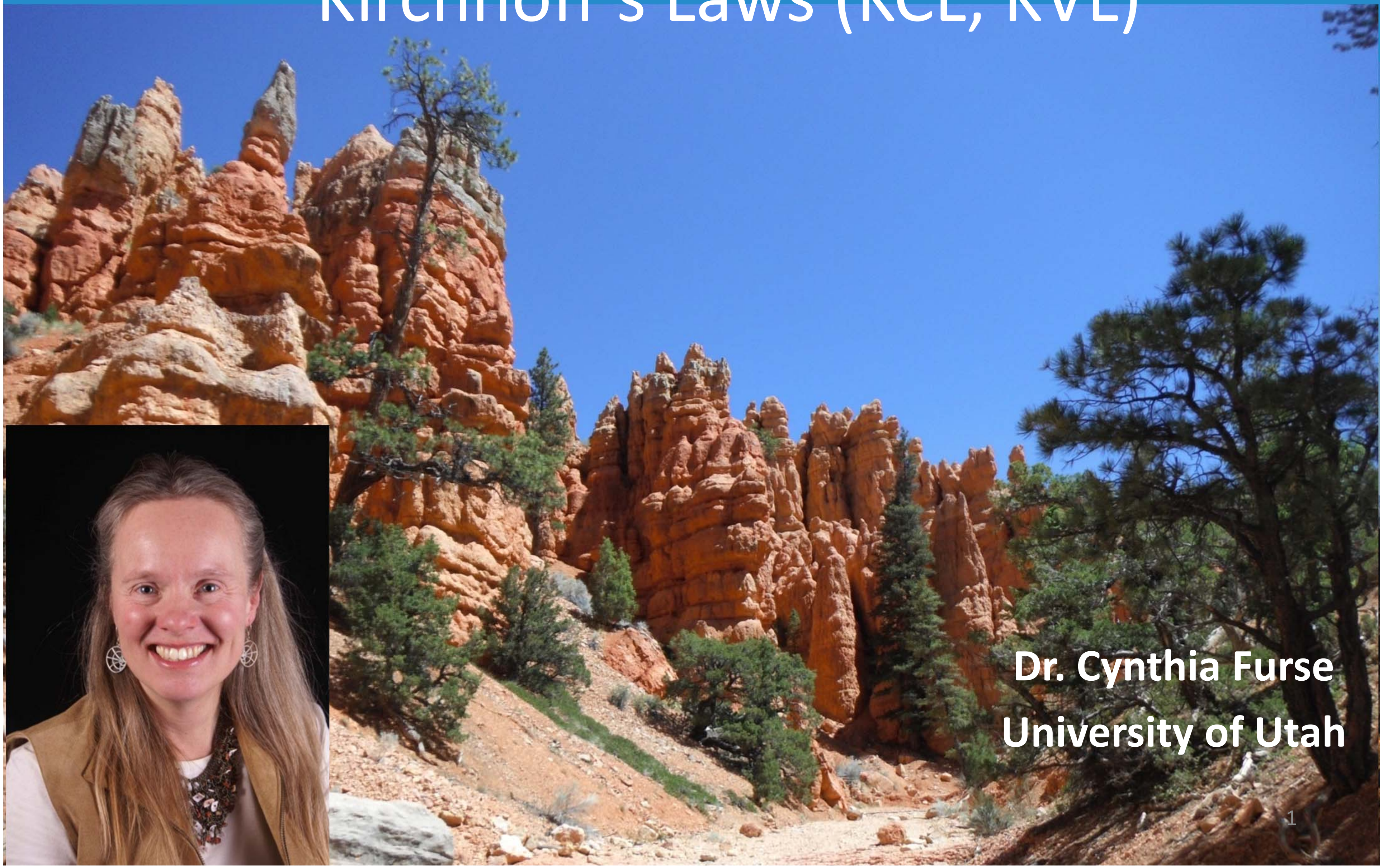
Introduction to Electrical & Computer Engineering

Kirchhoff’s Laws (KCL, KVL)



Dr. Cynthia Furse
University of Utah

Kirchhoff's Current Law (KCL)

Sum of currents entering a node =0

OR $$i_1 - i_2 - i_3 + i_4 = 0$$

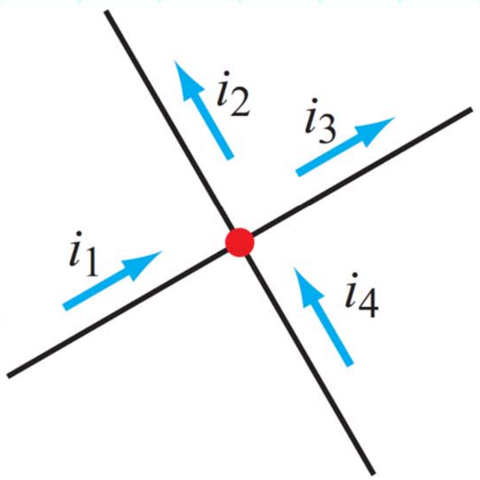
Sum of currents leaving a node = 0

OR $$-i_1 + i_2 + i_3 - i_4 = 0$$

Sum of currents leaving = sum of Currents entering a node

$$i_1 + i_4 = i_2 + i_3$$

$$\sum_{n=1}^{N} i_n = 0 \qquad \text{(KCL)},$$



Kirchhoff’s Voltage Law (KVL)

