Assignment 5

Basic Techniques in Computer Graphics
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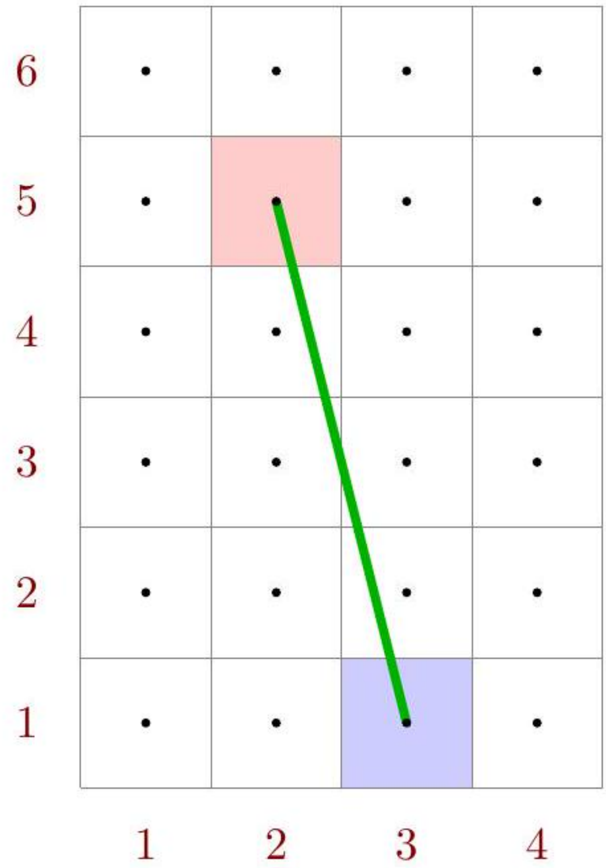
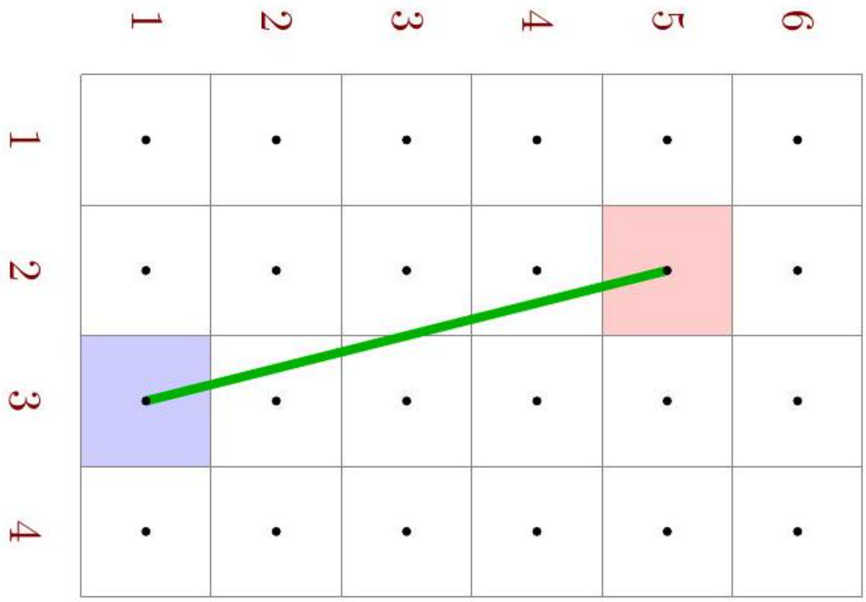
Exercise 1 Line Rasterization



