Measuring the diameter of the Sun

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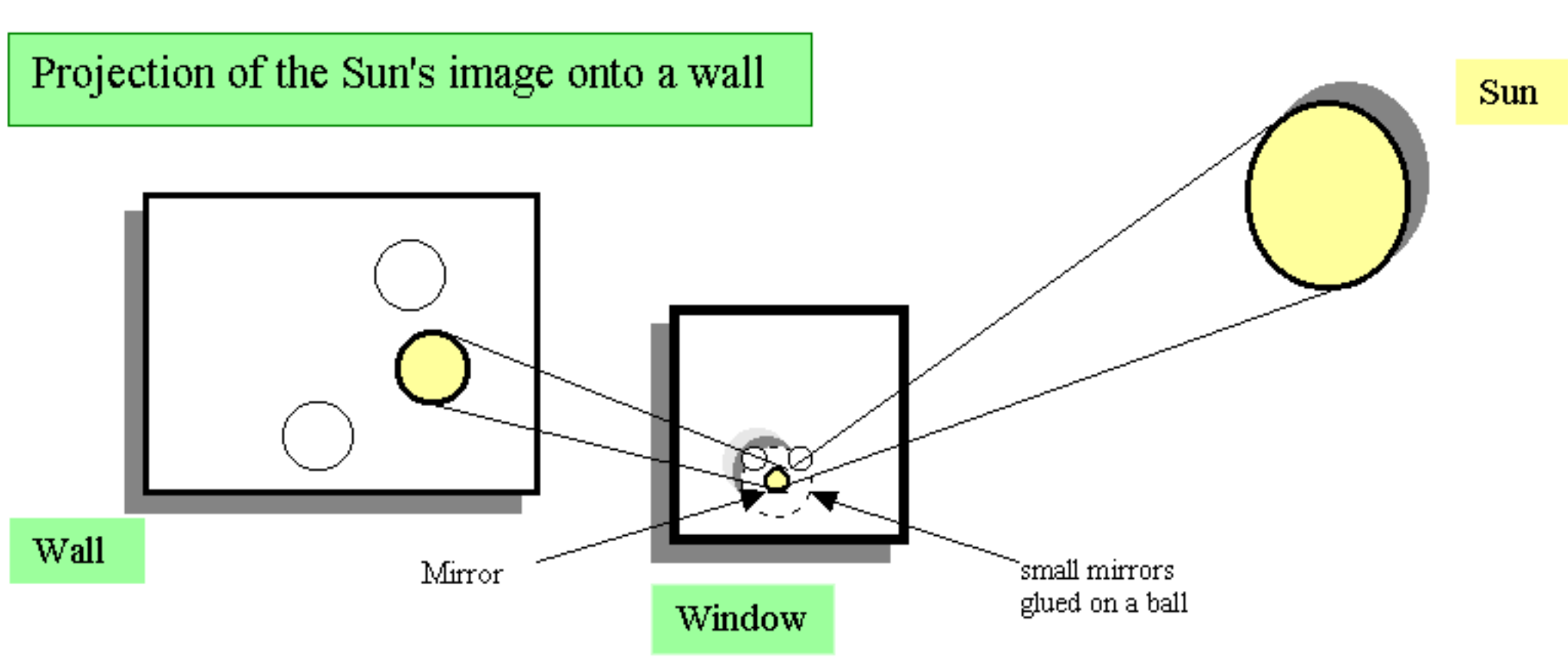
Abstract

A little sphere at which many small mirrors are glued can be used to observe the earth's rotation, to measure the angular diameter of the sun and the eccentricity of the elliptic orbit of the earth. When the sun is shining through a window the mirrors project the images onto a wall or onto a piece of white paper in the shade or into the far end of a darkened hallway through an opened door.

Knowing the radius of Earth's orbit and by measuring the apparent size of the sun we can easily determine the diameter of the sun and the eccentricity of the orbit of the Sun using trigonometry or simple geometry.



1. Introduction

The Sun's distance and its diameter was not known exactly until the British Captain James Cook observed the transit of Venus in 1769. A reasonable accurate value was derived from data in 1835 by the astronomer Enke. The actual distance between Earth and Sun varies from a minimum of 147 097 000 km to a maximum of 152 086 000 km because of Earth's elliptical orbit. We use 150 000 000 km (=1 AU) as the distance.

