Suspicion: Recognising and evaluating the effectiveness of extortion in the Iterated Prisoner's Dilemma

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Abstract

The Iterated Prisoner's Dilemma is a model for rational and evolutionary interactive behaviour, having applications in biology, the study of human social behaviour, and many other domains. It is often used to understand when and how a rational individual might accept an immediate cost to their own utility for the direct benefit of another.

Much attention has been given to a class of strategies called Zero Determinant strategies. It has been theoretically shown that these strategies can "extort" large classes of other strategies.

In this work, we describe an approach to identify if observed strategies are playing in an extortionate manner. This corresponds to a mathematical model of suspicion. Furthermore, experimental analysis of a large tournament with 204 strategies is considered. In this setting the most highly performing strategies do not play in an extortionate manner against each other but do against lower performing strategies. This suggests that whilst the theory of Zero Determinant strategies suggests that memory is not of fundamental importance to the evolution of cooperative behaviour, this narrative is incomplete.

By considering evolutionary frameworks, we use this measure of extortion to obtain a statistically significant correlation between the variability of behaviour and evolutionary stability. This shows that complex strategies that are able to adapt their behaviour for each interaction are more likely to survive the evolutionary process.

1 Introduction

Agent-based game-theoretic models have become a stalwart of the underpinning mathematics of interactive behaviours. One of the major pieces of work in this area is the pair of computer tournaments run by Robert Axelrod [2, 3]. These tournaments pitted submitted computer strategies against each other in plays of the Iterated Prisoner's Dilemma, which is a common game where agents can choose to pay a slight cost to their immediate utility in the hope of building a reputation. This has been used in economic and evolutionary game theory to understand the evolution of cooperative behaviour.

In [22] a class of strategies that can provably extort given opponents was described. These so called Zero-Determinant (ZD) strategies caused a substantial reaction in the game theoretic community. As stated in [10], the American Mathematical Society's news section stated that 'the world of game theory is currently on fire'. In [1, 8, 9, 10, 11, 16] some questions have already been asked about the true effectiveness of these strategies in an evolutionary setting. For example [1] showed that ZD strategies were not evolutionarily stable. Furthermore, in that work it was also postulated that 'evolutionarily successful ZD strategies can be designed that use longer memory to distinguish self from non-self'.

The main question being asked in this work is: is it possible to recognise this extortionate ZD behaviour? A mathematical procedure for suspicion is presented, reflecting the way that the continued actions of an extortionate individual might raise suspicion.

ZD strategies are defined by a constrained set of transition probabilities $p \in \mathbb{R}^4$ (so called memory-one strategies). In this work, a procedure is offered that will quantify if any strategy is acting in an extortionate way. A $p \in \mathbb{R}^4$ can be measured from actual interactions, regardless of whether or not the strategy is defined in such a way. In Section 2 a measure κ is defined that measures how close a measured $p \in \mathbb{R}^4$ is to being ZD. This allows for the recognition of extortionate behaviour even in the cases of numeric inexactitude that would otherwise imply a strategy was not extortionate. It is also possible to recognise sophisticated strategies that act extortionately against some opponents but not others. This was the concept suggested in [1] when speaking about being able to distinguish self from non-self.

This work makes use of the Axelrod Python library [15, 17] which contains a large number of Prisoner Dilemma strategies available for extensive numerical examples. The approach is presented in Section 2. All of the code and data discussed in Section 3 is open sourced, archived and written according to best scientific principles [28]. The data archive can be found at [13]. In Section 4, this large tournament is complemented with an evolutionary consideration that offers some insight in to the effectiveness of extortionate strategies.

2 Recognising Extortion

Zero Determinant (ZD) strategies are introduced in [22] in the context of matches between two memory-one strategies. Memory-one strategies are represented as elements of $\mathbb{R}^4$ mapping a state of $\{C, D\}^2$ to a probability of cooperating. A match between two such strategies creates a Markov chain with transient states $\{C, D\}^2$. The main result of [22] is that given two memory-one players $p, q \in \mathbb{R}^4$, a linear relationship between the players' scores can be forced by one of the players.

Using the notation of [22], the utilities for player p are given by $S_x = (R, S, T, P)$ and for player q by $S_y = (R, T, S, P)$ and the stationary scores of each player are given by S_X and S_Y respectively. The main result of [22] is that if

$$\tilde{p} = \alpha S_x + \beta S_y + \gamma \tag{1}$$

or

$$\tilde{q} = \alpha S_x + \beta S_y + \gamma \tag{2}$$

where $\tilde{p} = (1 - p_1, 1 - p_2, p_3, p_4)$ and $\tilde{q} = (1 - q_1, 1 - q_2, q_3, q_4)$ then:

$$\alpha S_X + \beta S_Y + \gamma = 0 \tag{3}$$

In [22] a particular type of ZD strategy is defined: extortionate strategies. If:

$$\gamma = -P(\alpha + \beta) \tag{4}$$

then the player can ensure they get a score χ times larger than the opponent. This extortion coefficient is given by:

$$\chi = \frac{-\beta}{\alpha} \tag{5}$$

Thus, if (4) holds and $\chi > 1$ a player is said to extort their opponent. Here, the reverse problem is considered: given a $p \in \mathbb{R}^4$ how does one identify α, β if they exist and is the strategy in fact acting in an extortionate way?

In this case constraints (1) and (4) correspond to:

$$\tilde{p}_1 = \alpha R + \beta R - P(\alpha + \beta) \tag{6}$$
$$\tilde{p}_2 = \alpha S + \beta T - P(\alpha + \beta) \tag{7}$$
$$\tilde{p}_3 = \alpha T + \beta S - P(\alpha + \beta) \tag{8}$$
$$\tilde{p}_4 = \alpha P + \beta P - P(\alpha + \beta) \tag{9}$$

Equation (9) ensures that $p_4 = \tilde{p}_4 = 0$. Equations (6-8) can be used to eliminate α, β, giving:

$$\tilde{p}_1 = \frac{(R - P)(\tilde{p}_2 + \tilde{p}_3)}{S + T - 2P} \tag{10}$$

with:

$$\chi = \frac{\tilde{p}_2(P - T) + \tilde{p}_3(S - P)}{\tilde{p}_2(P - S) + \tilde{p}_3(T - P)} \tag{11}$$

Given a strategy $p \in \mathbb{R}^4$ equations (9-11) can be used to check if a strategy is extortionate. The conditions correspond to:

$$p_1 = \frac{(R - P)(p_2 + p_3) - R + T + S - P}{S + T - 2P} \tag{12}$$
$$p_4 = 0 \tag{13}$$
$$1 > p_2 + p_3 \tag{14}$$

The algebraic steps necessary to prove these results are available in the supporting materials, and note that an equivalent formulation was obtained in [1].

