Responses to Reviews for Measuring the Price of Anarchy in Critical Care Unit Interactions

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Below we respond to each reviewer in detail.

0.1 Reviewer 1

Write the clarifications of notations $\lambda_h^{(l,h)}, \lambda_h^{(l,l)}$. One $h \in (NH, RG)$ other h denotes RG experiences high demand.

- Change one of the hs.

  Give explanation for figure 3.

- Add a sentence.

  Write clearly major contribution of the paper corresponding to the existing literature.

- Point out that this is there. Ask if there would be a better way of doing it?

  State clearly the objectives of the model.

- Not sure I understand, mention that have clarified the model for each player.

0.2 Reviewer 2

On page 5, the model is referred to as a Figure 6. This confuses and leaves a question on whether it is a figure or equations. I am unsure whether figure 6 is missing or authors are referring to the model on page 6.

- Mention that this has changed.

  The paper is unreadable. The unknowns used, some, are undefined, for example on page 3, h, ch, etc are not defined and it is very difficult to find definitions of those defined. I suggest that the authors provide a list of the parameters used somewhere in the paper where it is easy to refer.

- Highlight that this is all in the paper.

  On page 12, authors states that "It is also noted that as demand increases the effect of uncoordinated behaviour increases (and the recommended target also increases) as shown in Figure 13", is there any possible explanation to this finding.

- Suggest some explanations.

Are the Queuing and Game theoretic models never been implemented to solve such problems? There is a need to review literature that has information on application of the used models to hospital problems or related problems.

- This is in the paper.

The authors need to give a justification on the reason why they are implementing the models.

- Point to wider discussion invited by reviewer 3.

0.3 Reviewer 3

I recommend that the authors write a more complete and coherent Discussion and/or Conclusion which places their work in the context of the critcal care system. It should relate model limitations to the critical care system. The current way that limitations are discussed is too technical. A brief discussion of stakeholder feedback would also be useful.

- Add to discussion, point out that previous limitations were technical and now discuss ccu limitations. Mainly following from points suggested by reviewer 3.

At the beginning of Section 3.1, the authors claim that if neither CCU is able to admit patients, then admission to the CCU is cancelled and the patient is admitted to a general ward. In my experience modelling CCUs, it is highly unlikely that a critical patient would ever be admitted to a general ward. CCU beds are almost always equipped with ventilators and ward beds never. I believe that more likely courses of action are:

- *Sending the patient to another hospital's CCU. This would be a CCU outside of the model.*
- *"Bumping" a patient currently in the CCU to a ward bed, to free up a bed. This would normally be a patient who was nearly ready to be transferred to the ward anyway.*
- *Accommodating the patient in the hospital's post-anesthesia care unit (PACU), which sometimes functions as an overflow for the CCU.*

I recommend that the authors consult with their stakeholders and correct this comment in their paper, if necessary. This wouldn't have any impact on their model, but it is important to use the correct language.

- List all of above as options but use first as most related to model.

The axis labels on many of the graphs are in a very small font. The font size should be increased.

All axis labels have now got a bigger font. We appreciate this comment: if any other aspects of the graphics could be improved we would be happy to make the suggested changes.

The optimisation problem is shown as a figure (Figure 6). I suggest that this be separated from the text with a header such as "Optimisation Problem" (much like a theorem). Calling it a figure seems like a misnomer, and for me, actually made it harder to find.

This has been changed:

- The figure has been changed to a section with a header as suggested.
- A label (3) has been added for easier reference throughout the text.

In Figure 7, the caption should state which points are $f_N H$ and which are $F_R G$.

This is a good point, we have clarified this by noting 'crosses' and 'diamonds' in plain text in the caption.

I was puzzled as to whether the target, t, appeared in the definition of T^*. As I understand it, T^* is independent of t. Is this true? Also, for which values of K_{NH} and K_{RG} is the maximum achieved? Is my intuition correct that it is achieved at $K_{NH} = c_{NH}$ and $K_{RG} = c_{RG}$?

This is an excellent question. The reviewer understand correctly: T^* is not dependent of t.

Identifying the point at which throughput is maximised is however not immediate. When we began working on this paper we had the same intuition as the reviewer (that throughput would always be maximised when using the highest possible threshold), however this is not the case.

Indeed, for example for the second model, when using a system with parameters:

- $c_{\text{RG}} = 2, c_{\text{NH}} = 3$
- $\mu_{\text{RG}} = 0.22, \mu_{\text{NH}} = 1.66$
- $\lambda_{\text{RG}} = 3, \mu_{\text{NH}} = 1$

The throughputs for each cutoff pair are shown in the table below:

Throughput	Cut off pairs
1.704098	(2, 0)
1.843005	(1, 0)
1.857689	(0, 0)
3.27514	(2, 1)
3.472371	(1, 1)
3.51033	(0, 1)
4.291777	(2, 2)
4.488501	(1, 2)
4.549065	(0, 2)
4.767654	(2, 3)
4.877357	(1, 3)
4.916791	(0, 3)
4.928235	(2, 4)
4.961882	(1, 4)
4.969698	(0, 4)

We see that the optimal throughput is obtained when RG is in effect 'closed' and NH is never in a diverted state. This is due to the fact that the service rate for RG is very low and NH is not subject to high levels of demand. Through our investigations other examples occur that evidence this: the main idea that seems to pervade is that running a system at full capacity is not necessariy optimal.

If it is of interest the Sagemath code (making use of the models distributed with the previously made available code) to obtain the above table is:

```
capacity_list = [2, 3]
serviceratelist = [0.22, 1.66]
demandratelist = [3, 1]
model = Model2(capacity_list, serviceratelist, demandratelist)
model.sweepcutoffs()
tps = {model.data[pair].throughput1 + model.data[pair].throughput2: pair for pair in model.data}
for tp in sorted(tps.keys()):
    print round(tp,6), tps[tp]
```

A brief discussion of this has been added to the paper which now reads:

... This optimal throughput T^* is independent of t.

Note that intuitively it could be thought that T^* is obtained when $K_{\mathrm{NH}} = c_{\mathrm{NH}}$ and $K_{\mathrm{RG}} = c_{\mathrm{RG}}$ however this is not always the case (numerical experiments have been carried out to verify this).

> *The authors make extensive use of bullets in the text. While bullets can be useful to draw attention to key points, they can lead to lazy writing. In my view, converting some of the bullet lists would improve the clarity of the exposition.*

- Remove bullet points.