2. Projection-Methods for Sun observation

You can use binoculars mounted on a tripod pointing to the sun, one glass can be closed or you can hold a 2-liter bottle in a stand and glue a small piece of a mirror (less than 0,5 cm × 0,5 cm) near the middle of the bottle along the length of the cylinder. Fill the bottle with water before using it. Trace the image of the sun on a viewing screen, which is placed several meters away from the mirror. Never look directly at the sun.

The easiest way is to buy at Christmas time a sphere with many tiny hot glued mirrors, used as decoration for the tree. Fix the sphere in a stand near the window in a stand. If the sun is shining at the ball images of the sun appear onto the wall behind. The important points are that the mirror must be small compared to the distance of projection which should be at least 2 m, and you must get an image bright enough to measure. The projection from the mirror should be flat on to the surface of projection. If the image is round, then it is being projected correctly. Keep in mind that the projected image of the sun will be faint, so it needs to be projected into a darkened area. If the distance is too short, the image will to be faint. If it is too large, the edges of the image will be fuzzy. Look for an image which is round, sharp enough, and bright enough to measure.

3. Measuring the apparent angular diameter

Onto a piece of paper draw a circle bigger than the images of the sun. Trace the image in the middle of the circle. Using a stopwatch start timing at the first contact: circle / edge of the image. Stop timing when the image of the sun has moved completely out of the circle you have traced. See Fig. 1. Try this several times. Make at least 5 measurements.

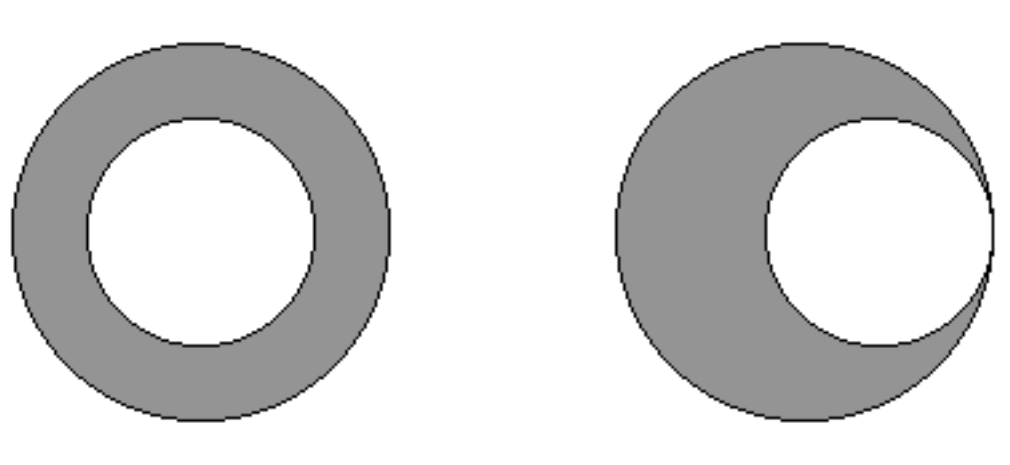


Fig. 1.

4. Explanations

The rotation of the Earth causes the image of the Sun to appear to move across the screen. The angular velocity of the daily rotation is easily determined. If the earth spins 360 degrees per 24 hours, then it spins 15 degrees per hour or 1/4 degrees per minutes. The time, it takes for the sun's image to move "one Sun diameter", is about 2 minutes. For larger or smaller images, the time will be constant, as it is a measure of an other constant, the spin of the earth. The apparent angular diameter of the sun can be found using the proportion:

$$15 \text{ degrees} / 60 \text{ minutes} = x \text{ degrees} / 2 \text{ minutes}$$

The small mirror acts like the hole of a big pinhole camera. The development of a normal pinhole camera projecting the image of the sun on a screen to a model reflecting the sun's light using a tiny mirror is shown in Fig. 2.

