Chapter 1 :

What exactly is a vector ?

Physics:
- length
- direction
- can be moved in parallel direction in 2D plane

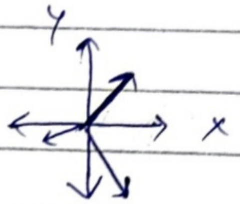
In cs :

It is like a list

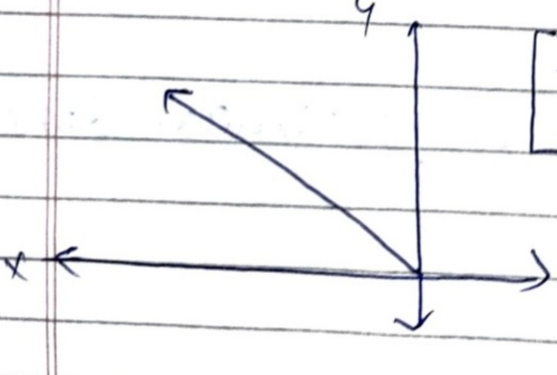
eg $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ here basically length of list = 2

In math :

operations (+ , x , -)



coordinates - how to get from tail to tip



$\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

For sum - use triangle law

Each vector represents a movement.

Scaling - stretch / squish a vector

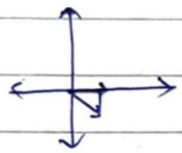
eg $3\vec{v}$, $1.8\vec{v}$

$$= 2\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

Chapter 2:

$\hat{i}$ - unit vector
$\hat{j}$ - ~~vect~~ unit vector

} basis vectors



$3\hat{i} - 2\hat{j}$

basis vectors - linearly independent vectors that span the full space

* One requires two vectors in X-Y plane ~~too~~ to obtain any vector in the plane

* adding vectors - linear combination
  $a\vec{v} + b\vec{w}$

* span of $\vec{v}$ & $\vec{w}$ is the set of all linear combination
  $a\vec{v} + b\vec{w}$
  all possible vectors achieved by scalar multiplication & vector addition

* collection of vectors can be seen as points

* In 3-D, linear combination is
$$a\vec{v} + b\vec{w} + c\vec{u}$$

* linearly dependent vectors; one vector can be expressed as a linear combination of the other, as it lies in the span of the other.

* linearly independent in In case of 2 vectors each contributes a dimension

Chapter 3: Essence of LA

* transformation is basically a function

* It is associated with movement of the vector.

* transformation is linear if: lines remain straight
origin is fixed
grid lines are parallel

* linear transformations - way to move around space
grid lines are parallel
origin is fixed.

* matrix - transformation of space

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where $\hat{j}$ lands.

where $\hat{i}$ lands

Chapter 4: Matrix Multiplication

2x2 Matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where $\hat{i}$ lands   where $\hat{j}$ lands

Linear transformation:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} a \\ c \end{bmatrix} + y\begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Composition = (linear transformation)$_1$ + (linear transformatio

Suppose you apply rotation to a matrix and then shear.

$$\xleftarrow{\text{R to L}}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{shear}}\left(\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{rotation}}\begin{bmatrix} x \\ y \end{bmatrix}\right) = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}}\begin{bmatrix} x \\ y \end{bmatrix}$$

Composition ↑ product

$$M_1 \times M_2 \neq M_2 \times M_1$$

Chapter 5: 3D Linear Transformation

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} + z \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

Chapter 6: The Determinant

The factor by which area of a given region increases or decreases.

Example: $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$\hat{i}$ scaled by factor - 3
$\hat{j}$ scaled by factor - 2

determinant = 0
$\Rightarrow$ space is squished into 1 line

determinant = -ve means space is flipped.

Chapter 7: Inverse matrices, column space and null space

Applications:

- solving a system of equations

Example:

$$2x + 5y + 3z = -3$$
$$4x + 0y + 8z = 0$$
$$x + 3y + 0z = 2$$

$$\underbrace{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}_{\vec{v}}$$

$$A\vec{x} = \vec{v}$$

↑ Some transformation due to which $\vec{x}$ lands on $\vec{v}$

A^{-1} - unique transformation with the property that if you first apply A & follow it by A^{-1} then you end up where you started.

$A^{-1} \times A = A \times A^{-1} = I$ = matrix corresponding to doing nothing.

