Playing Games: A Case Study in Active Learning Applied to Game Theory

Vince Knight

June 29, 2015

Abstract

A paper about active learning and using some example of this in a class on Game Theory

1 Introduction

Modern pedagogic theories as to how learning takes place such as constructivism and socialism [10, 11], indicate that an **active learning** approach is of benefit to student learning. As stated in [17] there are a variety of complementary definitions of active learning, however the general definition given in [17] is the one assumed in this paper:

> "Active learning is generally defined as any instructional method that engages students in the learning process. In short active learning requires students to do meaningful learning activities and think about what they are doing."

One could argue that all learning is active as students simply listening to a lecture are perhaps taking part in a 'meaningful learning activity', however as stated in [5] active learning is understood to imply that students:

- read, write, discuss, or engage in solving problems;
- engage in higher order tasks such as analysis, synthesis and evaluation.

A variety of studies have highlighted the effectiveness of active learning [8, 9, 17]. These two papers are in fact meta studies evaluating the effectiveness an active student centred approach. Note that the definition used in [8] corresponds to simply any pedagogic approach in which students are not passive consumers of a lecture during the class meeting. Some examples of active learning in a variety of subjects include:

- The flipped learning environment in a Physics class: [4].
- Inquiry based learning for the instruction of differential equations: [12].
- Using collaborative learning in a pharmacology class: [7].

The above sources (and references therein) generally discuss the pedagogic approach from a macroscopic point of view with regards to the course considered. This manuscript will give a detailed description of two particular active learning activities used in the instruction of Game Theoretic concepts:

- Section 2.1 will describe an in class activity and software package used to introduce students to the topic of best response dynamic [13].
- Section 2.2 will describe an implementation of Axelrod's tournament [2, 3].

These activities aim to introduce the student to the concepts and aspire to their curiosity as to the underlying mathematics. Note that if there is any doubt as to the effectiveness of active learning approaches, for example this paper (the only one that this author could identify) [1] identifies no such relationship are still beneficial to the students' learning. Indeed in [16] the greatest predictors of academic performance are identified not as general intelligence [19] but personality factors such as conscientiousness and openness.

2 An exemplar: a course in game theory

Game Theory as a topic is well suited to approaches that use activities involving students as players to introduce the concepts, rules and strategies for particular games and/or theorems presented.

In [6] one such activity is presented: a game that allow players to grasp the concept of common knowledge of rationality. Another good example is [15]: Yale's Professor Polak's course, the videos available at that reference (a YouTube playlist) all show that students are introduced to every concept through activity before discussing theory.

Just as the activity presented in [6] the activities presented here are both suited for as an early introduction to the concepts (although the activity of Section 2.2 is potentially better suited to being used at a later stage). Furthermore, these activities have also been used as outreach activities for high school students with no knowledge of further mathematics.

2.1 Best response dynamics

The first step in this activity and potentially before any prior description of Game Theory students are invited to answer the following simple question:

What is a game?

Through discussion the class will usually arrive at the following consensus:

- A game must have a certain number $N \geq 1$ of players;
- Each player must have available to them a certain number of strategies that define what they can do;
- Once all players have chosen their strategy, rules must specify what the outcome is.

This corresponds to the general definition of a strategic form game [13]. The main goal of this activity is to not only understand the vocabulary but also the important concept of response dynamics which aims to identify what is the best option given prior knowledge of all other players [13]. One particular game that can be analysed using base response dynamics is often referred to:

The two thirds of the average game.

A good description of the game and the human dynamics associated to the play is given in [14]. The use of this game in teaching is not novel in game theory [18] The rules are as follows:

- All players choose a number between 0 and 100;
- The player whose choice was closest to $\frac{2}{3}$ of the average of the choices wins.

To make use of this game in class as an introduction to the concept of best response dynamics students are handed a sheet of paper inviting them to write down a first play. After this initial play, a discussion is had that demonstrates that the equilibrium for this game is for all players to guess 0. This is shown diagrammatically in Figure 1.

Following this discussion students are invited to play again and write down their second guess. All of the results are collected, the author has used paper forms but an automated approach could also be used. In general the input and analysis of the data takes less than 10 minutes and can be done by a helper during another class activity. Following this, the result shown in Figure 3a are shown and discussed with the class.

The author has used this activity on a large number of occasions and at all times collected the data. Figure 3a shows the distribution of the guesses (depending on the round of play):

We see that the second round (after the rationalisation of play described in Figure 1) has guesses that are closer to the expected equilibrium behaviour. Figure 3b confirms this showing the linear relationship (albeit a weak one with $R^2 = .2$):

$$(\text{Second guess}) = .203 \times (\text{First guess}) + 9.45 \tag{1}$$

The fact that the coefficient of the relationship is less than one highlights that the second guess is in general lower than the first guess. As can be seen in Figure 2 not all students reduce their guess. Figure