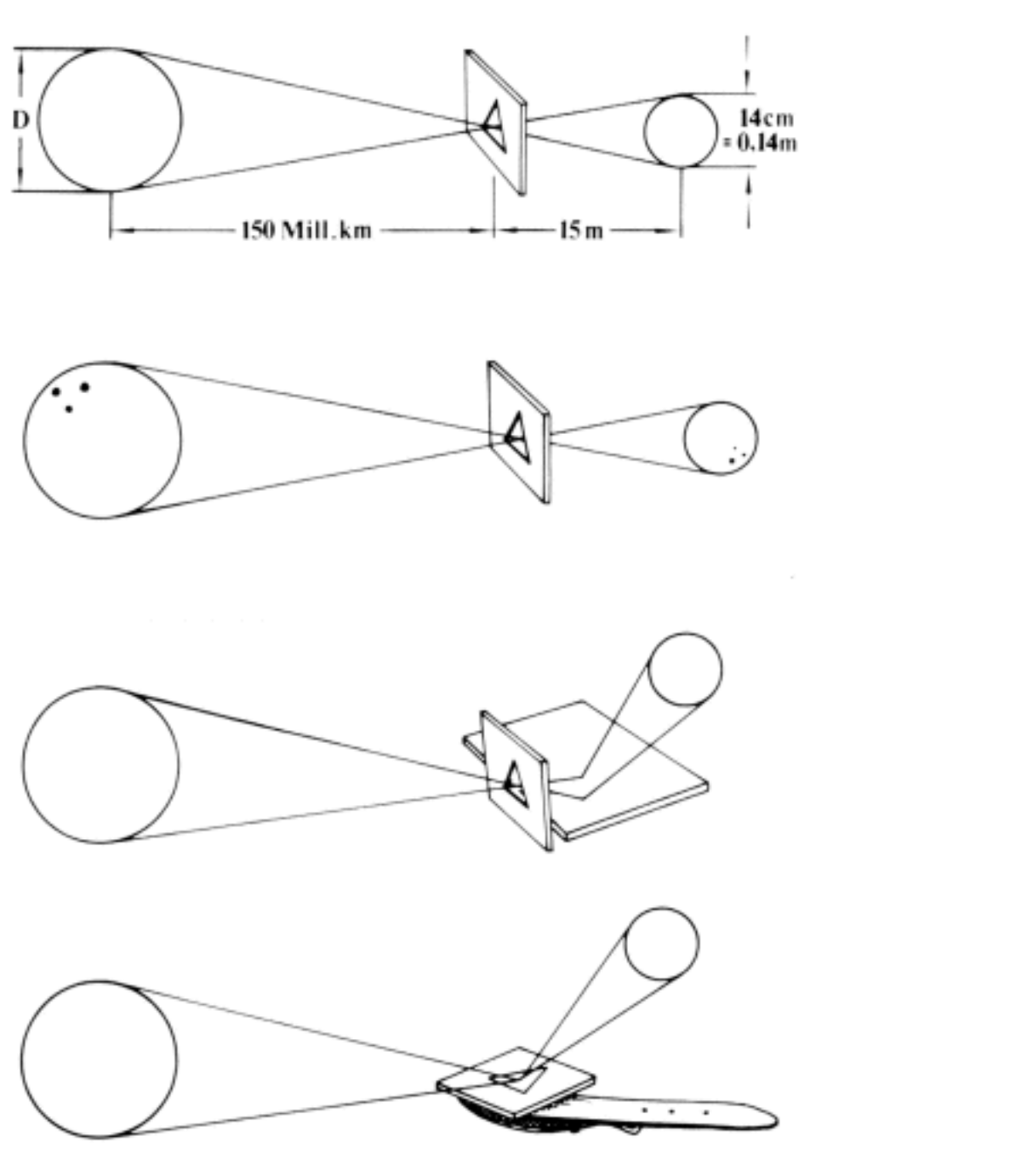


Fig. 2.

Locally you see the pictures of the rectangular mirrors.

Only at the equator the angular velocity of the sun is 1°/4 min.

If the sun stands above or below the celestial equator, its velocity is smaller. Polaris does not move, but the stars of Orion near the celestial equator move a lot concerning the spin of the earth.

As far as the declination of the sun d is concerned the path of the sun is given by $s = 1°/4 \text{ min} \times \cos \delta \times t$. Measuring t minutes for the passage of the sun you can determine the its diameter.

Because $-23{,}3° < \delta < 23{,}5°$ the mistake ignoring the position of the Sun is at maximum 9%. During the equinox the mistake is minimal.

The tiny mirror is reflecting light from the sun and is producing an image of the sun on the screen. There are two similar triangles in this experiment. One is an triangle whose base is the diameter of the sun, and whose congruent sides are rays from each side of the sun to your mirror. The base of the second triangle is the diameter of the sun's image on the screen. Its congruent sides are rays coming from the mirror. Since these are similar triangles, the angular diameter of the image on the screen is the same as the angular diameter of the sun in the sky.

5. Sun's Diameter in km

Younger students who are not yet able to use trigonometric functions can use similar triangles and proportions to determine the diameter in km. They draw a symmetric triangle: basis angle d = 10° at the top, length of the height (distance: basis - top of the triangle) d(E,S) = h = 15 cm. The basis b of the drawn triangle is then measured as 2,6 cm. See Fig. 3.

