Thesis Title

Thesis Subtitle

Author Name

B.Sc. Final Year Dissertation

Cardiff School of Mathematics

Acknowledgments

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Contents

List of Figures

Chapter 1

Introduction

1.1 Introduction

Chapter 2

Awesome theorems and stuff

2.1 Introduction

Note that I can refer to other chapters: see Chapter 1 and even specific equations in each chapter, this is an

Chapter 3

Conclusion

Chapter 4

Results

In this chapter, the implementation of a fingerprint function within the Axelrod-Python library will be examined. This includes the addition of two strategy transformers, the Dual and JossAnn as defined in **??**. Then several results will be presented where analytical fingerprints are compared with analytical ones. A discussion that compares different fingerprints of strategies within Axelrod-Python will also be given.

4.1 The Dual

The dual of a strategy is defined such that when when the original strategy and the dual are presented with identical histories they will return opposite actions. This relies on knowledge of how the original strategy would have behaved in a given situation, would be impractical to infer from the source code, however, the required behaviour can be achieved by having the original strategy as an attribute of the dual. Whenever the dual has to submit a move, it can first get the original strategy to suggest what move should it would have made, and then flip that action.

   Data: A strategy
   Result: The d of the strategy
1 if *First Turn* **then**
2 | create copy of original strategy;
3 end
4 simulate original strategy;
5 update original strategy's history/internal state;
6 return *Flip of original strategy's move*

Algorithm 1: The Dual of a Strategy

4.2   Implementation of Fingerprinting

As defined in section a fingerprint function is merely the expected score of a strategy when played against a Joss-Ann transformer of a probe with varying parameters. An approximation to this has been implemented within the Axelrod-Python library. It begins by taking a sample of the x, y values that may define the Joss-Ann Transformer. The strategy then plays a match against the a transformer with each of the sampled values. The average score per turn can be calculated at the end of each match which corresponds to the expected score required by the exact fingerprint function. The whole process can be repeated for reliability and the resulting scores plotted.

Whether the numerical fingerprint matches the analytical one relies heavily on the choice of parameters. The number of x, y values taken is determined by the `step` variable.

```python
def create_points(step):
    """Creates a set of Points over the unit square.
    A Point has coordinates (x, y). This function constructs points that are
    separated by a step equal to `step`. The points are over the unit
    square which implies that the number created will be (1/`step` + 1)^2.
    Parameters
    ----------
    step : float
        The separation between each Point. Smaller steps will produce more
        Points with coordinates that will be closer together.
    Returns
    ----------
    points : list
        of Point objects with coordinates (x, y)
    """
    num = int((1 / step) // 1) + 1
    points = [Point(j, k) for j in np.linspace(0, 1, num)
              for k in np.linspace(0, 1, num)]

    return points
```

Listing 1: Axelrod-Python code to create a sample of x, y points

4.3   Comparison of Analytical and Numerical Plots

In figure 4.1, several analytical fingerprints from previous literature are shown [1]. Colourings and shadings are used to make certain features stand out, and an attempt to replicate this behaviour was implemented in Axelrod-Python. The popular plotting library matplotlib has many options for different colour maps which are demonstrated in Appendix .

Using the analytical fingerprints from previous literature, and the fingerprint formulae provided alongside them, the most appropriate colour map was chosen. The colour map Seismic was selected due to its divergent

```python
def fingerprint(self, turns=50, repetitions=10, step=0.01, processes=None,
                filename=None, in_memory=False, progress_bar=True):
    """Build and play the spatial tournament.

    Creates the probes and their edges then builds a spatial tournament.
    When the coordinates of the probe sum to more than 1, the dual of the
    probe is taken instead and then the Joss-Ann Transformer is applied. If
    the coordinates sum to less than 1 (or equal), then only the Joss-Ann is
    applied, a dual is not required.

    Parameters
    ----------
    turns : integer, optional
        The number of turns per match
    repetitions : integer, optional
        The number of times the round robin should be repeated
    step : float, optional
        The separation between each Point. Smaller steps will
        produce more Points that will be closer together.
    processes : integer, optional
        The number of processes to be used for parallel processing
    progress_bar : bool
        Whether or not to create a progress bar which will be updated

    Returns
    ----------
    self.data : dictionary
        A dictionary where the keys are coordinates of the form (x, y) and
        the values are the mean score for the corresponding interactions.
    """
```

