# A review on Quantum Battery

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**Abstract**

Quantum Batteries or simply QB is a device which stores energy using the laws of quantum mechanics. Classical batteries work based on electrochemical properties of materials, but quantum battery is working on the basis of quantum phenomina. First it was introduced as a 2 level system which works on thebasis of quantum entanglement. Later it is identified that entanglement is not required for this. Nowadays, it is realized that it can be driven as a three level system, which uses three fields to charge. Through this review, we are focusing in the idea of quantum battery.

## 1 Introduction

Energy storage device is an unavoidable thing in modern world. What we need is more efficient and cheap ways to store energy which can haldle easily and safe. Nowadays we are uning classical batteries which is based on the principle of electrolysis and it is good for our daily needs.[1] Quantum Mechanics gives a way for that. It is called Quantum Battery or simply QB.
In 2013, Robert Alicki and Mark Fanes introduced the idea of Quantum battery which is based on entanglement.[2] They used ideas of quantum thermodynamics for that. Later it was identified that entanglement is not requitred for the working of QB[3]. In these days, the QB were introduced as a 2 level system. But later it the idea of 3 level system was introduced.[4]

Felix C Binder, Sai Vinjanampathym Kavan Modi and John Goold studied the problem of charging of quantum battery. It is not same as charging of classical batery. The state of the system will oscillate between ground state and fully charged state in several cases.[5]

# 2 Basic Idea of Quantum Battery

## 2.1 General Framework

The idea of passive state is defined in [6] A state is called passive if $Tr(\rho_0 U^\dagger H U) \geq Tr(\rho_0 H)$ for every unitary operators $U$ acting on $H$. That means, we cannot extract any energy from the system if the system is in passive state.

We choose a $d$ dimensional Hilbert space. Then, the Hamiltonian of the system can be writtern as

$$H = \sum_{j=1}^{d} \epsilon_j |j >< j| \qquad \epsilon_{j+1} = \epsilon_j \quad (1)$$

[2] The initial state of the system is defined using the density matrix $\rho$. Then, the time evolution is

$$i\hbar\dot{\rho} = [H(t), \rho(t)] \quad (2)$$

But $H(t)$ can be writtern as

$$H(t) = H + V(t)$$

where $V(t)$ is the time dependent field that drives the battery. Also, $V^\dagger(t) = V(t)$.[2]

The work that can be extracted using this process is

$$W = Tr(\rho H) - Tr(\rho(\tau)H) \quad (3)$$

$\tau$ is the time gone from the initial time. $\rho(\tau)$ is related to $\rho$ by the unitary transformation

$$\rho(\tau) = U^\dagger(\tau)\rho U(\tau) \quad (4)$$

where

$$U(\tau) = T \exp\left(-i \int_0^\tau dt[H + V(t)]\right)$$

From this, the ergotropy $C$ is

$$C = \max W = Tr(\rho H) - \min Tr(\rho(\tau)H) \quad (5)$$

[2]

In the following treatment, interaction picture is using. So, $\rho_{int}(t)U^\dagger\rho U$ Using this, the energy that transfered from Battery to Charging Hub can be writtern as

$$C(t) = Tr\left\{H\rho_{int}(t)\right\} - Tr(\rho H) \quad (6)$$

[7]

## 2.2 Charging of Quantum Battery

The Internal energy of a quantum system at a particulr time $t$ is given by the formula $Tr[\rho(t)H]$ where $\rho(t)$ is the density matrix is the state at the time $t$ and $H$ is the internal Hamiltonian. If the battery is charging, we can say that

$$Tr[(\rho(t) - \rho(0))H] \geq 0$$

and if the energy is draining out of the battery,

$$Tr[(\rho(t) - \rho(0))H] \leq 0$$

[5] We say that both charging and discharging is a cyclic unitary process which is achieved by applying the time dependent potential $V(t)$. We had described about $V$ above.

Let us call the passive state, which is the lowest energy state $\hat{\pi}$ and the highest energy state as $\hat{\omega}$, both of them are defined with respect to $H$. Then, these states are

$$\hat{\pi} = \sum_i p_i|\epsilon_i >< \epsilon_i| \qquad p_i \geq p_{i+1} \quad (7)$$

$$\hat{\omega} = \sum_i p_i|\epsilon_i >< \epsilon_i| \qquad p_i \leq p_{i+1} \quad (8)$$

Here, $\rho$ can be called as the generic state. The ergotropy will give the energy available in the battery. Here, one thing to note is if a battery is in the state $\hat{\omega}$, we cannot charge it further and if the state is we the state $\hat{\pi}$, we cannot drain out any energy from that further. $\hat{\pi}$, $\hat{\omega}$ and $\rho$ are related to each other through unitary transformation.[5]

In [5], there discussed with an array of $N$- qbits. There, a 2-level system was considered. There, entanglement was also used for the derivation. But in [3], they introduced a protocol for maximal work extraction without creating an entanglement in the ensemble during the runtime.

# 3 Implementation

Formally, the idea to make the idea practical as a two level system. There are two ways to implement the idea. One, as a Two Level System(TLS), other as a Quantum Harmonic Oscillator(QHO)[8]

Here, we can implement as different cases.Charging Hub and the battery can be choose as QHO or TLS. Treatment of each will be different.

**Both Charger and battery are TLSs**

Here, both charger and battery are treated as two level systems. The hamiltonian of Charging hub and battery were made up of pauli spin matrices. $\omega_0$ is the level spacing of each TLS and $g$ is coupling strength. The energy transfer is done by **Rabi Oscillations**[8]

In this case, the average energy, average power annd time of charging are given as follows.

$$\bar{E}_s = \hbar\omega_0$$

$$\bar{\tau} = \frac{\pi}{2g}$$

$$\bar{P}_s = \frac{2g\omega_0}{\pi}$$

**Charger is QHO and Battery is TLS**

Here the Hamiltonians are made up by ladder operators used in Harmonic oscillator and pauli spin matrices. There are 3 hamiltonains defined, and the total hamiltonian is the sum of these. This modes is called Jaynes-Cummings model. [9] That became the building block of more complicared many-body models such as Tavis-Cummings[10][11] and Dicke models[12]

Here, we define a quantity $K$ which is average number of excitations. The average energy and time of charging are

$$\bar{E} = \hbar\omega_0$$

$$\bar{\tau} = \frac{\pi}{2\sqrt{K}g}$$

$K$ can be defined using the ladder operators and Pauli-spin matrices.$K = a^\dagger a + \sigma_z/2$

### Both Battery and Charger are QHOs

Here, the hamiltonians are defined using the ladder operators for both CH and QB. The energy and power getting here are

$$E_s(\tau) = K\hbar\omega_0 \sin^2(g\tau) \tag{9}$$

$$P_s(\tau) = \frac{K\hbar\omega_0 \sin^2(g\tau)}{\tau} \tag{10}$$

Here, we can see that energy and power are oscillatory.

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