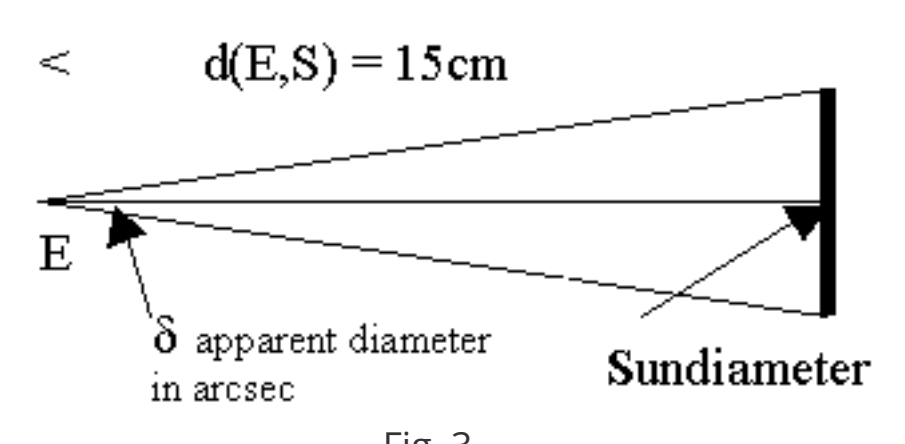


Fig. 3.

Because the distance D from the Earth to the Sun is nearly 150 Mil km, h is easily stretched to that scale. The goal is to change the value of h and also the triangle by calculation to the real distance D. First the angle has to be diminished to 1/2 degree by calculation. The steps are shown below.

Triangle determined by h = d(E,S)		in Deg	Length of the Basis b
15 cm	by drawing	10°	2,6 cm
15 cm	drawing not possible	1°	0,26 cm
15 cm		1/2°	0,13 cm
150 cm	by calculation	1/2°	1,3 cm
150 Mil km		1/2°	1,3 Mil km
distance (E;S)		Angular diameter	diameter of the Sun

Using trigonometry we get the same result: $\sin \delta/2 = \tan d/2 = \delta/2 / D(E,S)$.

The distance to the sun varies through the year, ranging from a minimum of 147 097 000 km (0,983 28 AU) to a maximum of 152 086 000 km (1,016 67 AU). You could find out the exact distance on the day of your observation, but the level of accuracy of this observation doesn't warrant it.

The semi-diameter has been observed since 1985 with the Tokyo Photoelectric Meridian Circle (Tokyo PMC) at Mitika. The mean value of the apparent diameter of 1 919,66 arcsec leads to an actual diameter of the sun of 1 391 000 km. The diameter variations are found to have amplitude larger than 0,08 arcsec. Most of them have a period longer than 130 days.

Although the Sun is spinning around its axis the difference between the radius from the equator to the centre and from the pole to the centre varies at maximum only some 10 kilometers. The sun is therefore nearly a perfect sphere. The sun's diameter is about 109 times the diameter of the earth.

6. The eccentricity of the Earth

The actual distances of planets from the sun are continually changing, because their orbits are ellipses governed by Kepler's laws. The first states that planets move in ellipses with the Sun in one focus.

It is convenient to start with the construction of ellipses by the gardeners. With two fixpoints and a string you can easily trace some ellipses on the blackboard. The dimensions and shape of an ellipse are described by the semimajor axis a, the eccentricity e and the numerical eccentricity e. Another useful quantity is the closest distance to the sun called perihelion distance. Figure 4 shows these quantities. In detail $e = e/a$, $r_P + r_A = 2a$.

