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ECE 3530

Homework 6

1.

$f(x,y)$	$x=0$	$x=1$
$Y=0$	0.1	0
$Y=1$	0.1	0.1
$Y=2$	0.1	0.2
$Y=3$	0	0.4

$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

a) $E(X \cdot Y) = \sum_x \sum_y xy\, P(x,y)$

ANSWER $\rho_{XY} = \frac{0.3}{(.46)(1)} = 0.65$

$\sigma_{xy} = E(X \cdot Y) - \mu_x \cdot \mu_y$

$\mu_X = \sum_x (x P(x))$ $P(x) = \sum_y P(x,y)$

$\sigma_x = \sqrt{\sigma_x^2}$ $\sigma_x^2 = E(x^2) - \mu_x^2$

$\sigma_y^2 = E(y^2) - \mu_y^2$

$\sigma_y = \sqrt{\sigma_y^2}$ $E(x^2) = \sum_x x^2 P(x)$ $\mu_Y = \sum_y (P_Y(y))$ $P(Y) = \sum_x P(x,Y)$

$g(x) = \sum_y f(x,y)$

$h(x) = \sum_x f(x,y)$

$\boxed{P(x=0) = 0.3 \quad P(X=1) = 0.7}$

$\boxed{\sigma_x = \sqrt{0.21} = \boxed{.46}}$

$\boxed{\sigma_y = \sqrt{1} = \boxed{1}}$

$P(Y=0) = 0.1$
$P(Y=1) = 0.2$
$P(Y=2) = 0.3$
$P(Y=3) = 0.4$

$\boxed{\mu_X = 0.3 \cdot 0 + 0.7 \cdot 1 = \boxed{0.7}}$

$\mu_Y = 0 \cdot 0.1 + \underbrace{1 \cdot 0.2}_{.2} + \underbrace{2 \cdot 0.3}_{.6} + \underbrace{3 \cdot 0.4}_{1.2} = \boxed{2}$

$E(X \cdot Y) = \underbrace{(0 \cdot 0 \cdot 0.1)}_{x=0\ y=0} + \underbrace{(0 \cdot 1 \cdot 0.1)}_{x=0\ y=1} + \underbrace{(0 \cdot 2 \cdot 0.1)}_{x=0\ y=2} + \underbrace{(0 \cdot 3 \cdot 0)}_{x=0\ y=3} + \underbrace{(1 \cdot 0 \cdot 0)}_{x=1\ y=0} + \underbrace{(1 \cdot 1 \cdot 0.1)}_{x=1\ y=1} + \underbrace{(1 \cdot 2 \cdot 0.2)}_{x=1\ y=2} + \underbrace{(1 \cdot 3 \cdot 0.4)}_{x=1\ y=3} = \boxed{1.7}$

0.1 + 0.4

1.2

$\boxed{\sigma x^2 = 0.7 - 0.49 = \boxed{.21}}$

$\boxed{\sigma y^2 = 5 - 4 = \boxed{1}}$

$\boxed{\sigma_{xy} = 1.7 - 0.7 \cdot 2 = \boxed{0.3}}$

$\boxed{E(y^2) = \sum_y y^2 P(y) = 0 \cdot 0.1 + 1 \cdot 0.2 + 4 \cdot 0.3 + 9 \cdot 0.4 = \boxed{5}}$

0 + 0.2 + 1.2 + 3.6

$\boxed{E(x^2) = \sum_x x^2 P(x) = 0 \cdot 0.3 + 1 \cdot 0.7 = \boxed{0.7}}$

b) $P(Y \geq 2, X=0) = \boxed{0.1}$

c) $P(Y \geq 2 \mid X=0) = P(y)(2|0) + P(y)(3|0)$

$= \frac{P(Y \geq 2, X=0)}{P(X)} + \frac{P(Y \geq 3, X=0)}{P(X)} = \frac{0.1}{0.3} + \frac{0}{0.3}$

$= \boxed{1/3}$

d) $\$(100X + 10Y^2) = v$

	X: 0	1
y: 0	0	100
1	10	110
2	40	140
3	90	190

$E(100X + 10Y^2) = \sum_x \sum_y v(x,y) f(x,y)$

$= (0 \cdot 0.1) + (10 \cdot 0.1) + (40 \cdot 0.1) + (90 \cdot 0)$
$+ (100 \cdot 0) + (110 \cdot 0.1) + (140 \cdot 0.2) + (190 \cdot 0.4)$

1, 4, 11, 28, 76

$= \$120$

e)

