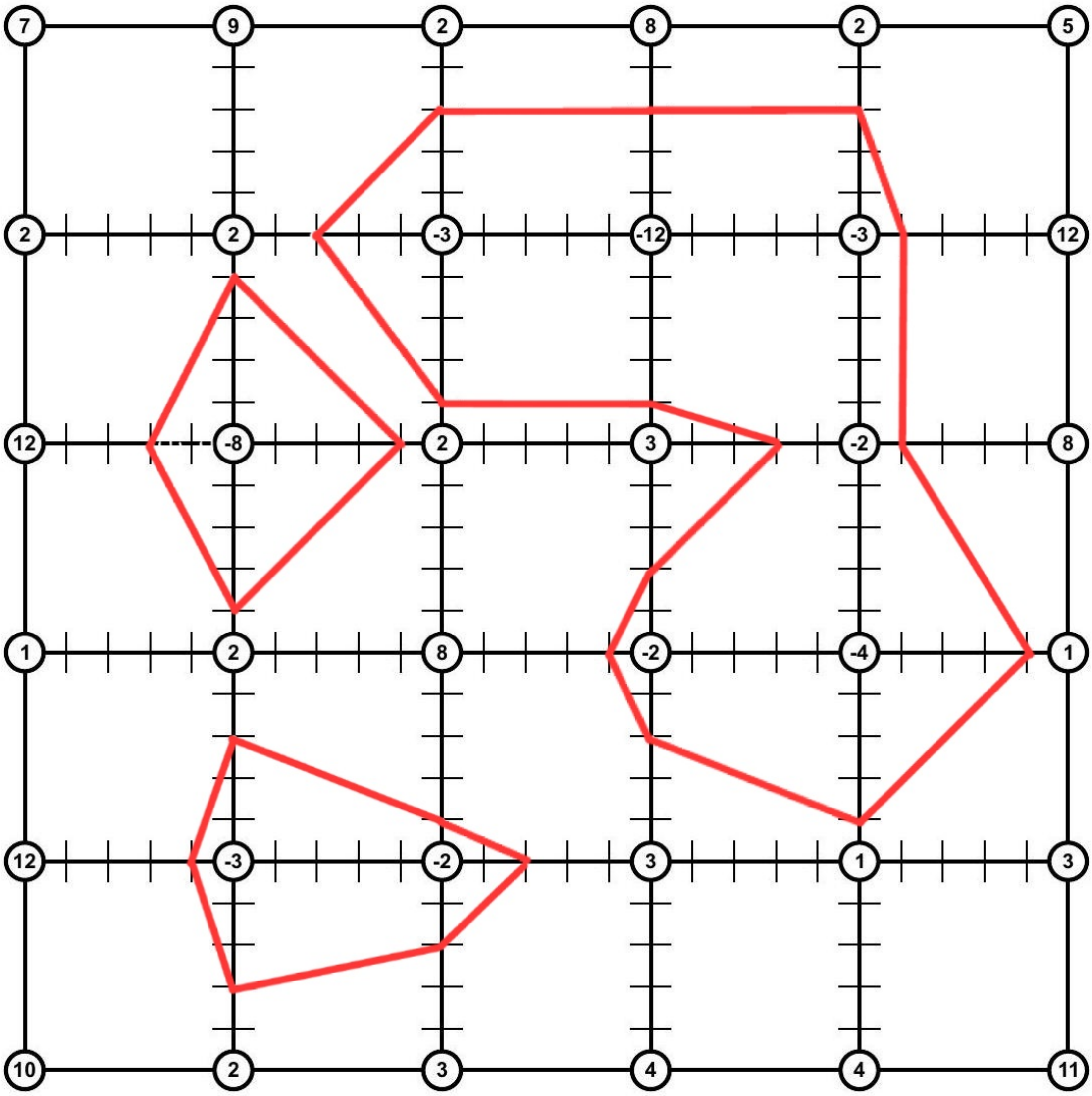
Assignment 10

Basic Techniques in Computer Graphics
WS 2022/2023
January 17, 2023

Frederik Muthers	412831
Tobias Broeckmann	378764
Debabrata Ghosh	441275
Martin Gäbele	380434