All extortionate strategies reside on a triangular (14) plane (12) in 3 dimensions (13). Using this formulation it can be seen that a necessary (but not sufficient) condition for an extortionate strategy is that it cooperates on average less than 50% of the time when in a state of disagreement with the opponent (14).

As an example, consider the known extortionate strategy $p = (8/9, 1/2, 1/3, 0)$ from [25] which is referred to as `Extort-2`. In this case, for the standard values of $(R, S, T, P) = (3, 0, 5, 1)$ constraint (12) corresponds to:

$$p_1 = \frac{2(p_2 + p_3) + 1}{3} = \frac{2(1/2 + 1/3) + 1}{3} = \frac{8}{9} \tag{15}$$

It is clear that in this case all constraints hold. As a counterexample, consider the strategy that cooperates 25% of the time: $p = (1/4, 1/4, 1/4, 1/4)$ obeys (14) but is not extortionate as:

$$p_1 \neq \frac{2(p_2 + p_3) + 1}{3} = \frac{2(1/4 + 1/4) + 1}{3} = \frac{2}{3} \tag{16}$$

Not all strategies are memory-one strategies but it is possible to measure a given p from any set of interactions between two strategies. This approach can then be used to confirm that a given strategy is acting in an extortionate manner even if it is not a memory-one strategy. However, in practice, if an exact form for p is not known but measured from observed plays of the game then measurement and/or numerical error might lead to an extortionate strategy not being confirmed as such.

As an example consider Table 1 which shows some actual plays of $p = (8/9, 1/2, 1/3, 0)$ against an alternating strategy ($p = (0, 0, 1, 1)$):

Turn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
(8/9, 1/2, 1/3, 0)	C	C	D	D	D	C	D	D	D	D	D	C	C	C	D	D	D	C	D	D
Alternator	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D

Table 1: A seeded play of 20 turns of two strategies.

In this particular instance the measured value of p for the known extortionate strategy would be: $(2/2, 1/5, 3/8, 4/4)$ which is does not fit the definition of a ZD strategy.

Note that measurement of behaviour might in some cases lead to missing values. For example the strategy $p = (8/9, 1/2, 1/3, 0)$ when playing against a cooperator will in fact never visit any state which would allow to measure p_3 and p_4. To overcome this, it is proposed that if s is a state that is not visited than p_s is approximated using a sensible prior. In Section 3 the overall cooperation rate is used. Another approach to overcoming this measurement error would be to measure our strategies in a minimalistically noisy environment.

Identifying if a strategy is ZD strategy can be written in the following linear algebraic form where $x = (\alpha, \beta)$ and $p^* = (\tilde{p}_1 - 1, \tilde{p}_2 - 1, \tilde{p}_3)$:

$$Cx = p^* \tag{17}$$

C corresponds to equations (6-8) and is given by:

$$C = \begin{bmatrix} R - P & R - P \\ S - P & T - P \\ T - P & S - P \end{bmatrix} \tag{18}$$

Note that in general, equation (17) will not necessarily have a solution. From the Rouché-Capelli theorem if there is a solution it is unique as $\text{rank}(C) = 2$ which is the dimension of the variable x. The best fitting x^* is defined by:

$$x^* = \text{argmin}_{x \in \mathbb{R}^2} \|Cx - p^*\|_2^2 \tag{19}$$

Known results [18, 24, 27] can now be applied:

$$x^* = \left(C^T C\right)^{-1} C^T p^* \tag{20}$$

This x^* corresponds to the nearest ZD strategy to the measured p. It is in fact a normal projection of p on to the plane defined by (12).

The squared norm of the remaining error is referred to as SS_{error}:

$$\text{SS}_{\text{error}} = \|Cx^* - p^*\|_2^2 \tag{21}$$

This gives expressions for α, β as $\alpha = x_1^*$ and $\beta = x_2^*$ thus the conditions for a strategy to be acting extortionately becomes:

$$-x_2^* < x_1^* \tag{22}$$

A further known result [18, 24, 27] gives an expression for SS_{error}:

$$\mathrm{SS}_{\mathrm{error}} = p^{*T} p^* - p^* C \left(C^T C\right)^{-1} C^T p^* = p^{*T} p^* - p^* C x^* \tag{23}$$

Using this approach, the memory-one representation $p \in \mathbb{R}^4$ of any strategy against any other can can be measured and if (22) holds then (23) can be used to identify if a strategy is acting extortionately.

For a measured p, $\mathrm{SS}_{\mathrm{error}}$ corresponds to the best fitting α, β, which omits the value of p_4 which is 0 if the strategy is extortionate (13). Suspicion of extortion then corresponds to a threshold on the following value κ:

$$\kappa = \mathrm{SS}_{\mathrm{error}} + p_4^2 \tag{24}$$

Comparing theoretic and actual plays of the IPD is not novel, see for example [23].

In the next section, this idea will be illustrated by observing the interactions that take place in a large computer based tournament of the IPD.

3 Numerical experiments

In [25] results from a tournament with 19 strategies, were presented with specific consideration given to ZD strategies. This tournament is reproduced here using the Axelrod-Python library [15]. To obtain a good measure of the corresponding transition rates for each strategy all matches have been run for 2000 turns and every match has been repeated 60 times. All of this interaction data is available at [13]. Note that in the interest of open scientific practice, [13] also contains interaction data for noisy and probabilistic ending interactions which are not investigated here.

Figure 1 shows the κ values for all the strategies in the tournament, as reported in [25] the extortionate strategy (which has an expected κ approximately 0) gains a large number of wins.

