



# Physics of Imaging Systems

## Basic Principles of Magnetic Resonance Imaging III

Prof. Dr. Lothar Schad

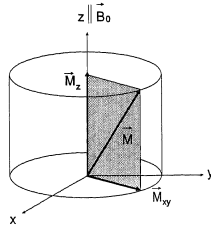


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[www.ma.uni-heidelberg.de/inst/cbtlm/ckm/](http://www.ma.uni-heidelberg.de/inst/cbtlm/ckm/)



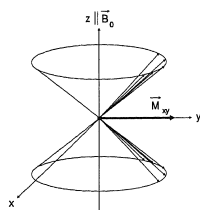
# Relaxation

## Magnetization: $M_z$ and $M_{xy}$



longitudinal magnetization:  $M_z$

transversal magnetization:  $M_{xy}$



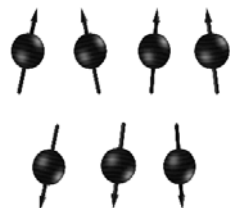
transversal magnetization:  $M_{xy}$

- phase synchronization after a  $90^\circ$ -pulse
- the magnetic moments  $\mu$  of the probe start to precess around  $B_1$  leading to a synchronization of spin packages  $\rightarrow M_{xy}$
- after  $90^\circ$ -pulse  $M_{xy} = M_0$

## Movie: $M_z$ and $M_{xy}$



Before exposure to RF radiation



the protons precess individually,  
i.e. out of phase, such that the individual  
transverse components compensate for each  
other and cancel out.  
The excess protons aligned parallel to the  $B_0$   
field result in a net magnetization in  
longitudinal direction.



After exposure to RF radiation



the protons precess synchronously,  
i.e. in phase. This results in a rotating  
component in the transverse direction.  
A longitudinal component is no longer present.



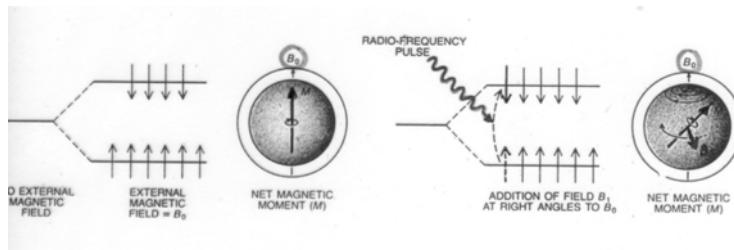
source: Schlegel and Mahr. "3D Conformal Radiation Therapy: A Multimedia Introduction to Methods and Techniques" 2007

## Longitudinal Relaxation Time: T1



thermal equilibrium

excited state



after  $90^\circ$ -pulse:

-  $N_{-1/2} = N_{+1/2}$  and  $M_z = 0$ ,  $M_{xy} = M_0$

after RF switched off:

- magnetization turns back to thermal equilibrium

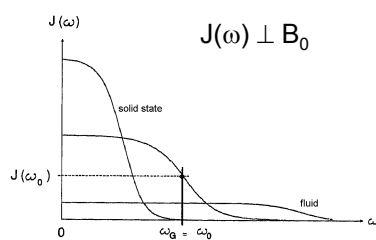
-  $M_z = M_0$ ,  $M_{xy} = 0$

→ T1 relaxation

longitudinal relaxation time T1

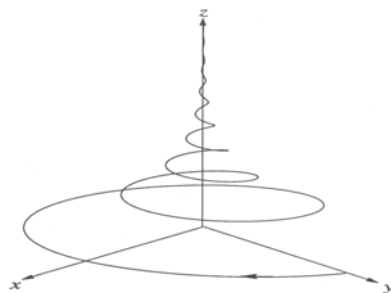
spin-lattice-relaxation time T1

## Physical Model of T1 Relaxation



- in a real spin system (tissue) every nuclei is surrounded by intra- and intermolecular magnetic moments
- thermal motion (rotation, translation, oscillation) leads to an additional fluctuating magnetic field  $B_{loc}(t)$  with typical spectral distribution  $J(\omega)$
- longitudinal components of  $J(\omega)$  at  $\omega_0$  allow energy transfer  $\hbar\omega_0$  from the spin system to the "lattice"

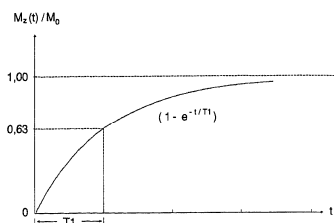
→ T1 relaxation



- trajectory of the tip of magnetization vector in the laboratory system

source: Liang and Lauterbur. "Principles of Magnetic Resonance Imaging" 2000

## Phenomological Description of T1 || B<sub>0</sub>

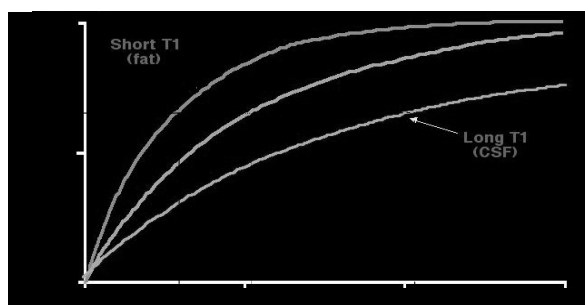


the longitudinal magnetization  $M_z$  relaxes exponential to the equilibrium state  $M_z = M_0$  with a typical time constant T1

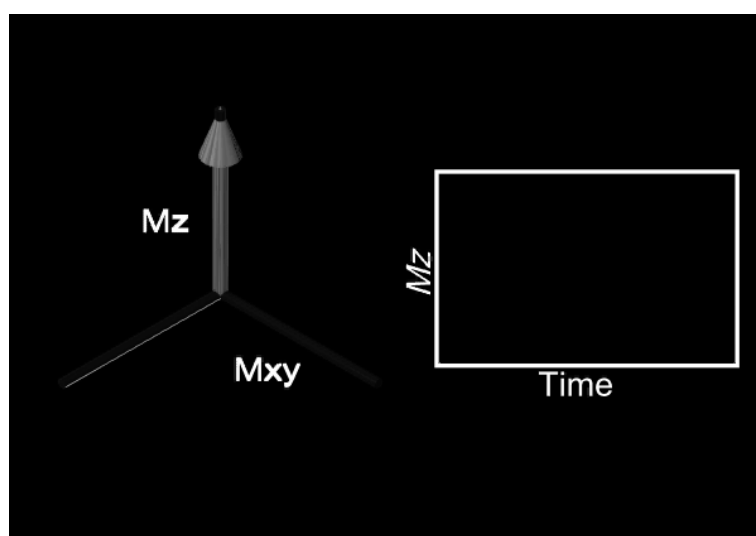
$$dM_z/dt = (\gamma \times B)_z + (M_0 - M_z)/T1 : \text{Bloch equation with T1}$$

with  $M_z = 0$  at  $t = 0$ :

