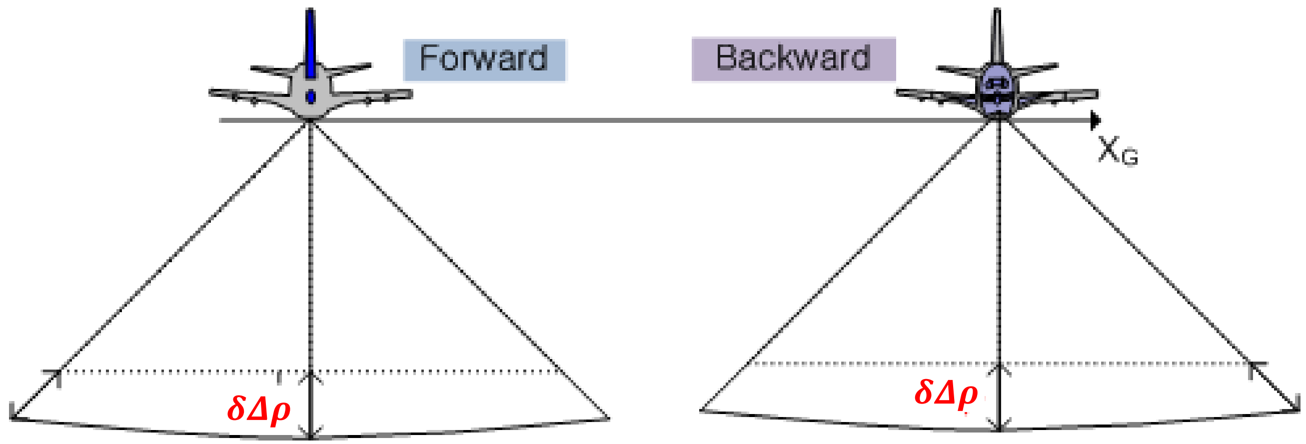
- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

Laser Beam Range Systematic Error ($\delta\Delta\rho$)

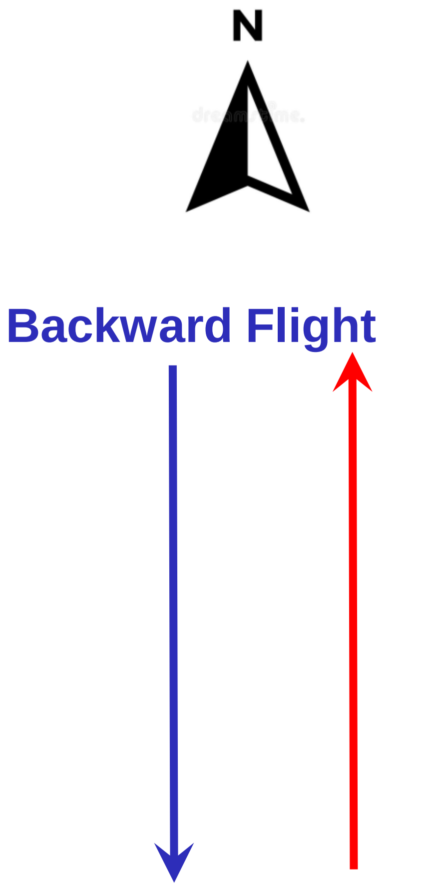
- **Linear Scanner**

Range Bias ($\delta\rho$)



Laser Beam Angular Systematic Error (δS)

- **Mathematical Analysis of the LiDAR Equation**



	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	**0**	$-x\ \beta\ \delta S$

- Y- axis is along the flight direction.

Top sign refers to the forward flight and the bottom sign refers to the backward flight.