Sum of voltages around a closed path =0

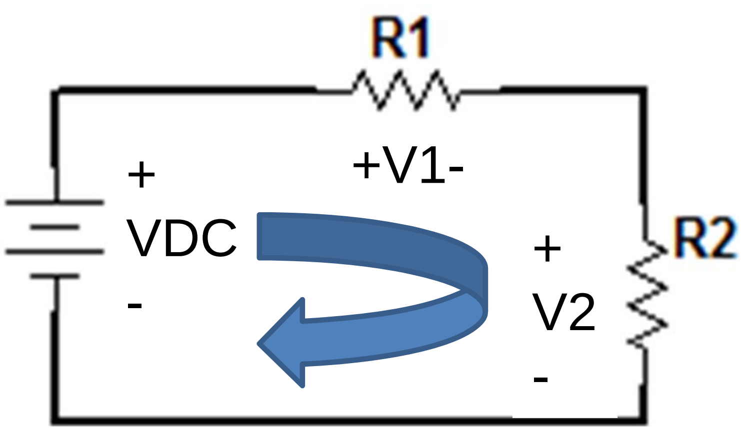
-VDC+V1+V2 = 0

OR

Sum of voltage drops = sum of voltage rises

VDC = V1+V2

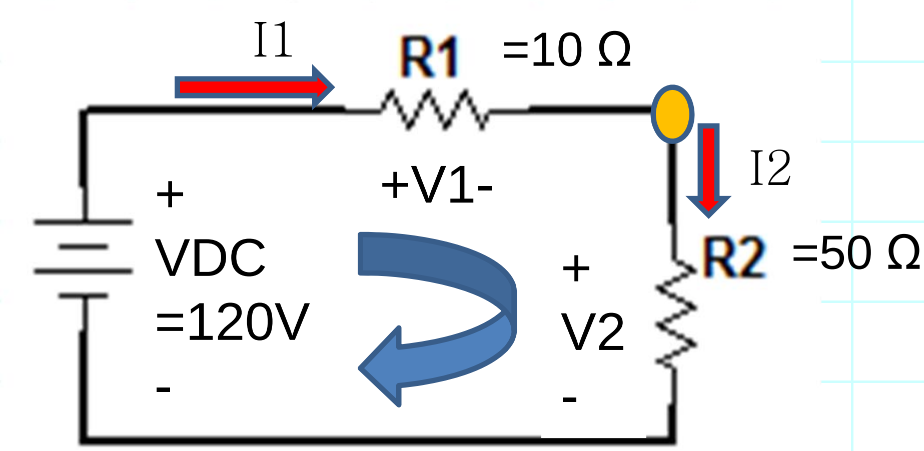
$$\sum_{n=1}^{N} v_n = 0 \qquad \text{(KVL)},$$



Sign Convention

- Add up the voltages in a systematic clockwise movement around the loop.
- Assign a positive sign to the voltage across an element if the (+) side of that voltage is encountered first, and assign a negative sign if the (−) side is encountered first.

Applying KCL and KVL



$$-VDC+V1+V2 = 0 \qquad (1)$$

What is KNOWN?

VDC=120V, R1=10Ω, R2=50Ω

What is Unknown?

V1,V2

2 Unknowns need 2 Equations

Ohm's Law gives 2 more equations

But 2 more unknowns (I1,I2)

$V1= (I1)(R1)$ and $V2=(I2)(R2)$ (2,3)

Node Equation gives one more eqn:

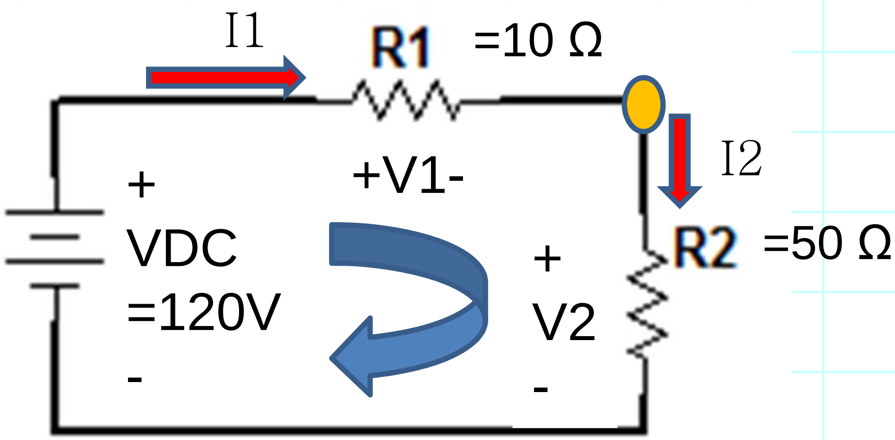
$I1 = I2$ (4)

Four unknowns (V1,V2,I1,I2)

Four Equations (1,2,3,4)

OK! Now we can solve it!

