Th02 Laws of Radiation & The Thermoelectric Effect
(Initial Report)

Matthew Evans

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1 Introduction

Just as conventional electrical current is the flow of positive charge around a circuit when a source emf is supplied, heat current is generated when a material is subject to a temperature gradient. Heat current, $\dot{Q}$, is the rate of heat flow and is given by

$$\dot{Q} = \frac{dQ}{dt} \tag{1}$$

where dQ is an infinitesimal amount of heat transferred in a time dt and the units of $\dot{Q}$ are $\mathrm{Js^{-1}}$. This experiment investigates two mechanisms of heat transfer: heat transfer in radiation and heat transfer in conduction.

A black body is a body which has the greatest absorption factor and also the highest emissivity for a given wavelength of electromagnetic radiation [1]. Assuming the object we are considering is a black body, for **radiation** the heat current is given by

$$\dot{Q} = \sigma A T^4 \tag{2}$$

where $\sigma = 5.67 \times 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$ is the Stefann-Boltzmann constant, A is the surface area of the object and T is the absolute temperature of the object. Equation (2) is known as the *Stefan-Boltmann Law.* This shows that the total radiation emitted is proportional to the temperature raised to the fourth power [1]. In this experiment we will investigate this dependence by using a burnished brass cylinder, as the black body, exposed to changing temperatures in an electric oven.

In addition to heat transfer via radiation, a heat current is also generated through a material when a temperature gradient is set-up. An example of this can be observed when heating one end of a metal rod causes the mobile charge carriers (for conductors usually electrons) [1] at the hotter end to gain more kinetic energy. This causes a net diffusion of electrons down through to the cold end of the bar. The heat current, for **conduction** is

$$\dot{Q} = \kappa A \frac{(T_H - T_C)}{L} \tag{3}$$

where κ is the thermal conductivity and is material dependent, A is the cross-sectional area of the rod/object, T_H and T_C are the temperatures of the hot and cold ends of the object respectively and L is the length between the two ends of the object. This experiment will be investigating the conduction via the diffusion of mobile charge carriers [2] in a semiconductor. In particular the **thermoelectric effect** will be inverstigated: the generation of an e.m.f due to a temperature gradient, known as the *Seedbeck Effect*, and the generation of a temperature gradient due to an electrical current current, known as the *Peltier Effect* [2].

2 Theory

By considering a black body with unit surface area, the Stefan-Boltzmann law, equation (2), can be expressed as

$$M_B = \sigma T^4 \tag{4}$$

where M_B is the irradiance or energy flux density, of the black body [1] and it's units are $\mathrm{Js^{-1}m^{-2}}$. However, if the black body has an absolute temperature T and is placed in surroundings with temperature T_0 then the surroundings will also be radiating [3]. Therefore, the black body will absorb some of this radiation from the surroundings and hence the *net* irradiance from the black body, M'_B is

$$M'_B = \sigma(T^4 - T_0^4) \tag{5}$$

Equation (5) is modified if we are not considering an ideal black body. If a 'real' object is considered, this will not absorb or emit radiation as well a black body. Equation (5) becomes

$$M'_B = \epsilon\sigma(T^4 - T_0^4) \tag{6}$$

where ϵ is the emissivity of the material. This is defined as

$$\epsilon = \frac{M}{M_B} \tag{7}$$

where M is the irradiance of the object [1]. For an ideal absorber (a black body) $\epsilon = 1$ and for an ideal reflector $\epsilon = 0$. In this experiment the object will be assumed to behave like a black body. Therefore, the concept of emissivity will not be investigated. However, if the interested reader wishes to find out more about this and further information about heat current in radiation then please refer to [3] for details.

Figure 1 [2] demonstrates how the Seedbeck effect occurs in a metal

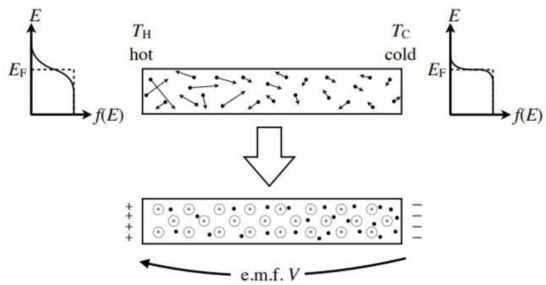

Figure 1: A demonstration of the Seedbeck effect. T_H and T_C represent the hot and cold ends of the metal respectively. The charge carriers move from the hot to the cold end due to different Fermi distributions of electrons. This sets up an e.m.f, V [2].

