mP2C – Shaded Area Probabilities

Directions: Read the explanation for each problem type mentioned below. Pay special attention to the methods and techniques used to solve the sample problems. Then, do the practice exercises that follow; use the appropriate method to solve each problem.

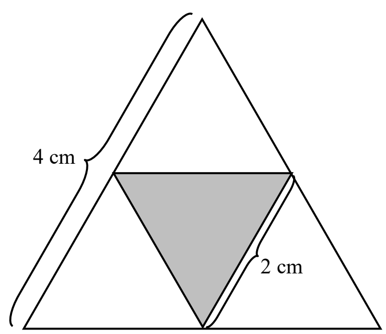
Topic Overview: Probability

Remember our probability equation? The total number of possibilities went on the bottom, and the number of desired possibilities went on top. Many questions will ask us about probabilities based on areas – these will require us to put our knowledge of geometry together with our knowledge of probability. Our formula for shaded area probability is:

$$\text{P(shaded)} = \frac{\text{shaded area}}{\text{total area}}$$

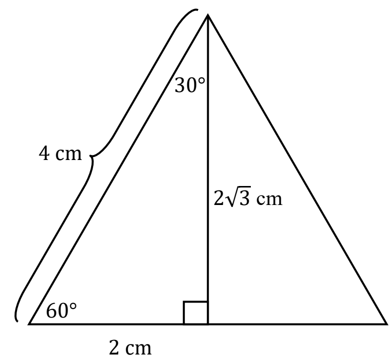
The tricky part about solving these questions will be finding the needed areas.

Sample Problem: Probability



The figure above shows a target shaped like an equilateral triangle. The "bull's-eye" is the shaded area, which is another equilateral triangle inscribed in the first. If a gun is fired at the target and hits a random point, what is the chance that it will hit the "bull's-eye"?

Because this is a shaded area question, we can use the formula from above. The challenge is to find the areas. To find the area of an equilateral triangle, we can use 30-60-90 relationships within the triangle to find a formula for it:



With those numbers, we can find the area by multiplying height, base and dividing by 2.

$$\text{Total Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$$

To find the area of the shaded region, just use the same right-triangle relationships in the smaller triangle:

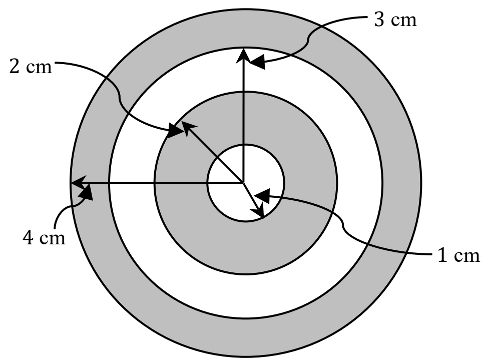
$$\text{Shaded Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2 \times \sqrt{3} = \sqrt{3}$$

Now we can use the formula:

$$\text{P(shaded)} = \frac{\text{shaded area}}{\text{total area}} = \frac{\sqrt{3}}{4\sqrt{3}} = \frac{1}{4} = 25\%$$

SAMPLE PROBLEM: PROBABILITY

Anthony throws a dart which hits a random point on the dartboard pictured below. What is the probability that his dart will hit a shaded region?



(A) $\frac{1}{4}$
(B) $\frac{2}{7}$
(C) $\frac{1}{2}$
(D) $\frac{5}{8}$
(E) $\frac{7}{8}$

First, we have to calculate some areas:

total area $= \pi r^2 = \pi(4^2) = 16\pi \text{ cm}^2$

The shaded area is a little trickier. We can subtract the white circle in the middle from the whole area first:

$16\pi - \pi = 15\pi \text{ cm}^2$

We still have to subtract the area of the white band in the middle, which is the area of the circle with radius of 3 cm minus the area of the circle with radius of 2 cm:

area of white band $= 9\pi - 4\pi = 5\pi \text{ cm}^2$

Therefore the shaded area is $15\pi - 5\pi = 10\pi \text{ cm}^2$

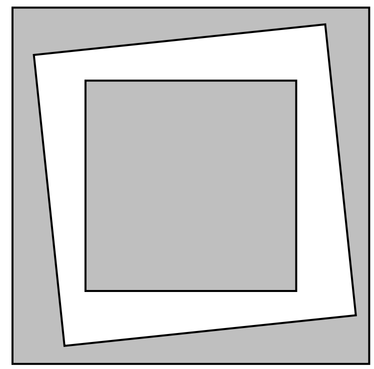
Now we can use our formula:

$\text{P(shaded)} = \frac{\text{desired area}}{\text{total area}} = \frac{10\pi}{16\pi} = \frac{5}{8}$

So the chance of Anthony hitting the shaded area is $\frac{5}{8}$, or 62.5%. The answer is (D).

Of course, in reality Anthony would likely be *aiming* for the shaded area, so hopefully he'd hit it more often than that!

SAMPLE PROBLEM: PROBABILITY



The three squares shown above have edges of lengths 3, 4, and 5, respectively. What is the probability that a dart thrown randomly at the design lands in a shaded region?