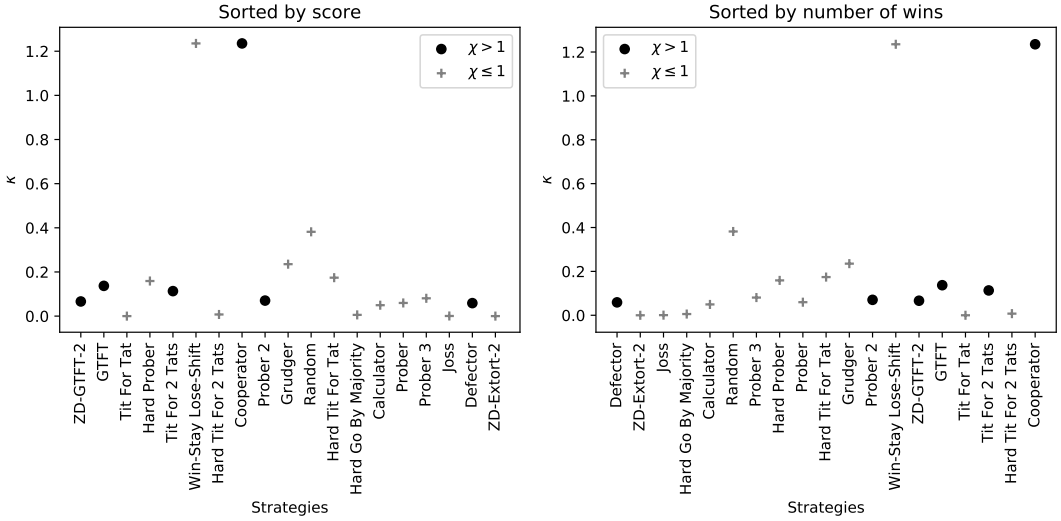

Figure 1: κ and state probabilities for the strategies of [25], ordered both by number of wins and overall score.

Here, the work of [25] is extended by investigating a tournament with 204 strategies. The results of this analysis are shown in Figure 2. The top ranking strategies by number of wins act in an extortionate way (but not against all strategies) and it can be seen that a small subgroup of strategies achieve mutual defection. All the top ranking strategies according to score achieve mutual cooperation and do not extort each other, however they **do** exhibit extortionate behaviour towards a number of the lower ranking strategies.

A detailed look at selected strategies is given in Table 2. The high scoring strategies presented have a large variation in κ whilst the ZD strategies have a low score but high probability of winning. This evidences an idea proposed in [1]: sophisticated strategies are able to recognise their opponent and defend themselves against extortion. The high ranking strategies were in fact trained to maximise score [7] which seems to have created strategies able to extort weaker strategies whilst cooperating with stronger ones. Indeed unconditional extortion is self defeating.

4 Evolutionary dynamics

From the large number of interactions a payoff matrix S can be measured where S_{ij} denotes the score (using standard values of $(R, S, T, P) = (3, 0, 5, 1)$) of the ith strategy against the jth strategy. Using this, the replicator equation describes the evolution of the system based on a population density fitness function:

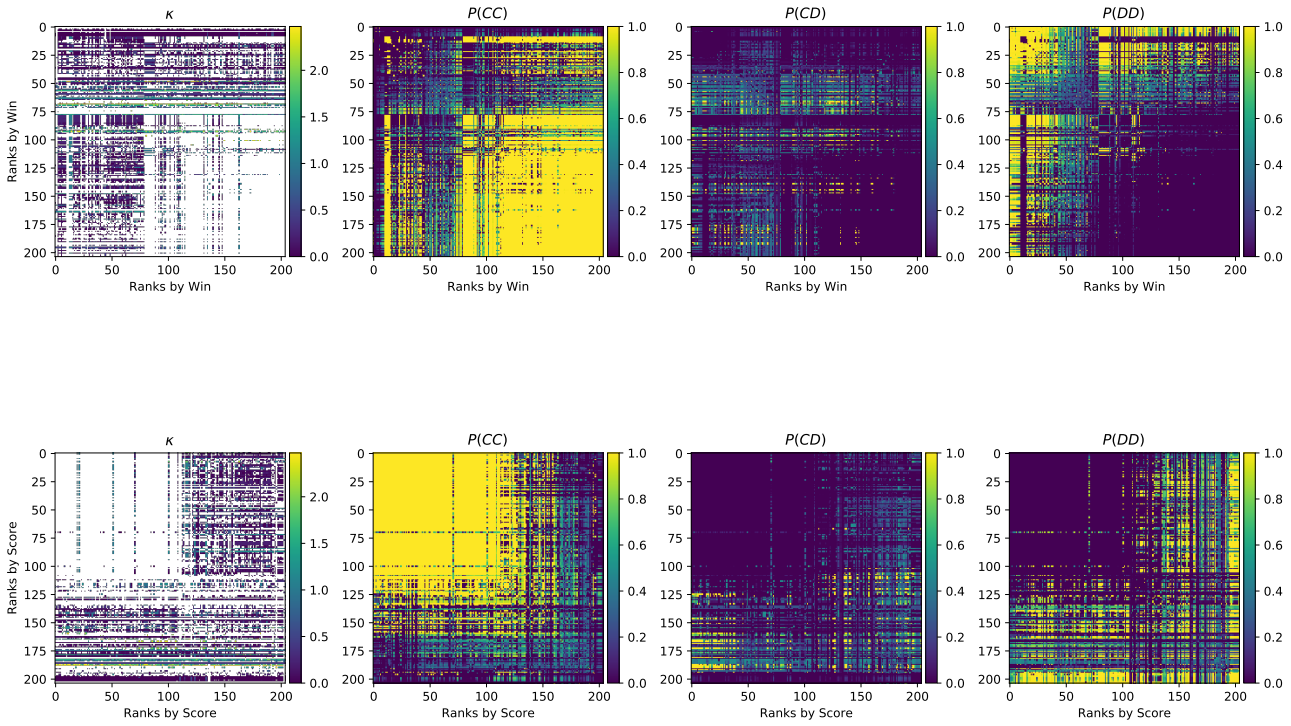

Figure 2: κ for the strategies for the full tournament. For the κ plot only strategy interactions for which $\chi > 1$ are displayed. Note that $P(DC)$ is not shown as it corresponds to the transpose of $P(CD)$.

