Machine Learning Lecture Series

UNIT: 02

Lecture: 03

Machine Learning

CSE21816

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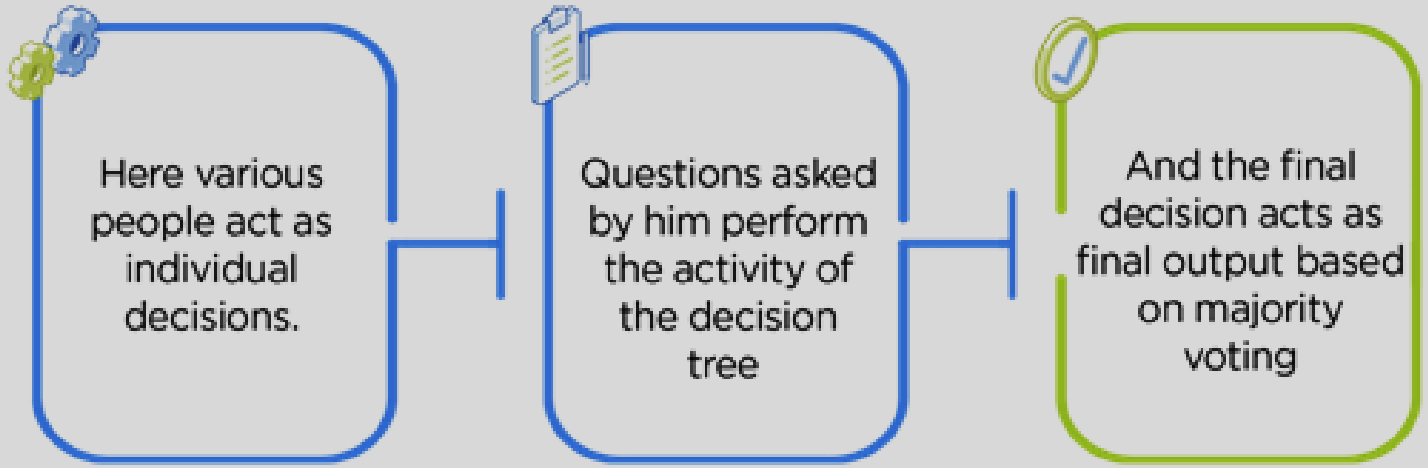
RANDOM FOREST

What is Random Forest Algorithm?

- A Random Forest is like a group decision-making team in machine learning. It combines the opinions of many “trees” (individual models) to make better predictions, creating a more robust and accurate overall model.
- The Random Forest Algorithm is widespread, and popular for its user-friendly nature and adaptability, enabling it to tackle both classification and regression problems effectively. The algorithm’s strength lies in its ability to handle complex datasets and mitigate overfitting, making it a valuable tool for various predictive tasks in machine learning.
- One of the most important features of the Random Forest Algorithm is that it can handle the data set containing continuous variables, as in the case of regression, and categorical variables, as in the case of classification. It performs better for classification and regression tasks. In this tutorial, we will understand the working of random forests and implement random forests on a classification task.

Real-Life Analogy of Random Forest

- Suppose a student named X wants to choose a course after his 10+2, and he is confused about the choice of course based on his skill set. So he decides to consult various people like his cousins, teachers, parents, degree students, and working people. He asks them varied questions like why he should choose, job opportunities with that course, course fee, etc. Finally, after consulting various people about the course he decides to take the course suggested by most people.



Working of Random Forest Algorithm

- Before understanding the workings of the random forest algorithm in machine learning, we must look into the ensemble learning technique. ***Ensemble*** simply means combining multiple models. Thus a collection of models is used to make predictions rather than an individual model.
- Ensemble uses two types of methods:

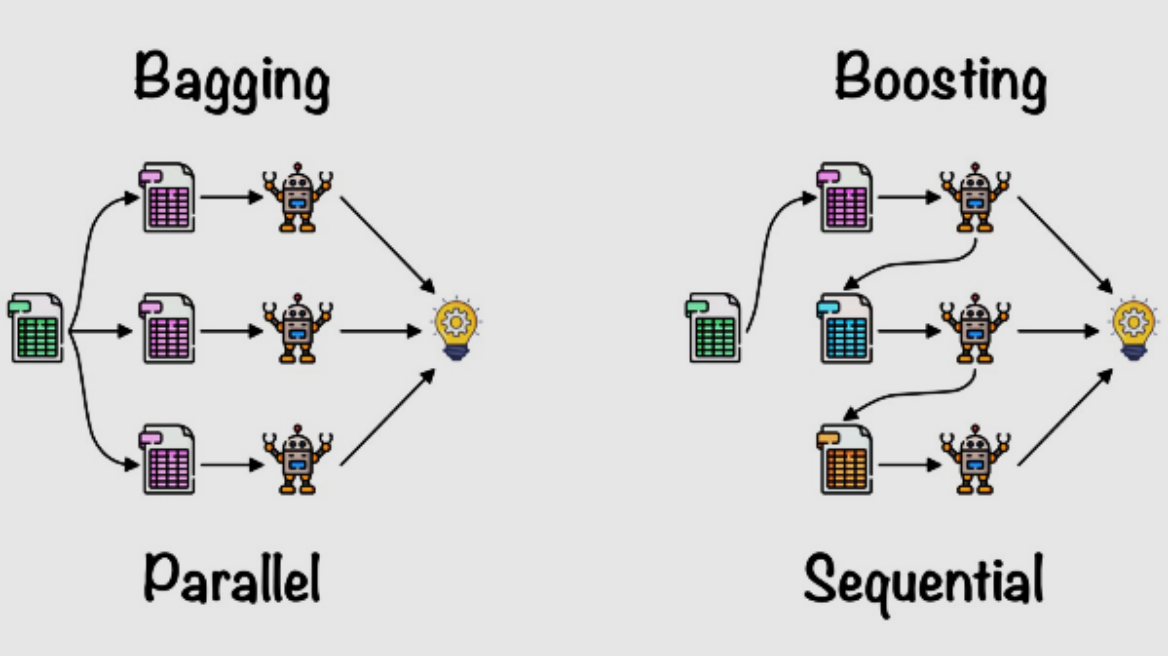
Bagging

- It creates a different training subset from sample training data with replacement and the final output is based on majority voting. For example, Random Forest.

Boosting

- It combines weak learners into strong learners by creating sequential models such that the final model has the highest accuracy. For example, ADA BOOST, XG BOOST.

Ensemble Learning Technique



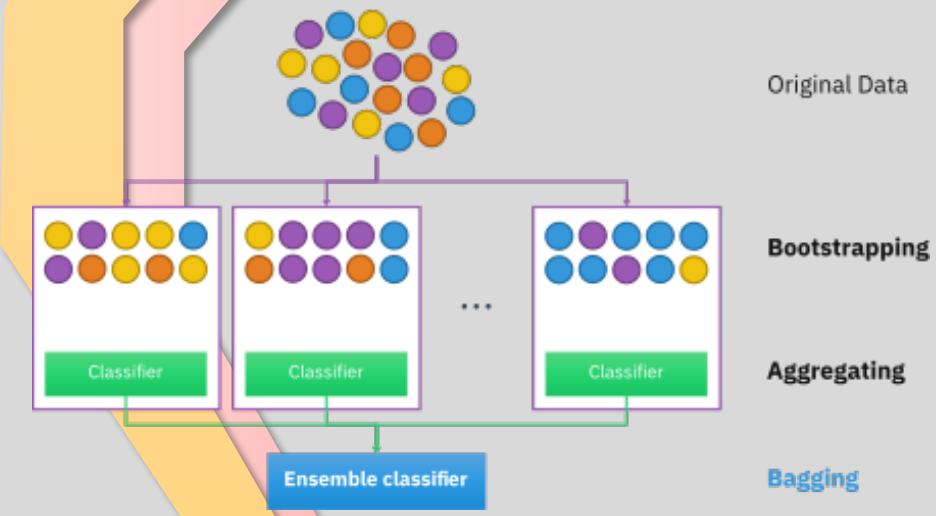
- As mentioned earlier, Random forest works on the Bagging principle. Now let’s dive in and understand bagging in detail.

Bagging

- Bagging, also known as Bootstrap Aggregation, serves as the ensemble technique in the Random Forest algorithm. Here are the steps involved in Bagging:

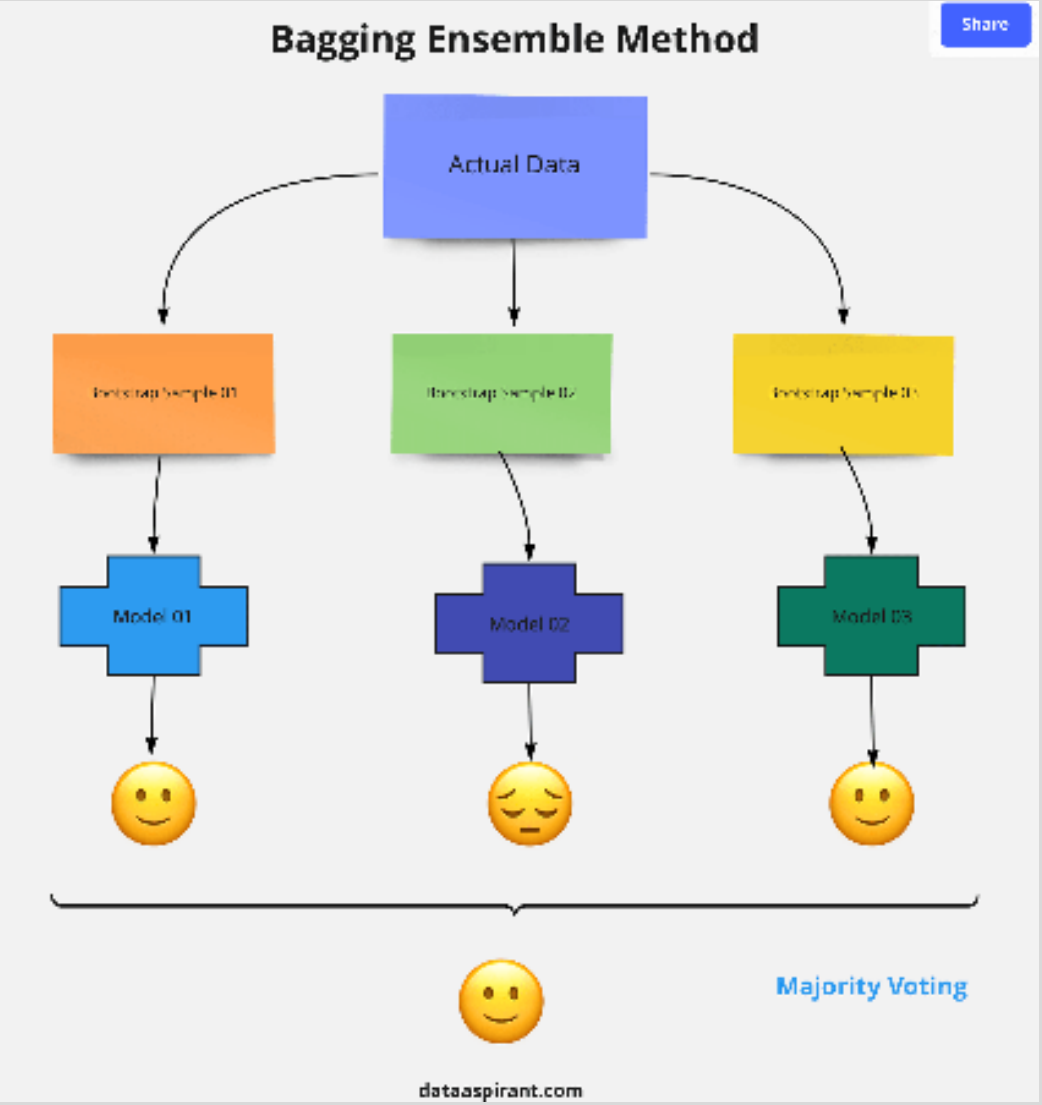
1. **Selection of Subset**: Bagging starts by choosing a random sample from the entire dataset.
2. **Bootstrap Sampling**: Each model is then created from these samples, called Bootstrap Samples, which are taken from the original data with replacement. This process is known as row sampling.
3. **Bootstrapping**: The step of row sampling with replacement is referred to as bootstrapping.
4. **Independent Model Training**: Each model is trained independently on its corresponding Bootstrap Sample. This training process generates results for each model.
5. **Majority Voting**: The final output is determined by combining the results of all models through majority voting. The most commonly predicted outcome among the models is selected.
6. **Aggregation**: This step, which involves combining all the results and generating the final output based on majority voting, is known as aggregation.

Bagging…



- Now let’s look at an example by breaking it down with the help of the following figure. Here the bootstrap sample is taken from actual data (Bootstrap sample 01, Bootstrap sample 02, and Bootstrap sample 03) with a replacement which means there is a high possibility that each sample won’t contain unique data. The model (Model 01, Model 02, and Model 03) obtained from this bootstrap sample is trained independently. Each model generates results as shown. Now the Happy emoji has a majority when compared to the Sad emoji. Thus based on majority voting final output is obtained as Happy emoji.

Bagging…


Bagging Ensemble Method
Share
Actual Data
Model 01
Model 02
Model 03
Majority Voting
dataaspirant.com

Boosting

- Boosting is one of the techniques that use the concept of ensemble learning. A boosting algorithm combines multiple simple models (also known as weak learners or base estimators) to generate the final output. It is done by building a model by using weak models in series.

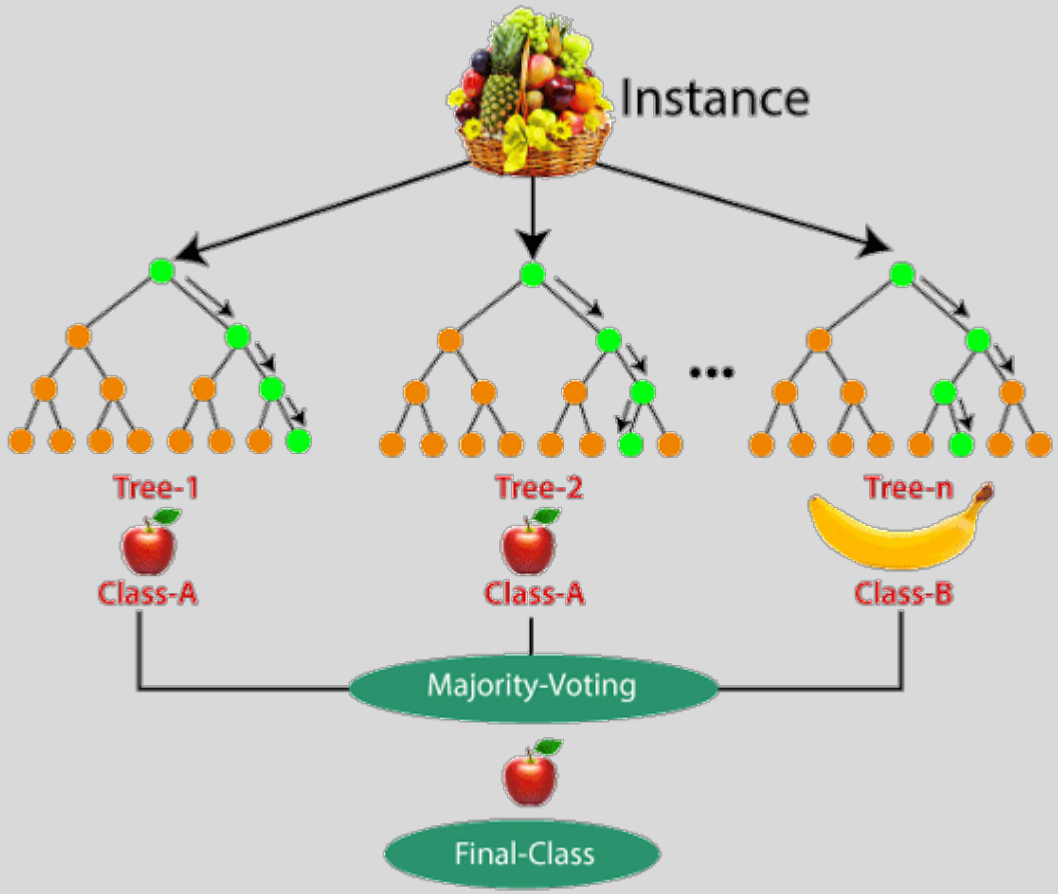
- There are several boosting algorithms; AdaBoost was the first really successful boosting algorithm that was developed for the purpose of binary classification. AdaBoost is an abbreviation for Adaptive Boosting and is a prevalent boosting technique that combines multiple “weak classifiers” into a single “strong classifier.”

Steps Involved in Random Forest Algorithm

- Step 1: In the Random forest model, a subset of data points and a subset of features is selected for constructing each decision tree. Simply put, n random records and m features are taken from the data set having k number of records.
- Step 2: Individual decision trees are constructed for each sample.
- Step 3: Each decision tree will generate an output.
- Step 4: Final output is considered based on Majority Voting or Averaging for Classification and regression, respectively.

Example

- Consider the fruit basket as the data as shown in the figure below. Now n number of samples are taken from the fruit basket, and an individual decision tree is constructed for each sample. Each decision tree will generate an output, as shown in the figure. The final output is considered based on majority voting. In the below figure, you can see that the majority decision tree gives output as an apple when compared to a banana, so the final output is taken as an apple.



Important Features of Random Forest

- **Diversity:** Not all attributes/variables/features are considered while making an individual tree; each tree is different.
- **Immune to the curse of dimensionality:** Since each tree does not consider all the features, the feature space is reduced.
- **Parallelization:** Each tree is created independently out of different data and attributes. This means we can fully use the CPU to build random forests.
- **Train-Test split:** In a random forest, we don't have to segregate the data for train and test as there will always be 30% of the data which is not seen by the decision tree.
- **Stability:** Stability arises because the result is based on majority voting/ averaging.

Difference Between Decision Tree and Random Forest

Decision trees	Random Forest
1. Decision trees normally suffer from the problem of overfitting if it's allowed to grow without any control.	1. Random forests are created from subsets of data, and the final output is based on average or majority ranking; hence the problem of overfitting is taken care of.
2. A single decision tree is faster in computation.	2. It is comparatively slower.
3. When a data set with features is taken as input by a decision tree, it will formulate some rules to make predictions.	3. Random forest randomly selects observations, builds a decision tree, and takes the average result. It doesn't use any set of formulas.

Important Hyperparameters in Random Forest

- Hyperparameters are used in random forests to either enhance the performance and predictive power of models or to make the model faster.

Hyperparameters to Increase the Predictive Power

- **n_estimators:** Number of trees the algorithm builds before averaging the predictions.
- **max_features:** Maximum number of features random forest considers splitting a node.
- **mini_sample_leaf:** Determines the minimum number of leaves required to split an internal node.
- **criterion:** How to split the node in each tree? (Entropy/Gini impurity/Log Loss)
- **max_leaf_nodes:** Maximum leaf nodes in each tree

Hyperparameters…

Hyperparameters to Increase the Speed

- ***n_jobs:*** it tells the engine how many processors it is allowed to use. If the value is 1, it can use only one processor, but if the value is -1, there is no limit.
- ***random_state:*** controls randomness of the sample. The model will always produce the same results if it has a definite value of random state and has been given the same hyperparameters and training data.
- ***oob_score:*** *OOB* means out of the bag. It is a random forest cross-validation method. In this, one-third of the sample is not used to train the data; instead used to evaluate its performance. These samples are called out-of-bag samples.