(A) $\frac{1}{8}$
(B) $\frac{7}{25}$
(C) $\frac{1}{2}$
(D) $\frac{13}{25}$
(E) $\frac{18}{25}$

To find the area of the shaded band around the outside of the figure, we need to subtract the areas of the largest and second-largest squares:

$\text{Area} = 5^2 - 4^2 = 9$

Now we need to add to that the area of the central shaded square, which is $3^2 = 9$. So the total shaded area is $9 + 9 = 18$. Now we can find the probability of hitting the shaded region using our formula:

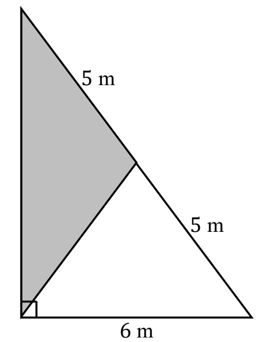
$\text{P(shaded)} = \frac{\text{desired area}}{\text{total area}} = \frac{18}{25} = 72\%$

The answer is (E)

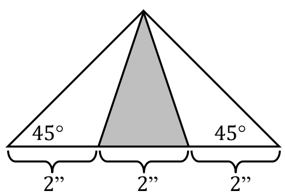
PRACTICE EXERCISES

For problems 1-4, in each of the following triangles, find the probability that a randomly selected point lies within the shaded area.

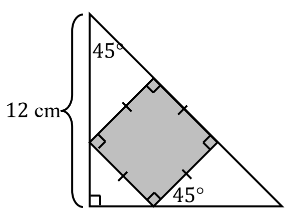
1.



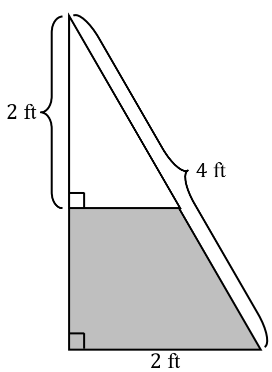
2.



3.



4.



For problems 5-8, a point (x, y) is randomly selected within the square with vertices $(0, 0)$, $(0, 10)$, $(10, 0)$, and $(10, 10)$. Find the probability that (x, y) satisfies the given inequality.

5. $y \geq x$

6. $y < 10 - 2x$

7. $x > 8$

8. $y \geq |x - 5|$

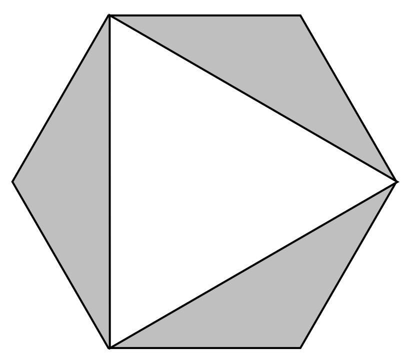
9. a. What is the probability that a dart randomly thrown at a dart board of radius 4 inches hits within 2 inches of the center?

   b. What is the probability that a dart randomly thrown at a dart board of radius 8 inches hits within 4 inches of the center?

   c. What is the probability that a dart randomly thrown at a dart board of radius 2 inches hits within 1 inch of the center?

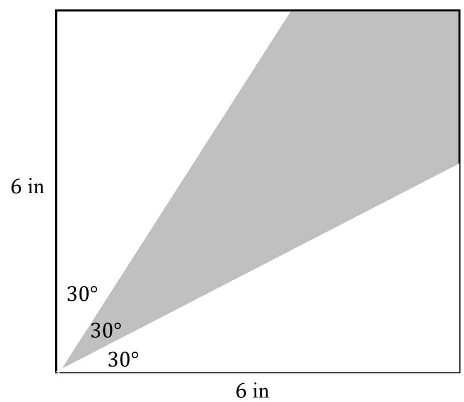
   d. What is the probability that a dart randomly thrown at a dart board of radius $2x$ inches hits within x inches of the center?

10. Every other vertex of a regular hexagon is connected to form an equilateral triangle. What is the probability that a randomly chosen point within the hexagon lies in one of the shaded areas outside the triangle?



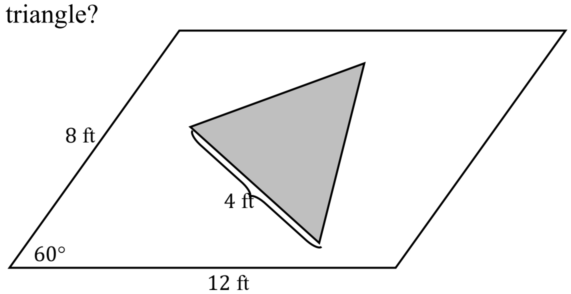
TEST EXERCISES (TIME: 10 MINUTES)

11. A square is depicted with side length 6 in. and its lower left vertex angle trisected. What is the probability that a randomly selected point within the square lies in the shaded region?



