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ECE 3530

Homework 8

1.

55, 56, 57, 58, 61, 72, 72, 72, 73

a)

Mean

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{9} 55+56+57+58+61+72+72+72+73$$

$$= 64$$

Median

$$\tilde{X} = 61$$

Mode = 72

b)

$$s^2 = \frac{1}{8} \sum_{i=1}^{9} (x_i - \bar{x})^2$$

$$= \frac{1}{8}\Big[\overset{81}{(55-64)^2} + \overset{64}{(56-64)^2} + \overset{49}{(57-64)^2} + \overset{36}{(58-64)^2} + \overset{9}{(61-64)^2} + \overset{64}{(72-64)^2} + \overset{64}{(72-64)^2} + \overset{64}{(72-64)^2} + \overset{81}{(73-64)^2}$$

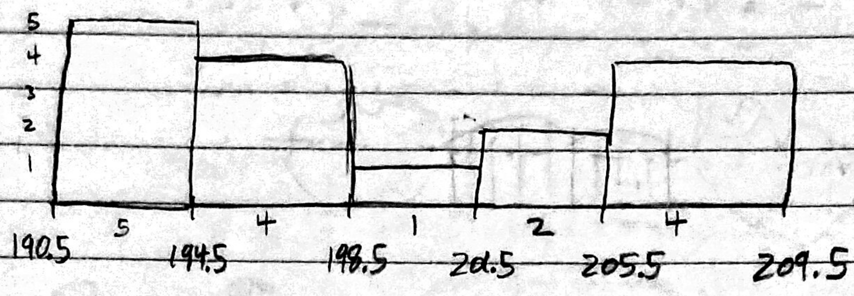
$$s^2 = 64$$

$$s = \sqrt{64} = 8$$

2.

192, 193, 193, 193, 194, 195, 195, 196, 197, 199, 205, 205, 206, 207, 207, 207

a)



b)

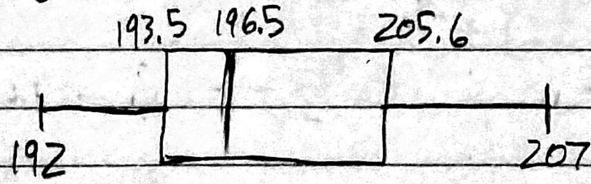
$n = 16$

Q_1: 193.5
Q_2: 196.5
Q_3: 205.6

$205.6 + 1.5(12.1) = 223.75$
$193.5 - 1.5(12.1) = 175.35$

outliers: none

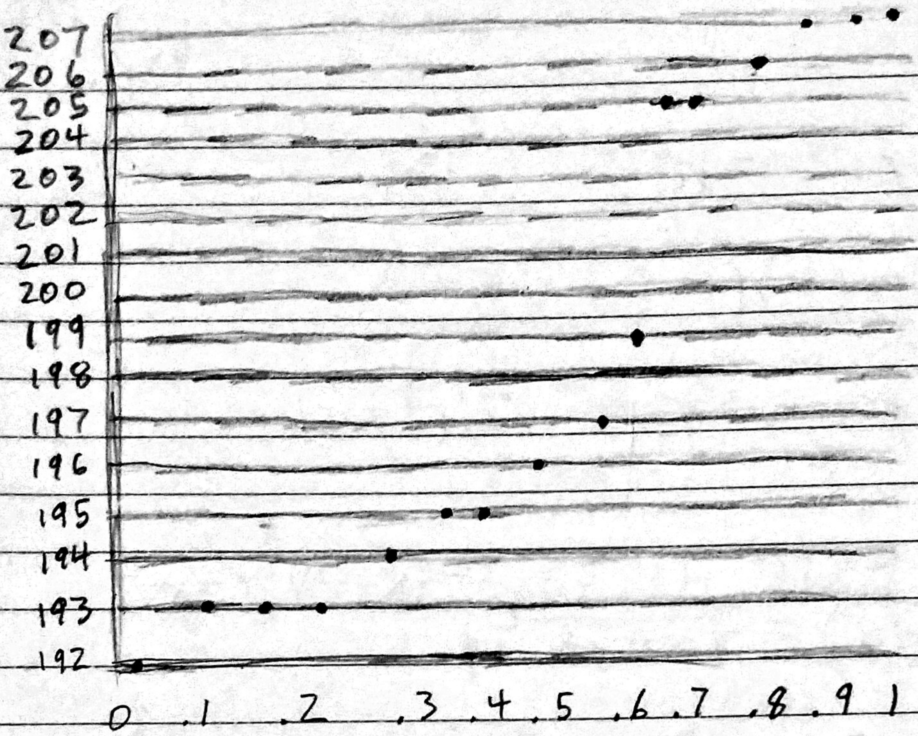
low: 192
high: 207



c)

$$q_i = \frac{i - \frac{3}{8}}{n + \frac{1}{4}}$$

.0384, .1, .162, .223, .285, .346, .408, .469, .553, .592, .654, .715, .777, .838, .9, .962



3.