Rank	Name	Score per turn	P(Win)	$P(CC)$	Overall κ	Min κ	5% CI κ	Median κ	Mean κ	Std κ	95% CI κ	Max κ
1	EvolvedLookerUp2_2_2	2.944	0.230	0.673	0.230	0.003	0.235	1.235	1.057	0.400	1.235	2.468
2	Evolved HMM 5	2.944	0.205	0.718	0.102	0.030	0.078	1.235	0.796	0.542	1.235	1.235
3	PSO Gambler 2_2_2	2.913	0.204	0.624	0.200	0.001	0.084	1.235	0.899	0.505	1.235	1.915
4	PSO Gambler Mem1	2.908	0.211	0.715	0.068	0.030	0.065	1.235	0.705	0.577	1.235	1.235
5	PSO Gambler 1_1_1	2.906	0.221	0.696	0.138	0.021	0.055	1.235	0.737	0.544	1.235	1.235
7	Evolved ANN 5	2.893	0.225	0.682	0.001	0.000	0.001	1.235	0.804	0.578	1.235	1.235
31	ZD-GTFT-2	2.721	0.000	0.806	0.066	0.020	0.064	1.235	0.786	0.537	1.235	1.235
45	ZD-GEN-2	2.689	0.016	0.801	0.016	0.003	0.015	1.235	0.694	0.599	1.235	1.235
47	Eatherley	2.682	0.000	0.828	0.084	0.000	0.007	1.235	0.895	0.474	1.235	1.235
69	Tit For Tat	2.638	0.000	0.723	0.000	0.000	0.000	1.235	0.773	0.549	1.235	1.235
75	Grumpy	2.630	0.075	0.793	0.000	0.000	0.000	1.235	0.978	0.495	1.235	1.235
88	Win-Stay Lose-Shift	2.616	0.099	0.649	1.235	0.059	1.000	1.235	1.172	0.164	1.235	1.235
103	Eventual Cycle Hunter	2.565	0.067	0.770	0.000	0.000	0.001	1.235	0.728	0.597	1.235	1.235
107	Tricky Level Punisher	2.537	0.062	0.828	0.000	0.000	0.000	1.235	0.710	0.595	1.235	1.235
127	Adaptive	2.272	0.500	0.363	0.000	0.000	0.000	0.063	0.084	0.098	0.274	0.529
169	Bully	1.970	0.381	0.141	1.529	0.059	0.200	1.529	1.373	0.375	1.529	2.469
179	Alternator	1.945	0.392	0.157	1.529	0.382	0.779	1.529	1.332	0.347	1.529	1.941
181	Negation	1.941	0.356	0.141	1.529	0.059	1.130	1.529	1.470	0.288	1.529	2.470
183	Cycler DC	1.931	0.324	0.149	1.529	0.382	0.382	1.529	1.279	0.374	1.529	1.941
188	Hopeless	1.908	0.352	0.261	2.471	1.191	1.235	2.471	2.247	0.372	2.471	2.471
194	Gradual Killer	1.892	0.354	0.439	0.000	0.000	0.000	0.173	0.254	0.326	1.004	1.004
196	Aggravater	1.879	0.930	0.087	0.235	0.000	0.059	0.059	0.163	0.256	1.023	1.101
200	ZD-Extort-2	1.821	0.851	0.179	0.000	0.000	0.000	0.000	0.019	0.094	0.010	0.583
201	ZD-Extort-4	1.820	0.865	0.106	0.000	0.000	0.000	0.000	0.021	0.069	0.204	0.407
202	ZD-Extort3	1.810	0.862	0.133	0.000	0.000	0.000	0.000	0.015	0.070	0.017	0.417
203	Defector	1.808	0.929	0.000	0.059	0.059	0.059	0.059	0.059	0.000	0.059	0.059
204	Handshake	1.806	0.870	0.046	0.000	0.000	0.000	0.059	0.126	0.288	1.200	1.720

Table 2: Summary of results for a selected list of strategies. The overall κ is computed by considering all transitions against all opponents.

$$\frac{dx_i}{dt} = x_i(S_i - x^T S x) \tag{25}$$

Equation (25) is solved numerically through an integration technique described in [21] and Figure 3 shows the evolution of the distribution of the system: the various strategies are ranked by scores. It is clear to see that only the high ranking strategies survive the evolutionary process (in fact, only 25 have a final long run probability value greater than 10^{-2}).

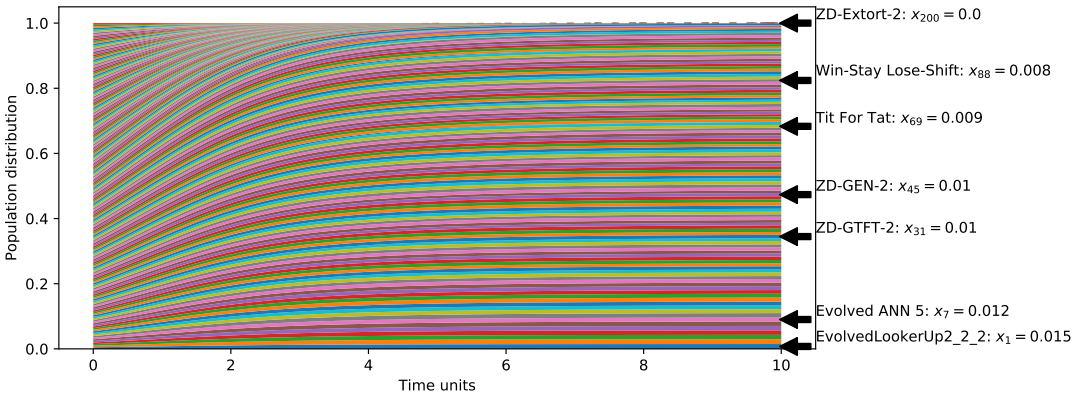

Figure 3: Numerical simulation of the replicator equation (25): strategies are ordered by score. Some selected strategies are highlighted with their long run population distribution.

In Figure 4 linear models are fitted to three summary measures of κ against the long run probabilities s of (25). In all cases the predictive power of these models is small (a low R^2) however, we note a significant positive correlation between the standard deviation of κ and s. Indeed: strategies that perform strongly according to equation (25) seem to be strategies that are able to modify their memory one representation depending on the opponent.

