2) a) $P: \quad \texttt{if}\ (x > 0)\ \{\ x := y + 1\ \}\ \texttt{else}\ \{\ \texttt{if}\ (x = 0)\ \{\ \texttt{skip}\ \}\ \texttt{else}\ \{\ x := x + 1\ \}\ \}$

using Backward Reasoning:

$$wp\,[\![\text{if}(x>0)\{x:=y+1\}\text{else}\{\text{if}(x=0)\{\text{skip}\}\text{else}\{x:x+1\}\}]\!](x>y)$$

$$= ((x>0) \wedge wp[\![x:=y+1]\!](x>y)) \vee ((x\leq 0) \wedge wp[\![\text{if}(x=0)\{\text{skip}\}\text{else}\{x:=x+1\}]\!](x>y))$$

$$= \underbrace{((x>0) \wedge \underbrace{(y+1>y)}_{\text{true}})}_{(x>0)} \vee ((x\leq 0) \wedge (((x=0) \wedge wp[\![\text{skip}]\!](x>y)) \vee ((x\neq 0) \wedge wp[\![x:=x+1]\!](x>y))))$$

$$= (x>0) \vee ((x\leq 0) \wedge (x=0) \wedge (x>y)) \vee ((x\neq 0) \wedge (x+1>y))$$

$$= (0>y) \vee ((x\neq 0) \wedge (x+1>y))$$

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b) $P: \quad \texttt{while}\ (x \geq 1)\ \{\ a := a * x;\ x := x - 1\ \}$

$f: \quad a = x!$

The loop charachteristic function $\Phi_F(f)$ is:

$$((x\geq 1) \wedge wp[\![a:=a*x\,;\, x:=x-1]\!](\ X\ )) \vee ((x<1) \wedge (a=x!))$$

we've seen in the lecture that $\Phi_F : \mathbb{P} \to \mathbb{P}$ is scott continuous.
using kleene's fixed foired theorem we get:

$$lfp\ \Phi_F = \sup_{n\in\mathbb{N}}\ \Phi_F^n(\underset{\uparrow\ \bot \text{ in } (\mathbb{P}, \Rightarrow)}{false})$$

$$\Rightarrow \Phi_f(False) = \Big(\underbrace{(x \geq 1) \wedge \underbrace{wp[\![a := a * x\ ;\ x := x-1]\!](false)}_{false}}_{false}\Big) \vee \Big((x < 1) \wedge (a = x!)\Big)$$

$$= (x < 1) \wedge (a = x!)$$

$$\Phi_f^2(false) = \Phi_f\big((x<1) \wedge (a = x!)\big) =$$

$$= \Big((x \geq 1) \wedge wp[\![a := a \cdot x\ ;\ x := x-1]\!]\big((x<1) \wedge (a = x!)\big)\Big) \vee \Big((x < 1) \wedge (a = x!)\Big)$$

$$= \Big((x \geq 1) \wedge wp[\![a := a \cdot x]\!]\big((x < 2) \wedge (a = (x-1)!)\big)\Big) \vee \big((x<1) \wedge (a = x!)\big)$$

$$= \Big((x \geq 1) \wedge \big((x<2) \wedge (ax = (x-1)!)\big)\Big) \vee \Big((x < 1) \wedge (a = x!)\Big)$$

$$= \Big((x = 1) \wedge (ax = (x-1)!)\Big) \vee \Big((x<1) \wedge (a = x!)\Big)$$

$$= (a = 1) \vee \Big((x<1) \wedge (a = x!)\Big)$$

$$\Phi^3(false) = \Phi_f\Big((a=1) \vee (x<1) \wedge (a = x!)\Big) =$$

$$= \Big((x \geq 1) \wedge wp[\![a := a \cdot x\ ;\ x := x-1]\!]\big((a=1) \vee (x<1) \wedge (a = x!)\big)\Big) \vee \Big((x < 1) \wedge (a = x!)\Big)$$

$$= \Big((x \geq 1) \wedge wp[\![a := a \cdot x]\!]\big((a=1) \vee ((x < 2) \wedge (a = (x-1)!))\big)\Big) \vee \Big((x<1) \wedge (a = x)\Big)$$

$$= \Big((x \geq 1) \wedge \big((ax = 1) \vee (x < 2) \wedge (ax = (x-1)!)\big)\Big) \vee \Big((x < 1) \wedge (a = x!)\Big)$$

$$= \Big((a = 1) \vee \big((x < 1) \wedge (a = x!)\big)\Big)$$

$\Longrightarrow$ Thus, by computing the loop-characteristing functional $\Phi_f(f)$, we found, using Kleeme's fixed point theorem that $wp(p, f)$ is in fact $(n=1) \vee ((x<1) \wedge (a = x!))$