Homework 2

1. A classic, difficult problem in computer science is the traveling salesperson problem. A salesperson must visit each of *n* cities exactly once each, and the goal is to find the shortest possible tour that starts and ends at the home city. That is, the tour forms a loop. We will only calculate the number of possible solutions involved, whereas the harder problem would be to find an optimal solution—efficiently. The traveling salesperson problem is equivalent to many other problems, including manufacturing problems where the cost of transitioning equipment from making one product to another depends on the products being made before and after the transition.

   Calculate the number of solutions that are possible for the following scenarios of the traveling salesperson problem.

   a) The starting/ending city is specified, and the salesperson must visit seven other cities.

   b) The starting/ending city is not specified, the salesperson must visit seven other cities, and tours that are the same when the tour is rotated to a different starting city are not considered to be different tours.

2. The following problem is from a classic paper on the method of simulated annealing for solving optimization problems [1]. Here, we only calculate the number of solutions that are involved in the problem. The method of simulated annealing is a clever way of finding an optimal solution.

   When the design of an integrated circuit becomes too complex, the circuitry must be placed on more than one chip. For simplicity, we consider using two chips. The goal then becomes to minimize the number of connections between chips in order to maximize speed and minimize pin count. For large circuits, we may identify circuit blocks and consider the problem of deciding how the blocks of circuitry should be allocated to two chips.

   Calculate the number of solutions that are possible for the following scenarios of the circuit-block allocation problem. Note that a solution that differs from another only in which chip is called chip 1 and which is called chip 2 is not a different solution. Also, having zero circuit blocks on one chip is not an allowed solution.

   a) 6 circuit blocks

   b) 6 circuit blocks but two of the blocks are identical

3. At the time of this writing, Wikipedia [2] lists 12 Republican presidential candidates as being listed in major polls, which presumably means they are viable candidates. (Five candidates who were in major polls have already dropped out.) When networks try to limit the number of candidates on stage for debates, there are many possible outcomes.

   a) How many ways are there to pick six candidates out of the 12 to be in a debate? Here, we are only concerned about which candidates are chosen.

   b) Once the six candidates are chosen, how many different ways are there to seat them in six chairs across the stage? Assume only one candidate sits in each chair, and assume the chairs are big enough to hold them.

   c) Once the six candidates are chosen, how many different ways are there to seat them in six chairs in a circle on a circular stage? Here, rotating a configuration does not make it a new configuration.

   d) If you were an engineering student who had no knowledge of politics, what is the probability that you could correctly guess the order that the five dropout candidates dropped out?

   e) If the Republican candidates for president and vice-president are chosen from the 12 current candidates, how many possible scenarios are possible for the top of the party ticket? Here, which of the two candidates is the presidential candidate does matter.

4. This problem deals with the number of passwords that are possible with given constraints.

   a) Your password must consist of eight characters. You must use only capital letters A-Z, and you must not repeat any characters. How many passwords are possible? Given that letters may not repeat, which would be more secure, having passwords of length 13 or of length 26?

   b) Your password must consist of two digits (0-9), five capital letters (A-Z), and one special character (chosen from a given set of 12 special characters). How many passwords are possible? Is this scheme more secure than the scenario in (a)? Justify your answer.

5. On a multiple-choice exam with four choices *a*, *b*, *c*, or *d* for each question, a correct answer receives one point. To discourage guessing, incorrect answers are penalized by subtracting 0.25 points.

a) If you can eliminate one incorrect answer and guess on the other three, what is the probability of getting one point? What is the probability of getting –0.25 points?

b) If the exam has four questions and you are told that each answer, *a*, *b*, *c*, or *d*, is the correct answer exactly once, what are the possible scores and the probabilities for those scores if you know the correct answer for the first problem and guess at the other three answers (but use each letter exactly once!)?

REF: [1] Kirkpatrick, S., Gelatt Jr, C. D., and Vecchi, M. P. (1983) "Optimization by Simulated Annealing", Science **220** (4598): 671-680.

[2] Republican Party presidential candidates, 2016. (2016, January 13). In *Wikipedia, The Free Encyclopedia*. Retrieved 07:19, January 20, 2016, from https://en.wikipedia.org/w/index.php?title=Republican_Party_presidential_candidates,_2016&oldid=699562940

ANS:

1.a) 5040 b) 5040

2.a) 31 b) 23

3.a) 924 b) 720 c) 120 d) 1/120 e) 66

4.a) $63 \bullet 10^9$ passwords, 26 letters is 6.2 billion times better than 13 b) $2.4 \bullet 10^{12}$

5.a) P(correct) = 1/3, P(incorrect) = 2/3 b) P(3 remaining all correct) = 1/6, P(1 remaining correct) = 1/2, P(0 remaining correct) = 1/3. Remember to determine the total points scored on the test for each of these scenarios (add 1 for one answer known at outset).