Nathan Donaldson
ECE/CS 3530

Homework #4

1.

a) $f(x) = \frac{d}{dx} F(x) = \begin{cases} 0 & -\infty < x \leq 0 \\ \frac{0.2}{3} & 0 < x \leq 3 \\ 0.4 & 3 < x \leq 5 \\ 0 & 5 < x < \infty \end{cases}$

b) $P(x \leq 4kW) = F(x = 4kW) = 0.6$

c) $P(1kW \leq x \leq 3kW) = F(3kW) - F(1kW)$

$= 0.2 - \frac{0.2}{3} = .133$

2.

a) $f(x) = \begin{cases} 1.5 - 2|x| & -1 \leq x \leq 1 \\ 0 & o.w. \end{cases}$

$f(x) \geq 0$ ✓

$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{0} 1.5 - 2|x|\,dx + \int_{0}^{1} 1.5 - 2|x|\,dx$

$1.5x - x^2 \Big|_{-1}^{0} + 1.5x - x^2 \Big|_{1}^{0}$

No, because $\int_{-\infty}^{\infty} f \neq 1$

$(-1.5 - 1) - (0) + (1.5 - 1) - (0)$

$-2.5 + .5 = -2$ X

b)

$$F(x) = 1+\sin(x)$$

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx \quad \text{for } -\infty < X < \infty$$

$\boxed{\text{* Does not increase, alternates between 1, and -1}}$

$$\int_{-1}^{x} 1+\sin(x) = x - \cos(x) \Big|_{-1}^{x}$$

$$x - \cos(x) - (-1 - \cos(-1))$$

$$x - \cos(x) + 1.54$$

c)

$$F(x) = \begin{cases} 0 & x<1 \\ 2-\frac{1}{x} & x \geq 1 \end{cases}$$

$$F(x) = P(X \leq x)$$

$$F(x) = \int_{1}^{x} 2-\frac{1}{x} = 2x - \log(x) \Big|_{1}^{x}$$

$$2x - \log(x) - 2 - 0$$

$$F(x) = \begin{cases} 0 & x<1 \\ 2x-\log(x) & x \geq 1 \end{cases}$$

$$2x - \log(x) - 2$$

$$(2(2) - \overset{b}{\log}(2)) - (2(\overset{a}{1}) - \log(1))$$

$$b > a \checkmark$$

d)

$$F(X)$$

$$f(x) = \begin{cases} 0.2 & x=0 \\ 0.6 & x=1 \\ 0.2 & x=2 \\ 0 & o.w. \end{cases}$$

$f(x) \geq 0$ ✓

$$\sum_x f(x) = 1$$

$0.2+0.6+0.2=1$ ✓

$P(X=x) = f(x)$ ✓

$0 = P(X=0)$

$1 = P(X=1)$

$2 = P(X=2)$

3.

a) $\int_0^1 f(x)dx = 1$, and $f(x) = f(1-x)$ for $0 \leq x \leq 1$

$$\int_0^1 1-x\,dx = x - \frac{x^2}{2}\Big|_0^1$$

$$\left(1-\frac{1}{2}\right) - 0$$

Has characteristics

$$\frac{1}{2}$$

$f(x)$ is symmetric around $1/2$ and 0 outside of $0 \leq x \leq 1$

A

b)

$$f(x)+f(-x)=1$$

$$\int_{-\infty}^{\infty} f(x)dx = \int_{0}^{\infty} f(-x)dx + \int_{0}^{\infty} f(x)dx$$

$$= \int_{0}^{\infty}(1-f(x))\,dx + \int_{0}^{\infty} f(x)\,dx$$

$$= \int_{0}^{\infty} 1\,dx = \infty$$

Total area must be 1

Does not have characteristics

c)

$$f(x) = \begin{cases} 0 & -\infty < x < -3\frac{5}{6} \\ 3 & -3\frac{5}{6} \le x \le -2\frac{5}{6} \\ 0 & -2\frac{5}{6} < x < \infty \end{cases}$$

3 between $-3\frac{5}{6}$ and $-2\frac{5}{6}$ and 0 everywhere else.

