

Signals

A **DC** (direct current) signal refers to a fixed voltage whose polarity never reverses. {Ex. 5V,-15V}

An **AC** (alternating current) occurs when charge carriers periodically reverse their direction of movement. {Ex. Sinusoid => $5\sin(10t)$, Square Waves, Sawtooth-shaped}

- The voltage of an AC power source changes from instant to instant in time.
- Wall plug is AC with a frequency of 60 hertz and 120V
 -
- RMS value =

Real signals such as your voice, environmental sensors, etc. are time-varying voltages or currents that carry information.

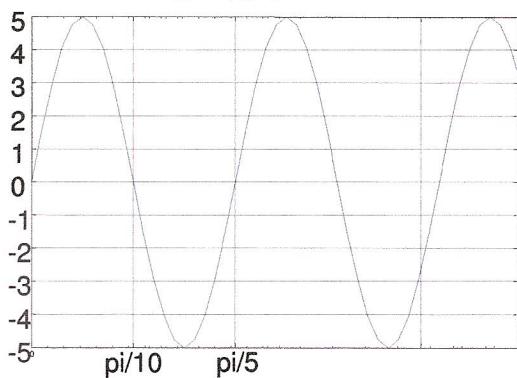
- Transducers transform one form of energy into another:
 - Ex: Microphone, Camera, Thermistor or other thermal sensor, Potentiometer, Light sensor, Computer, etc.
- Sine waves are "pretend" signals
 - Although sine waves are not *real* signals, we use them to simulate signals all the time, both in calculations and in the lab. This makes sense because all signals can be thought of as being made up of a spectrum of sine waves.

These types of signals can be hard to characterize mathematically. If a signal is periodic but arbitrary in amplitude, recall that it can be expressed by the Fourier series(a series of sinewaves of different frequencies and amplitudes).

Example #5

Sketch the following waveforms. Identify the dc component of the waveform and the ac component of the waveform.

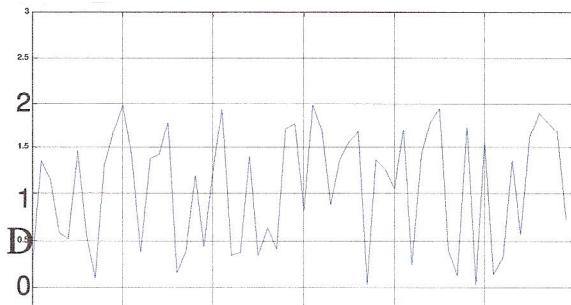
a. $V_s =$



b. $V_s =$

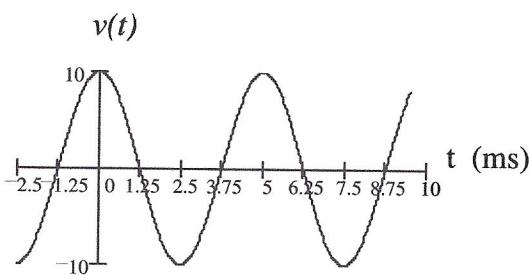


c. $V_s =$

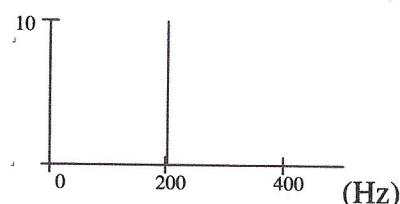


Sine wave:

Time domain:



Frequency domain:



amplitude :=

$$V_{\text{RMS}} := \sqrt{\frac{1}{T} \int_0^T v(t)^2 dt}$$

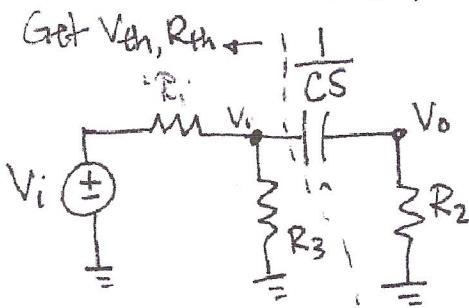
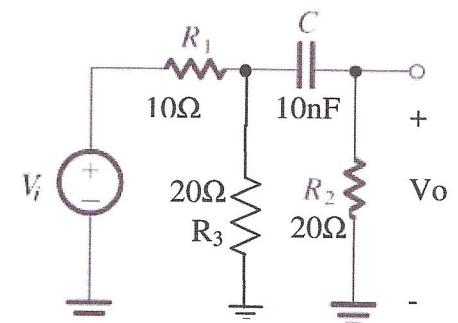
$$f = 200 \text{ Hz}$$

$$T := 5 \text{ ms}$$

$$f := \frac{1}{T} \quad \omega := 2\pi f$$

Example #6

When analyzing a time dependent element (capacitors), translate into frequency domain $\Rightarrow C = \frac{1}{j\omega C} = \frac{1}{sC}$ and then analyze the circuit using normal circuit analysis techniques. Analyze the circuit to the right to find the transfer function $\frac{V_o}{V_i}$. Solve the circuit symbolically first (with R_1, R_2, R_3, C) and then plug in their values.



$$V_{\text{th}} = \frac{R_3 \cdot V_i}{R_3 + R_1}, \quad R_{\text{th}} = R_1 \parallel R_3$$

$$\frac{V_o}{V_i} = \frac{R_2 C \cdot s \cdot R_3 \cdot V_i}{(R_1 + R_3)([R_2 + R_1 \parallel R_3](s + 1))}$$

$$= \frac{20(10n) s(20)}{(10 + 20)(266.7n \cdot 10n \cdot s + 1)}$$

$$\frac{V_o}{V_i} = \frac{4 \times 10^{-6} \cdot s}{30(266.7n \cdot s + 1)} = \boxed{\frac{(133n) \cdot s}{(266.7n \cdot s + 1)}}$$

What does this equation mean? By substituting $s = j\omega$ in the above equation. The magnitude of the equation is:

$$\left| \frac{133n(\omega)j}{(266.7n(\omega)j + 1)} \right| = \frac{|133n(\omega)j|}{|(266.7n(\omega)j + 1)|} = \frac{133n(\omega)}{\sqrt{(266.7n(\omega))^2 + 1^2}}$$

This magnitude can now be graphed with the x-axis as ω and the y axis as the calculated value. This is one of the graphs used for the Bode plots. To plot this, an understanding of dB is needed.

