ROB 538
Multiagent Systems
Homework 4: Game Theory

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I. Introduction

In this project, we explore the well-known *Bar Problem*, initially presented in Homework 3, where a set of 50 agents must decide on which nights to attend a bar. The bar has a capacity constraint, and each agent's goal is to maximize their individual reward, which is affected by the number of agents attending on the selected night. This scenario is framed within a multiagent system where agents must choose a night to attend based on different reward structures, which will be analyzed to identify stable distributions of agents.

The questions posed in this project include the following:

1) What is the Nash Equilibrium when each agent uses the *local reward* function, defined as

$$L(z) = x_k(z)e^{-x_k(z)/b},$$

where $x_k(z)$ represents the number of agents attending night k? We will compute the Nash Equilibrium for this reward structure and explain why this constitutes a stable equilibrium.

2) What is the Nash Equilibrium when each agent uses the *global reward* function, defined as

$$G(z) = \sum_{k=1}^{K} x_k(z)e^{-x_k(z)/b}?$$

We will compute the Nash Equilibrium for the global reward structure and determine whether the Nash Equilibria in parts (a) and (b) are the same. Additionally, we will discuss what this result implies about the expected performance of the local reward.

3) What is the Nash Equilibrium for the *difference reward* structure, given by

$$D_i(z) = x_i(z)e^{-x_i(z)/b} - (x_i(z) - 1)e^{-(x_i(z)-1)/b}?$$

In this case, we will calculate the Nash Equilibrium using this reward and explain why it represents a stable solution. We will also compare this equilibrium with those from parts (1) and (2) and analyze the expected performance of the difference reward.

In the following sections, we will compute the Nash Equilibria for each reward structure, providing algorithms, results, and analysis to answer the questions posed. The report will discuss the implications of each reward structure on the agents' behavior and the system's overall performance. The final analysis will help us understand how different reward structures influence agent coordination and equilibrium outcomes.

II. Methodology

The challenge lies in identifying a stable distribution of agents across nights, the Nash equilibrium. At equilibrium, no agent can improve their reward by switching to a different night. The method developed to achieve this equilibrium involves agents making decisions based on one of three reward structures: local, global, or difference rewards.

The algorithm iteratively guides agents through a process where they evaluate their payoffs and explore alternative nights, switching only if it leads to an improved reward. This method reliably converges to a Nash equilibrium because agents are naturally motivated to act in their self-interest. Through repeated iterations, the distribution of agents stabilizes, and no further improvements are possible.

The Nash equilibrium derived from each reward structure reflects how different incentives shape both individual decisions and the overall system's efficiency. Comparing these equilibrium states across the local, global, and difference rewards offers valuable insights into the effects of crowding, individual contribution, and performance within the system, revealing how different strategies impact collective behavior and resource allocation.

III. Results

A. Local Reward

1) Nash Equilibrium Distribution: The Nash equilibrium distributions of agents across the six nights using the local reward structure are:

```
Nash Equilibrium Distribution using
local reward:
(8, 8, 8, 8, 9, 9), (0, 0, 12, 12,
13, 13), (0, 10, 10, 10, 10, 10)
```

Algorithm 1 Payoff-Switching Method for Nash Equilibrium

```
Initialization:
Randomly assign n agents to k nights.
Track the number of agents attending each night.
Assume the system is in a stable state.

while system is not stable do
    for each agent i do
        Assume Stability:
        Each agent i evaluates their current situation.
        Assess whether switching to a different night yields a higher payoff.
        if agent i finds a better night then
            Move agent i to the new night.
            Mark system as unstable.
        else
            Remain on the current night.
        end if
    end for
end while

for each agent i do
    Agent Decision-Making:
    Calculate current payoff based on the reward function.
    Evaluate number of attendees on the current night.
    Compare current reward with potential reward from switching.
end for

for each agent i do
    Evaluating Payoffs:
    if switching nights improves payoff then
        Agent i switches to the new night.
        System is not at equilibrium.
    else
        Agent i remains on the current night.
        System stability is reinforced.
    end if
end for
```

