HOMEWORK #5 solution

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4. In a wind farm of 20 windmills, each windmill generates power 60 % of the time and is idle 40 % of the time, independent of what other windmills are doing. If each windmill generates 10 kW when running, find the probability that the total power output for the wind farm is at least 120 kW at any given instant.

SOL'N: We have Bernoulli trials where we may identify success as a windmill generating power, which happens $p = 0.6$ or 60 % of the time. We also have $n = 20$, since there are 20 windmills. 120 kW corresponds to 12 windmills operating. Thus, we are interested in finding the following probability:

$$P(m \geq 12 \text{ successes out of } n = 20 \text{ trials}) \equiv P(m \geq 12 \text{ for } n = 20)$$

Using the formula for Bernoulli trials, we may write this as follows:

$$P(m \geq 12 \text{ for } n = 20) = \sum_{m=12}^{20} C_m^{20} p^m q^{20-m}$$

A table of summations of binomial probabilities is available in [1]. These summations are for $m = 0$ to r, however. To use the table, we observe that the probability we seek is the complement of such a sum:

$$P(m \geq 12 \text{ for } n = 20) = 1 - \sum_{m=0}^{r=11} C_m^{20} p^m q^{20-m}$$

Thus, we use the table with $n = 20$, $r = 11$, and $p = 0.6$. The table value is 0.4044. Thus our answer is the complement of this value:

$$P(m \geq 12 \text{ for } n = 20) = 1 - 0.4044 = 0.5956$$

NOTE: An alternative approach is to let a stopped windmill be considered as "success" in the Bernoulli trials. In that case we are seeking the following probability that is found in the table directly, (using $r = 8$ and $p = 0.4$ now):

$$P(m \leq 8 \text{ for } n = 20) = \sum_{m=0}^{r=8} C_m^{20} p^m q^{20-m} = 0.5956$$

REF: [1] Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, and Keying Ye, *Probability and Statistics for Engineers and Scientists,* 8th Ed., Upper Saddle River, NJ: Prentice Hall, 2007.