Decibels

Your ears respond to sound logarithmically, both in frequency and in intensity.

Musical octaves are in ratios of two. "A" in the middle octave is 220 Hz, in the next, 440 Hz, then 880 Hz, etc...

It takes about ten times as much power for you to sense one sound as twice as loud as another.

10x power \approx 2x loudness

A bel is such a 10x ratio of power. Power ratio expressed in bels = $\log\left(\frac{P_2}{P_1}\right)$ bels The bel is named for Alexander Graham Bell.

It is a logarithmic expression of a unitless ratio (like gain).

The bel unit is never actually used, instead we use the decibel (dB, 1/10th of a bel).

$$\text{Power ratio expressed in dB} = 10 \cdot \log\left(\frac{P_2}{P_1}\right) \text{ dB}$$

dB are also used to express voltage and current ratios, which relate to power when squared. $P = \frac{V^2}{R} = I^2 \cdot R$

$$\text{Voltage ratio expressed in dB} = 10 \cdot \log\left(\frac{V_2^2}{V_1^2}\right) \text{ dB} = 20 \cdot \log\left(\frac{V_2}{V_1}\right) \text{ dB}$$

$$\text{Current ratio expressed in dB} = 20 \cdot \log\left(\frac{I_2}{I_1}\right) \text{ dB}$$

These are the most common formulas used for dB

Some common ratios expressed as dB

$$20 \cdot \log\left(\frac{1}{\sqrt{2}}\right) = -3.01 \text{ dB} \quad 10^{\frac{3}{20}} = 0.708 \quad 20 \cdot \log(\sqrt{2}) = 3.01 \text{ dB} \quad 10^{\frac{3}{20}} = 1.413$$

$$20 \cdot \log\left(\frac{1}{2}\right) = -6.021 \text{ dB} \quad 10^{\frac{-6}{20}} = 0.501 \quad 20 \cdot \log(2) = 6.021 \text{ dB} \quad 10^{\frac{6}{20}} = 1.995$$

$$20 \cdot \log\left(\frac{1}{10}\right) = -20 \text{ dB} \quad 10^{\frac{-20}{20}} = 0.1 \quad 20 \cdot \log(10) = 20 \text{ dB} \quad 10^{\frac{20}{20}} = 10$$

$$20 \cdot \log\left(\frac{1}{100}\right) = -40 \text{ dB} \quad 10^{\frac{-40}{20}} = 0.01 \quad 20 \cdot \log(100) = 40 \text{ dB} \quad 10^{\frac{40}{20}} = 100$$

We will use dB fairly commonly in this class, especially when talking about frequency response curves.

Example #7

The frequency domain expression for the output over the input of a circuit is solved to be

$$\frac{\text{output}}{\text{input}} = \frac{10^5(s+5)}{(s+1)(s+5000)} \quad \text{Substitute } s=j\omega \text{ into the above equation and calculate the magnitude(dB) and phase (degrees).}$$

Plug in values for ω equal to $10^{-1}, 0.8, 0.9, 10^{-0}, 2, 3, 4, 5, 6, 7, 10^1, 10^2, 10^3, 3000, 4000, 5000, 6000, 7000, 10^4, 10^5$ rad/sec and plot these values on a semilog graph for both magnitude and phase. Recall that magnitude, $|a+bj| = \sqrt{a^2 + b^2}$ and the phase, $\angle(a+bj) = \tan^{-1}(b/a)$

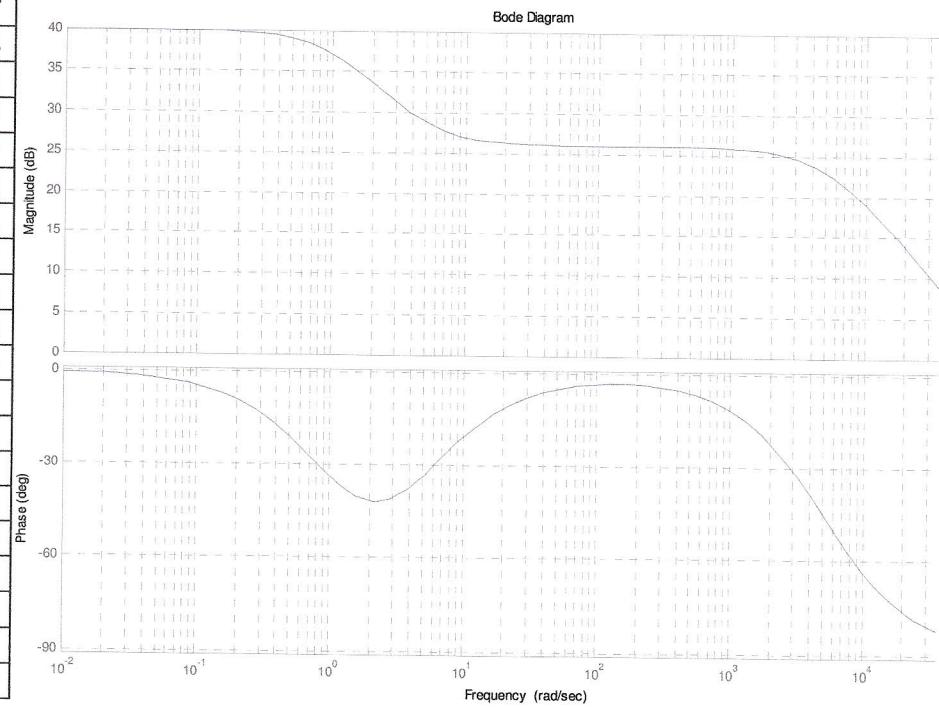
$$\text{Magnitude: } \left| \frac{\text{output}}{\text{input}} \right| = \left| \frac{10^5(j\omega+5)}{(j\omega+1)(j\omega+5000)} \right| = \frac{|10^5|(j\omega+5)}{|(j\omega+1)|(j\omega+5000)|} = \frac{\sqrt{(10^5)^2 + 0^2} \sqrt{5^2 + \omega^2}}{\sqrt{1^2 + \omega^2} * \sqrt{5000^2 + \omega^2}}$$

$$\text{Phase: } \angle\left(\frac{\text{output}}{\text{input}}\right) = \angle\left(\frac{10^5(j\omega+5)}{(j\omega+1)(j\omega+5000)}\right) = \frac{\angle 10^5 * \angle(j\omega+5)}{\angle(j\omega+1) * \angle(j\omega+5000)} =$$