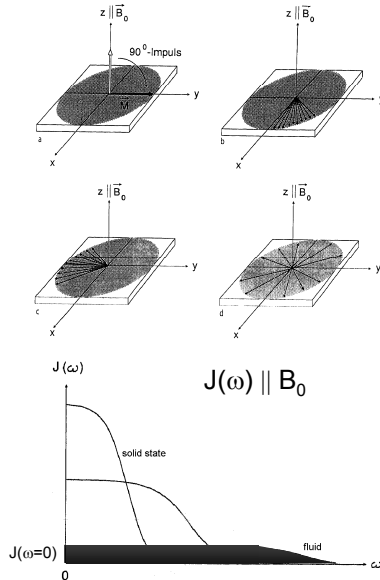
$$M_z(t) = M_0 (1 - \exp(-t/T1)) \rightarrow \text{solution of Bloch equation}$$



## Movie: T1 Relaxation



## Transversal Relaxation Time: T2



after  $90^\circ$ -pulse:

-  $N_{-1/2} = N_{+1/2}$  and  $M_z = 0$ ,  $M_{xy} = M_0$

after RF switched off:

- magnetization  $M_{xy}$  starts to rotate in the  $x,y$ -plane at Larmor frequency
- all transversal components  $J(\omega)$  of the fluctuating magnetic field  $B_{loc}(t)$  result in a dephasing of  $M_{xy} \rightarrow$  spin-spin interaction
- mainly static frequency components  $J(\omega)$  of the fluctuating magnetic field  $B_{loc}(t)$  at  $\omega = 0$  are contributing
- no energy transfer in the spin system (entropy  $\uparrow$ )
- no influence of T2 on T1, they are independent !

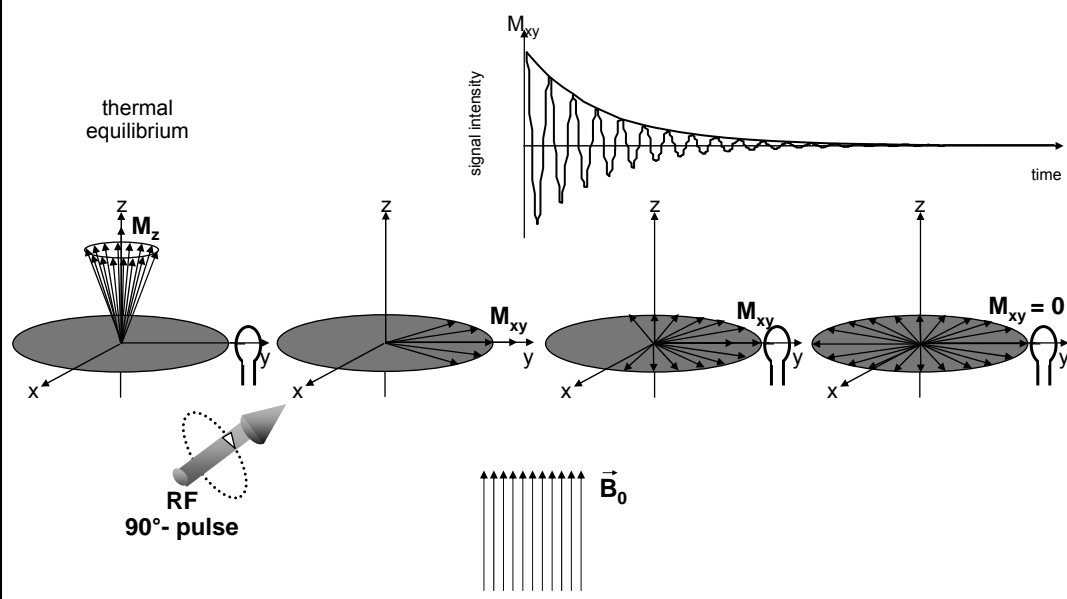
$\rightarrow$  T2 relaxation

transversal relaxation time T2

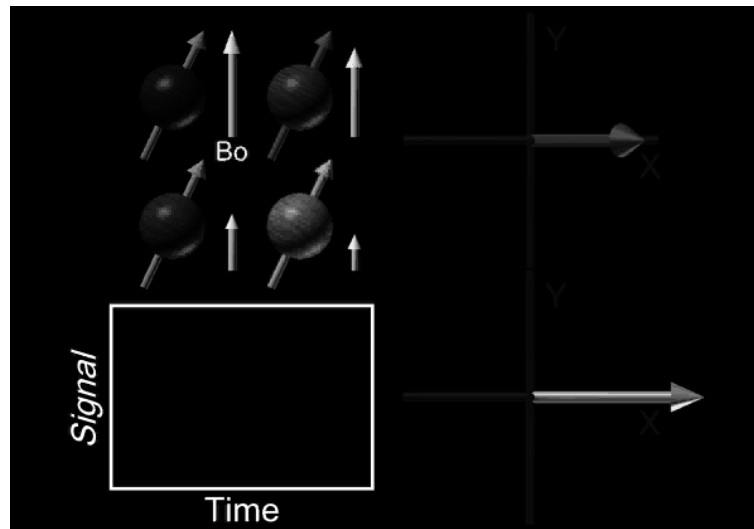
spin-spin-relaxation time T2

- although technical inhomogeneities of  $B_0$  cause dephasing of  $M_{xy} \rightarrow T2^*$  (effective relaxation)

## Physical Model of T2 Relaxation

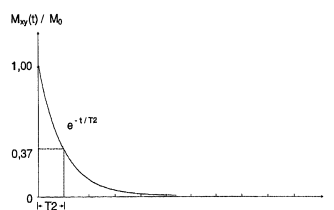


## Movie: Spin Dephasing



© Plewes DB, Plewes B, Kucharczyk W. The Animated Physics of MRI, University Toronto, Canada