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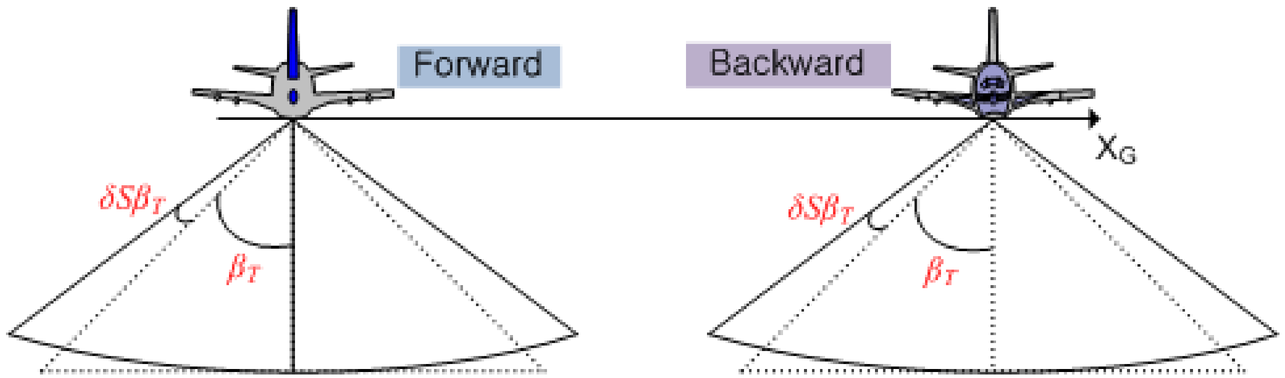
Laser Beam Angular Systematic Error (δS)

- **Linear Scanner**

Mirror Angle Scale Bias (δS)



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Error Sources: Systematic Errors

- **Biases will lead to systematic errors in the derived point cloud.**
- **Diagnostic hints:**
  - Lever-arm offset error:
    - Constant shift in the object space assuming constant attitude
    - Independent of the system parameters (height & look angle)
    - Planimetric effects depend on flight direction
  - Angular biases (attitude or mirror angles):
    - Planimetric coordinates are affected more than vertical coordinates.
    - Dependent on the system parameters (height & look angle, flight direction)
  - Range bias:
    - Mainly affects the vertical component
    - Independent of the system height and flight direction
    - Dependent on the system look angle

