

Lecture 3 sensory encoding

How does the nervous system represent sensory information?

What stimuli drive neurons, let's call them sensory 'features'?

How can a neural code be decoded?

How are the set of important sensory features allocated to neurons?

How are mechanisms used to construct a sensory representation?

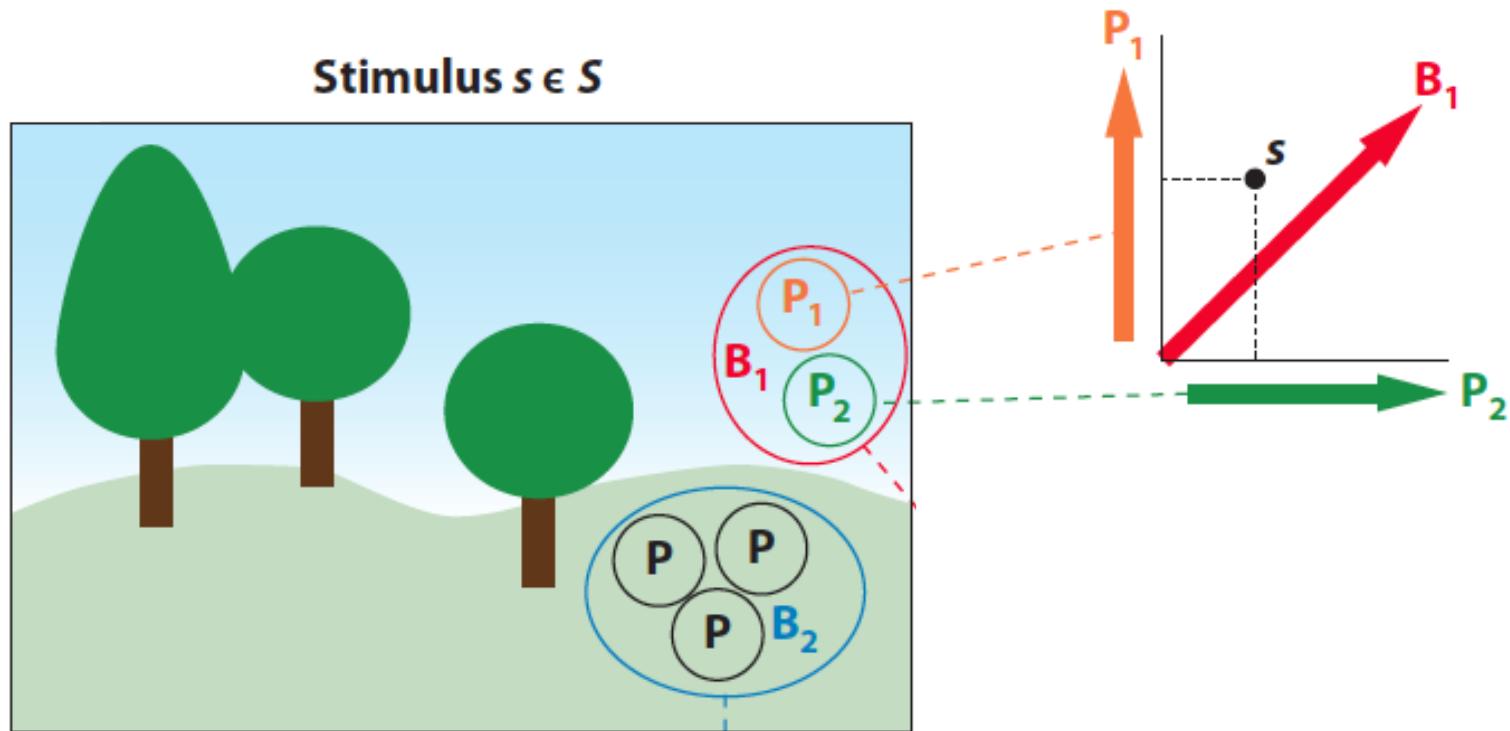
How do properties of sensory neurons influence behavior?

Why is a particular neural code advantageous (or optimal)?

Why are particular mechanisms advantageous?

What design principles influence sensory systems, can they be used for artificial systems?

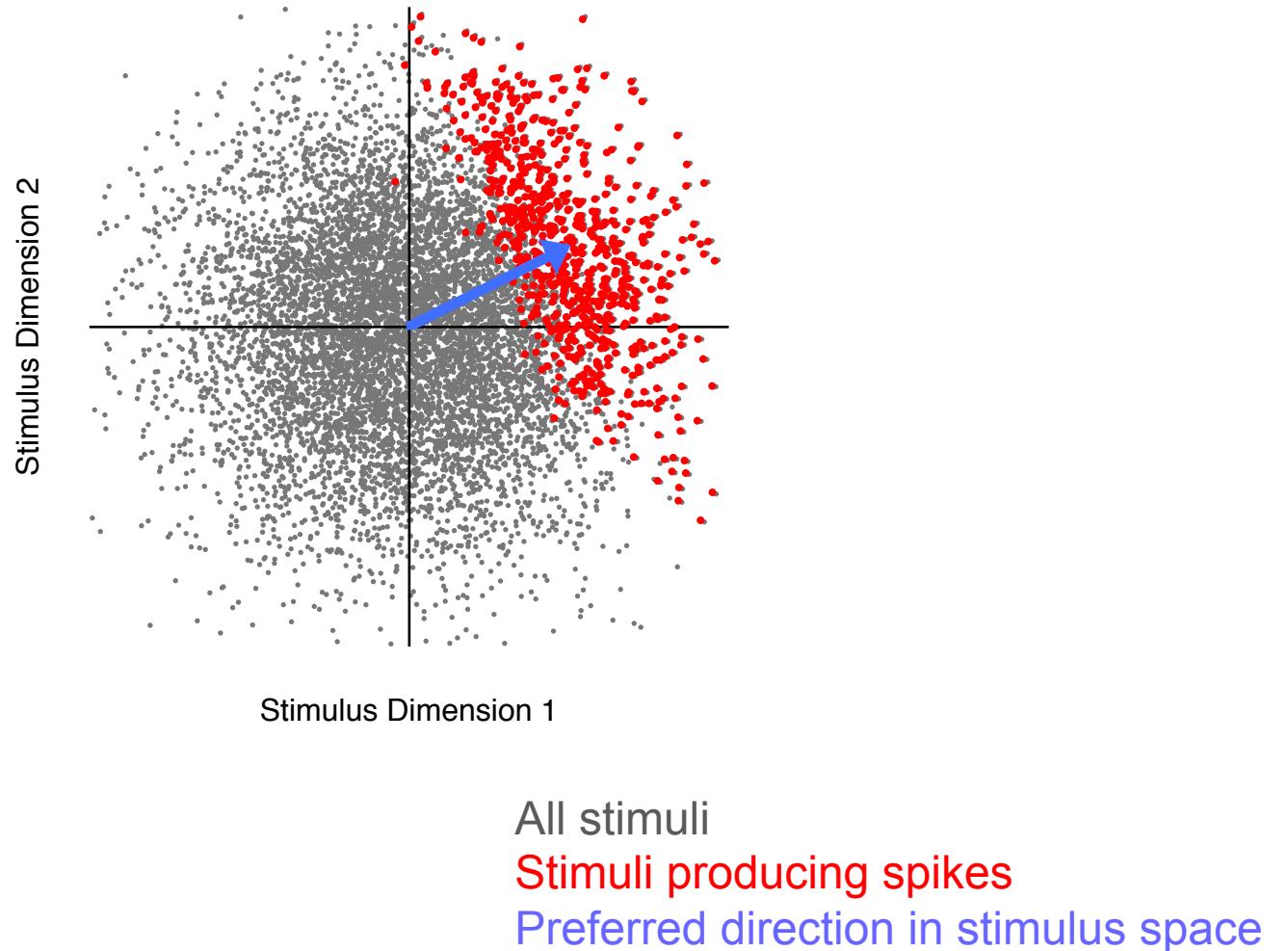
Stimulus space: A geometrical representation of input



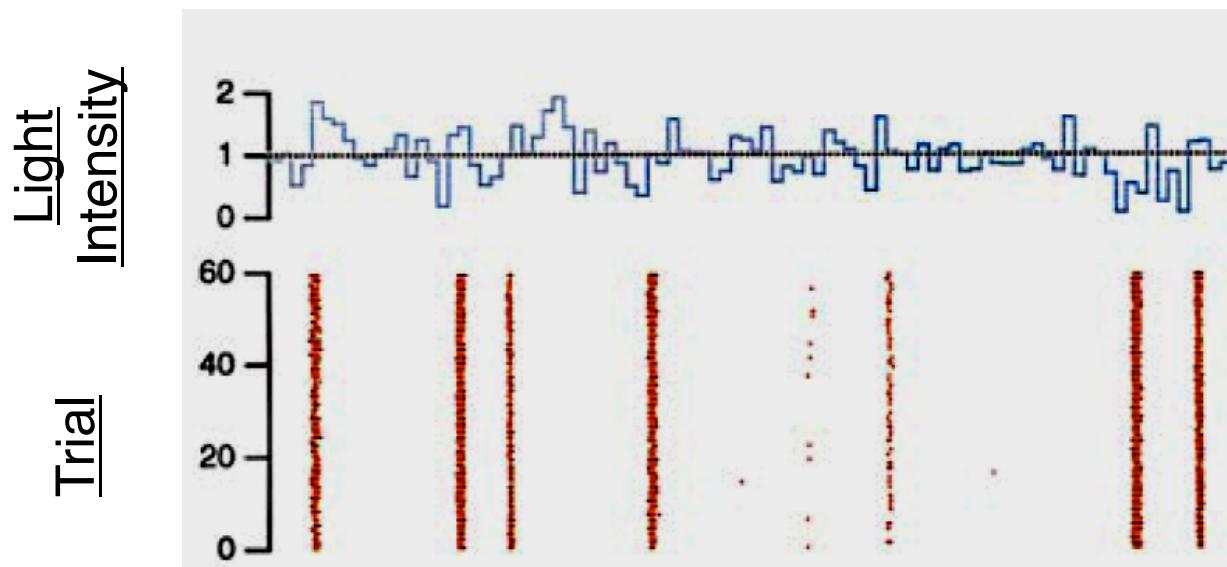
Not the same as “visual space”:
receptors are *dimensions*, not points

Stimulus space: A geometrical representation of input

Receptive field: a direction of sensitivity

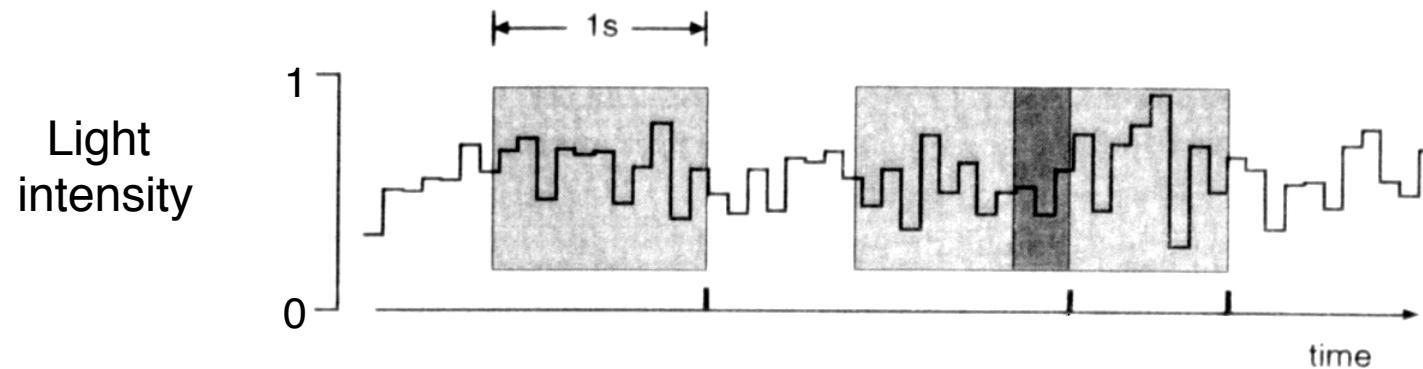


Receptive field: What stimuli makes the neuron respond?



Receptive field: What stimuli makes the neuron respond?

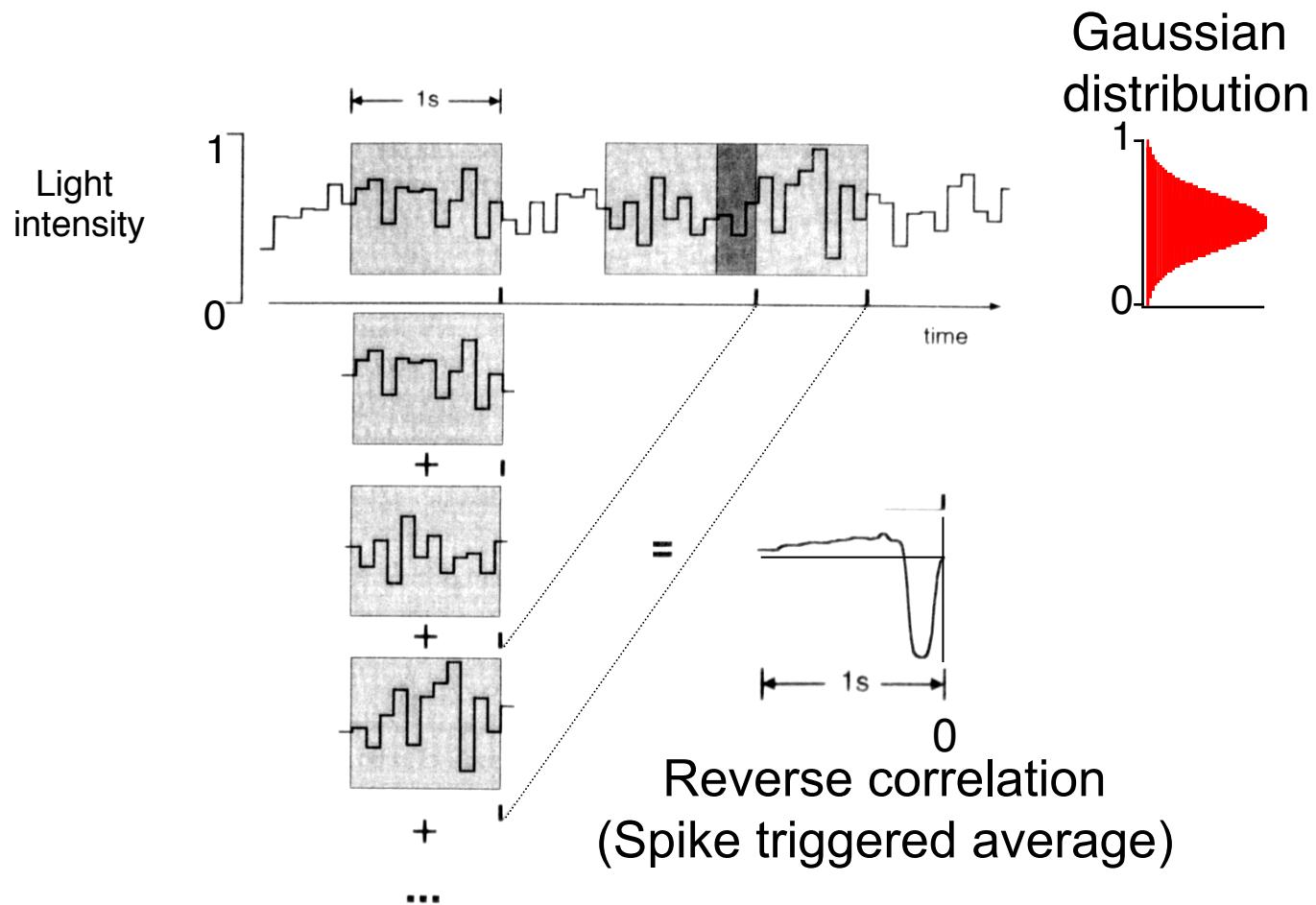
Reverse correlation for a spiking neuron



What stimulus makes the neuron respond?

