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ECE/CS 3530

Homework #5

1.

a) $_8C_1 = \frac{8!}{(8-1)!1!} = 8$

$_8C_2 = \frac{8!}{(8-2)!2!} = 28$

$_8C_3 = \frac{8!}{(8-3)!3!} = 56$

$8+28+56 = 92$ ways

b) $p = 0.2$ 3 or less

$$P(X \le 3) = \sum_{x=0}^{3} b(x, 8, 0.2) = \underset{56}{\frac{8!}{(5!)(3!)}}(.2)^3(.8)^5 + \underset{28}{\frac{8!}{(6!)(2!)}}(.2)^2(.8)^6 + \frac{8!}{(7!)(1!)}(.2)(.8)^7 + \frac{8!}{(8!)(0!)}(.2)^0(.8)^8$$

$$= .775 + \frac{8!}{(6!)(8!)}(.2)^0(.8)^8$$

$$= .9437$$

2. 500# p probability it works — K times sent back till working

$$f_K(k) = P(K=k) = \begin{cases} p(1-p)^{k-1}, & k = 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

$$\text{Cost} = 500 \cdot E(k) = \sum_{k=1}^{\infty} k\left(p(1-p)^{k-1}\right) \quad \boxed{\#500}$$

$$(1-x)^2 \sum_{n=1}^{\infty} n x^{n-1} = 1$$

$$1 - 2x + x^2 \sum_{n=1}^{\infty} n x^{n-1} = 1$$

$$\sum_{n=1}^{\infty} (1 - 2x + x^2) n x^{n-1}$$

$$\sum_{n=1}^{\infty} n x^{n-1} - 2n x^n + n x^{n+1} = 1$$

$1x^0 - 2x^1 + 1x^2$

$2x^1 - 2 \cdot 2x^2 + 2x^3$

$3x^2 - 2 \cdot 3x^3 + 3x^4$

only value that doesn't cancel is 1st 1, which equals 1

3.

a) $\mu = \frac{0.75+1.25}{2} = 1$

$\sigma^2 = \frac{(1.25-0.75)^2}{12} = 0.0208$

$\sigma = 0.1443$

b)

$$P(x<0.8) = \int_{0.75}^{0.8} 2\,dx = 2x\Big|_{0.75}^{0.8} = 0.1$$

c) $$P(0.875 < x < 1.125) = \int_{0.875}^{1.125} 2\,dx = 0.5$$

4.

a) $\mu = 500''$

$\sigma = 50''$

$$Z_1 = \frac{568-500}{50} = 1.36$$

$$Z_2 = \frac{432-500}{50} = -1.36$$

$.9131 - .0869 = .8262$

b)

$$3.49 = \frac{x-500}{50}$$

$$x = 674.5$$

c)

$P(X \geq 8) = .67$

$1 - .67 = .33$

$\sum_{x=8}^{10} b(x; 10, .33) = \frac{10!}{2!8!}(.33)^8(.67)^2 + \frac{10!}{1!9!}(.33)^9(.67)^1 + \frac{10!}{0!10!}(.33)^{10}(.67)^0$

$= .00284 + .00031 + .00001$

$= .00316$

$\mu = 10(.33) = 3.3$

$\sigma = \sqrt{10(.33)(.67)} = 1.49$

$z = \frac{522 - 500}{50} = .44$

$> 10.95\mu$

$= 15.31\mu$

5.

a)

$$f(x, \beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

with $\beta = 2$ years

a) $P(X \leq 5) = \int_0^5 \frac{1}{\beta} e^{-\frac{x}{\beta}}$

$\frac{d}{dx} e^{-\frac{x}{\beta}} = e^{-\frac{x}{2}} \cdot -\frac{1}{2}$

$= -\frac{1}{2} e^{-\frac{x}{2}}$

$= \frac{1}{2} \int_0^5 e^{-\frac{x}{2}}$

$= \int_0^5 -\frac{1}{2} e^{\frac{-x}{2}}$

$-e^{-\frac{x}{2}} \Big|_0^5 = \left(-e^{-\frac{5}{2}}\right) - \left(-e^{-\frac{0}{2}}\right)$

$= .918$

c)

$P(X \geq 8) = .67$

$1 - .67 = .33$ $\quad .00284$ $\quad \rightarrow 10.95\mu$

$$\sum_{x=8}^{10} b(x; 10, .33) = \frac{10!}{2!8!}(.33)^8(.67)^2 + \frac{10!}{1!9!}(.33)^9(.67)^1 + \frac{10!}{0!10!}(.33)^{10}(.67)^0$$

$$z = \frac{522 - 500}{50} = .44$$

$= 15.31\mu$

$= .00316$

5.

a)

$$f(x, \beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

with $\beta = 2$ years

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$$-e^{-\frac{x}{2}} \Big|_0^5 = \left(-e^{-\frac{5}{2}}\right) - \left(-e^{-\frac{0}{2}}\right)$$

$= .918$

b)

$P(X=6)$

$b(6;10,.918) = \frac{10!}{4!6!}(.918)^{6}(.082)^{4}$

$P(X=6) = .0057$

c)

$P(X=10)$

$b(10;10,.918) = \frac{10!}{0!10!}(.918)^{10}(.082)^{0}$

$P(X=10) = .425$