The effect of homophily in hierarchical systems

November 29, 2018

1 Mathematical formulation of model

- Given a hierarchical system with K levels.
- Level $0 \leq k < K$ has capacity C_k
- The first level ($k = 0$) has the most capacity and capacity is monotonically decreasing: $C_0 > C_1 > \cdots > C_{k-1}$.
- There are 2 types of agents: $j \in \{0, 1\}$.

Consider a state space $\mathcal{S}$:

$$\mathcal{S} = \mathbb{Z}_{\geq 0}^{K \times 2} \tag{1}$$

Where s_{ij} denotes the number of individuals of type j at level i.
For example,

- Let $K = 3$
- Let $C = (5, 3, 2)$

Then:

$$s = \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 1 \end{pmatrix}$$

corresponds to a system with 3 agents of first type and 1 of second type at the first level, 2 of first type and 1 of second type at the second level and 1 of each type at the 3rd level.

Given two elements $s^{(1)}, s^{(2)} \in \mathcal{S}$ the transition rates are given by:

$$Q_{s_1,s_2} = \begin{cases} \mu_{ij}, & \text{if } s^{(2)} - s^{(1)} = -e_{ij} \text{ and } s^{(1)}_{i0} + s^{(1)}_{i1} = C_i \\ rS_{ij} + S_{i,\bar{j}}, & \text{if } s^{(2)} - s^{(1)} = e_{ij} - e_{i-1,j} \text{ and } s^{(1)}_{i0} + s^{(1)}_{i1} < C_i \text{ and } i > 0 \\ \lambda_j, & \text{if } s^{(1)}_{00} + s^{(1)}_{01} = C_0 - 1 \end{cases} \tag{2}$$

Where:

- μ_{ij} is the retirement rate of agents of type j at level i.
- $r > 1$ is a constant that reflects the homophily effect.
- λ_j is the hiring rate of individuals of type j.