White noise analysis

For a Gaussian white-noise stimulus, the reverse correlation gives the input that is transmitted with greatest sensitivity



What stimulus makes the neuron respond?

Reverse correlation

Continuous

$$C(\tau) = \int_0^T r(t)s(t - \tau)dt$$

Discrete

$$C(\tau) = \sum_{t=0}^T r(t)s(t - \tau)\Delta t$$

For spikes, this is proportional to the spike-triggered average

These equations give a mathematical definition for reverse correlation. If we think about signals as being continuous, then it is appropriate to use the integral. However in modern neuroscience, all continuous signals are sampled and digitized, and then integrals are computed by summing over digitized values. Even if you take a photograph on a film camera, the journal will digitize when they publish it. Thus it is appropriate to consider the ‘discrete’ form of this equation, where the summation (Σ) sign is used. This is also easier to translate into something like a computer program or spreadsheet.

$r(t)$: the response as a function of time

$s(t)$: the stimulus as a function of time

T: The length of the experiment

Δt : How much points are separated in time, for example 1 s, or 1 ms.

τ : The time delay of the stimulus relative to the response. When

$$t=0, \text{ the equation becomes: } C(0) = \sum_{t=0}^T r(t)s(t)\Delta t$$

In other words, for $C(0)$, the response, $r(t)$, is multiplied by the stimulus, $s(t)$ that occurs at the same time for every time point. All results are then summed. Unless there is an instantaneous effect between $s(t)$ and $r(t)$, this $C(0)$ should be close to zero.

$$\text{Likewise, when } \tau = 1, \text{ the equation is: } C(1) = \sum_{t=0}^T r(t)s(t-1)\Delta t$$

In this case the response at time t , $r(t)$, is multiplied with the stimulus that occurred one time point in the past, $s(t-1)$, for every time point. In other words, the stimulus is shifted one time point before multiplying. If the stimulus caused the response with a delay of one time point, there will likely be a positive or negative correlation.

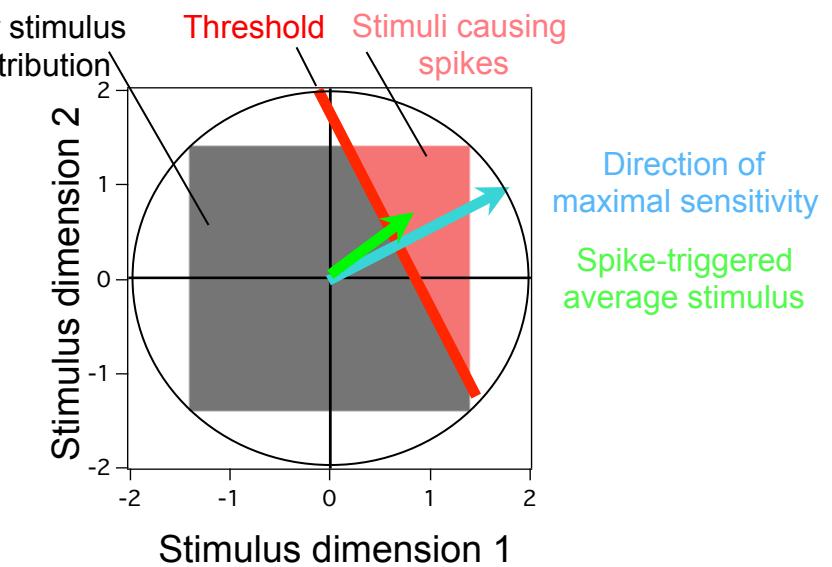
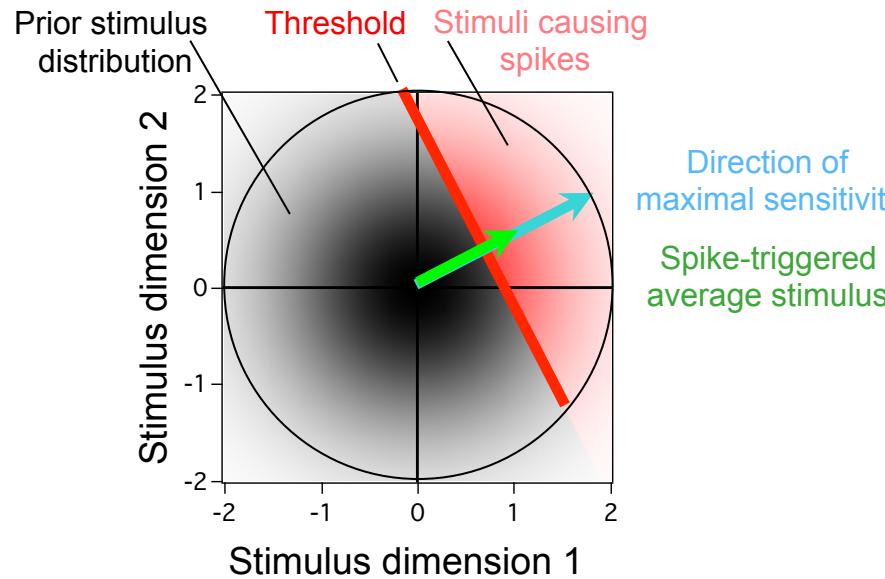
$$\text{When } \tau = -1, \text{ the equation is: } C(-1) = \sum_{t=0}^T r(t)s(t+1)\Delta t$$

If there is no correlation between the stimulus at one point of time and another point in time, the response should not be correlated with the stimulus in the future, and so the reverse correlation $C(\tau)$ for $\tau < 0$ should be close to zero. This is a good reality check on ones experiments and analysis.

Why correlation?

A correlation can be thought of as an average over many products. If two signals $x(t)$ and $y(t)$ are positively correlated, then $x(t)$ will be positive when $y(t)$ is positive, and $x(t)$ multiplied by $y(t)$ will be > 0 . If $x(t)$ and $y(t)$ are unrelated, then on average, $x(t)$ multiplied by $y(t)$ will be zero. If $x(t)$ and $y(t)$ are negatively correlated, $x(t)$ multiplied by $y(t)$ will be < 0 .

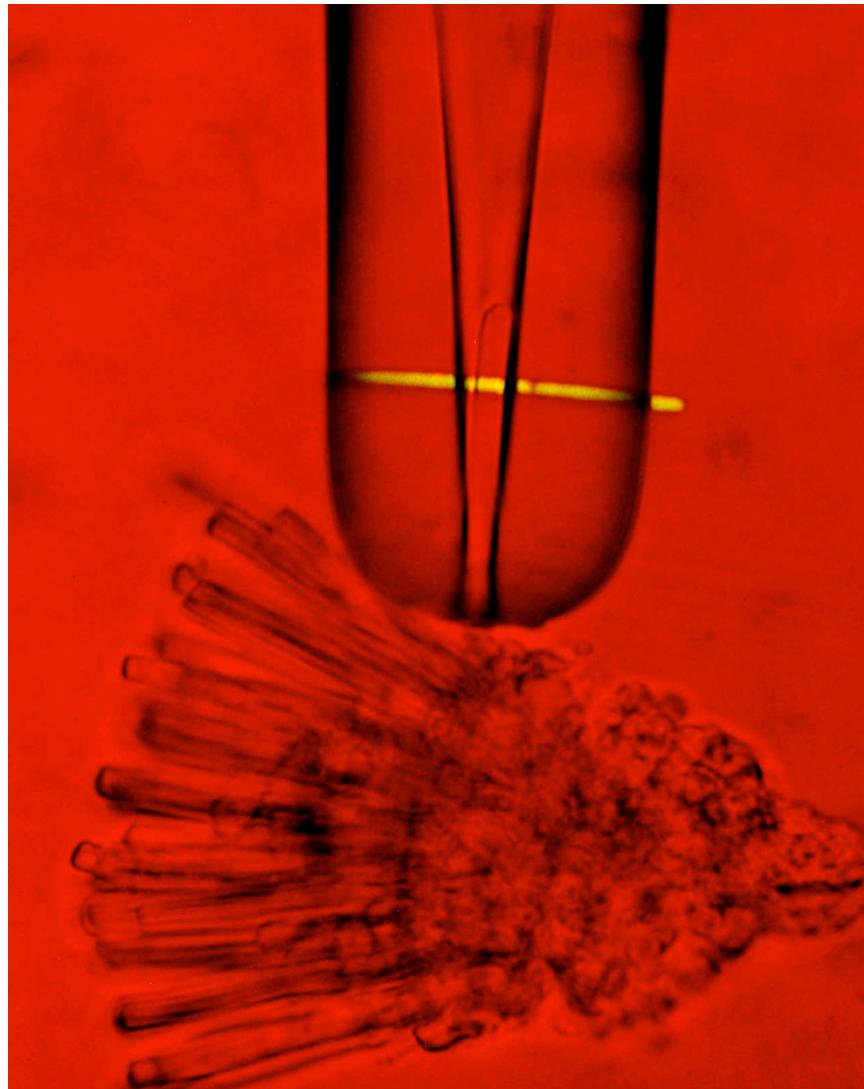
Computing the most effective stimulus (Bussgang's theorem)



If an input is Gaussian white noise, correlation of the input with the output yields the most effective input , even if there is a distortion of the signal' s amplitude such as a threshold or saturation.

Bussgang (1952)

How will a neuron respond to a stimulus?

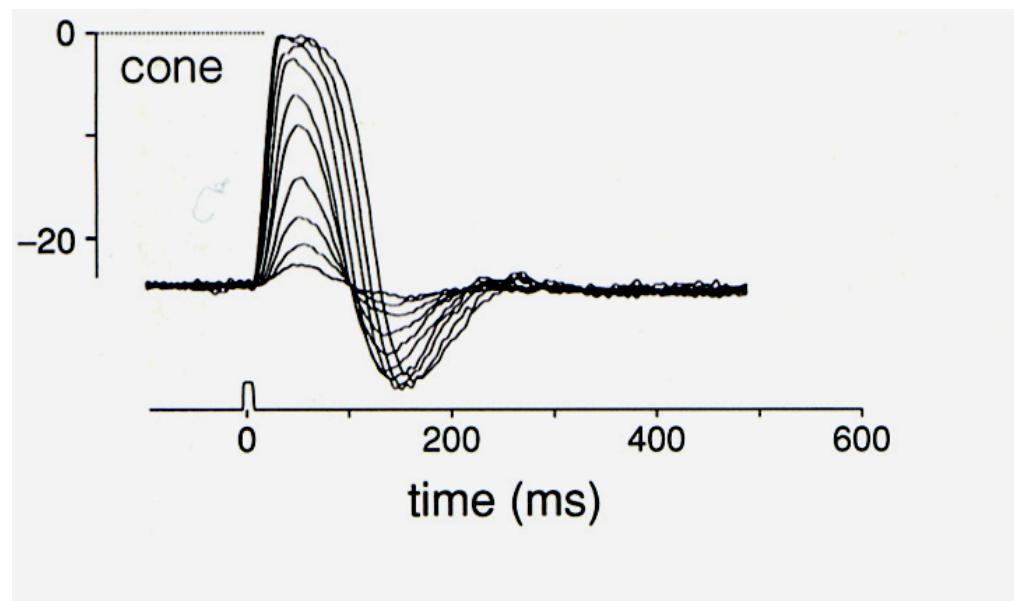


Baylor, Lamb and
Yau (1979)

How will a neuron respond to a stimulus?

