Thesis Title

Thesis Subtitle

Author Name

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Cardiff School of Mathematics

Acknowledgments

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Chapter 1

Introduction

1.1   Introduction

There is this data that is pretty awesome, I'm going to plot it and show it to you in sections 1.2 and 1.3.

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1.2   The data

In Figure 1.1 we see data that was generated using (1.2):

$$x \in \{x \in \mathbb{Z} | 1 \leq x \leq 1999\} \tag{1.1}$$

$$y = 2(1 + \epsilon)x + 5 \tag{1.2}$$

where $\epsilon \in (-0.5, 0.5)$ is a random number.

Figure 1.1: The great data

1.3 The distribution of the data
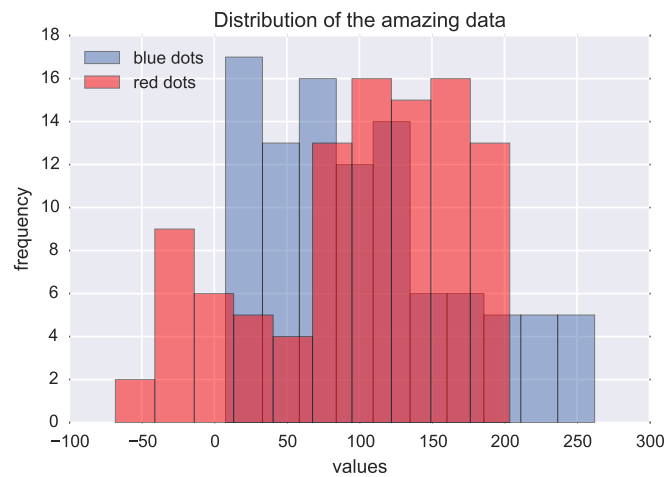
Figure shows the distribution of the data.

Figure 1.2: The distribution of the great data

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1.4   Conclusion

This chapter was amazing, here is a reference to a paper [14].

Chapter 2

Literature Review

The Prisoner's Dilemma is a very popular model in game theory and there have been many papers written about the subject. The game has been applied to many different research areas is often used to model systems in biology [16], sociology [11], psychology [13], and economics [8]. The start of this chapter will give a brief overview of the literature and particularly relevant work will be highlighted. This is followed by an outline of how Axelrod's work is currently being reproduced by an open-source community. Finally, an introduction to fingerprinting and some necessary definitions and theorems are given at the end of the chapter.

2.1 Background

The political scientist Robert Axelrod held the first IPD tournament in 1980 [3]. Many well-known game theorists were invited to submit strategies that would compete against each other in a round robin style format. All strategies also competed against a random strategy (that would randomly choose between 'C' and 'D') and a copy of themselves. All strategies knew that the length of each game was 200 moves, and the whole tournament was repeated 5 times for reliability. Out of the 13 strategies that were entered, TitForTat was announced as the winner and was submitted by Professor Anatol Rapoport from the Department of Psychology of the University of Toronto.

TitForTat is a very simple strategy (see Section 2.2.1) and as explained in [4], it won because of three defining characteristics:

- 'Niceness' - A strategy is said to be nice if it is not the first to defect.

- 'Provocability' - Immediately after an opponent defects, the strategy should defect in retaliation.

- 'Forgiveness' - The strategy is willing to continue with mutual cooperation even after some defections.

Axelrod's second tournament [4] saw a dramatic increase in terms of size, with 62 strategies being entered from 6 different countries. The contestants ranged from a 10-year-old computer hobbyist to professors of

computer science, economics, psychology, mathematics, sociology, political science and evolutionary biology. The countries represented were the United States, Canada, Great Britain, Norway, Switzerland, and New Zealand. Despite the fact that all contestants had full knowledge of the previous tournament, TitForTat was the overall winner once again. One large difference in the mechanics of the first and second tournament was that the second tournament did not specify how many moves a game would last. Instead, the game ended probabilistically with a 0.00346 chance of finishing on any given move. This parameter was chosen so that the median length of a game would be 200 moves (in line with the first tournament).

