# Week 6 - Algebra

Now that we have a good knowledge of programming we can take a closer look at a Mathematics package. There are a variety of such packages: [Maple](http://www.maplesoft.com/products/maple/), [Mathematica](http://www.wolfram.com/products/) and others. The mathematics package we will use is called [Sage](http://sagemath.org/), as stated on the Sage website ([sagemath.org](http://sagemath.org/)):

Sage is a free open-source mathematics software system licensed under the GPL. It combines the power of many existing open-source packages into a common Python-based interface.

Sage being an open-source program means that it is being developed by other mathematicians all over the world. It is also completely free (so you can download a copy on your own computer).

Sage allows us to solves equations, differentiate expressions, plot graphs and do various other mathematical operations.

**Sage is based on Python so you can use all the general programming techniques you have learnt up until now in Sage.** Sage has all of the commands and functions you are used to in Python although some might do a bit more. Sage also has a lot of built in objects for specific mathematical operations.

1. **TICKABLE** Open Sage

* Navigate to [sage.maths.cf.ac.uk](https://sage.maths.cf.ac.uk/) where you can setup a user account following these instructions:
  1. Click on 'Sign up for a new Sage Notebook account';
  2. **Use your cardiff university username as the username for Sage**;
  3. This will create an account, you will be taken to the login screen again, use the above login details to login.
* Take a look at the following video that goes through this process as well as shows you how to create your first Sage worksheet.
* [Video hint](http://www.youtube.com/watch?v=3E9LvXV_zrA&feature=youtu.be)

1. **TICKABLE** It is very easy to get help in Sage. Simply type any command followed by a ? to get a help file for it:

* cos?  
  sum?
* [Video hint](http://youtu.be/Fbq1GNiF85k)

1. We can use Sage as an advanced calculator. Try out the following commands:

* cos(3)  
  cos(3.)  
  sqrt(4)  
  sqrt(8)  
  sqrt(8.)
* Sage uses I to denote the imaginary constant:
* I \*\* 2  
  sqrt(-53.)

1. **TICKABLE** Obtain an exact form for . Once you have an expression, experiment with the factor, simplify and expand functions.

* [Video hint](http://youtu.be/kJwDWY6Wd7s)

1. Sage can also be used to obtain prime factors of numbers:

* factor(234398)
* We can do this as above using a function but also using the factor method on the object: 234398:
* 234398.factor()
* To obtain the number of prime factors that a number has we can use the len function or method:
* f = 234398.factor()  
  len(f)
* Obtain the mean number of factors for the first 1000 integers (you can use the mean sage function).
* [Video hint](http://youtu.be/5e0wAaO4I84)

1. Using the is\_prime method investigate the following claim:

* All numbers of the form are prime for and .

1. **TICKABLE** A very important aspect of Sage is it's ability to handle symbolic computation. In other words it is capable of simplifying:

* To do so we need to first declare symbolic variables:
* y = var('y')
* (We could also declare x but x is a default variable in Sage that is always declared). Once we have done that we can assign the above expression to a variable:
* myexp = x \*\* 2 - 5 \* x \*\* 2 + 12 \* x \* y - 9 \* y \*\* 2
* Note that here I'm using the exponentiation used in python: \*\* but in Sage we can also use ^:
* myexp = x ^ 2 - 5 \* x ^ 2 + 12 \* x \* y - 9 \* y ^ 2
* Once we have done that we can factorise the expression using the factor method:
* myexp.factor()
* [Video hint](http://youtu.be/_mmHrQ7FzaY)

1. We can also define functions in Sage. The following code defines :

* f(x) = x^3 + pi\*x^2 - 23/2\*x^2 - 23/2\*pi\*x + 15\*x+ 15\*pi
* To take a look at we can use the plot function:
* plot(f(x), x, -15, 15)
* Experiment with the arguments in that expression.
* To idenfity the 3 (visible on the plot) roots of our function we can use the roots method on :
* f.roots()
* We can also try to factorise :
* f.factor()
* Finally we can use the solve function:
* solve(f(x) == 0, x)
* [Video hint](http://youtu.be/UA7LVwmMmfk)

1. **TICKABLE** Using Sage obtain the solution to the following equations:
   1. $x^2 - 53 x + 2 a = 0 $
   2. (Investigate the Sage function find\_root)
   3. $x^5 + sin(x) - 2 \* x = .5 $

* [Video hint](http://youtu.be/GVEbkBXBmTw)

1. It is also possible to solve systems of equations using Sage. In this case we pass a list of equations as arguments to the solve function. The following code gives a solution to this system of equation:

* y, z = var('y', 'z')  
  solve([x + y == z, 3\*x - y == 0, y + z ==1], [x, y, z])