Has characteristics

4.

a) $E(X) = (0.6 \cdot \$12) + (0.2 \cdot \$15) + (0.05 \cdot \$18) + (0.15 \cdot \$24)$

$= \$14.7$

b)

$$30 \le x \le 40 = \frac{1}{80}$$

$$40 \le x \le 60 = \frac{1}{40} - \frac{1}{80} = \frac{2}{80} - \frac{1}{80} = \frac{\frac{1}{80}}{20} = \frac{1}{1600}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x)dx = \int_{30}^{40} x \frac{1}{80} dx + \int_{40}^{60} x \frac{1}{1600}(x-20)dx - \int_{60}^{100} x \frac{1}{1600}(x-100)dx$$

kHz

$$= \frac{1}{80}\frac{x^2}{2}\Big|_{30}^{40} + \frac{1}{1600}\left(\frac{x^3}{3} - 20\frac{x^2}{2}\right)\Big|_{40}^{60} - \frac{1}{1600}\left(\frac{x^3}{3} - 100\frac{x^2}{2}\right)\Big|_{60}^{100}$$

$$= \left[\left(\frac{1}{80}\cdot\frac{40^2}{2}\right) - \left(\frac{1}{80}\cdot\frac{30^2}{2}\right)\right] + \left[\left(\frac{1}{1600}\left(\frac{60^3}{3} - 20\frac{60^2}{2}\right)\right) - \left(\frac{1}{1600}\left(\frac{40^3}{3} - 20\frac{40^2}{2}\right)\right)\right]$$

$$- \left[\left(\frac{1}{1600}\left(\frac{100^3}{3} - 100\frac{100^2}{2}\right)\right) - \left(\frac{1}{1600}\left(\frac{60^3}{3} - 100\frac{60^2}{2}\right)\right)\right]$$

$$= 60.208 \text{ kHz}$$

5.

a) $$\int_{110}^{140} f(x)\,dx = \int_{110}^{120} \frac{1}{40}\,dx + \int_{120}^{160} -\frac{1}{40^2}(x-160)\,dx$$

$$= \left[\frac{1}{40x}\right]\Big|_{110}^{120} + \left[-\frac{1}{40^2}\left(\frac{x^2}{2} - 160x\right)\right]\Big|_{120}^{160}$$

$$= \left[\frac{1}{(40)(120)} - \frac{1}{40(110)}\right] + \left[\left(-\frac{1}{40^2}\left(\frac{160^2}{2} - 160(160)\right)\right) - \left(\frac{1}{40^2}\left(\frac{120^2}{2} - 160(120)\right)\right)\right]$$

$$= \frac{5}{8}$$

b) $$\int_{110}^{120} x\,\frac{1}{40}\,dx + \int^{160} x\left(-\frac{1}{40^2}\right)(x-160)\,dx$$

$$\left[\frac{1}{40}\left(\frac{x^2}{2}\right)\right]\Big|_{110}^{120} + \left[\left(-\frac{1}{40^2}\right)\left(\frac{x^3}{3} - \frac{160x^2}{2}\right)\right]\Big|_{120}^{160}$$

$$\left[\frac{1}{40}\left(\frac{120^2}{2}\right) - \frac{1}{40}\left(\frac{110^2}{2}\right)\right] + \left[\left(-\frac{1}{40^2}\left(\frac{160^3}{3} - \frac{160(160^2)}{2}\right)\right) - \left(-\frac{1}{40^2}\left(\frac{120^3}{3} - \frac{160(120^2)}{2}\right)\right)\right]$$

$$= \frac{365}{3}\ \mu m$$

c)

$$\sigma_X^2 = E(X^2) - \mu_X^2 = \int_{100}^{120} x^2 \frac{1}{40} dx + \int_{120}^{160} x^2 \left(-\frac{1}{40^2}\right)(x-160)\, dx - \left(\frac{365}{3}\right)^2$$

$$\sigma_X^2 = \frac{1}{40} \cdot \frac{x^3}{3} \Big|_{100}^{120} - \frac{1}{40^2} \cdot \left(\frac{x^4}{4}\right) \Big|_{120}^{160} + \frac{1}{40^2} \cdot 160 \frac{x^3}{3} \Big|_{120}^{160} - \left(\frac{365}{3}\right)^2$$

$$\sigma^2 x = \frac{1775}{9}$$

$$\sigma_X = \frac{\sqrt{1775}}{3} = 14.04\, \mu m$$