* $\det(A) \neq 0$
$\Rightarrow A^{-1}$ exists

* rank = no. of dimensions in the output of a matrix

* column space - set of all possible outputs $A\vec{v}$

$$\underbrace{\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}}_{\text{span of columns}}$$

span of columns
$\Updownarrow$
column space

* null space / Kernel - vectors that become $0 \Rightarrow$ null

Chapter 8:

2d input $\xrightarrow{\text{transformation}}$ 3d output

$$\begin{array}{c c} & \overbrace{\hphantom{2 \quad 0}}^{\text{2 columns}} \\ \text{3 rows} & \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ -2 & 1 \end{bmatrix} \end{array} \leftarrow \text{coordinates } \hat{j} \text{ lands}$$

↑ coordinates where basis vectors land

$$\begin{bmatrix} 3 & 1 & 5 \\ 4 & 2 & 6 \end{bmatrix} \Rightarrow \text{coordinates where they land in 2D space}$$

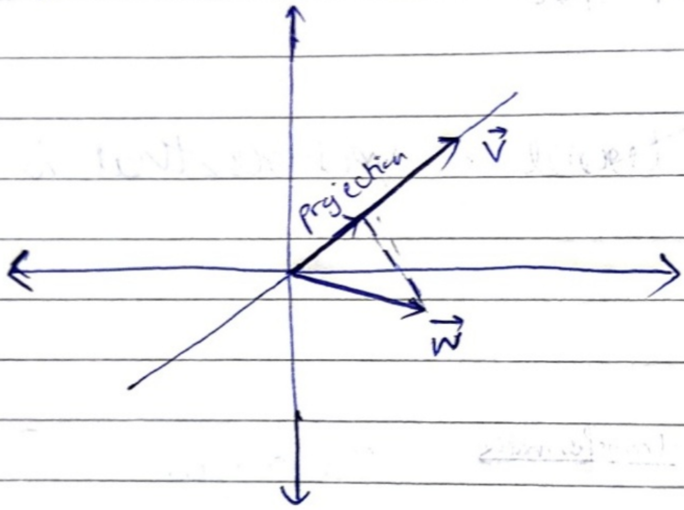
↑ ↑ ↑
basis vectors

Chapter 9: Dot Products & Duality

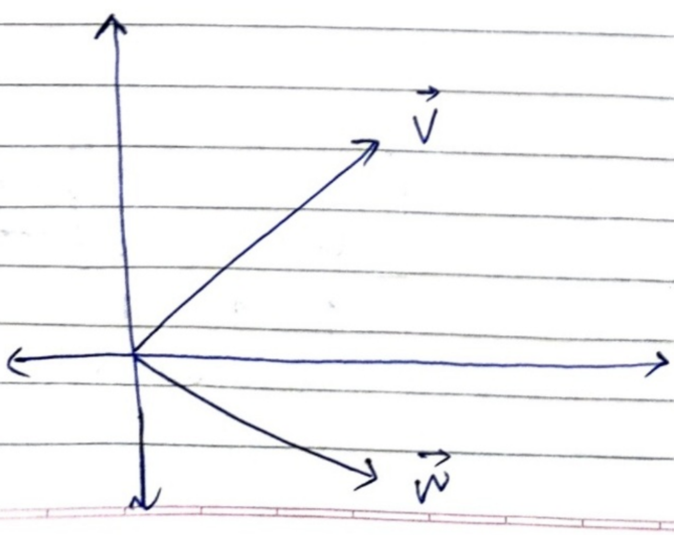
Example:

$$\begin{bmatrix}6\\2\\8\\3\end{bmatrix} \cdot \begin{bmatrix}1\\8\\5\\3\end{bmatrix} = 6+16+40+9 = 71$$

$$\underbrace{\begin{bmatrix}4\\1\end{bmatrix}}_{\vec{v}} \cdot \underbrace{\begin{bmatrix}2\\-1\end{bmatrix}}_{\vec{w}}$$



Chapter 10: Cross Products



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$\vec{v} \times \vec{w}$ = area of parallelogram
Orientation matters

$\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$

anticlockwise (+ve)
clockwise - (-ve)

Example: $\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\vec{w} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\vec{v} \times \vec{w} = \det\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\right)$