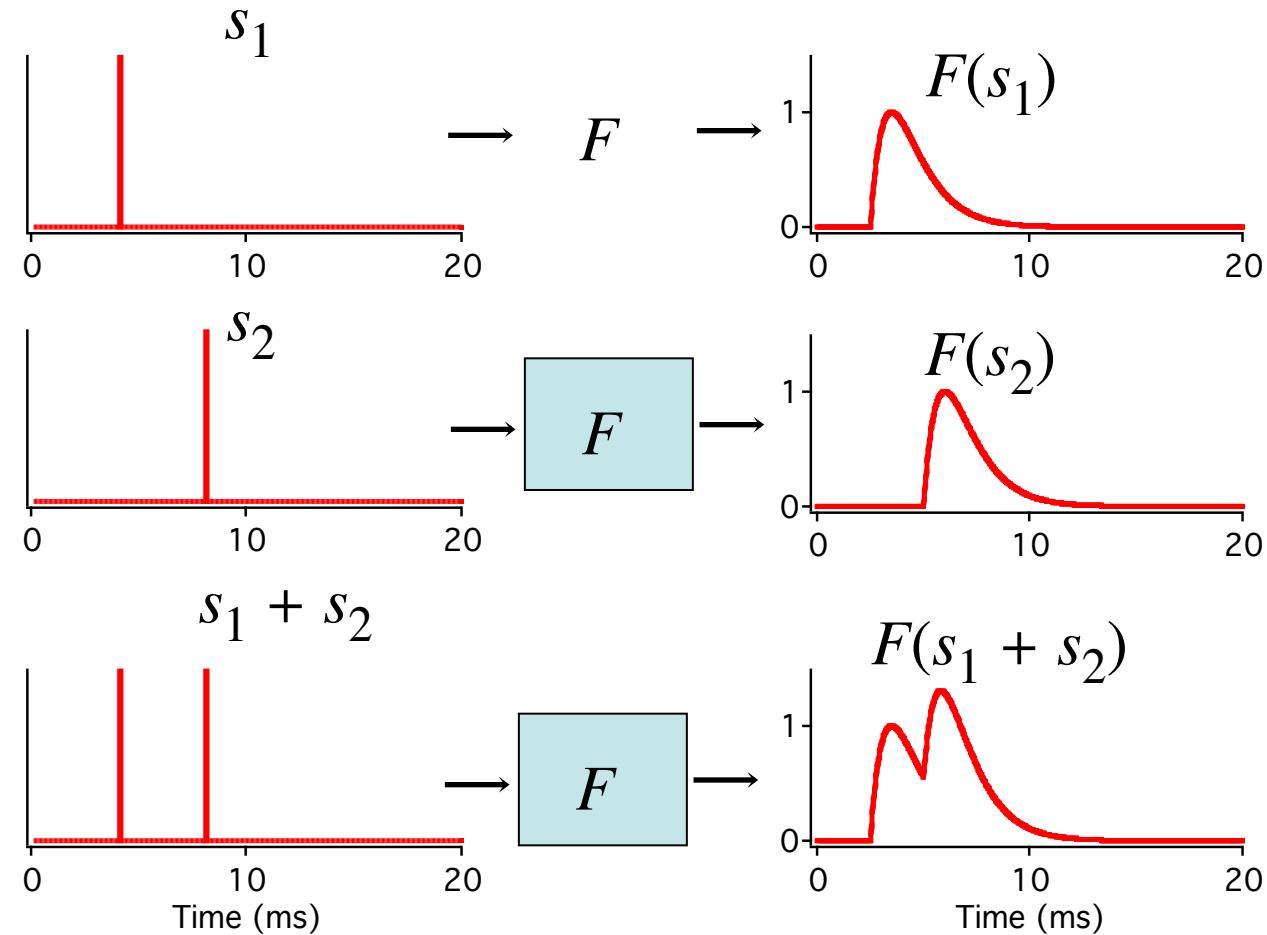
Single photoreceptor responses to a flash of light

Photocurrent
pA



Baylor (1987)

A linear system



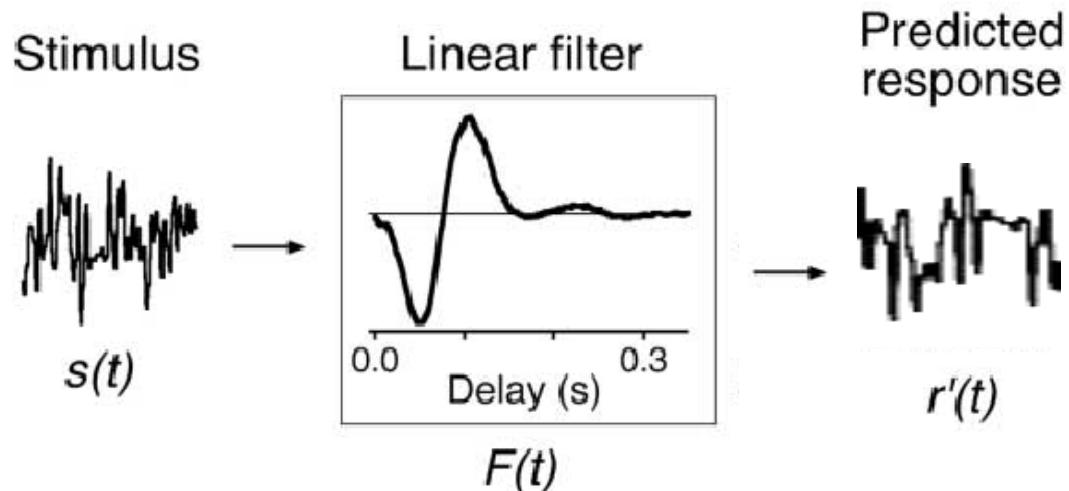
The Superposition Principle

When two inputs are presented to a linear system, their effects sum.

$$F(s_1 + s_2) = F(s_1) + F(s_2)$$

A linear model of the cell's response

A



Convolution

$$r'(t) = \int s(\tau)F(t - \tau)d\tau \quad r'(t) = \sum s(\tau)F(t - \tau)\Delta\tau \\ = F * S$$

$F(t)$ is called:

- Impulse response function
- Linear kernel
- First-order kernel
- First-order Wiener kernel

Like we did with the reverse correlation above, let's take the discrete convolution, it's easier to do calculations by hand. Here, we're considering that the length of the linear temporal filter is 1 second.

$$r'(t) = \sum_{\tau=t-1}^t s(\tau)F(t-\tau)\Delta\tau$$

t: time, the variable that indicates the time during the experiment, so if the experiment is 60 s long, t varies between 0 and 60 s.

τ : the summation variable that indicates time relative to the filter. Because the filter is 1 s long, τ varies between t-1 and t second.

$r'(t)$: predicted response at time t

$S(\tau)$: stimulus at time point τ . Note that since τ varies between t-1 and t, $s(\tau)$ is summed over the previous one second to compute $r'(t)$.

$F(t-\tau)$: The Filter at time $t-\tau$. Since τ varies between t-1 and t, $F(t-\tau)$ is actually summed between $F(1)$ and $F(0)$.

To compute $r'(t)$, the sum is taken over $\tau=t-1$ and $\tau=t$. Thus the sum is computed across :

$$s(t-1)F(t-(t-1)) = s(t-1)F(1)$$

and

$$s(t)F(t-t) = s(t)F(0)$$

So the stimulus is multiplied by *the time-reverse of the filter*. Then all points are summed. In this way, the filter weights the stimulus over the previous one second, according to the filter's time course. It's instructive to work out a very simple example. Let's say the stimulus is five time points long $s=(0,1,1,1,0)$, i.e. $s(0)=0, s(1)=s(2)=s(3)=1, s(4)=0$.

The filter is two time points long $(1,-1)$, i.e. $F(0)=1, F(1)=-1$. τ goes from t-1 to t. Then,

Stimulus	Filter reversed in time	Response
$s(0)=0$	$F(1)=-1$	
$s(1)=1$	$F(0)=1$	$r(1) = s(1)*F(0)+s(0)*F(1) = 1$
$s(2)=1$		$r(2) = s(2)*F(0)+s(1)*F(1) = 0$
$s(3)=1$		$r(3) = s(3)*F(0)+s(2)*F(1) = 0$
$s(4)=0$		$r(4) = s(4)*F(0)+s(3)*F(1) = -1$

Note that this filter has essentially taken the derivative of the stimulus, taking $(0,1,1,1,0)$ as an input, generating $(1,0,0,-1)$ as an output

Filters and step responses

Monophasic

Biphasic

Two different algorithms for convolution

Impulse response

Dot product with time-reverse of filter

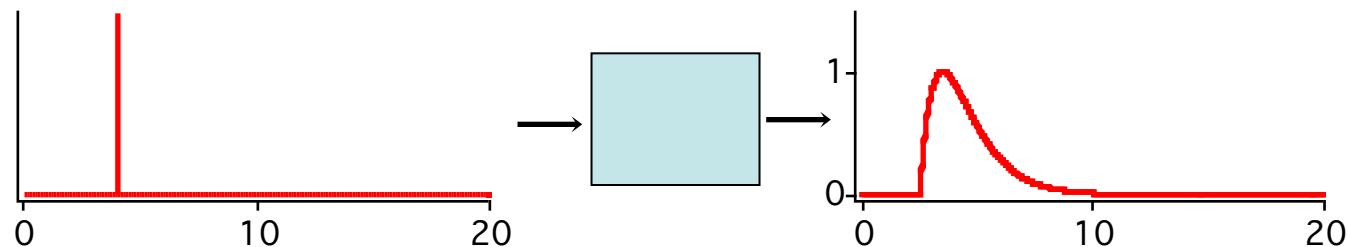
How do you find the filter?

What stimulus produces the largest response?

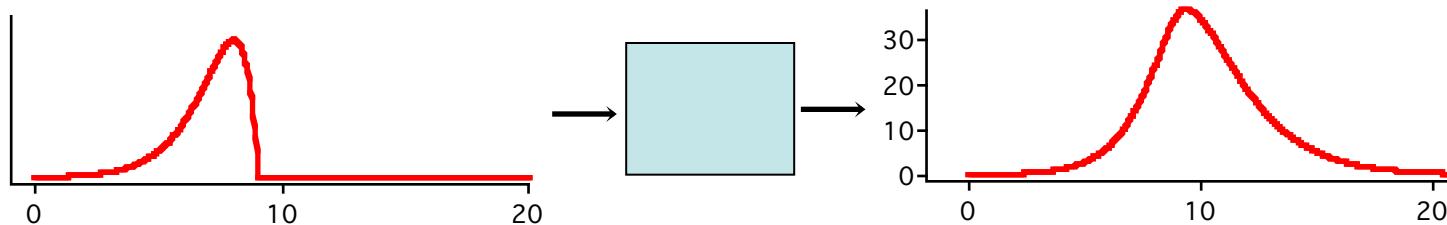
geometrical interpretation

Summary of relationship between the response to an impulse and the most effective input.

For a cell (or any system) responding in the linear range, the response to a brief input (an impulse):

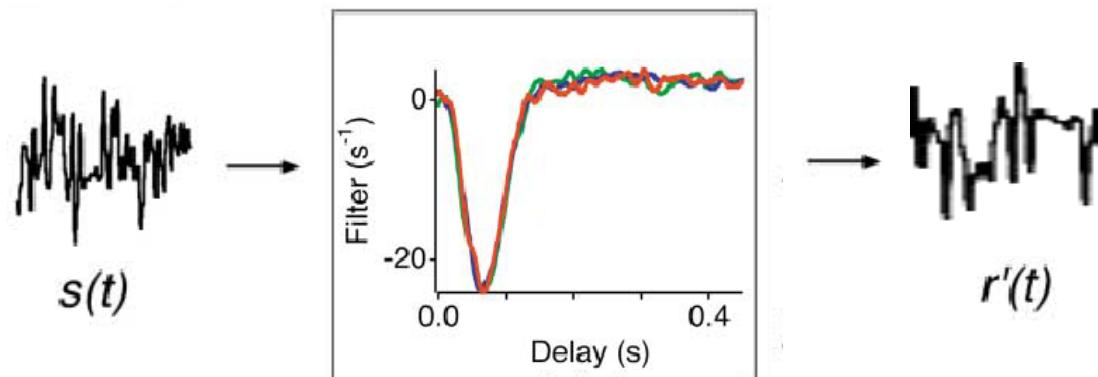


yields the time-reverse of the sequence of inputs that the cell is most sensitive to:

