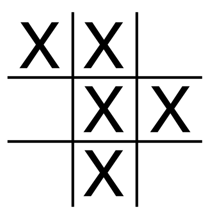
EX: How many ways are there to place 5 X's and 4 O's on a tic-tac-toe board? Only the final pattern matters in this problem. Note that, since a game could end before all squares on the tic-tac-toe board are filled with an X or O, this is not the same as asking how many possible final patterns could arise in a tic-tac-toe game. Here, the question is simplified by ignoring the order in which X's and O's are placed on the board.

Assuming all tickets are equally likely to be picked, find the probability that Ann, Ben, and Cam win the three prizes.

SOL'N: Since there are only two symbols, we need only specify where the X's are located on the tic-tac-toe board. There are nine possible locations, and we are picking five of them for the X's.



Since we only care about the final patterns, and not the order in which the X's are placed on the board, we use a combinatoric coefficient for 9 items taken 5 at a time:

$$_9C_5 = \frac{9!}{(9-5)!5!} = \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1} = 126.$$

This is our answer but, in a more general setting, we would multiply 126 by the number of ways of placing other symbols, namely the O's. As a check, we calculate the number of ways to choose the four locations for the O's given the four squares remaining. (Since the order of placing the O's doesn't matter, there should be only one way.) Using a combinatoric coefficent, we have

$$_4C_4 = \frac{4!}{(4-4)!4!} = \frac{4!}{0!4!} = 1.$$

Note that $0! = 1$.

Our final answer, as expected, is $126\cdot 1 = 126$.