Applying KCL and KVL



-VDC+V1+V2 = 0

VDC=120V, R1=10Ω, R2=50Ω

V1= (I1)(R1) and V2=(I2)(R2)

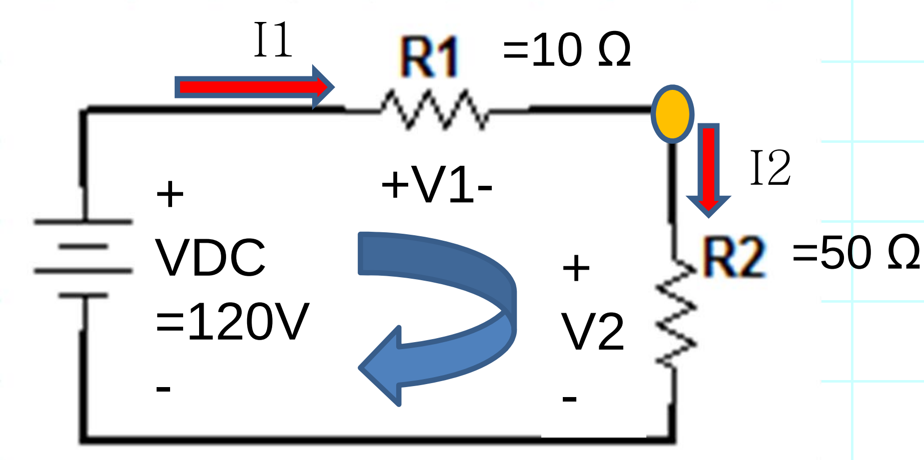
I1 = I2 = I

Substituting:

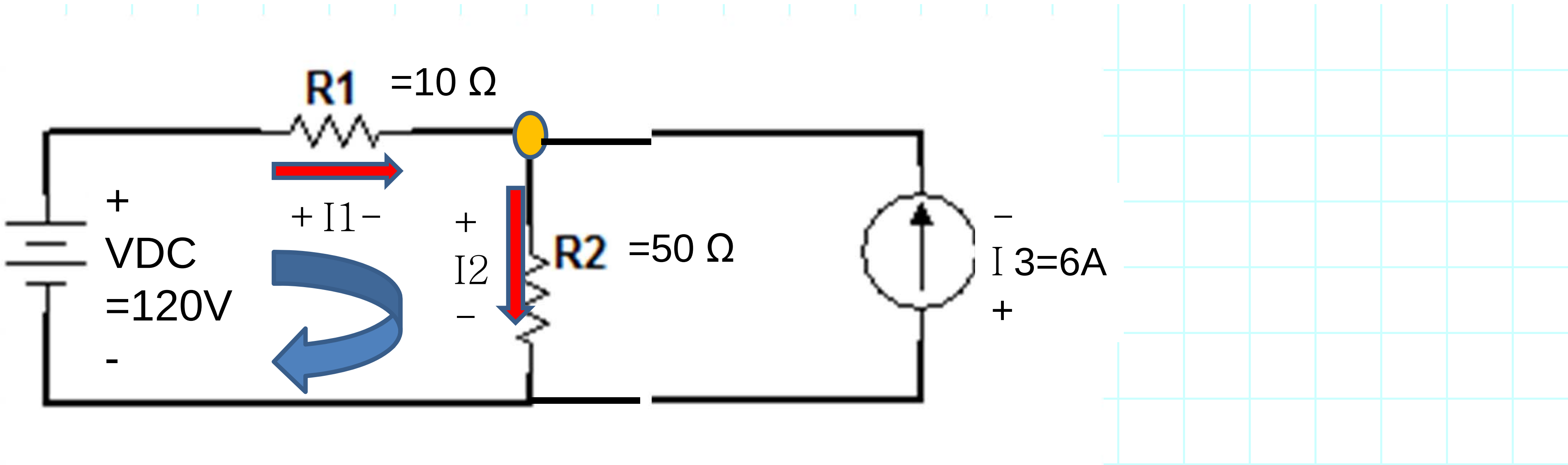
-120V + (I) (10Ω) + (I) (50Ω) = 0
I = 120V / (10 + 50 Ω) = 2A

Then
V1=(2A)(10Ω) = 20V
V2=(2A)(50Ω) = 100V
Check: VDC = ? V1+V2 Yes!

KCL and KVL Cookbook



1. Write all KNOWN values
2. Keep track of all unknown values (Need as many independent equations as unknowns)
3. Write all KVL (Loop) equations. Each loop must pick up at least one new element. Current sources can't be in loops.
4. Apply Ohm's Law (I prefer to do this 'as I go', for convenience)
5. Apply as many KCL (node) equations as needed to fill in unknowns. Each KCL equation must pick up at least one new current.
6. Solve for the unknowns



1. Write all KNOWN values   VDC=120V, R1=10Ω, R2=50Ω, I3=6A

2. Keep track of all unknown values   I1, I2
(Need as many independent equations as unknowns)

3. Write all KVL (Loop) equations.   -VDC + (I1)(R1)+(I2)(R2)=0   (1)
Each loop must pick up at least ONE new element. Current sources are not counted as 'new' elements.
4. Apply Ohm's Law (I prefer to do this 'as I go', for convenience)

5. Apply as many KCL (node) equations as needed to fill in unknowns. Each KCL equation must pick up at least ONE new current. I1 + I3 = I2   (2)
6. Solve for the unknowns