## Phenomenological Description of $T_2 \perp B_0$

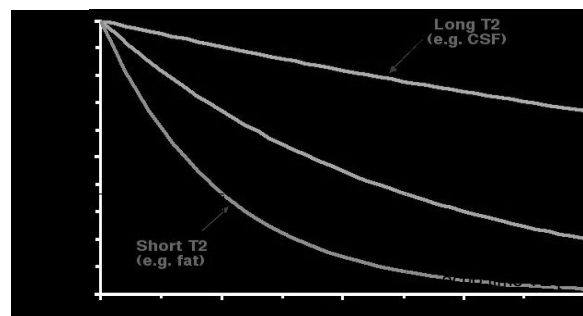


the transversal magnetization  $M_{xy}$  relaxes exponential to  $M_{xy} = 0$  with a typical time constant  $T_2$

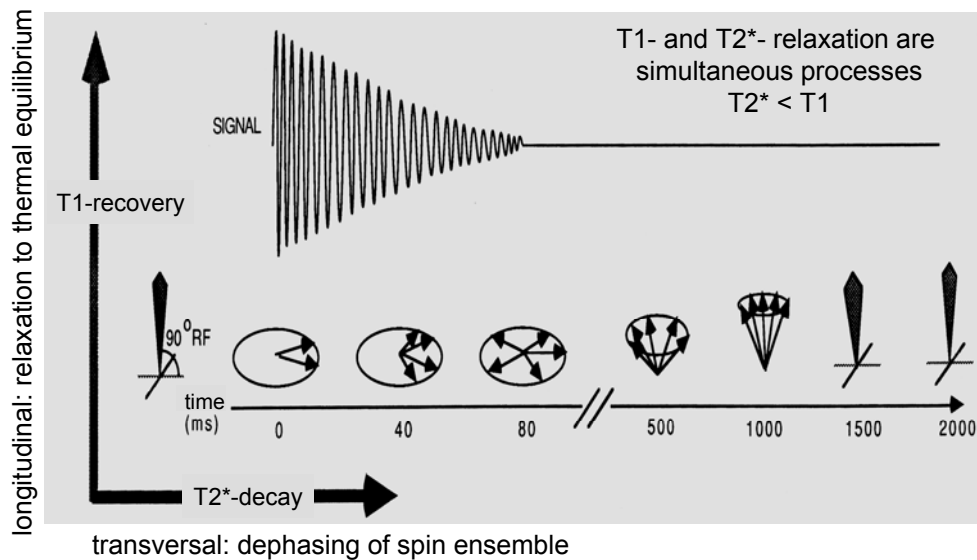
$dM_{xy}/dt = (\gamma \times B)_{xy} - M_{xy}/T_2$  : Bloch equation with  $T_2$

with  $M_{xy} = M_0$  at  $t = 0$ :

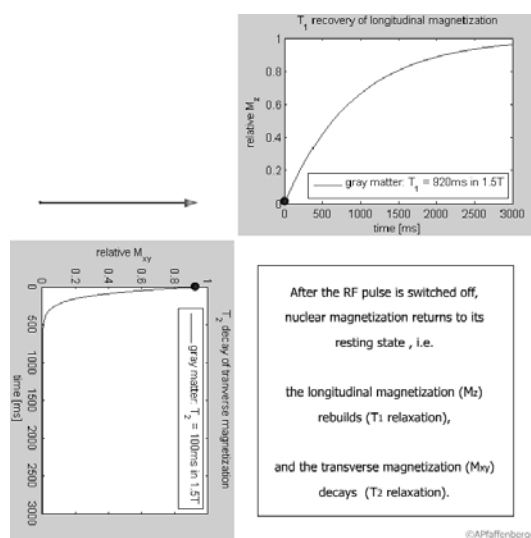
$M_{xy}(t) = M_0 \exp(-t/T_2)$  → solution of Bloch equation



## Simultaneous T1 and T2 Relaxation



## Movie: T1 and T2 Relaxation



source: Schlegel and Mahr. "3D Conformal Radiation Therapy: A Multimedia Introduction to Methods and Techniques" 2007

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## T1 and T2 Relaxation Times *in-vivo*

tissue	T2 [ms]	T1 [s] at 0.5 T	T1 [s] at 1.0 T	T1 [s] at 1.5 T
skeletal muscle	47 ± 13	0,55 ± 0,10	0,73 ± 0,13	0,87 ± 0,16
myocardium	57 ± 16	0,58 ± 0,09	0,75 ± 0,12	0,87 ± 0,14
liver	43 ± 14	0,33 ± 0,07	0,43 ± 0,09	0,50 ± 0,11
kidney	58 ± 24	0,50 ± 0,13	0,59 ± 0,16	0,65 ± 0,18
spleen	62 ± 27	0,54 ± 0,10	0,68 ± 0,13	0,78 ± 0,15
fat <sup>a</sup>	84 ± 36	0,21 ± 0,06	0,24 ± 0,07	0,26 ± 0,07
grey matter	101 ± 13	0,66 ± 0,11	0,81 ± 0,14	0,92 ± 0,16
white matter	92 ± 22	0,54 ± 0,09	0,68 ± 0,12	0,79 ± 0,13

<sup>a</sup> more than one exponential component

- T1 increases with  $B_0$
- T2 nearly independent of  $B_0$

Bottomley et al. Med Phys 1984

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## NMR History: Relaxation Times

Harvard  
1948

Nicolaas Bloembergen

Robert Pound

Edward Purcell

- characterized the relaxation times of the nuclear response signal in detail

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## Comparison: CT and MRI

**CT**

WM:	1025 Hu	} Δ = 1%
GM:	1035 Hu	
CSF:	1000 Hu	

**MRI**

	<b>T2</b>	<b>T1</b>	} Δ = 100%
WM:	90 ms	550 ms	
GM:	100 ms	1000 ms	
CSF:	>1000 ms	2000 ms	

patient: astrocytoma grade II

- no bones
- + best soft tissue contrast
- + no radiation

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## Bloch Equations with T1 and T2

$$\frac{dM_z}{dt} = (\gamma \times B)_z + (M_0 - M_z)/T_1$$

$$\frac{dM_{xy}}{dt} = (\gamma \times B)_{xy} - M_{xy}/T_2$$

rotating system:

$$\frac{d\vec{M}(t)}{dt} = \gamma \cdot \vec{M} \times \vec{B}(t) - \frac{M_x(t) \vec{i} + M_y(t) \vec{j}}{T_2} - \frac{(M_z(t) - M_z^0) \vec{k}}{T_1}$$

laboratory system:

