# Thesis Title

Thesis Subtitle

**Author Name**

B.Sc. Final Year Dissertation

Cardiff School of Mathematics

CARDIFF UNIVERSITY

PRIFYSGOL CAERDYDD

# Acknowledgments

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# Chapter 1

# Introduction

## 1.1 Introduction

There is this data that is pretty awesome, I'm going to plot it and show it to you in sections 1.2 and 1.3.

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## 1.2 The data

In Figure 1.1 we see data that was generated using (1.2):

$$x \in \{x \in \mathbb{Z} | 1 \leq x \leq 1999\} \tag{1.1}$$

$$y = 2(1 + \epsilon)x + 5 \tag{1.2}$$

where $\epsilon \in (-0.5, 0.5)$ is a random number.

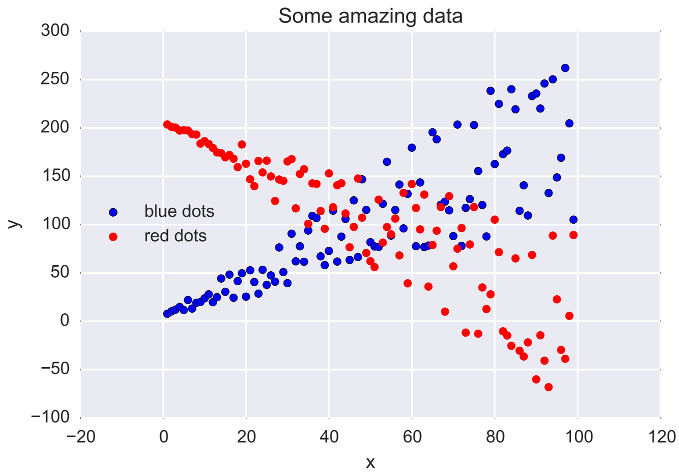


Figure 1.1: The great data

## 1.3 The distribution of the data

Figure shows the distribution of the data.

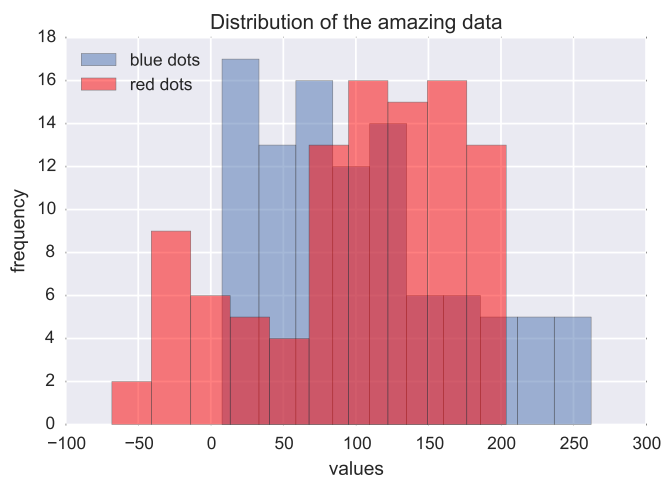


Figure 1.2: The distribution of the great data

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## 1.4 Conclusion

This chapter was amazing, here is a reference to a paper [7].

# Chapter 2

# Literature Review

Prisoner's Dilemma is a very popular problem in game theory and there have been many papers written about the subject. In this chapter a brief overview of the available literature will be given and particularly relevant work will be highlighted. This is followed by an outline of how Axelrod's work is currently being reproduced by an open-source community. Finally, an introduction to fingerprinting and some necessary defenitions and theorems are given at the end of the chapter.

## 2.1 Background

The political scientist Robert Axelrod held the first IPD tournament in 1980. Many well-known game theorists were invited to submit strategies that would compete against each other in a round robin style format. All strategies also competed against a random strategy (that would randomly choose between $C$ and $D$) and a copy of themselves. All strategies knew that the length of each game was 200 moves, and the whole tournamtent was repeated 5 times for reliability. Out of the 13 strategies that were entered, TitForTat was announced as the winner and was submitted by Professor Anatol Rapoport from the Department of Psychology of the University of Toronto.

TitForTat is a very simple strategy that begins by playing $C$ and then plays the same move as it opponent did on the previous turn. As explained in [3], TitForTat won because of three defining characteristics:

- 'Niceness' - A strategy is said to be nice if it is not the first to defect.
- 'Provocability' - Immediately after an opponent defects, the strategy should defect in retaliation.
- 'Forgiveness' - The strategy is willing to continue with mutual cooperation even after some defections.

