

# Playing Games: A Case Study in Active Learning Applied to Game Theory

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## Abstract

A paper about active learning and using some example of this in a class on Game Theory

## 1 Introduction

Modern pedagogic theories as to how learning takes place such as constructivism and socialism [9, 10], indicate that an **active learning** approach is of benefit to student learning. As stated in [16] there are a variety of complementary definitions of active learning, however the general definition given in [16] is the one assumed in this paper:

“Active learning is generally defined as any instructional method that engages students in the learning process. In short active learning requires students to do meaningful learning activities and think about what they are doing.”

One could argue that all learning is active as students simply listening to a lecture are perhaps taking part in a ‘meaningful learning activity’, however as stated in [4] active learning is understood to imply that students:

- read, write, discuss, or engage in solving problems;
- engage in higher order tasks such as analysis, synthesis and evaluation.

A variety of studies have highlighted the effectiveness of active learning [7, 8, 16]. These two papers are in fact meta studies evaluating the effectiveness an active student centred approach. Note that the definition used in [7] corresponds to simply any pedagogic approach in which students are not passive consumers of a lecture during the class meeting. Some examples of active learning in a variety of subjects include:

- The flipped learning environment in a Physics class: [3].
- Inquiry based learning for the instruction of differential equations: [11].
- Using collaborative learning in a pharmacology class: [6].

The above sources (and references therein) generally discuss the pedagogic approach from a macroscopic point of view with regards to the course considered. This manuscript will give a detailed description of two particular active learning activities used in the instruction of Game Theoretic concepts:

- Section 2.1 will describe an in class activity and software package used to introduce students to the topic of best response dynamic [12].
- Section 2.2 will describe an implementation of Axelrod’s tournament [2, 2].

These activities aim to introduce the student to the concepts and aspire to their curiosity as to the underlying mathematics. Note that if there is any doubt as to the effectiveness of active learning approaches, for example this paper (the only one that this author could identify) [1] identifies no such relationship are still beneficial to the students’ learning. Indeed in [15] the greatest predictors of academic performance are identified not as general intelligence [18] but personality factors such as conscientiousness and openness.

## 2 An exemplar: a course in game theory

Game Theory as a topic is well suited to approaches that use activities involving students as players to introduce the concepts, rules and strategies for particular games and/or theorems presented.

In [5] one such activity is presented: a game that allow players to grasp the concept of common knowledge of rationality. Another good example is [14]: Yale's Professor Polak's course, the videos available at that reference (a YouTube playlist) all show that students are introduced to every concept through activity before discussing theory.

Just as the activity presented in [5] the activities presented here are both suited for as an early introduction to the concepts (although the activity of Section 2.2 is potentially better suited to being used at a later stage). Furthermore, these activities have also been used as outreach activities for high school students with no knowledge of further mathematics.

### 2.1 Best response dynamics

The first step in this activity and potentially before any prior description of Game Theory students are invited to answer the following simple question:

#### **What is a game?**

Through discussion the class will usually arrive at the following consensus:

- A game must have a certain number  $N \geq 1$  of players;
- Each player must have available to them a certain number of strategies that define what they can do;
- Once all players have chosen their strategy, rules must specify what the outcome is.

This corresponds to the general definition of a strategic form game [12]. The main goal of this activity is to not only understand the vocabulary but also the important concept of response dynamics which aims to identify what is the best option given prior knowledge of all other players [12]. One particular game that can be analysed using best response dynamics is often referred to:

#### **The two thirds of the average game.**

A good description of the game and the human dynamics associated to the play is given in [13]. The use of this game in teaching is not novel in game theory [17] The rules are as follows:

- All players choose a number between 0 and 100;
- The player whose choice was closest to  $\frac{2}{3}$  of the average of the choices wins.

To make use of this game in class as an introduction to the concept of best response dynamics students are handed a sheet of paper inviting them to write down a first play. After this initial play, a discussion is had that demonstrates that the equilibrium for this game is for all players to guess 0. This is shown diagrammatically in Figure 1. Following this discussion students are invited to play again and write down their second guess.

### 2.2 Repeated and random games

- The theory
- Tournaments:
  - Basic type.
  - Infinitely repeated game.
  - Markov games.

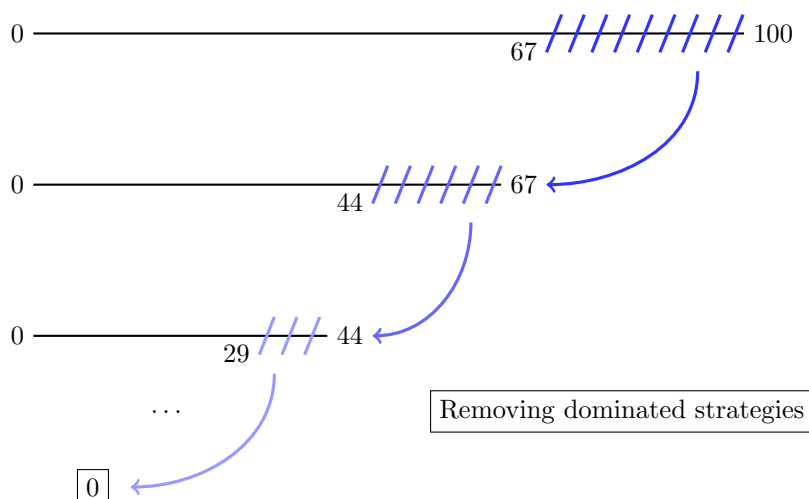


Figure 1: Equilibrium behaviour in the two thirds of the average game

### 3 Summary

- Give some examples of feedback.
- Mention how methods could be applied to other courses.
- Certain class management ideas (mainly that I will not speak first a lot of the time) :- Not sure if this is useful.