X_1: 11
X_2: 9
X_3: 11
X_4: 8
X_5: 10
X_6: 11
X_7: 11
X_8: 12
X_9: 11
X_{10}: 11
X_{11}: 5
X_{12}: 14
X_{13}: 14
X_{14}: 9
X_{15}: 13
X_{16}: 12
X_{17}: 6
X_{18}: 5
X_{19}: 8
X_{20}: 11

$$\bar{X} = \frac{1}{20}(11+9+11+8+10+11+11+12+11+11+5+14+14+9+13+12+6+5+8+11)$$
$$= 10.65$$

$$s^2 = \frac{1}{19}((11-10.65)^2+(9-10.65)^2+(8-10.65)^2+(10-10.65)^2+(11-10.65)^2+(11-10.65)^2+(12-10.65)^2+(11-10.65)^2+(11-10.65)^2+(5-10.65)^2+(14-10.65)^2+(14-10.65)^2+(9-10.65)^2+(13-10.65)^2+(12-10.65)^2+(6-10.65)^2+(5-10.65)^2+(8-10.65)^2+(11-10.65)^2)$$

$$\downarrow$$

$$\frac{1}{19}(.1225+2.7225+7.0225+.4225+.1225+.1225+1.8225+.1225+.1225+31.9225+11.2225+11.2225+2.7225+5.5225+1.8225+21.6225+31.9225+7.0225+.1225)$$
$$= 7.248$$

$\mu = 20 \cdot \frac{1}{2} = 10$

$\sigma^2 = 20(\frac{1}{2})(1-\frac{1}{2}) = 5$

They seem to be fairly close.

b) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$

5, 8, 10

$z = \frac{5.5-10}{2.236} = -2.012$

Area = .0222

$z = \frac{4.5-10}{2.236} = -2.45$

Area = .0071

$P(x=5) = .0222 - .0071 = .0147$

$z = \frac{8.5-10}{2.236} = -.6708$

Area = .2514

$z = \frac{7.5-10}{2.236} = -1.118$

Area = .1335

$P(x=8) = .2514 - .1335 = .12013$

$$Z = \frac{10.5 - 10}{2.236} = .2236$$

Area $= .5871$

$$Z = \frac{9.5 - 10}{2.236} = -.2236$$

Area $= .4129$

$$P(X=10) = .5871 - .4129 = .1762$$

4.

$E[X] = 1$, $E[Y] = 5$
(means)

$E[X^2] = 5$, $E[Y^2] = 41$
(second Moments)

mean and variance?

a)

$2X+20$ $\mu = E(X)$ or $E(2X+20)$

$$\mu = E(2X+20) = 2E(X) + 20 = 2+20 = 22$$

$$\sigma^2_{ax+by} = a^2\sigma^2_X + b^2\sigma^2_Y = 2^2(5-1) + 20^2(0) = 16$$

b)

$$\mu = E(X+Y) = E(X) + E(Y) = 6$$

c)

$\mu = E(X-Y) = E(X) - E(Y) = -4$

d)

$\mu = E(10X+3Y) = 10E(X) + 3E(Y)$

$= 10+15 = 25$

e)

$\mu = E(10X+3Y^2) = 10E(X) + 3E(Y^2)$

$= 133$

5.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

a)
no, $n < 30$ and distribution unknown

b)

$$z = \frac{210 - 200}{12/\sqrt{9}} = 2.5$$

Area $= .9938$

$$z = \frac{190 - 200}{12/\sqrt{9}} = -2.5$$

Area $= .0062$

$P(190 < X < 210) = .9938 - .0062 = .9876$

c)

$$\mu = \sum x f(x) = 0 \cdot \frac{3}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} = 1$$

$$\sigma^2 = E[X^2] - \mu_x^2$$

$$E[X^2] = (0 \cdot \frac{3}{8} + 1 \cdot \frac{3}{8} + 4 \cdot \frac{1}{8} + 9 \cdot \frac{1}{8} = 2$$

$\sigma^2 = 2 - 1 = 1$

$$z = \frac{1.2 - 1}{1/\sqrt{100}} = 2$$

$\sigma = \sqrt{\sigma^2} = 1$

Area $= .9772$

$P(X > 1.2) = 1 - .9772 = .0228$