mP3C – COMPLEX AVERAGE CONCEPTS

Directions: Read the explanation for each problem type mentioned below. Pay special attention to the methods and techniques used to solve the sample problems. Then, do the practice exercises that follow; use the appropriate method to solve each problem.

TOPIC OVERVIEW: COMPLEX AVERAGE QUESTIONS

The ACT will test our knowledge of certain statistical measures. In an earlier lesson we looked at the idea of *average*, which on the ACT always means arithmetic mean, or the average that we're used to seeing. In this lesson we'll look at some more complicated problems with average as well as *median*, *mode*, and *range*.

Many questions will gives us a set of numbers and then ask us to find the value of one or more of these quantities. Let's define them first and then look at some examples.

The *average* is the sum of the numbers divided by how many numbers we're given – it's likely you've done this many times before.

The *median* is the middle number of the set (if there is an odd number of numbers) after you have put the numbers in increasing numerical order. If there is an even number of numbers in the set, then the median is the average of the middle two.

The *mode* is the number that appears the most often in a set. If two or more numbers appear the most often, then each one is a mode (there can be more than one mode).

The *range* is the difference between the largest number and the smallest number.

SAMPLE PROBLEM: AVERAGES

The daily sales for the last two weeks at Jimmy's Jumping Jackknife Jamboree are as follows:

$$2, 10, 14, 9, 2, 20, 4, 21, 10, 2, 11, 14, 19, 27$$

Find the sum of the median, mode and range of the data.

    (A) 38
    (B) 37.5
    (C) 35
    (D) 28
    (E) 25

The first step in questions like this is to put the numbers in order:

$$2, 2, 2, 4, 9, 10, 10, 11, 14, 14, 19, 20, 21, 27$$

The median is the middle number – in this case we have 14 numbers, so there's not one in the middle. We'll take the average of the two numbers in the middle instead, 10 and 11. The median is 10.5.

The mode is the number that appears the most often and in this set of data, 2 appears the most (three times). The mode is 2.
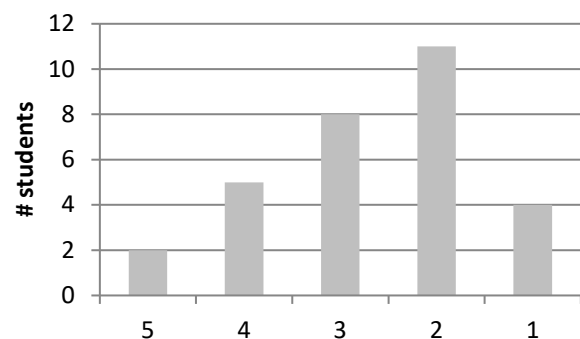
The range is the difference between the largest and smallest numbers, and is $27 - 2 = 25$.

The sum of these numbers is $10.5 + 2 + 25 = 37.5$, so the answer is (B).

SAMPLE PROBLEM: AVERAGES

The table below shows the distribution of missing assignments for the 30 students in Mr. Kang's Chemistry class. Use this data to answer the next three questions.

Missed assignments

1. What is the average number of missed assignments in the class?

We know how to find average in a general way, but here the data is presented in a different way. The first bar (above the 5), really represents two students who are missing 5 assignments each. So if we're making a list of the students, there would be two 5's in the list. But we don't need to make a list to find the average, we need to find the total number of missing assignments, and divide it by the number of students. The first bar represents $5 \times 2 = 10$ missing assignments. In a similar manner, we can find the numbers that the other four bars represent:

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$5 \times 2 + 4 \times 5 + 3 \times 8 + 2 \times 11 + 1 \times 4 = 80.$

Now we can find the average by dividing:

Average $= \frac{80}{30} = 2\frac{2}{3}$. So, the average is $2\frac{2}{3}$.

2. What is the median number of missed assignments in the class?

To find the median, we need to know which two students are in the middle of the distribution. The trouble is that the way the data is presented groups the students, so it's difficult to find the two students in the middle. We can solve this by thinking about the students as numbers. Imagine the students all standing in a row, arranged according to the number of assignments they've missed. The 1st and 2nd students are the ones missing five assignments (the first bar on the graph). The 3rd through 7th students are the ones missing four assignments (the second bar). We need to know when we get to 15, and we can see that is happens right at the end of the third bar $(2 + 5 + 8 = 15)$. Therefore the 15th student is missing 3 assignments, and the 16th student is missing 2. The median is the average of those two numbers, 2.5.