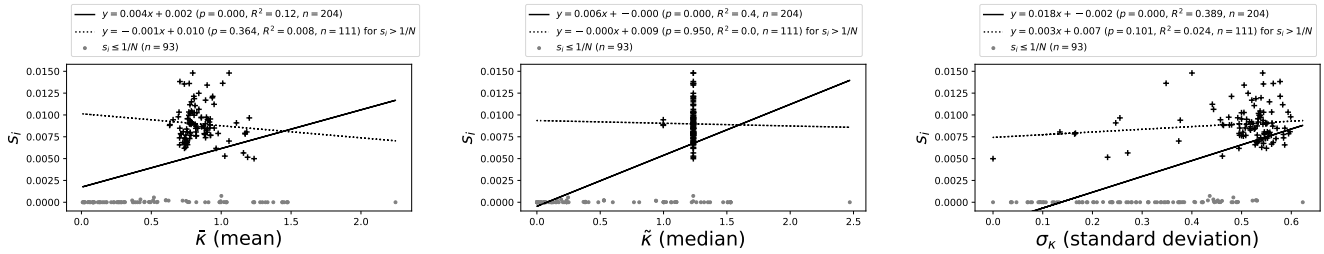

Figure 4: Linear regression analysis of long run probabilities of (25) against the mean, median and standard deviation of κ (for a given strategy).

In [16] a large data set of pairwise fixation probabilities in the Moran process is made available at [14] Figure 5 shows linear models fitted to three summary measures of κ and the mean (over population size N and opponents) value of $x_1 \cdot N$. This specific measure of fixation is chosen as x_1 is usually compared to the neutral fixation probability of $1/N$. As was noted in [16], the specific case of $N = 2$ differs from all other population sizes which is why it is presented in isolation. Similarly to the conclusions from Figure 4 we note that there is a significant relationship between the variability of κ and the ability for a strategy to become fixed.

Note that in both Figures 4 and 5, there is a large proportion of strategies with $\tilde{\kappa} \approx 1.23$. This corresponds to $p = (1, 1, 1, 1)$ (a cooperator) and indicates cooperative behaviour.

These findings confirm the work of [16] in which sophisticated strategies resist evolutionary invasion of shorter memory strategies. This also confirms the work of [1, 10] which proved that ZD strategies where not evolutionarily stable due to the fact that they score poorly against themselves.

The work also provides strong evidence to the importance of adaptability: strategies that offer a variety of behaviours corresponding to a higher standard deviation of κ are significantly more likely to survive the evolutionary process. This corresponds to the following quote of [4]:

> "It is not the most intellectual of the species that survives; it is not the strongest that survives; but the species that survives is the one that is able to adapt to and to adjust best to the changing environment in which it finds itself."

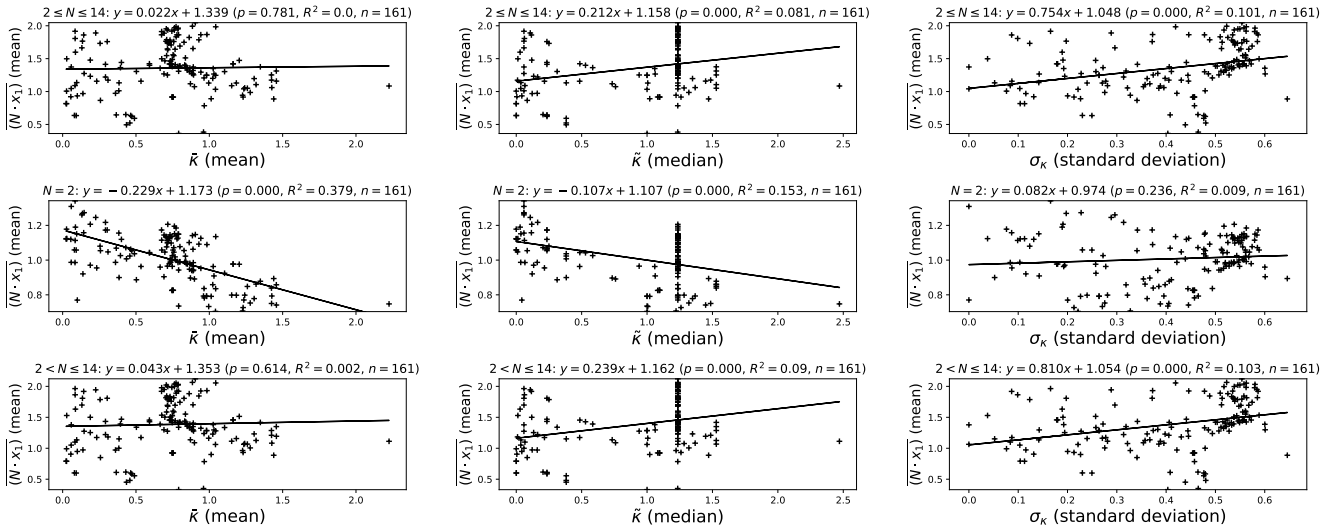

Figure 5: Linear regression analysis of pairwise fixation probabilities from [16] against the mean, median and standard deviation of κ (for a given strategy).

5 Conclusion

This work defines an approach to measure whether or not a player is playing a strategy that corresponds to an extortionate strategy as defined in [22]: a mathematical model for suspicion. All extortionate strategies have been classified as lying on a triangular plane. This rigorous classification fails to be robust to small measurement error, thus a statistical approach is proposed approximating the solution of a linear system. Using this, a large number of pairwise interactions is simulated.

The work of [22], whilst showing that a clever approach to taking advantage of another memory-one strategy exists: this is not the full story. Though the elegance of this result is very attractive, just as the simplicity of the victory of Tit For Tat in Axelrod's original tournaments was, it is incomplete. Extortionate strategies achieve a high number of wins but they do not achieve a high score and fail to be evolutionarily stable.

Instead, it is in fact the more sophisticated strategies that are able to adapt to their opponent and act extortionately against weaker strategies and cooperate with like minded strategies that perform well.