6. Solve:

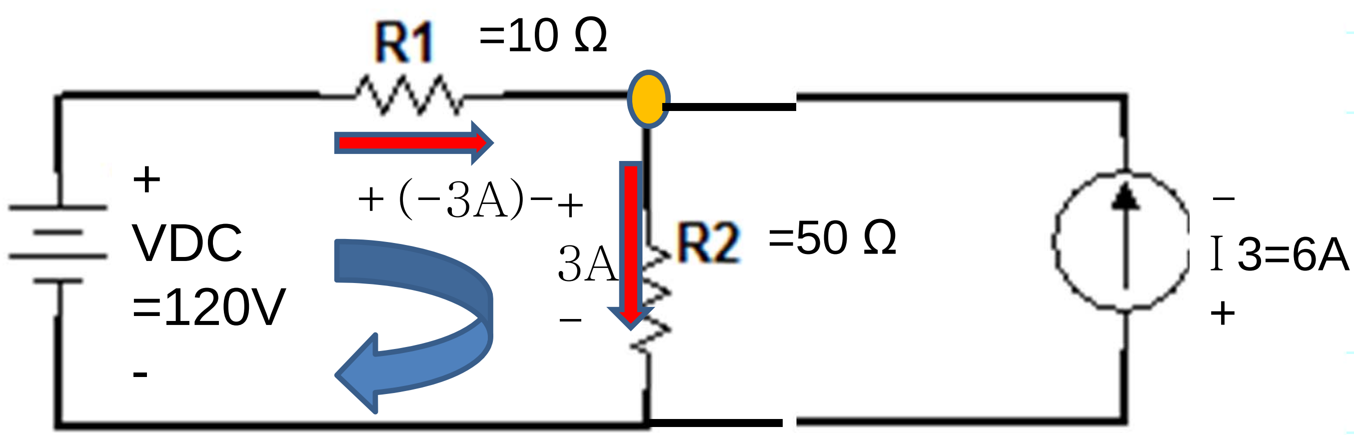
$-120V + (I1)(10\Omega)+(I2)(50\Omega)=0$
$I1 + 6A = I2$

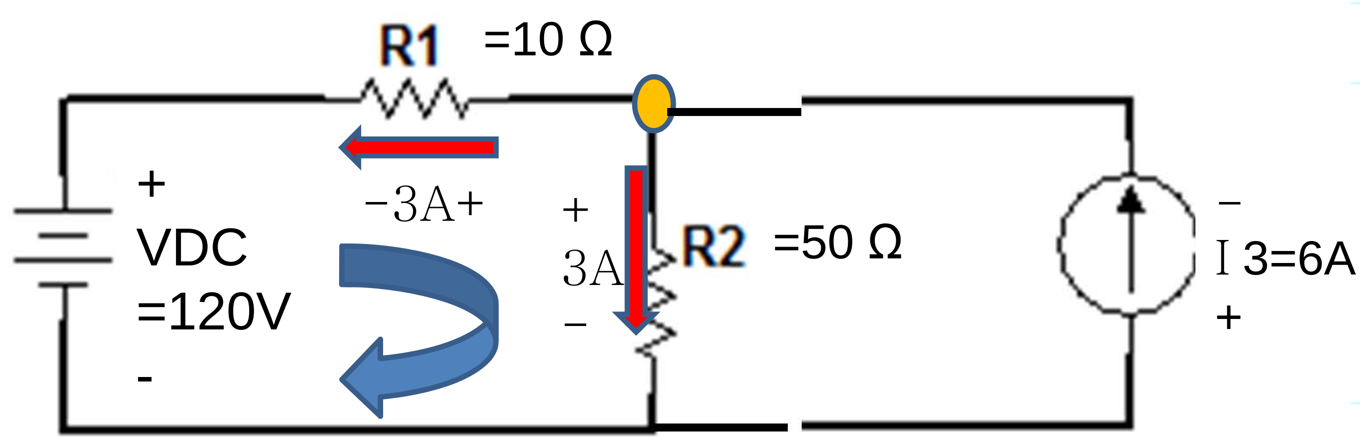
Substitute to remove variables:
$-120V + (I1)(10\Omega)+(I1+6A)(50\Omega)=0$
$-120V + (I1)(10\Omega)+(I1)(50\Omega)+(6A)(50\Omega)=0$

Solve for remaining variable:
$I1=(120V - (6A)(50\Omega)) / (10\Omega + 50\Omega) = -3A$

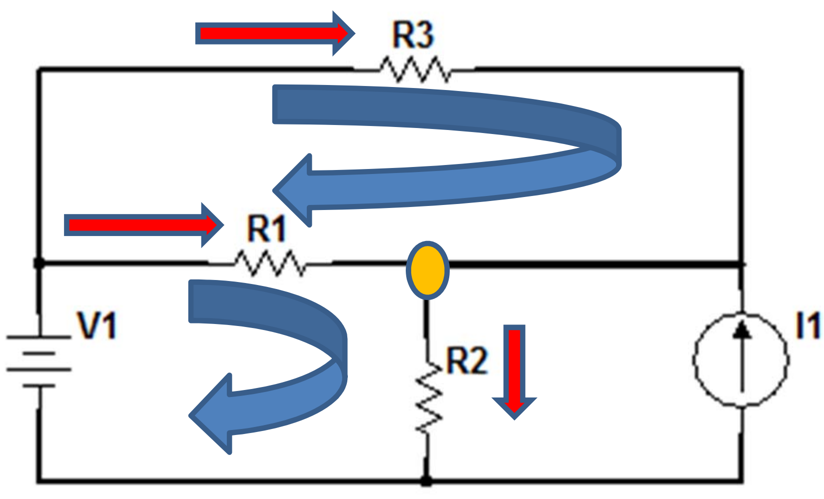
Go back to original equations to find other variables
$I2 = -3A + 6A = 3A$