$$M_x(t) = e^{-\frac{t}{T_2}} \cdot (M_x(0) \cos(\omega_0 \cdot t) + M_y(0) \cdot \sin(\omega_0 \cdot t))$$

$$M_y(t) = e^{-\frac{t}{T_2}} \cdot (M_y(0) \cos(\omega_0 \cdot t) - M_x(0) \cdot \sin(\omega_0 \cdot t))$$

$$M_z(t) = M_0 \cdot (1 - e^{-\frac{t}{T_1}}) + M_z(0) \cdot e^{-\frac{t}{T_1}} \quad \omega_0 = \gamma \cdot B_0$$

complex signal:

$$M_{\perp} = M_x + iM_y$$

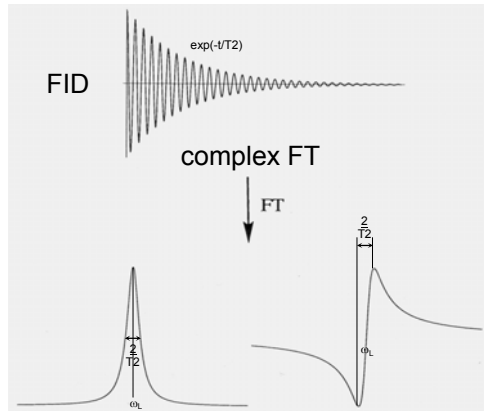
$$M_{\perp} = M_0 \exp(-i\omega_L t - t/T_2)$$

## Complex Signal: Simulated FID



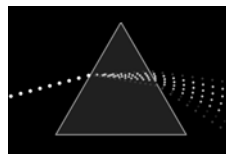
absorption  
 $M_x$ : real part FT

$$M_x(\omega) = M_0 \frac{T_2}{1 + (\omega - \omega_L)^2 T_2^2}$$



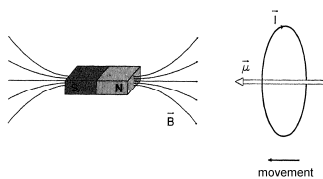
dispersion  
 $M_y$ : imaginary part FT

$$M_y(\omega) = M_0 \frac{T_2^2 (\omega - \omega_L)}{1 + (\omega - \omega_L)^2 T_2^2}$$



source: Liang and Lauterbur. "Principles of Magnetic Resonance Imaging" 2000

## Macroscopic Effect: Diamagnetism



- Lenz's law: the induced current produces an own magnetic moment  $\mu$  in a conductor opposite to  $B_0$

- most of biological tissues have diamagnetic properties since the electron magnetization  $M_e$  of the electron sheath is opposite to  $B_0$  due to Lenz's law:

$$B = \mu_0(H + M_e)$$

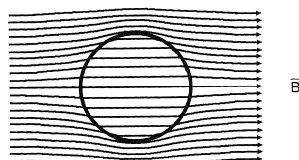
$$M_e = \chi H$$

$$\text{with } \mu_0 = 1.257 \cdot 10^{-6} \text{ Vs/A}$$

magnetic field constant

$$\chi_{H_2O} = -0.72 \cdot 10^{-6}$$

magnetic susceptibility

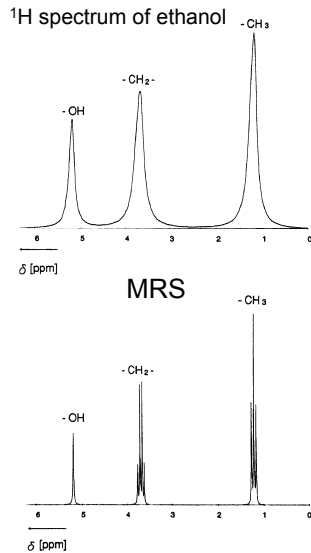


- weaker B-field inside a diamagnetic sphere due to e<sup>-</sup>-shielding which is very effective since

$$\gamma_{e^-} = 658 \gamma_p$$

- intersection of different tissues creates additional local field inhomogeneities of  $B_0$   
can be "homogenized" by additional shim coils

## Microscopic Effect: Chemical Shift



- precession frequency of nuclei bound in a specific molecule is determined by the local magnetic field  $B_{loc}$ :

$$B_{loc} = B - \delta B$$

$$\omega_{loc} = \gamma B_{loc} = \gamma(1 - \delta)B$$

with  $\delta = 10^6 (\omega - \omega_{ref})/\omega_0$   
the relative chemical shift [ppm]

-  $\delta \sim 10$  ppm for <sup>1</sup>H  
 $\delta \sim 100$  ppm for <sup>13</sup>C, <sup>19</sup>F, and <sup>31</sup>P

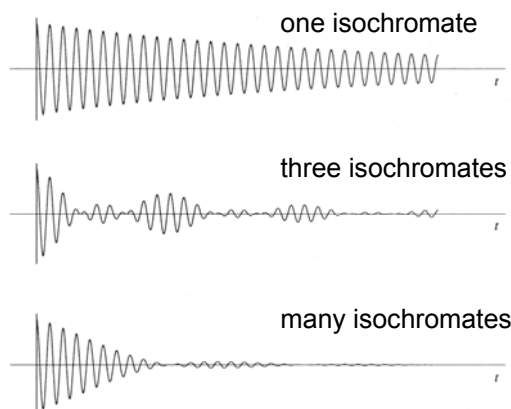
- high resolution spectrum at  $B_0 > 1.5$  T with  $\Delta B/B_0 < 0.1 - 0.5$  ppm show multiplet splitting due to spin-spin coupling  
 → domain of MRS

- in MRI only protons of water are imaged, chemical shift is not relevant!  
 exception:  $\delta_{fat} = 3.5$  ppm (220 Hz) at 1.5 T

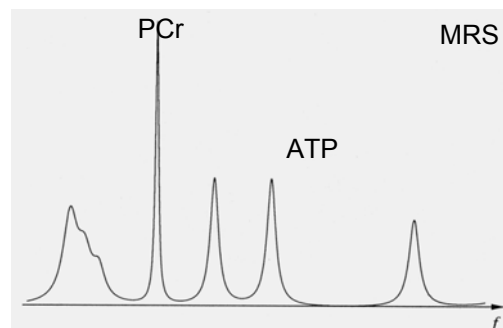
## Summary: FID and MRS



simulated isochromates



simulated <sup>31</sup>P absorption spectrum



source: Liang and Lauterbur. "Principles of Magnetic Resonance Imaging" 2000

## Summary: FID and MRI



FID signal is the transient response of a spin system after RF excitation;  
FID is a complex signal with amplitude and phase