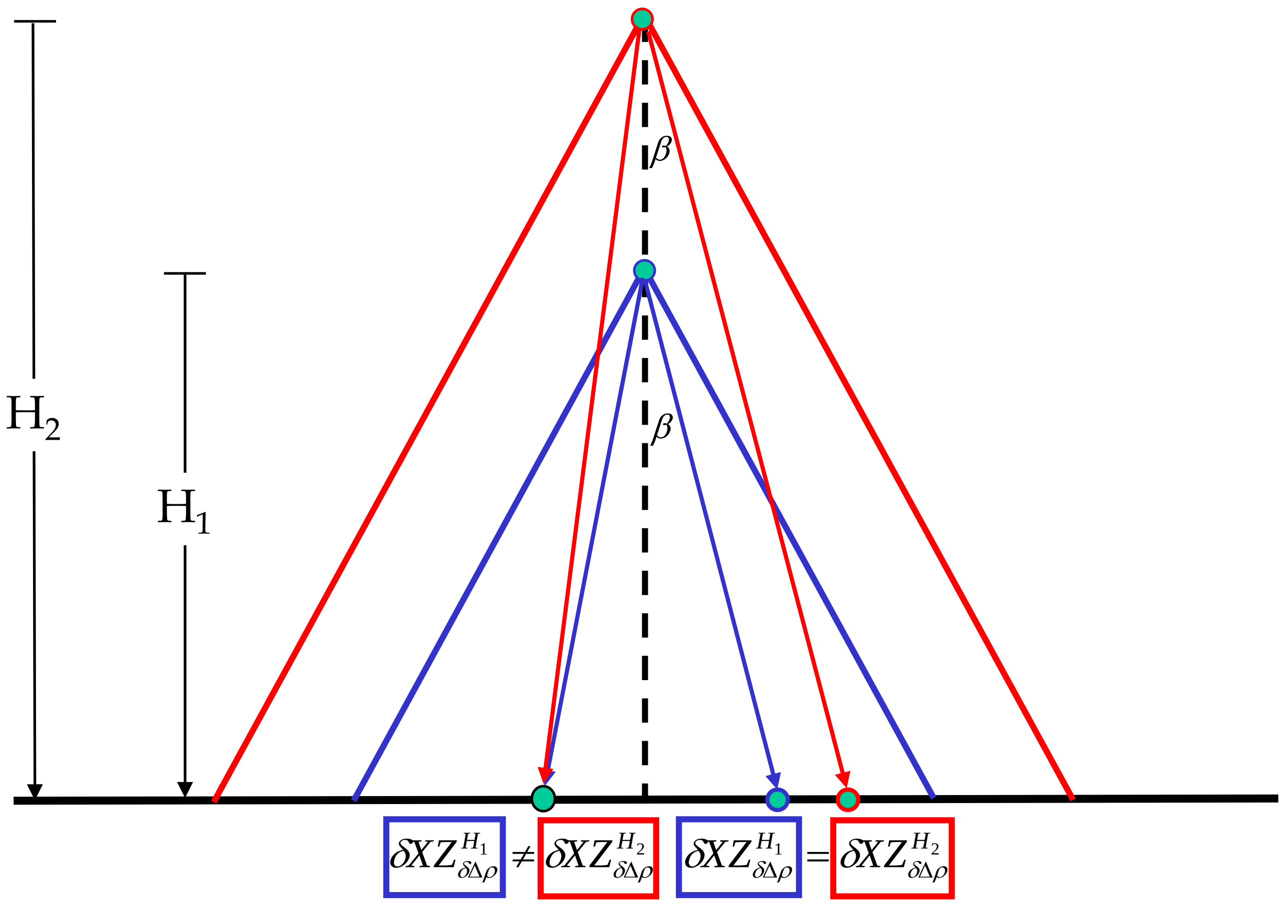
Error Sources: Systematic Errors

- **Mathematical Analysis of the LiDAR Equation for range bias:**

	δX_m	δY_m	δZ_m
$\delta\Delta X$	$\pm\delta\Delta X$	0	**0**
$\delta\Delta Y$	0	$\pm\delta\Delta Y$	**0**
$\delta\Delta Z$	0	0	$\delta\Delta Z$
$\delta\Delta\omega$	0	$\mp z\ \delta\Delta\omega$	**0**
$\delta\Delta\varphi$	$\pm z\ \delta\Delta\varphi$	0	$-x\ \delta\Delta\varphi$
$\delta\Delta\kappa$	0	$\pm x\ \delta\Delta\kappa$	**0**
$\delta\Delta\rho$	$\mp sin(S\beta)\ \delta\Delta\rho$	0	$-\ cos(S\beta)\ \delta\Delta\rho$
δS	$\pm z\ \beta\ \delta S$	**0**	$-x\ \beta\ \delta S$

