"Guess who" example

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Let's see if you have grasped the concept of marginal, joint, conditional distributions, and Bayes theorem. There is a game called "Guess Who". In this game, you have to find out what person your opponent has picked. To do this, you can ask yes-or-no questions regarding the characteristics of this person (e.g., is this person bald?). You can see all potential people in the figure below.

Let's look at a couple of probabilities:

$$p(notbald) = 19/24$$

 $p(bald) = 5/24$
 $p(mustache|bald) = 0/5$
 $p(mustache|notbald) = 5/19$

These results show that p(bald) = 1 - p(notbald) but that $p(mustache|bald) \neq 1 - p(mustache|notbald)$.

Here is a tricky question. Say that our opponent lies with 1/6 probability. Then, what is the probability that the person we selected is John given that my opponent has answered that his person has a mustache? I will denote the answer by A.

$$\begin{split} p(Jon|A) &= p(Jon, A\,true|A) + p(Jon, A\,lie|A) \\ &= p(Jon|A\,true, A)p(A\,true|A) + p(Jon|A\,lie, A)p(A\,lie|A) \\ &= p(Jon|mustache)p(A\,true) + p(Jon|notmustache)p(A\,lie) \\ &= 0 \times (5/6) + (1/19) \times (1/6) \end{split}$$



Figure 1: People from "Guess Who" game