=

magnitude=99.5V/V=20*log(99.5V/V)=39.96dB; phase=0+1.15-5.7-0.001=-4.6 degrees

ω (rad/sec)	Mag(dB)	Phase(Degrees)
.1	39.95852	-4.56598
0.8	37.96134	-29.5787
0.9	37.56169	-31.7936
1	37.16003	-33.7015
2	33.65488	-41.6565
3	31.33539	-40.6357
4	29.84395	-37.3498
5	28.86056	-33.7474
6	28.19187	-30.412
7	27.72321	-27.4878
10	26.94647	-20.969
100	26.02927	-3.43523
1000	25.85037	-11.5391
3000	24.68522	-31.0402
4000	23.87217	-38.7171
5000	23.0103	-45.0458
6000	22.1467	-50.2326
7000	21.30768	-54.4951
10000	19.0309	-63.4579
100000	-0.01084	-87.1399
1000000	-20.0001	-89.7138



Frequency response

The "response" of a system or circuit is the output for a given input.

A "transfer function" is a mathematical description of how the output is related to the input.

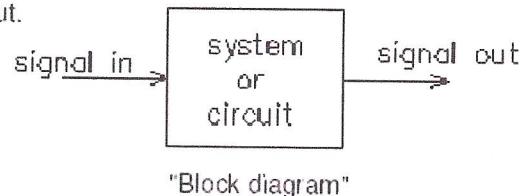
$$\text{output} = \text{Transfer function} \times \text{input}$$

$$\text{or... Transfer function} = \frac{\text{output}}{\text{input}}$$

No real system or circuit treats all frequencies the same, so all real transfer functions are functions of frequency.

$$\text{Transfer function} = H(\omega) \text{ or } H(f) \text{ or, Transfer function} = H(s)$$

The transfer function can be used to describe the "frequency response" of a circuit. That is, how does the circuit respond to inputs of different frequencies.



"Block diagram"

Bode Plots

- 2 plots – both have logarithm of frequency on x-axis
 - y-axis magnitude of transfer function, $H(s)$, in dB
 - y-axis phase angle, in degrees

The plot can be used to interpret how the input affects the output in both magnitude and phase over frequency. To sketch the graphs, the circuit is first analyzed to find output/input (transfer function). This equation is used as the basis for the plots. The equation is analyzed for magnitude and phase as shown in the previous example (#5)

Is there a shortcut to graph the Bode plots?

MAGNITUDE PLOT:

- Determine the Transfer Function of the system:

$$H(s) = \frac{K(s+z_1)(s+z_2)\dots}{(s+p_1)(s+p_2)\dots}$$

- Rewrite it by factoring both the numerator and denominator into the standard form

$$H(s) = \frac{Kz_1 z_2 \dots (\frac{s}{z_1} + 1)(\frac{s}{z_2} + 1)\dots}{p_1 p_2 \dots (\frac{s}{p_1} + 1)(\frac{s}{p_2} + 1)\dots}$$

where the z s are called zeros and the p s are called poles.

- Replace s with $j\omega$. Then find the Magnitude of the Transfer Function.

$$H(j\omega) = \frac{Kz_1 z_2 \dots (\frac{j\omega}{z_1} + 1)(\frac{j\omega}{z_2} + 1)\dots}{p_1 p_2 \dots (\frac{j\omega}{p_1} + 1)(\frac{j\omega}{p_2} + 1)\dots}$$

If we take the \log_{10} of this magnitude and multiply it by 20 it takes on the form of

$$\begin{aligned} 20 \log_{10}(H(j\omega)) &= 20 \log_{10} \left(\frac{Kz_1 z_2 \dots (\frac{j\omega}{z_1} + 1)(\frac{j\omega}{z_2} + 1)\dots}{p_1 p_2 \dots (\frac{j\omega}{p_1} + 1)(\frac{j\omega}{p_2} + 1)\dots} \right) = \\ &20 \log_{10}|K| + 20 \log_{10}|z_1| + 20 \log_{10}|z_2| + \dots + 20 \log_{10}\left|\left(\frac{j\omega}{z_1} + 1\right)\right| + 20 \log_{10}\left|\left(\frac{j\omega}{z_2} + 1\right)\right| + \dots - 20 \log_{10}|p_1| \\ &- 20 \log_{10}|p_2| - \dots - 20 \log_{10}\left|\left(\frac{j\omega}{p_1} + 1\right)\right| - 20 \log_{10}\left|\left(\frac{j\omega}{p_2} + 1\right)\right| - \dots \end{aligned}$$

Recall => $\log(a*b) = \log(a) + \log(b)$ and $\log(a/b) = \log(a) - \log(b)$

You can see from this expression that each term contributes a number to the final value at a specific frequency. Therefore, each of these individual terms is very easy to show on a logarithmic plot. The entire Bode log magnitude plot is the result of the superposition of all the straight line terms. This means with a little practice, we can quickly sketch the effect of each term and quickly find the overall effect. To do this we have to understand the effect of the four different types of terms.

