

2.0 As stated in the hint the volume of the d -dimensional ball is:

The acceptance rate of a sample is proportional to the ratio of volumes of the d -dimensional unit box ($1^d = 1$) and ball $\left(\frac{2\pi^{\frac{d}{2}}}{d \cdot (\frac{d}{2} - 1)!} \right)$

\Rightarrow meaning The acceptance probability is:

$$p = \frac{V_0}{V_B} = \frac{2\pi^{\frac{d}{2}}}{d \cdot (\frac{d}{2} - 1)!}$$

• For $d = 10$:

$$p = \frac{2\pi^{2.5}}{10 \cdot (4!)} = \frac{34.98}{240} = 0.145$$

\Rightarrow as seen in the lecture, following the geometric distribution Expectation, the Expected number of samples to obtain 1 accepted one is: $E = \frac{1}{p}$

\Rightarrow meaning to obtain 1000 accepted samples

we will need: $1000E = \frac{1000}{p} = \frac{1000}{0.145} = 6859.7$

$= 6860$ samples

⇒ similarly, for $d = 20$

$$P = \frac{2\pi^{10}}{20 \cdot 9!} = 0.0258$$

$$E = 38.749$$

$$1000E = 38,749,34 \approx 38,750 \text{ samples}$$

⇒ and for $d = 40$:

$$P = \frac{2\pi^{20}}{40 \cdot 19!} = 3.6 \cdot 10^{-9}$$

$$E = 277,413,226.2$$

$$1000E = 2.77 \cdot 10^{11} \text{ samples}$$