SYMPLECTIC GROUPOIDS FOR CLUSTER MANIFOLDS

Abstract. We prove some things. And we are happy! It seems to work!

Outline

(1) Intro to Poisson manifolds, symplectic groupoid and Poisson spray
(2) Intro to cluster algebra and compatible Poisson structures
(3) Cluster symplectic groupoid
(4) Totally positive cluster manifolds (definition of manifolds with corners [check D Joyce], associahedron of type A and generalized associahedron)
(5) Symplectic topology of the groupoid, and examples

1. Introduction

2. Poisson Geometry

3. Cluster Algebras

4. Cluster Symplectic Groupoids

In this section we give an integration to a symplectic groupoid $\mathcal{G}(\Sigma)$ of the Poisson structure on a cluster manifold $M(\Sigma)$.

5. Totally Positive Cluster Manifolds

In this section we show that the totally nonnegative part $M_{\geq 0}(\Sigma)$ of a cluster manifold is a manifold with corners in the sense of [**?**]. Moreover, we show that the nonnegative cluster manifold is a union of symplectic leaves for any compatible Poisson structure on $\mathcal{A}(\Sigma)$. Here there is a unique dense symplectic leaf and the boundary of $M_{\geq 0}(\Sigma)$ is again a union of symplectic leaves of lower dimension where the Poisson structure degenerates.

Theorem 5.1. *Let Σ be a seed. The 1-skeleton of $M_{\geq 0}(\Sigma)$ given by 0-dimensional and 1-dimensional symplectic leaves identifies with the exchange graph of $\mathcal{A}(\Sigma)$. Moreover, if Σ is a seed of finite-type, then $M_{\geq 0}(\Sigma)$ provides a realization of the generalized associahedron with the same Cartan type as Σ.*

6. Symplectic Topology of the Nonnegative Cluster Groupoid

Let $\mathcal{G}_{\geq 0}(\Sigma)$ denote the symplectic groupoid over $M_{\geq 0}(\Sigma)$. In this section we introduce a Poisson spray which may be used to construct $\mathcal{G}_{\geq 0}(\Sigma)$ and apply a Moser argument to show that up to symplectomorphism $\mathcal{G}_{\geq 0}(\Sigma)$ can be identified with $T^*M_{\geq 0}(\Sigma)$.