Playing Games: A Case Study in Active Learning Applied to Game Theory

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Abstract

A paper about active learning and using some example of this in a class on Game Theory

1 Introduction

Modern pedagogic theories as to how learning takes place such as constructivism and socialism [11, 12], indicate that an **active learning** approach is of benefit to student learning. As stated in [18] there are a variety of complementary definitions of active learning, however the general definition given in [18] is the one assumed in this paper:

> "Active learning is generally defined as any instructional method that engages students in the learning process. In short active learning requires students to do meaningful learning activities and think about what they are doing."

One could argue that all learning is active as students simply listening to a lecture are perhaps taking part in a 'meaningful learning activity', however as stated in [5] active learning is understood to imply that students:

- read, write, discuss, or engage in solving problems;
- engage in higher order tasks such as analysis, synthesis and evaluation.

A variety of studies have highlighted the effectiveness of active learning [9, 10, 18]. These two papers are in fact meta studies evaluating the effectiveness an active student centred approach. Note that the definition used in [9] corresponds to simply any pedagogic approach in which students are not passive consumers of a lecture during the class meeting. Some examples of active learning in a variety of subjects include:

- The flipped learning environment in a Physics class: [4].
- Inquiry based learning for the instruction of differential equations: [13].
- Using collaborative learning in a pharmacology class: [7].

The above sources (and references therein) generally discuss the pedagogic approach from a macroscopic point of view with regards to the course considered. This manuscript will give a detailed description of two particular active learning activities used in the instruction of Game Theoretic concepts:

- Section 2.1 will describe an in class activity and software package used to introduce students to the topic of best response dynamic [14].
- Section 2.2 will describe an implementation of Axelrod's tournament [2, 3].

These activities aim to introduce the student to the concepts and aspire to their curiosity as to the underlying mathematics. Note that if there is any doubt as to the effectiveness of active learning approaches, for example this paper (the only one that this author could identify) [1] identifies no such relationship are still beneficial to the students' learning. Indeed in [17] the greatest predictors of academic performance are identified not as general intelligence [22] but personality factors such as conscientiousness and openness.

2 An exemplar: a course in game theory

Game Theory as a topic is well suited to approaches that use activities involving students as players to introduce the concepts, rules and strategies for particular games and/or theorems presented.

In [6] one such activity is presented: a game that allow players to grasp the concept of common knowledge of rationality. Another good example is [16]: Yale's Professor Polak's course, the videos available at that reference (a YouTube playlist) all show that students are introduced to every concept through activity before discussing theory.

Just as the activity presented in [6] the activities presented here are both suited for as an early introduction to the concepts (although the activity of Section 2.2 is potentially better suited to being used at a later stage). Furthermore, these activities have also been used as outreach activities for high school students with no knowledge of further mathematics.

2.1 Best response dynamics

The first step in this activity and potentially before any prior description of Game Theory students are invited to answer the following simple question:

What is a game?

Through discussion the class will usually arrive at the following consensus:

- A game must have a certain number $N \geq 1$ of players;
- Each player must have available to them a certain number of strategies that define what they can do;
- Once all players have chosen their strategy, rules must specify what the outcome is.

This corresponds to the general definition of a strategic form game [14]. The main goal of this activity is to not only understand the vocabulary but also the important concept of response dynamics which aims to identify what is the best option given prior knowledge of all other players [14]. One particular game that can be analysed using base response dynamics is often referred to:

The two thirds of the average game.

A good description of the game and the human dynamics associated to the play is given in [15]. The use of this game in teaching is not novel in game theory [21] The rules are as follows:

- All players choose a number between 0 and 100;
- The player whose choice was closest to $\frac{2}{3}$ of the average of the choices wins.

To make use of this game in class as an introduction to the concept of best response dynamics students are handed a sheet of paper inviting them to write down a first play. After this initial play, a discussion is had that demonstrates that the equilibrium for this game is for all players to guess 0. This is shown diagrammatically in Figure 1.

Following this discussion students are invited to play again and write down their second guess. All of the results are collected, the author has used paper forms but an automated approach could also be used. In general the input and analysis of the data takes less than 10 minutes and can be done by a helper during another class activity. Following this, the result shown in Figure 3a are shown and discussed with the class.

