Assignment 12

Basic Techniques in Computer Graphics
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Exercise 1 Bernstein Polynomials and Bézier Curves

(a) Derivatives of Bernstein Polynomials

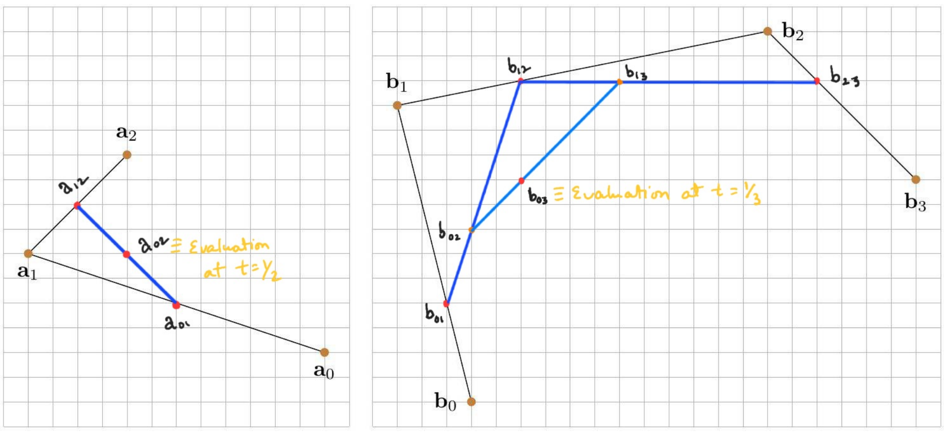
$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} B_i^n(t) &= \frac{\mathrm{d}}{\mathrm{d}t} \binom{n}{i} t^i (1-t)^{n-i} \\
&= \frac{\mathrm{d}}{\mathrm{d}t} \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i} \\
&= \frac{n!}{i!(n-i)!} \frac{\mathrm{d}}{\mathrm{d}t} t^i (1-t)^{n-i} \\
&= \frac{n!}{i!(n-i)!} [it^{i-1}(1-t)^{n-i} + t^i \cdot (n-i)(1-t)^{n-i-1} \cdot (-1)] \\
&= \frac{n!}{i!(n-i)!} \cdot it^{i-1}(1-t)^{n-i} - \frac{n!}{i!(n-i)!} \cdot (n-i)t^i(1-t)^{n-i-1} \\
&= \frac{n \cdot (n-1)!}{(i-1)!((n-1)-(i-1))!} \cdot t^{i-1}(1-t)^{(n-1)-(i-1)} - \frac{n \cdot (n-1)!}{i!((n-1)-i)!} \cdot (n-i)t^i(1-t)^{(n-1)-i} \\
&= n \left[\frac{(n-1)!}{(i-1)!((n-1)-(i-1))!} \cdot t^{i-1}(1-t)^{(n-1)-(i-1)} - \frac{(n-1)!}{i!((n-1)-i)!} \cdot (n-i)t^i(1-t)^{(n-1)-i} \right] \\
&= n \left(B_{i-1}^{n-1}(t) - B_i^{n-1}(t) \right)
\end{aligned}
$$

(b) Derivatives of Bézier Curves Given a Bézier curve $\mathbf{b}(t)$ of degree n with control points $\mathbf{p}_0, \ldots, \mathbf{p}_n$, we can derive the derivative $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b}(t)$ as follows:

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=0}^{n} p_i B_i^n(t) \\
&= \sum_{i=0}^{n} p_i \frac{\mathrm{d}}{\mathrm{d}t} B_i^n(t) \\
&= \sum_{i=0}^{n} p_i \cdot n\left(B_{i-1}^{n-1}(t) - B_i^{n-1}(t)\right) \quad \text{[Using the identity from task (a)]} \\
&= n\sum_{i=0}^{n} p_i \left(B_{i-1}^{n-1}(t) - B_i^{n-1}(t)\right) \\
&= n\sum_{i=0}^{n} p_i B_{i-1}^{n-1}(t) - n\sum_{i=0}^{n} p_i B_i^{n-1}(t) \\
&= n\sum_{i=1}^{n} p_i B_{i-1}^{n-1}(t) - n\sum_{i=0}^{n-1} p_i B_i^{n-1}(t) \\
&= n\sum_{i=0}^{n-1} p_{i+1} B_i^{n-1}(t) - n\sum_{i=0}^{n-1} p_i B_i^{n-1}(t) \\
&= n\sum_{i=0}^{n-1} (p_{i+1} - p_i) B_i^{n-1}(t) \\
&= \sum_{i=0}^{n-1} n(p_{i+1} - p_i) B_i^{n-1}(t) \\
&= \mathbf{b}'(t)
\end{aligned}
$$

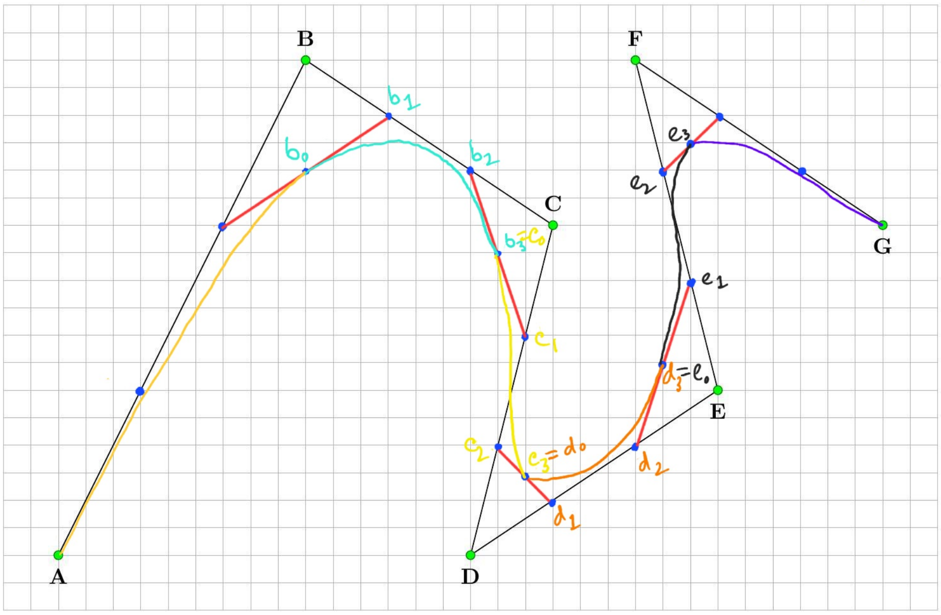
which shows that $\mathbf{b}'(t)$ is a Bézier curve of degree $n-1$ with control points $n(p_{i+1} - p_i), i = 0, \ldots, n-1$ i.e. $n(p_1 - p_0), n(p_2 - p_1), \ldots, n(p_n - p_{n-1})$.

(c) De Casteljau Algorithm



Exercise 2 Splines

(a) B-Spline to Bézier Spline Conversion



(b) Interpolating Bézier Spline Construction