## 2.2 Axelrod Python Library

- Work to reproduce Axelrod's tournament [7]

| Year | Reference | Number of Strategies | Type |
|---|---|---|---|
| 1979 | | 13 | Standard |
| 1979 | | 64 | Standard |
| 1984 | | 64 | Evolutionary |
| 1991 | | 13 | Noisy |
| 2005 | | 223 | Varied |
| 2012 | | 13 | Standard |

Table 2.1: An overview of published tournaments

is often used to model systems in biology [8], sociology [5], psychology [6], and economics [4].

## 2.3 Fingerprinting

A method for differentiating between strategies in a graphical manner is first given in [2] by Ashlock. Several definitions, theorems and proofs concerning the construction of a fingerprint are outlined. This is then followed by a few examples. The examples presented are of low quality and the only probe strategy (see section/definition etc) used is TitForTat.

Ashlock then extends his fingerprinting work further in [1]. Far more examples are presented, and many different fingerprint functions are listed. A large number of the analytical fingerprint functions use a probe that is not TitForTat. Fingerprinting is then used to to assess how three evolutionary algorithms produce different populations. The evolutionary methods involved are finite-state machines, lookup tables and feedforward neural nets.

**Definition 1** *If $A$ is a strategy for playing the iterated prisoner's dilemma, then the **Joss-Anne of A**, $JA(A, x, y)$ is a transformation of that strategy. Instead of the original behaviour, it makes move $C$ with probablility $x$, move $D$ with probability $y$, and otherwise uses the response appropriate to strategy $A$ (if $x + y < 1$).*

The notation JA comes from the initials of the names Joss and Anne. Joss was a strategy submitted to one of Axelrods original tournaments and it would occasionally defect without provocation in the hopes of a slight improvement in score. Anne is the first name of A. Stanley who suggested the addition of random cooperation (refs from ashlock paper) instead of random defection [1]. When $x+y = 1$, the original strategy is not used, and the resulting behavior is a random strategy with probabilities $(x, y)$. In more general terms, a JA strategy is an alteration of a strategy $A$ that causes the strategy to be played with random noise inserted into the responses.

**Definition 2** *A **Fingerprint** $F_A(S, x, y)$ with $0 \leq x, y \leq 1$, $x + y \leq 1$ for strategy $S$ and probe $A$, is the function that returns the expected score of strategy $S$ against $JA(A, x, y)$ for each possible $(x, y)$.*

The fingerprint function can often be found quite easily (see Chapter 3 for an example construction) however it is the plot of this function that is of interest. This is because an approximation of the plot can be found in cases where the analytical function would be too difficult or time consuming to compute.

**Definition 3** *The* ***Double Fingerprint*** $F_{AB}(S, x, y)$ *with* $0 \leq x, y \leq 1$ *returns the expected score of strategy* $S$ *against* $JA(A, x, y)$ *if* $x + y \leq 1$*, and* $JA(B, 1 - y, 1 - x)$ *if* $x + y \geq 1$.

The double fingerprint allows us to extend the idea of a fingerprint over the full unit square. A unit square is preferable because it is more easily manipulated by a computer, is more easily viewed by humans, and uses paper more efficiently.

**Definition 4** *Strategy* $A'$ *is said to be the* ***Dual*** *of strategy* $A$ *if* $A$ *and* $A'$ *can be written as finite-state machines that are identical except that their responses are reversed.*

An alternative wording is that, given a history for an opponent, the responses of the original strategy and the dual would be opposite. It's important to note that this is different to taking a strategy and flipping it's responses. The dual relies on knowledge of the underlying state of the original strategy, whereas the flip does not This is shown in Table 2.2.

| Pavlov | Dual | Flip | Opponent |
|---|---|---|---|
| C | D | D | C |
| C | D | C | D |
| D | C | C | D |
| C | D | C | C |
| C | D | D | C |
| C | D | C | D |
| D | C | C | C |
| D | C | D | D |
| C | D | D | |

Table 2.2: The different responses of Pavlov, Pavlov's Dual and Flipped Pavlov

The subtle difference between Dual and Flip can be highlighted further by inspecting each row individualy.

<u>Row 1</u> - Pavlov always plays $C$ on the first go. Flip will change this to $D$. Dual knows that Pavlov always plays $C$ and so swaps to $D$.

<u>Row 2</u> - In the previous round for Pavlov the strategies played $(C, C)$, and so Pavlov plays $C$ again. For Flip, the preceding interaction was $(D, C)$, in this instance Pavlov would play $D$ again, so this gets flipped to $C$. The previous turn for Dual was $(D, C)$ so it infers that Pavlov had $(C, C)$. It knows that Pavlov would play $C$ and so plays $D$.