Interpret / Reality Check:





KCL and KVL Cookbook



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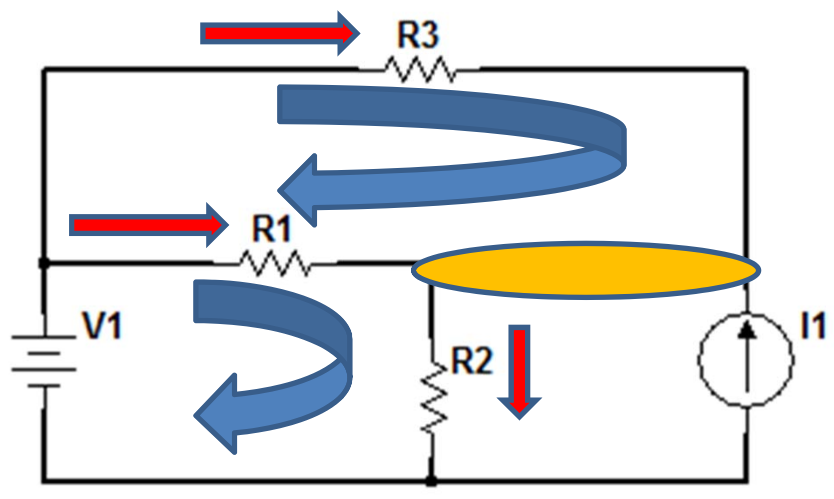
KCL and KVL Cookbook



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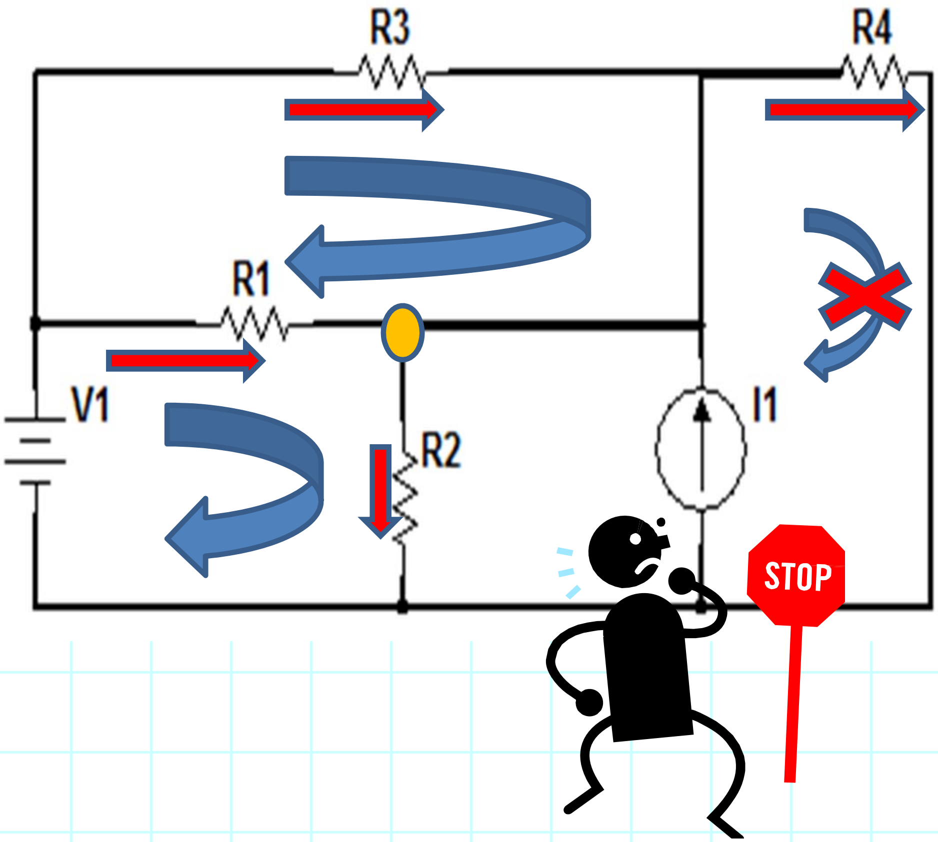