Year	Reference	Number of Strategies	Type
1979	[3]	13	Standard
1979	[4]	64	Standard
1984	[5]	64	Evolutionary
1991	[7]	13	Noisy
2005	[9]	223	Varied
2012	[17]	13	Standard

Table 2.1: An overview of published tournaments

2.2   Strategies of Particular Interest

Throughout this report many different strategies will be discussed and used in small examples. This section will provide a description of some of these strategies in order to aid the reader, as a strategy's name is not always particularly descriptive.

2.2.1   TitForTat

TitForTat is the most well known strategy for playing Prisoner's Dilemma due to it winning both of Axelrod's first two tournaments. It starts with a cooperative move and proceeds to play the same as the opponent did on the previous move [4, 12].

2.2.2   Pavlov

In [15], the strategy Pavlov is introduced (sometimes referred to as Win-Stay Lose-Shift). Pavlov plays by repeating its previous move if it was successful (received payoff T or R), and swapping the move if it was unsuccessful (received payoff P or S). The paper explains how this allows it take advantage of strategies that cooperate unconditionally and can also correct occasional mistakes.

2.2.3   Gradual

The strategy Gradual is present in [6]. It begins the game by cooperating, then after the first defection of the other player, it defects one time and cooperates twice. After the second defection of the opponent, it

Figure 2.1: Ranked violin plot of the mean payoff for each player

Figure 2.2: Matrix plot of pair wise payoffs for each player

defects two times and cooperates twice. After the nth defection it reacts with n consecutive defections and then two cooperations. In [6] it is claimed that Gradual outperforms TitForTat and this has been confirmed by work done through the Axelrod-Python library [14].

2.3 Axelrod-Python Library

The Axelrod-Python library [10] is an open source Python package for creating reproducible research into the Prisoner's Dilemma. The original aim was to recreate Axelrod's tournaments as described in section 2.1 and verify their results. As the library has grown, so has the overall aim. The goal now, is "to provide a resource, with facilities for the design of new strategies and interactions between them, as well as conducting tournaments and ecological simulations for populations of strategies" [14]. For many of the tournaments that have been described the original source code is not available, and in the few cases where access was available there were no tests and minimal documentation.

The Axelrod-Python library is equipped with an extensive tests suite and all code is comprehensively documented. The documentation can be found at `http://axelrod.readthedocs.io/en/latest/` along with several tutorials designed to help introduce researchers to the library. In total the library now consists of 147 strategies (at the time of writing) including classical ones from the literature and and some exciting original contributions. It is interesting that the original winner, TitForTat, is currently ranked 49th overall. The library also has the capability to run ecological simulations or construct tournaments on alternative topologies (not just the original round robin format).

An example implementation of a strategy and a demonstration of using the library will now be given, followed by examples of some of the many plots the library can produce. The source code for TitForTat is shown in Listing 1.

Strategies are implemented as classes with a `strategy` method. The strategy method contains the logic that defines how the strategy will behave as can be seen in listing 1 lines 28 - 36. If the player does not have a history then it must be the first move and it chooses to play 'C'. On all subsequent moves the player checks whether the opponent played 'D' on it's previous move. If that statement is true the player also plays 'D', otherwise it will play 'C'.

In listing 2 an example of how to produce a simple tournament is shown. Lines 7 - 10 create two plots. The first is a ranked violin plot of the mean payoff for each player and the second is a matrix plot of pair wise payoffs for each player. These plots can be seen in figures 2.1 and 2.2.

```python
class TitForTat(Player):
    """
    A player starts by cooperating and then mimics the previous action of the
    opponent.

    Note that the code for this strategy is written in a fairly verbose
    way. This is done so that it can serve as an example strategy for
    those who might be new to Python.

    Names:

    - Rapoport's strategy: [Axelrod1980]_
    - TitForTat: [Axelrod1980]_
    """

    # These are various properties for the strategy
    name = 'Tit For Tat'
    classifier = {
        'memory_depth': 1,  # Four-Vector = (1.,0.,1.,0.)
        'stochastic': False,
        'makes_use_of': set(),
        'long_run_time': False,
        'inspects_source': False,
        'manipulates_source': False,
        'manipulates_state': False
    }

    def strategy(self, opponent):
        """This is the actual strategy"""
        # First move
        if not self.history:
            return C
        # React to the opponent's last move
        if opponent.history[-1] == D:
            return D
        return C
```

