

3. c.

$$E(X_{\max}) = \sum_{x=1}^f x \cdot P(X_{\max} = x) = \sum_{x=1}^f x \cdot \frac{(3x(x-1)+1)}{f^3}$$

hint $\nearrow x=1$

$$= \frac{1}{f^3} \cdot \sum_{x=1}^f x \cdot (3x^2 - 3x + 1) = \frac{1}{f^3} \left(3 \sum_{i=1}^f x^3 - 3 \sum_{x=1}^f x^2 + \sum_{x=1}^f x \right)$$

$$= \frac{1}{f^3} \left(3 \cdot \frac{f^2(f+1)^2}{4} - \left(\frac{f(f+1)(f+2)}{6} \right) + \frac{f(1+f)}{2} \right)$$

$$= \frac{f(f+1)}{2f^3} \left[\frac{3}{2} f(f+1) - \frac{f+2}{3} + 1 \right]$$

$$= \frac{f+1}{2f^2} \left[\frac{3}{2}(f^2+f) - \frac{1}{3}(f+2) + 1 \right]$$

\Rightarrow

$$\lim_{f \rightarrow \infty} \frac{E(X_{\max})}{f} = \lim_{f \rightarrow \infty} \frac{f+1}{2f^3} \left[\frac{3}{2}(f^2+f) - \frac{1}{3}(f+2) + 1 \right]$$

$$= \underbrace{\frac{\frac{3}{2}(f^2+f)(f+1)}{2f^3}}_{\rightarrow 0} - \underbrace{\frac{\frac{1}{3}(f+2)(f+1)}{2f^3}}_{\rightarrow 0} + \underbrace{\frac{(f+1)}{2f^3}}_{\rightarrow 0}$$

$$= \boxed{\frac{3}{4}}$$