Wo01 Waves in a Ripple Tank & Diffraction and Interference

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1 Introduction

This experiment investigates the various properties and characteristics of waves. These physical phenomena are at the heart of many transmission processes; such as the propagation of sound through a medium responsible for audio communication and the transmission of electromagnetic waves in a vaccuum which enables visible light from the Sun to travel to Earth. In addition, quantum mechanics has made the connection between the wave-like nature exhibited by particles and the particle-like nature of waves. Interesting developments have been made due to the study of waves and this is at the forefront of modern technologies.

A wave can be thought of as a time-varying disturbance that propagates through a medium or a vacuum. This can be described mathematically as a *wave function* $\psi(x,t)$

$$\psi(x,t) = \psi_0 cos(kx - \omega t) \tag{1}$$

where, ψ_0 is the wave amplitude, x is the spatial position of the propagating wave at time t, k is the wave vector related to the wavelength, λ by $k = \frac{2\pi}{\lambda}$, and ω is the angular frequency related to the time period, T, of the wave by $\omega = \frac{2\pi}{T}$. Equation (1) only considers the real part of the propagating wave. In most instances, waves also have an imaginary part. From Euler's identity equation (1) can be re-expressed as a complex exponential

$$\psi(x,t) = \psi_0 e^{i(kx - \omega t)} \tag{2}$$

and all the symbols have their usual meaning as defined for equation (1).

The various characteristics of waves is the same for different types of waves [1]. Making use of this physical concept, the first of these experiments investigates wave phenomena using water waves in a ripple tank. In particular, this will study reflection, refraction, diffraction and interference. The latter two phenomena will be studied further in the second experiment where a laser light will be diffracted through single and double slits to investigate interference and diffraction patterns. Furthermore, extensions to these experiments will look into other wave phenomena such as the Doppler effect, multiple slit diffraction and interference and how water wave speed depends on the depth.

2 Theory

When a wave front approaches at an angle to a boundary *reflection* occurs. This is when the angle of incidence, θ_i is equal to the angle of reflection, θ_r at the boundary

$$\theta_i = \theta_r \tag{3}$$

Figure 1 describes this pictorially.

If the wave passes between two different media and changes speed, as a consequence, it will therefore change direction. This is known as *refraction*. Figure 1 shows a physical representation for this phenomena.

Refraction of a wave between two different media is described by Snell's Law

$$n_i sin\theta_i = n_t sin\theta_t \tag{4}$$

where n_i and n_t are the refactive indices of the inital and final media respectively, θ_i is the angle of incidence and θ_t is the angle of refraction between two different media. This is usually used when considering the refraction of electromagnetic waves at a boundary. However, the physics of waves is the same for different types of waves and, more recently, it has been found [4] that water waves, when refracted obey Snell's Law (4).

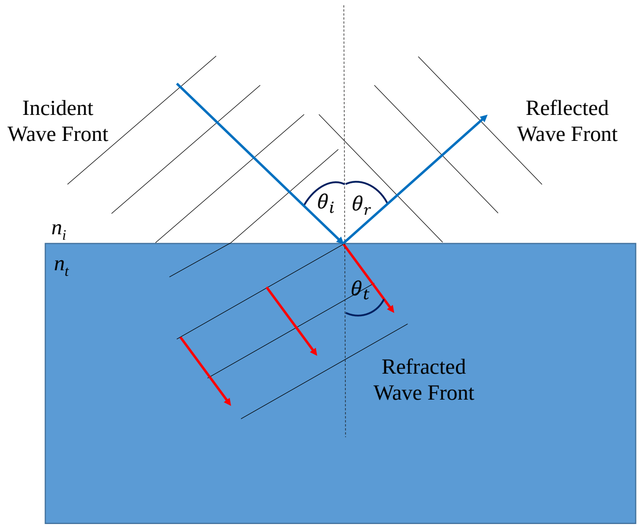

Figure 1: A diagram demonstrating the physical processes of reflection and refraction as an incident wave front encounters a boundary. The direction of travel for the incident and reflected waves are shown by the black arrows, whilst the direction of travel for the refracted wave is shown by the red arrow. The two media have refractive indices n_i and n_t. The angles, θ_i, θ_r and θ_t correspond to the angles of incidence, reflection and refraction respectively between direction of travel and the normal to the boundary.