The median is 2.5.

3. What is the mode of the data?

Finally, a question that makes use of the way the data is presented. Here we don't need to rearrange anything – the data is already in order, and even better, the tallest bar is the one that repeats the most.

The mode is 2.

Another problem type we may encounter on the ACT deals with *weighted averages*. You may have seen these in school, in classes where grades are determined by multiple types of assignments. For example, a teacher may assign a greater value to test grades than she assigns to homework grades or attendance.

The general rule for finding weighted averages is to multiply the weight of an item by the value for that item, and then summing all those values.

SAMPLE PROBLEM: AVERAGES

In Mr. Kang's Chemistry class, final grades are determined by assigning the following weights:

Participation: 10%
Homework: 15%
Quizzes: 20%
Tests: 30%
Final: 25%

If Peyton's averages are as listed below, what is the minimum score he can get on the final to get a 90 in the class?

Participation: 100%
Homework: 90%
Quizzes: 90%
Tests: 85%

Using the strategy mentioned earlier, we'll multiply the weights of each type of grade by Peyton's averages, and then add them together. Because the weights are given as percentages, we'll have to convert them to decimals before we multiply. Remember, we want the final grade to be a 90:

Participation: $(0.10)(100) = 10$

Homework: $(0.15)(90) = 13.5$

Quizzes: $(0.20)(90) = 18$

Tests: $(0.30)(85) = 25.5$

Final: $(0.25)(x) = 0.25x$

Now we add all these together, set them equal to 90 and solve for x:

$10 + 13.5 + 18 + 25.5 + 0.25x = 90$

$67 + 0.25x = 90$

$0.25x = 23$

$x = 92$

Peyton needs a 92 on the final to get a 90 in the class.

SAMPLE PROBLEM: AVERAGES

Sameer is trying to make the elite bowling league at MarveLanes. He needs an average of 220 in five games to make the league, and in his first four games he bowls 198, 221, 185, and 223. What does he need to bowl in the fifth game to make the team?

Always remember that if we're given an average and a number of numbers, we can find the sum by multiplying – in this case Sameer has five games and needs an average of 220 in each. That makes the total he needs $5 \times 220 = 1100$. Because we know his scores so far, we can just find the difference – this will be what he needs in his last game to make the team:

Total $= 198 + 221 + 185 + 223 = 827$

$1100 - 827 = 273$

Sameer needs 273 in his final game.

PRACTICE EXERCISES

1. In the set $\{1, 2, 3, 4, 5, 5, 7\}$, if the mode is a, the range is b, and the median is c, what is the sum of $b - a$ and $a - c$?

2. In the set $\{1, 2, 303\}$, what is the difference between the average and the median?

3. To increase the mean of a set of six numbers by 5, by how much would the sum of the six numbers have to increase?

4. If one number is removed from a set of numbers and the average is unchanged, what can we say about the number that was removed?

5. In a math class, the boys average 75 on the final exam while the girls average 85. Can we conclude that the class average is 80? Why or why not?

6. Find a set of three distinct positive integers whose average, median, and range are all equal, if the smallest integer is 5.

7. There is a short cut for computing the average of a set of numbers if the numbers all comprise an arithmetic sequence—that is, they are evenly spaced. See if you can guess it after finding the average of:

    a. $\{3, 4, 5, 6, 7\}$
    b. $\{-6, -4, -2, 0, 2, 4, 6\}$
    c. $\{22, 27, 32, 37, 42\}$

8. How does the short cut from Problem 8 work if there is an even number of elements in the set? Find the average of:

    a. $\{2, 4, 6, 8\}$
    b. $\{-20, -10, 0, 10, 20, 30\}$
    c. $\{571, 573, 575, 577, 579, 581, 583, 585\}$

9. If the median of a set of three numbers is the same as their average, what does that tell you about the three numbers? Explain.

10. What is the average of 1000 consecutive integers, the smallest one of which is 2012?

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TEST EXERCISES (TIME: 10 MINUTES)

11. A total of 999 donors have given an average of $1000 each to a candidate's campaign for governor. The 1000th donor is a multimillionaire who gives $51,000 to the campaign. By how many dollars does the average donation per donor increase as a result of the wealthy donor's gift?