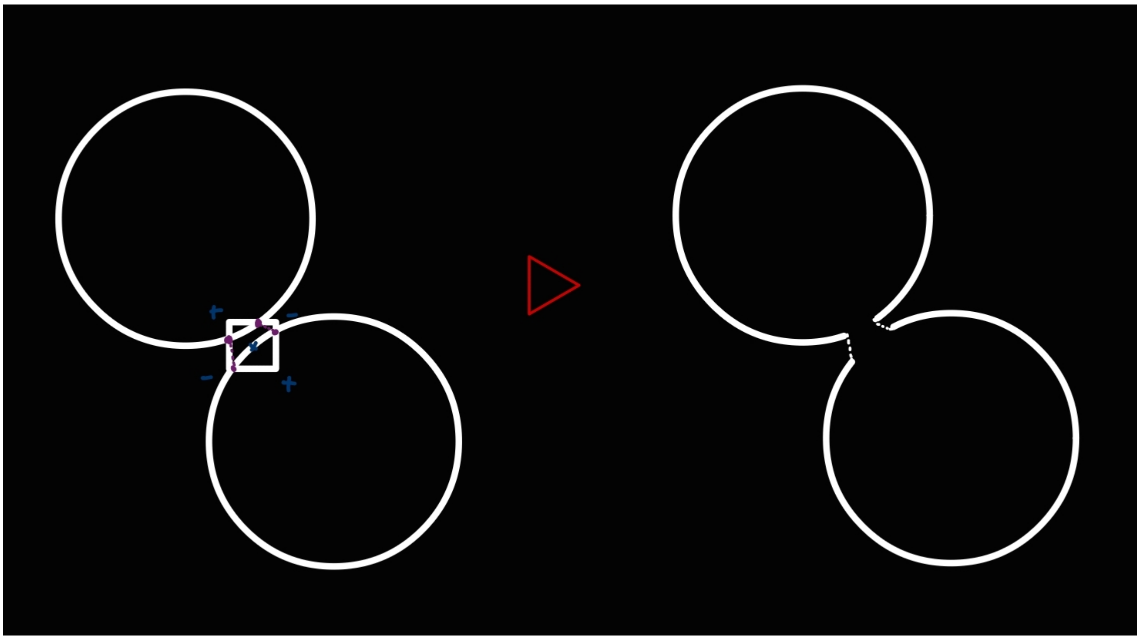
Exercise 1 Indirect Rendering of Implicit and Volumetric Geometry

(a) Surface Extraction Executing the Marching Squares algorithm (the 2D equivalent of MC) on the discrete implicit data given we extract a zero-level representation as shown below:



The get the geometry correct along with the topology, linear interpolation has been used to find the intersection points.