Diffraction occurs when a wave 'bends' as it passes through a gap (or slit) between two obstacles or when the wave travels around a barrier. An illustration of the diffraction of light is given in Figure 2 when incident light waves encounter a single slit.

The intensity of diffracted light from a single slit into an angle θ [2] is given by

$$I = I_0 \frac{sin^2\alpha}{\alpha^2} \tag{5}$$

where $\alpha = \frac{\pi a sin\theta}{\lambda}$, λ is the wavelength of the light and a is the slit width.

For two slits, the intensity of diffracted light into an angle θ [2] is

$$I = I_0 \frac{sin^2\alpha}{\alpha^2} cos^2\delta \tag{6}$$

where $\delta = \frac{\pi d sin\theta}{\lambda}$ and d is the separation distance between the two slits. A derivation for equations (5) and (6) involving a geometrical approach and phasor diagrams can be found by [3]. These equations can also be dervied by considering Fourier transforms and the convolution theorem this approach can be found from [4].

Using equation (5), the intensity minima for a single slit occurs when α is a multiple of π. This therefore means that

$$sin\theta = \frac{m\lambda}{a} \qquad (m = \pm1, \pm2, ...) \tag{7}$$

If L'Hôpital's rule [3] is applied to equation (5) in the limit as the intensity at $\alpha \to 0$ is found to be $I = I_0$ as expected. When the detector to slit distance is big, the small angle approximation can be used. Equation (7) can then be approximated to

$$\theta \approx \frac{m\lambda}{a} \qquad (m = \pm1, \pm2, ...) \tag{8}$$

The intensity maxima of a single slit diffraction pattern can be approximately found by using equation (5) and realising that they occur when the sine function is a maximum (±1) [3]. In other words, when

$$\alpha = \pm\left(m + \frac{1}{2}\right)\pi \qquad (m = 0, 1, 2, ...) \tag{9}$$

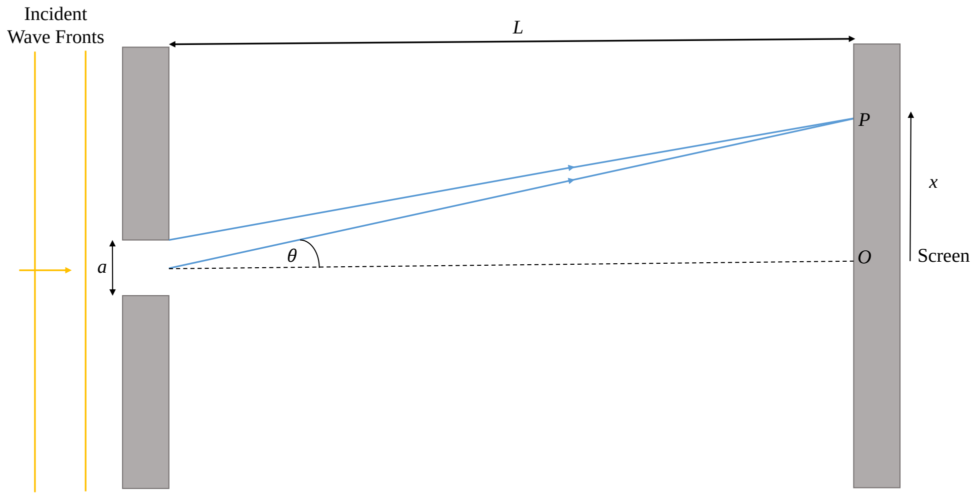

Figure 2: An illustration demonstrating the diffraction of light from a single slit and defining some terms. The incident light wave approaches a slit of width a and two beams are considered from the Huygen's wavelets [3] formed as a result of passing through the slit. These two beams arrive at a point P creating an interference pattern on the screen and the bottom ray is diffracted through by an angle θ. The distance between the plane of the slit and the screen is L and the vertical distance between the origin, O and point P is x.