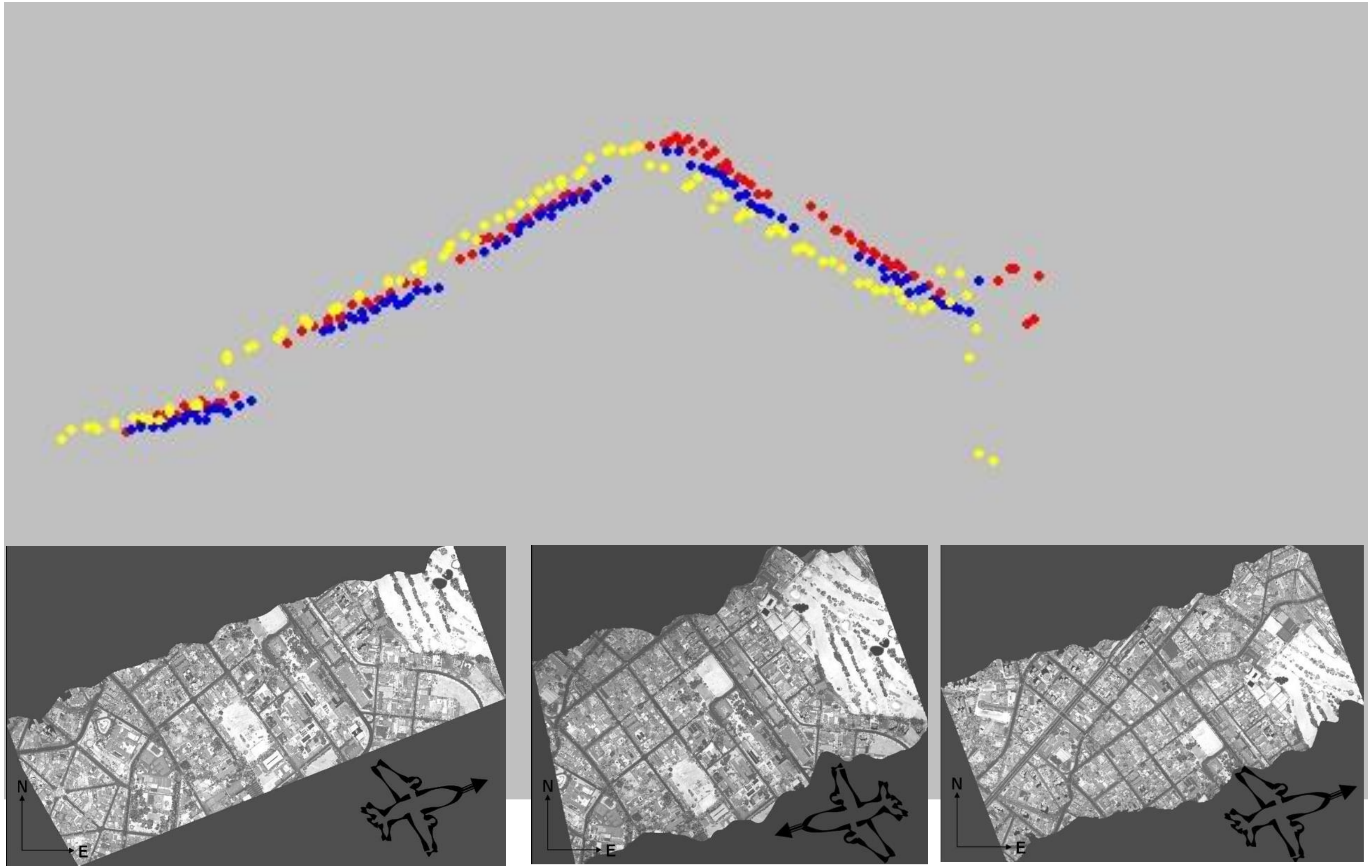
Error Sources: Systematic Errors

- **Mathematical Analysis of the LiDAR Equation**



Impact of the range systematic error on strips captured at different flying heights

Error Sources: Systematic Errors



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Error Sources: Systematic Errors

	Flying Height	Flying Direction	Look Angle
Lever-Arm Offset Bias	Effect is independent of the Flying Height	Effect is dependent on the Flying Direction (Except δZ_m)	Effect is independent of the Look Angle
Boresight Angular Bias	Effect increases with the Flying Height	Effect is dependent on the Flying Direction	Effect changes with the Look Angle (Except δX_m)
Laser Beam Range Bias	Effect is independent of the Flying Height	Effect is independent of the Flying Direction	Effect changes with the Look Angle (Except δY_m)
Laser Beam Angular Bias	Effect increases with the Flying Height	Effect is independent of the Flying Direction	Effect changes with the Look Angle (Except δY_m)

- Assumption:
  - Linear Scanner
  - Constant Attitude & Straight Line Trajectory
  - Flying Direction Parallel to the Y axis
  - Flat horizontal terrain