$(\text{3D vector})_1 \times (\text{3D vector})_2 = \text{3D vector}$

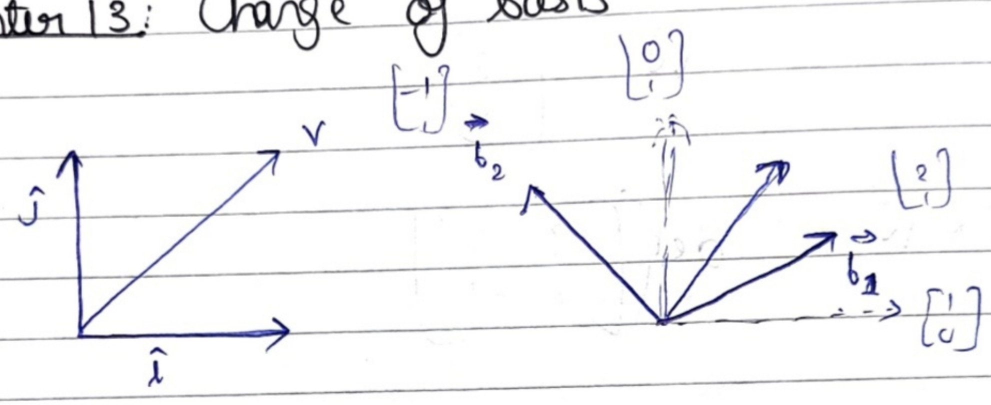
$\vec{v} \times \vec{w} = \vec{p}$

direction is given by right hand rule

Chapter 11 : Cross Products in the Light of Linear Transformation

duality - anytime you have a linear transformation from some space to the number line its associated with a unique vector in that space in the sense that performing the linear transformation is the same as taking dot product with that vector.

Chapter 13: Change of basis



$$\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}-4\\1\end{bmatrix}$$

jen's basis vectors in our coordinates — jen's vector — regular grid

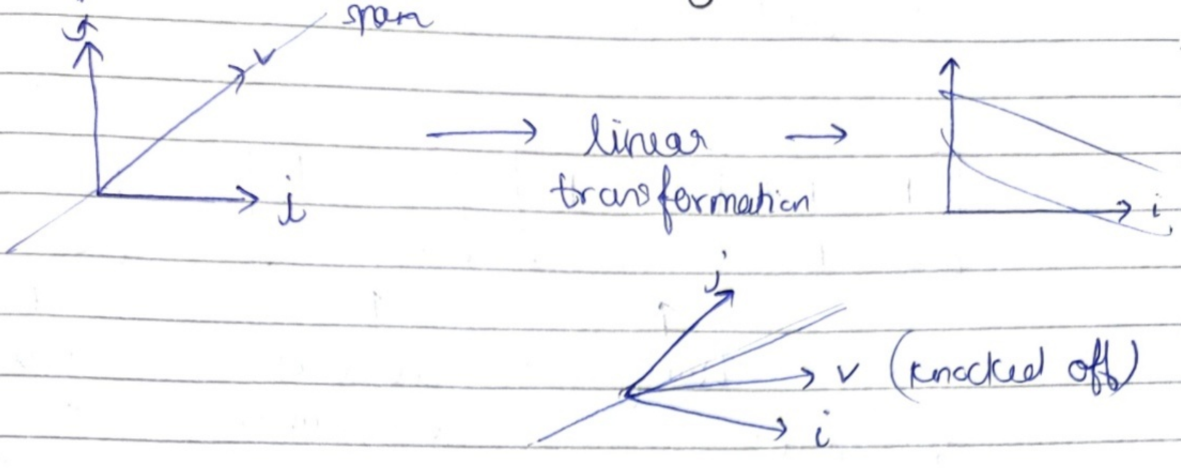
$$\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}5/3\\1/3\end{bmatrix}$$

jen's language

How to translate a matrix

$$\underbrace{\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 1\end{bmatrix}}_{\text{transformation matrix in her language}}\vec{v}$$

Chapter 14: Eigen values & eigen vectors.



some vectors lie on their span and linear transformation is like some scalar multiplication these are called eigen vectors.
the factor by which they are scaled is called eigen value.
if eigen value is negative then the vector is flipped & squished by 1/2.

matrix vector multiplication

$$A\vec{v} = \underbrace{\lambda\vec{v}}_{\text{scalar multiplication}}$$

$$A\vec{v} = (\lambda * I)\,\vec{v}$$

$$(A - \lambda I)\,\vec{v} = \vec{0}$$

$$\therefore \det(A - \lambda I) = 0$$

diagonal matrix: $\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$

x & y – eigen values.

Date

eigen basis - basis vectors which are ~~diagonal~~ eigen vectors.

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

↑ change of basis matrix

↑ transformation

↑ change of basis matrix

↑ matrix represent transformation according to new basis

Chapter 16: Abstract Space

Formal definition of linearity:

Additivity: $L(\vec{v}+\vec{w}) = L(\vec{v}) + L(\vec{w})$

Scaling: $L(c\vec{v}) = cL(\vec{v})$

Our current space: All polynomials

Basis functions:

$b_0(x) = 1$

$b_1(x) = x$

$b_2(x) = x^2$

$b_3(x) = x^3$

$\vdots$

Example: $5 + 3x + x^2$

$$\begin{array}{c} 5 \\ 3x \\ x^2 \\ \\ \\ \\ \end{array} \begin{bmatrix} 5 \\ 3 \\ 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$