These include: 1) Constant terms

$$K$$

2) Poles and Zeros at the origin

$$|j\omega|$$

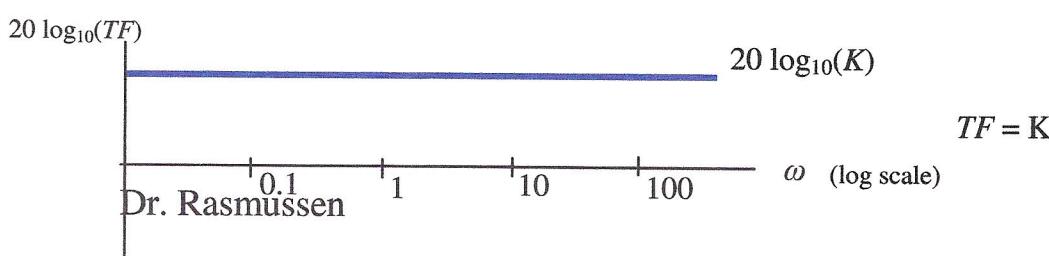
3) Poles and Zeros not at the origin

$$\left|1 + \frac{j\omega}{p_1}\right| \text{ or } \left|1 + \frac{j\omega}{z_1}\right|$$

4) Complex Poles and Zeros (not addressed at this time)

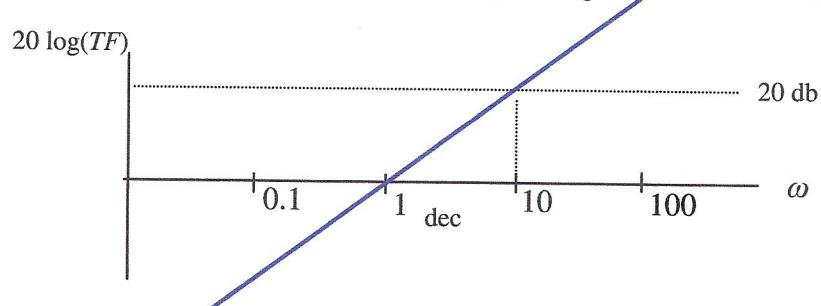
Effect of Constant Terms:

Constant terms such as K contribute a straight horizontal line of magnitude $20 \log_{10}(K)$ (not dependent on frequency)



Effect of Individual Zeros and Poles at the origin:

A zero at the origin occurs when there is an s or $j\omega$ multiplying the numerator. Each occurrence of this causes a positively sloped line passing through $\omega = 1$ with a rise of 20 db over a decade.



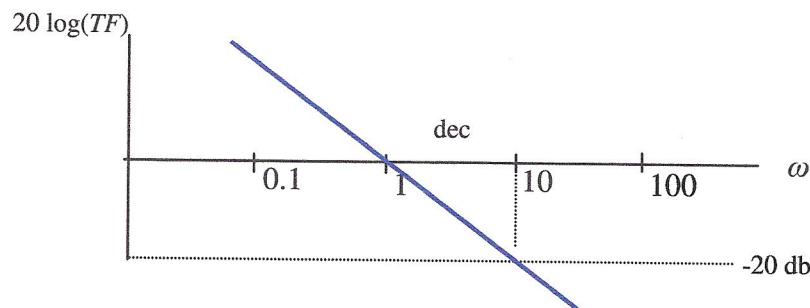
$$TF = |j\omega|$$

$$\text{If } \omega=0.1 \Rightarrow |j0.1|=0.1V/V=20\log(0.1)=$$

$$\text{If } \omega=1 \Rightarrow |j1|=1V/V=20\log(1)=$$

$$\text{If } \omega=10 \Rightarrow |j10|=10V/V=20\log(10)=$$

A pole at the origin occurs when there are s or $j\omega$ multiplying the denominator. Each occurrence of this causes a negatively sloped line passing through $\omega = 1$ with a drop of 20 db over a decade.



$$TF = \frac{1}{j\omega}$$

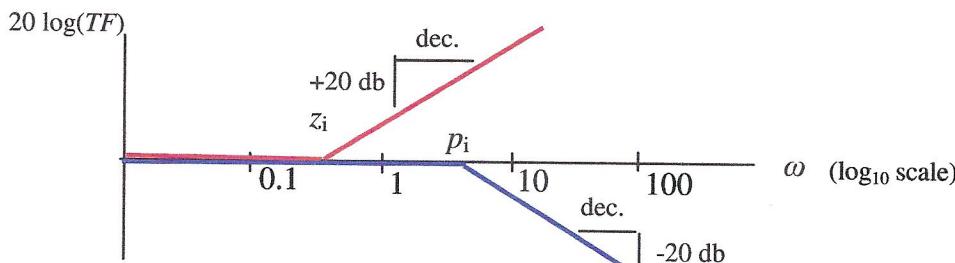
$$\text{If } \omega=0.1 \Rightarrow |\frac{1}{j0.1}|=10V/V=20\log(10)=$$

$$\text{If } \omega=1 \Rightarrow |\frac{1}{j1}|=1V/V=20\log(1)=$$

$$\text{If } \omega=10 \Rightarrow |\frac{1}{j10}|=0.1V/V=20\log(0.1)=$$

Effect of Individual Zeros and Poles Not at the Origin

Zeros and Poles not at the origin are indicated by the $(1+j\omega/z_i)$ and $(1+j\omega/p_i)$. The values z_i and p_i in each of these expression is called a **break frequency**. Below their break frequency these terms do not contribute to the log magnitude of the overall plot. Above the break frequency, they represent a ramp function of 20 db per decade. Zeros give a positive slope. Poles produce a negative slope.



$$TF = \frac{\left| 1 + \frac{j\omega}{z_i} \right|}{\left| 1 + \frac{j\omega}{p_i} \right|}$$

Before looking at the effect of the 2nd order terms, let's learn how to plot with the three terms already described. We will work several examples where we show how the Bode log magnitude plot is sketched. To complete the **log magnitude vs. frequency plot** of a Bode diagram, **superposition** all the lines of the different terms on the same plot.

PHASE PLOT:

For our original transfer function,

$$H(j\omega) = \frac{K z_1 z_2 \cdots (j\omega/z_1 + 1)(j\omega/z_2 + 1) \cdots}{p_1 p_2 \cdots (j\omega/p_1 + 1)(j\omega/p_2 + 1) \cdots}$$

the cumulative phase angle associated with this function are given by

$$\angle H(j\omega) = \frac{\angle K \angle z_1 \angle z_2 \cdots \angle(j\omega/z_1 + 1) \angle(j\omega/z_2 + 1) \cdots}{\angle p_1 \angle p_2 \cdots \angle(j\omega/p_1 + 1) \angle(j\omega/p_2 + 1) \cdots}$$

Then the cumulative phase angle as a function of the input frequency may be written as

$$\angle H(j\omega) = \angle[K + z_1 + z_2 + \cdots - p_1 - p_2 - \cdots] + \tan^{-1}(j\omega/z_1) + \tan^{-1}(j\omega/z_2) + \cdots - \tan^{-1}(j\omega/p_1) - \tan^{-1}(j\omega/p_2) - \cdots$$

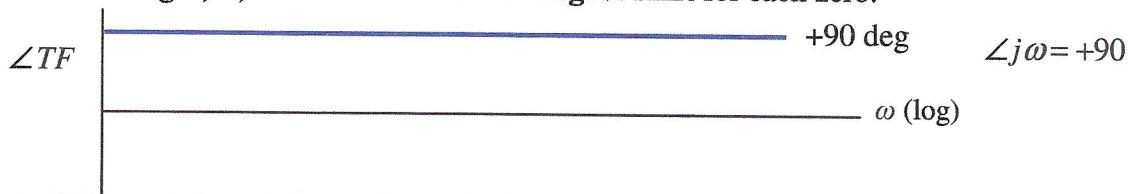
Once again, to show the phase plot of the Bode diagram, lines can be drawn for each of the different terms. Then the total effect may be found by superposition.