Error Sources: Random Errors

- **The effect of random errors can be analyzed through variance-covariance propagation**
  - Use the law of error propagation to evaluate the accuracy (noise level) of the derived point cloud as it is determined by the accuracy (noise level) in the LiDAR measurements

$$r_I^m = r_b^m(t) + R_b^m(t)\, r_{lu}^b + R_b^m(t)\, R_{lu}^b\, R_{lb}^{lu}(t)\, r_I^{lb}(t)$$

$$r_I^m = f(\vec{y})$$

$$\vec{y} = (r_b^m(t), R_b^m(t), \alpha(t), \beta(t), \rho(t))$$

$$B = {\partial f}/{\partial \vec{y}}$$

$$\sum r_I^m = B \sum \vec{y}\, B^T$$

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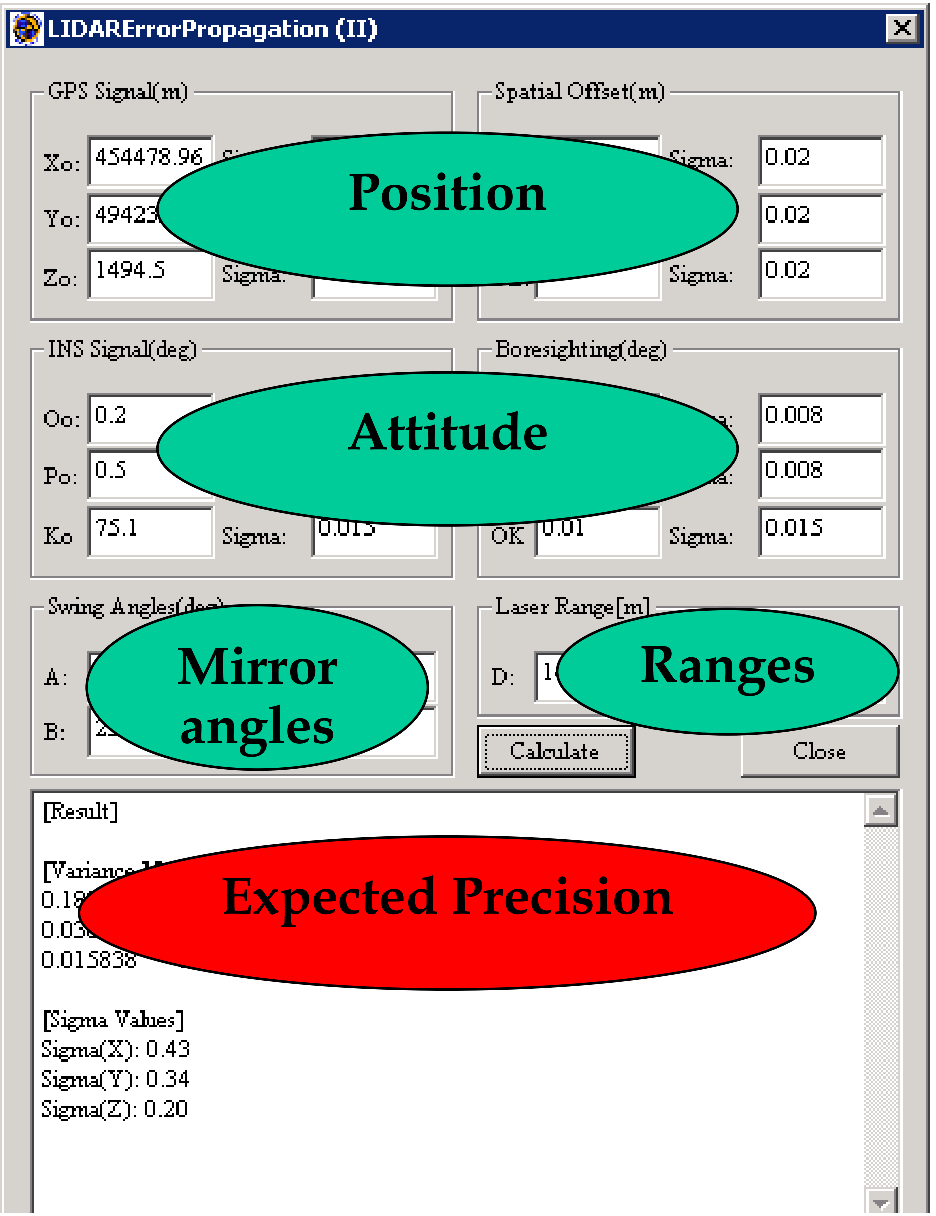
Error Sources: Random Errors