Following Axelrod's seminal work [2, 3], it was commonly thought that evolutionary cooperation required strategies that followed a simple set of rules. The discovery/definition of extortionate strategies [22] seemingly showed that complex strategies could be taken advantage of. In this manuscript it has been shown that not only is it possible to detect and prevent extortionate behaviour but that more complex strategies can be evolutionary stable. The complex strategies in question were obtained through reinforcement learning approaches [7, 16]. This work demonstrates the possibility for the evolution of cooperation through suspicion.

Acknowledgements

The following open source software libraries were used in this research:

- The Axelrod [15, 17] library (IPD strategies and tournaments).
- The sympy library [19] (verification of all symbolic calculations).
- The matplotlib [6] library (visualisation).
- The pandas [26], dask [5] and NumPy [20] libraries (data manipulation).
- The SciPy [12] library (numerical integration of the replicator equation).

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Proof of algebraic condition for extortionate strategies

The defining equations for an extortionate strategy are:

$$\tilde{p}_1 = \alpha(R-P) + \beta(R-P) \tag{1}$$
$$\tilde{p}_2 = \alpha(S-P) + \beta(T-P) \tag{2}$$
$$\tilde{p}_3 = \alpha(T-P) + \beta(S-P) \tag{3}$$
$$\tilde{p}_4 = 0 \tag{4}$$

Using equation (2), α is isolated

$$\alpha = \frac{-\beta(P-T) - \tilde{p}_2}{P-S} \tag{5}$$

Substituting this value in to equation (3), β is isolated:

$$\beta = -\frac{P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1}{(S-T)(2P-S-T)} \tag{6}$$

Substituting this back in to (5) gives:

$$\alpha = \frac{-\tilde{p}_2 + (P-T)(P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1)}{(S-T)(2P-S-T)(P-S)} \tag{7}$$

Substituting equations (6-7) in to equation (1) gives the required expression for p_1.
Taking the ratio of equations (6-7) gives the required expression for χ.
Finally, the condition $\chi > 1$ corresponds to:

$$\tilde{p}_2(P-T) + \tilde{p}_3(S-P) > \tilde{p}_2(P-S) + \tilde{p}_3(T-P) \tag{8}$$

which can be simplified to:

$$\tilde{p}_2 < -\tilde{p}_3 \tag{9}$$

recalling that $\tilde{p}_2 = p_2 - 1$ and $\tilde{p}_3 = p_3$ gives the required result.

List of all strategies used from [21]

1. Adaptive - Deterministic - Memory length: ∞ - [25]
2. Adaptive Tit For Tat: 0.5 - Deterministic - Memory length: ∞ - [39]
3. Aggravater - Deterministic - Memory length: ∞ - [21]
4. Alexei: (D,) - Deterministic - Memory length: ∞ - [43]
5. ALLCorALLD - Stochastic - Memory length: 1 - [2]
6. Alternator - Deterministic - Memory length: 1 - [31, 11]
7. Alternator Hunter - Deterministic - Memory length: ∞ - [21]
8. AntiCycler - Deterministic - Memory length: ∞ - [21]
9. Anti Tit For Tat - Deterministic - Memory length: 1 - [18]
10. AON2 - Deterministic - Memory length: 2 - [19]
11. Adaptive Pavlov 2006 - Deterministic - Memory length: ∞ - [24]
12. Adaptive Pavlov 2011 - Deterministic - Memory length: ∞ - [25]
13. Appeaser - Deterministic - Memory length: ∞ - [21]
14. Arrogant QLearner - Stochastic - Memory length: ∞ - [21]
15. Average Copier - Stochastic - Memory length: ∞ - [21]
16. BackStabber: (D, D) - Deterministic - Memory length: ∞ - [21]
17. Better and Better - Stochastic - Memory length: ∞ - [29]
18. Black - Stochastic - Memory length: 5 - [10]
19. Borufsen - Deterministic - Memory length: ∞ - [10]
20. Bully - Deterministic - Memory length: 1 - [32]
21. Bush Mosteller: 0.5, 0.5, 3.0, 0.5 - Stochastic - Memory length: ∞ - [20]
22. Calculator - Stochastic - Memory length: ∞ - [29]
23. Cautious QLearner - Stochastic - Memory length: ∞ - [21]
24. Cave - Stochastic - Memory length: ∞ - [10]
25. Champion - Stochastic - Memory length: ∞ - [10]
26. Colbert - Deterministic - Memory length: 4 - [10]
27. CollectiveStrategy - Deterministic - Memory length: ∞ - [26]
28. Contrite Tit For Tat - Deterministic - Memory length: 3 - [42]
29. Cooperator - Deterministic - Memory length: 0 - [31, 11, 34]
30. Cooperator Hunter - Deterministic - Memory length: ∞ - [21]
31. Cycle Hunter - Deterministic - Memory length: ∞ - [21]
32. Cycler CCCCCD - Deterministic - Memory length: 5 - [21]

33. Cycler CCCD - Deterministic - Memory length: 3 - [21]
34. Cycler CCD - Deterministic - Memory length: 2 - [31]
35. Cycler DC - Deterministic - Memory length: 1 - [21]
36. Cycler DDC - Deterministic - Memory length: 2 - [31]
37. Cycler CCCDCD - Deterministic - Memory length: 5 - [21]
38. Davis: 10 - Deterministic - Memory length: ∞ - [9]
39. Defector - Deterministic - Memory length: 0 - [31, 11, 34]
40. Defector Hunter - Deterministic - Memory length: ∞ - [21]
41. Desperate - Stochastic - Memory length: 1 - [41]
42. Delayed AON1 - Deterministic - Memory length: 2 - [19]
43. DoubleCrosser: (D, D) - Deterministic - Memory length: ∞ - [21]
44. Doubler - Deterministic - Memory length: ∞ - [29]
45. DoubleResurrection - Deterministic - Memory length: 5 - [15]
46. EasyGo - Deterministic - Memory length: ∞ - [25, 29]
47. Eatherley - Stochastic - Memory length: ∞ - [10]
48. EugineNier: (D,) - Deterministic - Memory length: ∞ - [43]
49. Eventual Cycle Hunter - Deterministic - Memory length: ∞ - [21]
50. Evolved ANN - Deterministic - Memory length: ∞ - [21]
51. Evolved ANN 5 - Deterministic - Memory length: ∞ - [21]
52. Evolved ANN 5 Noise 05 - Deterministic - Memory length: ∞ - [21]
53. Evolved FSM 4 - Deterministic - Memory length: 4 - [21]
54. Evolved FSM 16 - Deterministic - Memory length: 16 - [21]
55. Evolved FSM 16 Noise 05 - Deterministic - Memory length: 16 - [21]
56. EvolvedLookerUp1_1_1 - Deterministic - Memory length: ∞ - [21]
57. EvolvedLookerUp2_2_2 - Deterministic - Memory length: ∞ - [21]
58. Evolved HMM 5 - Stochastic - Memory length: 5 - [21]
59. Feld: 1.0, 0.5, 200 - Stochastic - Memory length: 200 - [9]
60. Firm But Fair - Stochastic - Memory length: 1 - [16]
61. Fool Me Forever - Deterministic - Memory length: ∞ - [21]
62. Fool Me Once - Deterministic - Memory length: ∞ - [21]
63. Forgetful Fool Me Once: 0.05 - Stochastic - Memory length: ∞ - [21]
64. Forgetful Grudger - Deterministic - Memory length: 10 - [21]
65. Forgiver - Deterministic - Memory length: ∞ - [21]
66. Forgiving Tit For Tat - Deterministic - Memory length: ∞ - [21]
67. Fortress3 - Deterministic - Memory length: 3 - [7]
68. Fortress4 - Deterministic - Memory length: 4 - [7]
69. GTFT: 0.33 - Stochastic - Memory length: 1 - [17, 33]