Advantages and Disadvantages of Random Forest

Advantages

- It can be used in classification and regression problems.
- It solves the problem of overfitting as output is based on majority voting or averaging.
- It performs well even if the data contains null/missing values.
- Each decision tree created is independent of the other; thus, it shows the property of parallelization.
- It is highly stable as the average answers given by a large number of trees are taken.
- It maintains diversity as all the attributes are not considered while making each decision tree though it is not true in all cases.
- It is immune to the curse of dimensionality. Since each tree does not consider all the attributes, feature space is reduced.
- We don’t have to segregate data into train and test as there will always be 30% of the data, which is not seen by the decision tree made out of bootstrap.

Advantages and Disadvantages of Random Forest

Disadvantages

- Random forest is highly complex compared to decision trees, where decisions can be made by following the path of the tree.
- Training time is more than other models due to its complexity. Whenever it has to make a prediction, each decision tree has to generate output for the given input data.

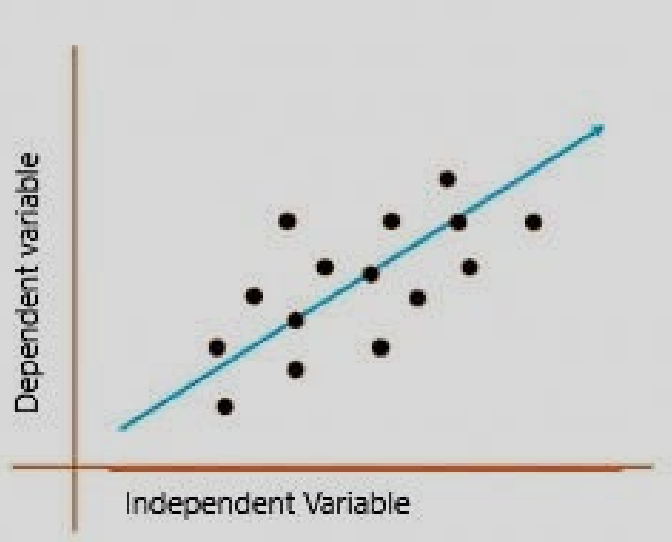
REGRESSION

Linear Regression

- Linear regression predicts the relationship between two variables by assuming a linear connection between the independent and dependent variables.
- It seeks the optimal line that minimizes the sum of squared differences between predicted and actual values.
- Applied in various domains like economics and finance, this method analyzes and forecasts data trends.
- It can extend to multiple linear regression involving several independent variables and logistic regression, suitable for binary classification problems
- Linear regression shows the linear relationship between the independent(predictor) variable i.e. X-axis and the dependent(output) variable i.e. Y-axis, called linear regression.

Simple Linear Regression

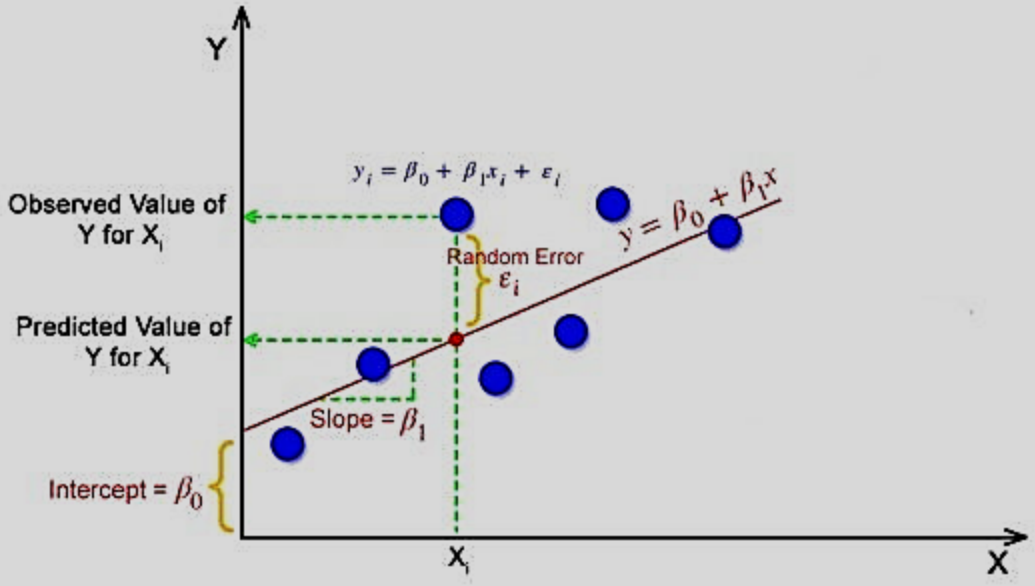
- If there is a single input variable X(independent variable), such linear regression is simple linear regression.
- In a simple linear regression, there is one independent variable and one dependent variable.
- The model estimates the slope and intercept of the line of best fit, which represents the relationship between the variables.
- The slope represents the change in the dependent variable for each unit change in the independent variable, while the intercept represents the predicted value of the dependent variable when the independent variable is zero.



Simple Regression Calculation

- To calculate best-fit line linear regression uses a traditional slope-intercept form which is given below,
- $Y = \beta_0 + \beta_1 X$
- where Y_i = Dependent variable, β_0 = constant/Intercept, β_1 = Slope/Intercept, X_i = Independent variable.

This algorithm explains the linear relationship between the dependent (output) variable y and the independent(predictor) variable X using a straight line $Y = \beta_0 + \beta_1 X$.



Simple Regression Calculation

- The goal of the linear regression algorithm is to get the **best values for β_0 and β_1** to find the best-fit line. The best-fit line is a line that has the least error which means the error between predicted values and actual values should be minimum.

Random Error(Residuals)

- In regression, the difference between the observed value of the dependent variable($\mathbf{y_i}$) and the predicted value(**predicted**) is called the residuals.
- $\varepsilon_i = y_{predicted} - y_i$
- **where** $y_{predicted} = \beta_0 + \beta_1 X_i$

What is the best-fit line?

- In simple terms, the best-fit line is a line that fits the given scatter plot in the best way. Mathematically, the best-fit line is obtained by minimizing the Residual Sum of Squares(RSS).

Cost Function for Linear Regression

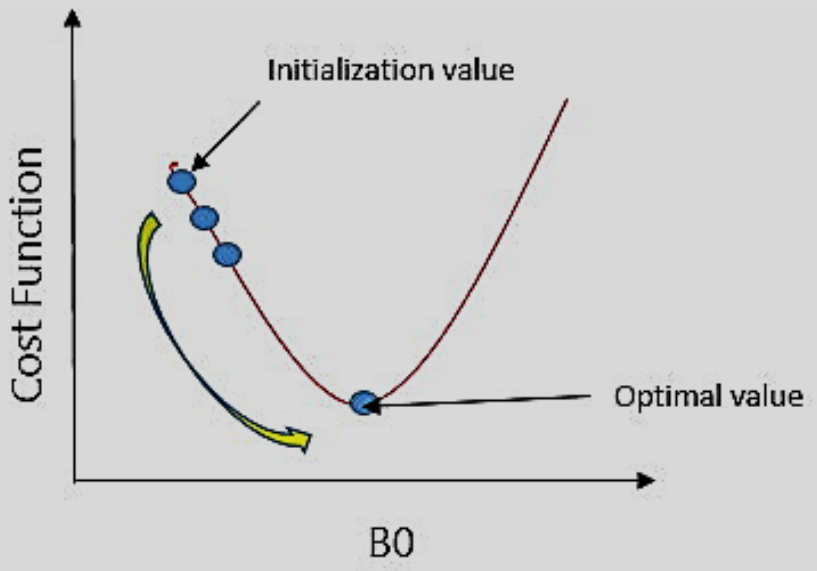
- The cost function helps to work out the optimal values for β_0 and β_1, which provides the best-fit line for the data points.
- In Linear Regression, generally **Mean Squared Error (MSE)** cost function is used, which is the average squared error that occurred between the $y_{predicted}$ and y_i.
- We calculate MSE using the simple linear equation y=mx+b:

$$MSE = \frac{1}{N}\sum_{i=1}^{n}(y_i - (B1 x_i + B0))^2$$

- Using the MSE function, we'll update the values of β_0 and β_1 such that the MSE value settles at the minima. These parameters can be determined using the gradient descent method such that the value for the cost function is minimum.

Gradient Descent for Linear Regression

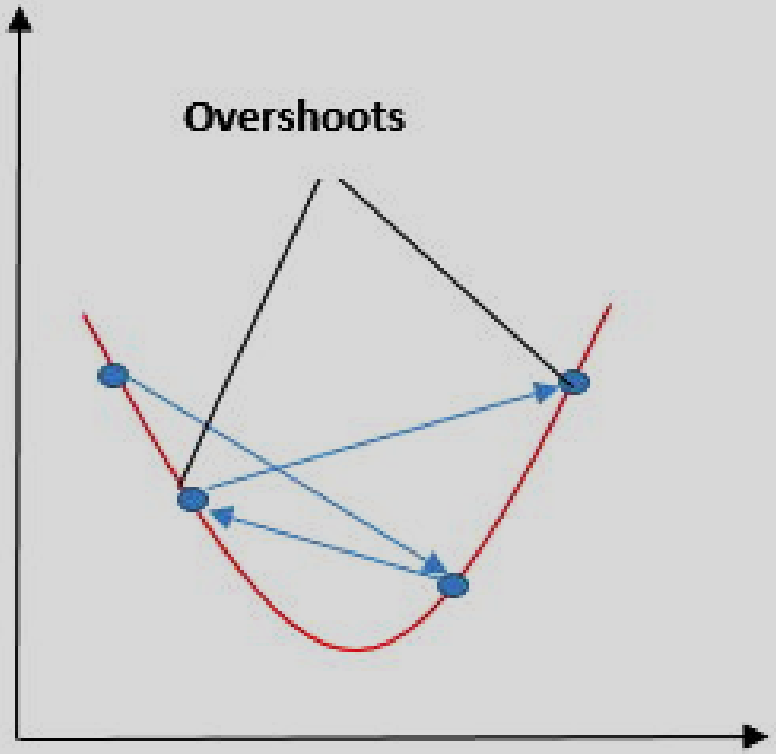
- Gradient Descent is one of the optimization algorithms that optimize the cost function(objective function) to reach the optimal minimal solution. To find the optimum solution we need to reduce the cost function(MSE) for all data points. This is done by updating the values of B_0 and B_1 iteratively until we get an optimal solution.
- A regression model optimizes the gradient descent algorithm to update the coefficients of the line by reducing the cost function by randomly selecting coefficient values and then iteratively updating the values to reach the minimum cost function.