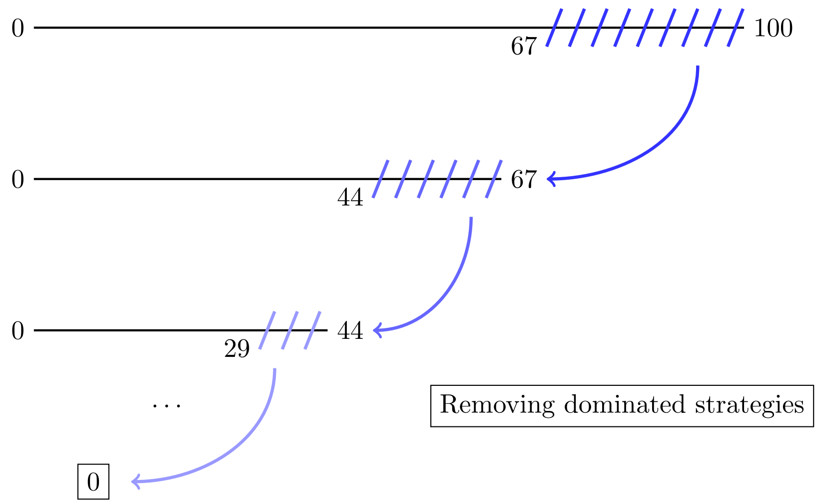
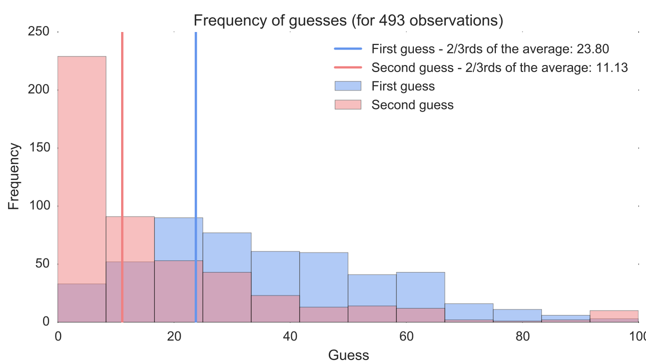
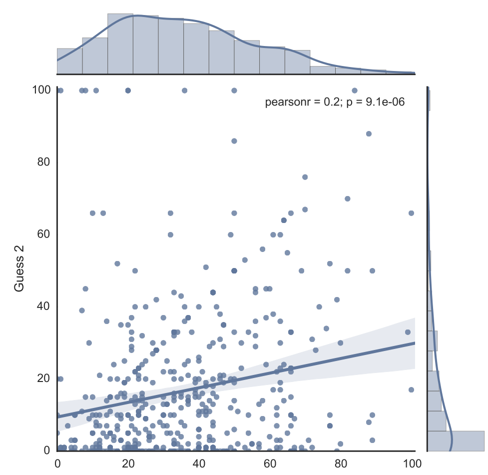


Figure 1: Equilibrium behaviour in the two thirds of the average game



(a) Frequency of guesses depending on the round of play.

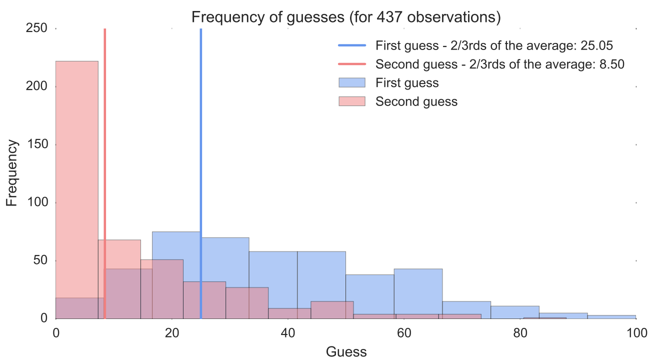


(b) Linear relationship between guesses of each round of play.

Figure 2: Results from all data collected.

3 shows the results when removing these irrational moves. In this particular case the linear relationship is in fact stronger $R^2 = .43$:

$$\text{(Second guess)} = .33 \times \text{(First guess)} + 0.20 \tag{2}$$



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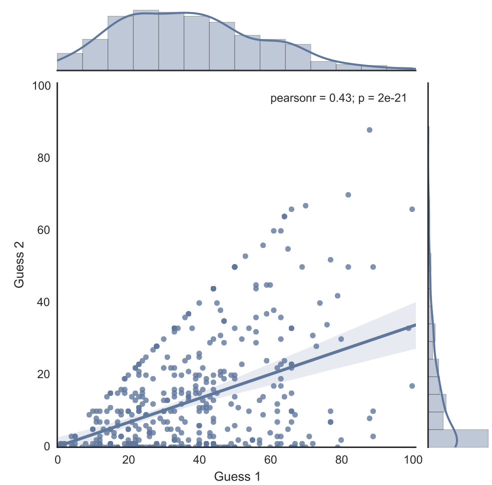

(b) Linear relationship between guesses of each round of play.

Figure 3: Results from data when removing increasing guesses.

2.2 Repeated and random games

- The theory
- Tournaments:
  - Basic type.
  - Infinitely repeated game.
  - Markov games.

At the end of the activity, students are shown the graphical results and a discussion had about why the theoretic equilibrium was not the winner. This discussion usually revolves around the observation that not everyone acted rationally and second that some participants felt like they should 'spoil' the game by guessing larger in the second round.

Finally, if time permits (and depending on the level of the participants), the linear relationship of (1) is used to discuss what would happen if more rounds were to be played. In particular it is possible to discuss ideas of convergence when generalising (1) to be:

$$\text{Guess}_{n+1} = .203 \times \text{Guess}_n + 9.45 \tag{3}$$

To summarise this activity:

1. Participants are explained the rules and play one round of the two thirds of the average game.
2. A rationalisation and explanation of equilibrium behaviour is described.
3. Participants play another round.
4. Results are analysed and discussed.

3 Summary

- Give some examples of feedback.
- Mention how methods could be applied to other courses.
- Certain class management ideas (mainly that I will not speak first a lot of the time) ¡- Not sure if this is useful.

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