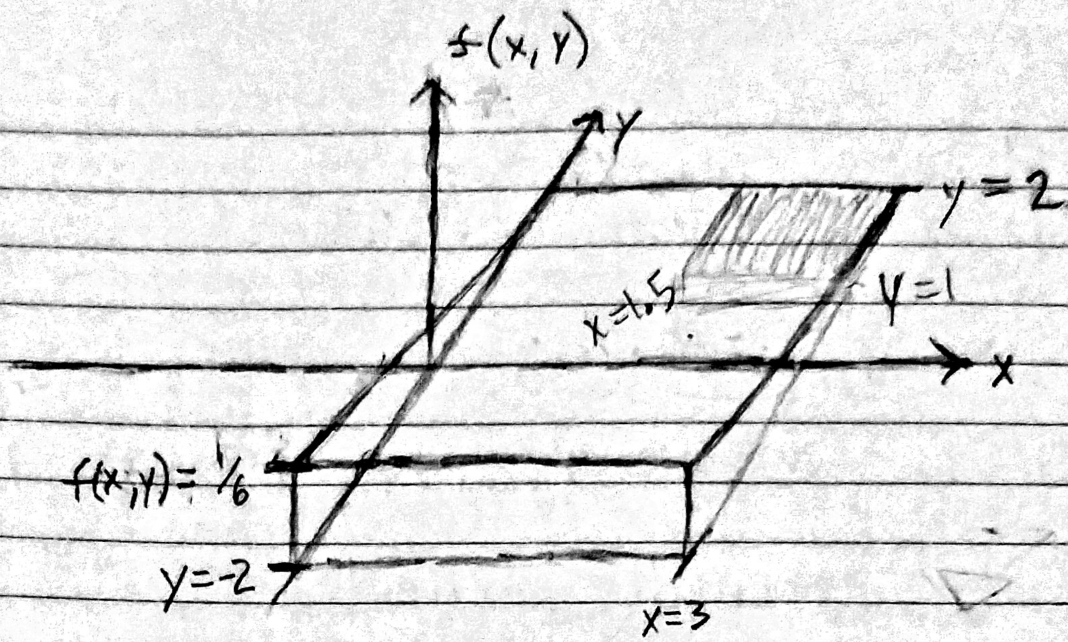
$\mu_Z = 1 \quad \sigma_Z^2 = 0.5 \quad E[2Y - 4Z] = 2\mu_Z - 4\mu_Y$

$2Y - 4Z \qquad = 4 - 4 = \boxed{0}$

$Var[2Y - 4Z] = 2^2 \sigma_Y^2 + (-4)^2 \sigma_Z^2$

$= 12$

2.



a)

$$0 \le x \le 3 \qquad -2 \le y \le 2$$

$$f(x,y) = \begin{cases} -\frac{1}{24}(y-2) & 0 \le x \le 3 \text{ and } -2 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

b)

$$\int_1^2 \int_{1.5}^3 -\frac{1}{24}(y-1)\,dx\,dy$$

$$\int_1^2 -\frac{1}{24}(y-2)x \Big|_{1.5}^3 dy$$

$$\int_1^2 -\frac{1}{24}(y-2)(1.5)\,dy$$

$$P(X>1.5 \text{ and } Y>1) = \frac{-1}{16}\left(\frac{3}{2}-2\right) = \boxed{\frac{1}{32}}$$

3.

keep this $\{P(10 \text{ years}) = 0.99$

$\mu_x = 10$ years

$\sigma^2 = 2.5$ years $\quad \sigma^2 = .5, 1.5, 2.5$

normally distributed

\$100 to reduce σ^2 to 1.5 years

or

\$200 to reduce to 0.5 years

\$100 per year to increase mean of lifetime

$$z = \frac{x - \mu}{\sigma}$$

$$-2.33 = \frac{10 - \mu}{0.5}$$

$$x = 11.165$$

$$\$100(11.165 - 10) + 200 = \$316.5$$

4.

$$g(x) = \begin{cases} 1/4, & x = 1, 2 \\ 1/8, & x = 3, 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

$$h(y) = \begin{cases} 1/3, & y = 1, 2 \\ 1/12, & y = 3, 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

a)

X

y \ x	1	2	3	4	5	6
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$	$1/24$	$1/24$
2	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{1}{24}$	$1/24$	$1/24$
3	$\frac{1}{48}$	$\frac{1}{48}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$
4	$\frac{1}{48}$	$\frac{1}{48}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$
5	$\frac{1}{48}$	$\frac{1}{48}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$
6	$\frac{1}{48}$	$\frac{1}{48}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$	$\frac{1}{96}$

y

$$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12}$$

$$1/8 + 1/12 = \frac{3}{24} + \frac{2}{24}$$

b)

$$P(x+y>8) = \frac{1}{96} \cdot 10 = \frac{10}{96} = \frac{5}{48}$$

5.

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2_{XY}}} e^{-(x^2 - 2\rho_{XY} \cdot xy + y^2)/2(1-\rho^2_{XY})}$$

a)

$$\frac{1}{2\pi\sqrt{1}} e^{\frac{-(x^2+y^2)}{2(1)}}$$

$$\frac{1}{2}\pi e^{-(x^2+y^2)/2}$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu_x)/2\pi_x^2}$$

$$\frac{1}{\sqrt{2\pi}} e^{(-x-\mu)^2/2\sigma^2} = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$\frac{1}{\sqrt{2\pi}} e^{(-y-\mu)^2/2\sigma^2} = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}$$

$$\left(\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\right)\left(\frac{1}{\sqrt{2\pi}} e^{-y^2/2}\right)$$

$$\frac{1}{2\pi} e^{(-x^2/2)(-y^2/2)}$$

$$\boxed{\frac{1}{2\pi} e^{-(x^2+y^2)/2}}$$

b) $\frac{1}{2\pi\sqrt{1-(1/3)^2}} e^{-(x^2-2(1/3)\cdot 3x+9)/2(1-(1/3)^2)}$

$\frac{1}{2\pi\sqrt{8/9}} e^{-(x^2-2x+9)/2(8/9)}$

$\sigma^2 = 8/9$

$$\frac{-(x^2-2x+1)+10}{2(8/9)}$$

$$\frac{-(x-1)^2+10}{2(8/9)}$$

$$\left(e^{\frac{-(x-1)^2}{2(8/9)}}\right)\left(e^{10/16/9}\right)$$

$$\boxed{\begin{array}{l} K = \sqrt{2\pi}\, e^{45/8} \\ \mu = 1 \end{array}}$$