Review of:
Measuring the Price of Anarchy in Critical Care Unit Interactions
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Critical Care Units (CCUs) in hospitals provide specialised care to patients who are critically ill. These units are typically quite small compared to other hospital wards and are expensive to operate, because of their high staffing levels and specialised equipment. Therefore, hospitals are often only able to accommodate fluctuations in CCU demand by diverting patients to CCUs in other hospitals.

This paper examines the "Price of Anarchy" associated with decentralised decision-making in the administration of the CCUs. The CCUs of two hospitals are examined. The administrator of each CCU attempts to achieve a prescribed bed occupancy target by adjusting their cutoff above which they start to divert patients to the other CCU. The price of anarchy is defined to be the ratio of CCU patient throughput with coordinated decision-making to patient throughput with decentralised decision-making.

The authors CCU flow model is a state-dependent Markov queueing model, which they assume to be at stochastic equilibrium. Their de-centralised decision model is defined as a pure strategy game theory problem. Combining queueing models with game theory is an important contribution to health care operations research. Hospital administrators and managers do not operate with perfect information and decisions are frequently made in response to pressure from other actors in the system. In my view, this area of research is both very interesting and very important.

The computations in this paper rigorous and generally will-explained. Although the Theorem proven is not directly used in their analysis, it none-the-less provides useful insight into the problem.

The weakest part of the paper is Section 4, the Conclusion. Although the authors did this work with the Aneurin Bevan University Health Board, and used data from two hospitals, there is no discussion of feedback from stakeholders on their results. Has their paper resulted in any consideration on the part of stakeholders or policy makers about whether bed occupancy is the most reasonable target for CCUs?

A limitation of their work, which is not discussed in Section 4, is that they do not consider different levels of acuity for patients. I believe that stratifying CCU inflow by acuity levels would be an important extension to their model. This relates to their comment in the fourth bullet that patient length of stay could be given by the service rate of the original CCU. In reality, length of stay will be most influenced by acuity. The difference in mean length of stay between the two hospitals that they study is most probably related to a difference in average patient acuity between the two CCUs.

Another limitation of their work is that not all CCUs are equal. For example, only some will have a neurosurgeon available. This limits which transfers are medically recommended. These considerations would come into play more, if a network of CCUs were considered. The authors allude to this in their second bullet point.

I recommend that the authors write a more complete and coherent Discussion and/or Conclusion which places their work in the context of the critcal care system. It should relate model limitations to the critical care system. The current way that limitations are discussed is too technical. A brief discussion of stakeholder feedback would also be useful.

The authors should be commended making their software available on the internet.

Specific Comments

1.

At the beginning of Section 3.1, the authors claim that if neither CCU is able to admit patients, then admission to the CCU is cancelled and the patient is admitted to a general ward. In my experience modelling CCUs, it is highly unlikely that a critical patient would ever be admitted to a general ward. CCU beds are almost always equipped with ventilators and ward beds never. I believe that more likely courses of action are:

- Sending the patient to another hospital's CCU. This would be a CCU outside of the model.

- "Bumping" a patient currently in the CCU to a ward bed, to free up a bed. This would normally be a patient who was nearly ready to be transferred to the ward anyway.

- Accommodating the patient in the hospital's post-anesthesia care unit (PACU), which sometimes functions as an overflow for the CCU.

I recommend that the authors consult with their stakeholders and correct this comment in their paper, if necessary. This wouldn't have any impact on their model, but it is important to use the correct language.

2.

The axis labels on many of the graphs are in a very small font. The font size should be increased.

3.

The optimisation problem is shown as a figure (Figure 6). I suggest that this be separated from the text with a header such as "Optimisation Problem" (much like a theorem). Calling it a figure seems like a misnomer, and for me, actually made it harder to find.

4.

In Figure 7, the caption should state which points are f_{NH} and which are F_{RG}.

5.

I was puzzled as to whether the target, t, appeared in the definition of T^*. As I understand it, T^* is independent of t. Is this true? Also, for which values of K_{NH} and K_{RG} is the maximum achieved? Is my intuition correct that it is achieved at $K_{NH} = c_{NH}$ and $K_{RG} = c_{RG}$?

6.

The authors make extensive use of bullets in the text. While bullets can be useful to draw attention to key points, they can lead to lazy writing. In my view, converting some of the bullet lists would improve the clarity of the exposition.