NEPR 209 HW 3: Probability

Please submit your write-up as a PDF, either LaTeXed or handwritten and scanned.

Exercise 1: COVID tests and Bayes rule

You tested positive for COVID-19! How worried should you be? The probability of a positive test given that you have the disease (true positive) is 0.95, and the probability of a negative test given that you don't have the disease (true negative) is 0.98. Luckily, the prevalence rate of COVID-19 at Stanford currently is about 1 in 100. relatively lower than the California state prevalence rate of 15 in 100. What are the chances that you actually have COVID-19 while at Stanford? How does that change if you consider your risk as equaling the prevalence rate of California?

(Note: The true positive and true negative rates here were somewhat arbitrarily chosen, and may not reflect those of the tests available at Stanford. The prevalence rates were pulled from the Stanford COVID-19 Dashboard for the week of 1/31/2022.)

Exercise 2: Inter-spike Intervals and Beyond

Consider a neuron whose mean firing rate is μ Hz. Let's assume that the spikes of the neuron follow a Poisson process. Then, the duration before the first spike follows an exponential distribution:

$$\Pr(T_1 > t; \mu) = e^{-\mu t}.$$

(Note: recall from lecture that we can't characterize $\Pr(T_1 = t)$, because the probability that a continuous random variable is exactly equal to a value is 0. So instead, we can only characterize T_1 as being greater than t.)

(Note 2: $\Pr(T_1 > t; \mu)$ is not a probability density function. Its complement, $\Pr(T_1 \le t) = 1 - \Pr(T_1 > t)$ is the cumulative density function.)

(Note 3: Because events in a Poisson process are independent from one another, one event does not influence the timing of the next event. Therefore, T_1 is not only the the duration before the first event but also the duration between any two events, i.e. inter-spike interval.)

What if we are interested in T_1, T_2, ..., T_k (i.e. the duration before k spikes)? T_k follows a *gamma* distribution with parameters k and μ.

Derive $\Pr(T_k > t; k, \mu)$ of the gamma distribution. Explain your reasoning.

Hint: your answer may contain a sum notation.

Exercise 3: Communication through Noisy Neurons

A signal encoded in spikes at discrete time steps are sent through a series of connected neurons and their synapses. Unfortunately, the neurons are a little bit noisy and do not transmit the signal with perfect fidelity. Let's represent the signal as being composed of symbols of 0s and 1s (absence and presence of spikes). At each time point, there is no spike (symbol 0) with probability p and there is a spike (symbol 1) with probability $1-p$. Each symbol is transmitted incorrectly with probability ϵ_0 for symbol 0 (0 sent, 1 received) and ϵ_1 for symbol 1 (1 sent, 0 received). Each time step is independent from other time steps.

3(a)

What is the probability that a symbol (either 0 or 1) is transmitted correctly through the neurons?

3(b)

What is the probability that the message 1001 is transmitted correctly through the neurons?

3(c)

To improve reliability, the source neuron repeats each symbol three times and the received message is decoded by majority rule. In other words, a 0 is sent as 000 and a 1 is sent as 111, and it is decoded correctly if the received string of three letters has at least two 0s and two 1s, respectively. What is the probability that a 0 is correctly decoded?

3(d)

Suppose the three-symbol scheme is used. What is the probability that a symbol was 0 given that the received string is 101?

Exercise 4: Working with Discrete Representations of Probability Distributions (Programming)

Please find the Colab notebook exercise on the course website.