FID amplitude is dependent on many parameters like: flip angle, number of spins, and magnetic field strength

FID timing is dependent on the grade of local magnetic field inhomogeneities characterized by  $T2^*$ :

$$1/T2^* = 1/T2 + \gamma\Delta B_z \text{ with } T2^* < T2$$

$T2^*$ : the effective (local)  $T2$  relaxation time

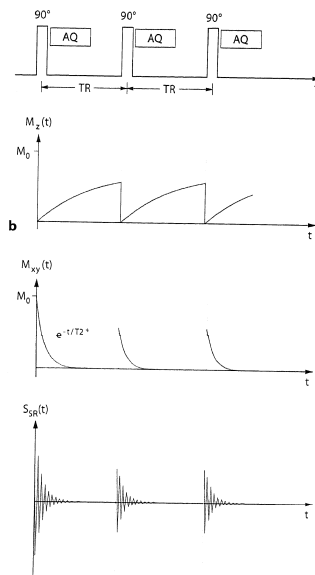
$T2$ : the true  $T2$  relaxation time

## Standard Techniques for $T1$ and $T2$



Saturation-Recovery Sequence  
Inversion-Recovery Sequence  
Spin-Echo Sequence

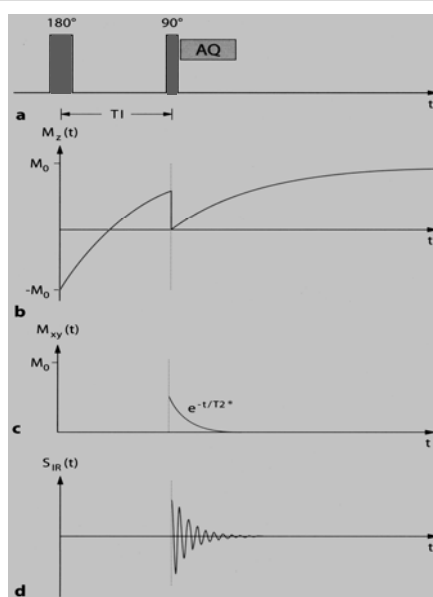
## Saturation-Recovery Sequence



- saturation-recovery sequence
- 90°-pulse moves the longitudinal magnetization  $M_0$  to the x-, y – plane → FID
- transversal magnetization  $M_{xy}$  decays with  $T2^*$
- longitudinal magnetization starts to recover to thermal equilibrium →  $M_z \uparrow$  with  $T1$
- after TR actual (reduced) magnetization  $M_z$  is moved to the x-, y – plane → FID
- repeat measurement with different TR →  $T1$  determination by

$$S \sim \rho [1 - \exp(-TR / T1)] \quad \text{with } TR \gg T2^*$$

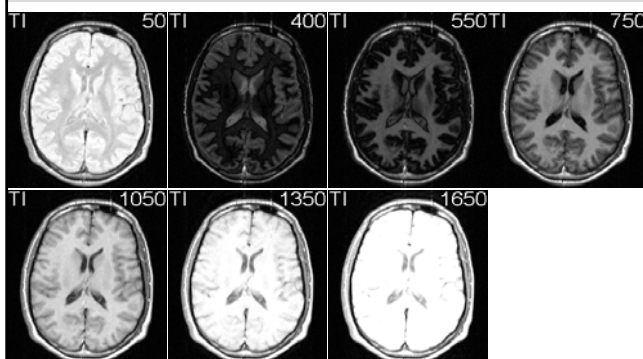
## Inversion-Recovery Sequence



- inversion-recovery sequence
- 180°-pulse invert the longitudinal magnetization  $M_0$  to  $-M_0$  at the z-axes
- longitudinal magnetization starts to recover to thermal equilibrium →  $M_z \uparrow$  with  $T1$
- inversion time  $TI$
- after  $TI$  90°-pulse moves the actual (reduced) longitudinal magnetization  $M_z$  to the x-, y – plane → FID
- transversal magnetization  $M_{xy}$  decays with  $T2^*$
- repeat measurement with different  $TI$  →  $T1$  determination by

$$S \sim \rho [1 - 2 \exp(-TI / T1)] \quad \text{with } TR > 5 T1$$

## T1 Measurement: Inversion Recovery



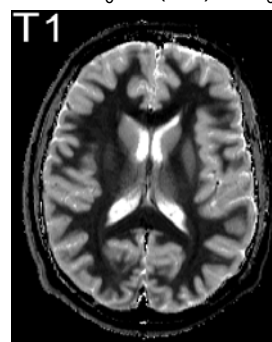
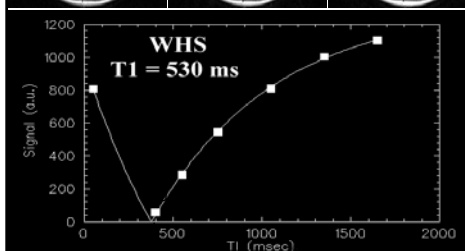
inversion recovery ( $M_z(0) = -M_0$ ):

$$M_z(t) = M_0 (1 - 2 \exp(-TI/T1))$$

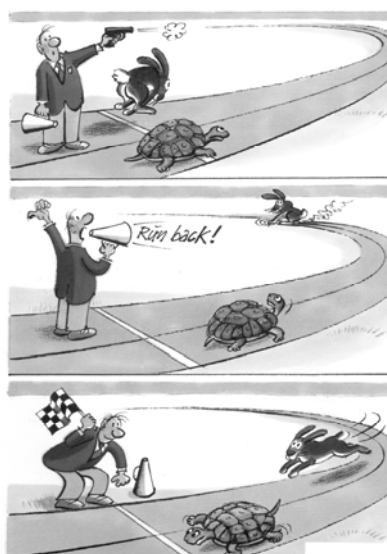
with  $M_z = 0$  at  $TI = TI_0$ :

$$0.5 = \exp(-TI_0/T1)$$

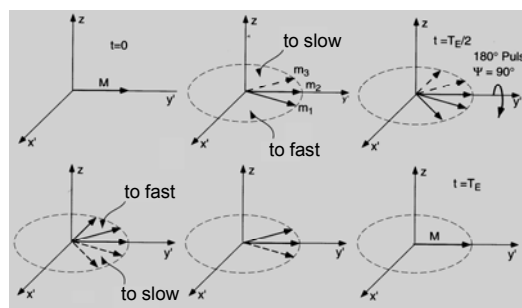
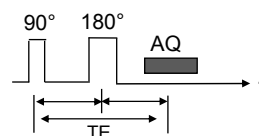
$$\rightarrow T1 = -TI_0 / \ln(0.5) = TI_0 / 0.7$$