Effect of Constants on Phase:

A positive constant, $K > 0$, has no effect on phase. A negative constant, $K < 0$, will set up a phase shift of $\pm 180^\circ$.

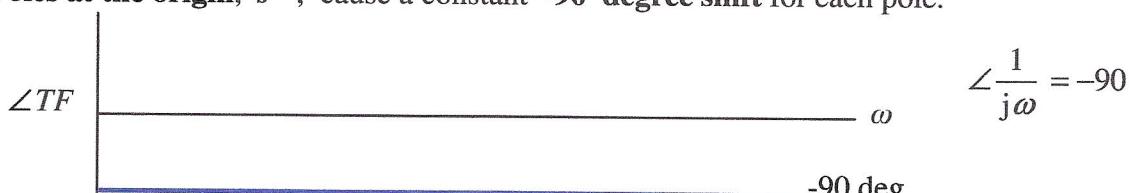
Effect of Zeros at the origin on Phase Angle:

Zeros at the origin, s , cause a constant $+90^\circ$ degree shift for each zero.



Effect of Poles at the origin on Phase Angle:

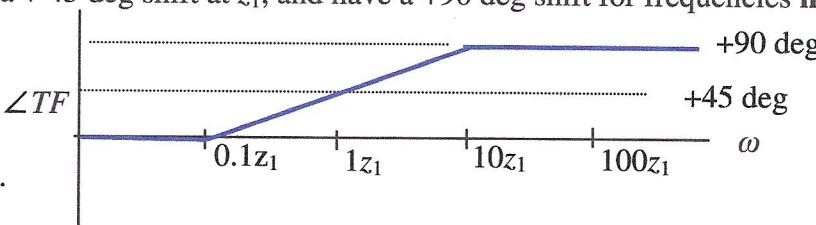
Poles at the origin, s^{-1} , cause a constant -90° degree shift for each pole.



Effect of Zeros not at the origin on Phase Angle:

Zeros not at the origin, like $1 + \frac{j\omega}{z_1}$, have no phase shift for frequencies much lower than z_1 , have

a $+45^\circ$ shift at z_1 , and have a $+90^\circ$ shift for frequencies much higher than z_1 .



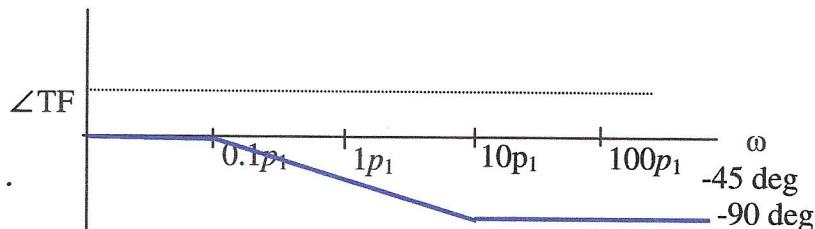
If $z_1=1 \Rightarrow \angle(1+j\omega) = \tan^{-1}(\frac{\omega}{1})$
If $\omega=0.1 \Rightarrow \tan^{-1}(0.1)=$
If $\omega=1 \Rightarrow \tan^{-1}(1)=$
If $\omega=10 \Rightarrow \tan^{-1}(10)=$
If $\omega=100 \Rightarrow \tan^{-1}(100)=$

To draw the lines for this type of term, the transition from 0° to $+90^\circ$ is drawn over 2 decades, starting at $0.1z_1$ and ending at $10z_1$.

Effect of Poles not at the origin on Phase Angle:

Poles not at the origin, like $\frac{1}{1 + \frac{j\omega}{p_1}}$, have no phase shift for frequencies much lower than p_1 ,

have a -45 deg shift at p_1 , and have a -90 deg shift for frequencies much higher than p_1 .



To draw the lines for this type of term, the transition from 0° to -90° is drawn over 2 decades, starting at $0.1p_1$ and ending at $10p_1$.

When drawing the phase angle shift for not-at-the-origin zeros and poles, first locate the break frequency of the zero or pole. Then start the transition 1 decade before, following a slope of $\pm 45^\circ/\text{decade}$. Continue the transition until reaching the frequency one decade past the break frequency.

SUMMARY OF STRAIGHT-LINE APPROXIMATION PROCEDURE STEPS(No COMPLEX):

(Note that a decade is a multiple of 10 – 1,10,100,1000,etc)

1. Rearrange the equation into standard form:

$$H(s) = \frac{Kz_1 z_2 \cdots (\frac{s}{z_1} + 1)(\frac{s}{z_2} + 1)\cdots}{p_1 p_2 \cdots (\frac{s}{p_1} + 1)(\frac{s}{p_2} + 1)\cdots}$$

where K, z_1 , z_2 , etc are all constant values.

2. Determine the poles and zeros.

*Note: If there are more than one poles/zeros at the same break frequency(say there are r), just multiply the slope/phase changes by r. (ex. $(1+s/10)^2 \Rightarrow$ it is a negative zero(numerator) and so it will change the slope by $2*20\text{dB/dec}$ and have a $2*45^\circ$ slope/dec.*

3. Draw the magnitude plot:

a. Determine starting value:

Case 1: No pole or zero at the origin:

$$\text{starting value} = 20 \log_{10}\left(\frac{Kz_1 z_2 \cdots}{p_1 p_2 \cdots}\right)$$

Case 2: A pole or zero at the origin:

- Pick a frequency value less than the lowest pole or zero value.