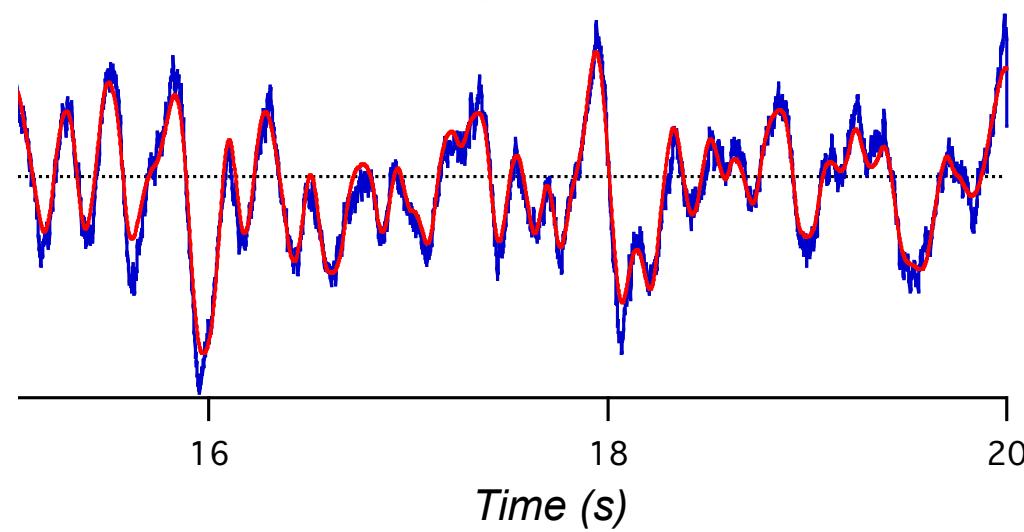


A linear model of the cell's response

Stimulus Linear filter Predicted response

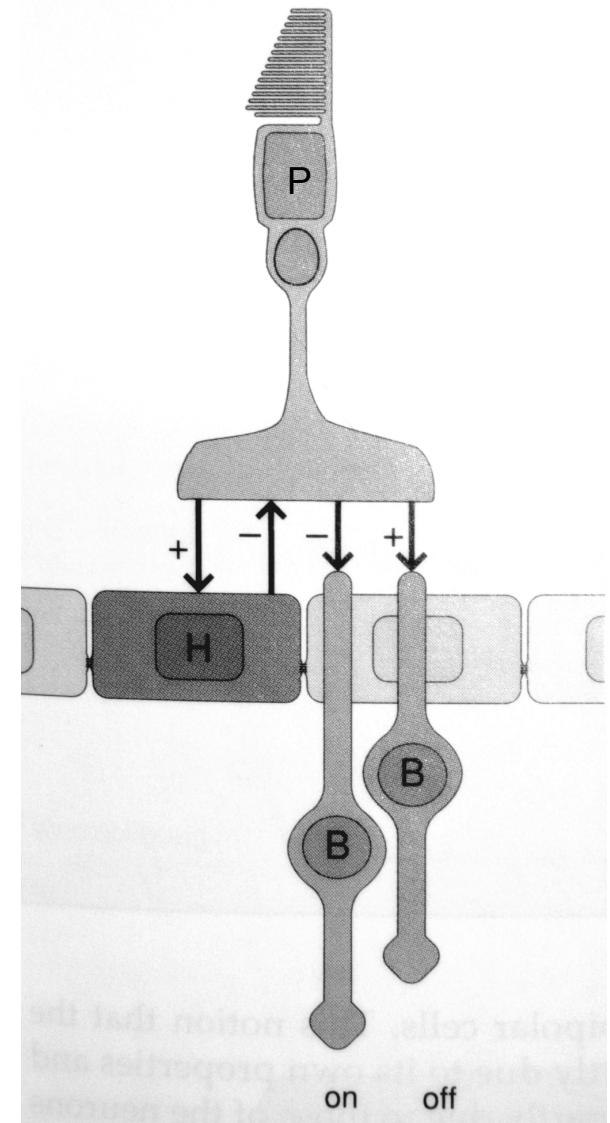
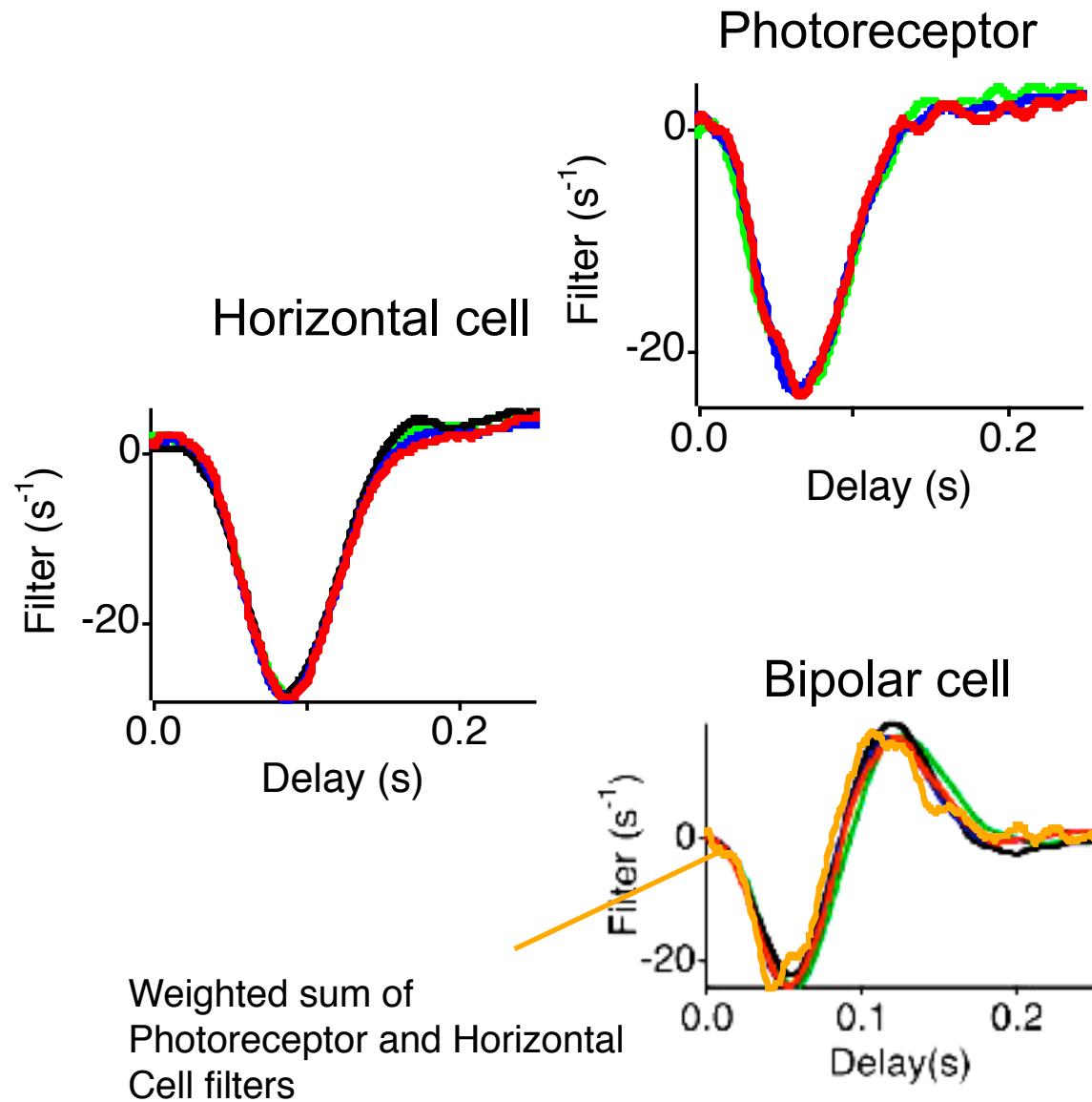


$F(t)$

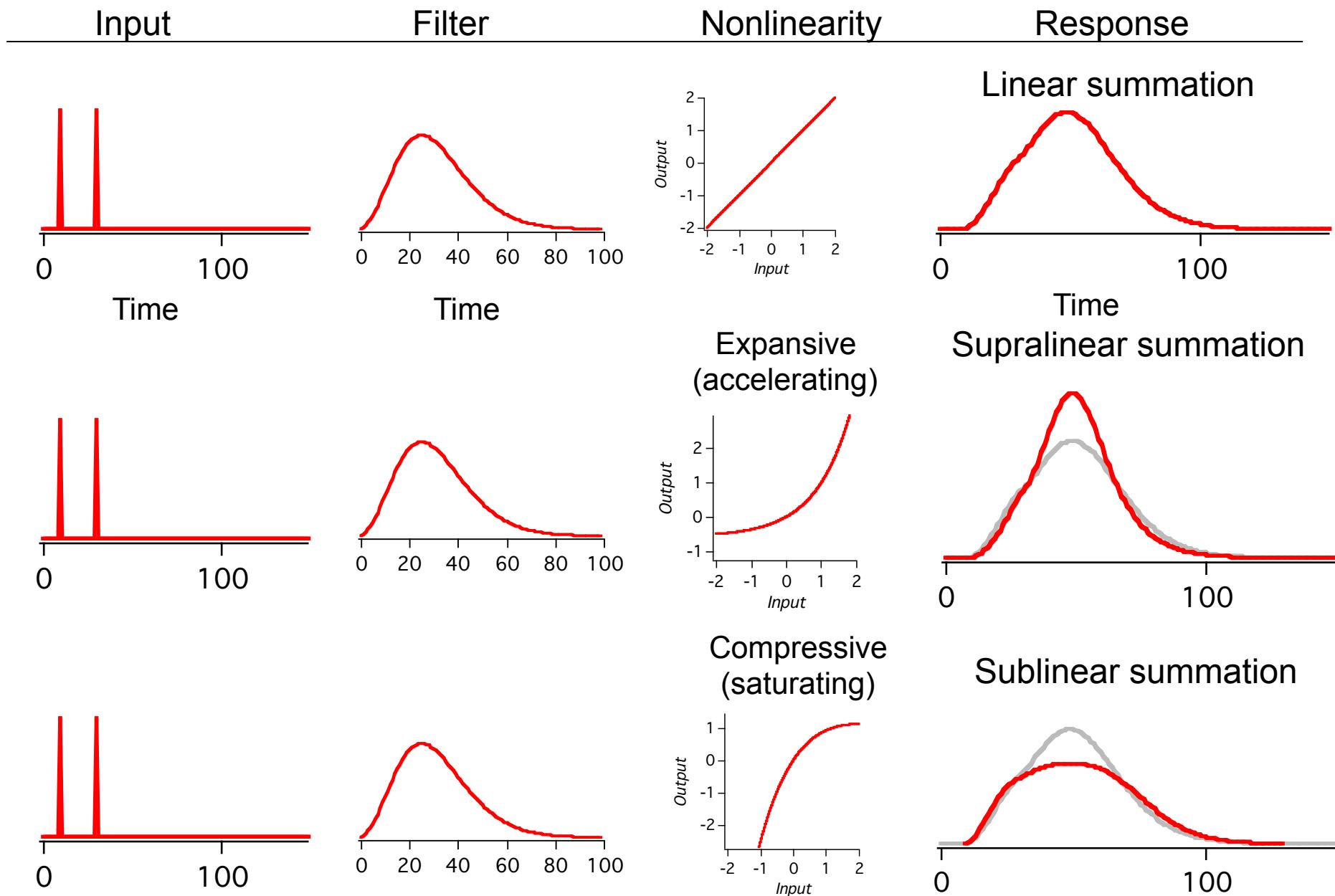


— Photoreceptor
— Linear model

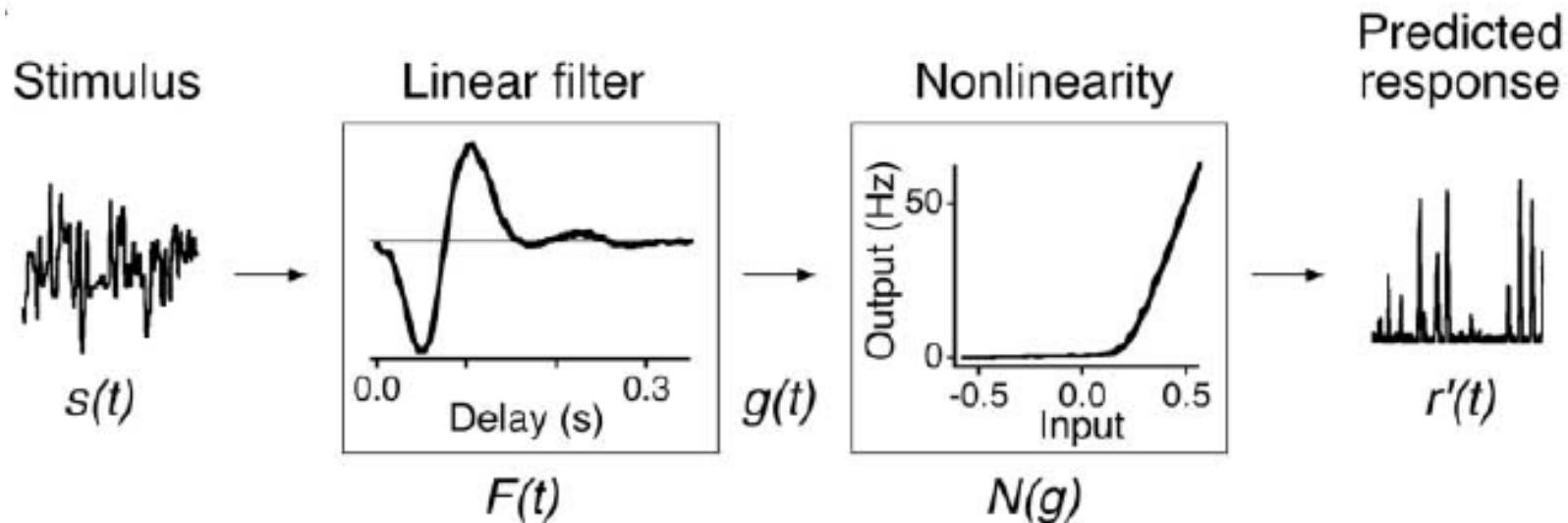
Linear filters in the early visual system



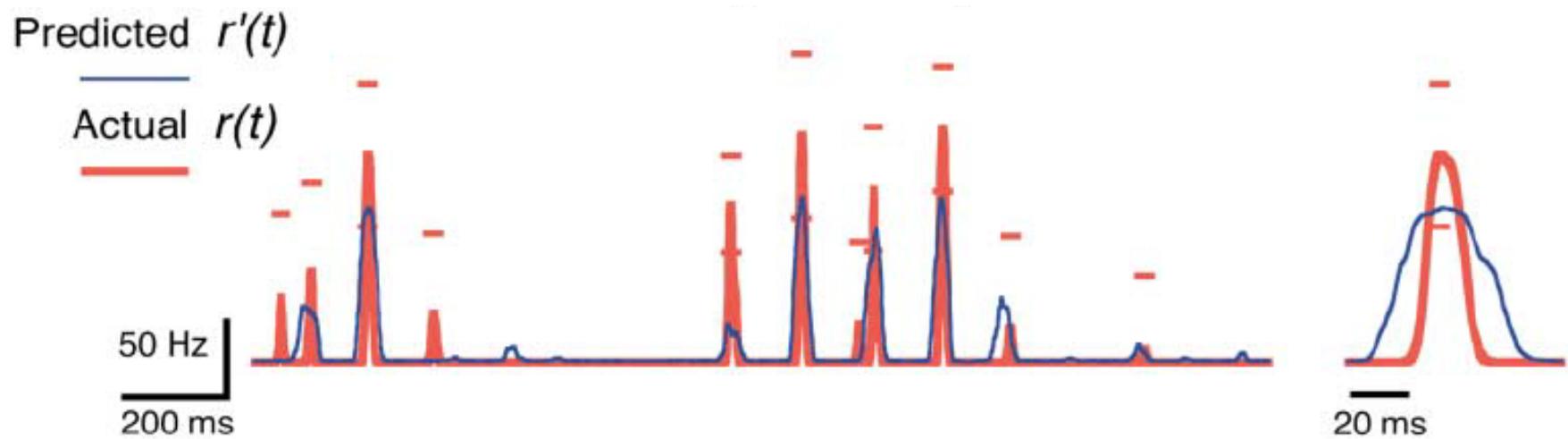
Nonlinear summation of inputs by a static nonlinearity



Linear - Nonlinear “LN” model of a “Feature Selective Neuron”



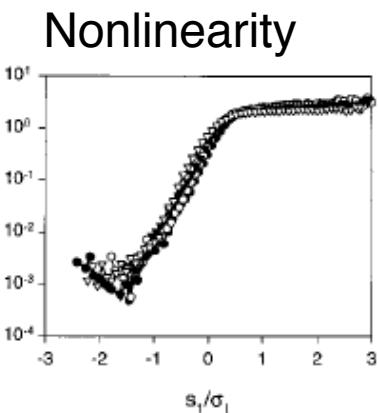
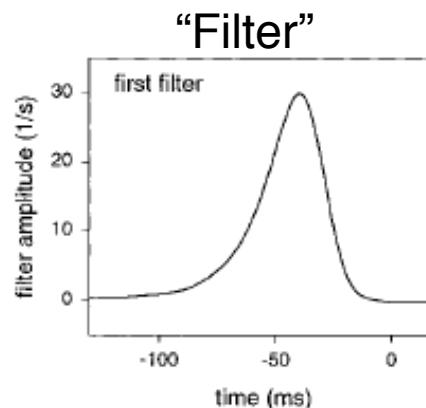
Retinal ganglion cell firing rate and LN model



LN models in the nervous system

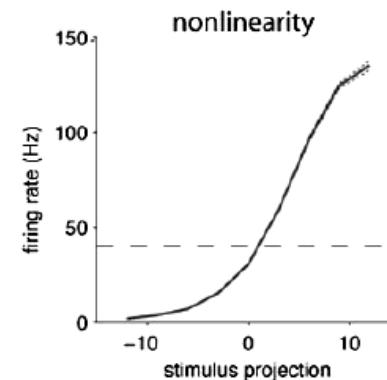
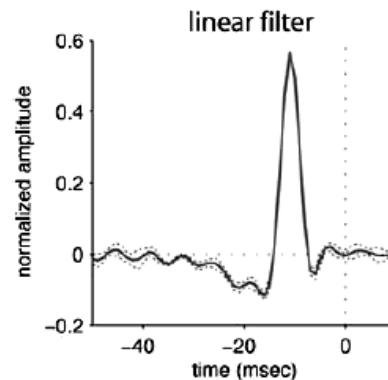
Fly H1 Motion sensitive neuron