70. General Soft Grudger: n=1,d=4,c=2 - Deterministic - Memory length: ∞ - [21]
71. Getzler - Stochastic - Memory length: ∞ - [10]
72. Gladstein - Deterministic - Memory length: ∞ - [10]
73. Soft Go By Majority - Deterministic - Memory length: ∞ - [31, 11, 10]
74. Soft Go By Majority: 10 - Deterministic - Memory length: 10 - [21]
75. Soft Go By Majority: 20 - Deterministic - Memory length: 20 - [21]
76. Soft Go By Majority: 40 - Deterministic - Memory length: 40 - [21]
77. Soft Go By Majority: 5 - Deterministic - Memory length: 5 - [21]
78. ϕ - Deterministic - Memory length: ∞ - [21]
79. GraaskampKatzen - Deterministic - Memory length: ∞ - [10]
80. Gradual - Deterministic - Memory length: ∞ - [13]
81. Gradual Killer: (D, D, D, D, D, C, C) - Deterministic - Memory length: ∞ - [29]
82. Grofman - Stochastic - Memory length: ∞ - [9]
83. Grudger - Deterministic - Memory length: 1 - [12, 25, 13, 41, 9]
84. GrudgerAlternator - Deterministic - Memory length: ∞ - [29]
85. Grumpy: Nice, 10, -10 - Deterministic - Memory length: ∞ - [21]
86. Handshake - Deterministic - Memory length: ∞ - [35]
87. Hard Go By Majority - Deterministic - Memory length: ∞ - [31]
88. Hard Go By Majority: 10 - Deterministic - Memory length: 10 - [21]
89. Hard Go By Majority: 20 - Deterministic - Memory length: 20 - [21]
90. Hard Go By Majority: 40 - Deterministic - Memory length: 40 - [21]
91. Hard Go By Majority: 5 - Deterministic - Memory length: 5 - [21]
92. Hard Prober - Deterministic - Memory length: ∞ - [29]
93. Hard Tit For 2 Tats - Deterministic - Memory length: 3 - [38]
94. Hard Tit For Tat - Deterministic - Memory length: 3 - [40]
95. Harrington - Stochastic - Memory length: ∞ - [10]
96. Hesitant QLearner - Stochastic - Memory length: ∞ - [21]
97. Hopeless - Stochastic - Memory length: 1 - [41]
98. Inverse - Stochastic - Memory length: ∞ - [21]
99. Inverse Punisher - Deterministic - Memory length: ∞ - [21]
100. Joss: 0.9 - Stochastic - Memory length: 1 - [38, 9]
101. Kluepfel - Stochastic - Memory length: ∞ - [10]
102. Knowledgeable Worse and Worse - Stochastic - Memory length: ∞ - [21]
103. Level Punisher - Deterministic - Memory length: ∞ - [15]
104. Leyvraz - Stochastic - Memory length: 3 - [10]
105. Limited Retaliate: 0.1, 20 - Deterministic - Memory length: ∞ - [21]
106. Limited Retaliate 2: 0.08, 15 - Deterministic - Memory length: ∞ - [21]

107. Limited Retaliate 3: 0.05, 20 - Deterministic - Memory length: ∞ - [21]
108. Math Constant Hunter - Deterministic - Memory length: ∞ - [21]
109. Naive Prober: 0.1 - Stochastic - Memory length: 1 - [25]
110. MEM2 - Deterministic - Memory length: ∞ - [27]
111. Michaelos: (D,) - Stochastic - Memory length: ∞ - [43]
112. Mikkelson - Deterministic - Memory length: ∞ - [10]
113. MoreGrofman - Deterministic - Memory length: 8 - [10]
114. More Tideman and Chieruzzi - Deterministic - Memory length: ∞ - [10]
115. Negation - Stochastic - Memory length: 1 - [40]
116. Nice Average Copier - Stochastic - Memory length: ∞ - [21]
117. N Tit(s) For M Tat(s): 3, 2 - Deterministic - Memory length: 3 - [21]
118. Nydegger - Deterministic - Memory length: 3 - [9]
119. Omega TFT: 3, 8 - Deterministic - Memory length: ∞ - [37]
120. Once Bitten - Deterministic - Memory length: 12 - [21]
121. Opposite Grudger - Deterministic - Memory length: ∞ - [21]
122. π - Deterministic - Memory length: ∞ - [21]
123. Predator - Deterministic - Memory length: 9 - [7]
124. Prober - Deterministic - Memory length: ∞ - [25]
125. Prober 2 - Deterministic - Memory length: ∞ - [29]
126. Prober 3 - Deterministic - Memory length: ∞ - [29]
127. Prober 4 - Deterministic - Memory length: ∞ - [29]
128. Pun1 - Deterministic - Memory length: 2 - [6]
129. PSO Gambler 1_1_1 - Stochastic - Memory length: ∞ - [21]
130. PSO Gambler 2_2_2 - Stochastic - Memory length: ∞ - [21]
131. PSO Gambler 2_2_2 Noise 05 - Stochastic - Memory length: ∞ - [21]
132. PSO Gambler Mem1 - Stochastic - Memory length: 1 - [21]
133. Punisher - Deterministic - Memory length: ∞ - [21]
134. Raider - Deterministic - Memory length: 3 - [8]
135. Random: 0.5 - Stochastic - Memory length: 0 - [39, 9]
136. Random Hunter - Deterministic - Memory length: ∞ - [21]
137. Random Tit for Tat: 0.5 - Stochastic - Memory length: 1 - [21]
138. Remorseful Prober: 0.1 - Stochastic - Memory length: 2 - [25]
139. Resurrection - Deterministic - Memory length: 5 - [15]
140. Retaliate: 0.1 - Deterministic - Memory length: ∞ - [21]
141. Retaliate 2: 0.08 - Deterministic - Memory length: ∞ - [21]
142. Retaliate 3: 0.05 - Deterministic - Memory length: ∞ - [21]
143. Revised Downing: True - Deterministic - Memory length: ∞ - [9]