However, upon further analysis of applying differentiation to equation (5) and setting equal to zero to find the maxima it is found that the there is no maxima at $m = 0$ [3]. Therefore, when equation (9), with $m \neq 0$, is subsituted into equation (5) the intensity maxima, I_m is approximately given by [3]

$$I_m \approx \frac{I_0}{\left(m + \frac{1}{2}\right)^2 \pi^2} \qquad (m = \pm 1, \pm 2, ...) \tag{10}$$

3 Method

3.1 Waves in a Ripple Tank

The ripple tank was placed on an up-turned crate and the camera was carefully mounted on the tripod directly above the ripple tank. These factors were needed to ensure that the camera set-up was in the best possible position for obtaining the required images of the water waves. Then the height of the tripod was noted along with the camera 'zoom' focus in order to keep consistency when taking photographs of the waves generated. The tray was then partially filled with water up to the sloping part of the foam [1] to reduce reflections from the edges. Then a stroboscopic LED was used illuminate this from underneath the tray. The stroboscopic LED 'froze' the movement of the waves so that these can be captured by the camera and the resulting images can then be analysed. Figure 3 shows this experimental setup and Figure 4 shows the various ripple tank components.

The frequency of the vibrator was first set to 20 Hz. The height of the dipper was carefully adjusted so that the best quality images were obtained from the camera. These images were then converted to distances in cm using an image processing program. A calibration image was taken of a rule in by the camera to get a conversion between image distance and 'real' distance. The uncertainties in the image processing program in the distances were noted and these were also converted to real distances. Figure 5 shows the calibration image used.

By using the x and y image program distances the length of the red line that corresponds to 1cm in real distance in Figure 5, was determined using

$$d' = \sqrt{x^2 + y^2} \tag{11}$$

where d' is the image program distance. The conversion factor was then found to be 0.66 ± 0.1cm corresponded to 1cm in real distance. Therefore, all image distances were divided by 0.66 i.e.

$$d = \frac{d'}{0.66} \tag{12}$$

to gain the real image distance, d, in centimetres.

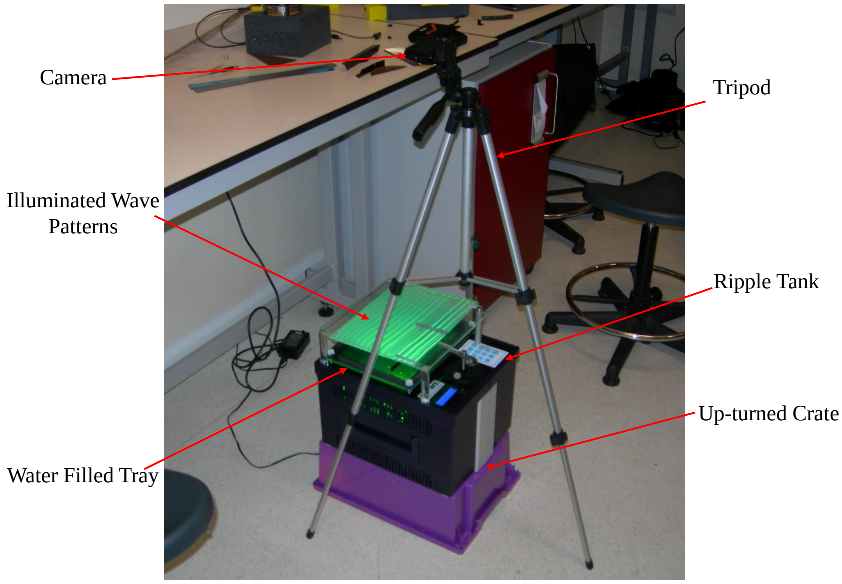

Figure 3: The experimental set-up and equipment for the Waves in a Ripple Tank experiment.