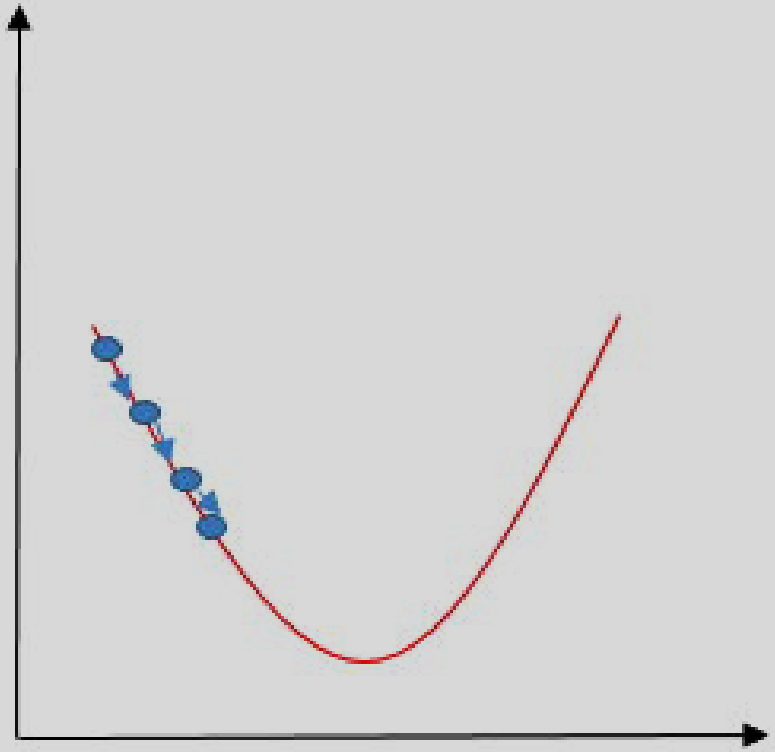
Gradient Descent Example

- Imagine a U-shaped pit and you are standing at the uppermost point in the pit, and your motive is to reach the bottom of the pit.
- Suppose there is a treasure at the bottom of the pit, and you can only take a discrete number of steps to reach the bottom. If you opted to take one step at a time, you would get to the bottom of the pit in the end but, this would take a longer time. If you decide to take larger steps each time, you may achieve the bottom sooner but, there's a probability that you could overshoot the bottom of the pit and not even near the bottom.
- In the gradient descent algorithm, the number of steps you're taking can be considered as the **learning rate**, and this decides how fast the algorithm **converges** to the minima.

Gradient Descent Example



High learning rate



Low learning rate

Gradient Descent Example

- To update B_0 and B_1, we take gradients from the cost function. To find these gradients, we take partial derivatives for B_0 and B_1.
- We need to minimize the cost function J. One of the ways to achieve this is to apply the batch gradient descent algorithm, the values are updated in each iteration. (The last two equations show the updating of values)
- The partial derivatives are the gradients, and they are used to update the values of B_0 and B_1. Alpha is the learning rate.

$$J = \frac{1}{n}\sum_{i=1}^{n}(B_0 + B_1 \cdot x_i - y_i)^2$$

$$\frac{\partial J}{\partial B_0} = \frac{2}{n}\sum_{i=1}^{n}(B_0 + B_1 \cdot x_i - y_i)$$

$$\frac{\partial J}{\partial B_1} = \frac{2}{n}\sum_{i=1}^{n}(B_0 + B_1 \cdot x_i - y_i) \cdot x_i$$

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$$B_0 = B_0 - \alpha \cdot \frac{2}{n}\sum_{i=1}^{n}(pred_i - y_i)$$

$$B_1 = B_1 - \alpha \cdot \frac{2}{n}\sum_{i=1}^{n}(pred_i - y_i) \cdot x_i$$

Evaluation Metrics for Linear Regression

- The strength of any linear regression model can be assessed using various evaluation metrics. These evaluation metrics usually provide a measure of how well the observed outputs are being generated by the model.
- The most used metrics are,

1.Coefficient of Determination or R-squared (R2)

2.Root Mean Squared Error (RSME) and Residual Standard Error (RSE)

Coefficient of Determination or R-squared (R2)

- R-squared is a number that explains the amount of variation that is explained/captured by the developed model. It always ranges between 0 and 1 . Overall, the higher the value of R-squared, the better the model fits the data.
- Mathematically it can be represented as,
- $$R^2 = 1 – (RSS/TSS)$$

Metrics…

- **Residual sum of Squares (RSS)** is defined as the sum of squares of the residual for each data point in the plot/data. It is the measure of the difference between the expected and the actual observed output.

$$\text{RSS} = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$

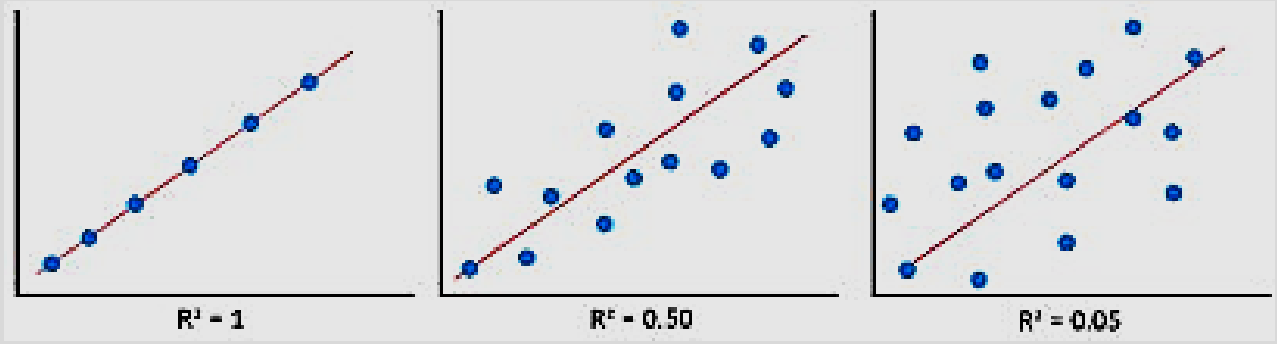
- **Total Sum of Squares (TSS)** is defined as the sum of errors of the data points from the mean of the response variable. Mathematically TSS is,

$$TSS = \sum (y_i - \bar{y}_i)^2$$

- where y hat is the mean of the sample data points.

Metrics…

- The significance of R-squared is shown by the following figures,



- The **Root Mean Squared Error** is the square root of the variance of the residuals. It specifies the absolute fit of the model to the data i.e. how close the observed data points are to the predicted values. Mathematically it can be represented as,

$$RMSE = \sqrt{\frac{RSS}{n}} = \sqrt{\sum_{i=1}^{n} \left(y_i^{Actual} - y_i^{Predicted}\right)^2 / n}$$

Metrics…

- To make this estimate unbiased, one has to divide the sum of the squared residuals by the **degrees of freedom** rather than the total number of data points in the model. This term is then called the **Residual Standard Error(RSE)**. Mathematically it can be represented as,

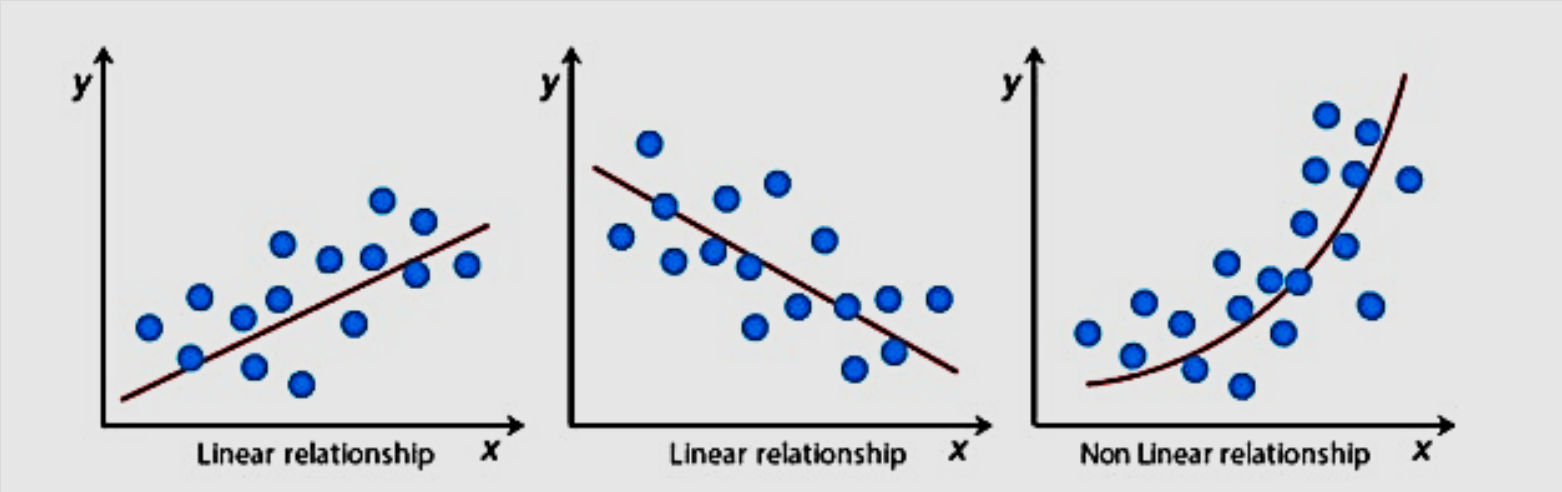
$$RSE = \sqrt{\frac{RSS}{df}} = \sqrt{\sum_{i=1}^{n} \left(y_i^{Actual} - y_i^{Predicted}\right)^2 / (n-2)}$$

- R-squared is a better measure than RSME. Because the value of Root Mean Squared Error depends on the units of the variables (i.e. it is not a normalized measure), it can change with the change in the unit of the variables.

