Assignment 7

Basic Techniques in Computer Graphics
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Exercise 1 Lighting Models

(a) Phong vs. Blinn-Phong We use Phong and Blinn-Phong lighting models to calculate the specular term to model the reflection occurring on shiny surfaces (glossy reflection), more specifically the cosine factor associated with the deviation from the mirror-like reflected vector of the incoming light.

In Phong lighting model, we take into account the angle between the reflected vector and the viewing direction to calculate the cosine factor. For light source l, viewing point v, point p on the surface with normal n, we first have to calculate the reflected vector r as: $r = (2nn^\top - I_d)(l - p)$, where I_d is the identity matrix. We then use that to calculate the cosine factor for Phong model as $\left(\frac{r^\top(v-p)}{||r||\cdot||v-p||}\right)^s$ with shining factor s. which involves quite a lot of computation in terms of matrix multiplication to the least.

In the Blinn-Phong model, we eliminate the computationally costly calculation to alleviate such problems. We use the angle between the halfway vector (unit vector computed from the addition of light source and viewpoint with respect to the point on the surface) h and normal vector n. We can get h by using simple additive calculations and normalization as $u_l = \frac{l-p}{||l-p||}, u_v = \frac{v-p}{||v-p||}, h = \frac{u_l+u_v}{||u_l+u_v||}$. The cosine factor is then calculated in a straightforward manner as $(n^\top h)^s$ with shining factor s. We don't need to calculate the reflected vector in this model.

Consequently, Blinn-Phong model is more useful for real-time graphics application where we need faster computation without much deviation from realistic aspects.

(b) Comparison

- Phong (and Blinn-Phong) lighting model:
  - Advantage: Phong model (and Blinn-Phong) can be easily used in practice as it's fast enough to be implemented in the rendering pipeline and requires only local information.
  - Disadvantage: In such a model, the lighting only depends on the angle with the normal vector, but not on the angle with the tangential plane which essentially makes it isotropic and therefore can't capture more complex anisotropic effects.
- Cook-Torrance lighting model:
  - Advantage: Uses a microfacet surface theory to calculate the specular term which incorporates BRDF (Bidirectional Reflectance Distribution Function) and anisotropic structures making the lighting much more realistic.
  - Disadvantage: Computationally quite heavy to be used in practice heavily.

(c) Cook-Torrance The main assumption about surfaces in the Cook-Torrance model is that any surface at a microscopic scale (when zoomed in with a microscope) can be described by tiny reflective mirrors called microfacets. The way light is reflected depends on the distribution of those microfacets which can differ a lot depending on the roughness.

For the case of diffuse reflections, the surface behaves like a Lambertian reflector which means the distribution of the facet orientations in the surface has to be uniform so that incoming light is reflected onto any outgoing direction with the same probability.

The term G in the formula for Cook-Torrance model provides a way to calculate the geometric term associated with self-shadowing or self-occlusion (probably in case of light coming in a very flat angle) where one microfacet lies in the shadow of the another microfacet depending on the distribution of the microfacets.

Exercise 2 Local Lighting

(a) In Phong Lighting model, we calculate the specular term as: $C_{sp}(\mathbf{p}, \mathbf{n}, l, \mathbf{v}) = C_l \cdot \alpha_{sp} \cdot \left(\frac{r^\top(\mathbf{v}-\mathbf{p})}{||r||\cdot||\mathbf{v}-\mathbf{p}||}\right)^s$, where $r = (2\mathbf{n}\mathbf{n}^\top - I_d)(l - \mathbf{p})$.

We have, $\mathbf{n} = (0, 1, 0)^T, l = (0, 0, 0)^T, \mathbf{p} = (3, -12, -4)^T$. Therefore,

$$
\begin{aligned}
r &= (2\mathbf{n}\mathbf{n}^\top - I_d)(l - \mathbf{p}) \\
&= \left(2\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0 & 1 & 0\end{pmatrix} - \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right)\left(\begin{pmatrix}0\\0\\0\end{pmatrix} - \begin{pmatrix}3\\-12\\-4\end{pmatrix}\right) \\
&= \left(\begin{pmatrix}0&0&0\\0&2&0\\0&0&0\end{pmatrix} - \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right)\left(\begin{pmatrix}0\\0\\0\end{pmatrix} - \begin{pmatrix}3\\-12\\-4\end{pmatrix}\right) \\
&= \begin{pmatrix}-1&0&0\\0&1&0\\0&0&-1\end{pmatrix}\left(\begin{pmatrix}0\\0\\0\end{pmatrix} - \begin{pmatrix}3\\-12\\-4\end{pmatrix}\right) \\
&= \begin{pmatrix}-1&0&0\\0&1&0\\0&0&-1\end{pmatrix}\begin{pmatrix}-3\\12\\4\end{pmatrix} \\
&= \begin{pmatrix}3\\12\\-4\end{pmatrix}
\end{aligned}
$$