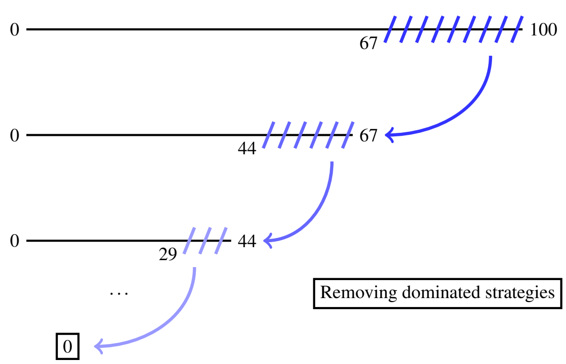
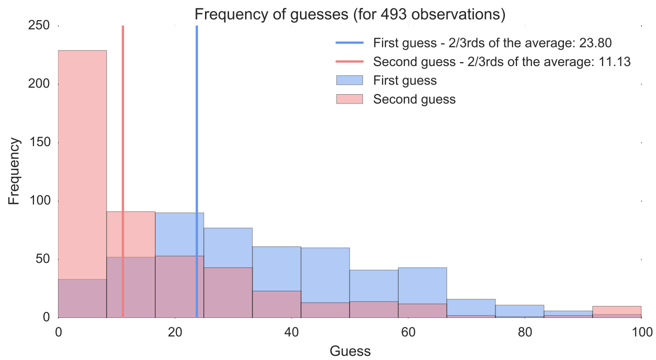


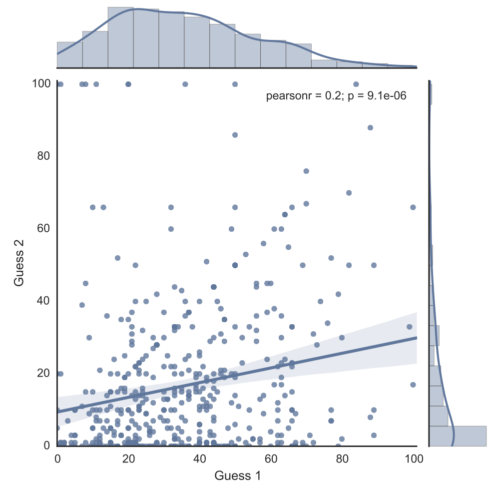
Figure 1: Equilibrium behaviour in the two thirds of the average game

The author has used this activity on a large number of occasions and at all times collected the data. Figure 3a shows the distribution of the guesses (depending on the round of play):

We see that the second round (after the rationalisation of play described in Figure 1) has guesses that are closer to the expected equilibrium behaviour. Figure 3b confirms this showing the linear relationship (albeit a weak one with $R^2 = .2$):



(a) Frequency of guesses depending on the round of play.



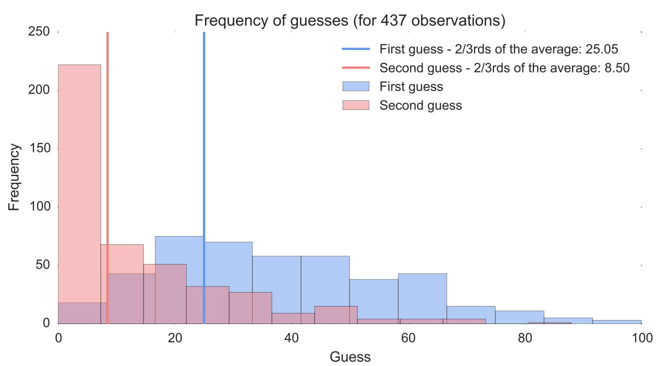
(b) Linear relationship between guesses of each round of play.

Figure 2: Results from all data collected.

$$(\text{Second guess}) = .203 \times (\text{First guess}) + 9.45 \tag{1}$$

The fact that the coefficient of the relationship is less than one highlights that the second guess is in general lower than the first guess. As can be seen in Figure 2 not all students reduce their guess. Figure 3 shows the results when removing these irrational moves. In this particular case the linear relationship is in fact stronger $R^2 = .43$:

$$(\text{Second guess}) = .33 \times (\text{First guess}) + 0.20 \tag{2}$$



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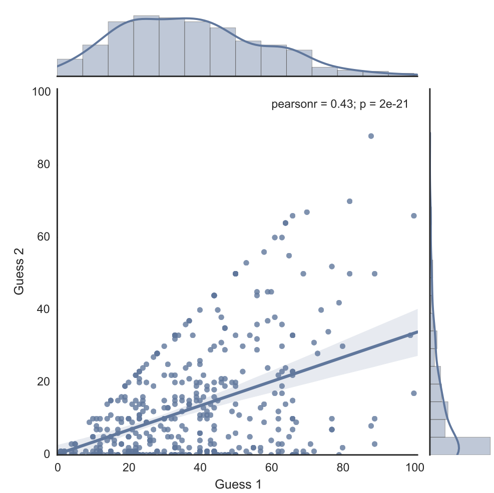

(b) Linear relationship between guesses of each round of play.