Assumptions of Linear Regression

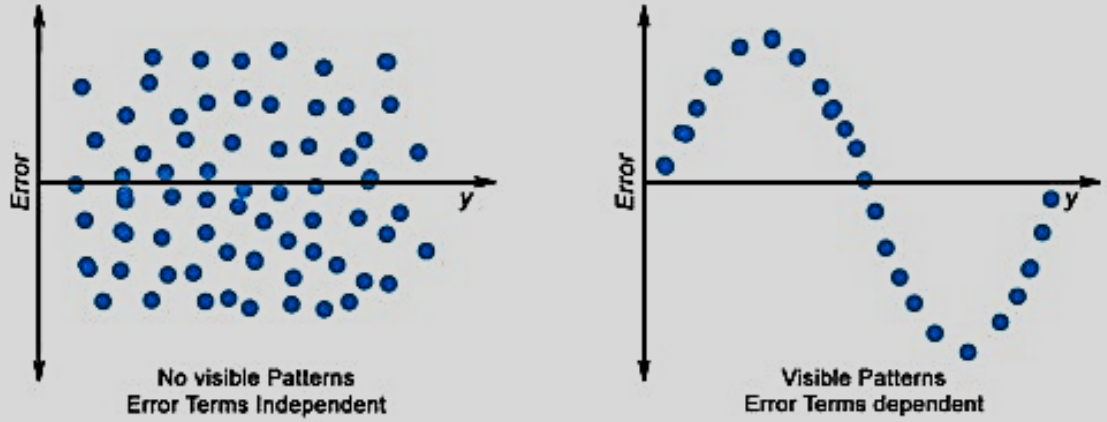
- Regression is a parametric approach, which means that it makes assumptions about the data for the purpose of analysis. For successful regression analysis, it's essential to validate the following assumptions.

1. **Linearity of residuals**: There needs to be a linear relationship between the dependent variable and independent variable(s).



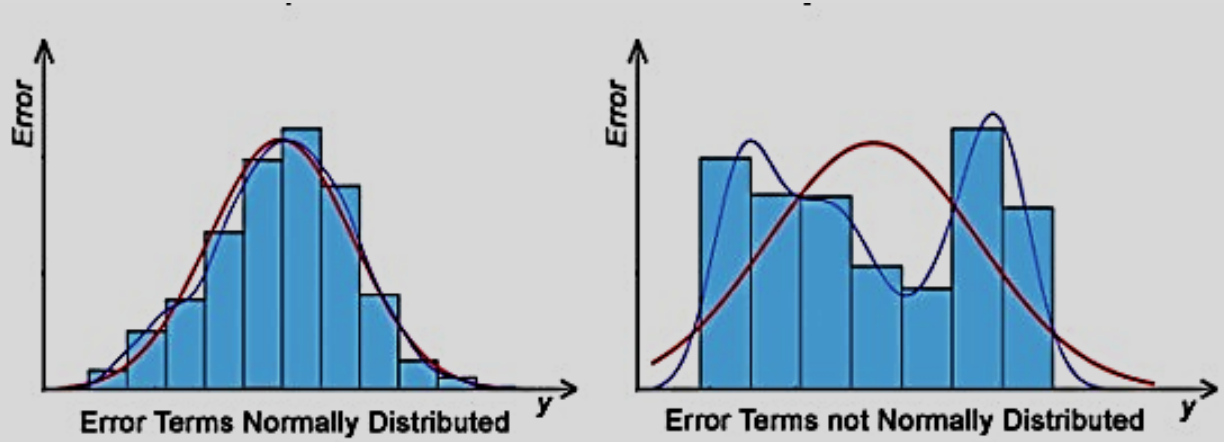
Assumptions…

- . **Independence of residuals:** The error terms should not be dependent on one another (like in time-series data wherein the next value is dependent on the previous one). There should be no correlation between the residual terms. The absence of this phenomenon is known as **Autocorrelation.**
- There should not be any visible patterns in the error terms.



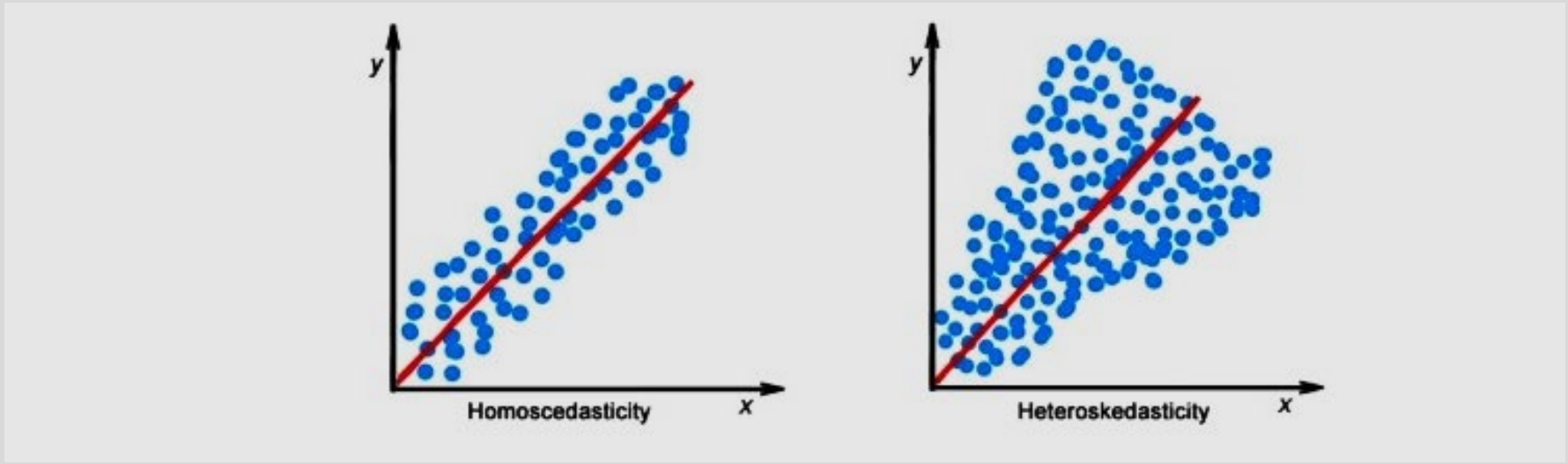
Assumptions…

- **Normal distribution of residuals:** The mean of residuals should follow a normal distribution with a mean equal to zero or close to zero. This is done in order to check whether the selected line is actually the line of best fit or not.
  If the error terms are non-normally distributed, suggests that there are a few unusual data points that must be studied closely to make a better model.



Assumptions…

- **The equal variance of residuals:** The error terms must have constant variance. This phenomenon is known as **Homoscedasticity**.
- The presence of non-constant variance in the error terms is referred to as **Heteroscedasticity**. Generally, non-constant variance arises in the presence of *outliers or extreme leverage values*.



Hypothesis in Linear Regression

- Once you have fitted a straight line on the data, you need to ask, "**Is this straight line a significant fit for the data?**" Or "Is **the beta coefficient explain the variance in the data plotted?"** And here comes the idea of **hypothesis testing** on the beta coefficient. The Null and Alternate hypotheses in this case are:
- H_0: $B_1 = 0$
- H_A: $B_1 \neq 0$
- To test this hypothesis we use a **t-test,** test statistics for the beta coefficient is given by,

$$t = \frac{m - \mu}{s/\sqrt{n}}$$

Assessing the model fit

- Some other parameters to assess a model are:

1. **t statistic:** It is used to determine the p-value and hence, helps in determining whether the coefficient is significant or not
2. **F statistic**: It is used to assess whether the overall model fit is significant or not. Generally, the higher the value of the F-statistic, the more significant a model turns out to be.

Multiple Linear Regression

- Multiple linear regression is a technique to understand the relationship between a *single* dependent variable and *multiple* independent variables.
- The formulation for multiple linear regression is also similar to simple linear regression with
- the small change that instead of having one beta variable, you will now have betas for all the variables used. The formula is given as:
- $Y = B_0 + B_1X_1 + B_2X_2 + \ldots + B_pX_p + \boldsymbol{\varepsilon}$

Considerations of Multiple Linear Regression

- All the four assumptions made for Simple Linear Regression still hold true for Multiple Linear Regression along with a few new additional assumptions.