- **Random errors will lead to noise in the derived point cloud.**
- **Diagnostic hints:**
  - GNSS/INS-position noise:
    - Similar noise level in derived point cloud
    - Independent of the system parameters (height & look angle)
  - Angular noise (GNSS/INS-attitude or mirror angles):
    - Planimetric coordinates are affected more than vertical coordinates.
    - Dependent on the system parameters (height & look angle)
    - The magnitude of the introduced noise increases with an increase in the flying height and off-nadir angle.
  - Range noise:
    - Mainly affects the vertical component
    - Independent of the system height
    - Dependent on the system look angle

LiDAR Random Error Propagation Calculator

- **The calculator allows one to enter specific values for each of the input measurements/parameters for a certain LiDAR point and to enter the noise level (precision) for each of the measurements/parameters.**
- **The program then determines the precision of the ground coordinates of the point.**
- **Conversely, if the user requires a specific precision in the final ground coordinates, the program can be used to determine the measurements' precision that would be required for the input components through a trial and error process.**

LiDAR Random Error Propagation Calculator



http://ilmbwww.gov.bc.ca/bmgs/pba/trim/specs

LiDAR Random Error Propagation Calculator

- Accuracy of the system components

System Model	GNSS (m) Post-Processed	IMU (deg) Post-Processed			Scan Angle (deg)	Laser Range (cm)
		Roll	Pitch	Heading		
ALTM 2050	0.05 – 0.3	0.008	0.008	0.015	0.009	~ 2
ALTM 3100	0.05 – 0.3	0.005	0.005	0.008	0.009	~ 2

- System Manufacturer Specification (Optech: ALTM 2050 and ALTM 3100)
    - Horizontal accuracy : 1/2000 x altitude
    - Vertical accuracy : <15 cm at 1200 m
      : <25 cm at 2000 m