(a) Case Reduction



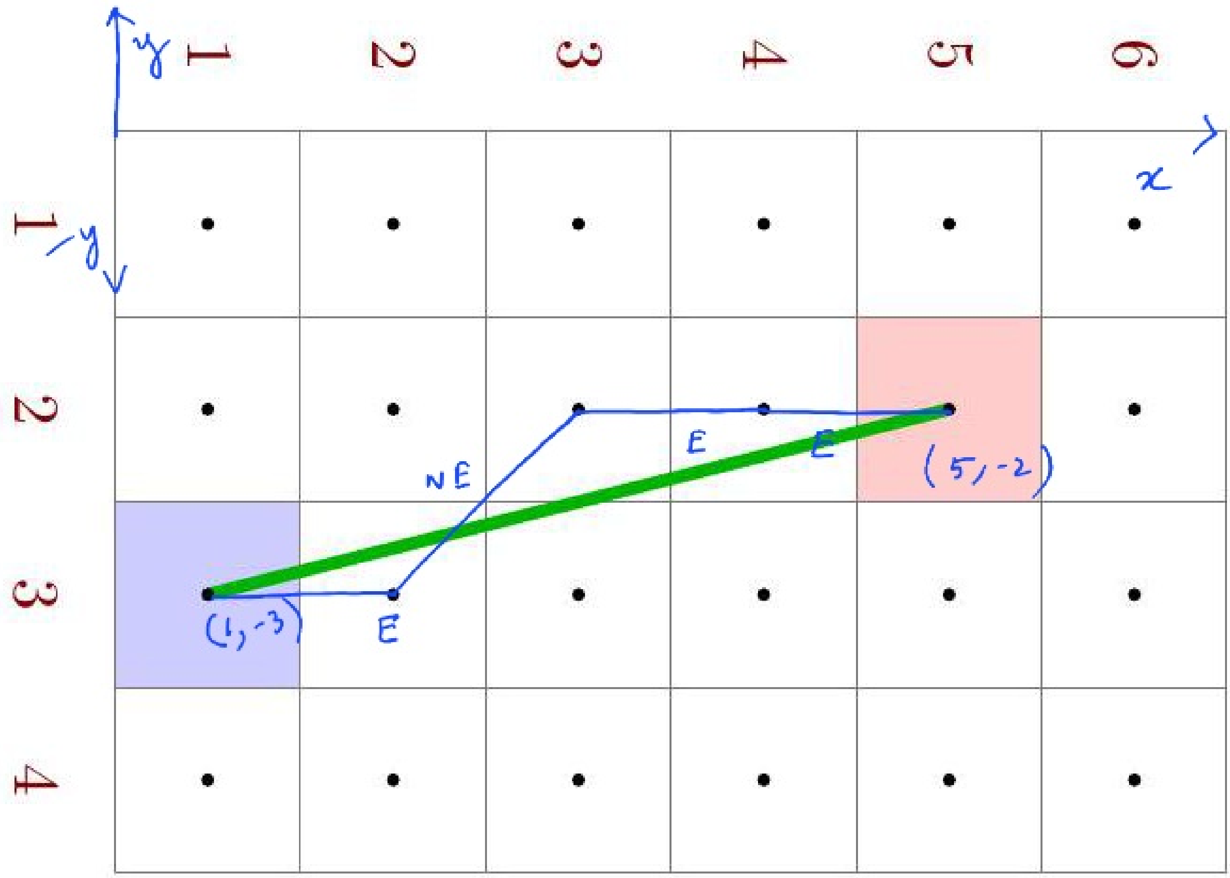
In the original co-ordinate system, the line segment starts at $x_0 = (3,1)^\top$ and ends at $x_1 = (2,5)^\top$. We need to specify the map $(x,y)^\top \to (\dots,\dots)^\top$ that transforms the line into a setting with slope $m \in [0,1]$. If we flip the x and y coordinates we get the new coordinates of the points as: $x_0 = (1,3)^\top$ and $x_1 = (5,2)^\top$. Now we change the sign of the new y coordinate to get: $x_0 = (1,-3)^\top$ and $x_1 = (5,-2)^\top$ with $m = \frac{-2-(-3)}{5-1} = \frac{1}{4} \in [0,1]$. So our required map is: $(x,y)^\top \to (y,-x)^\top$ which gave us $x_0 = (1,-3)^\top$ and $x_1 = (5,-2)^\top$.

(b) Bresenham Algorithm Using the new mapping in part (a), we get: $\Delta x = 5 - 1 = 4$ and $\Delta y = -2 - (-3) = 1$. So, the initial value of $d = 2 * \Delta y - \Delta x = 2 * 1 - 4 = -2$, $\Delta_E = 2 * \Delta y = 2 * 1 = 2$ and $\Delta_{NE} = 2 * \Delta y - 2 * \Delta x = 2 * 1 - 2 * 4 = -6$.