As mentioned in Section 1, for conductors the charge carriers are normally electrons with stationary postively charged ions as shown in Figure 1. However, for semiconductors depending on whether it is n-type doped (adding extra electrons) or p-type doped (reducing electrons) the charge carriers will either be electrons or positively charged holes respectively. Where doping means adding impurities to the semiconductor to obtain the desired electron amounts. By analysing Figure 1 we can see that the charge carriers on the left of the figure have more kinetic energy and therefore move to the cooler end at temperature T_C. This net diffusion causes the negative charge carriers to congregate at one end leaving postively charged ions at the other [2]. This therefore induces an e.m.f, V, between the two ends of the metal and is known as the Seedbeck effect [2]. This effect is described by the following relation [2]

$$S = \frac{dV}{dT} \tag{8}$$

where S is the Seedbeck coefficient, dV is a small voltage change and dT is a small temperature change.

The opposite of the Seedbeck effect is when a current (of charge carriers) flows through a material and delivers heat from one end to the other and is known as the *Peltier Effect*. The relation

$$\dot{Q} = \Pi I \tag{9}$$

relates the rate of removal of heat $\dot{Q}$ to the current, I [2] where Π is the Peltier coefficient.

These two phenomena can be investigated further by considering heat engines and heat pumps. A heat engine is a device that uses input heat and converts it into work, with the residual deposited into a cold reservoir. Whereas a heat pump is a device that takes heat from a cold reservoir and deposits it into a warmer place using a net input of work to do so. These concepts are demonstrated in Figure 2 [2].

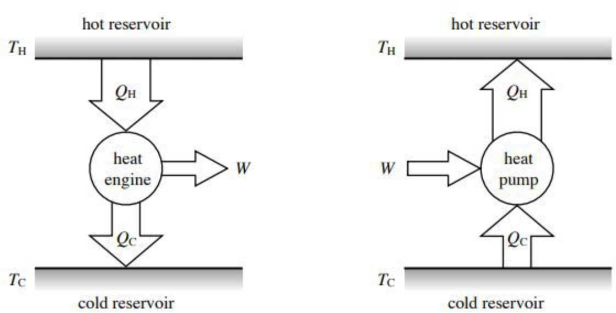

Figure 2: Left: visual representation of a heat pump. Right: visual representation of a heat engine. [2].

By inspecting Figure 2 and considering the conservation of energy (1st Law of Thermodynamics) [2] the following equation can be constructed

$$Q_C + W = Q_H \tag{10}$$

where Q_C is the heat either the heat rejected (heat engine) or heat removed (heat pump), W is the output/input work done and Q_H is the input heat (heat engine) or heat leaving (heat pump). The efficiency, η, of a heat engine is

$$\eta = \frac{W}{Q_H} \tag{11}$$

for the ideal case $Q_H = W$ and the efficiency would be 1. However, in practice some heat is always dissipated and this is never achieved [4]. The efficiency of an ideal reversible Carnot engine, η_{ideal} is given by

$$\eta_{ideal} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} \tag{12}$$

where T_C and T_H are the cold and hot reservoir temperatures respectively. Equation (12) shows that the efficiency of a Carnot engine is large when the difference between the two temperatures is large and it is small when the tempeartures are similar [4]. For a more detailed derivation of equation (12) and for more details about heat engines and heat pumps see [4].

We can define the coefficient of performance, k, for a heat pump in two different ways, depending on its purpose. When extracting energy from the cold reservoir (refrigeration mode) the coefficient of performance is

$$k = \frac{Q_C}{W} \tag{13}$$

whereas when the heat pump is delivering energy to the hot reservoir (heating mode) the coefficient of performance is given by

$$k = \frac{Q_H}{W} \tag{14}$$

3 Method

3.1 Laws of Radiation

First, the experimental will be set-up as shown in Figure 3 [1] making all relevant connections to the CASSY-Sensor computer interface. The themperautre sensor (thermocouple) and thermopile shown in Figure 3 will be connected to the CASSY-senor by a theromocouple adapter and μV box respectively [1].