<u>Row 3</u> - In the previous round for Pavlov the strategies played $(C, D)$, and so Pavlov would change to play $D$. For Flip, the preceding interaction was $(C, D)$, in this instance Pavlov would change to $D$, so this gets flipped to play $C$ again. The previous turn for Dual was $(D, D)$ so it infers that Pavlov had $(C, D)$. It knows that Pavlov would play $D$ in this instance and so plays $C$.

**Theorem 1** *If* $A$ *and* $A'$ *are dual strategies, then* $F_{AA'}(S, x, y)$ *is identical to the function* $F_A(S, x, y)$ *extended over the unit square.*

Proof from [2]:

**Proof 1** *The Markov chain for the dual strategy $A'$ will have the same transitions as the Morkov chain for the strategy $A$. However, each entry for $x$ corresponds to the probability that the strategy $J(A, x, y)$ will randomly choose $C$ when it would not normally do so. For strategy $A'$, this will occur whenever $J(A', x, y)$ does not randomly respond $D$, which has probability $1 - y$.*

*Similarly, each $y$ corresponds to the probability that the strategy $J(A, x, y)$ will randomly choose $D$ when it would usually respond $C$. For strategy $A'$, this will occur whenever $J(A', x, y)$ does not randomly respond $C$, which has probability $1 - x$.*

*Thus the Markov chain for $J(A', x, y)$ is the Markov chain for $J(A, x, y)$ with the mapping $(x, y) \to (1-y, 1-x)$. Therefore $F_{AA'}(S, x, y)$ extends to the remainder of the unit square the function given by $F_A(S, x, y)$.* ■

# Chapter 3

# Theory

## 3.1 Analytical Fingerprints

There are several steps to constructing the Fingerprint of a strategy and basic knowledge of Markov Chains is required. An outline of the steps is as follows:

1. Build the markov chain for IPD between the strategy and probe strategy.
2. Construct the corresponding transition matrix.
3. Find the steady state distribution.
4. Calculate the overall expected score by taking the dot product of the steady state distribution with the payoff vector given in .
5. Plot the resulting function.

We will now apply this process in order to obtain a fingerprint for the strategy Win-Stay-Lose-Shift (sometimes referred to as Pavlov) when probed by Tit-For-Tat.

**Step 1** - Build the markov chain.

**Step 2** - Construct the transition matrix.

$$T = \begin{array}{c} \\ (C,C) \\ (C,D) \\ (D,C) \\ (D,D) \end{array} \begin{array}{c} \begin{array}{cccc} (C,C) & (C,D) & (D,C) & (D,D) \end{array} \\ \begin{pmatrix} 1-y & 0 & 0 & x \\ y & 0 & 0 & 1-x \\ 0 & 1-y & x & 0 \\ 0 & y & 1-x & 0 \end{pmatrix} \end{array} \tag{3.1}$$

**Step 3** - Find the steady state distribution.

$$\pi = \begin{bmatrix} \frac{x(1-x)}{2y(1-x)+x(1-x)+y(1-y)}, \\ \frac{y(1-x)}{2y(1-x)+x(1-x)+y(1-y)}, \\ \frac{y(1-y)}{2y(1-x)+x(1-x)+y(1-y)}, \\ \frac{y(1-x)}{2y(1-x)+x(1-x)+y(1-y)} \end{bmatrix} \tag{3.2}$$

**Step 4** - Calculate the expected score.

$$F = \pi \cdot \begin{bmatrix} 3 \\ 0 \\ 5 \\ 1 \end{bmatrix} = \frac{3x(1-x)+y(1-x)+5y(1-y)}{2y(1-x)+x(1-x)+y(1-y)} \tag{3.3}$$

**Step 5** - Plot the resulting function.

## 3.2 Finite State Machines

Figure 3.1 and figure 3.2 show the Finite State Machine (FSM) representations for Tit-For-Tat and Pavlov respectively. Nodes represent the previous action taken by the strategy and the opponent, ie node $(D, C)$ implies that is the preceding turn, the strategy chose to Defect and the opponent chose to co-operate. Arcs represent the choice made by the opponent at the current turn, and lead us to the state of the next turn.

It should be noted that these figures are not necessarily the simplest representation of their corresponding strategy. For example, Tit-For-Tat requires no knowledge of its own previous moves, but they have been included for completeness.

In figure 3.3 we have a more complex FSM for the strategy Majority which plays in the following way:

- If the opponent has cooperated the majority of the time, Majority will cooperate
- If the opponent has defected the majority of the time, Majority will defect
- Note - the strategy shown is technically Soft Majority, if the opponents cooperations and defections are equal it will cooperate. Hard Majority would defect in this situation.