Listing 1: Source code for TitForTat

```
1   >>> import axelrod as axl
2   >>> axl.seed(0)  # Set a seed
3   >>> players = [s() for s in axl.strategies]  # Create players
4   >>> tournament = axl.Tournament(players)  # Create a tournament
5   >>> results = tournament.play()  # Play the tournament
6   >>> plot = axl.Plot(results)
7   >>> p = plot.boxplot()
8   >>> p.show()
9   >>> q = plot.payoff()
10  >>> q.show()
```

Listing 2: Example code to produce a simple tournament

2.4 Fingerprinting

It is easy to produce large numbers of strategies, however their analysis can be time consuming. Even the simple question 'Are these two strategies the same?' does not always have an obvious answer. This is especially true when considering source code, as two identical strategies can be coded in different ways.

A method for comparing strategies is first given in [2]. Ashlock outlines several definitions, theorems and proofs concerning the construction of a fingerprint. These are then followed by some examples, however they are of low quality and the only probe (see section/definition etc) used is TitForTat.

Ashlock then extends his fingerprinting work further in [1]. More examples are presented, and many different fingerprint functions are listed. Also, a large number of the analytical fingerprint functions use a probe that is not TitForTat. Fingerprinting is then used to to assess how three evolutionary algorithms produce different populations. The evolutionary methods involved are finite-state machines, lookup tables and feed forward neural nets.

Definition 1 *If A is a strategy for playing the iterated prisoner's dilemma, then the **Joss-Anne of A**, $JA(A, x, y)$ is a transformation of that strategy. Instead of the original behaviour, it makes move 'C' with probablility x, move 'D' with probability y, and otherwise uses the response appropriate to strategy A (if $x + y < 1$).*

The notation JA comes from the initials of the names Joss and Anne. Joss was a strategy submitted to one of Axelrods original tournaments and it would occasionally defect without provocation in the hopes of a slight improvement in score. Anne is the first name of A. Stanley who suggested the addition of random cooperation (refs from ashlock paper) instead of random defection [1]. When $x + y = 1$, the original strategy is not used, and the resulting behaviour is a random strategy with probabilities (x, y). In more general terms, a JA strategy is an alteration of a strategy A that causes the strategy to be played with random noise inserted into the responses.

Definition 2 *A **Fingerprint** $F_A(S, x, y)$ with $0 \leq x, y \leq 1$, $x + y \leq 1$ for strategy S and probe A, is the function that returns the expected score of strategy S against $JA(A, x, y)$ for each possible (x, y).*

13

Definition 3 *The **Double Fingerprint** $F_{AB}(S, x, y)$ with $0 \leq x, y \leq 1$ returns the expected score of strategy S against $JA(A, x, y)$ if $x + y \leq 1$, and $JA(B, 1 - y, 1 - x)$ if $x + y \geq 1$.*

Definition 4 *Strategy A' is said to be the **Dual** of strategy A if A and A' can be written as finite-state machines that are identical except that their responses are reversed.*

An alternative wording is that, given a history for an opponent, the responses of the original strategy and the dual would be opposite. It is important to note that this is different to taking a strategy and flipping it's responses. The dual relies on knowledge of the underlying state of the original strategy, whereas the flip does not. This is shown in Table 2.2.

Opponent	Pavlov	Dual	Flip
C	C	D	D
D	C	D	C
D	D	C	C
C	C	D	C
C	C	D	D
D	C	D	C
C	D	C	C
D	D	C	D
	C	D	D

Table 2.2: The different responses of Pavlov, Pavlov's Dual and Flipped Pavlov

The subtle difference between Dual and Flip can be highlighted further by inspecting each row individually.