Brenner et al., *Neuron* (2000)



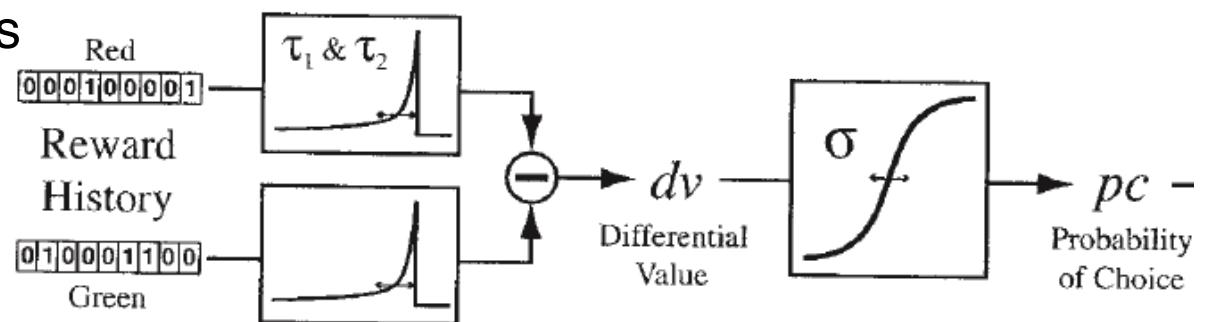
Songbird auditory forebrain neuron

Nagel & Doupe, *Neuron* (2007)

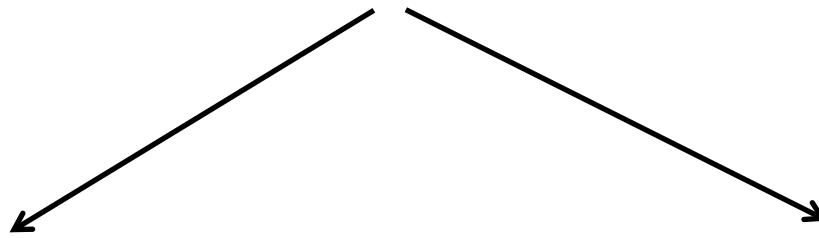


Whole monkey making decisions

Corrado, Sugrue, Seung & Newsome, *J. Exp. Anal. Behav.* (2005)



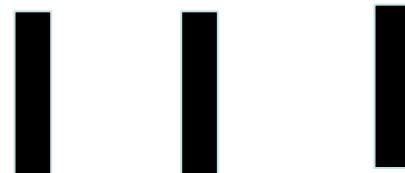
Properties of feature detection



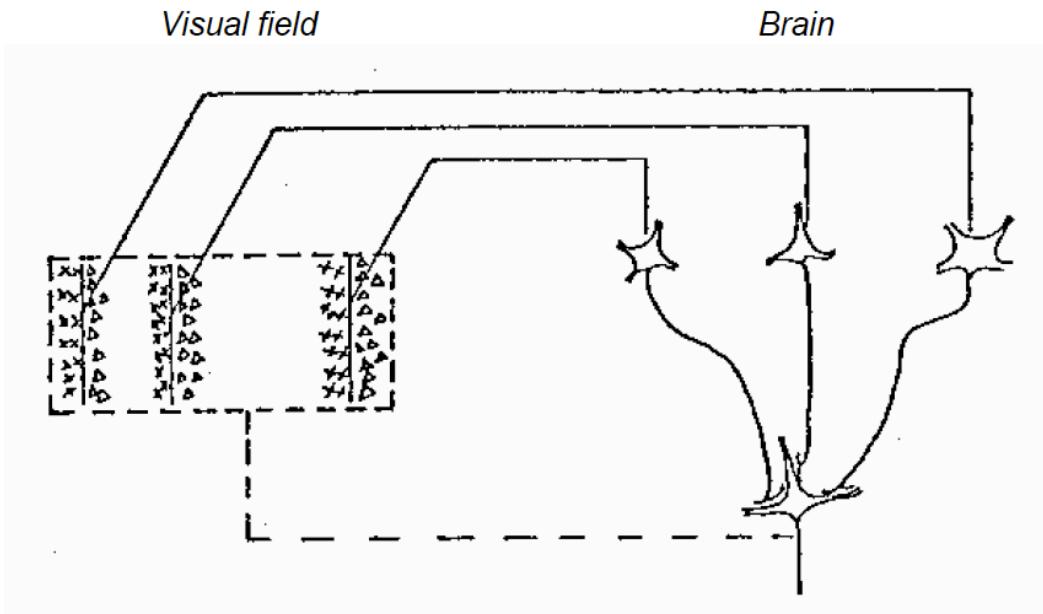
Selectivity



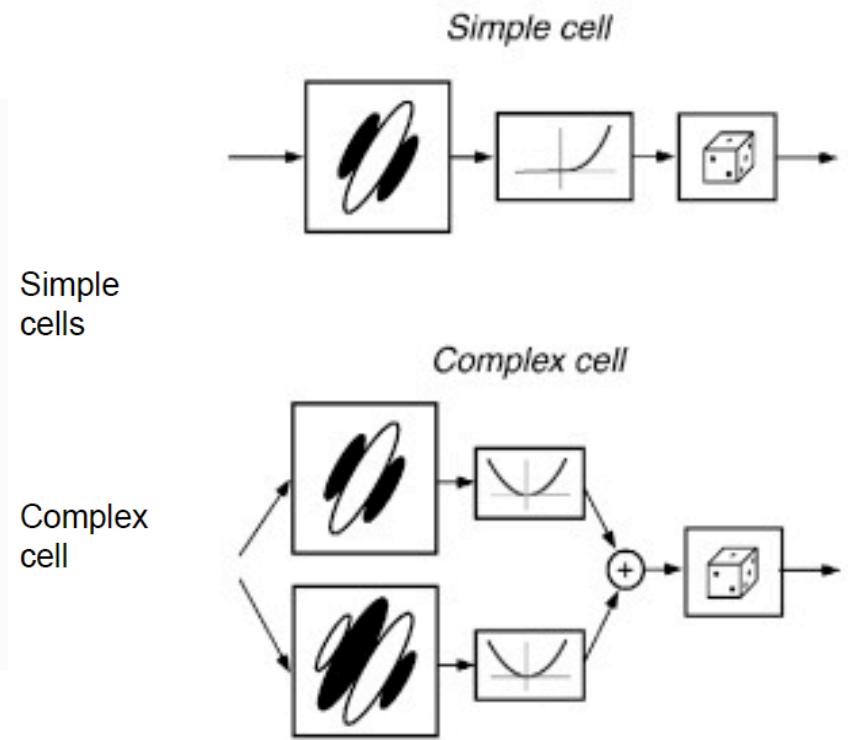
Invariance



Models of selectivity and invariance in primary visual cortex

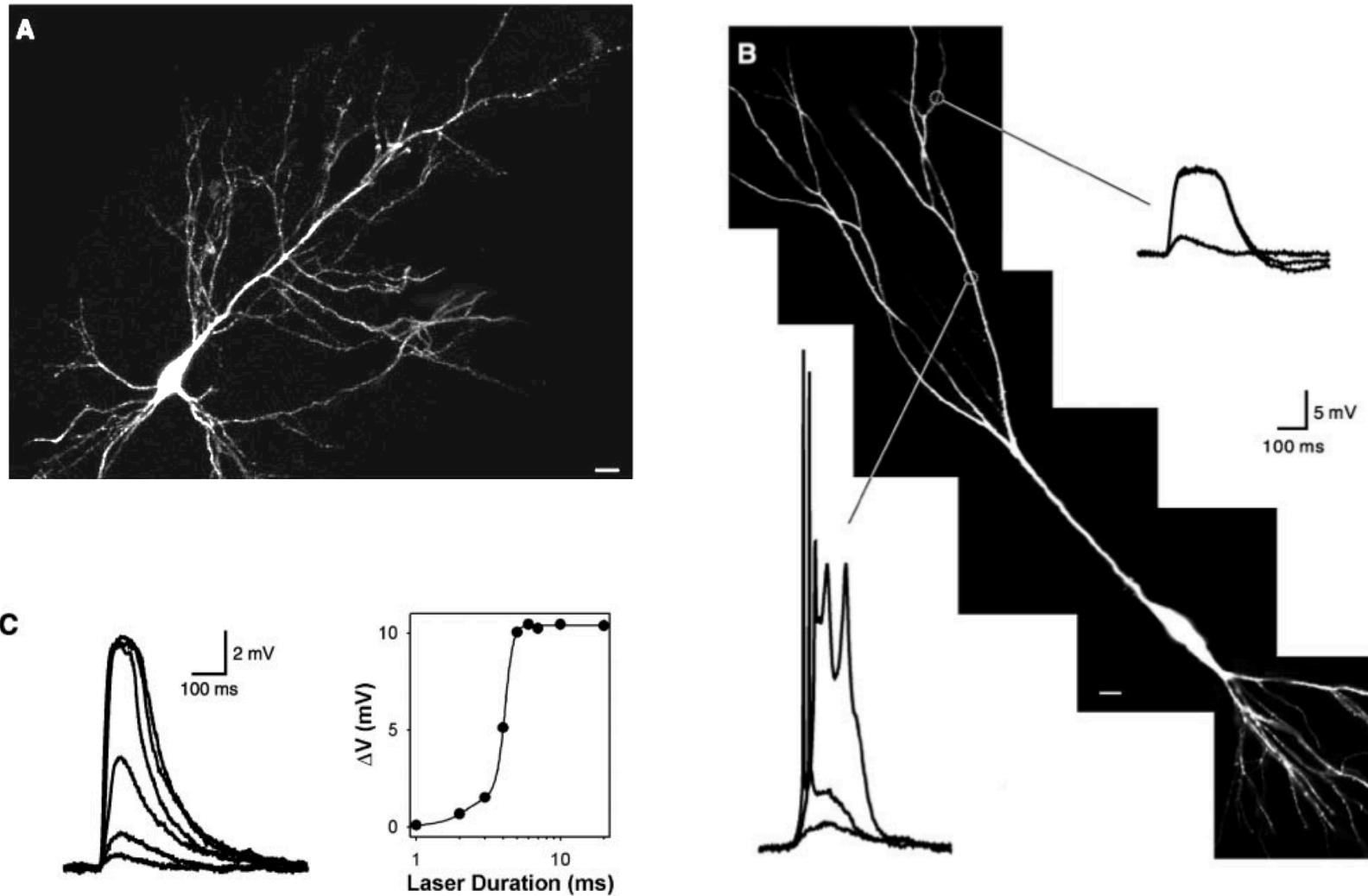


Hubel & Wiesel, 1963



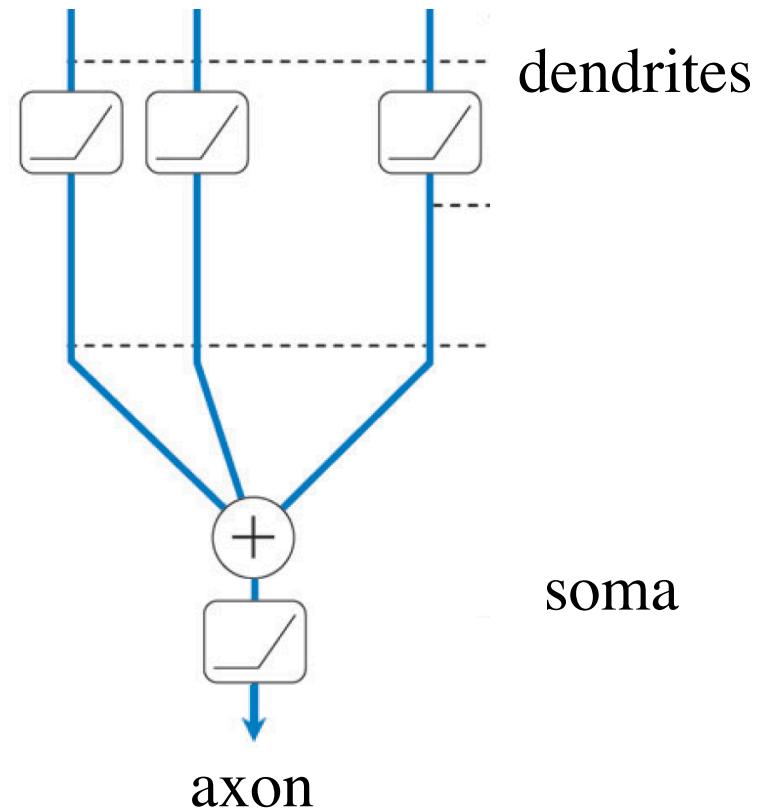
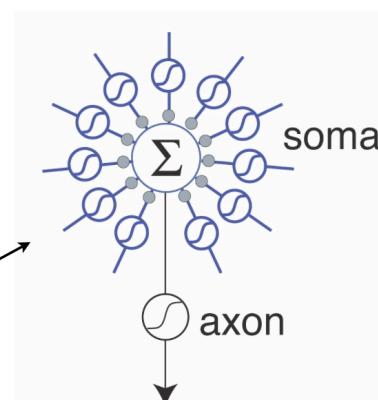
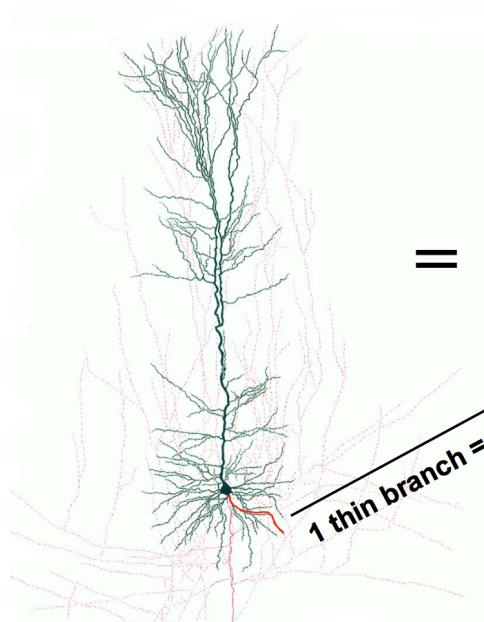
Rust et al., 2005

All or none calcium action potentials in pyramidal cell dendrites



Wei et al., 2001. Compartmentalized and Binary Behavior of Terminal Dendrites in Hippocampal Pyramidal Neurons