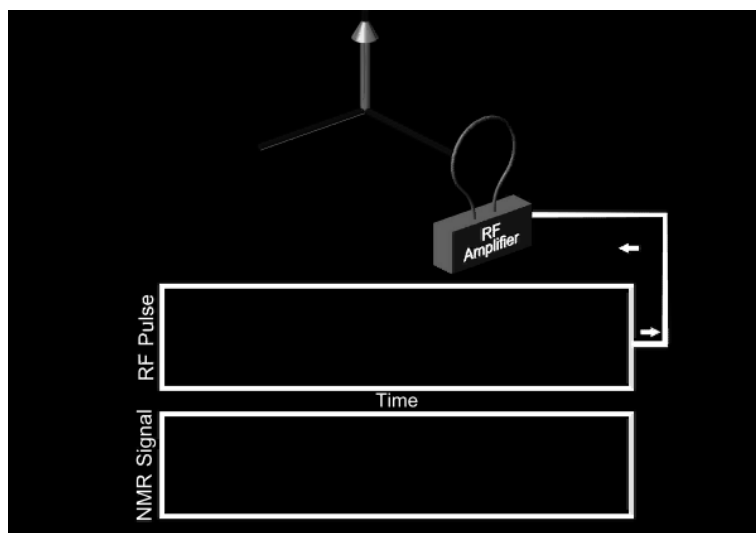
## How to get rid of "Scanner's" Dephasing ?



180° refocusing pulse

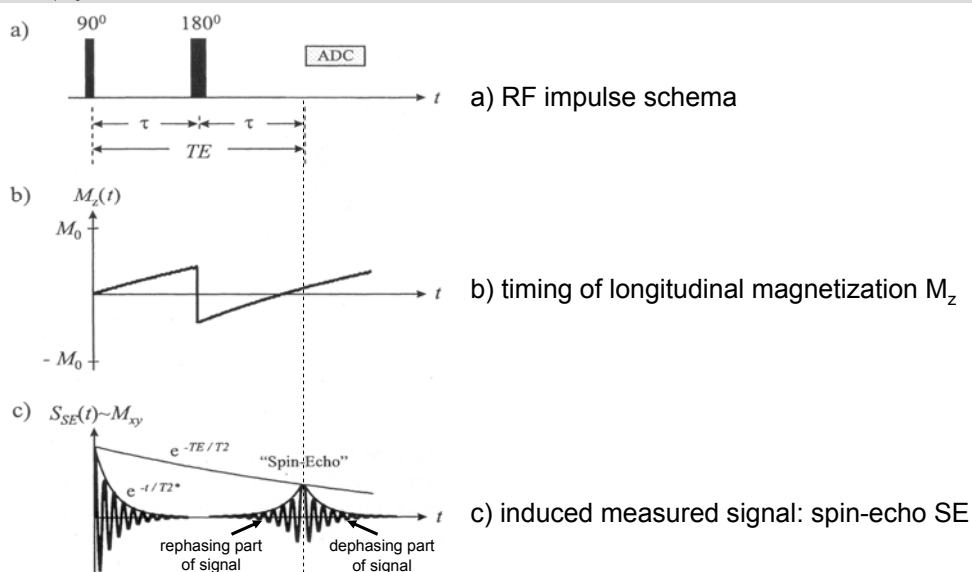


## Movie: Spin-Echo I



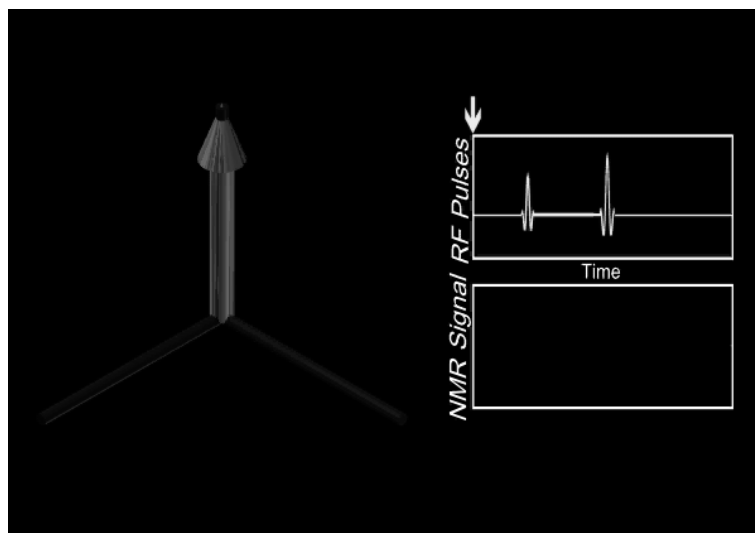
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## Spin-Echo Schema



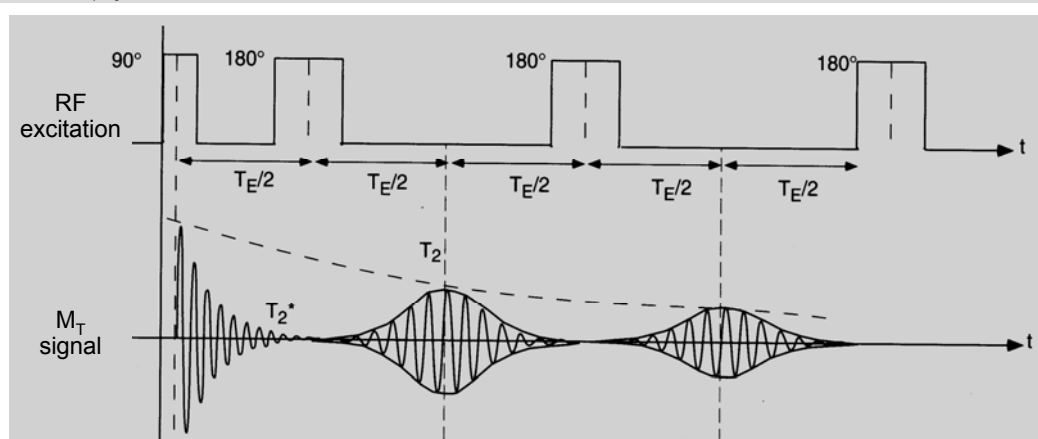
source: Schlegel and Bille. "Medizinische Physik Bd. 2" 2002