<u>Row 1</u> - Pavlov always plays 'C' on the first go. Flip will change this to 'D'. Dual knows that Pavlov always plays 'C' and so swaps to 'D'.

<u>Row 2</u> - In the previous round for Pavlov the strategies played (C, C), and so Pavlov plays 'C' again. For Flip, the preceding interaction was (D, C), in this instance Pavlov would play 'D' again, so this gets flipped to 'C'. The previous turn for Dual was (D, C) so it infers that Pavlov had (C, C). It knows that Pavlov would play 'C' and so plays 'D'.

<u>Row 3</u> - In the previous round for Pavlov the strategies played (C, D), and so Pavlov would change to play 'D'. For Flip, the preceding interaction was (C, D), in this instance Pavlov would change to 'D', so this gets flipped to play 'C' again. The previous turn for Dual was (D, D) so it infers that Pavlov had (C, D). It knows that Pavlov would play 'D' in this instance and so plays 'C'.

Theorem 1 *If A and A' are dual strategies, then $F_{AA'}(S, x, y)$ is identical to the function $F_A(S, x, y)$ extended over the unit square.*

A proof for this theorem will now be given which was first presented in [2]:

Proof 1 *The Markov chain for the dual strategy A' will have the same transitions as the Markov chain for the strategy A. However, each entry for x corresponds to the probability that the strategy $JA(A, x, y)$ will randomly choose 'C' when it would not normally do so. For strategy A', this will occur whenever $JA(A', x, y)$ does not randomly respond 'D', which has probability $1 - y$.*

Similarly, each y corresponds to the probability that the strategy $JA(A, x, y)$ will randomly choose 'D' when it would usually respond 'C'. For strategy A', this will occur whenever $JA(A', x, y)$ does not randomly respond 'C', which has probability $1 - x$.

Thus the Markov chain for $JA(A', x, y)$ is the Markov chain for $JA(A, x, y)$ with the mapping $(x, y) \to (1 - y, 1 - x)$. Therefore $F_{AA'}(S, x, y)$ extends to the remainder of the unit square the function given by $F_A(S, x, y)$. ∎

Theorem 1 allows the fingerprint function to naturally extend over the unit square. Figure 2.3 shows that in the bottom left region of the plot, the strategy plays against $JA(A, x, y)$ and in the top right region it plays against $JA(A', 1 - y, 1 - x)$.

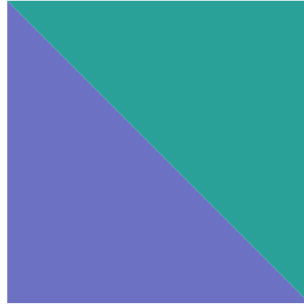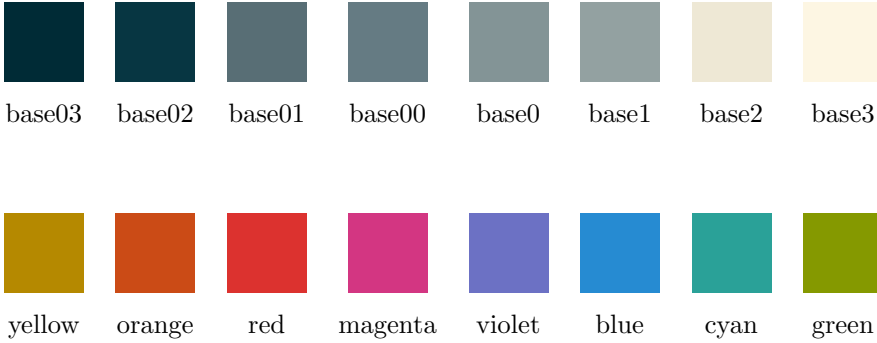Figure 2.3: Whether the Strategy is probed by the Dual or not

base03   base02   base01   base00   base0   base1   base2   base3

yellow   orange   red   magenta   violet   blue   cyan   green

Chapter 3

Theory

3.1 Analytical Fingerprints

There are several steps to constructing the Fingerprint of a strategy and basic knowledge of Markov Chains is required. An outline of the steps is as follows:

1. Build a Markov chain model of an IPD between the strategy and probe strategy.

2. Construct the corresponding transition matrix.

3. Find the steady state distribution.

4. Calculate the overall expected score by taking the dot product of the steady state distribution with the payoff vector given in section **??** to obtain the fingerprint function.