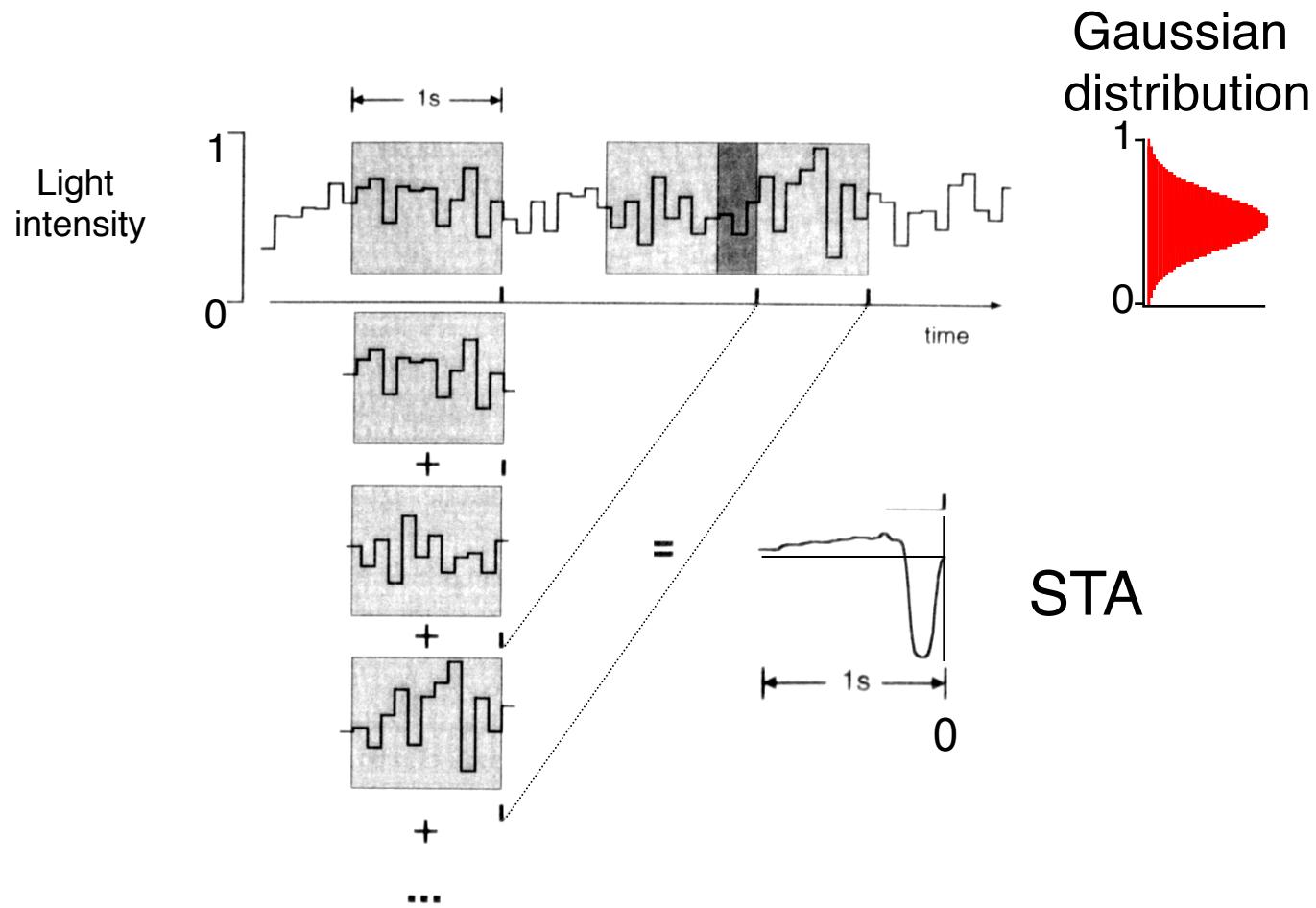
“Two layer” model of active dendrites



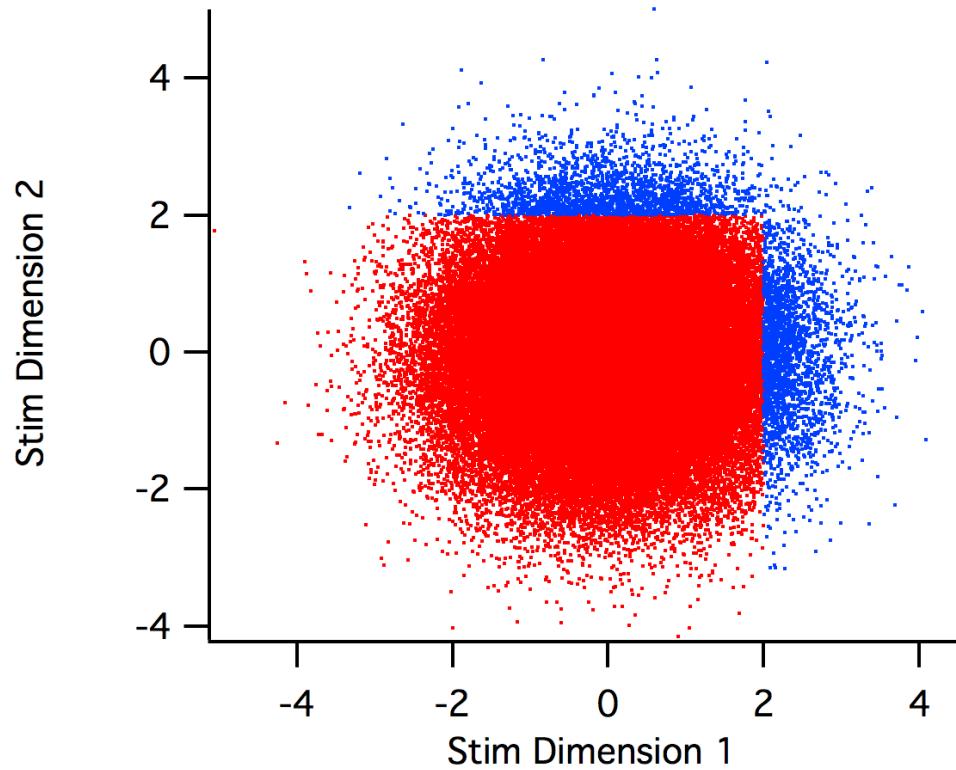
Poirazi, Brannon & Mel, 2003

Bartlett Mel

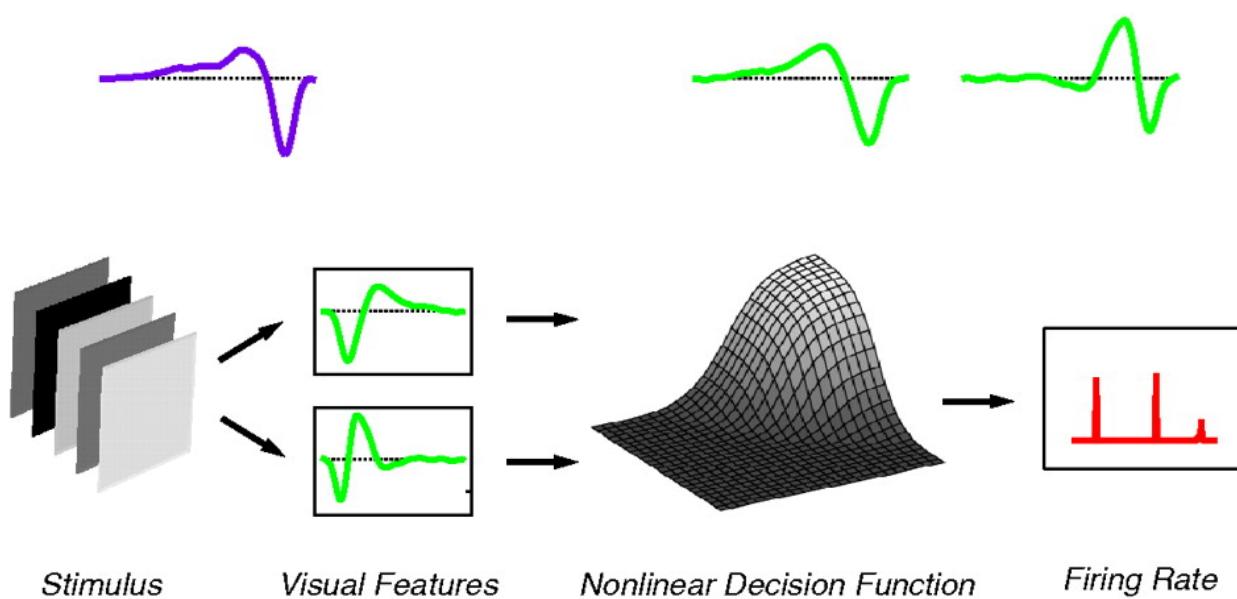
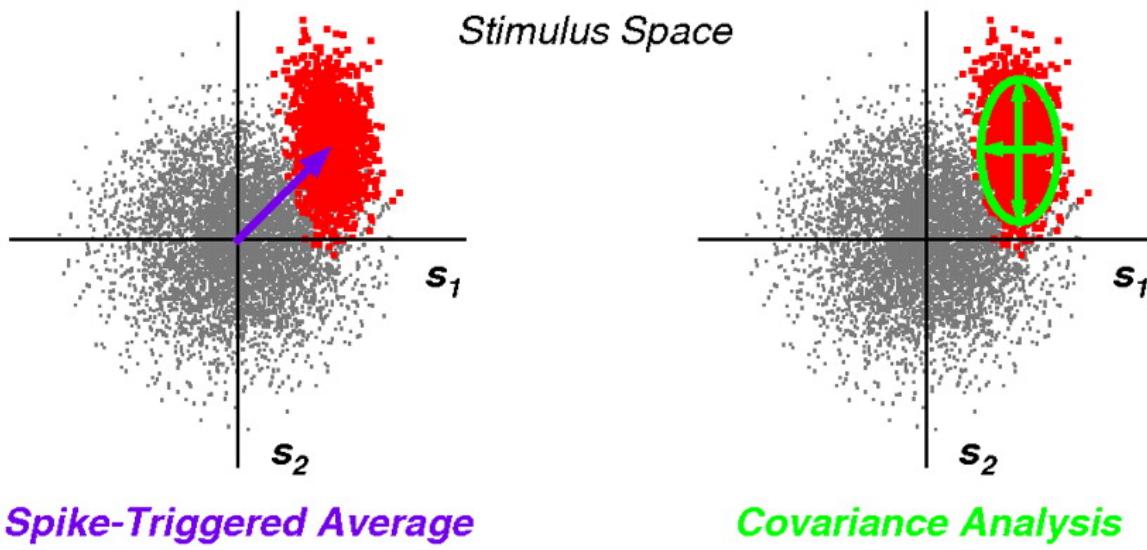
Analyzing the spike-triggered stimulus ensemble



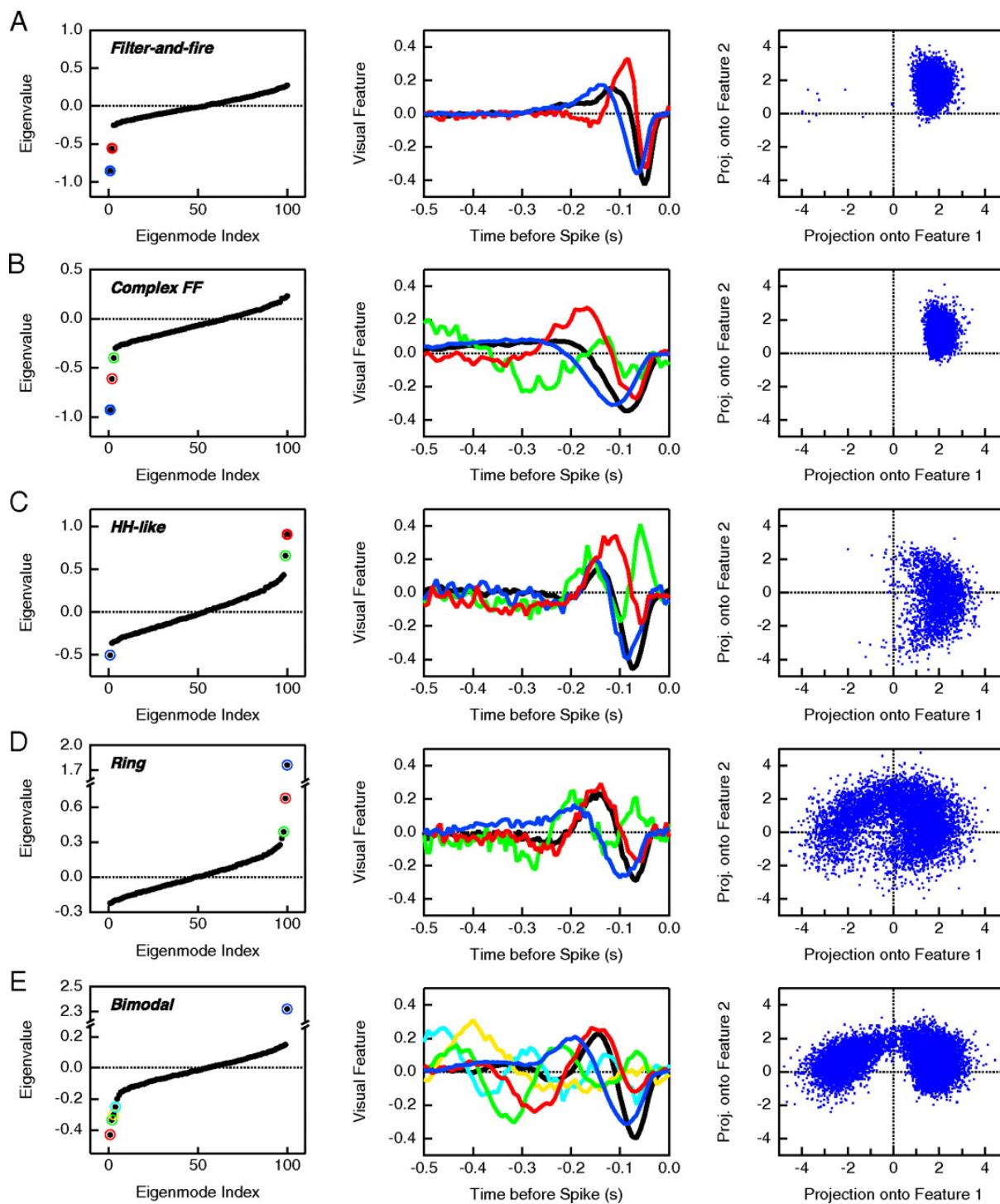
Sensitivity to multiple stimulus dimensions