1. **Overfitting**: When more and more variables are added to a model, the model may become far too complex and usually ends up memorizing all the data points in the training set. This phenomenon is known as the overfitting of a model. This usually leads to high training accuracy and very low test accuracy.
2. **Multicollinearity**: It is the phenomenon where a model with several independent variables, may have some variables interrelated.
3. **Feature Selection:** With more variables present, selecting the optimal set of predictors from the pool of given features (many of which might be redundant) becomes an important task for building a relevant and better model.

Multicollinearity

- As multicollinearity makes it difficult to find out which variable is contributing towards the prediction of the response variable, it leads one to conclude incorrectly, the effects of a variable on the target variable. Though it does not affect the precision of the predictions, it is essential to properly detect and deal with the multicollinearity present in the model, as random removal of any of these correlated variables from the model causes the coefficient values to swing wildly and even change signs.
- Multicollinearity can be detected using the following methods.

1. **Pairwise Correlations:** Checking the pairwise correlations between different pairs of independent variables can throw useful insights into detecting multicollinearity.

Multicollinearity…

- **Variance Inflation Factor (VIF):** Pairwise correlations may not always be useful as it is possible that just one variable might not be able to completely explain some other variable but some of the variables combined could be ready to do this. Thus, to check these sorts of relations between variables, one can use VIF. VIF explains the relationship of one independent variable with all the other independent variables. VIF is given by,

$$VIF = \frac{1}{1-R^2}$$

- *where i* refers to the i^{th} variable which is represented as a linear combination of the rest of the independent variables.
- The common heuristic followed for the VIF values is if VIF > 10 then the value is high and it should be dropped. And if the VIF=5 then it may be valid but should be inspected first. If VIF < 5, then it is a good VIF value.

Overfitting and Underfitting in Linear Regression

- There have always been situations where a model performs well on training data but not on the test data. While training models on a dataset, overfitting, and underfitting are the most common problems faced by people.
- Before understanding overfitting and underfitting one must know about bias and variance.

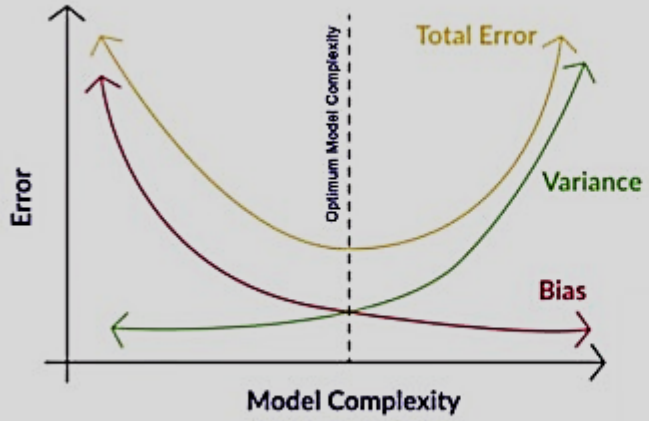
Bias:

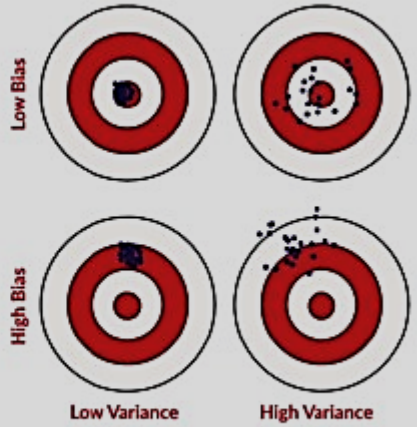
- Bias is a measure to determine how accurate is the model likely to be on future unseen data. Complex models, assuming there is enough training data available, can do predictions accurately. Whereas the models that are too naive, are very likely to perform badly with respect to predictions. Simply, Bias is errors made by training data.

Overfitting…

- Generally, linear algorithms have a high bias which makes them fast to learn and easier to understand but in general, are less flexible. Implying lower predictive performance on complex problems that fail to meet the expected outcomes.
- **Variance:**
- Variance is the sensitivity of the model towards training data, that is it quantifies how much the model will react when input data is changed.
- Ideally, the model shouldn't change too much from one training dataset to the next training data, which means that the algorithm is good at picking out the hidden underlying patterns between the inputs and the output variables.
- Ideally, a model should have lower variance which means that the model doesn't change drastically after changing the training data(it is generalizable). Having a higher variance will make a model change drastically even on a small change in the training dataset.
- Let's understand what a bias-variance tradeoff is.

Bias Variance Tradeoff





- In the pursuit of optimal performance, a supervised machine learning algorithm seeks to strike a balance between low bias and low variance for increased robustness.
- In the realm of machine learning, there exists an inherent relationship between bias and variance, characterized by an inverse correlation.
  - Increased bias leads to reduced variance.
  - Conversely, heightened variance results in diminished bias.

Bias…

- Finding an equilibrium between bias and variance is crucial, and algorithms must navigate this trade-off for optimal outcomes.
- In practice, calculating precise bias and variance error terms is challenging due to the unknown nature of the underlying target function.
- Now, let’s delve into the nuances of overfitting and underfitting.

Overfitting

- When a model learns each and every pattern and noise in the data to such extent that it affects the performance of the model on the unseen future dataset, it is referred to as ***overfitting***. The model fits the data so well that it interprets noise as patterns in the data.
- When a model has low bias and higher variance it ends up memorizing the data and causing overfitting. Overfitting causes the model to become specific rather than generic. This usually leads to high training accuracy and very low test accuracy.
- Detecting overfitting is useful, but it doesn't solve the actual problem. There are several ways to prevent overfitting, which are stated below:
    - Cross-validation
    - If the training data is too small to train add more relevant and clean data.
    - If the training data is too large, do some feature selection and remove unnecessary features.
    - Regularization

Underfitting

- Underfitting is not often discussed as often as overfitting is discussed. When the model fails to learn from the training dataset and is also not able to generalize the test dataset, is referred to as ***underfitting***. This type of problem can be very easily detected by the performance metrics.
- When a model has high bias and low variance it ends up not generalizing the data and causing underfitting. It is unable to find the hidden underlying patterns from the data. This usually leads to low training accuracy and very low test accuracy. The ways to prevent underfitting are stated below,
- Increase the model complexity
- Increase the number of features in the training data
- Remove noise from the data.

Hands on

- Let’s take the dataset as an example,

Month	Money Spent (X)	Avg(X) – X	[Avg(X) – X]^2	Sales (Y)	Avg(Y) – Y	[Avg(X)-X * Avg(Y)-Y]
1	2000	1300	1,690,000	5000	7600	11,400,000
2	3000	300	90,000	10000	2600	780,000
3	4000	-700	490,000	15000	-2400	1,680,000
4	2500	800	640,000	8000	4600	3,680,000
5	5000	-1700	2,890,000	25000	-12400	21,080,000
AVG(X)	3300			12600		
		SUM XX	5,800,000		SUM YY	37,918,000

Hands on…

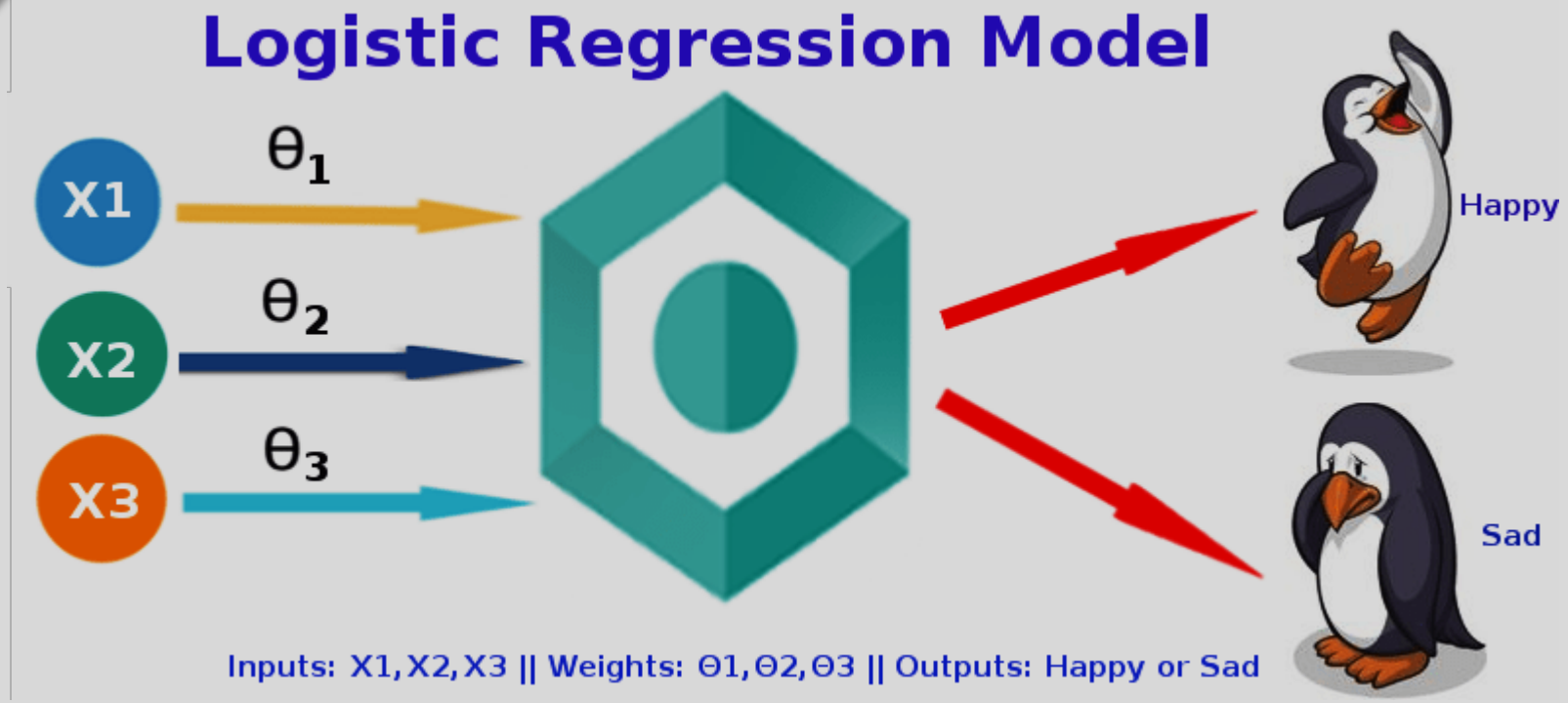
- Find the average of the
- X variable, in this case, it is the Money Spent = 3300
- Y variable, in this case, it is Sale = 12600
- Measure the difference between the Average X and individual X
- Square the difference and add the result: **SUM XX = 5, 800,000**
- Multiple the between Avg(X)-X and Avg(Y)-Y and add the results: **SUM XY = 37,918,000**
- To calculate the Slope of the Line, divide the SUM XY by SUM XX
- b (Slope of the line) = 37,918,000 / 5,800,000 = 6.53758621
- To calculate the y-intercept subtract Avg(Y) from Slope * AVG(X)
- a (y-intercept) = 12600 – 6.53758621* 3300 = 8,974.0345
- So, now you have a Simple Linear Regression
- **Y = a + bX**
- **Y = 8,974.0345 + 6.53758621 * X**