## Movie: Spin-Echo II



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## Multi Spin-Echoes



$$1/T_2^* = 1/T_2 + 1/T_2'$$

T<sub>2</sub>\* : „effective“ relaxation with T<sub>2</sub>\* < T<sub>2</sub>  
T<sub>2</sub> : „true“ relaxation due to irreversible dephasing  
T<sub>2</sub>' : „scanner“ relaxation due to static and constant field inhomogeneities

source: Dössel. „Bildgebende Verfahren in der Medizin“ 2000



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# NMR History: Spin-Echo

Illinois

1949

Erwin Hahn

- discovered a "second" nuclear resonance signal, the spin echo
- achieved T1 and T2 weighting

The first observed spin echo by  
E. Hahn (1950)

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# T2 Measurement by Multi Spin-Echo

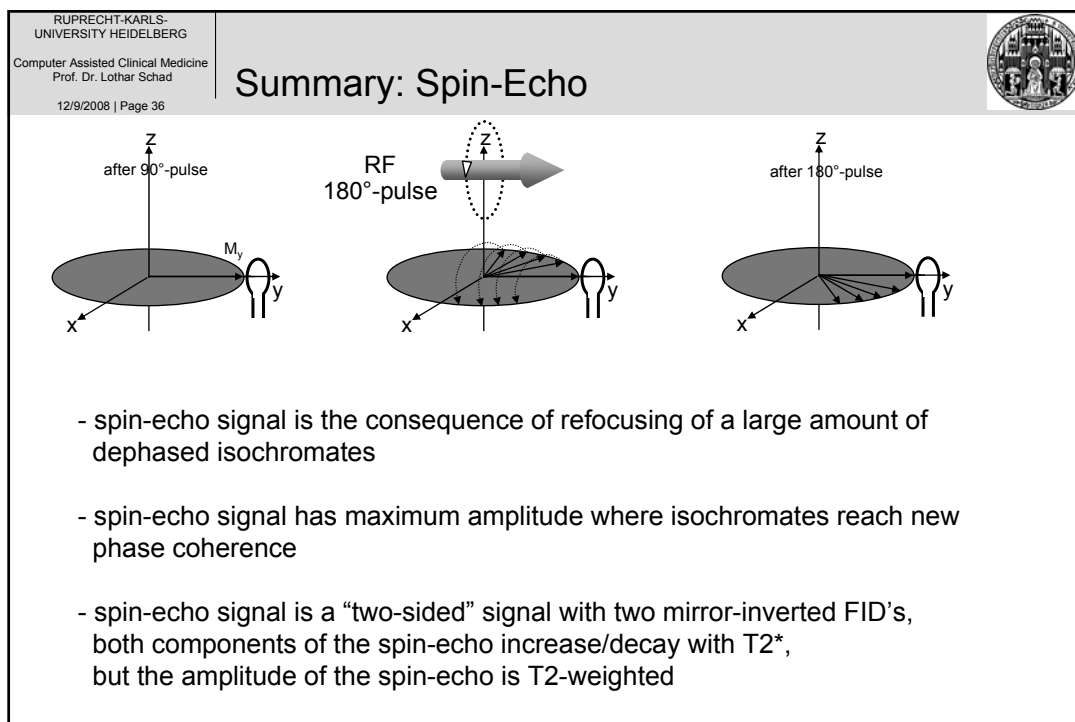
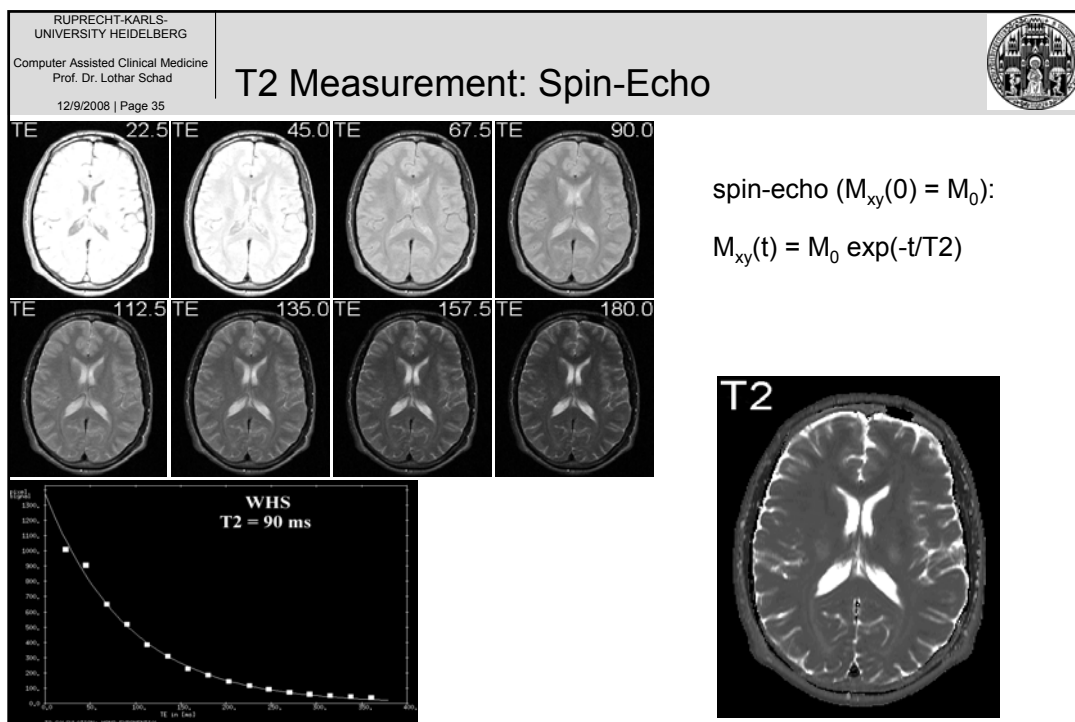
multi spin-echo technique

