Prof. Dr. Leif Kobbelt
Patrick Schmidt, Joe Jakobi

Basic Techniques in Computer Graphics

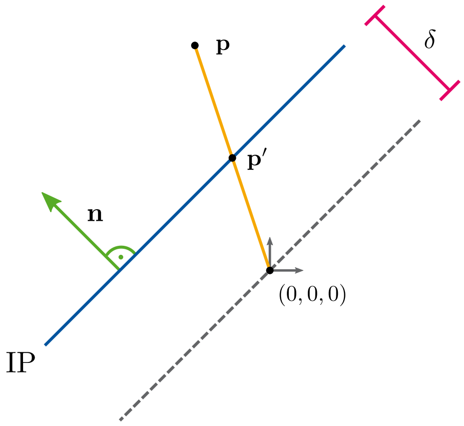
Assignment 3

Date Published: November 8th 2022, Date Due: November 15th 2022, 10:15AM

- All assignments (programming and theory) have to be completed in teams of 3–4 students. Teams with fewer than 3 or more than 4 students will receive no points.
- Hand in **one solution per team per assignment**.
- Every team must work independently. Teams with identical solutions will receive no points.
- Solutions are due on November 15th 2022, 10:15AM via Moodle. Late submissions will receive zero points. No exceptions!
- Instructions for **programming assignments**:
  - Make sure you are part of a Moodle group with 3-4 members. See "Group Management" in the Moodle course room.
  - Download the solution template (a zip archive) through the Moodle course room.
  - Unzip the archive and populate the `assignmentXX/MEMBERS.txt` file. The names and student ids listed in this file **must match** your moodle group **exactly**.
  - Complete the solution.
  - Prepare a new zip archive containing your solution. It must contain exactly the files that you changed. **Only change the files you are explicitly asked to change in the task description.** The directory layout must be the same as in the archive you downloaded. (At the very least it must contain the `assignmentXX/MEMBERS.txt`.)
  - One team member uploads the zip archive through Moodle before the deadline, using the group submission feature.
  - Your solution must compile and run correctly **on our lab computers** by only inserting your **assignment.cc** and **shader files** into the Project. If it does not compile on our machines, you will receive no points. If in doubt you can test compilation in the virtual machine provided on our website.
- Instructions for **text assignments**:
  - Prepare your solution as a single pdf file per group. Submissions on paper will not be accepted.
  - If you write your solution by hand, write neatly! Anything we cannot decipher will receive zero points. No exceptions!
  - Add the names and student ID numbers of all team members to every pdf.
  - Unless explicitly asked otherwise, always justify your answer.
  - Be concise!
  - Submit your solution via Moodle, together with your coding submission.

Exercise 1 Projective Geometry **[24 Points]**

Consider the following sketch of a 3D scene with the camera located at the origin $(0, 0, 0)^\top$. The image plane (IP) is defined by the normal vector $\mathbf{n}$ with $||\mathbf{n}|| = 1$ and the focal distance δ. The point $\mathbf{p}'$ is the projection of $\mathbf{p}$ onto the image plane.



(a) Projection Matrix **[6 Points]**

Assume that $\mathbf{n} = (0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})^\top$. Use the theorem of similar triangles to **derive** the projection matrix $\mathbf{P} \in \mathbb{R}^{4\times4}$ that maps $\mathbf{p}$ to $\mathbf{p}'$ depending on δ. Both points are expressed in homogeneous coordinates, i.e. $\mathbf{p} \in \mathbb{R}^4$ and $\mathbf{p}' \in \mathbb{R}^4$. Don't forget to explain your derivation.

(b) Vanishing Points Derivation **[6 Points]**

From now on, assume we picked a value for δ such that the projection matrix becomes

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix}.$$

Given a line $\mathbf{L}(\lambda) = \mathbf{o} + \lambda\mathbf{d}$, **derive** a formula for its vanishing point under the projection $\mathbf{P}$. Since we did not define a 2D coordinate system on the image plane yet, you can specify the vanishing point in 3D.
Hint: Note that we are not in the standard projection setting.

(c) Existence of Vanishing Points **[2 Points]**

In which case does a vanishing point exist in this setting? **Write down** a formula for this condition.