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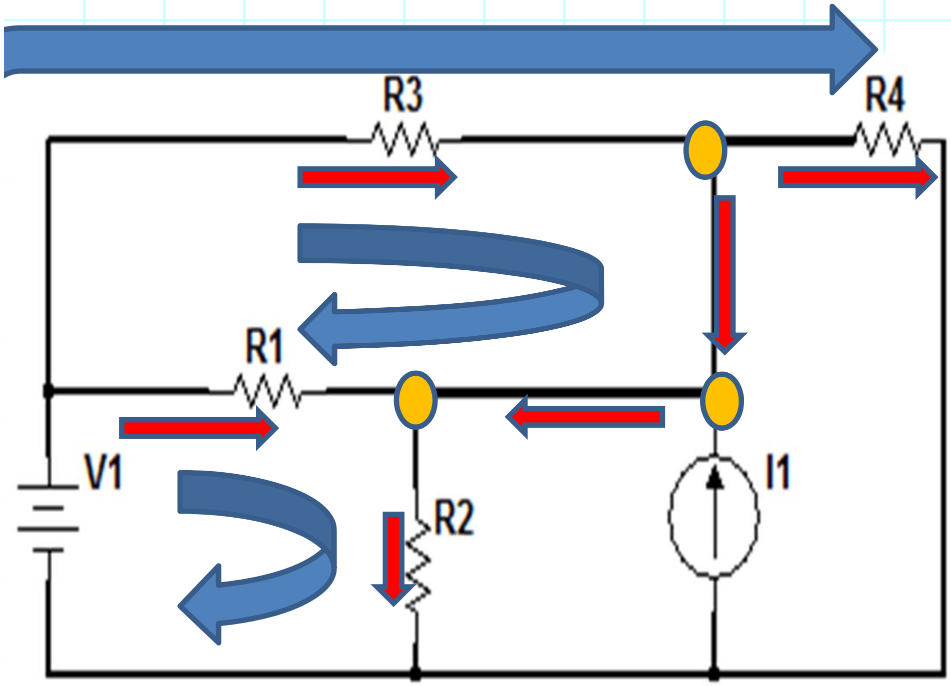

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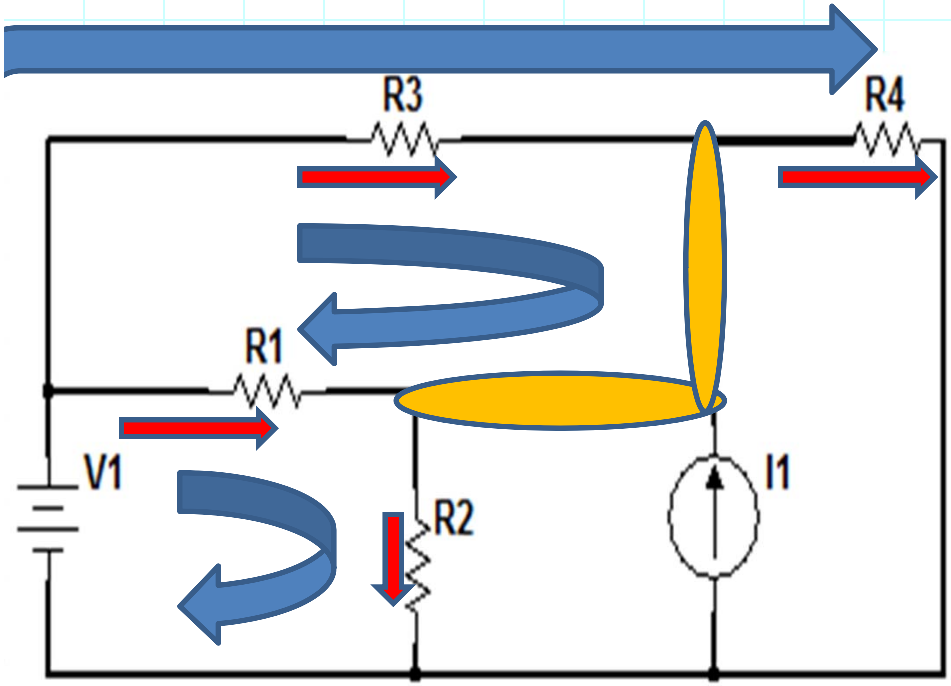

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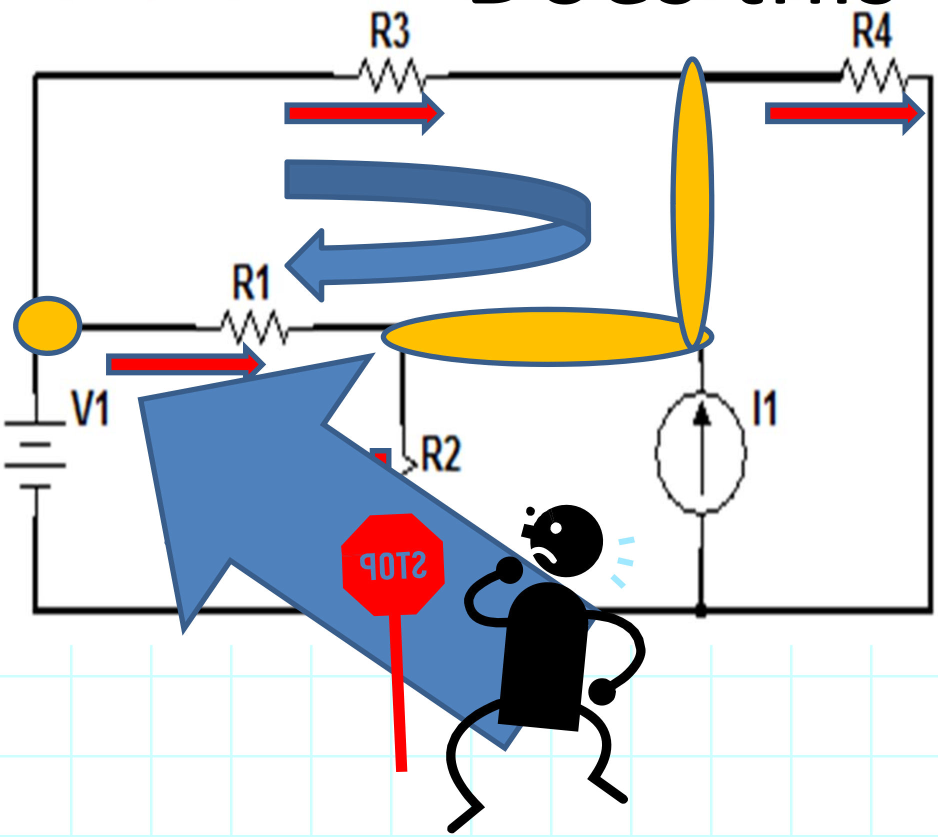
KCL and KVL Cookbook