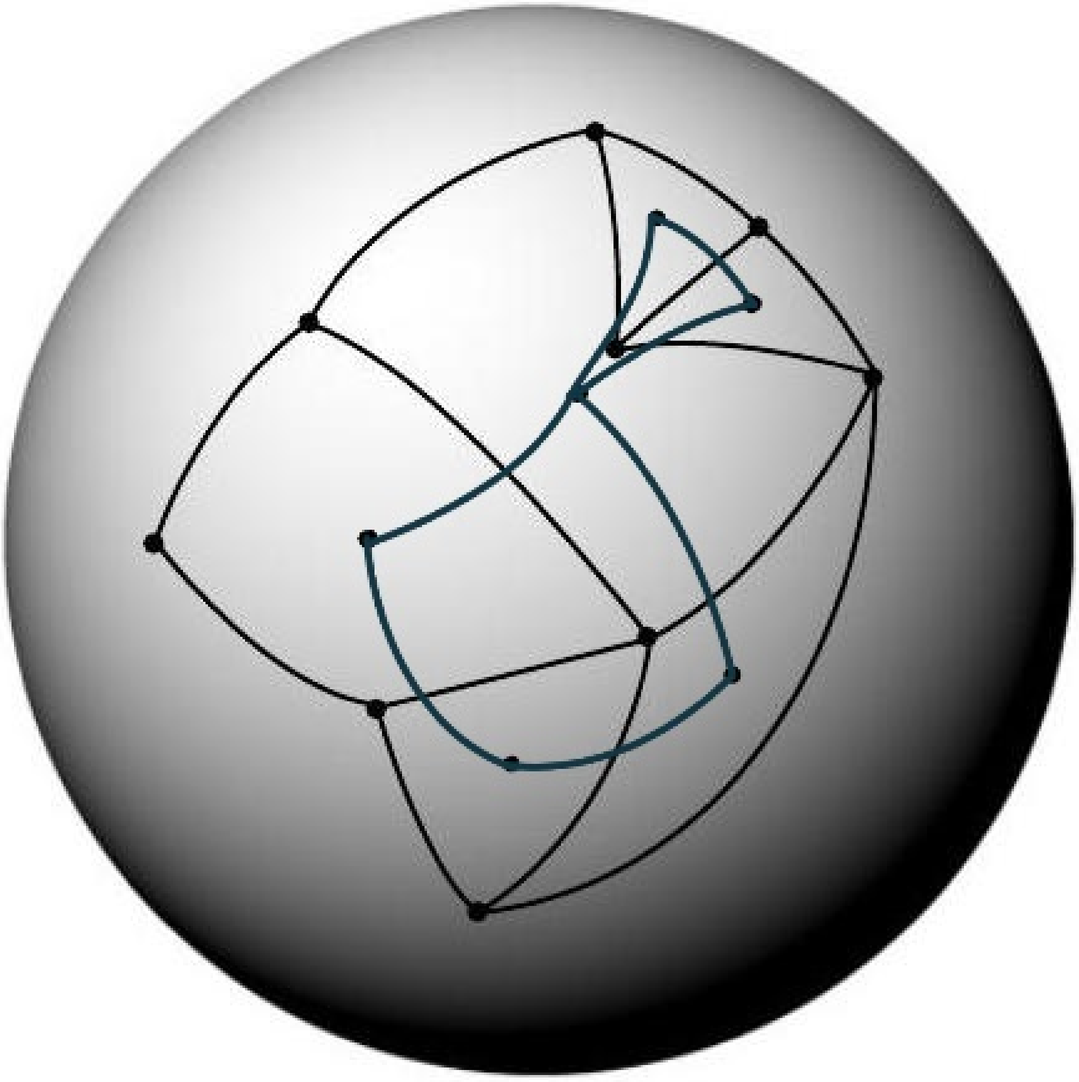
(b) Topological Equivalence The zero-level representation extracted from the Marching Squares algorithm might not always be topologically equivalent to the original implicitly defined object depending on the methods used to handle ambiguous cases. If we take the sign of square-center to determine the way we draw the resulting line segments connecting the intersection points, we might run into cases like in the following figure:



In this scene, the square center lies inside one of the circles and we get a zero-level representation using the positive value of the square center which is topologically different from the original objects (we get a connected curve rather than two disjoint circles). But if we find a way to better decide the way we handle ambiguity, the marching squares algorithm can be perfectly aligned topologically with the original implicitly defined object.

Exercise 2 Meshes

(a) The edges of the dual mesh are in dark blue color.