(d) Compute Vanishing Points **[4 Points]**

Consider the triangle spanned by $\mathbf{p}_0 = (-1, 2, 2)^\top$, $\mathbf{p}_1 = (-2, 4, 0)^\top$ and $\mathbf{p}_2 = (2, 0, -2)^\top$. **Compute** the vanishing points of its edges under $\mathbf{P}$, or argue why they do not exist. Again, you can specify the vanishing points as 3D points on the image plane. The normal vector of the im is still given as $\mathbf{n} = (0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})^\top$.

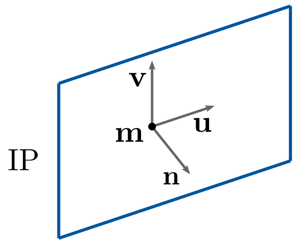
(e) Geometric Construction of Vanishing Points **[2 Points]**

As an alternative to the formula you derived in part (b), briefly **describe** a geometric construction that also gives the vanishing point of the line $\mathbf{L}(\lambda) = \mathbf{o} + \lambda\mathbf{d}$ without using the projection matrix $\mathbf{P}$.

(f) Local Coordinate Systems **[4 Points]**

So far, we expressed points on the image plane in 3D. This is because in this exercise the viewing direction is not aligned with one of the standard axes, so we cannot simply drop one coordinate.

We are now given a 3D point $\mathbf{m}$ and two orthogonal 3D vectors $\mathbf{u}$ and $\mathbf{v}$, all lying within the image plane. In homogeneous coordinates, **derive** the matrix $\mathbf{M} \in \mathbb{R}^{3\times 4}$ that expresses a 3D point $(x, y, z, w)^\mathsf{T}$ on the image plane as a 2D point $(\alpha, \beta, w)^\mathsf{T}$ within the local coordinate system with the origin $\mathbf{m}$ and main axes $\mathbf{u}$ and $\mathbf{v}$.



You can assume, that $\mathbf{n}$, $\mathbf{u}$ and $\mathbf{v}$ form an orthonormal frame, i.e. $\mathbf{n}^\mathsf{T}\mathbf{u} = \mathbf{n}^\mathsf{T}\mathbf{v} = \mathbf{u}^\mathsf{T}\mathbf{v} = 0$ and $\mathbf{n}^\mathsf{T}\mathbf{n} = \mathbf{u}^\mathsf{T}\mathbf{u} = \mathbf{v}^\mathsf{T}\mathbf{v} = 1$.

Hints:

- Derive the translation matrix that moves $\mathbf{m}$ to the origin.
- Derive a **linear** basis transformation from the standard basis to the basis formed by $\mathbf{u}, \mathbf{v}, \mathbf{n}$.
- Derive the matrix that drops the coordinate in $\mathbf{n}$ direction.

Don't forget to explain your derivation.

Exercise 2 Transformations and Normals **[12 Points]**

Consider a triangle with the three vertices $\mathbf{p_1} = (8, -10, 2)^T$, $\mathbf{p_2} = (-4, -10, -2)^T$ and $\mathbf{p_3} = (6, 4, 6)^T$. The triangle is first rotated by $45°$ around the y-axis, then scaled by $(1, 4, 16)$ (in x-, y-, and z-direction) and finally translated by 10 units along the x-axis.

(a) **[4 Point]**

Derive the transformation matrix $\mathbf{M} \in \mathbb{R}^{4\times 4}$ that can be used to transform the triangle.
Hint: $\cos(45°) = \sin(45°) = 1/\sqrt{2}$

(b) **[2 Point]**

As seen in the lecture, every 3D affine transformation can be represented by a 4×4 matrix using extended coordinates. For which kinds of three-dimensional affine transformations can the corresponding transformation matrices be used as-is (without any modifications) to also transform normals? You can assume that normals are explicitly normalized after they are transformed. Explain your answer.

(c) **[6 Point]**

Derive a matrix that correctly transforms the normal of the triangle. Compute the normal of the triangle before and after the transformation. You have to normalize again after the transformation. When normalizing, you can use a calculator and round to three decimal digits.