- Plug in the frequency in the standard form equation above and take the magnitude. *This value is for that frequency only.* There is a constant slope going through this point.
+20dB/dec slope if the location is a zero. -20dB/dec slope if the location is a pole.
- b. Begin at the starting point. Start with the slope (0 slope if a constant, +20dB/dec slope if zero at origin, -20dB/dec slope if pole at origin). From left to right, at each zero add +20dB/dec to the current slope and at each pole -20dB/dec. Continue through each frequency.

4. Draw the phase plot:

- Determine the starting value:

Case 1: No pole or zero at the origin:

If constant>0 then starting value = 0°

If constant<0 then starting value = $\pm 180^\circ$

Case 2: A pole or zero at the origin:

starting value = $+90^\circ$ if zero at origin

starting value = -90° if pole at origin

- b. Label each range of frequency according to the following(suggest putting on graph):

zero => from 1 decade before frequency to 1 decade after frequency: $+45^\circ$ slope/dec

pole => from 1 decade before frequency to 1 decade after frequency: -45° slope/dec

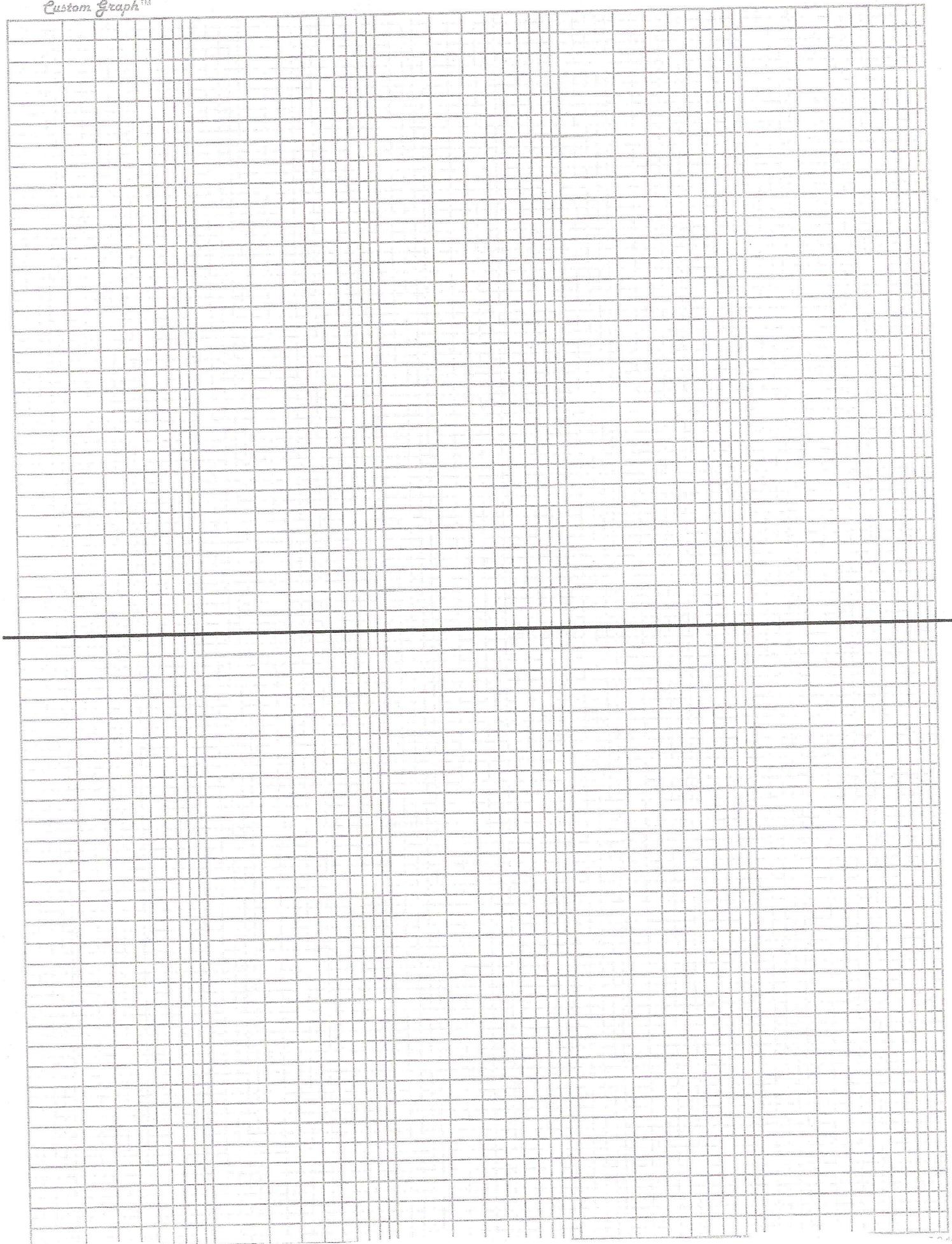
(eg if $\omega=10$ and is a pole then range is $1 < \omega < 100$ with a slope of -45° slope/dec)

- c. Look at each frequency range that has a slope. Add all slopes within that region. From left to right: start with starting value and slope of 0, continue until first region of change. Add all slopes within that region. Continue until the end is met. If no slope during a region the slope is constant (0).

Example:

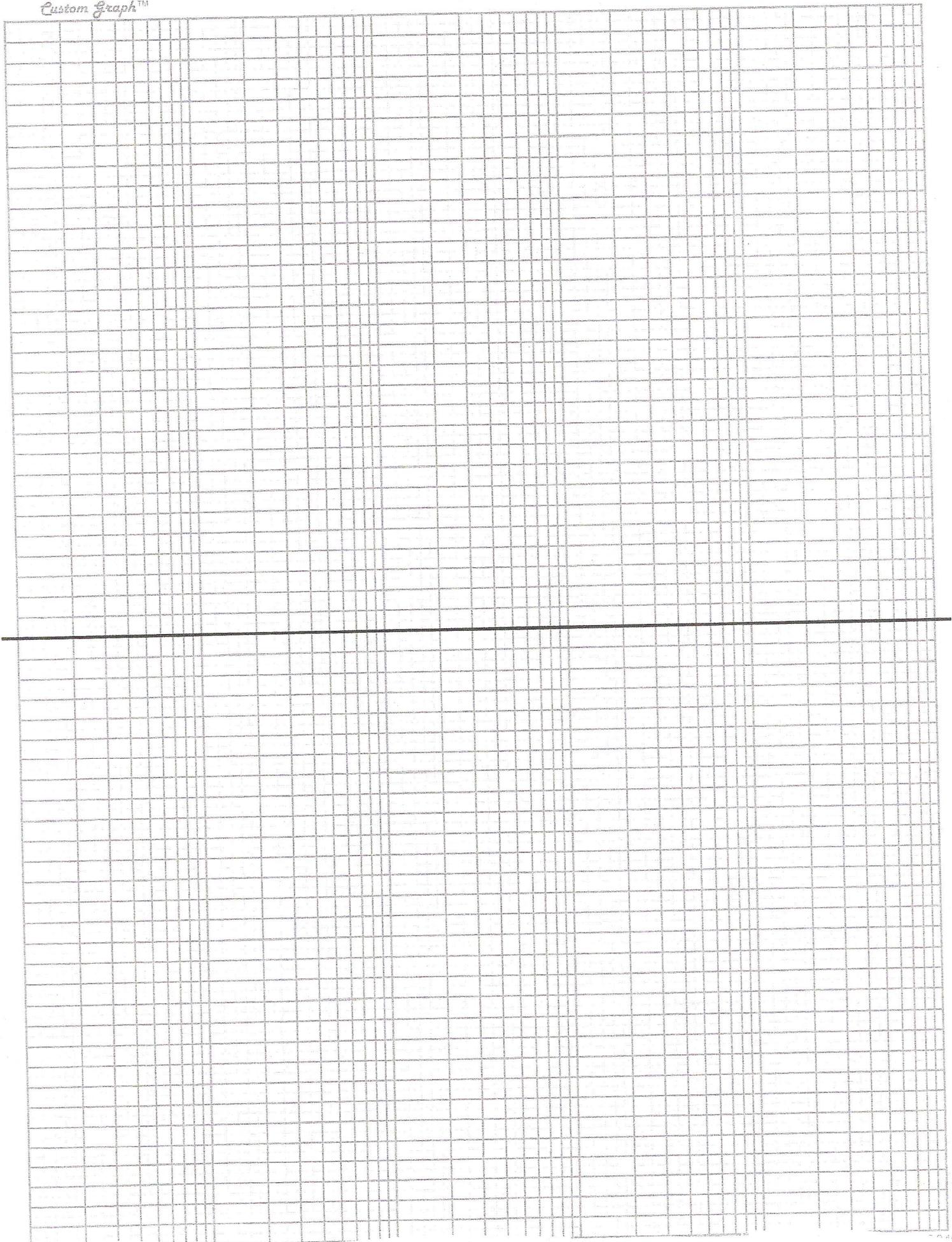
$$H(s) = \frac{(s+100)(s+1k)}{(s+10)(s+10k)}$$

Custom Graph™



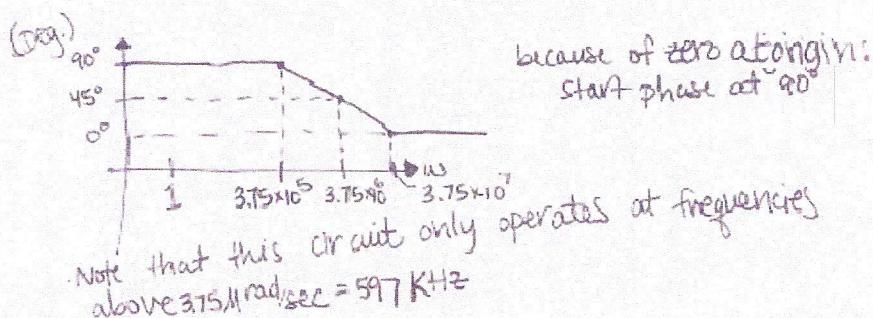
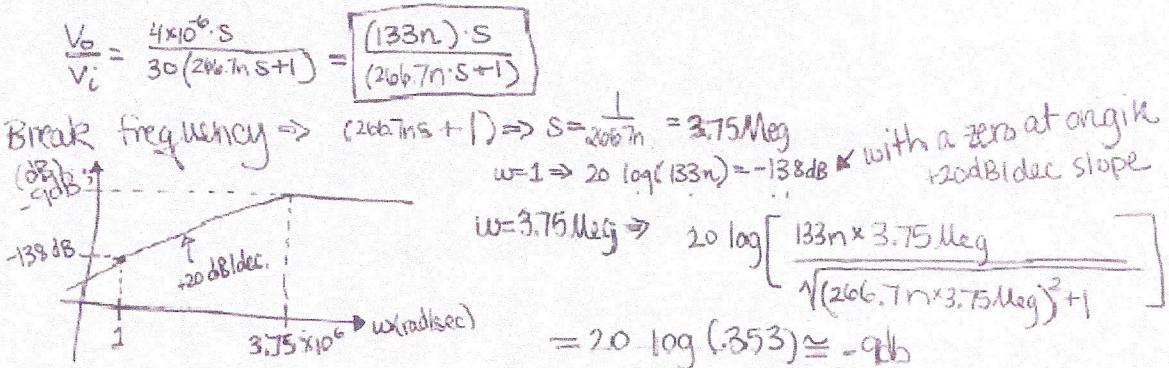
Example

$$H(s) = \frac{100(s+100)(s+10)}{s^2(s+10k)}$$

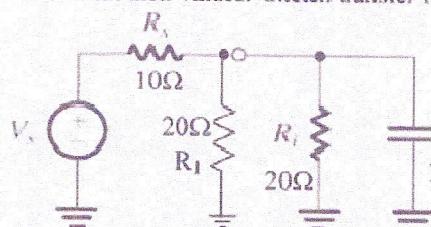


Example

From circuit before: $\frac{V_o}{V_i} = \frac{133n \cdot s}{(266.7ns + 1)}$

Example #10 Bode Plots;Example 11:

Analyze the following circuit to find the transfer function V_i/V_s . Solve the circuit symbolically first and then with their values. Sketch transfer function using a straight-line approximation procedure.