Does this work? NO!



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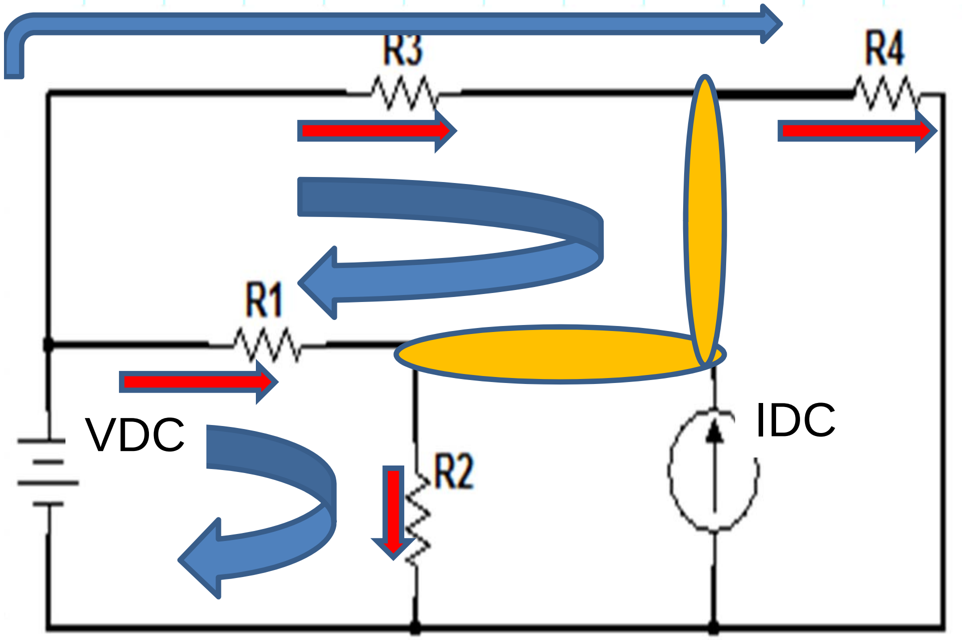
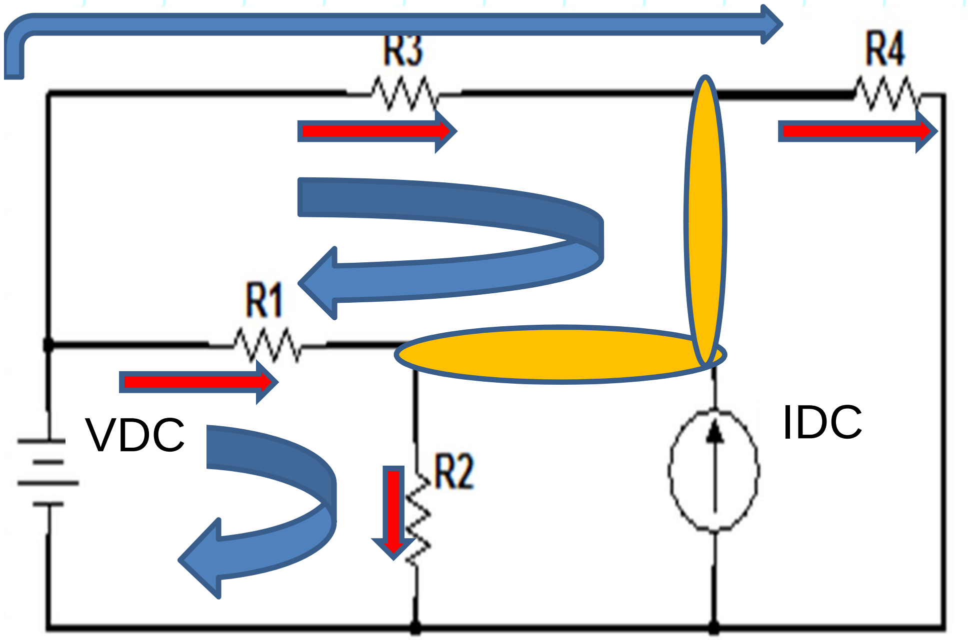
6. Solve for the unknowns.