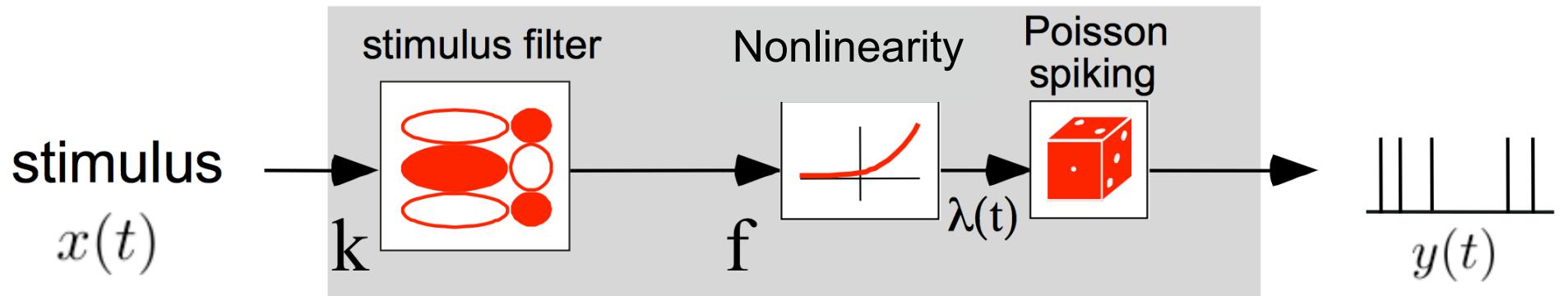
Spike-triggered Covariance



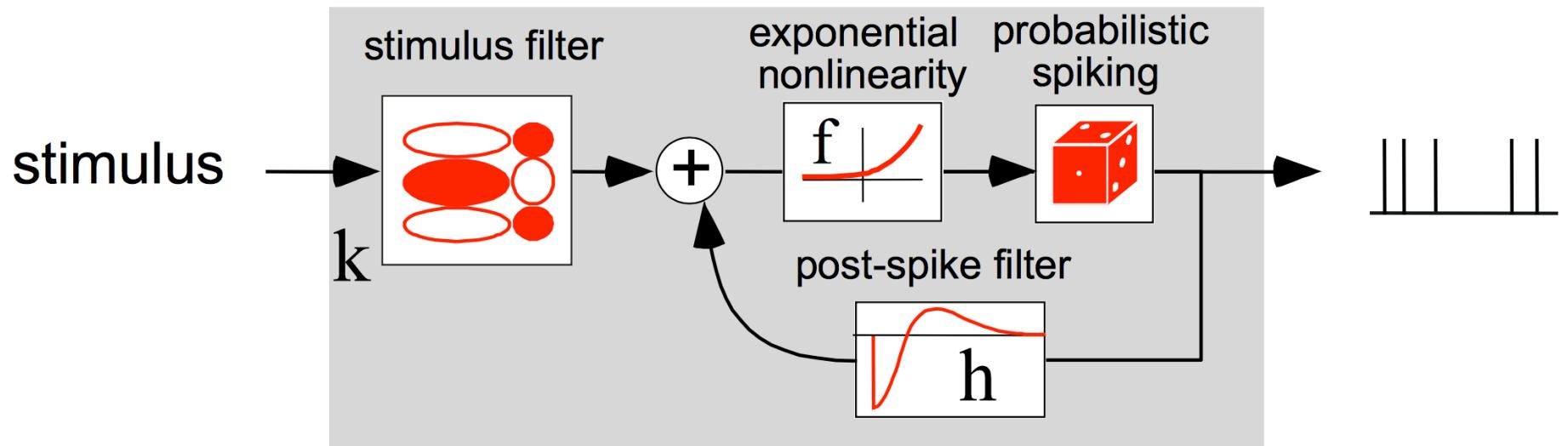
Fairhall et al, 2006



Linear-Nonlinear-Poisson



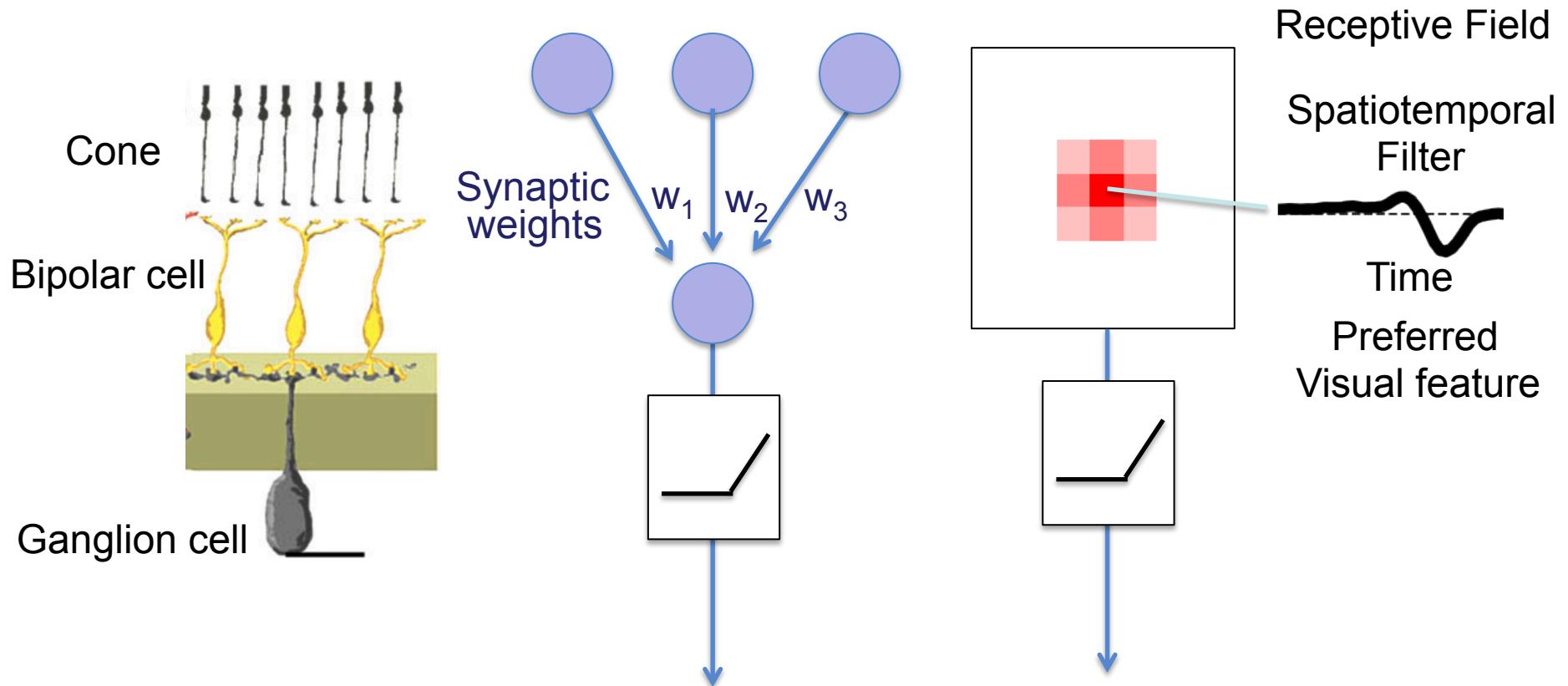
Generalized Linear Model



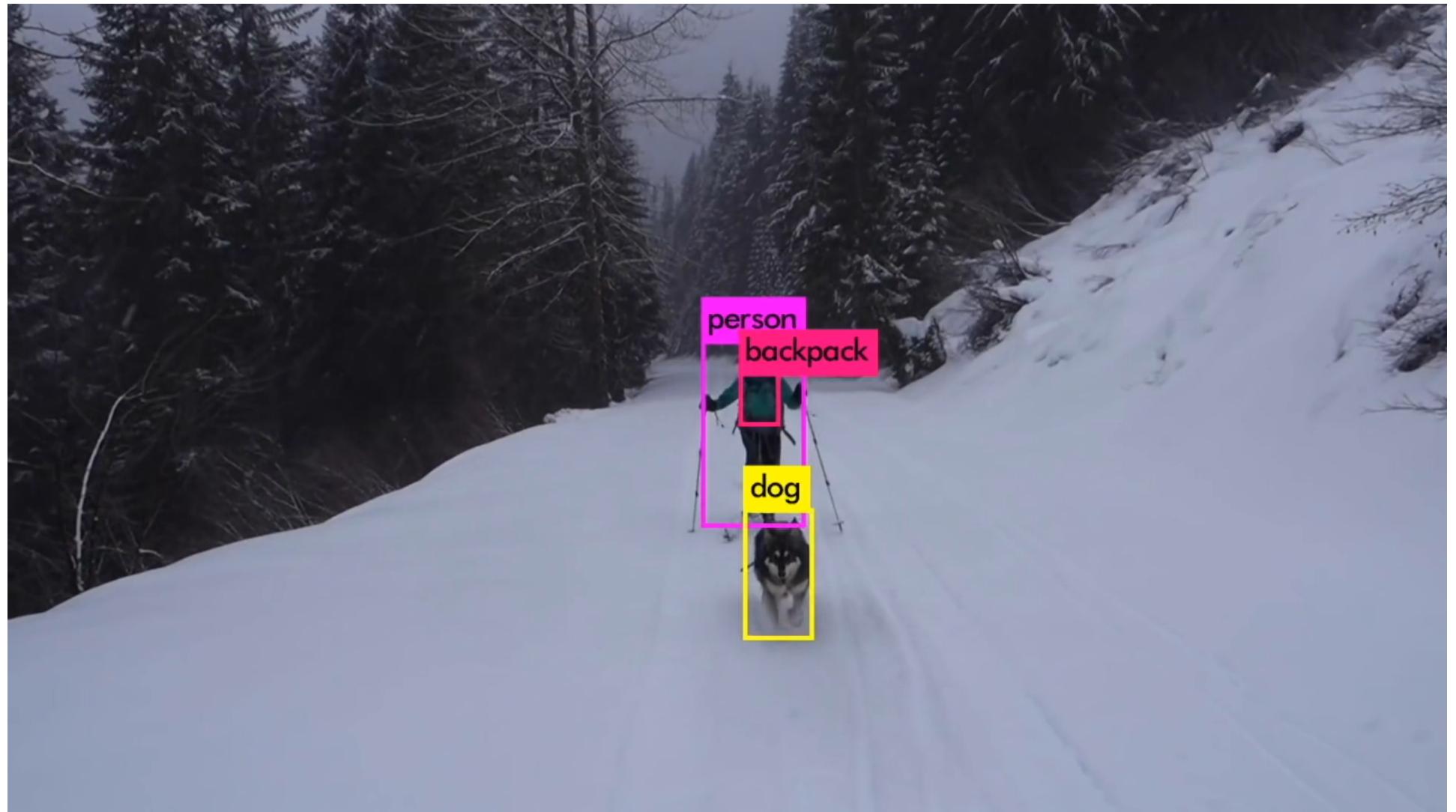
A form of LN model
Can be optimized, don't need Gaussian input
Is “convex”, meaning there are no local minima
Only works with “exponential family” distribution of responses, restricted nonlinearities

A Simple Model of Visual Responses

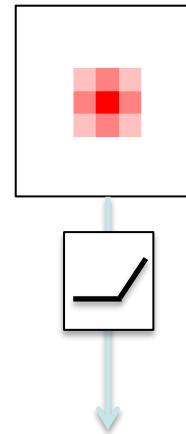
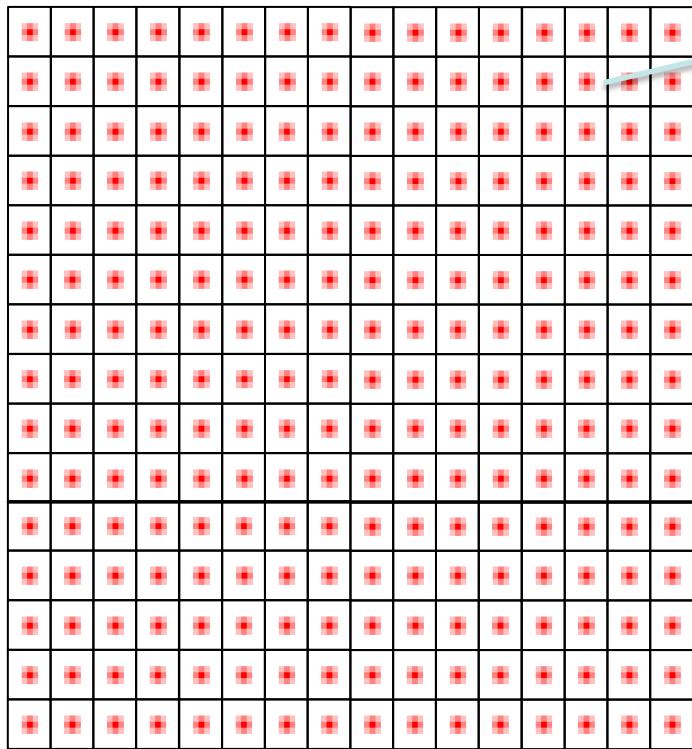
Linear-Nonlinear (LN) Model