Figure 3: Results from data when removing increasing guesses.

At the end of the activity, students are shown the graphical results and a discussion had about why the theoretic equilibrium was not the winner. This discussion usually revolves around the observation that not everyone acted rationally and second that some participants felt like they should 'spoil' the game by guessing larger in the second round.

Finally, if time permits (and depending on the level of the participants), the linear relationship of (1) is used to discuss what would happen if more rounds were to be played. In particular it is possible to discuss ideas of convergence when generalising (1) to be:

$$\text{Guess}_{n+1} = .203 \times \text{Guess}_n + 9.45 \tag{3}$$

To summarise this activity:

1. Participants are explained the rules and play one round of the two thirds of the average game.
2. A rationalisation and explanation of equilibrium behaviour is described.
3. Participants play another round.
4. Results are analysed and discussed.

This activity is still quite passive in terms of physical activity (participants are seated throughout). Nevertheless it allows the data used for the discussion of the theory to come directly from the participants and further more all students are active participants and there are no difficulties with regards to encouraging participation (references to these are discussed in [19]).

2.2 Repeated and random games

This activity is used to introduce students to the concepts of repeated games [14]. The mathematical details can be omitted from the initial description of the activity to the participants but for completeness it they are included here:

A repeated game is played over discrete time periods. Each time period is indexed by $0 < t \leq T$ where T is the total number of periods. In each period N players play a static game referred to as the **stage game** independently and simultaneously selecting actions. Players make decisions in full knowledge of the **history** of the game played so far (ie the actions chosen by each player in each previous time period). The payoff is defined as the sum of the utilities in each stage game for every time period.

One of the most renowned repeated games is referred to as **Axelrod's tournament** which is what is recreated in this activity [2, 3].

Initially a description of the prisoner's dilemma [14] is given. The prisoner's dilemma is a simple two player game that is often used to introduce the very basic notions of game theory. It is described by the following two matrices:

$$A = \begin{pmatrix} 3 & 5 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix}$$

The row player has utility given by A and the column player has utilities given by B. The strategies available to each player are to cooperate: C or to defect: D. Playing C corresponds to players choosing their first row/column and D, the second row/column.

Thus if both players cooperate they both receive a utility of 3, if one player defects, the defector gets a utility of 5 and the cooperator a utility of 0. Finally if both players defect they receive a utility of 1. As players (in this framework) aim to maximise their score, the Nash equilibrium for this game is for both players to defect.

After describing this activity and in particular explaining the simple mathematical idea of **dominated strategy** (which is what is used in the activity of Section 2.1) participants are made aware of the concept of Nash equilibrium. This in turn can lead to a brief description of the tragic yet brilliant life of John Nash.

At this point the activity is described:

1. All participants will form four groups/teams;
2. Teams will *duel* each other in repetitions of 5 to 8 rounds (depending on available time).
3. All teams will play in a round robin tournament with cumulative scores being recorded.
4. The victorious team will be the team with the highest total score.

The tournament is run with all participants present (even those not watching). All participants are invited to stand and confer in their teams. The importance of standing (as a physical activity) is noted in [8] (whilst that reference is mainly concerned with the impact of activity on physical well being it also describes advantages in terms of concentration). Before every round of every duel opposing teams are encouraged to discuss strategies, after which they do not face each other and following a prompt hold up a card indicating either C or D. Duels are recorded on the white board in a table similar to the one shown in Table 1, in which two strategies constantly cooperate (thus obtaining a utility of 3 in each round). Table 2 shows an example where a strategy that is alternating plays against a strategy that always defects.

Table 1: Playing Tit For Tat against Cooperator

3	6	9	12	15
3	6	9	12	15

Table 2: Playing Alternator against Defector

5	6	11	12	17
0	1	1	2	2

Figure 4 shows a photo of a final board for a particular implementation of this activity.