5. This can then be plotted as a heatmap to make it easier to visualize.

As an example, this process will now be applied to obtain a fingerprint for the strategy Win-Stay-Lose-Shift (sometimes referred to as Pavlov) when probed by Tit-For-Tat.

Step 1 - Build the markov chain.

Step 2 - Construct the transition matrix.

$$
T = \begin{array}{c} \\ (C,C) \\ (C,D) \\ (D,C) \\ (D,D) \end{array}
\begin{array}{cccc} (C,C) & (C,D) & (D,C) & (D,D) \end{array}
\begin{pmatrix}
1-y & 0 & 0 & x \\
y & 0 & 0 & 1-x \\
0 & 1-y & x & 0 \\
0 & y & 1-x & 0
\end{pmatrix}
\tag{3.1}
$$

Step 3 - Find the steady state distribution.

$$\pi = \begin{bmatrix} \dfrac{x(1-x)}{2y(1-x)+x(1-x)+y(1-y)}, \\[2mm] \dfrac{y(1-x)}{2y(1-x)+x(1-x)+y(1-y)}, \\[2mm] \dfrac{y(1-y)}{2y(1-x)+x(1-x)+y(1-y)}, \\[2mm] \dfrac{y(1-x)}{2y(1-x)+x(1-x)+y(1-y)} \end{bmatrix} \tag{3.2}$$

Step 4 - Calculate the expected score.

$$F = \pi \cdot \begin{bmatrix} 3 \\ 0 \\ 5 \\ 1 \end{bmatrix} = \frac{3x(1-x)+y(1-x)+5y(1-y)}{2y(1-x)+x(1-x)+y(1-y)} \tag{3.3}$$

Step 5 - Plot the resulting function.

3.2 Finite State Machines

Figure 3.1 and figure 3.2 show FSM (Finite State Machine) representations for Tit-For-Tat and Pavlov respectively. However, it should be noted that FSM representations are not necessarily unique. Nodes represent the previous action taken by the strategy and the opponent, ie. node (D, C) implies that on the preceding turn, the strategy chose to Defect and the opponent chose to co-operate. Arcs represent the choice made by the opponent at the current turn, and lead us to the state for the next turn.

These are not necessarily the simplest FSM representation of the strategies. For example, Tit-For-Tat requires no knowledge of its own previous moves, but they have been included for completeness.

In figure 3.3 we have a more complex FSM for the strategy Majority for a game with 4 turns. Majority plays in the following way:

- If the opponent has cooperated the majority of the time, Majority will cooperate

- If the opponent has defected the majority of the time, Majority will defect

- Note - the strategy shown is technically Soft Majority, if the opponents cooperations and defections are equal it will cooperate. Hard Majority would defect in this situation.

Clearly this means that the strategy Majority requires knowledge of all previous states. It has been stated previously that this implies that Majority could not be represented as a FSM. However in Theorem 2 it is shown that if the number of turns in a game is known, any strategy can be represented as a FSM. This will be proved in two different ways:
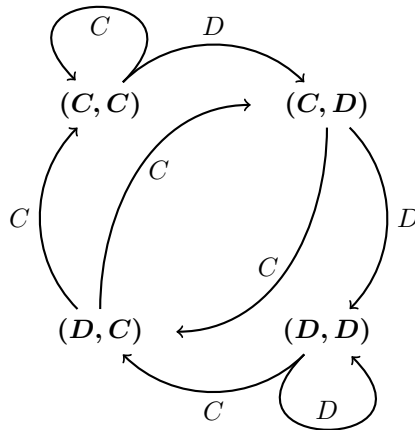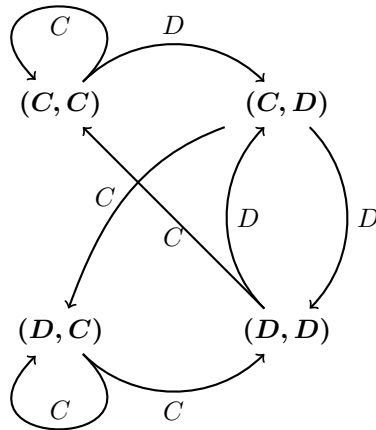
Figure 3.1: FSM for TitForTat