LOGISTIC REGRESSION

What is Logistic Regression?

- Logistic regression estimates the relationship between a dependent variable and one or more independent variables and predicts a categorical variable or a continuous one.
- Logistic regression used for classification tasks.
- It is a predictive analytic technique based on the probability idea.
- The dependent variable in logistic regression is binary (coded as 1 or 0).
- It defines relationships among characteristics and a specific outcome.
- Logistic regression uses a more sophisticated cost function called the “Sigmoid function” or “logistic function” instead of a linear function.
- The logistic regression hypothesis limits the cost function to a value between 0 and 1, making linear functions unsuitable for this task.
- Logistic regression finds applications in various fields such as finance, marketing, healthcare, and social sciences, where it is employed to model and predict binary outcomes.

Logistic Regression

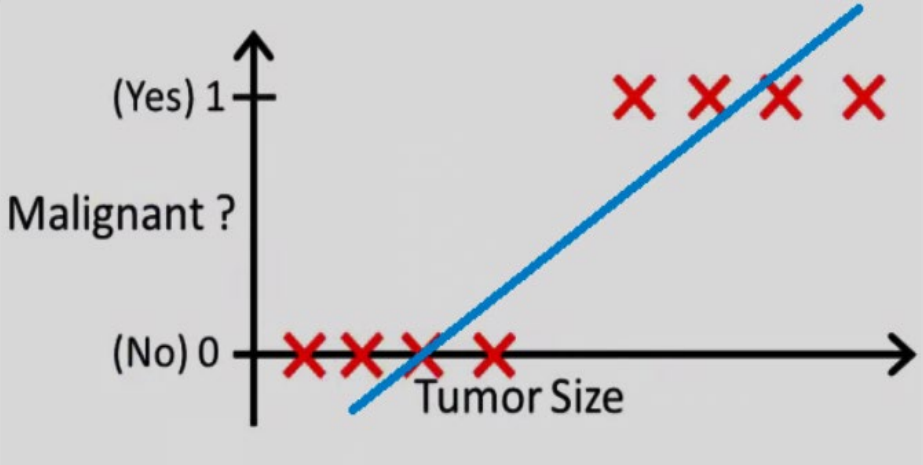


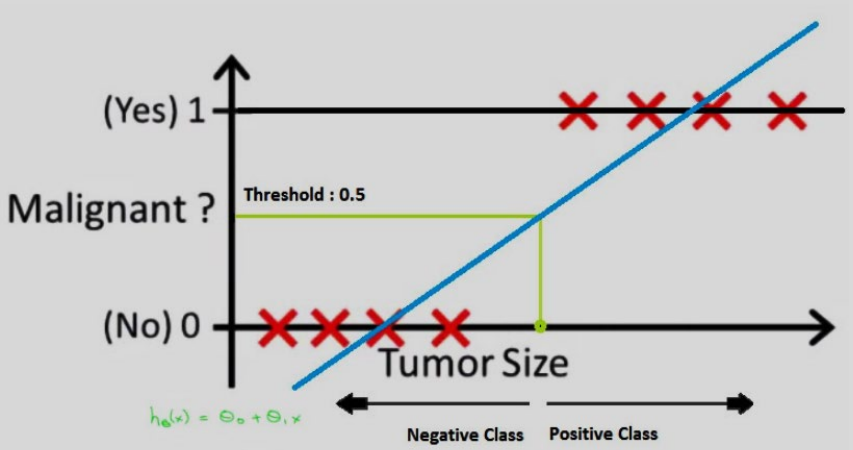
- Logistic Regression is considered a regression model that also predicts the likelihood that a given data entry belongs to the category labeled “1.” Logistic regression models the data using the sigmoid function.

Why the Name Logistic Regression?

- We call it logistic regression because of its special trick, the sigmoid function.
- Think of it as a secret formula that turns numbers into probabilities, helping us decide between two outcomes. Or, we can say ‘Logistic Regression’ since the technique behind it is quite similar to Linear Regression.
- The name “Logistic” originates from the Logit function associated with the standard logistic distribution, which plays a central role in this categorization approach.

Why Regression can't be used instead of logistics regression.





- Let's assume we have information about tumor size and malignancy. Because this is a classification issue, we can see that all the values will fall between 0 and 1. And, by fitting the best-found regression line and assuming a threshold of 0.5.
- We can select a point on the x-axis where all values to the left are considered negative, and all values to the right are regarded as positive.

Contd...

- But what if the data contains an outlier... For 0.5 thresholds, even if we fit the best-found regression line, we won't be able to determine any point where we can distinguish classes. It will insert some instances from the positive class into the negative class. The green dotted line (Decision Boundary) separates malignant and benign tumors, however, it should have been a yellow line that separates the positive and negative cases. As a result, even a single outlier can throw the linear regression estimates off.

Logit function to Sigmoid Function

- Moving from the logit function to the sigmoid function is like turning raw data into something we can use, sort of like turning a block of wood into a finely crafted sculpture.
- Logistic Regression can be expressed as,

$$log\left(\frac{p(X)}{1-p(X)}\right) = \beta 0 + \beta 1X$$

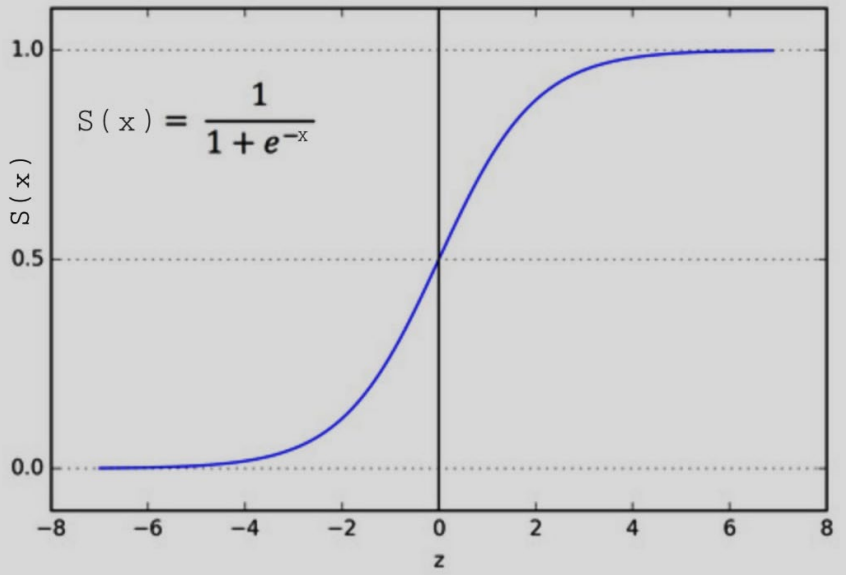
- where p(x)/(1-p(x)) is termed odds, and the left-hand side is called the **logit or log-odds function**. The odds are the ratio of the chances of success to the chances of failure. As a result, in Logistic Regression, a linear combination of inputs is translated to log(odds), with an output of 1.

Contd...

- The following is the inverse of the aforementioned function,

$$p(X) = \frac{e^{\beta 0+\beta 1X}}{1 + e^{\beta 0+\beta 1X}}$$

- This is the Sigmoid function, which produces an S-shaped curve. It always returns a probability value between 0 and 1. The Sigmoid function is used to convert expected values to probabilities. The function converts any real number into a number between 0 and 1. We utilize sigmoid to translate predictions to probabilities in machine learning.



Types of Logistic Regression

- Just like there are different types of pizza, there are different types of logistic regression: binary, multinomial, and ordinal. Each one serves a different purpose.

1. Binary Logistic Regression – two or binary outcomes like yes or no
2. Multinomial Logistic Regression – three or more outcomes like first, second, and third class or no class degree
3. Ordinal Logistic Regression – three or more like multinomial logistic regression but here with the order like customer rating in the supermarket from 1 to 5

Requirements for Logistic Regression

- To use logistic regression, you need clean data, no big surprises between data points, and a straight line that shows the relationship between variables.
  - This model can work for all the datasets, but still, if you need good performance, then there will be some assumptions to consider,
  - The dependent variable in binary logistic regression must be binary.
  - Only the relevant variables should be included.
  - The independent variables must be unrelated to one another. That is, there should be minimal or no multicollinearity in the model.
  - The log chances are proportional to the independent variables.
  - Large sample sizes are required for logistic regression.