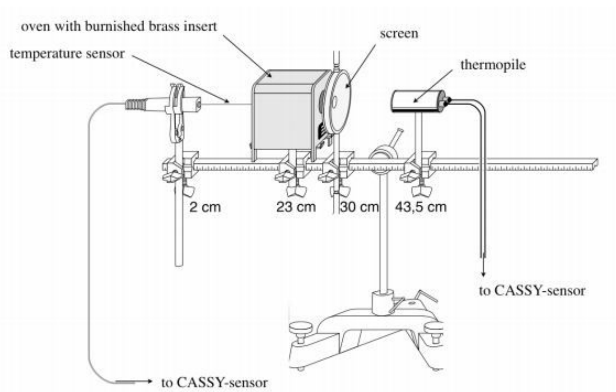

Figure 3: Experimental set-up for investigating the Stefan-Boltmann law using an electric oven [1].

The glass window of the thermopile will be used. The thermocouple will then be used to measure the room temperature, T_0 and the thermopile will be used to measure the corresponding output voltage V_0. The uncertainties will also be noted. V_0 is an offset measurement and will therefore be subtracted from all the subsequent measured values of the voltage, V [1] giving

$$V_{cal} = V - V_0 \tag{15}$$

where V_{cal} is the calibrated voltage measurement.

Next, the oven is switched on and the voltage readings will be taken every 25 C up to a final temperature of 450 C [1]. After this, the oven will be turned off and the voltage readings will also be recorded during the cool down process. Once the temperature falls below 100 C [1] the voltage measurements will be stopped.

Once all the measurements have been made, the room temperature values will be re-checked by removing the thermocouple from the oven and the thermopile will be covered.

The calibrated voltage measurements will be made using (15) along with the assocaited uncertainties determined. These measurements and uncertaintes will be converted to irradiance values, M'_B using data given by [1] and appropriate propagation of errors respectively. Then a graph of irradiance, M'_B *vs* temperature difference, $T^4 - T_0^4$ can be plot and by using equation (4) the Stefan-Boltzmann constant, σ can be determined. This can then be compared with the theoretical value given by [1].

3.2 The Thermoelectric Effect

There will be three parts to this experiment: the Seedbeck coefficient, conservation of energy and the efficiency of the Peltier heat engine as a function of load (resistance). All of these will be investigated using the thermoelectric module as shown in Figure 4 [2].

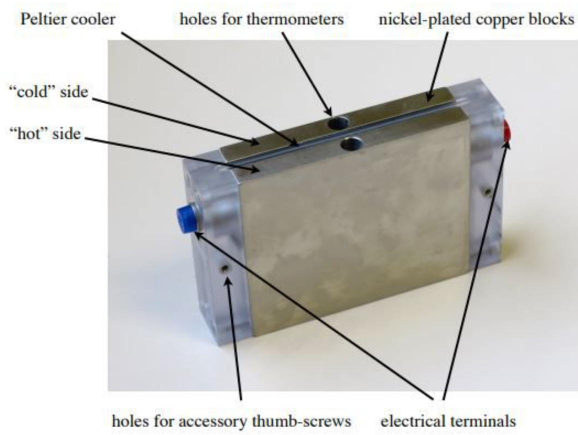

Figure 4: The thermoelectric module used to investigate various properties of the thermoelectric effect. A conventional current enters the device through the red terminal and leaves at the blue terminal [2].

The current passing through the module causes the hot side to heat up and the cold side to cool down. The module shown in Figure 4 consists of 142 Peltier cooling elements [2] which generates these temperatures in the copper blocks. One such element is shown in Figure 5 [2].

