Homework 1

1. A coin is tossed 4 times. We use H to denote heads and T to denote tails. For instance, HTHH means we got the sequence heads, tails, heads, heads.

   a) Write down the elements of the sample space. *Hint: Using a tree diagram can be helpful.*

   b) List the outcomes included in each of the following events. Also compute the probabilities for each event. Assume that the coin is fair.
   - Event A: Number of times the coin came up heads is 0.
   - Event B: Number of times the coin came up heads is 1.
   - Event C: The first two tosses came up heads.
   - Event D: Number of times the coin came up heads is greater than 1.

   c) Answer YES or NO to the following questions:
   - Are the events A and B mutually exclusive?
   - Are the events B and C mutually exclusive?
   - Are the events C and D mutually exclusive?

   d) Compute the probability $P(A \cup D)$.

   e) Compute the probability $P(C \cup D)$.

2. You are given the sample space, $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, and the events X = even numbers, Y = numbers greater than or equal to 6, $Z = \{2, 4\}$. You are told that $P(0) = P(2) = P(4) = P(6) = P(8)$ and $P(1) = P(3) = P(5) = P(7)$. Furthermore, you are told that odd numbered outcomes are twice as likely as the even numbered outcomes.

   (a) Draw the Venn diagram showing the sample space, S, and events X, Y, Z. Note: The areas of the events in a Venn diagram don't have to be proportional to their probabilities, but you should correctly show the relationship of the events. For instance, if two events are mutually exclusive, they should not intersect in the Venn diagram.

   (b) List the elements of the each of the following and compute their probability.
   - $Z \cup Y$
   - $X \cup Y$
   - $X' \cap Z$
   - $(X \cap Z)'$
   - $(X \cap Z) \cup Y$

3. Among 110 students in a probability class. On a three-question exam, 29 get question 1 incorrect, 39 get question 2 incorrect, and 22 get question 3 incorrect. Further, 19 students get both questions 1 and 2 incorrect, 13 get both questions 2 and 3 incorrect, and 7 get both questions 1 and 3 incorrect. In addition, 6 get all three questions incorrect. Find the probability that a randomly selected student:

   a) Gets question 1 incorrect but gets question 2 correct.

   b) Gets questions 2 and 3 incorrect but gets question 1 correct.

   c) Gets all three questions correct.

4. Let S represent the sample space of all houses in zip code 84106. You are also given the events:

   - A = Houses built after 1990
   - B = Houses with a 2-car garage
   - C = Houses with lead-based paint

   $P(A) = 0.1, P(B) = 0.6, P(C) = 0.6,$

   $P(A \cap C) = 0, P(A \cap B) = 0.1,$ and $P(B \cap C) = 0.4$.

   a) Compute the probability $P(A \cap B \cap C)$. *Hint:* $A \bigcap B \bigcap C = (A \bigcap C) \bigcap B$

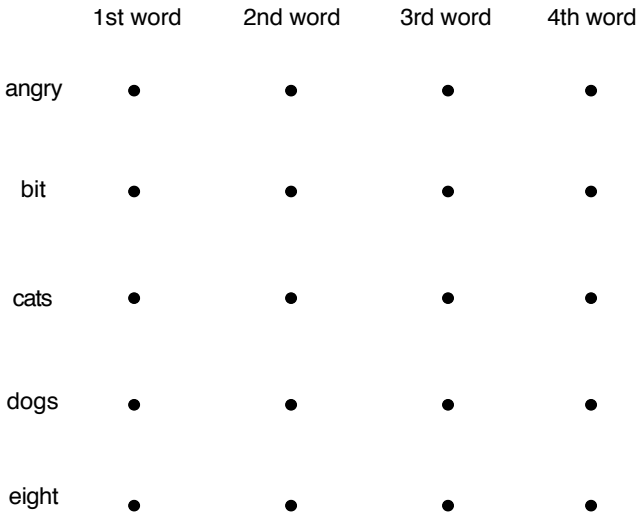
   b) Compute the probability $P(A \bigcup B \bigcup C)$.

   c) Using rules of probability and the probabilities you've been given above, show that it is not possible to find a house in this zip code built after 1990 that doesn't have a 2-car garage. *Hint: First write this event as an intersectio of two other events or their complements.*

5. Consider the following vocabulary words:

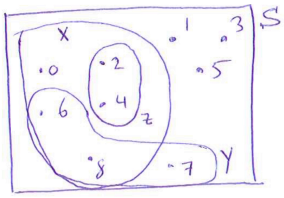
angry bit cats dogs eight

a) Using a diagram like the one shown below, draw paths for all grammatically correct 4-word sentences containing both the words "cats" and "dogs". Note that the same word may be used more than once. (Answers may vary, according to what you consider to be grammatically correct.)



b) In speech recognition, large documents are analyzed to estimate probabilities of certain words following other words. By counting path segments in your diagram, estimate the probability, P(bit, cats), that a transition from one word to the next, chosen at random from all those that are possible, will be the transition "bit cats". Assume all sentences in your diagram are equally likely. (Note that this is *not* the same as saying that all transitions are equally likely, since some transitions occur in multiple sentences.) Note: answers may vary, according to what you considered grammatically correct in (a), but your answer to (b) must be consistent with your answer to (a).

ANS:

1. a) S={HHHH, HHHT, HHTH, and 13 more outcomes}, b) 1/4, 1/4, 11/16, c) yes, yes, no
2. a)  b) 6/13, 7/13, 0, 11/13, 6/13
3. a) 1/11, b) 7/110, c) 53/110
4. a) $P(A \cap B \cap C) = 0$, b) 0.8, c) 0
5. Answers vary.