Responses to Reviews for Measuring the Price of Anarchy in Critical Care Unit Interactions

Vincent Knight*   Izabela Komenda   Jeff Griffiths

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Below we respond to each reviewer in detail.

0.1 Reviewer 1

> *Write the clarifications of notations $\lambda_h^{(l,h)}, \lambda_h^{(l,l)}$. One $h \in (NH, RG)$ other h denotes RG experiences high demand.*

Thank you to the reviewer for noticing this detail. We have changed all occurrences of h denoting an element of NH, RG to be H. This clarifies the paper.

> *Give explanation for figure 3.*

Another pointer in the text to the Figure has been added ("The transition rates are given by (2) and are illustrated diagrammatically in Figure 3"). This Figure is meant to illustrate diagrammatically the state space and transitions.

> *Write clearly major contribution of the paper corresponding to the existing literature.*

> *State clearly the objectives of the model.*

The following paragraph is in the introduction of the paper: if the reviewer does not find it sufficient we would gladly clarify it further. At present we feel that it very clearly places the contribution of the paper (as highlighted by the 3rd reviewer) and also highlights what the main finding/objective of the model is.

" The work presented in this paper contributes to the growing body of literature by applying state dependent queueing models in a game theoretical context to CCU interaction. In particular this consideration allows for the investigation of targets imposed by central control [4]. The findings of this work justify and identify a choice of targets that align the interests of the individual hospitals with social welfare. "

0.2 Reviewer 2

> *On page 5, the model is referred to as a Figure 6. This confuses and leaves a question on whether it is a figure or equations. I am unsure whether figure 6 is missing or authors are referring to the model on page 6.*

This has now been fixed: the optimisation problem that defines the game is now in a section with title: 'Optimisation problem' and is referred to by a labelling equation number. This was suggested by another reviewer and we feel improves the paper.

The paper is unreadable. The unknowns used, some, are undefined, for example on page 3, h, ch, etc are not defined and it is very difficult to find definitions of those defined. I suggest that the authors provide a list of the parameters used somewhere in the paper where it is easy to refer.

We were disappointed by this comment. To suggest that the paper is unreadable is an exaggeration. Perhaps the reviewer did not have time to read page 2 which contained a table that clearly listed the variables they mention?

If we have misunderstood and that table is not sufficient we would gladly list the variables with their definitions in a format more pleasing to the referee. Although we note that the other two reviewers, one of which has clearly read the paper carefully did not find the paper unreadable.

On page 12, authors states that "It is also noted that as demand increases the effect of uncoordinated behaviour increases (and the recommended target also increases) as shown in Figure 13", is there any possible explanation to this finding.

At present it is merely an observation from the emergent behaviour of the model however a sentence has been added:

" It is also noted that as demand increases the effect of uncoordinated behaviour increases (and the recommended target also increases) as shown in Figure 12. This is potentially due to the fact that as demand increases there is the scope for larger discrepancy between optimal and sub optimal behaviours. "

Are the Queuing and Game theoretic models never been implemented to solve such problems? There is a need to review literature that has information on application of the used models to hospital problems or related problems.

The authors need to give a justification on the reason why they are implementing the models.

It seems that the reviewer did indeed not read page 2 of the manuscript. As highlighted by the 3rd reviewer a literature review is present. This literature review highlights that there is not much work at the intersection of game theory, queueing theory and healthcare.

Furthermore the contributions and goals of the model are very clearly stated on page 2 and again highlighted in the conclusion section. If the reviewer could clarify there expectations if this is not sufficient we would gladly highlight the contributions of the paper (as recognized by the referees that have read it) further.

" The work presented in this paper contributes to the growing body of literature by applying state dependent queueing models in a game theoretical context to CCU interaction. In particular this consideration allows for the investigation of targets imposed by central control [4]. The findings of this work justify and identify a choice of targets that align the interests of the individual hospitals with social welfare. "

0.3 Reviewer 3

I recommend that the authors write a more complete and coherent Discussion and/or Conclusion which places their work in the context of the critcal care system. It should relate model limitations to the critical care system. The current way that limitations are discussed is too technical. A brief discussion of stakeholder feedback would also be useful.

This has been addressed, each technical bullet point is now complemented by a sentence putting the limitation in the context of critical care. Furthermore a discussion has been added that describes the feedback and further emphasises the contribution of the work:

" Despite these mathematical limitations the work presented here gives a strong analytical evidence as to the use of policies in a decentralised healthcare environment.