    (A)   1
    (B)   50
    (C)   51
    (D)   499
    (E) 50000

12. The average of the numbers in a set is 30. However, while trying to calculate it, Peter accidentally divides by one less than the correct number of elements in the set, and as a result gets 35 as the average instead. What is the sum of the numbers in the set?

    (A) 150
    (B) 175
    (C) 180
    (D) 210
    (E) 245

13. A set contains an even number of distinct integers. An integer is added to the set which is greater than its median. As a result, the median of the set:

    (A) decreases.
    (B) increases.
    (C) stays the same.
    (D) is now equal to the number that was added.
    (E) may increase or decrease; it cannot be determined from the given information.

The following chart pertains to Questions 14 and 15.
It shows the number of participants in after school sports at Springfed High School, on both Varsity and Junior Varsity teams.

| Sport	Gender	Level	Number of Players
Soccer	Boys	Varsity	15
		JV	23
	Girls	Varsity	21
		JV	29
Baseball / Softball	Boys	Varsity	18
		JV	21
	Girls	Varsity	20
		JV	26
Track & Field	Boys	Varsity	16
		JV	34
	Girls	Varsity	13
		JV	27
Football	Boys	Varsity	28
		JV	40
Field Hockey	Girls	Varsity	18
		JV	0

14. What is the average number of students on a girls' varsity team?

    (A) 17
    (B) 18
    (C) 19
    (D) 20
    (E) 21

15. If there are a total of 78 male athletes at Springfed High (some in multiple sports), what is the average number of sports played per male athlete, to the nearest tenth?

    (A) 1.4
    (B) 1.5
    (C) 2.0
    (D) 2.4
    (E) 2.5

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16. Jeffrey, Walter, and Donny are bowling. The highest possible individual score in bowling is a 300. What is the least number of points Donny can score such that it is still possible for the three to have an average score of 250?

    (A) 50
    (B) 100
    (C) 150
    (D) 200
    (E) 250

17. Eleven test listeners were asked to listen to a new, unreleased song and rate it on a 0 to 10 scale. The average 0 to 10 score was less than 6, and the median score was the same as the mode. Which of the following could NOT be the median?

    (A) 7
    (B) 8
    (C) 9
    (D) 10
    (E) Any of the above could be the median.

18. The line graph below shows Maddie's scores on 3 tests in her Spanish class:
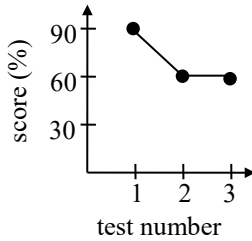
Which of the following graphs shows Maddie's cumulative average score after each test?

(A)

(B)

(C)

(D)

(E)
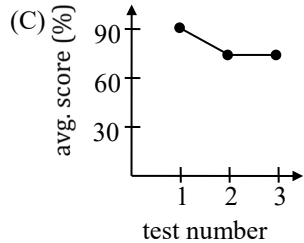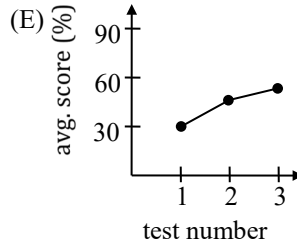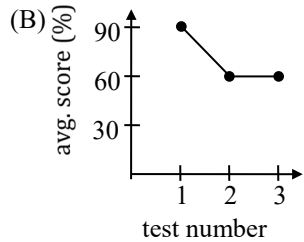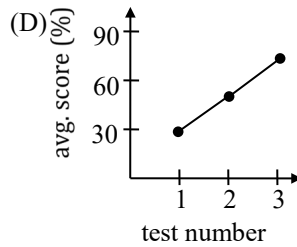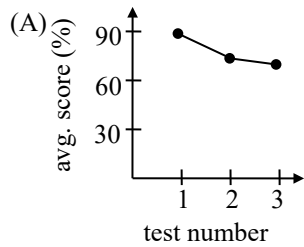
19. Twenty students take a 100-point test; all receive integer scores. If the average of their scores is 78, at most how many students could have received scores of 90 or higher?

(A) 9
(B) 10
(C) 15
(D) 17
(E) 19

20. In Mr. Khan's algebra class, four tests were given, counting for 10%, 20%, 30%, and 40% of the final grade, respectively. Alina scored 8 points higher on each test than on the preceding one. Her final grade in the class was a 92. What was her score on the first test?

(A) 68
(B) 72
(C) 76
(D) 84
(E) 92

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