

ECE 3530/CS 3130: Engineering Probability & Stats

Lecture Notes Spring 2015

Lecture #4

1 Counting

This section covers how to determine the size, *i.e.*, the number of outcomes, in the sample space or in an event.

Example: Tablet Specs

A tablet manufactured by Pear Computer Co can have 64 GB, 32 GB, or 16 GB of memory, and can either have 4G connectivity or not. How many different versions of tablet does Pear make?

Solution: The solution is 6: (64, 4G), (64, No), (32, 4G), (32, No), (16, 4G), (16, No).

1.1 Multiplication Rule

What is the general formula for this, so we don't have to list outcomes?

Def'n: *Multiplication Rule*

If one operation can be performed in n_1 ways, and for each of those ways, a second operation can be performed in n_2 ways, then there are $n_1 n_2$ ways to perform the two operations together.

Example: Form Entry

A web form asks for 1) a person's state of birth (or n/a if not born in the US) and 2) a pin code between 0 and 9999. How many different entries are possible?

Solution: There are 50 states and 1 n/a, for a total of 51 ways to answer the first question, and 10,000 ways to answer the second question. By the multiplication rule, there are $51(10^4) = 5.1 \times 10^5$ different entries possible.

We don't need to stop at two "operations" – many problems involve more.

Def'n: *Multiplication Rule*

If a first operation can be performed in n_1 ways, and for each of those ways, a second operation can be performed in n_2 ways, and for each of those ways to do both operations together, there are n_3 ways to do a third operation, and so forth through the k th operation, then there are $n_1 n_2 \cdots n_k$ ways to perform the k operations together.

Example: Phonics testing

The state of Utah tests early elementary students on their ability in phonics by presenting them with a (potentially fake) three letter word and asking them to pronounce it. The first letter is a consonant, the second a vowel (excluding y), and the third a consonant. The consonant is allowed to repeat. How many different questions could there be?

Solution: There are 26 letters, five of them vowels (a e i o u). Thus there are 21 consonants. The first letter must be one of 21, the second letter must be one of five vowels, and the third letter is one of 21 consonants again. The answer is $(21)(5)(21) = 2205$.

Example: Turn in order

There are 115 students in a class taking an exam. Consider the first, second, and third person to turn in their exam. How many ways can this happen? Assume order is important, and each person can turn in their exam only once.

Solution: The first person can be any of the 115. The second person can't be the same as the 1st, so is one of the remaining 114. The third is similarly one of the remaining 113. Thus there are $(115)(114)(113) = 1,481,430$ ways for this order to happen.

1.2 Permutations

In general, when we take elements of a set (such as the students in a class, above) and order a few (or all) of them, it is called a *permutation*.

The number of ways to put r elements in order, from a set of n unique possibilities, is denoted ${}_nP_r$, and is calculated as:

$${}_nP_r = \frac{n!}{(n-r)!} = n \cdot (n-1) \cdot (n-2) \cdots (n-r+1) \quad (1)$$

Note $0! = 1! = 1$.

If you're going to order ALL of the elements of the set, then $r = n$, so there are $n \cdot (n-1) \cdot (n-2) \cdots 1 = n!$ ways to order them.

Example: Cards in the deck

Order is important in a card deck because it affects which player will get each card.

1. How many ways are there to order the 52 cards in a standard card deck?
2. How many ways are there to pick 4 cards, in order which you pick them, from a standard card deck?

Example: Three pills

You are sick and there are three pills in front of you, one which will do nothing (N), one which will kill you (K), and one which will cure you (C).

You have no idea which one is which. You will put the three pills in order, and take one at a time, and stop when you either get cured or killed.

1. How many ways are there to put the pills in order from first to last? (You might stop before you get to the end, but just answer how many ways there are to put them in order.
2. How many ways are there to order the three pills that will cure you?