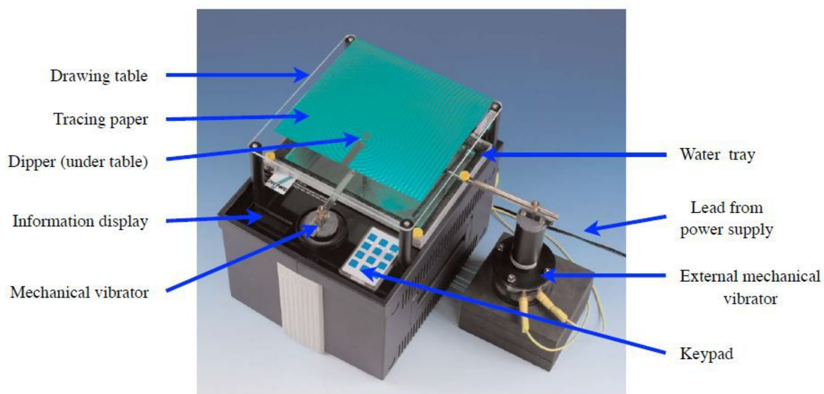

Figure 4: A labelled diagram showing the various parts to the ripple tank. Modified from [1].

Various different objects and obstacles were placed in the water enabling various wave phenomena to be investigated:

1. **The speed of water waves:** Plane waves generated from the vibrator were investigated to determine the wavelength of the waves at different frequencies. Three images were obtained for each frequency to gain more reliable results. The wavelengths were then found by using the image processing program in conjunction with equation (11). Then, an average of the three wavelengths for each frequency was obtained, along with propagation of the associated uncertainties, to gain a more reliable value for the wavelength. These wavelengths were then converted to real distances using equation (12) and the uncertainties were also obtained. The speed of the water waves was then determined by plotting a graph of wavelength, λ against inverse frequency (time), $1/f$..

2. **Reflection:** A barrier was placed in the tray at an angle to the incident wave fronts. The angle of incidence, θ_i and angle of reflection, θ_r was then measured (using an image processing program) to verify relation (3) for 20 Hz waves. In addition, lower frequency wave fronts were also investigated qualitatively to see how reflection changed at different frequencies.

3. **Refraction:** The velocity of water waves depend on the depth. Therefore, an object was be placed underwater and this resulted to change the of velocity of the water waves as they pass over it. This caused a change in direction of the waves and hence refraction could be investigated.

4. **Single-slit interference:** A barrier with a gap was placed in front of the dipper [1]. The gap width was then changed and the camera was used to take pictures at these different widths. In addition, the frequency was also adjusted to investigate this further. Images of the the minima from the resulting interference patterns were be studied.

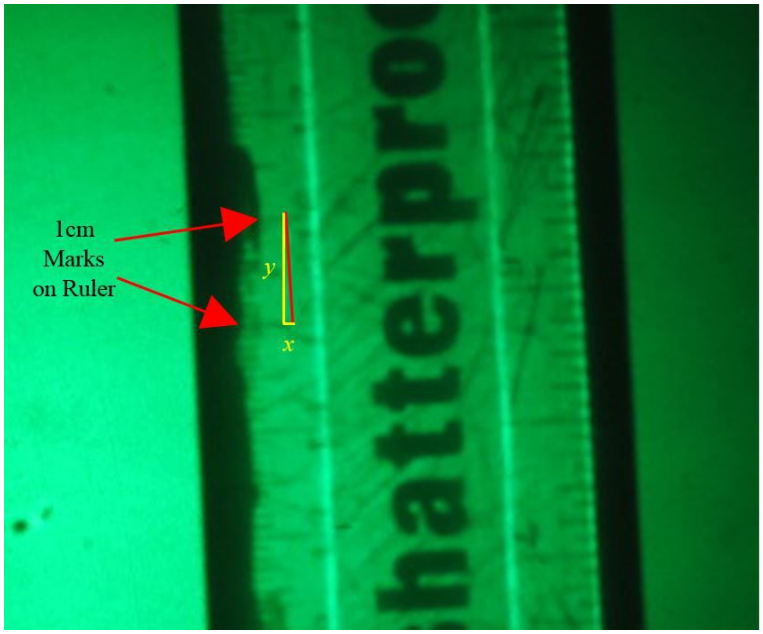

Figure 5: A photograph of a ruler was used to convert the image distances to real distances from the camera. The points between a 1cm mark on the ruler were measured in the image processing program and the distance noted. Then this was used as a conversion factor to find the wavelengths of subsequent images taken of the wave fronts in the experiment. x and y are the image distances and these were used to determine the length of the red line which corresponds to 1cm in real distance.