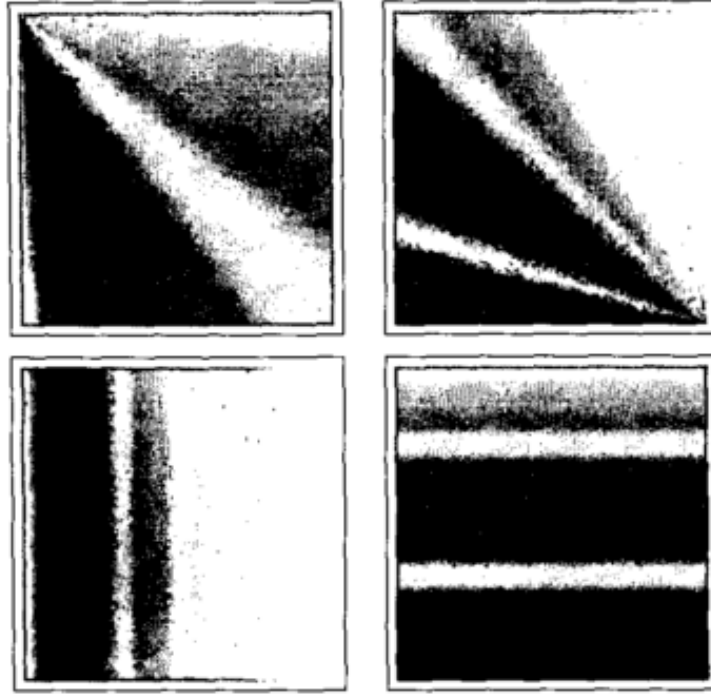Listing 2: Axelrod-Python docstring for the fingerprint function

Figure 4.1: Shaded plots of the fingerprint functions for the strategies TitForTat, Psycho, AllD and AllC, in reading order.
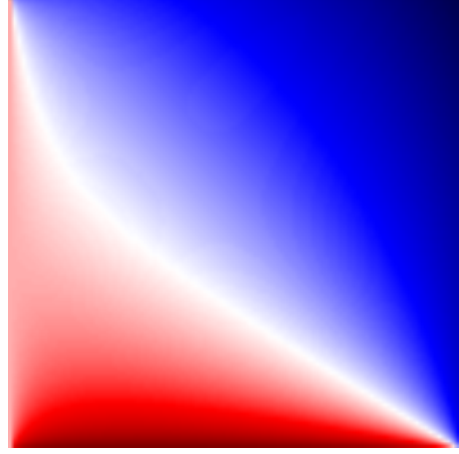
properties (although all colour maps are available within the library). With divergent colour maps, all extreme values (high or low) are coloured, whilst mid range values are left white. This highlights areas of interest, and in Figure 4.2 it can be seen that this matches previous work very well.

With the knowledge that the choice of colourmap is appropriate, a comparison can now be made between analytical fingerprints and numerical ones obtained via the Axelrod-Python library. Table 4.1 gives the exact fingerprint functions of several well known strategies that will then be used to validate the numerical versions.

Figures 4.3 4.4 4.5 4.6 4.7 compare plots of known exact fingerprint functions with analytical approximations obtained with the Axelrod-Python library. The analytical plots were created with the code seen in listing 3. The parameters `turns`=500, `repetitions`=200, `step`=0.01 are as described in section 4.2. The parameter `processes`=0 ensures that the function will use the maximum number of cores available on the computer.
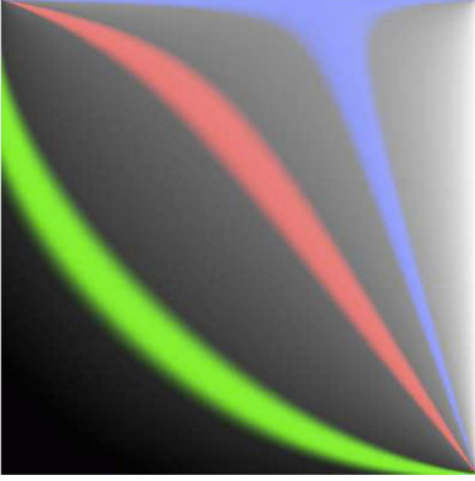
(a) WSLS fingerprint from previous literature　　　(b) Analytical WSLS fingerprint demonstrating Seismic colouring