As discussed above, reducing the decision making of critical care managers to rational reactions to capacity targets is not without limitations. However, this quantitative model of behaviour was described as insightful and informative ABUHB. In practice, stakeholders describe a rule of 80% capacity. Whilst this is not only at times impossible it is also not evidence based and in particular does not take in to account interactions between CCUs. This is a common theme in practice and the literature which this manuscript aims to address. ”

> *At the beginning of Section 3.1, the authors claim that if neither CCU is able to admit patients, then admission to the CCU is cancelled and the patient is admitted to a general ward. In my experience modelling CCUs, it is highly unlikely that a critical patient would ever be admitted to a general ward. CCU beds are almost always equipped with ventilators and ward beds never. I believe that more likely courses of action are:*
>
> - *Sending the patient to another hospital's CCU. This would be a CCU outside of the model.*
> - *"Bumping" a patient currently in the CCU to a ward bed, to free up a bed. This would normally be a patient who was nearly ready to be transferred to the ward anyway.*
> - *Accommodating the patient in the hospital's post-anesthesia care unit (PACU), which sometimes functions as an overflow for the CCU.*
>
> *I recommend that the authors consult with their stakeholders and correct this comment in their paper, if necessary. This wouldn't have any impact on their model, but it is important to use the correct language.*

This is a very good point, after discussion with our stakeholders we confirm all the suggestions made by the reviewer and have modified the text:

“ This model assumes that if the bed occupancy level at both Units exceeds a predetermined threshold, then the admission to the CCU is cancelled. This cancellation could correspond, to sending the patient to a completely different CCU (outside of the model), moving one of the current patients (ready to be dismieed) in the CCU to another ward and/or using the post-anethesisa care unit as a temporary overflow measure. This models corresponds to the first of those possibilities: the patient is lost (from the point of view of this model). ”

- List all of above as options but use first as most related to model.

> *The axis labels on many of the graphs are in a very small font. The font size should be increased.*

All axis labels have now got a bigger font. We appreciate this comment: if any other aspects of the graphics could be improved we would be happy to make the suggested changes.

> *The optimisation problem is shown as a figure (Figure 6). I suggest that this be separated from the text with a header such as "Optimisation Problem" (much like a theorem). Calling it a figure seems like a misnomer, and for me, actually made it harder to find.*

This has been changed:

- The figure has been changed to a section with a header as suggested.
- A label (3) has been added for easier reference throughout the text.

> *In Figure 7, the caption should state which points are f_{NH} and which are F_{RG}.*

This is a good point, we have clarified this by noting ‘crosses’ and ‘diamonds’ in plain text in the caption.

> *I was puzzled as to whether the target, t, appeared in the definition of T^*. As I understand it, T^* is independent of t. Is this true? Also, for which values of K_{NH} and K_{RG} is the maximum achieved? Is my intuition correct that it is achieved at $K_{NH} = c_{NH}$ and $K_{RG} = c_{RG}$?*

This is an excellent question. The reviewer understand correctly: T^* is not dependent of t.

Identifying the point at which throughput is maximised is however not immediate. When we began working on this paper we had the same intuition as the reviewer (that throughput would always be maximised when using the highest possible threshold), however this is not the case.

Indeed, for example for the second model, when using a system with parameters:

- $c_{\text{RG}} = 2, c_{\text{NH}} = 3$
- $\mu_{\text{RG}} = 0.22, \mu_{\text{NH}} = 1.66$
- $\lambda_{\text{RG}} = 3, \mu_{\text{NH}} = 1$

The throughputs for each cutoff pair are shown in the table below:

Throughput	Cut off pairs
1.704098	(2, 0)
1.843005	(1, 0)
1.857689	(0, 0)
3.27514	(2, 1)
3.472371	(1, 1)
3.51033	(0, 1)
4.291777	(2, 2)
4.488501	(1, 2)
4.549065	(0, 2)
4.767654	(2, 3)
4.877357	(1, 3)
4.916791	(0, 3)
4.928235	(2, 4)
4.961882	(1, 4)
4.969698	(0, 4)

We see that the optimal throughput is obtained when RG is in effect 'closed' and NH is never in a diverted state. This is due to the fact that the service rate for RG is very low and NH is not subject to high levels of demand. Through our investigations other examples occur that evidence this: the main idea that seems to pervade is that running a system at full capacity is not necessariy optimal.

If it is of interest the Sagemath code (making use of the models distributed with the previously made available code) to obtain the above table is:

```
capacity_list = [2, 3]
serviceratelist = [0.22, 1.66]
demandratelist = [3, 1]
model = Model2(capacity_list, serviceratelist, demandratelist)
model.sweepcutoffs()
tps = {model.data[pair].throughput1 + model.data[pair].throughput2: pair for pair in model.data}
for tp in sorted(tps.keys()):
    print round(tp,6), tps[tp]
```

A brief discussion of this has been added to the paper which now reads:

" ... This optimal throughput T^* is independent of t.

Note that intuitively it could be thought that T^* is obtained when $K_{\text{NH}} = c_{\text{NH}}$ and $K_{\text{RG}} = c_{\text{RG}}$ however this is not always the case (numerical experiments have been carried out to verify this). "

The authors make extensive use of bullets in the text. While bullets can be useful to draw attention to key points, they can lead to lazy writing. In my view, converting some of the bullet lists would improve the clarity of the exposition.

3 of the 5 bullet point lists have been changed to be paragraphs. We felt that this left a nice balance but if the reviewer feels strongly about this, we would be happy to change the remaining bullet point lists.