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ECE 3530

Homework 10

1.

a) $x_1 = 1$ $\quad y_1 = 0$
$x_2 = 2$ $\quad y_2 = 7$
$x_3 = 3$ $\quad y_3 = 6$
$x_4 = 4$ $\quad y_4 = 14$
$x_5 = 5$ $\quad y_5 = 11$
$x_6 = 6$ $\quad y_6 = 10$

x_i	$\bar{x}$	y_i	$\bar{y}$	$x_i - \bar{x}$	$y_i - \bar{y}$	SXY = 35	SXX = 17.5
1	3.5	0	8	-2.5	-8	20	6.25
2	"	7	"	-1.5	-1	1.5	2.25
3	"	6	"	-.5	-2	1	.25
4	"	14	"	.5	6	3	.25
5	"	11	"	1.5	3	4.5	2.25
6	"	10	"	2.5	2	5	6.25

$$b = \frac{SXY}{SXX} = \frac{35}{17.5} = 2$$

$$a = \bar{y} + b\bar{x} = 8 + 2(3.5) = 15$$

$$\hat{y} = 15 + 2x_i$$

b)
$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

$$SST = 64 + 1 + 4 + 36 + 9 + 4 = 118$$

$$SSE = (0-15-2(1))^2 + (7-15-2(2))^2 + (6-15-2(3))^2$$
$$+ (14-15-2(4))^2 + (11-15-2(5))^2 + (10-15-2(6))^2$$
$$= 289 + 144 + 225 + 81 + 196 + 289 = 1224$$
$$R^2 = 1 - \frac{1224}{118} = -9.37$$

2.

a)

Aiden is right, least squares method finds optimal coefficients for poly models

b)

2 points

3.

a)

$H_0: \sigma^2 = 1$
$H_1: \sigma^2 \neq 1$

$$\frac{(61-1)s^2}{\chi^2_{.025}} < \sigma^2 < \frac{(61-1)s^2}{\chi_{.975}}$$

$$\frac{(60)s^2}{83.298} < 1 \qquad \frac{(60)s^2}{40.482} > 1$$

$s < 1.3883$, $s > .6747$

b)

$H_0: \mu^2 = 0$
$H_1: \mu^2 = 0$

$$\bar{x} - z_{\alpha/2}\frac{1}{\sqrt{61}} < 0 < \bar{x} + z_{\alpha/2}\frac{1}{61}$$

$$\bar{x} - z_{.025}\left(\frac{1}{\sqrt{61}}\right) < 0 < \bar{x} + z_{.025}\left(\frac{1}{61}\right)$$

$$\bar{x} - .975\left(\frac{1}{\sqrt{61}}\right) < 0 < \bar{x} + .975\left(\frac{1}{\sqrt{61}}\right)$$

$\bar{x} < .125$ $\qquad\qquad$ $x > -.125$

4.

$$\bar{x} = \frac{1}{27}(65+65+65+65+64+67+68+68+58+65+65+65+64+64+66+63+61+72+64+64+63+63+67+67+67+60+70)$$

$= 65$

$$S^2 = \frac{1}{26}\big(\cancel{(65-65)}^2+\cancel{(65-65)}^2+\cancel{(65-65)}^2+\cancel{(65-65)}^2+\cancel{(64-65)}^2+\cancel{(67-65)}^2+\cancel{(68-65)}^2+\cancel{(68-65)}^2+\cancel{(58-65)}^2+\cancel{(65-65)}^2+\cancel{(65-65)}^2+\cancel{(65-65)}^2+\cancel{(64-65)}^2+\cancel{(64-65)}^2+\cancel{(66-65)}^2+\cancel{(63-65)}^2+\cancel{(61-65)}^2+\cancel{(72-65)}^2+\cancel{(64-65)}^2+\cancel{(64-65)}^2+\cancel{(63-65)}^2+\cancel{(63-65)}^2+\cancel{(67-65)}^2+\cancel{(67-65)}^2+\cancel{(67-65)}^2+(60-65)^2+(70-65)^2\big)$$

$$= 1(5)+4(4)+9(2)+49+1(1)+4(3)+16(1)+49+25+25$$

$= 216$

$s = \sqrt{216} = 14.7$

5.

$x_1 = 350$ $y_1 = 31.4$
$x_2 = 360$ $y_2 = 35.6$
$x_3 = 370$ $y_3 = 41.8$
$x_4 = 380$ $y_4 = 51$
$x_5 = 390$ $y_5 = 56.8$
$x_6 = 400$ $y_6 = 62.8$
$x_7 = 410$ $y_7 = 67.4$

a)

x_i	$\bar{x}$	y_i	$\bar{y}$	$x_i - \bar{x}$	$y_i - \bar{y}$	$S_{xy} = 1774$	$S_{xx} = 2800$
350	380	31.4	49.54	-30	-18.14	544.2	900
360	"	35.6	"	-20	-13.94	278.8	400
370	"	41.8	"	-10	-7.74	77.4	100
380	"	51	"	0	1.46	0	0
390	"	56.8	"	10	7.26	72.6	100
400	"	62.8	"	20	13.26	265.2	400
410	"	67.4	"	30	17.86	535.8	900

$\bar{x} = \frac{1}{7}(350 + 360 + 370 + 380 + 390 + 400 + 410)$

$\bar{y} = \frac{1}{7}(31.4 + 35.6 + 41.8 + 51 + 56.8 + 62.8 + 67.4)$

$b = \frac{1774}{2800} = .633$

$a = 49.54 - .633(380) = -191$

$\hat{y} = -191 + .633x$

b)

$\hat{y} = -191 + .633(430)$

$= 81.2337$

c)

$v = 5, \quad 1-\alpha = .95$

$\alpha = .05$

$.633 - 2.571\left(\frac{s}{\sqrt{2800}}\right) < \beta < .63 + 2.571\left(\frac{s}{\sqrt{2800}}\right)$

$SSE = (31.4 + 191 - .633(350))^2 + (35.6 + 191 - .633 \cdot 360)^2 + (41.8 + 191 - .633(370))^2 + (51 + 191 - .633(380))^2 + (56.8 + 191 - .633(390))^2 + (62.8 + 191 - .633(400))^2 + (67.4 + 191 - .633(410))^2$

$SSE = .7225 + 1.6384 + 1.9881 + 2.1316 + 137.5929 + .36 + 1.2769$

$= 145.7104$

s^2 can also be $\frac{s_{yy} - b\,s_{xy}}{n-2}$

$s^2 = \frac{145.7104}{5} = 29.14208$

$s = \sqrt{29.14208} = 1.3403$

$.633 - 2.571\left(\frac{1.3403}{\sqrt{2800}}\right) < \beta < .633 + 2.571\left(\frac{1.3403}{\sqrt{2800}}\right)$

$0.5084 < \beta < 0.6987$

d)