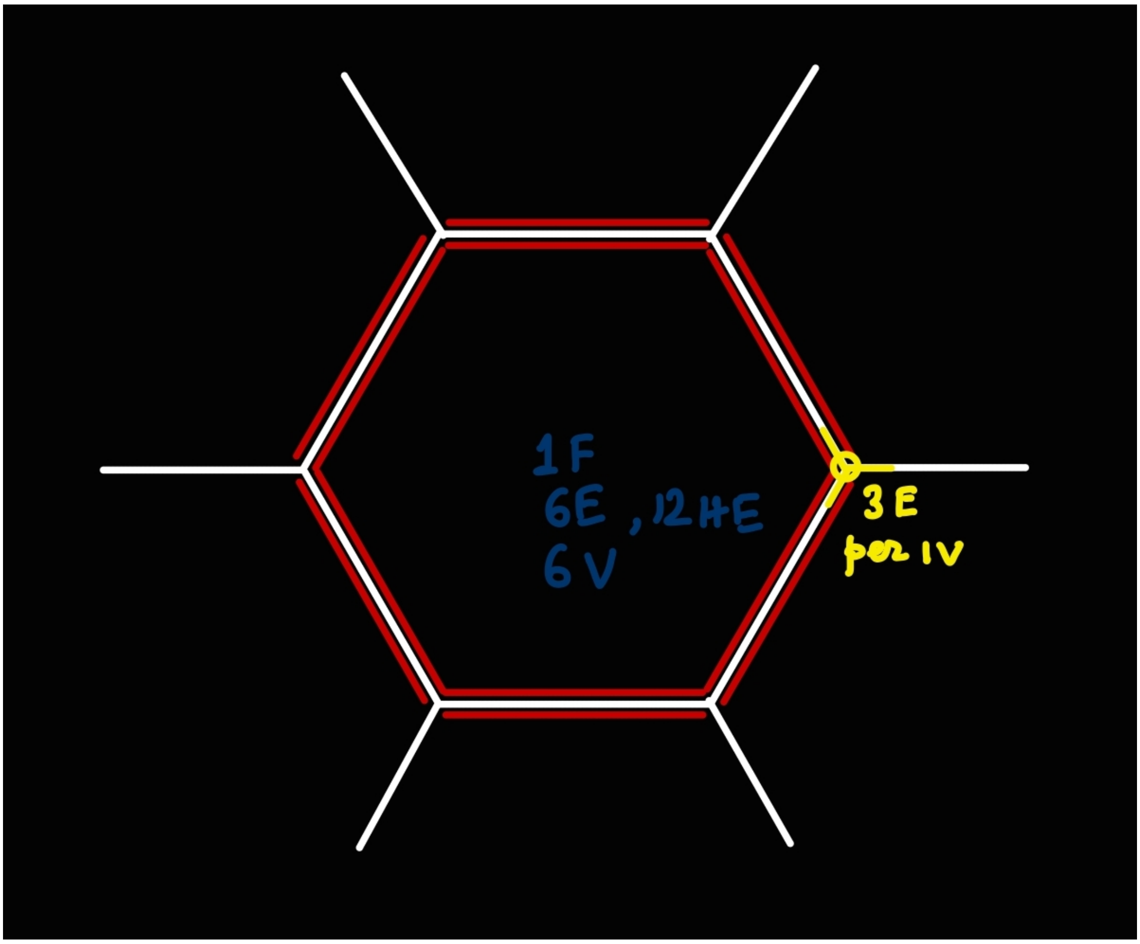


(b) We know that the Euler formula not only holds for triangle meshes but also for arbitrary polygon meshes. The usual Euler formula with number of vertices V, number of edges E and number of faces F in a mesh is given by

$$V - E + F = 2(1 - g), \text{ where g = genus}$$

Now, for a closed hexagon mesh of genus 1, we have

$$V - E + F = 0 \tag{1}$$



As seen in the lecture, we can again consider each edge as a pair of half edges (HEs) since every edge can belong to 2 adjacent hexagons with each hexagon associated with one of the 2 half edges. As we can see from the figure above each edge (in white) is associated with 2 half-edges (in red color), we have:

$$HE = 2E \tag{2}$$

We can also see that each hexagon is then associated with 6 half edges meaning each face possesses 6 HEs on average which gives us:

$$HE = 6F \tag{3}$$

From (2) and (3) we have:

$$2E = 6F \implies E = 3F \tag{4}$$

Firstly, we can multiply both sides of (1) by 6 and use equation (2) to get:

$$6V - 3HE + 6F = 0 \tag{5}$$

Now we use (3) in equation (5):

