Report on "Dominance regions for rank two cluster algebras"
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One of the main issues in cluster algebra theory is to construct "good bases" for a cluster algebra or the associated upper cluster algebra. Recently Qin [12] studied the possible good bases in an unified way by a certain partial order (called the dominance order) for $\mathbf{g}$-vectors, where each good basis is parametrized by $\mathbf{g}$-vectors. The main result of [12] is briefly summarized as follows:

- Any good basis contains all cluster monomials.
- A pair of good bases are "triangular" to each other with respect to the above dominance order.

In particular, knowing the explicit description of the dominance order is useful, for example, to construct another basis from a known one. In general, however, it is a difficult problem to describe the dominance order concretely because it is defined by infinitely many inequalities associated with all mutations of $\mathbf{g}$-vectors.

Having the above result in mind, the paper study the dominance order on $\mathbf{g}$-vectors in the rank 2 case. In this case the region representing dominance order (he dominance region) is defined in Definition 3.1. The first main result is Theorem 1.2, where the dominance region is explicitly described as polygons. The proof is based on Lemma 4.3 and the limit formula for Chebyshev polynomials (Lemma 2.3). The second main result is Theorem 1.6, where the maximum possible support of any basis element is given. The proof is based on Theorem 1.2 and results from [12] and [7,17].

These results are very useful to understand the result of [12] more concretely. Also, the paper is well-written with complete proofs. So, I strongly recommend the paper for publication in your journal.

Comments.

1. p.1 Figure: Isn't it better to put the figure number and the caption?
2. p.8 In the first paragraph: Maybe it is a good idea to clarify here (or somewhere else) the relation between what you mean by $\mathbf{g}$-vectors (defined in (1)) in this paper and the ones in Fomin-Zelevinsky's paper (Cluster Algebras IV) more explicitly.
3. p.8 Eq.(3): It might be better to describe where this transformation comes from. (for example., more explicit reference)
4. p.8 l.30: The second k should be in math symbol.
5. p.17 Somewhere after l.18: It is better to mention that the filled and empty circles in the figure correspond to λ and λ' (maybe also in the caption of figures).
6. p.18 l.41: in our figures $\to$ in the figures after Theorem 1.6 (or, better put the figure number for the figure)