144. RichardHufford - Deterministic - Memory length: ∞ - [10]
145. Ripoff - Deterministic - Memory length: 2 - [5]
146. Risky QLearner - Stochastic - Memory length: ∞ - [21]
147. SelfSteem - Stochastic - Memory length: ∞ - [14]
148. ShortMem - Deterministic - Memory length: 10 - [14]
149. Shubik - Deterministic - Memory length: ∞ - [9]
150. Slow Tit For Two Tats 2 - Deterministic - Memory length: 2 - [29]
151. Sneaky Tit For Tat - Deterministic - Memory length: ∞ - [21]
152. Soft Grudger - Deterministic - Memory length: 6 - [25]
153. Soft Joss: 0.9 - Stochastic - Memory length: 1 - [29]
154. SolutionB1 - Deterministic - Memory length: 3 - [4]
155. SolutionB5 - Deterministic - Memory length: 5 - [4]
156. Spiteful Tit For Tat - Deterministic - Memory length: ∞ - [29]
157. Stalker: (D,) - Stochastic - Memory length: ∞ - [14]
158. Stein and Rapoport: 0.05: (D, D) - Deterministic - Memory length: ∞ - [9]
159. Stochastic Cooperator - Stochastic - Memory length: 1 - [1]
160. Stochastic WSLS: 0.05 - Stochastic - Memory length: 1 - [3]
161. Suspicious Tit For Tat - Deterministic - Memory length: 1 - [13, 18]
162. Tester - Deterministic - Memory length: ∞ - [10]
163. TF1 - Deterministic - Memory length: ∞ - [21]
164. TF2 - Deterministic - Memory length: ∞ - [21]
165. TF3 - Deterministic - Memory length: ∞ - [21]
166. ThueMorse - Deterministic - Memory length: ∞ - [21]
167. ThueMorseInverse - Deterministic - Memory length: ∞ - [21]
168. Thumper - Deterministic - Memory length: 2 - [5]
169. Tideman and Chieruzzi - Deterministic - Memory length: ∞ - [9]
170. Tit For Tat - Deterministic - Memory length: 1 - [9]
171. Tit For 2 Tats - Deterministic - Memory length: 2 - [11]
172. Tranquilizer - Stochastic - Memory length: ∞ - [9]
173. Tricky Cooperator - Deterministic - Memory length: 10 - [21]
174. Tricky Defector - Deterministic - Memory length: ∞ - [21]
175. Tricky Level Punisher - Deterministic - Memory length: ∞ - [15]
176. Tullock: 11 - Stochastic - Memory length: 11 - [9]
177. Two Tits For Tat - Deterministic - Memory length: 2 - [11]
178. VeryBad - Deterministic - Memory length: ∞ - [14]
179. Weiner - Deterministic - Memory length: ∞ - [10]
180. White - Deterministic - Memory length: ∞ - [10]

181. Willing - Stochastic - Memory length: 1 - [41]
182. Winner12 - Deterministic - Memory length: 2 - [30]
183. Winner21 - Deterministic - Memory length: 2 - [30]
184. Win-Shift Lose-Stay: D - Deterministic - Memory length: 1 - [25]
185. Win-Stay Lose-Shift: C - Deterministic - Memory length: 1 - [38, 33, 22]
186. WmAdams - Stochastic - Memory length: ∞ - [10]
187. Worse and Worse - Stochastic - Memory length: ∞ - [29]
188. Worse and Worse 2 - Stochastic - Memory length: ∞ - [29]
189. Worse and Worse 3 - Stochastic - Memory length: ∞ - [29]
190. Yamachi - Deterministic - Memory length: ∞ - [10]
191. ZD-Extortion: 0.2, 0.1, 1 - Stochastic - Memory length: 1 - [36]
192. ZD-Extort-2: 0.1111111111111111, 0.5 - Stochastic - Memory length: 1 - [38]
193. ZD-Extort3: 0.11538461538461539, 0.3333333333333333, 1 - Stochastic - Memory length: 1 - [34]
194. ZD-Extort-2 v2: 0.125, 0.5, 1 - Stochastic - Memory length: 1 - [23]
195. ZD-Extort-4: 0.23529411764705882, 0.25, 1 - Stochastic - Memory length: 1 - [21]
196. ZD-GTFT-2: 0.25, 0.5 - Stochastic - Memory length: 1 - [38]
197. ZD-GEN-2: 0.125, 0.5, 3 - Stochastic - Memory length: 1 - [23]
198. ZD-Mem2 - Stochastic - Memory length: 2 - [28]
199. ZD-Mischief: 0.1, 0.0, 1 - Stochastic - Memory length: 1 - [36]
200. ZD-SET-2: 0.25, 0.0, 2 - Stochastic - Memory length: 1 - [23]
201. e - Deterministic - Memory length: ∞ - [21]
202. Dynamic Two Tits For Tat - Stochastic - Memory length: ∞ - [21]
203. Meta Hunter: 6 players - Deterministic - Memory length: ∞ - [21]
204. Meta Hunter Aggressive: 7 players - Deterministic - Memory length: ∞ - [21]

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