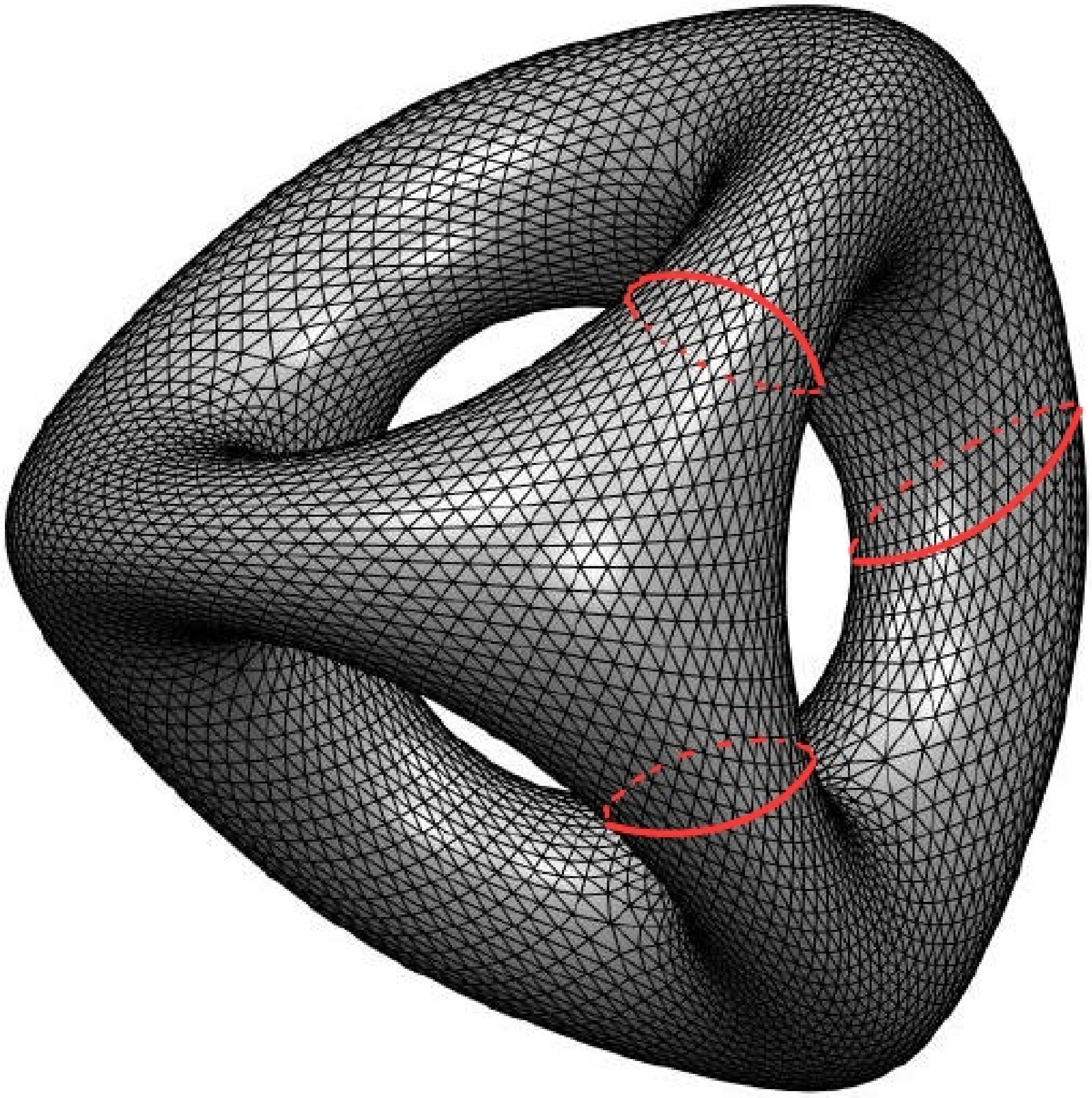
$$6V - 2HE = 0 \implies HE = 3V \tag{6}$$

So, we can see that each vertex is associated with 3 half edges and that means the average vertex valence of a closed hexagon mesh (of genus 1) is **3**.

Finally, using (4) in equation (1) we have:

$$V - 3F + F = 0 \implies V = 2F \tag{7}$$

(c)



We can use the definition of the genus as the maximum number of cuts possible through an object so that the object is still connected. In the figure above, we see that we can have a maximum of 3 cuts and any more cuts applied will split the object into pieces. So the genus of the above object is **3**.