$||r|| = \sqrt{(3^2 + 12^2 + (-4)^2)} = \sqrt{9 + 144 + 16} = \sqrt{169} = 13.$

We also have, $\mathbf{v} = \left(\frac{93}{11}, -\frac{72}{11}, -\frac{114}{11}\right)^T, \alpha_{sp} = \text{diag}\left(\frac{143}{236}, \frac{143}{236}, \frac{143}{236}\right), C_l = \left(1, \frac{2}{3}, 0\right)$. We first calculate $(\mathbf{v} - \mathbf{p}) = \left(\frac{93}{11}, -\frac{72}{11}, -\frac{114}{11}\right)^T - (3, -12, -4)^T = \left(\frac{60}{11}, \frac{60}{11}, -\frac{70}{11}\right)$ and $||\mathbf{v} - \mathbf{p}|| = \frac{10}{11}\sqrt{6^2 + 6^2 + (-7)^2} = \frac{10}{11}\sqrt{36 + 36 + 49} = \frac{10}{11}\sqrt{121} = \frac{10}{11} \times 11 = 10$. For $s = 1$, we then have,

$$
\begin{aligned}
C_{sp}(\mathbf{p},\mathbf{n},l,\mathbf{v}) &= C_l \cdot \alpha_{sp} \cdot \left(\frac{r^\top(\mathbf{v}-\mathbf{p})}{||r|| \cdot ||\mathbf{v}-\mathbf{p}||}\right)^s \\
&= \left(1,\frac{2}{3},0\right) \cdot \mathrm{diag}\left(\frac{143}{236},\frac{143}{236},\frac{143}{236}\right) \cdot \left(\frac{(3,12,-4)\begin{pmatrix}\frac{60}{11}\\ \frac{60}{11}\\ -\frac{70}{11}\end{pmatrix}}{13 \times 10}\right) \\
&= \left(1,\frac{2}{3},0\right) \cdot \mathrm{diag}\left(\frac{143}{236},\frac{143}{236},\frac{143}{236}\right) \cdot \left(\frac{\frac{1180}{11}}{130}\right) \\
&= \left(1,\frac{2}{3},0\right) \cdot \mathrm{diag}\left(\frac{143}{236},\frac{143}{236},\frac{143}{236}\right) \cdot \left(\frac{118}{143}\right) \\
&= \left(1,\frac{2}{3},0\right) \cdot \begin{pmatrix}\frac{143}{236} & 0 & 0\\ 0 & \frac{143}{236} & 0\\ 0 & 0 & \frac{143}{236}\end{pmatrix} \cdot \left(\frac{118}{143}\right) \\
&= \left(1,\frac{2}{3},0\right) \cdot \begin{pmatrix}\frac{1}{2} & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{2}\end{pmatrix} \\
&= \left(\frac{1}{2},\frac{1}{3},0\right)
\end{aligned}
$$

Given $\alpha_d = \text{diag}\left(\frac{13}{48}, \frac{13}{48}, \frac{13}{48}\right)$, we can calculate the diffuse term as follows

$$
\begin{aligned}
C_d(\mathbf{p}, \mathbf{n}, l) &= C_l \cdot \alpha_d \cdot \frac{\mathbf{n}^T(l-\mathbf{p})}{||l-\mathbf{p}||} \\
&= \left(1, \frac{2}{3}, 0\right) \cdot \text{diag}\left(\frac{13}{48}, \frac{13}{48}, \frac{13}{48}\right) \cdot \frac{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -12 \\ -4 \end{pmatrix}\right)}{||\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -12 \\ -4 \end{pmatrix}||} \\
&= \left(1, \frac{2}{3}, 0\right) \cdot \text{diag}\left(\frac{13}{48}, \frac{13}{48}, \frac{13}{48}\right) \cdot \frac{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} -3 \\ 12 \\ 4 \end{pmatrix}}{||\begin{pmatrix} -3 \\ 12 \\ 4 \end{pmatrix}||} \\
&= \left(1, \frac{2}{3}, 0\right) \cdot \text{diag}\left(\frac{13}{48}, \frac{13}{48}, \frac{13}{48}\right) \cdot \frac{12}{13} \\
&= \left(1, \frac{2}{3}, 0\right) \cdot \begin{pmatrix} \frac{13}{48} & 0 & 0 \\ 0 & \frac{13}{48} & 0 \\ 0 & 0 & \frac{13}{48} \end{pmatrix} \cdot \frac{12}{13} \\
&= \left(1, \frac{2}{3}, 0\right) \cdot \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix} \\
&= \left(\frac{1}{4}, \frac{1}{6}, 0\right)
\end{aligned}
$$

Since there is no ambient light, we can finally calculate the perceived color of the point p as $C(\mathbf{p}, \mathbf{n}, l, \mathbf{v}) = C_d(\mathbf{p}, \mathbf{n}, l) + C_{sp}(\mathbf{p}, \mathbf{n}, l, \mathbf{v}) = \left(\frac{1}{4}, \frac{1}{6}, 0\right) + \left(\frac{1}{2}, \frac{1}{3}, 0\right) = \left(\frac{3}{4}, \frac{1}{2}, 0\right)$.