5. **Double-slit interference:** Two gaps were created from barriers in the tray and the Young's fringes generated from the interference patterns were investigated.

6. **Double-source Interference:** Two choerent sources will be observed using the double dipper or mechanical vibrators [1]. With the double dipper, Young's diffraction pattern will be observed and with the mechanical vibrator standing waves will be generated for analysis.

7. **Extensions:** Various other wave properties will potentially be investigated:

   - Doppler Effect by moving an external vibrator as it is generating waves.
   - Speed of water waves as a function of depth.
   - Diffraction due to waves passing *around* a barrier.

3.2 Diffraction and Interference

Figure 6 shows the apparatus used for investigating diffraction and interference of light.

A laser of wavelength 650nm will be used to illuminate one slit on the rotating wheel. This will then be detected by the light sensor mounted on the linear translator. The resulting data collected from the sensor and its position are then stored on the computer. The data can then be analysed using the given software.

The interface box will be turned on. The "table and graph" option will be selected [2]. This is necessary in order to analyse the results obtained from the experiment quantitatively. Then Hardware set-up will be selected and the interface will be chosen. The interface box will then be connected to the computer.

Then on the displayed image of the interface box, the inputs 1 and 2 will be selected. In order to correspond with the experimental equipment as shown in Figure 6, the "Rotary Motion Sensor" will then be chosen [2]. Next, "light intensity %" and "position, m" [2] will be selected in order to obtain intensity and position measurements obtained from the light sensor and linear translator respectively. During these steps not any exclamation marks indiciating problems with the set-up [2].

Finally, "Start" will be chosen to measure intensity, I vs position, x [2]. The resulting .txt files can then be exported to analyse these measurements further.

Once all of the necessary adjustments have been made to the computer software the laser will be carefully aligned to ensure it runs parallel to the optical rail. The slit wheel will be removed at first from the experimental set-up shown in Figure 6. This is so the sensor receiving close to 100% of the laser light [2] and the computer can then record this. This acts as a check to see if the software is performing as expected.

Figure 6: A laser will be used to illuminate a rotating slit wheel. This is then diffracted and the light sensor mounted on the linear translator then detects this. The adjustable slits are used to adjust the resolution of the light received by the sensor. The readings obtained by the sensor along with the position of the linear translators are then sent to the interface box so they can be stored on the computer. Image modified from [2].

The slit wheel will then be returned to examine various diffraction patterns. Different slit widths will also be used to see qualitatvely [2] the diffraction patterns. These will then be compared with theory. Initially, a single slit will be studied with the following conditions, an initial slit separation of $a = 0.04$mm, a 1mm slit in front of the sensor and a wheel position of $L = 0.5$m from the sensor [2].

Further diffraction experiments will then follow: single-slit, double-slit and multiple-slit patterns will be studied. In addition, different shape apertures will also be used to study generated diffraction patterns.

References

[1] College of Engineering Mathematics and Physical Sciences, University of Exeter, PHY2026, *Waves in a Ripple Tank Worksheet* (Accessed 8$^{\text{th}}$ February 2019).

[2] College of Engineering, Mathematics and Physical Sciences, University of Exeter, PHY2026, *Diffraction and Interference Worksheet* (Accessed 8$^{\text{th}}$ February 2019).

[3] Young, Hugh D and Freedman, Roger A, *University Physics*, 13$^{\text{th}}$ Edition, Chapter 36, pages 1312 - 1322, 2014.

[4] `https://physicsworld.com/a/working-with-water-waves/` (Accessed 9$^{\text{th}}$ February 2019.)

[5] `http://people.ucalgary.ca/~lvov/471/labs/fraunhofer.pdf` (Accessed 9$^{\text{th}}$ Febraury 2019.)