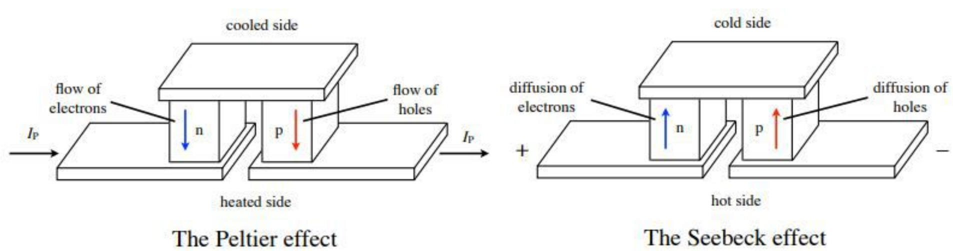

Figure 5: A diagram showing one of the Peltier cooling elements. The charge carriers are electrons in the n-type semiconductor and positive holes in the p-type. In the Peltier effect, when the conventional current, I_P, is applied, the charge carriers in both blocks move from the top to the bottom of the device [2] cooling the top and heating the bottom. In the Seedbeck effect, the charge carriers move from the hot to the cold side, creating an electric field, and hence setting up an e.m.f as shown [2].

To investigate these concepts, several accessories are provides and these are shown in Figure 6 [2].

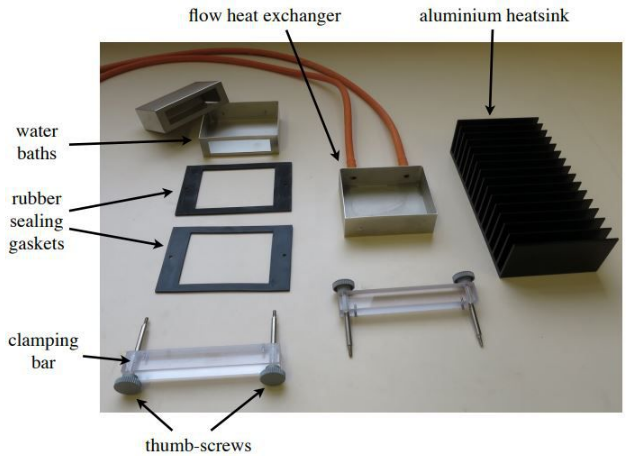

Figure 6: An assortment of accessories which can be attached to the thermoelectric module to investigate various principles of the thermoelectric effect [2].

Before any further steps are taken the dimensions of the copper block, the length l, width w and depth d will be measured and uncertainties noted. This information will be needed for the second part of the experiment - see Section 3.2.2 equation (18).

3.2.1 Part 1

To investigate the Seedbeck coefficient, the voltage across the terminals of the thermoelectric module must first be measured using a voltmeter. This should be below 0.5mV [2], however if it is above this then the module should be left to equilibrate in temperature. Once the desired voltage has been measured, two thermometers with a range of -10 to $+50$ C should be inserted into the holes in the module shown in Figure 4. The temperatures in both should be identical. If not, note the difference and this should be used to correct the temperature of one of the thermometers for all the following measurements.

Next, the thermometer in the hot side should be carefully replaced with one which has a range of 0 to 100 C and the voltmeter should be connected to the terminals of the module. Now, the water baths can be attached to both sides of the module using the appropriate accessories shown in Figure 6. The cold bath is filled with tap water and the hot one filled with boiled water from a kettle. Measurements will immediately be taken to record the temperatures at suitable intervals with corresponding output voltages. Once all the measurements have been obtained, the experiment is repeated reversing the temperatures of the two water baths [2]. Then the results for both parts of the experiment can be plotted as output voltage, V, *vs* temperature difference, $T_H - T_C$. The gradient can be obtained and using relation (8), the Seedbeck coefficient can be obtained and compared to the known value for the device given in [2].

The thermoelectric module is then disconnected, the contents of the water baths are emptied and removed from the module.

3.2.2 Part 2

First, the circuit will be connected as shown in Figure 7 [2]. The rheostat load resistor will be adjusted to a resistance of 5 Ω and then the input current, I_P, will be set-up to have a value of 2.5 A [2]. Throughout the next two parts of the experiment, care will be taken to ensure that the external ammeter is **not** connected to the Tenma power supply [2].

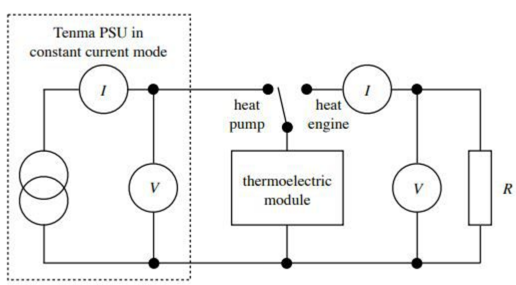

Figure 7: Circuit diagram for Part 2 and Part 3 of the experiment [2]. In this case the switch will be unplugging a lead from the Tenma power supply unit and plugging into an ammeter leaving the connection at the other end in the red terminal of the thermoelectric module.

