Homework 6

1. Discrete random variables X and Y have the following joint probability distribution

$f(x, y)$	$x = 0$	$x = 1$
$Y = 0$	0.1	0
$Y = 1$	0.1	0.1
$Y = 2$	0.1	0.2
$Y = 3$	0	0.4

a) Compute the correlation coefficient, ρ_{XY}.

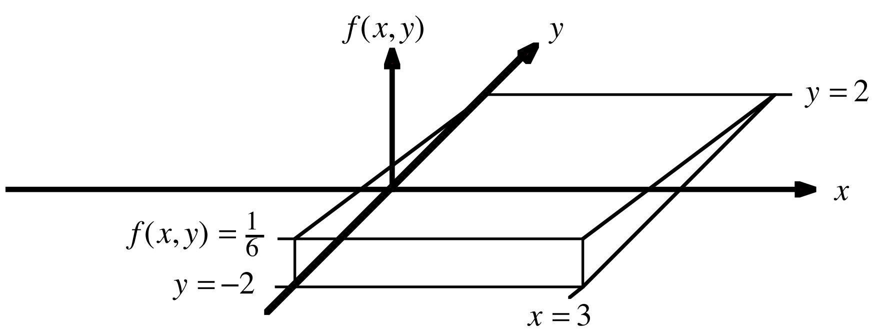
b) Compute the probability $P(Y \geq 2, X = 0)$.

c) Compute the probability $P(Y \geq 2 \mid X = 0)$.

d) If this is a game, and you win \$$(100X + 10Y^2)$ each time you play this game, what is the expected amount you win per game?

e) There is a third random variable, Z, that is independent from Y. The mean and variance for Z are given as $\mu_Z = 1$ and $\sigma_Z^2 = 0.5$. Compute the mean and variance of the linear combination $2Y - 4Z$.

2. Consider the pdf, $f(x, y)$, for jointly distributed random variables X and Y, as plotted below:



a) Write the definition of $f(x, y)$ in equation form. In other words, fill in the right-hand side of the defining equation $f(x, y) = ...$

b) Find $P(X > 1.5 \text{ and } Y > 1)$. Hint: The probability you are calculating is the volume of $f(x, y)$ to the right of $x = 1.5$ and further into the page than $y = 1$. Use the plot of $f(x, y)$ and geometry to find the requested volume without having to perform an integration.

3. Your job is to make sure that a washing machine lasts more than 10 years, the warranty period, with probability 0.99. As it is, the mean lifetime is 10 years with standard deviation 2.5 years, and it is gaussian (normally) distributed. You can leave the standard deviation the same for no cost. To reduce the standard deviation, it costs \$100 to reduce the standard deviation to 1.5 years, or \$200 to reduce it to 0.5 years. It costs \$100 per year (for any positive real value of years) you wish to increase the mean of the lifetime. (For example, if you want the machine to have a mean lifetime of 11.5 years with a standard deviation of 1.5 years, the cost would be \$100(11.5 – 10) = \$150, plus \$100 to reduce the standard deviation to 1.5, for a total cost of \$250. However, this configuration would not have probability 0.99 of having a lifetime greater than 10 years.) Find the lowest cost way to achieve the engineering design goals. Consider only solutions with standard deviation of 0.5, 1.5, or 2.5 years. For the mean, use as many digits of accuracy as Table A.3 in the text will accommodate.

4. A game consists of rolling a pair of dice, one red and one blue. Let X denote the outcome of the red dice and let Y denote the outcome of the blue dice. X and Y are independent random variables. Both dice are rigged and they have the following marginal distributions:

$$g(x) = \begin{cases} 1/4, & x = 1,2 \\ 1/8, & x = 3,4,5,6 \\ 0, & \text{otherwise} \end{cases} \quad h(y) = \begin{cases} 1/3, & y = 1,2 \\ 1/12, & y = 3,4,5,6 \\ 0, & \text{otherwise} \end{cases}$$

a) Make a table showing the joint distribution $f(x, y)$.

b) Find the probability that the sum of the two dice is greater than 8.

5. The probability density function for a 2-dimensional gaussian (or normal) distribution is described by the following formula:

$$f(x,y) = \frac{1}{2\pi\sqrt{1-\rho_{XY}^2}} e^{-(x^2 - 2\rho_{XY} \cdot xy + y^2)/2(1-\rho_{XY}^2)}$$

where $\rho_{XY} \equiv$ correlation coefficient for X and Y

a) Show that $f(x, y)$, may be written as a product of two 1-dimensional normal (or gaussian) pdf's when $\rho_{XY} = 0$.

b) For $y = 3$ and $\rho_{XY} = 1/3$, write $f(x, 3)$ as a constant, k, times a 1-dimensional normal (or gaussian) pdf. Find the values of the following: k, μ, and σ^2.

ANS:

1.

2.

3. $\mu = 11.165$ and $\sigma = 0.5$ gives minimal cost of \$316.50

4.

5.