Markov Chains Sage Sheet

Last updated: October 14, 2012.

1. Discrete Markov Chains
   Navigate to the Sage code snippet at: `http://interact.sagemath.org/node/39` and perhaps start a new Sage worksheet so that you can try a few of your own Sage commands.

   WARNING: The convention used in this code snippet is different to the notes. The columns sums add to 1 not the row sums. Keep this in mind as you work through it.

   (a) Using the default inputs, what is the steady state distribution associated with this Markov chain (try and use the Sage "solve" command to verify this)?

   (b) How long does it seem to take to arrive at that state (try and use Sage to verify this)?

2. Continuous Markov Chains
   The evolution of a Continuous Markov Chain obeys the following differential equation:

$$\frac{d\pi(t)}{dt} = \pi(t)Q$$

   which has solution:

$$\pi(t) = \pi(0)e^{Qt}$$

   This requires the calculation of the exponential of a Matrix. This is not straightforward! One approach is to use the following formula:

$$e^M = \mathbb{I} + \sum_{k=1}^{\infty} \frac{M^k}{k!}$$

   Navigate to the Sage code snippet at `http://interact.sagemath.org/node/71` and experiment with the code to see how good this approach is.

   (a) Consider the following Markov Chain:

$$Q = \begin{pmatrix} -3 & 3 & 0 & 0 \\ 4 & -7 & 3 & 0 \\ 0 & 4 & -7 & 3 \\ 0 & 0 & 4 & -4 \end{pmatrix}$$

   (b) Use the Sage "solve" command to obtain the steady state probabilities for this Markov Chain.