

Proof of algebraic condition for extortionate strategies

The defining equations for an extortionate strategy are:

$$\tilde{p}_1 = \alpha(R - P) + \beta(R - P) \quad (1)$$

$$\tilde{p}_2 = \alpha(S - P) + \beta(T - P) \quad (2)$$

$$\tilde{p}_3 = \alpha(T - P) + \beta(S - P) \quad (3)$$

$$\tilde{p}_4 = 0 \quad (4)$$

Using equation (2), α is isolated

$$\alpha = \frac{-\beta(P - T) - \tilde{p}_2}{P - S} \quad (5)$$

Substituting this value in to equation (3), β is isolated:

$$\beta = -\frac{P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1}{(S - T)(2P - S - T)} \quad (6)$$

Substituting this back in to (5) gives:

$$\alpha = \frac{-\tilde{p}_2 + (P - T)(P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1)}{(S - T)(2P - S - T)(P - S)} \quad (7)$$

Substituting equations (6-7) in to equation (1) gives the required expression for p_1 .

Taking the ratio of equations (6-7) gives the required expression for χ .

Finally, the condition $\chi > 1$ corresponds to:

$$\tilde{p}_2(P - T) + \tilde{p}_3(S - P) > \tilde{p}_2(P - S) + \tilde{p}_3(T - P) \quad (8)$$

which can be simplified to:

$$\tilde{p}_2 > -\tilde{p}_3 \quad (9)$$

recalling that $\tilde{p}_2 = p_2 - 1$ and $\tilde{p}_3 = p_3$ gives the required result.