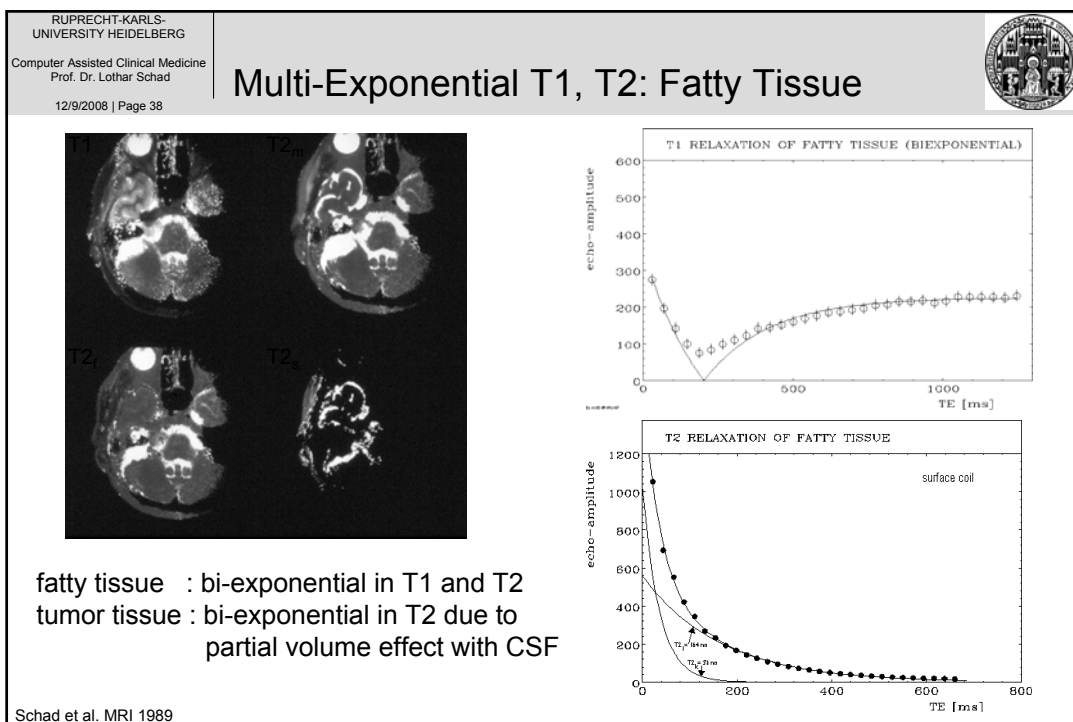
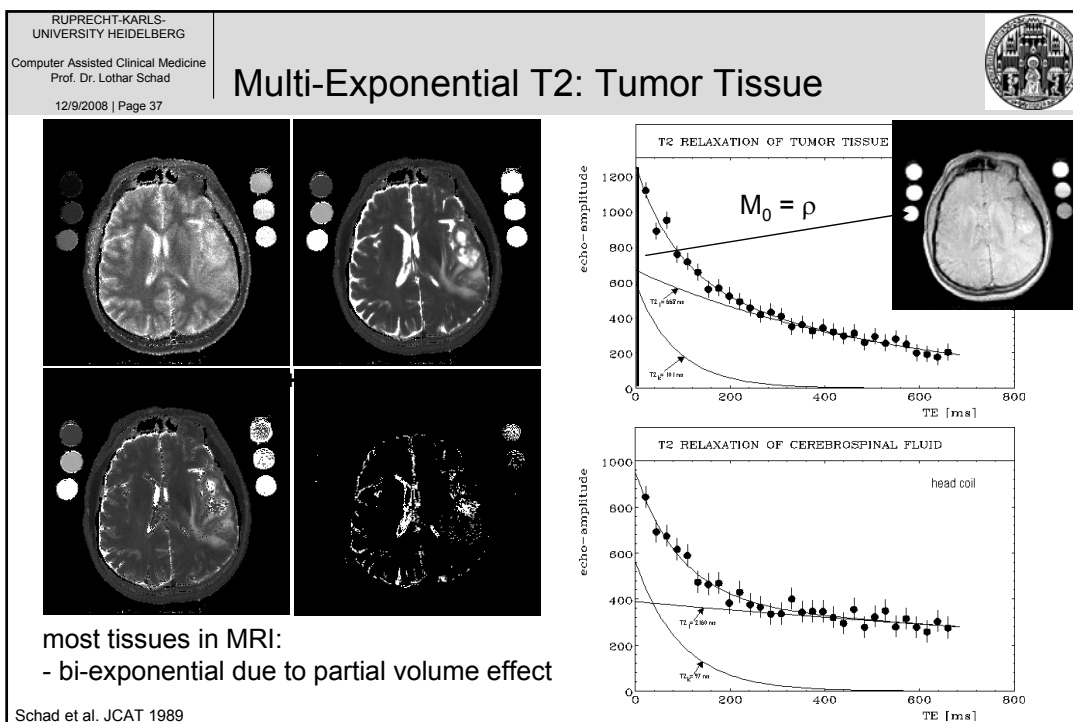
TR: repetition time  
TE: spin-echo time

SE-signal:

$$SI \sim M_{xy} = M_{xy} e^{-t/T_2}$$

WM:  $T_2 \cong 90 \text{ ms}$   
GM:  $T_2 \cong 100 \text{ ms}$   
CSF:  $T_2 > 500 \text{ ms}$





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MRI – Based Therapy Planning ?

tissue	p [N]	T2 [msec]	FB [N]
muscle	13,9 ± 1,4	< 2	82,4
	3,7 ± 0,5	5,5 ± 1,1	
	59,4 ± 0,7	30,4 ± 0,6	
	23,0 ± 1,0	105,0 ± 2,5	
fat	11,9 ± 2,6	< 2	88,1
	38,8 ± 1,5	53,7 ± 1,3	
	49,3 ± 1,7	193,9 ± 4,3	
grey matter	15,6 ± 0,4	< 2	84,4
	84,4 ± 0,4	90,1 ± 0,2	
white matter	24,3 ± 0,8	< 2	69,9
	4,7 ± 0,3	4,5 ± 0,5	
	69,9 ± 0,7	65,5 ± 0,2	
liver	19,3 ± 0,3	< 2	80,7
	80,7 ± 0,3	52,4 ± 0,1	
kidney	16,1 ± 0,4	< 2	83,9
	83,9 ± 0,4	60,0 ± 0,1	

T2 RELAXATION OF MUSCLE

tissues in NMR:

- multi-exponential with very short T2 relaxation components → invisible at conventional MRI
- measurement of proton densities (e.g. for n-dosimetry, HI-therapy) is not possible !

Brix. Dissertation, Heidelberg 1988

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T1-, T2 – Based Tissue Segmentation

tissue characterization

tissue segmentation

Schad et al. ZMP 1992

Friedlinger et al. Comp Med Ima Graph 1995