We now use the formula that if $d < 0$, we go East(E) and increase x by 1, d by Δ_E and else we go North-East(NE) and increase both x and y by 1 and d by Δ_{NE} to fill the table given as:

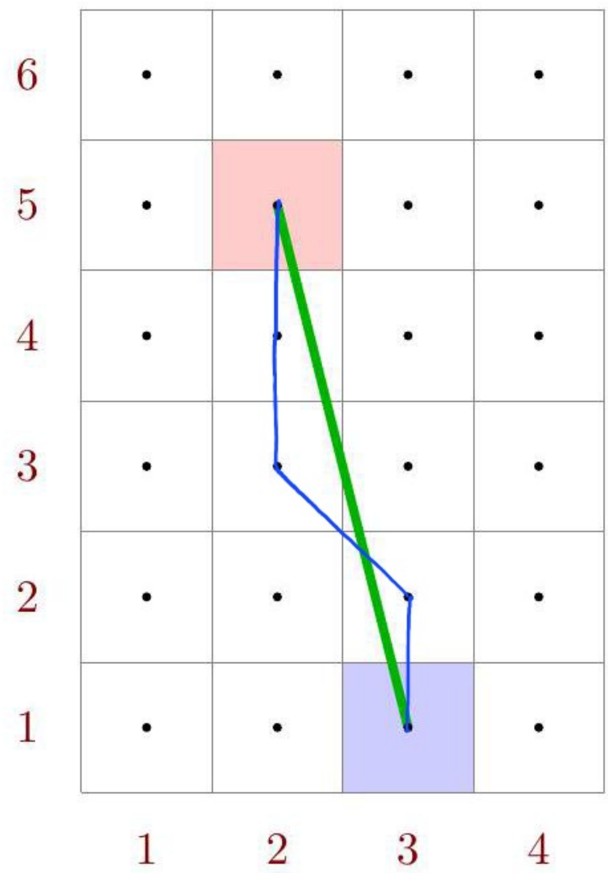
Step	x	y	d	decision (E or NE)
0	1	-3	-2	E
1	2	-3	0	NE
2	3	-2	-6	E
3	4	-2	-4	E
4	5	-2	-2	End

In our transformed coordinate system applying Bresenham algorithm gives us the following figure:



Now if we apply the inverse transformation $((y, -x)^\top \to (x, y)^\top$ or $(x, y)^\top \to (-y, x))$ from part (a) to each point we get the following coordinates and figure as per the original:

Step	x	y	d	decision (E or NE)
0	3	1	-2	E
1	3	2	0	NE
2	2	3	-6	E
3	2	4	-4	E
4	2	5	-2	End


6
5
4
3
2
1
1
2
3
4

Exercise 2 Rasterization of Quadratic Polynomials

(a) Decision Variable Similar to the linear case we can compute $d(x, y)$ as follows:

$$d(x, y) = F(x + 1, y + 0.5)$$

We can write $F(x, y)$ as $F(x, y) = ax^2 + bx + c - y$. We need to choose $F(x, y) = f(x) - y$ as $0 \leq f'(x) \leq 1 \forall x \in \{x_0, .., x_0 + n\}$. Thus we get a formula for $d(x, y)$:

$$d(x, y) = F(x + 1, y + 0, 5) = ax^2 + 2ax + a + bx + b + c - y - 0.5$$

Similar to the linear case, we decide to go east (E) if $d < 0$ and northeast (NE) otherwise.

(b) Decision Variable Updates For $\Delta_E(x, y)$ we calculate:

$$\Delta_E(x, y) = d(x + 1, y) - d(x, y) = F(x + 2, y + 0.5) - F(x + 1, y + 0.5)$$

$$= ax^2 + 4ax + 4a + bx + 2b + c - y - 0.5 - (ax^2 + 2ax + a + bx + b + c - y - 0.5)$$

$$= ax^2 - ax^2 + 4ax - 2ax + bx - bx + 4a - a + 2b - b + c - c - y + y - 0.5 + 0.5$$

$$= 2ax + 3a + b$$

For $\Delta_{NE}(x, y)$ we see:

$$\Delta_{NE}(x, y) = F(x + 2, y + 1.5) - F(x + 1, y + 0.5) = \Delta_E(x, y) - 1 = 2ax + 3a + b - 1$$