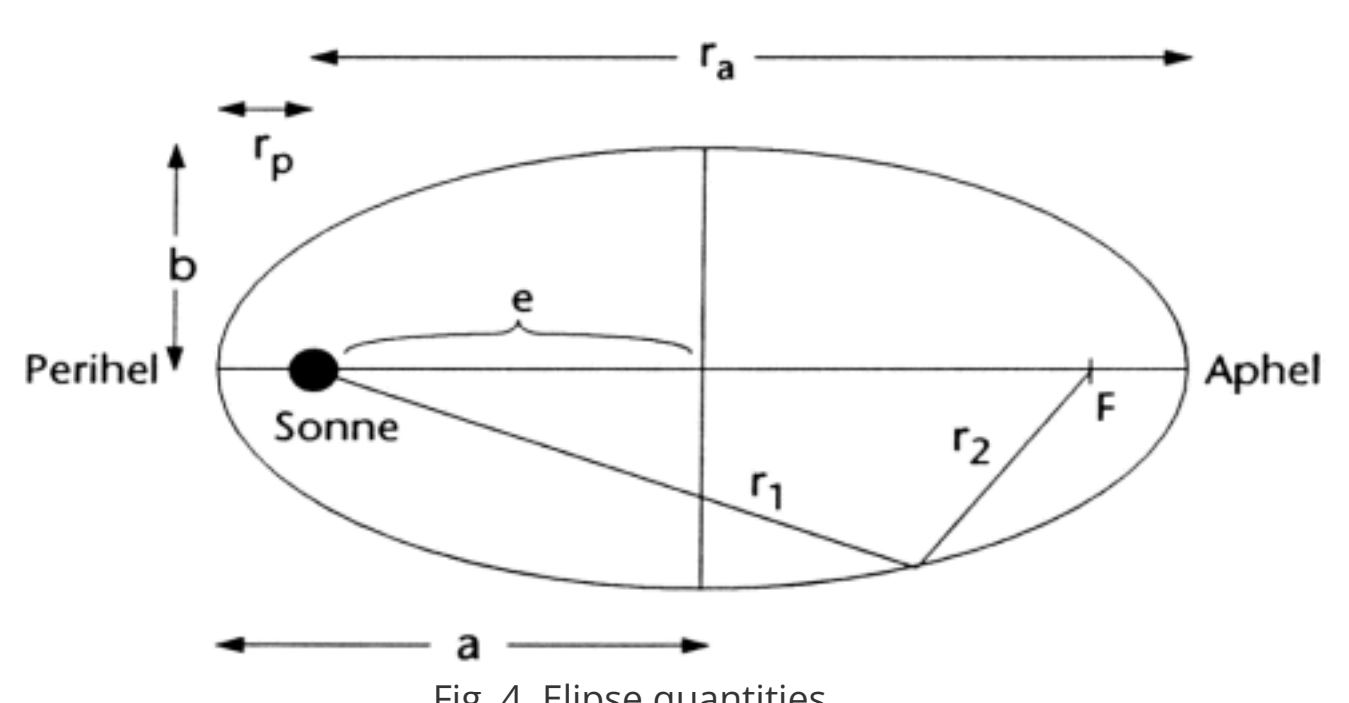


Fig. 4. Elipse quantities.

The changes in the distance to sun can be determined by the different angular diameters of the sun. For good results series of continuous observations are to be made during one year. In the easiest case two observation weeks are needed: the first week in January and first week in July.

Remember the sun stands in general above or below the celestial equator. The fact is described by the declination of the sun d. Therefore $t_{real} = t \times \cos \delta / 4$ min. (In January d = -23° and in July d = +23°). The mistake ignoring the position of the Sun is max 9%.

Figure 4 shows measurements the apparent radius of Sun (closed line astronomical reference book and crosses observations of the students during one year time using a telescope).

The observations show:

1 - The orbit of the Earth is no circle with the Sun in the middle. Because the apparent angle diameter f is very small, the relation is $f_A \times r_A = f_P \times r_P$.

r_A: aphel distance, r_P: perihel distance, f_A and f_P are the angle diameter at aphel and perihel.

2 - If the earth's orbit is elliptic the Sun can not stand in the centre of the ellipse (different diameters in January and July).

3 - By comparing the maximum and minimum diameter the numeric eccentricity can be estimated.

$$\varepsilon = e/a = (r_P - r_A)/(r_P - r_A) = \frac{1-(r_P/r_A)}{1-(r_P/r_A)} = \frac{1-(\phi_P/\phi_A)}{1+(\phi_P/\phi_A)} \rightarrow \varepsilon = 0{,}017$$

The exact value is e (Earth) = 0,0167

Proposals for didactical activities: Sun's Diameter

Task 1

Measure the apparent diameter of the sun by measuring the transit time of the sun's image.

Nr.	Time in sec
1	
2	
3	
4	
5	
6	
7	

Mean value ______________sec

Calculate the apparent diameter f in arcsec.

Task 2

Determine the diameter in km by using trigonometric relations.

Task 3

Determine the eccentricity of the earth using the given data of the sun in the year 1999.

Diagram: month versa apparent diameter f

$$\varepsilon = e/a = \frac{1-(\phi_P/\phi_A)}{1+(\phi_P/\phi_A)}$$

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