The experiment will first be run in heat pump mode for 60 s taking appropriate current, I_P and voltage V_P readings over suitable time intervals from the Tenma power supply. Whilst in this mode, the work done is

$$W = \int_0^{t_1} I_P V_P dt \tag{16}$$

Once all the measurements and associated uncertainties have been made in heat pump mode over this time interval, $t_1 = 60$ s, a plot of power as a function of time is generated and the area of the graph obtained in order to determine the work done in heat pump mode.

Heat is extracted from the cold side and the flow of heat, ΔQ, accompanying this temperature change is

$$\Delta Q = mc\Delta T \tag{17}$$

where $c = 384$ J kg^{-1} K^{-1} is the specific heat capacity of copper [2], m and ΔT are the mass and temperature change of the copper block respectively. By using the measurements made at the start of the experiment, as mentioned in Section 3.2, for the dimensions of the copper block equation (17) can be re-expressed as

$$\Delta Q = \rho lwdc\Delta T \implies Q_H - Q_C = \rho lwdc(T_H - T_C) \tag{18}$$

and hence by using the result obtained for the work done, W, the 1st Law of Thermodynamics can (10) can be checked.

At $t_1 = 60$ s the circuit in Figure 7 will be changed to heat engine mode. he thermoelectric module now drives a load resistance. The output currents and voltages will now be measured using the external ammeter and voltmeter shown in Figure 7 and uncertainties noted. The work done is now

$$W = \int_{t_1}^{\infty} I_E V_E dt \tag{19}$$

I_E and V_E are the current and voltage measurements respectively. The measurements for I_E and V_E will be stopped when the power readings are negligible compared with the value made at time t_1 [2]. Again, the area of a plot of power *vs* time can be obtained to find the work done for a heat engine. A similar approach to the heat pump will be taken using equation (18) to obtain the heat extracted from the hot side and absorbed by the cold. Then equation (10) can be tested once again for heat engine mode.

Once this had been achieved, equation (11) will be used to calculate the efficiency of the thermoelectric module. Then, by using appropriate temperatures that represent the obtained data equation (12) will be used to obtain the efficiency of an ideal Carnot engine. Finally, these two efficiencies will be compared.

3.2.3 Part 3

For the final part of the experiment, Section 3.2.2 for the heat engine will be repeated using different load resistances in the range from 1Ω to 20Ω [2]. The efficiencies will be determined and a graph will then be plot of efficiency *vs* load resistance. The **maximum power transfer theorem** will be investigated. This states that the maximum power transferred to a load will be when the load resistance, R is equal to the internal resistance r of the voltage source, r [2]; in other words when $R = r$. Then, by analysing at what load resistance the efficiency will be a maximum this theorem can be investigated further.

3.3 Safety

Both experiments there are many sensitive electrical components used; such as an electric oven adapters, power supplies and circuit components. Therefore, no food or drink should be consumed whilst working in the laboratory and care should be taken when using switches, especially in Section 3.2.2. Another important point is that the Peltier device current must not go above 5 A [2].

In addition to this, much of the equipment used is at high temperatures; for example the electric oven, the thermoelectric module and boiling water for use in water baths. Great care should be taken when transporting hot water around the laboratory and contact should be avoided with hot objects. Furthermore, "the temperature of either side of the thermoelectric module must not exceed 100 C" [1].

References

[1] College of Engineering Mathematics and Physical Sciences, University of Exeter, *Laws of Radiation Worksheet* (Accessed 19$^{\text{th}}$ December 2018).

[2] College of Engineering, Mathematics and Physical Sciences, University of Exeter, *The Thermoelectric Effect Worksheet* (Accessed 19$^{\text{th}}$ December 2018).

[3] Young, Hugh D and Freedman, Roger A, *University Physics*, 13$^{\text{th}}$ Edition, Chapter 17, pages 634 - 637, 2014.

[4] Young, Hugh D and Freedman, Roger A, *University Physics*, 13$^{\text{th}}$ Edition, Chapter 20, pages 728 - 740, 2014.