KCL and KVL Equations



Loops:
$-VDC+I1R1+I2R2=0$
$I3R3 – I1R1 =0$
$-VDC + I3R3 + I4R4 = 0$
Node:
$I3 – I4+ I1 – I2 + IDC = 0$

KCL and KVL Matrix Math



-VDC+I1R1+I2R2=0
I3R3 – I1R1 =0
-VDC + I3R3 + I4R4 = 0
I3 – I4+ I1 – I2 + IDC = 0

R1	R2	0	0	I1	=	VDC
-R1	0	R3	0	I2		0
0	0	R3	R4	I3		VDC
1	-1	1	-1	I4		-IDC

Next, use Gaussian Elimination
OR
Matlab matrix solution

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OR $$i_1 - i_2 - i_3 + i_4 = 0$$

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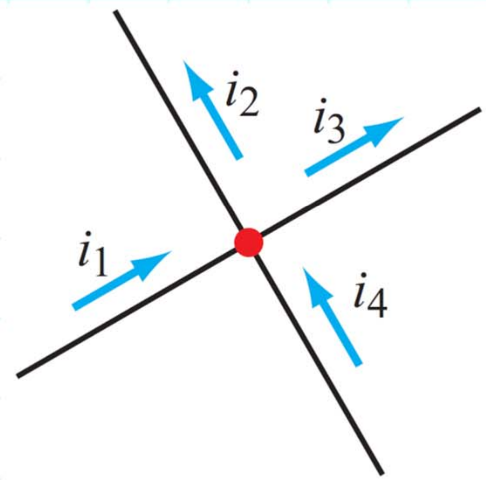
OR $$-i_1 + i_2 + i_3 - i_4 = 0$$

Sum of currents leaving = sum of Currents entering a node

$$i_1 + i_4 = i_2 + i_3$$

$$\sum_{n=1}^{N} i_n = 0 \qquad \text{(KCL)},$$



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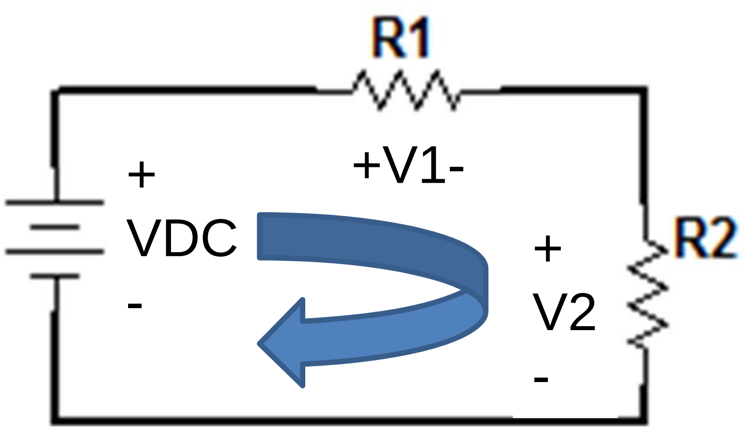

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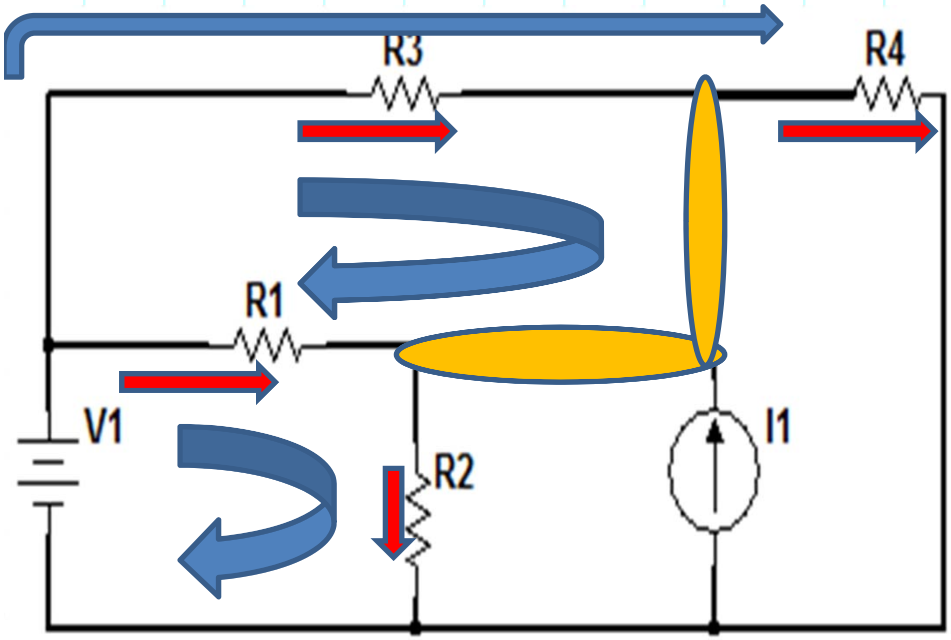
KCL and KVL Cookbook



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Another Way …

Red Canyon (Bottom of Thunder Mountain)
Near Bryce Canyon, Utah