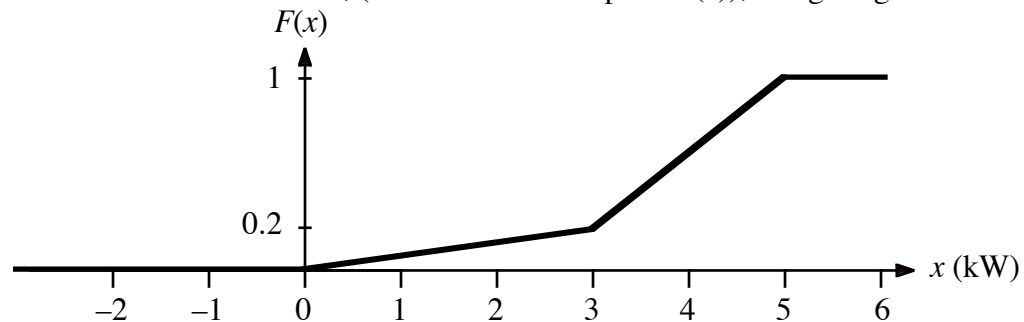1. As shown below, a lighting company has determined the cumulative distribution function, $F(x)$, of the power level chosen by users of a lecture hall equipped with dimmer switches for two levels, (hence the two ramps in $F(x)$), of lighting.

a) Determine the probability density function, $f(x)$, for the power level.
b) Find the probability that the power level is less than 4 kW.
c) Find the probability that the power level is between 1 kW and 3 kW.

2. (From Tolga Tasdizen)
    (a) The function
    $$f(x) = \begin{cases} 1.5 - 2|x|, & -1 \le x \le 1 \\ 0, & o.w. \end{cases}$$
    is not a valid probability density function. Why not?

    (b) The function $F(x) = 1 + \sin(x)$ is not a valid cumulative distribution function. Why not?

    (c) The function
    $$F(x) = \begin{cases} 0, & x < 1 \\ 2 - \frac{1}{x}, & x \ge 1 \end{cases}$$
    is not a valid cumulative distribution function. Why not?

    (d) Is the following valid probability mass function for a discrete random variable? If it is not, state the reason.
    $$f(x) = \begin{cases} 0.2, & x = 0 \\ 0.6, & x = 1 \\ 0.2, & x = 2 \\ 0, & o.w. \end{cases}$$

3. Determine whether a probability density function, $f(x)$, can have the characteristics listed in (a). Repeat the exercise for the characteristics listed in (b) and then (c).
    a) $\int_0^1 f(x)dx = 1$, and $f(x) = f(1-x)$ for $0 \le x \le 1$.
    b) $f(x) = 1 - f(-x)$ for $-\infty < x < \infty$.
   c) $f(-3) = 3$, and $f(x) = 0$ for $0 \le x < \infty$.

4.   A manufacturer called Ligent, ("we're half intelligent"), makes microprocessors that are pin-for-pin equivalents to chips made by a well-known manufacturer. Ligent sells four (and only four) chip models for the following prices:

   Model A $12   Model T $15   Model X $18   Model Z $24

When customers buy chips, they choose the various models with the following probabilities:

   $P(A) = 0.6$ $P(T) = 0.2$ $P(X) = 0.05$ $P(Z) = 0.15$

a)   A market analyst wishes to calculate the average price of chips that customers purchase. Calculate this average or mean price.

b)   The company has measured the maximum sustainable clock speed of an infinite number of model T chips. The plot below shows the results as a probability density function for maximum sustainable clock speed. Find the expected value of the maximum sustainable clock speed.
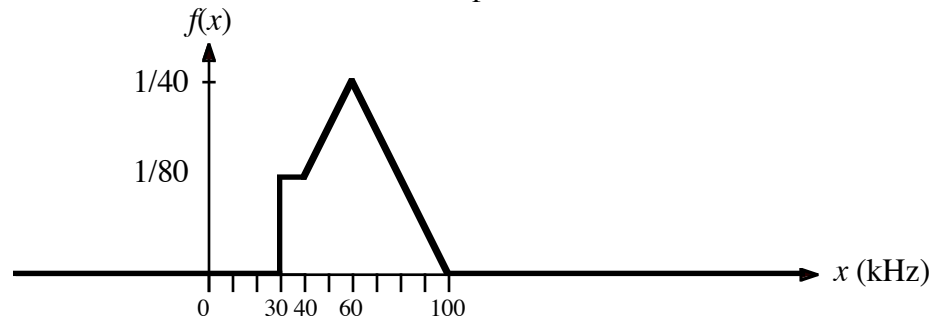
5.   The thickness of cladding on an optical fiber varies randomly along its length. The probability density function, $f(x)$, shown below, describes the thickness variations. A consistent thickness, (i.e., a low variance of thickness), is desirable, as the fiber is then less likely to become stuck when being pulled through conduits.
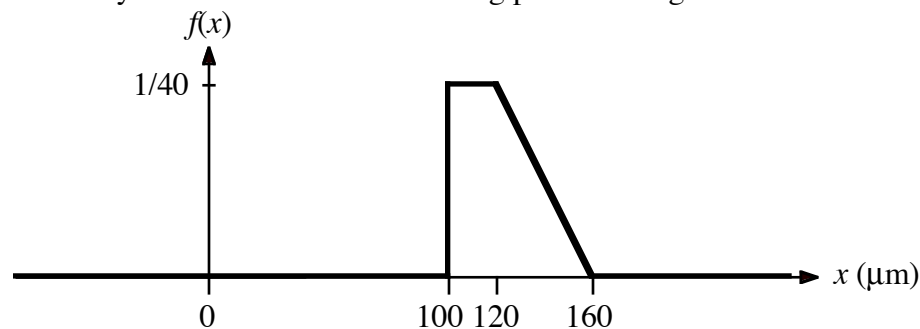
a)   Find the probability, $P(110 < x \le 140)$, that the thickness of the cladding is between 110 and 140 μm.

b)   Determine the mean thickness, $E(x)$, of the cladding in μm.

c)   Determine the standard deviation, σ, of the thickness of the cladding in μm.

ANS:

1.

2.

3.

4.

5.