Decision Boundary – Logistic Regression

- The decision boundary in logistic regression is like a fence that separates the cats from the dogs. It's where we say, "This side is for cats, and that side is for dogs."

- We can establish a threshold to predict the class to which a given data point belongs. The estimated probability obtained is then classified based on this threshold.

- If the predicted value is less than 0.5, categorize the particular student as a pass; otherwise, label it as a fail. There are two types of decision boundaries: linear and non-linear. To provide a complicated decision boundary, the polynomial order can be raised.

Why can’t we use the cost function used for linearity for logistic regression?

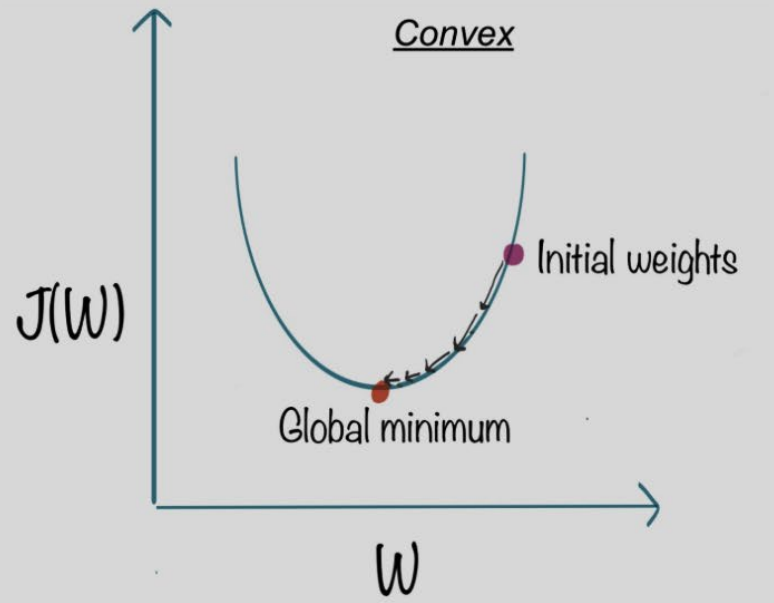
- The cost function for linear regression is a mean squared error. If we use it for logistic regression, the parameter function will become non-convex. Only if the function is convex will gradient descent lead to a global minimum.

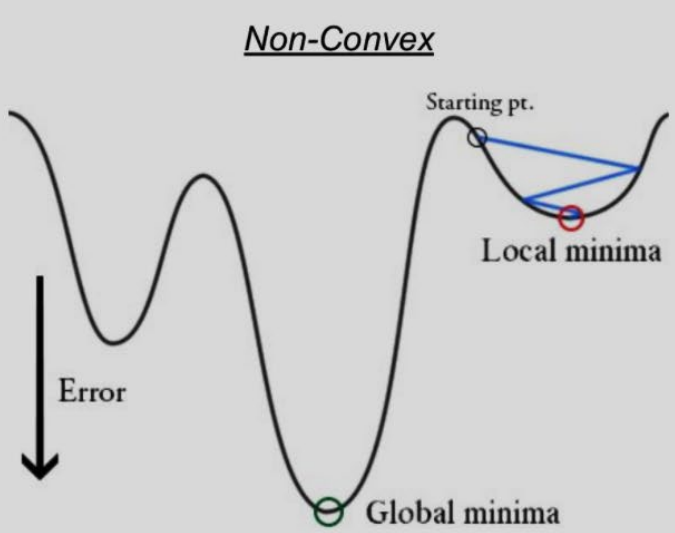
Cost function – Linear Regression Vs Logistic Regression

- In linear regression, we measure how far our guesses are from the real thing. In logistic regression, we measure how close our guesses are to the probabilities we want.
- Linear regression employs the Least Squared Error as the loss function, which results in a convex network, which we can then optimize by identifying the vertex as the global minimum. For logistic regression, however, it is no longer a possibility. Also, modifying the hypothesis results in a non-convex graph with local minimums when calculating Least Squared Error using the sigmoid function on raw model output.
- What is cost function? Cost functions are used in machine learning to estimate how poorly models perform. Simply put, a cost function is a measure of how inaccurate the model is in estimating the connection between X and y. This is usually stated as a difference or separation between the expected and actual values. A machine learning model's goal is to discover parameters, weights, or a structure that minimizes the cost function.

Contd...

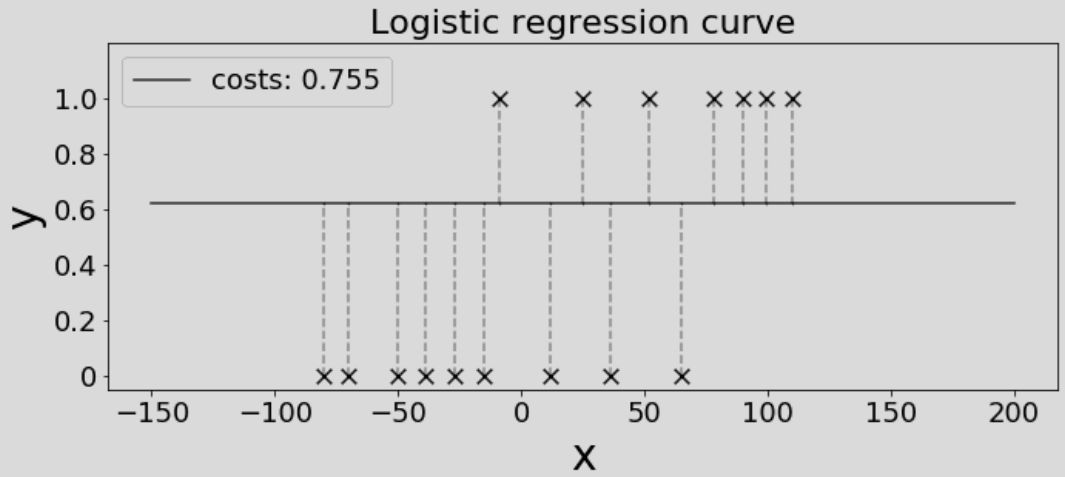
- A convex function indicates there will be no intersection between any two points on the curve, but a non-convex function will have at least one intersection. In terms of cost functions, a convex type always guarantees a global minimum, whereas a non-convex type only guarantees local minima.

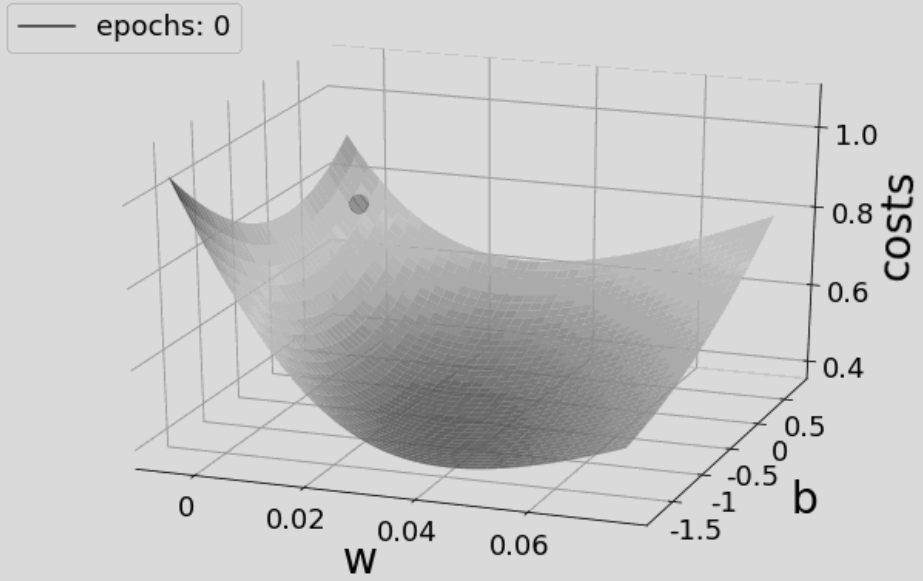




How to Reduce Cost Function? – Gradient Descent

- Gradient descent is like taking small steps down a hill to find the shortest path. It helps us tweak our model to make it as accurate as possible.
- The challenge now is: how can we lower the cost value? Gradient Descent can be used to accomplish this. Gradient descent's main objective is to reduce the cost value.





Regularization

- Regularization is like adding guardrails to a race track. It stops our model from going off track and overfitting, keeping it in check.
- Let's also discuss Regularization quickly for reducing the cost function to match the parameters to training data. L1 (Lasso) and L2 (Lasso) are the two most frequent regularization types (Ridge). Instead of simply maximizing the aforementioned cost function, regularization imposes a limit on the size of the coefficients in order to avoid overfitting. L1 and L2 use distinct approaches to defining upper limits for coefficients, allowing L1 to conduct feature selection by setting coefficients to 0 for less relevant characteristics and reducing multi-collinearity, whereas L2 penalizes extremely large coefficients but does not set any to 0. There's also a parameter that regulates the constraint's weight, λ, to ensure that coefficients aren't penalized too harshly, resulting in underfitting.
- It's a fascinating topic to investigate why L1 and L2 have different capacities owing to the 'squared' and 'absolute' values, and how λ affects the weight of regularized and original fit terms. We won't go into everything here, but it's well worth your time and effort to learn about. The steps below demonstrate how to convert an original cost function to a regularized cost function.

$$L1: \quad J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)}) \quad s.t. \quad \|\theta\|_1 <= C$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)}) + \frac{\lambda}{m}\sum_{j=1}^{n} |\theta_j|$$

$$L2: \quad J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)}) \quad s.t. \quad \|\theta\|_2^2 <= C^2$$

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)}) + \frac{\lambda}{2m}\sum_{j=1}^{n} \theta_j^2$$

$m = number\ of\ samples, \quad n = number\ of\ features$