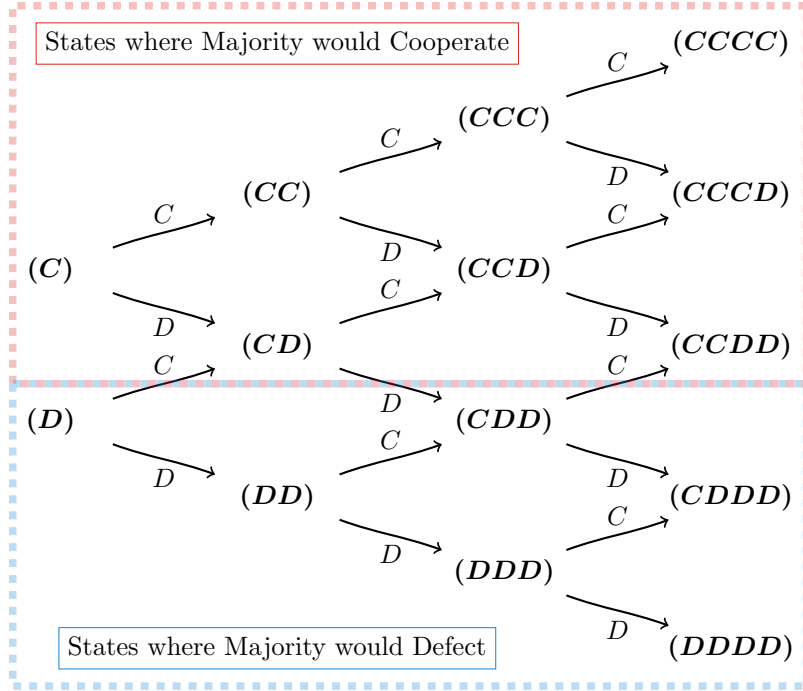Figure 3.2: FSM for Pavlov (Win-Stay Lose-Shift)

Figure 3.3: FSM for Majority in a game with 4 Turns

- Firstly, by altering the definition of an automaton

-

A formal defintion of a Finite State Machine is given by Definition 5 but first we will outline some motivating key characteristics of a system that can be modeled with a FSM:

- The system must be describable by a finite set of states.

- The system must have a finite set of inputs that can trigger transitions between states.

- The behavior of the system at a given point in time depends upon the current state and the input that occurs at that time.

- For each state the system may be in, behavior is defined for each possible input.

- The system has a particular initial state.

Definition 5 *A **Deterministic Finite State Machine** M is a tuple $(S, \sigma, \delta, s_0, F)$ where*

- σ *is the set of symbols representing the input of M.*

- S *is the set of states of M.*

- $s_0 \in S$ *is the starting state.*

- $F \subseteq S$ *is the set of final states of M.*

- $\delta : S \times \sigma \to S$ *is the transition function.*

Theorem 2 *Given a deterministic strategy α and 2 histories h_1, h_2, then for all games of length $n \in \mathbb{N}$ there exists a FSM such that $\alpha(h_1, h_2)$ can be obtained from the FSM.*

Proof 2 *Let $\sigma = \{C, D\}$ and*

$$S = \bigcup_{i=0}^{n+1} \{C, D\}^i \times \{C, D\}^i \delta((h_1, h_2), a) = ()$$

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