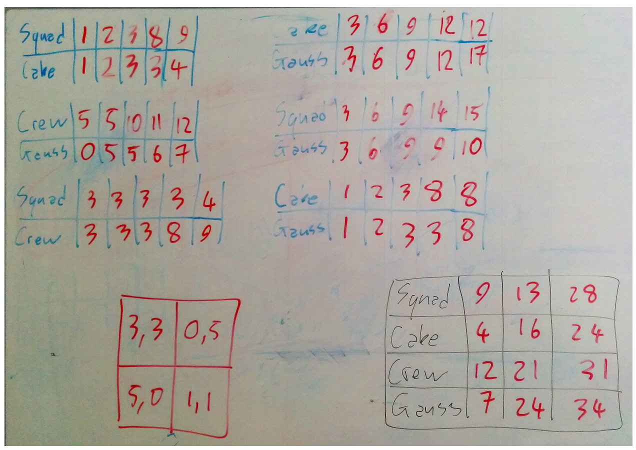

Figure 4: A photo of an actual implementation of the tournament.

The names of the strategies shown in Tables 1 and 2 are strategies that were used in the original tournaments run by Axelrod [2, 3]. The interesting fact of repeated games and one that becomes apparent to participants through the activity is that whilst repeating the stage Nash equilibrium (always defect) is indeed a Nash equilibrium for the repeated games, this equilibrium is not unique as reputation now has a part to play.

Note that if students do not realise this it is important to remind them that the goal is not to win each duel but to obtain a high score. Often during the tournament one team will (during the pre round discussion) exclaim:

> "We will cooperate until you defect, at which point we will defect throughout"

Without realising it the participants have described a well known strategy (**Grudger**) which takes in to account the entire history of play.

This activity can be complemented with a demonstration of software that allows for the rapid simulation of Axelrod's tournament [20]. Figure 5 shows the performance of the strategies when put in an evolutionary context.

One of the weaknesses of this approach is that all participants observe the play by all the teams. Whilst from a mathematical perspective reputation is inferred to mean the reputation gained during a particular duel, this has the effect of teams being able to observe how other teams seem to play. A true replication of Axelrod's tournament would not allow for this. One possibility would invite participants to leave the room which might be logistically constrained. From a pedagogic point of view however, having participants observe the duels often leads to a much more engaged discussion (after as well as during the activity).

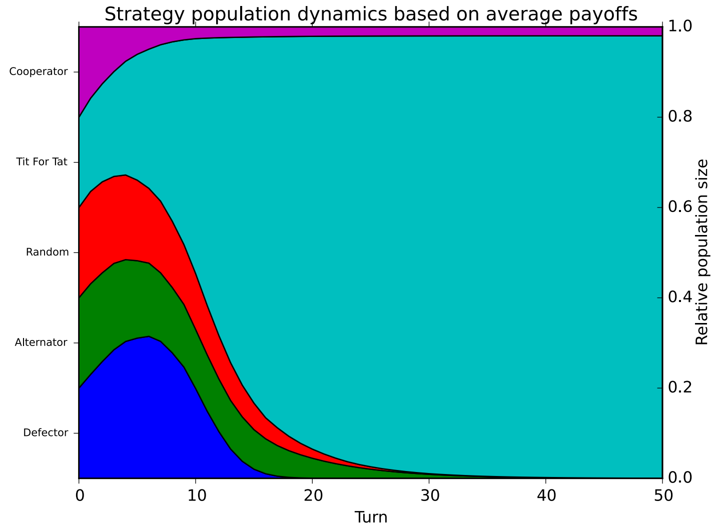

Figure 5: Repeated games in an evolutionary context.

This activity is usually very enjoyable and leads to a lively discussion. Further to the fun had by participants, the theoretic discussion about repeated games can be placed in the exact context of the tournament that was just played.

This tournament can be used to introduce further game theoretic topics with slight modifications:

- **Infinitely repeated games with discounting**: the discount factor can be interpreted as a probability of the duel continuing for another round (this can be randomly sampled).
- **Markov games**: the two random game states can be a true game and an absorbing game so that this corresponds to an infinite game with discounting.
- **Evolutionary games**: the two random game states can be a true game and an absorbing game so that this corresponds to an infinite game with discounting.

3 Summary

- Give some examples of feedback.
- Mention how methods could be applied to other courses.
- Certain class management ideas (mainly that I will not speak first a lot of the time) ¡- Not sure if this is useful.

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