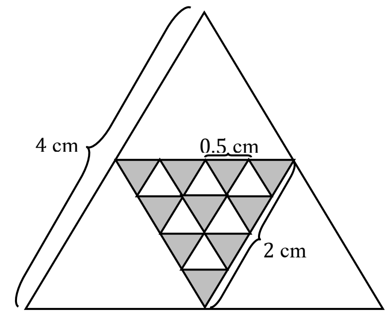
(A) $\frac{1}{3}$
(B) $\frac{5}{12}$
(C) $\frac{3-\sqrt{3}}{3}$
(D) $\frac{\sqrt{3}-1}{3}$
(E) $\frac{\sqrt{3}}{3}$

12. In the diagram below, an equilateral triangle of side length 4 feet lies entirely within a parallelogram with sides 8 and 12 feet and a base angle of 60°. What is the probability that a randomly chosen point within the parallelogram lies within the triangle?



(A) $\frac{1}{16}$
(B) $\frac{1}{12}$
(C) $\frac{\sqrt{3}}{12}$
(D) $\frac{\sqrt{3}}{6}$
(E) $\frac{1}{8}$

13. What is the probability that a randomly selected point within the large equilateral triangle below lies in one of the small shaded equilateral triangles?



(A) $\frac{1}{6}$
(B) $\frac{1}{4}$
(C) $\frac{5}{32}$
(D) $\frac{5}{24}$
(E) $\frac{3}{8}$

14. What is the smallest integer x such that a randomly selected point in the circle centered at $(0, 0)$ with radius 10 has a greater than 25% chance of also falling on or within the circle centered at $(0, 0)$ with radius x?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

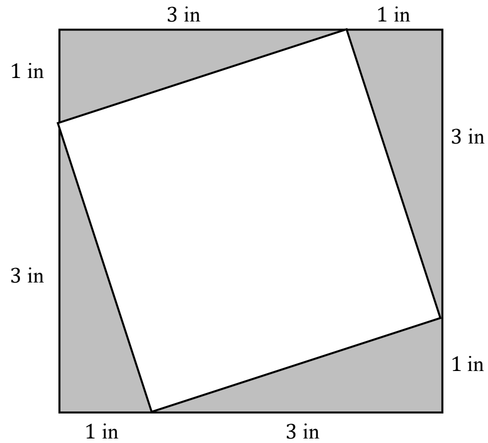
15. The solution to the following system of inequalities is graphed in the coordinate plane:

$$x + y < 4$$
$$x > 0$$
$$y > 0$$

A point is then randomly selected within the solution region. What is the probability that the point satisfies the inequality $x > 3$?

(A) $\frac{1}{16}$
(B) $\frac{1}{12}$
(C) $\frac{1}{9}$
(D) $\frac{3}{16}$
(E) $\frac{1}{4}$

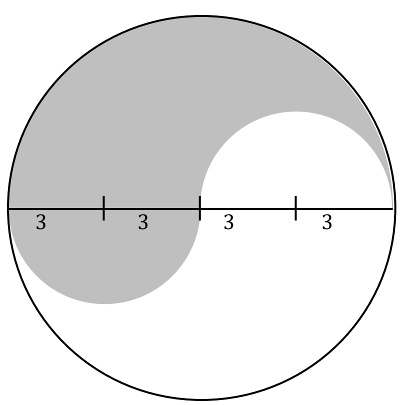
16. What is the probability that a randomly selected point within the square of side length 4 inches depicted below falls within one of the shaded triangles?



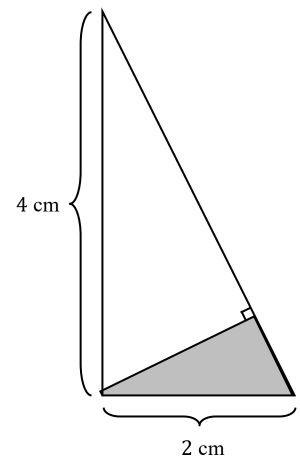
(A) $\frac{3}{16}$
(B) $\frac{3}{10}$
(C) $\frac{3}{8}$
(D) $\frac{7}{10}$
(E) $\frac{3}{4}$

17. What is the probability that a randomly chosen point within a circle of radius 10 lies within 3 units of its center?

(A) 3%
(B) 9%
(C) 12%
(D) 30%
(E) 36%



18. In a right triangle with legs 4 cm and 2 cm, an altitude is drawn from the right angle to the hypotenuse. What, to the nearest integer percent, is the probability that a randomly chosen point within this right triangle falls within the smaller, shaded right triangle, as shown below?



(A) 20%
(B) 22%
(C) 25%
(D) 35%
(E) 45%

19. Two real numbers x and y (not necessarily integers) are randomly selected between 0 and 10. What is the probability that $x + y < 5$?

(A) 12.5%
(B) 25%
(C) 37.5%
(D) 50%
(E) 75%

20. The yin-yang figure to the left is made up of a large circle of radius 6 cm, as well as two semicircles of radius 3 cm whose diameters lie end to end on the large circle's diameter, as shown. What is the probability that a dart randomly thrown at this figure will land in the shaded area?

(A) $\frac{\pi}{8}$
(B) $\frac{1}{\pi}$
(C) $\frac{3}{8}$
(D) $\frac{1}{2}$
(E) $\frac{5}{8}$