Now, in the object shown in the figure above, the depicted closed triangle mesh has 19968 edges which is a large number compared to the genus and the term $2(1-g)$ in the Euler formula. So we can use the approximation used in the lecture:

$$V - E + F \approx 0$$

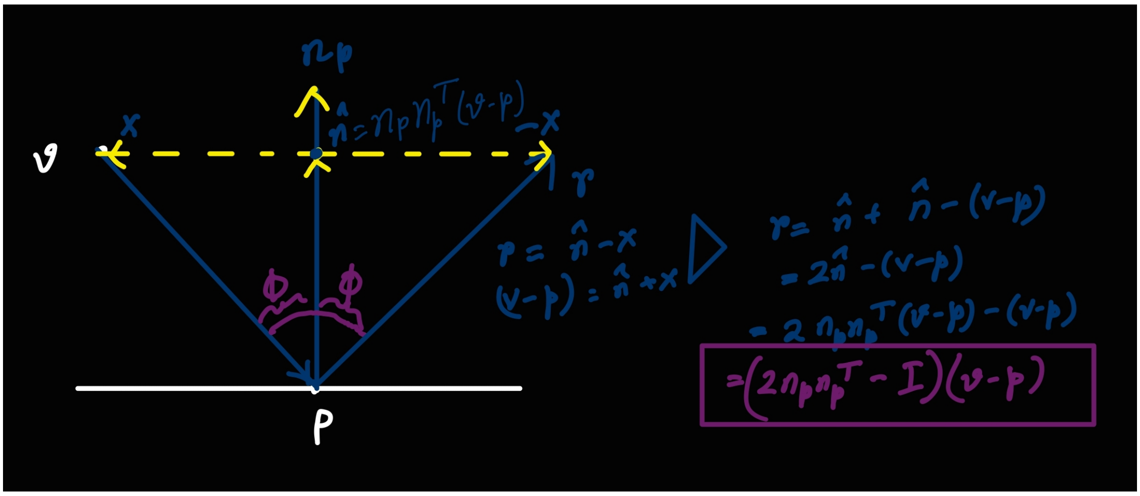
Using the above assumption, we already derived in class that for a triangle mesh: $2E = 3F$. which gives us, $2 \times 19968 = 3F \implies F = \frac{39936}{3} = 13312$.

We also derived that $F = 2V$. giving us, $13312 = 2V \implies V = \frac{13312}{2} = 6656$.

So the mesh above has got approximately 6656 vertices and 13312 triangles.

Exercise 3 Environment Mapping

(a)



For an object surrounded by a cube map, the viewer is positioned at $v = \begin{pmatrix} 2 \\ 1 \\ 8 \end{pmatrix}$ and looking at point $p = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$ with the normal $n_p = \begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{pmatrix}$ at that point. To calculate the reflected view vector r, we use the following formula as derived in the figure above:

$$
\begin{aligned}
r &= (2n_p n_p^\top - \mathbf{I_3})(v - p) \\
&= \left(2 \begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{pmatrix} \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \left(\begin{pmatrix} 2 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix} \right) \\
&= \left(2 \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix} \\
&= \left(\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) \begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix} \\
&= \begin{pmatrix} -4 \\ -1 \\ -2 \end{pmatrix}
\end{aligned}
$$

(b) Given a sufficiently large scene/room and relatively small reflection surface, the parallax effect is very small. This means that a change of the point on the reflection surface the camera points at while maintaining the same reflection vector only has a very small effect on the point that should be visible in the reflection. This means it is a sensible simplification to only consider the reflection vector and not the point on the reflection surface the viewer looks at.

(c) We have to consider the reflection vector $r = \begin{pmatrix} -4 \\ -1 \\ -2 \end{pmatrix}$ and select the largest component by absolute value. In this case that is $r_x = -4$ meaning the side of the cube we have to sample is negX.

(d) To calculate the texel coordinates we first compute $\begin{pmatrix} \frac{r_y}{r_x} \\ \frac{r_z}{r_x} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ \frac{1}{2} \end{pmatrix}$. The texel coordinates are then given by

$$\begin{pmatrix} (\frac{1}{4} + 1) \times 512 \\ (\frac{1}{2} + 1) \times 512 \end{pmatrix} = \begin{pmatrix} 512 + 128 \\ 512 + 256 \end{pmatrix} = \begin{pmatrix} 640 \\ 768 \end{pmatrix}$$

In total the texel we need to access is at pixel coordinates $\begin{pmatrix} 640 \\ 768 \end{pmatrix}$ on the negX side of our cube-map.