This implies that the strategy Majority requires knowledge of all previous states, and therefore could not be represented as an FSM. However in Theorem 2 it is shown that if the number of turns in a game is known, any strategy can be represented as an FSM. A formal defintion of a Finite State Machine is given by Definition 5 but first we will outline some motivating key characteristics of a system that can be modeled with a FSM:

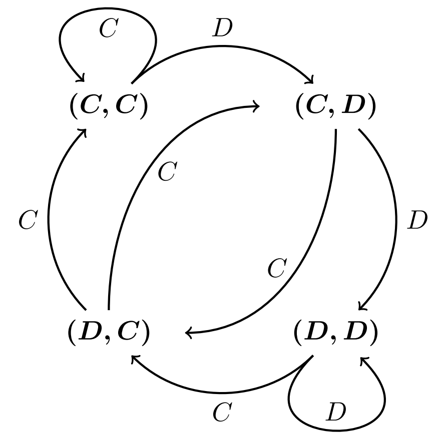


Figure 3.1: FSM for TitForTat

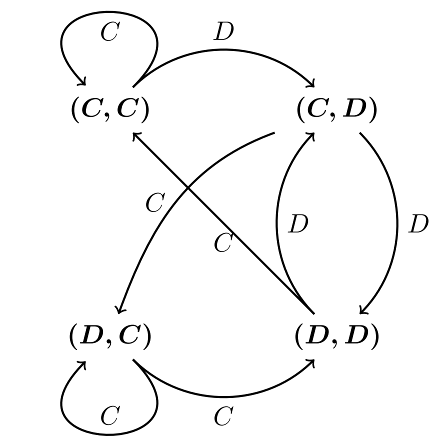


Figure 3.2: FSM for Pavlov (Win-Stay Lose-Shift)

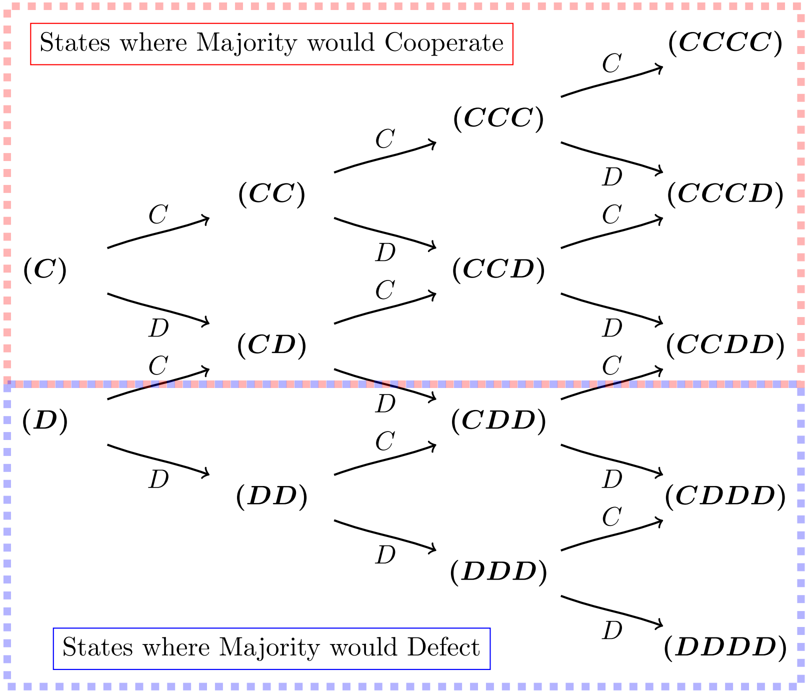


Figure 3.3: FSM for Majority in a game with 4 Turns

- The system must be describable by a finite set of states.
- The system must have a finite set of inputs that can trigger transitions between states.
- The behavior of the system at a given point in time depends upon the current state and the input that occurs at that time.
- For each state the system may be in, behavior is defined for each possible input.
- The system has a particular initial state.

**Definition 5** *A* ***Deterministic Finite State Machine*** *$M$ is a tuple $(S, \sigma, \delta, s_0, F)$ where*

- *$\sigma$ is the set of symbols representing the input of $M$.*
- *$S$ is the set of states of $M$.*
- *$s_0 \in S$ is the starting state.*
- *$F \subseteq S$ is the set of final states of $M$.*
- *$\delta : S \times \sigma \rightarrow S$ is the transition function.*

**Theorem 2** *Given a determenistic strategy $\alpha$ and 2 histories $h_1, h_2$, then for all games of length $n \in 1, 2, 3, \ldots$ there exists a FSM such that $\alpha(h_1, h_2)$ can be obtained from the FSM.*

**Proof 2** *Let $\sigma = \{C, D\}$ and*

$$S = \bigcup_{i=0}^{n+1} \{C, D\}^i \times \{C, D\}^i \delta((h_1, h_2), a) = ()$$

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