### References

- [1] T. M. Andrews et al. "Active learning not associated with student learning in a random sample of college biology courses". In: *CBE Life Sciences Education* 10.4 (2011), pp. 394–405. ISSN: 19317913. DOI: 10.1187/cbe.11-07-0061.
- [2] R. Axelrod. "More Effective Choice in the Prisoner's Dilemma". In: *Journal of Conflict Resolution* 24.3 (1980), pp. 379–403. ISSN: 0022-0027. DOI: 10.1177/002200278002400301.
- [3] P. S. Bates and R. Galloway. "The inverted classroom in a large enrolment introductory physics course : a case study ." In: ().
- [4] C. C. Bonwell and J. a. Eison. *Active Learning: Creative Excitement in the Classroom*. 191 ASHE-ERIC Higher Education Reports. 1991, p. 121. ISBN: 1878380087.
- [5] A. J. Brokaw and T. E. Merz. "Active Learning with Monty Hall in a Game Theory Class". In: *The Journal of Economic Education* 35.3 (2004), pp. 259–268. ISSN: 0022-0485. DOI: 10.3200/JECE.35.3.259-268.
- [6] I. Depaz. "Using Peer Teaching to Support Co-operative Learning in Undergraduate Pharmacology". In: *Bioscience Education e-Journal* 11.June (2008). ISSN: 14797860. DOI: 10.3108/beej.11.8.
- [7] S. Freeman et al. "Active learning increases student performance in science, engineering, and mathematics." In: *Proceedings of the National Academy of Sciences of the United States of America* 111.23 (2014), pp. 8410–5. ISSN: 1091-6490. DOI: 10.1073/pnas.1319030111. URL: <http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=4060654%5C&tool=pmcentrez%5C&rendertype=abstract>.

- [8] R. R. Hake. “Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses”. In: *American Journal of Physics* 66.1 (1998), p. 64. ISSN: 00029505. DOI: 10.1119/1.18809.
- [9] K. Illeris. *Contemporary theories of learning : learning theorists – in their own words*. 2009, p. 244. ISBN: 9780415473439. DOI: 10.1080/0158037X.2011.577173.
- [10] a. Jordan, O. Carlile, and a. Stack. *Approaches To Learning: A Guide For Teachers: A Guide for Educators*. 2008, p. 278. ISBN: 0335226701, 9780335226702. URL: <http://books.google.com/kw/books?id=C82nud-9W6MC>.
- [11] O. N. Kwon, K. Allen, and C. Rasmussen. “Students’ Retention of Mathematical Knowledge and Skills in Differential Equations”. In: *School Science and Mathematics* 105.5 (2005), pp. 227–240. ISSN: 00366803. DOI: 10.1111/j.1949-8594.2005.tb18163.x. URL: <http://www.questia.com/PM.qst?a=o%5C&se=gglsc%5C&d=5009565207>.
- [12] M. Maschler, E. Solan, and S. Zamir. *Game theory*. Cambridge University Press, 2013, p. 1003. ISBN: 9781107005488. DOI: <http://dx.doi.org/10.1017/CB09780511794216>. URL: <http://www.cambridge.org/gb/academic/subjects/economics/economics-general-interest/game-theory>.
- [13] R. Nagel. *Unraveling in guessing games: An experimental study*. 1995. DOI: <http://www.aeaweb.org/aer/>. URL: <http://www.jstor.org/stable/2950991>.
- [14] B. Polak. *Game Theory with Ben Polak*. 2008. URL: <https://www.youtube.com/watch?v=nM3rTU927io%5C&list=PL6EF60E1027E1A10B> (visited on 06/21/2015).
- [15] A. E. Poropat. “Other-rated personality and academic performance: Evidence and implications”. In: *Learning and Individual Differences* 34 (2014), pp. 24–32. ISSN: 17447682. DOI: 10.1016/j.lindif.2014.05.013. URL: <http://dx.doi.org/10.1016/j.lindif.2014.05.013>.
- [16] M. Prince. “Does Active Learning Work ? A Review of the Research”. In: *Journal of Engineering Education* 93.July (2004), pp. 223–231. ISSN: 1069-4730. DOI: 10.1002/j.2168-9830.2004.tb00809.x.
- [17] The Economics Network. *The Handbook for Economics Lecturers*. 2013. URL: <http://www.economicsnetwork.ac.uk/handbook/experiments/3> (visited on 06/21/2015).
- [18] W. R. Wright. *General Intelligence, Objectively Determined and Measured*. 1905. DOI: 10.1037/h0065005.