2) Local Rewards per Night: The local rewards for each night at the Nash equilibrium are:

```
Local Rewards (per night) at Nash
Equilibrium:
[2.9430, 2.9430, 2.9430, 2.9430,
2.9219, 2.9219]
[0.0, 0.0, 2.6776, 2.6776, 2.5599,
2.5599]
[0.0, 2.8650, 2.8650, 2.8650,
2.8650, 2.8650]
```

B. Global Reward

1) Nash Equilibrium Distribution: The Nash equilibrium distribution of agents across the six nights for the global reward structure is:

```
Nash Equilibrium Distribution using
global reward:
(8, 8, 8, 8, 9, 9)
```

2) Global Reward: The global reward at the Nash equilibrium is:

```
Global Rewards at Nash Equilibrium:
17.6159
```

C. Difference Reward

1) Nash Equilibrium Distribution: The Nash equilibrium distribution of agents across the six nights for the difference reward structure is:

```
Nash Equilibrium Distribution using
difference reward:
(8, 8, 8, 8, 9, 9)
```

2) Difference Rewards per Night: The difference rewards for each night at the Nash equilibrium are:

```
Difference Rewards (per night) at
Nash Equilibrium:
[0.0250, 0.0250, 0.0250, 0.0250,
-0.0212, -0.0212]
```

IV. Conclusion

In this analysis, we examined the Nash equilibrium under three different reward structures—local, global, and difference rewards—for the bar problem with 50 agents, a bar capacity $b = 4$, and $k = 6$ nights. The goal was to analyze how these different reward structures affect agent behavior and system performance.

Under the local reward structure, agents aim to maximize their individual payoff by selecting nights based on crowd levels. This results in a distribution where agents attempt to avoid overcrowding while still maximizing their personal rewards. The Nash equilibrium configurations, such as `(8, 8, 8, 8, 9, 9)`, reflect this balance, though some slight disparities in attendance remain. We also see equilibrium states of `(0, 0, 12, 12, 13, 13)`, `(0, 10, 10, 10, 10, 10)`. While the local reward leads to relatively balanced distributions, agents primarily focus on minimizing their own penalties without considering the system as a whole, resulting in a less coordinated but still functional equilibrium.

The global reward structure, by contrast, incentivizes agents to consider the overall system's performance, resulting in a more balanced distribution across all nights. Equilibrium distributions, like `(8, 8, 8, 8, 9, 9)`, show that agents in the global reward structure are more evenly spread out, aiming to optimize the total collective reward rather than their individual outcomes. This leads to a more efficient allocation of agents across the nights, reducing crowding and increasing the system-wide payoff. In comparison to the local reward structure, the global reward encourages greater coordination among agents, leading to a more efficient Nash equilibrium.

The difference reward structure focuses on the marginal effect of attending a particular night, encouraging agents to consider how their attendance impacts their individual reward. The resulting distribution, such as `(8, 8, 8, 8, 9, 9)`, shows that the equilibrium is similar to the global reward structure, but agents still prioritize their personal contributions to the system rather than its overall efficiency. While the difference reward structure can lead to higher individual rewards, it may not always produce the most efficient outcome at the system level.

In conclusion, the results demonstrate that the choice of reward structure significantly influences both agent behavior and system performance. The global reward structure promotes a more efficient Nash equilibrium by encouraging agents to coordinate and balance their choices across nights. In contrast, the local and difference reward structures emphasize individual incentives, leading to less efficient but more personalized outcomes. Ultimately, the global reward structure proves to be the most effective in fostering a balanced and coordinated system-wide performance.

V. Acknowledgments

I would like to acknowledge the assistance of ChatGPT, an AI language model developed by OpenAI, for providing support in refining grammar and enhancing sentence structure throughout this document [1].

References

[1] OpenAI, "ChatGPT: A language model for conversational AI," 2023. [Online]. Available: https://www.openai.com/chatgpt. [Accessed: 10-Nov-2024].