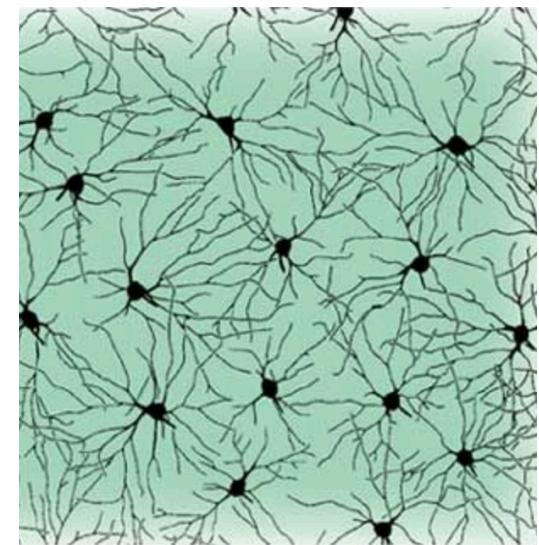
Deep Learning Object Recognition



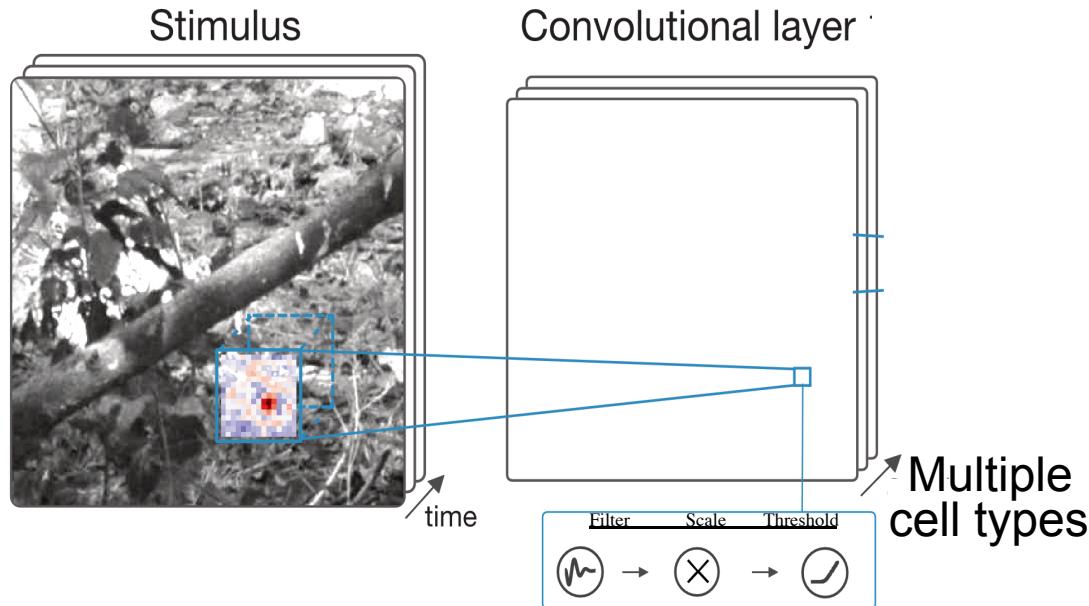
“Convolutional” layer



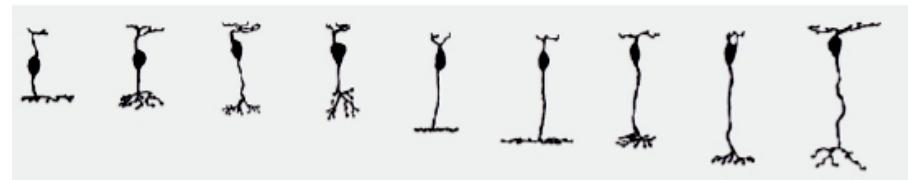
Like a mosaic
of retinal neurons



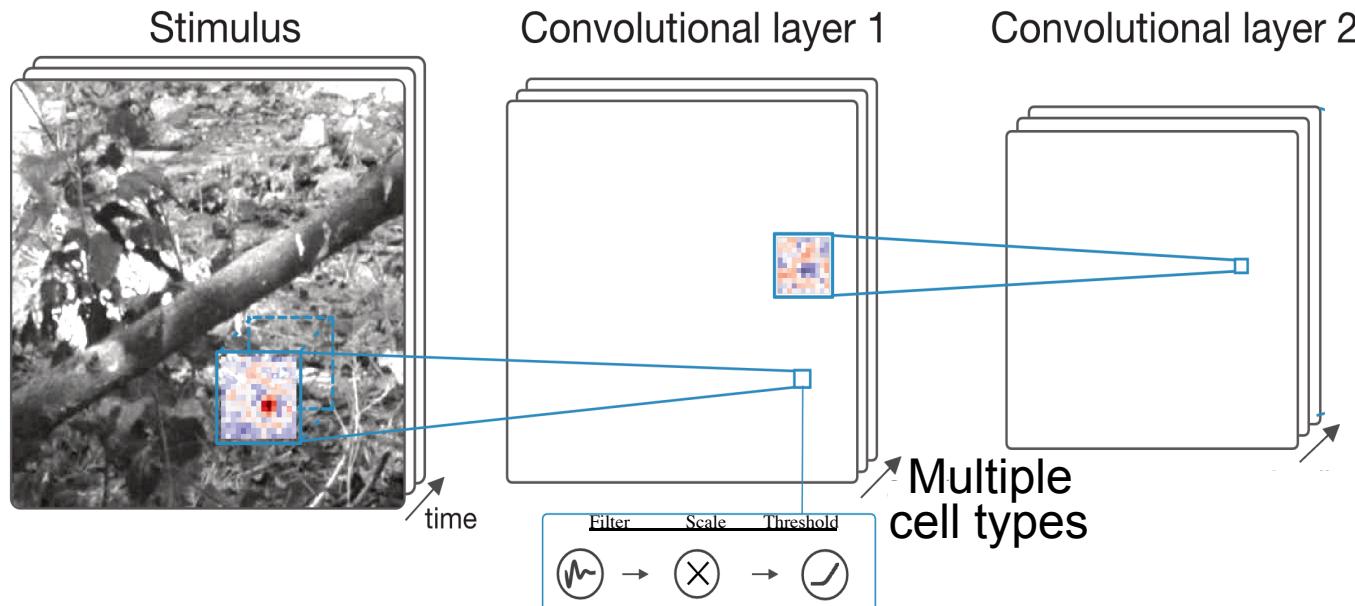
Multiple cell types



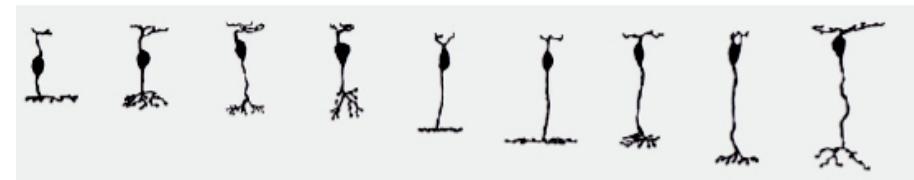
Like the multiple cell types in the retina



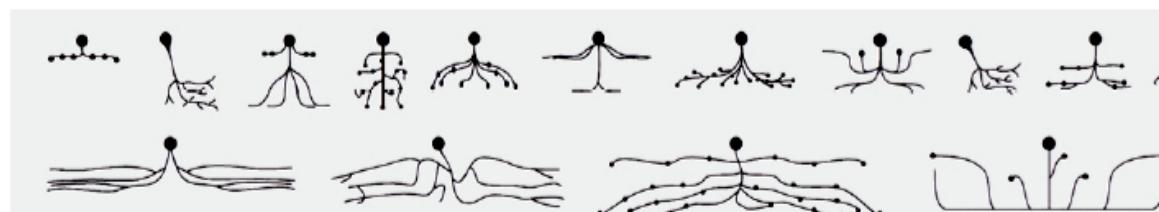
Multiple Layers



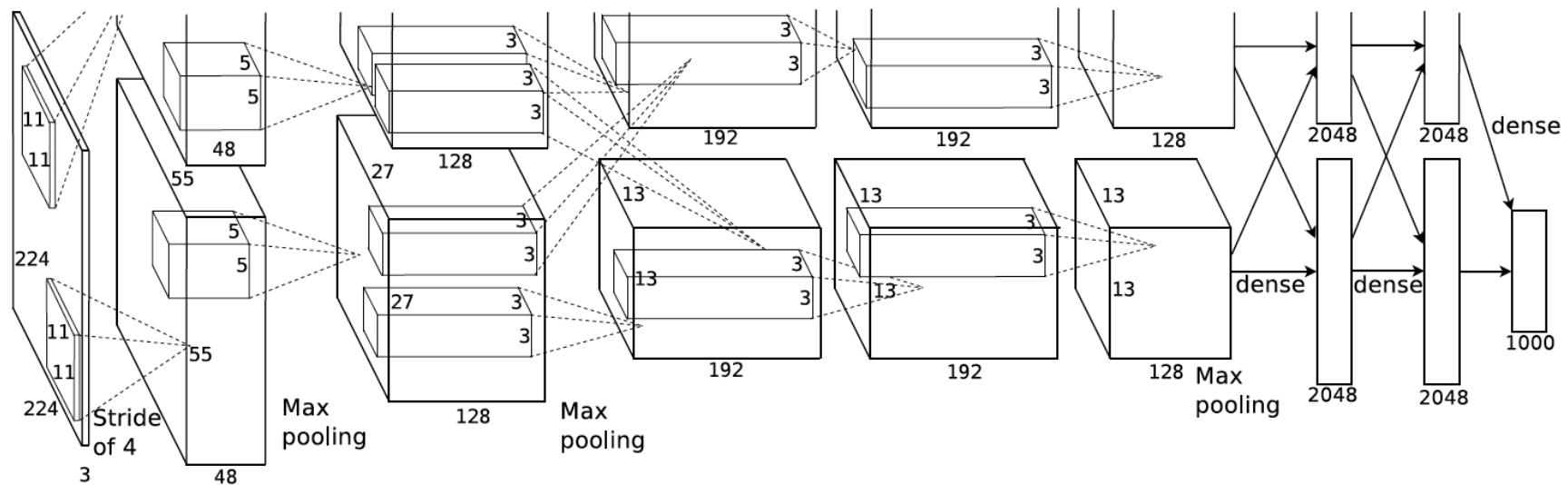
Like the multiple cell types in the retina



Like the hierarchy of retinal circuitry

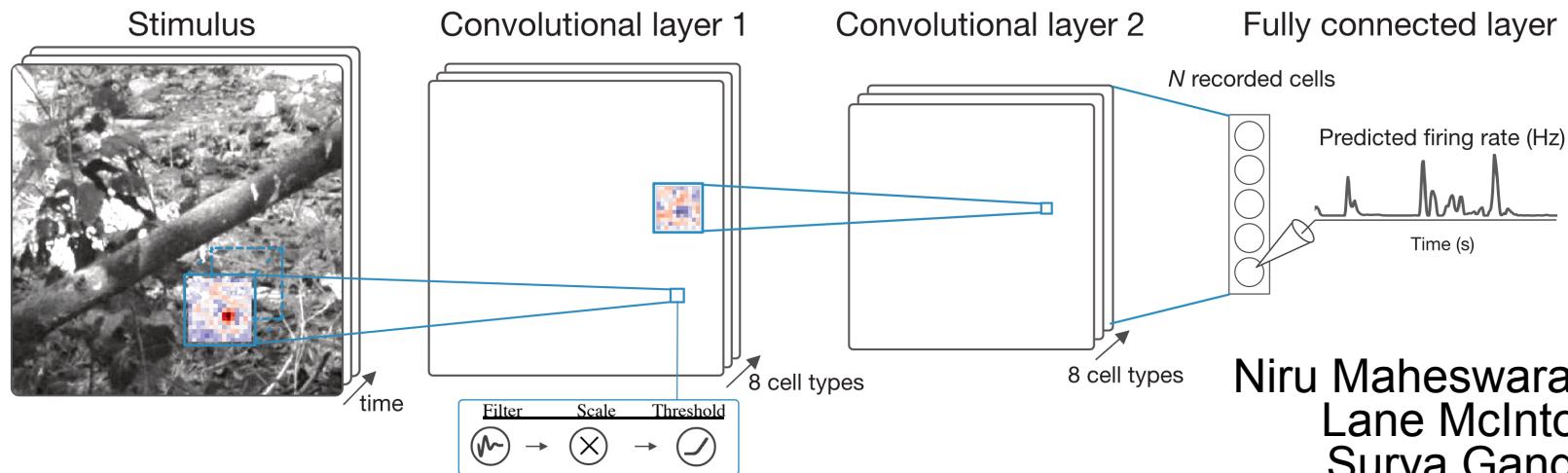


Object Recognition Deep Network

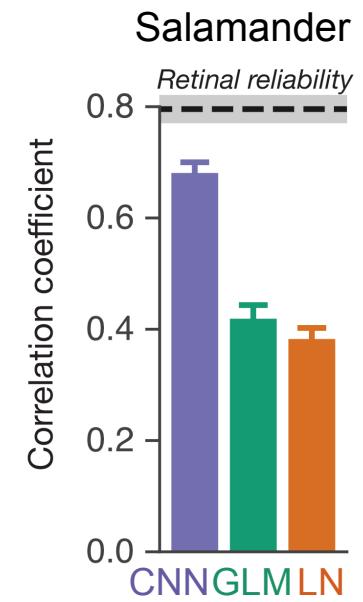
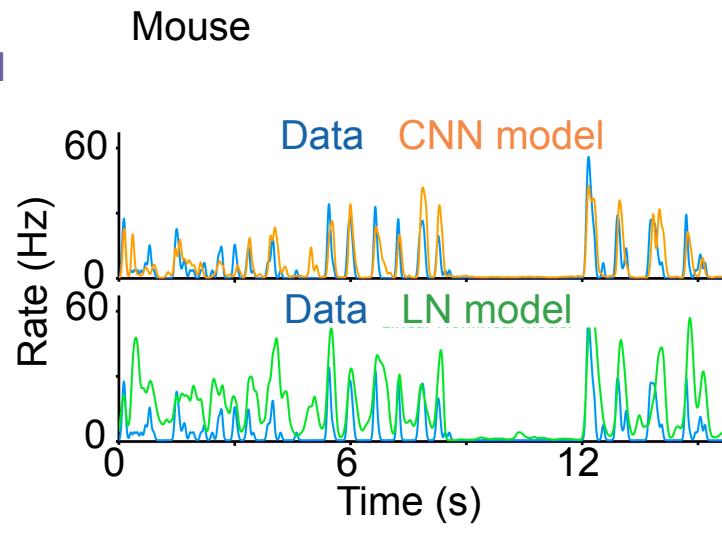
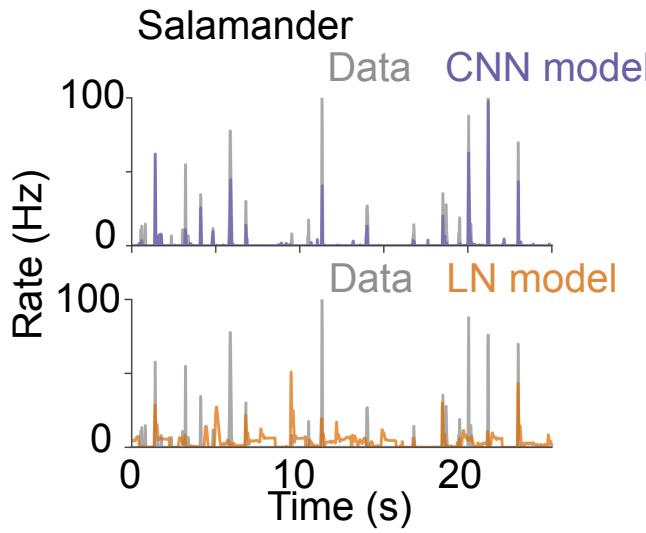


Convolutional Network Model of the Retina

Model schematic



Niru Maheswaranathan
Lane McIntosh
Surya Ganguli



McIntosh, L., et al., (NIPS 2016), Maheswaranathan et al., (*biorxiv* 2018).