(b) For $\mathbf{n} = (0,1,0)^T, l = (0,0,0)^T, \mathbf{p} = (3,-12,-4)^T$ and new viewing position $\mathbf{v}' = (6,0,-8)^T$, we use the Blinn-Phong model to calculate the specular term. First we compute, $u_l = \frac{l-\mathbf{p}}{||l-\mathbf{p}||} = \frac{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -12 \\ -4 \end{pmatrix}}{||\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -12 \\ -4 \end{pmatrix}||} = \frac{\begin{pmatrix} -3 \\ 12 \\ 4 \end{pmatrix}}{||\begin{pmatrix} -3 \\ 12 \\ 4 \end{pmatrix}||} = \frac{(-3,12,4)^T}{13}, u_{\mathbf{v}'} = \frac{\mathbf{v}'-\mathbf{p}}{||\mathbf{v}'-\mathbf{p}||} =$
$\frac{\begin{pmatrix} 6 \\ 0 \\ -8 \end{pmatrix} - \begin{pmatrix} 3 \\ -12 \\ -4 \end{pmatrix}}{||\begin{pmatrix} 6 \\ 0 \\ -8 \end{pmatrix} - \begin{pmatrix} 3 \\ -12 \\ -4 \end{pmatrix}||} = \frac{\begin{pmatrix} 3 \\ 12 \\ -4 \end{pmatrix}}{||\begin{pmatrix} 3 \\ 12 \\ -4 \end{pmatrix}||} = \frac{(3,12,-4)^T}{13}$.

Now we can calculate the halfway vector as $h = \frac{u_l + u_{\mathbf{v}'}}{||u_l + u_{\mathbf{v}'}||} = \frac{\frac{(-3,12,4)^T}{13} + \frac{(3,12,-4)^T}{13}}{||\frac{(-3,12,4)^T}{13} + \frac{(3,12,-4)^T}{13}||} =$

$\frac{\left(0,\frac{24}{13},0\right)^T}{||\left(0,\frac{24}{13},0\right)^T||} = \frac{\left(0,\frac{24}{13},0\right)^T}{\frac{24}{13}} = (0,1,0)^T$. From this, we get the specular term with $s = 1$ as

$$
\begin{aligned}
C_{sp}(\mathbf{p},\mathbf{n},l,\mathbf{v}') &= C_l \cdot \alpha_{sp} \cdot \left(\mathbf{n}^\top h\right)^s \\
&= \left(1,\frac{2}{3},0\right) \cdot \operatorname{diag}\left(\frac{143}{236},\frac{143}{236},\frac{143}{236}\right) \cdot (0,1,0)(0,1,0)^T \\
&= \left(1,\frac{2}{3},0\right) \cdot \begin{pmatrix} \frac{143}{236} & 0 & 0 \\ 0 & \frac{143}{236} & 0 \\ 0 & 0 & \frac{143}{236} \end{pmatrix} \cdot 1 \\
&= \left(1,\frac{2}{3},0\right) \cdot \begin{pmatrix} \frac{143}{236} & 0 & 0 \\ 0 & \frac{143}{236} & 0 \\ 0 & 0 & \frac{143}{236} \end{pmatrix} \\
&= \left(\frac{143}{236},\frac{143}{354},0\right)
\end{aligned}
$$

Finally the perceived color of the point p is, $C(\mathbf{p},\mathbf{n},l,\mathbf{v}') = C_d(\mathbf{p},\mathbf{n},l) + C_{sp}(\mathbf{p},\mathbf{n},l,\mathbf{v}') = \left(\frac{1}{4},\frac{1}{6},0\right) + \left(\frac{143}{236},\frac{143}{354},0\right) = \left(\frac{101}{118},\frac{101}{177},0\right)$.

Exercise 3 Shading Models

(a) Motivation Using lighting models,we only decide the colors of individual points but in the rendering pipeline we need to calculate the colors for all the points inside a polygon (say triangle). Via the shading models such as Flat, Gouraud, or Phong shading models we address the problem of interpolating the color or brightness values of the points inside a polygon.

(b) Flat Shading Since we use planar polygons in the Flat Shading model, the normal remains the same for all points which leads to constant brightness or radiance inside the polygon in terms of color computation. As a whole it leads to piecewise constant coloring or illumination.

Mach Banding is the phenomenon of perceived increased differences between colors near boundaries of two polygons (e.g. triangle faces) due to the enhanced contrast created by the retina of our eye and the brain associated with piecewise constant coloring in flat shading.

(c) Gouraud Shading In the Gouraud Shading model, we interpolate the colors of the pixels inside a polygon by using the colors of the vertices calculated using a lighting model. For any point inside the polygon, we interpolate the color value using the barycentric coordinates of the point. On such case, we get rid of the piecewise constant coloring that is the characteristics of the Flat Shading model. So Gouraud Shading model leads to much smoother results.

(d) Phong Shading In Phong Shading model, we first do the geometric interpolation by computing the normal for each pixel within the polygon by linearly interpolating the normal vectors of the vertices of the polygon. We can use the barycenteric coordinates of a point to calculate the interpolated normal. Then we compute the color value using Phong lighting model pixel-wise from the interpolated normal vector. Therefore, with the smooth transition of the normals, it manages to get rid of the faceting artifacts (at least in the interior region) we see in Gouraud shading coming from the linear interpolation of the color space.