Homework 5

1. Error correction schemes for transmission of binary bits add extra bits so errors can be detected and corrected. For example, an extra bit could be included to make the number of 1's in a transmission be an even number. A one-bit error may be detected at the receiving end by counting how many 1's are in the received number.

   More sophisticated error-correction schemes map binary numbers to longer binary numbers that differ from one another in as many bits as possible so multiple bit flips may be detected. (Actually, things are a bit more involved than this description, but this basic description will suffice here.) An error correction coding scheme maps received binary messages to the nearest binary codeword as measured by Hamming distance (i.e., the number of bits that differ for two binary numbers.) This works perfectly unless too many bits are erroneous, which causes the received pattern to be closer to an incorrect codeword.

   a) Consider a codeword consisting of 8 bits. Determine how many other 8-bit binary patterns are within a Hamming distance of 3 or less of this codeword. Calculations of this sort tell us how many bit errors must occur before the error correction scheme fails.

   b) If the probability of error for each bit is $p = 0.2$, independent of other bits, find the probability that 3 or less bits out of 8 will be erroneous. Calculations of this sort tell us the probability of errors occurring in the decoding process.

2. Small batch board manufacturers have a frustratingly low probability of successfully delivering a working board to you, and they charge you for the board regardless of if it works or not. Assume that the charge per manufacture is \$500, and the probability of it working is p. Assume that the number of times until you need to send the board for manufacture until a working board comes back, random variable K, has the geometric probability mass function (or probability density function), which happens to be a good assumption:

$$f_K(k) = P(K = k) = \begin{cases} p(1-p)^{k-1}, & k = 1,2,\ldots \\ 0, & \text{otherwise} \end{cases} .$$

   We need to budget for the cost of board manufacturing: What is the expected value of the money we will spend on the board manufacturing? Note: you might need that

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2} .$$

3. The internet connection speed at any time from your home can depend on the amount of overall internet traffic at that time. Let the random variable X denote the speed of connection in megabits per second (MBPS). Assuming X is uniformly distributed on the interval 0.75 to 1.25 MBPS, answer the following questions:

a) Find the mean connection speed and standard deviation.

b) What is the probability that the connection speed will be less than 0.8 MBPS at any given time?

c) What is the probability that the connection speed will be between 0.875 MBPS and 1.125 MBPS at any given time?

4. Let the random variable X denote the annual snowfall amount at a well-known Utah ski resort. X has a normal distribution with mean 500 inches and standard deviation 50 inches.

a) What is the probability that in a given year the snowfall will be between 432 and 568 inches?

b) Find a value d such that X is in the range $500 \pm d$ with probability 0.999.

c) What is the probability that at least 8 out of 10 consecutive years will have annual snowfall amount greater than 522 inches?

5. Let T be a random variable that is the time to failure (in years) of a certain type of electrical component. T has an exponential probability density (or mass) function

$$f(x;\beta) = \begin{cases} \dfrac{1}{\beta} e^{-\frac{x}{\beta}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

with $\beta = 2$ years.

a) Compute the probability that a given component will fail in 5 years or less.

b) A laboratory uses 10 of these components. Let X be the number of components out of the 10 that have failed in 5 years or less. Compute the probability that 6 components have failed in 5 years or less.

c) For the same laboratory, compute the probability that all 10 components have failed in 5 years or less.

ANS:

1. See Conceptual Tools: Probability: Binomial Dist/Bernoulli: Example 4

2. Patwari HW.

3. Tasdizen HW

4. Tasdizen HW

5. Tasdizen HW