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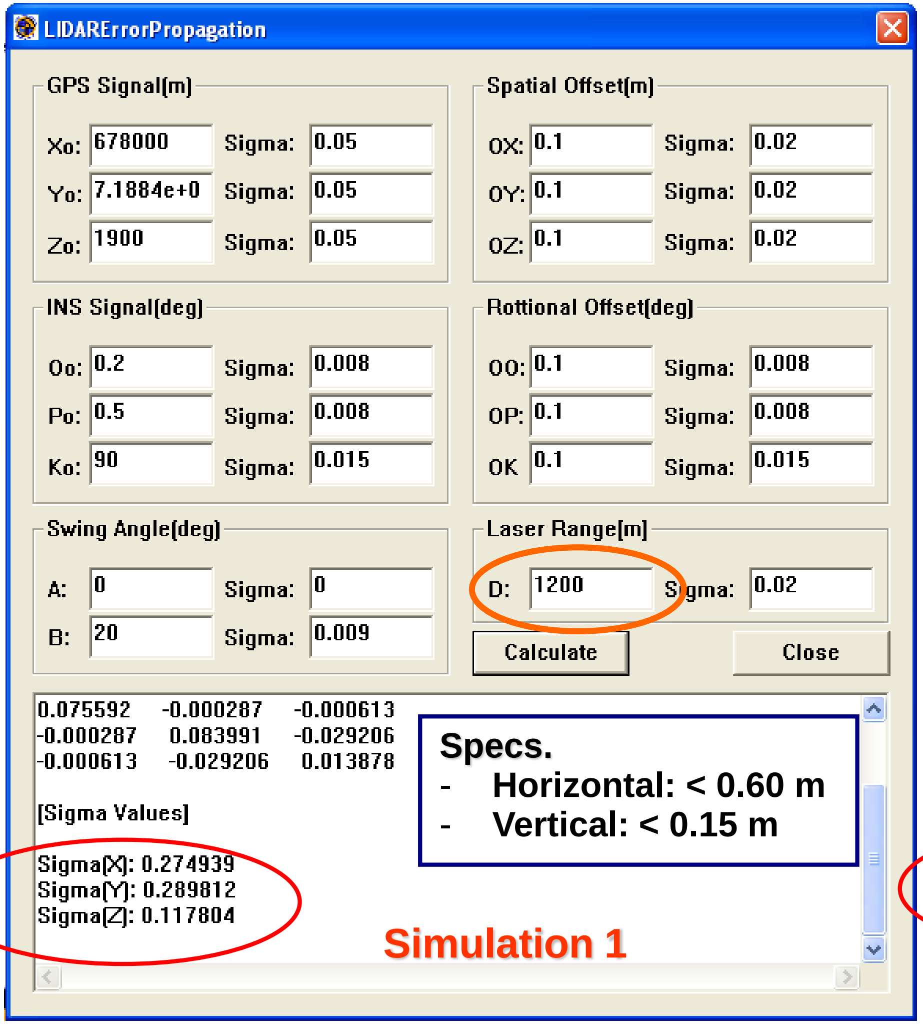
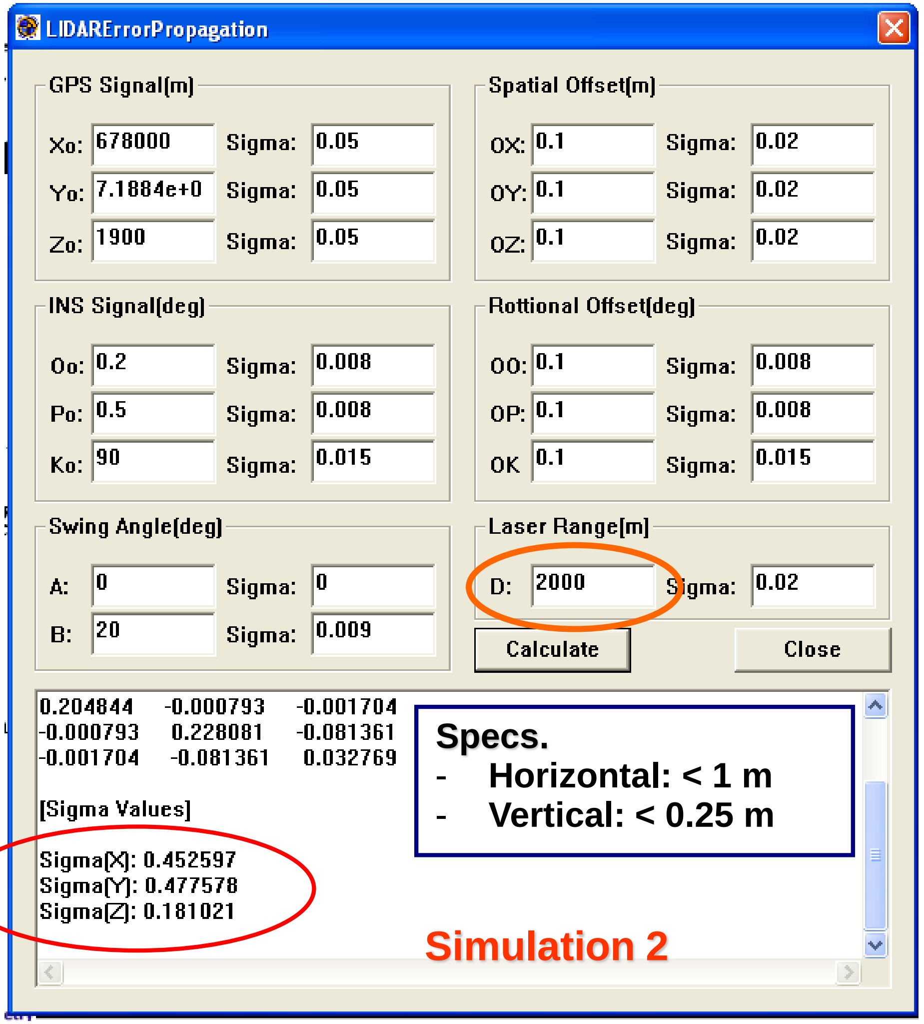
LiDAR Random Error Propagation Calculator

- Expected accuracy (assuming flat solid surface) of the ground coordinates as derived from the error propagation – ALTM 2050





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