q1) $\sigma(x) = \dfrac{1}{1+e^{-x}}$

a) Show $\sigma(-x) = 1 - \sigma(x)$

$$\frac{1}{1+e^{x}} = 1 - \frac{1}{1+e^{-x}}$$

$$\frac{1}{1+e^{x}} - 1 = -\frac{1}{1+e^{-x}}$$

$$1+e^{-x}\left(\frac{1}{1+e^{x}} - 1\right) = -1$$

$$\frac{1}{1+e^{x}} - 1 + \frac{e^{-x}}{1+e^{x}} - e^{-x} = -1$$

$$\frac{1+e^{-x}}{1+e^{x}} = e^{-x}$$

$$1+e^{-x} = e^{-x}(1+e^{x})$$

$$1+\cancel{e^{-x}} = \cancel{e^{-x}} + e^{-x}e^{x}$$

$$1 = e^{-x}e^{x}$$

$$1 = \frac{\cancel{e^{x}}}{\cancel{e^{x}}} = 1$$

better solution

$$\sigma(-x) + \sigma(x) = 1$$

$$\frac{1}{1+e^{x}} + \frac{1}{1+e^{-x}} = 1$$

$$\frac{1+e^{-x} + 1+e^{x}}{(1+e^{x})(1+e^{-x})} = 1$$

$$\frac{e^{-x}+e^{x}+2}{1+e^{x}+e^{-x}+\cancel{e^{x}e^{-x}}^{\,1}} = 1$$

$$\frac{e^{-x}+\cancel{e^{x}+2}}{e^{-x}+\cancel{e^{x}+2}} = 1$$

$$1 = 1$$

b) Show $\dfrac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$

$$\frac{d}{dx}\left(\frac{1}{1+e^{-x}}\right) = \frac{(1+e^{-x})(0) - 1(-e^{-x})}{(1+e^{-x})^2}$$

$$= \frac{e^{-x}}{(1+e^{-x})(1+e^{-x})}$$

$$(1+e^{-x})(1+e^{-x})$$

and $\sigma(x)(1-\sigma(x))$

$$= \frac{1}{1+e^{-x}}\left(1 - \frac{1}{1+e^{-x}}\right)$$

$$= \frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^2}$$

$$= \frac{1+e^{-x}-1}{(1+e^{-x})^2}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

$$\therefore \frac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$$