$$\frac{V_i}{V_s} = \frac{(R_i || R_i || \frac{1}{C_i s}) \cdot V_s}{R_s + (R_i || R_i || \frac{1}{C_i s})}$$

$$\frac{V_i}{V_s} = \frac{R_i || R_i || \frac{1}{C_i s}}{\frac{1}{R_i} + \frac{1}{R_i} + \frac{1}{C_i s}} = \frac{1}{\frac{R_i}{R_i R_i} + \frac{R_i}{R_i R_i} + \frac{C_i s}{R_i R_i}}$$

$$= \frac{R_i R_i}{R_i + R_i + C_i R_i R_i s}$$

$$\frac{V_i}{V_s} = \frac{R_i R_i}{(R_i + R_i + C_i R_i R_i s) + R_i R_i}$$

$$(R_i + R_i + C_i R_i R_i s)$$

$$\frac{V_i}{V_s} = \frac{R_i R_i}{(R_i R_i + R_s R_i + R_s R_i + C_i R_i R_i R_s s)}$$

$$\frac{V_i}{V_s} = \frac{R_i}{(1 + \frac{s}{20 \text{ MHz}})}$$

This circuit only
operates below
 $20 \text{ Meg rad/sec} \cong 3.2 \text{ MHz}$

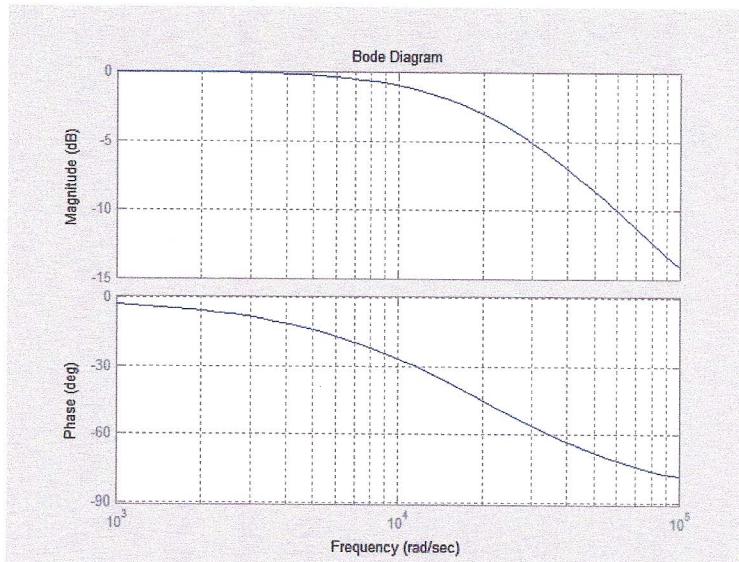
BODE PLOTS IN MATLAB

Examples using three different methods applied to the transfer function from Prelab 1:

$$TF = \frac{20000}{s + 20000}$$

Method 1: Easiest (If you have the Control Toolbox in Matlab)

```
s=tf('s');
H = (20000/(s+20000));
Bode(H)
grid on
```



Method 2: Annalisa's Way (With no Control Toolbox...)

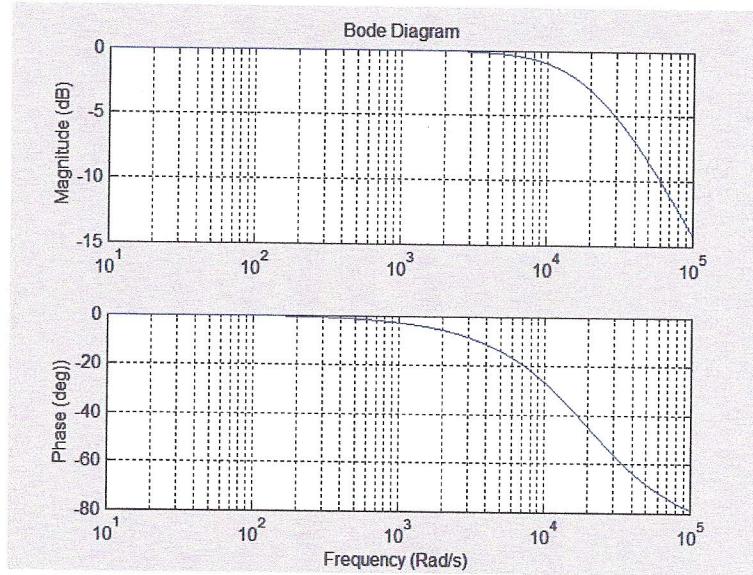
%Expand the numerator and denominator of your transfer function by multiplying out the terms. Then
%make an array of the coefficients of the numerator and denominator of the transfer function in descending
%order of powers. Example: if numerator is As^2+Bs+C, array will be num=[A B C]. Note that the arrays
%for the numerator and denominator must be equal in length.

```
numTF=[0 20000];
denomTF=[1 20000];
w=0:10:10e4;
```

```
%Function 'freqs' gives the frequency response in the s-domain
Y=freqs(numTF,denomTF,w);
y1=abs(Y);
y2=angle(Y);
```

```
subplot(2,1,1)
semilogx(w,20*log10(y1))
grid on
ylabel('Magnitude (dB)')
title('Bode Diagram')
```

```
subplot(2,1,2)
semilogx(w,y2*(180/pi))
grid on
ylabel('Phase (deg)')
xlabel('Frequency (Rad/s)')
```



Method 3: Dr. Rasmussen's Way (With no Control Toolbox....)

```
%Function 'logspace' creates an array of 200 points from -1 to 10^5 spaced logarithmically
```

```
w=logspace(-1,5,200);
```

```
MagH=sqrt(0^2+20000^2)./sqrt(w.^2+20000^2);
```

```
MagHdb=20*log10(MagH);
```

```
PhaseHRad=-atan(w/20000);
```

```
PhaseHDeg=PhaseHRad*180/pi;
```

```
subplot(2,1,1)
semilogx(w, MagHdb)
ylabel('20 log10(|TF|) [dB]')
title('Bode Diagram')
grid on
```

```
subplot(2,1,2)
semilogx(w, PhaseHDeg)
xlabel('frequency [rad/s]')
ylabel('Phase Angle [deg]')
grid on
```

