Q1

(a) Here, P diverges with probability $\frac{2}{3}$. However, weakest pre-expectations only consider f on termination.

Therefore,

$$wp(P, x^2) \text{ for } P := \{x := 2\} [1/3] \{diverge\}$$

$$= \frac{1}{3} \cdot wp [\![x := 2]\!](x^2) + \frac{2}{3} wp [\![diverge]\!](x^2)$$

$$= \left(\frac{1}{3} \cdot 2^2\right) + 0$$

$$= 4/3 \text{ (Answer)}$$

Now, since P diverges, the weakest liberal pre-condition is —

$$wlp(P, x^2)$$

$$= \frac{1}{3} \cdot 2^2 + \text{Prob.}(P \text{ diverges})$$

$$= 4/3 + 2/3$$

$$= 6/3 = 2 \text{ (Answer)}$$

(b) Let us calculate $wp [\![P_1]\!](x)$ and $wp [\![P_2]\!](x)$ and compare them.

We have —

$$wp [\![P_1]\!](x) = [y < 0]_s wp [\![skip]\!](x) + [y \geq 0]_s wp [\![\{x := 1\} [1/2] \{skip\}]\!](x)$$

$$= 0 + 1 \cdot wp [\![\{x := 1\} [1/2] \{skip\}]\!](x) \quad \text{, since } s(y) = 5$$

$$= \frac{1}{2} wp [\![x := 1]\!](x) + \frac{1}{2} wp [\![skip]\!](x)$$

$$= \frac{1}{2} \cdot 1 + 0$$

$$= \frac{1}{2} \text{ (Answer)}$$

Now,

$$wp[\![P_2]\!](x) = \frac{1}{2} wp[\![\{x := x+3\}[1/3]\{x := x\}]\!](x) + \frac{1}{2} wp[\![x := 0]\!](x)$$

$$= \frac{1}{2} wp\left[\!\!\left[\frac{1}{3} wp[\![\{x := x+3\}]\!](x) + \frac{2}{3} wp[\![x := x]\!](x)\right]\!\!\right](x) + \left(\frac{1}{2} \cdot 0\right)$$

$$= \cancel{\frac{1}{2} wp\left[\!\!\left[\frac{1}{3}(x+3) + \frac{2}{3}x\right]\!\!\right](x)}$$

$$= \frac{1}{2} \cdot \frac{1}{3} wp[\![x := x+3]\!](x) + \frac{1}{2} \cdot \frac{2}{3} wp[\![x := x]\!](x)$$

$$= \frac{1}{6}(x+3) + \frac{1}{3}x = \frac{1}{2}x + \frac{1}{2} = \frac{x+1}{2}$$

Therefore, P_1 and P_2 are equivalent w.r.t post-expectation $f = x$ if $s(x) = 0$ for P_2. In all other cases, they are <u>not</u> equivalent.

(c) We again compute $wp[\![P_1]\!](x)$ and $wp[\![P_2]\!](x)$ and compare them.

$wp[\![P_1]\!](x)$

$= wp[\![\text{while}(x \neq x)\{\{x := y+1\}[1/2]\{x := y-1\}\}]\!](x)$

$\therefore \Phi_x(Y) = \underbrace{[x \neq x]}_{=0} wp[\![\{x := y+1\}[1/2]\{x := y-1\}; Y]\!] + [x = x].x$

$= x$

Therefore, by Kleene's fixpoint theorem,

$\Phi_x^k(\underline{0}) = x \quad \Rightarrow \lim_{k \to \infty} \Phi^k(\underline{0}) = x$

$\therefore \; wp[\![P_1]\!](x) = x$ ——— (i)

Now,

$wp[\![P_2]\!](x) = wp[\![\text{while}(\text{true})\{\text{skip}\}]\!](x)$

$\therefore \Phi_x(Y) = \text{true}.\, wp[\![\text{skip}; Y]\!](x) + \text{false}.\, x$

$= \text{true}.\, wp[\![\text{skip}; Y]\!](x) + 0$

$\therefore$ It never terminates.

Hence, P_1 and P_2 are <u>not</u> equivalent w.r.t. $f = x$.