Figure 4.2: A comparison of a fingerprint plot from previous literature to asses the suitability of the Seismic colour map

| Strategy	Analytical Fingerprint Function
TitForTat	$\dfrac{y^2 + 5xy + 3x^2}{(x+y)^2}$
Psycho (Anti TitForTat	$\dfrac{4(y-1)(x-1) + 5(y-1)^2}{2(y-1)(x-1) + (x-1)^2 + (y-1)^2}$
WinStayLoseShit (Pavlov)	$\dfrac{(3x+y)(x-1) + 5y(y-1)}{(x+2y)(x-1) + y(y-1)}$
AllC (Cooperator)	$3 - 3y$
AllD (Defector)	$4x + 1$

Table 4.1: A selection of exact fingerprint functions for well known strategies. The probe used is TitForTat.

```
1  import axelrod as axl
2  strats = [axl.TitForTat, axl.WinStayLoseShift, axl.AntiTitForTat,
3           axl.Cooperator, axl.Defector]
4  for s in strats:
5      probe = axl.TitForTat
6      af = axl.AshlockFingerprint(s, probe)
7      data = af.fingerprint(turns=500, repetitions=200, step=0.01, processes=0)
8      p = af.plot()
9      p.savefig('{}-Numerical.pdf'.format(s.name))
```
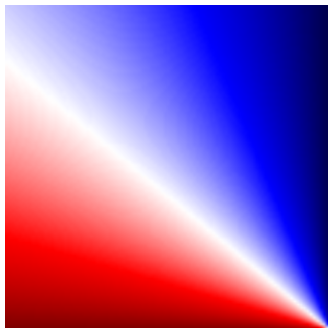
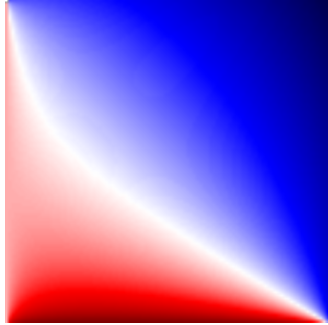Listing 3: Code to create the numerical plots for several strategies
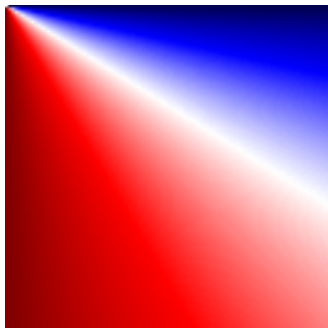
(a) Exact analytical fingerprint

Figure 4.3: A comparison of the analytical fingerprint of TitForTat and the numerical version produced by Axelrod-Python library.
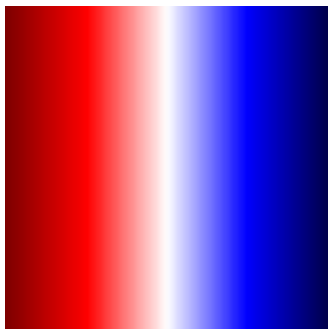

(a) Exact analytical fingerprint

Figure 4.4: A comparison of the analytical fingerprint of Psycho and the numerical version produced by Axelrod-Python library.

Figure 4.5: A comparison of the analytical fingerprint of WinStayLoseShit and the numerical version produced by Axelrod-Python library.

Figure 4.6: A comparison of the analytical fingerprint of Cooperator and the numerical version produced by Axelrod-Python library.

(a) Exact analytical fingerprint

Figure 4.7: A comparison of the analytical fingerprint of Defector and the numerical version produced by Axelrod-Python library.
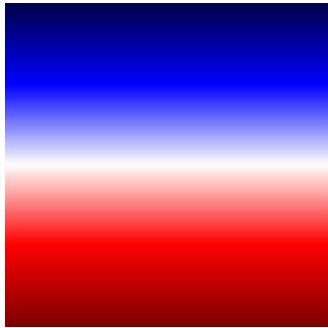
Chapter 5

Appendix

The end.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis

augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

Bibliography

[1]   Dan Ashlock, Eun-Youn Kim, and W.K. VonRoeschlaub. "Fingerprints: enabling visualization and auto-
matic analysis of strategies for two player games". In: *Proceedings of the 2004 Congress on Evolutionary
Computation (IEEE Cat. No.04TH8753)* (2004), pp. 381–387. DOI: 10.1109/CEC.2004.1330882. URL:
http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=1330882 (cit. on p. 8).