Recognising and evaluating the effectiveness of extortion in the Iterated Prisoner's Dilemma

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Abstract

Since the introduction of zero-determinant strategies, extortionate strategies have received considerable interest. While an interesting class of strategies, the definitions of extortionate strategies are algebraically rigid, apply only to memory-one strategies, and require complete knowledge of a strategy (memory-one cooperation probabilities). We describe a method to detect extortionate behaviour from the history of play of a strategy. When applied to a corpus of 204 strategies this method detects extortionate behaviour in well-known extortionate strategies as well others that do not fit the algebraic definition. The highest performing strategies in this corpus are able to exhibit selectively extortionate behavior, cooperating with strong strategies while exploiting weaker strategies, which no memory-one strategy can do. These strategies emerged from an evolutionary selection process and their existence contradicts widely-repeated folklore in the evolutionary game theory literature: complex strategies can be extraordinarily effective, zero-determinant strategies can be outperformed by non-zero determinant strategies, and longer memory strategies are able to outperform short memory strategies. Moreover, while resistance to extortion is critical for the evolution of cooperation, the extortion of weak opponents need not prevent cooperation between stronger opponents, and this adaptability may be crucial to maintaining cooperation in the long run.

1 Introduction

The Iterated Prisoner's Dilemma is a model for rational and evolutionary interactive behaviour, having applications in biology, the study of human social behaviour, and many other domains. Since the introduction of zero-determinant strategies in [**press2012**], extortionate strategies have received considerable interest in the literature [**hilbe2015partners**]. These strategies "enforce" a difference in stationary payouts between themselves and their opponents. The definition requires a precise algebraic relationship between the probabilities of cooperation given the outcome of the previous round of play and slight alterations to these probabilities can cause a strategy to no longer satisfy the necessary equations.

In [**adami2013evolutionary**, **Hilbe2013**, **hilbe2013adaptive**, **hilbe2015partners**, **ichinose2018zero**, **Moran1707**] the true effectiveness of these strategies in an evolutionary setting was discussed. For example [**adami2013evolutionary**] showed that ZD strategies were not evolutionarily stable. Furthermore, in that work it was also postulated that 'evolutionarily successful ZD strategies could be designed that use longer memory to distinguish self from non-self'.

The algebraic relationships of extortion define a subspace of $p \in \mathbb{R}^4$ which can be used broaden the definition of an extortionate strategy by requiring only that the defining cooperation probabilities of a strategy are close to an algebraically extortionate strategy, by the usual technique of orthogonal projection. Moreover, given the history of play of a strategy in an actual matchup, we can empirically observe its four cooperation probabilities, measure the distance to the subspace of extortionate strategies, and use this distance as a measure of the extortionality of a strategy. This method can be applied to any strategy regardless of the memory depth and avoids the algebraic rigidity issues.

We apply this method to the largest known corpus of strategies for the iterated prisoner's dilemma (the Axelrod Python library [**Knight2016**, **Knight2018**]) and show empirically that the method in fact detects extortionate strategies. A large tournament with 204 strategies demonstratee that sophisticated strategies do in fact recognise extortionate behaviour and adapt to their opponents. Further, statistical analysis of these strategies in the context of evolutionary dynamics demonstrates the importance of adaptability to achieve evolutionary stability. All of the code and data discussed in Section 3 is open sourced, archived, and written according to best scientific principles [**Wilson2014**]. The data archive can be found at [**vincent˙knight˙2018˙1297075**]. In Section 4, this large tournament is complemented with evolutionary dynamics that offer some insight in to the effectiveness of extortionate strategies.

Several theoretical insights emerge from this work. Infamously, extortionate strategies do not play well with themselves. In [**press2012**], Press and Dyson claim that a player with a "theory of mind" would rationally chose to cooperate against an opponent that also has knowledge of zero determinant strategies to avoid sustained mutual defection. While not possible for memory-one strategies, we show that this behavior is exhibited by relatively simple longer memory strategies which previously emerged from an evolutionary selection process. Similarly, in [**adami2013evolutionary**], Adami and Hintze suggest that there may exist strategies that are able to selectively behave extortionately to some opponents and cooperatively to others. We show that this is indeed the case for the same evolved strategies. It seems that humans have trouble explicitly creating such strategies but evolution is able to simply by optimizing for total payoff in IPD interactions. Accordingly, while resistance to extortionate behavior appears critical to the evolution of cooperation, there is no prohibition on selectively extorting weaker opponents, even in population dynamics, and this behavior is evolutionarily advantageous.

2 Recognising Extortion

Zero Determinant (ZD) strategies are a special case of memory-one strategies, which are defined by elements of $\mathbb{R}^4$ mapping a state of $\{C, D\}^2$, corresponding to the prior round of play, to a probability of cooperating in the next round. A match between two such strategies creates a Markov chain with transient states $\{C, D\}^2$. The main result of [**Press2012**] is that given two memory-one players $p, q \in \mathbb{R}^4$, a linear relationship between the players' scores can, in some cases, be forced by one of the players for specific choices of these probabilities.

Using the notation of [**Press2012**], the utilities for player p are given by $S_x = (R, S, T, P)$ and for player q by $S_y = (R, T, S, P)$ and the stationary scores of each player are given by S_X and S_Y respectively. The main result of [**Press2012**] is that if

$$\tilde{p} = \alpha S_x + \beta S_y + \gamma \tag{1}$$

or

$$\tilde{q} = \alpha S_x + \beta S_y + \gamma \tag{2}$$

where $\tilde{p} = (1 - p_1, 1 - p_2, p_3, p_4)$ and $\tilde{q} = (1 - q_1, 1 - q_2, q_3, q_4)$ then:

$$\alpha S_X + \beta S_Y + \gamma = 0 \tag{3}$$

Extortionate strategies are defined as follows. If this relationship is satisfied

$$\gamma = -P(\alpha + \beta) \tag{4}$$

then the player can ensure they get a score χ times larger than the opponent. This extortion coefficient is given by:

$$\chi = \frac{-\beta}{\alpha} \tag{5}$$

Thus, if (4) holds and $\chi > 1$ a player is said to extort their opponent. First, the reverse problem is considered: given a $p \in \mathbb{R}^4$ can one determine if the associated strategy is attempting to act in an extortionate way?

2.1 Subspace of Extortionate Strategies

Constraints (1) and (4) correspond to:

$$\tilde{p}_1 = \alpha R + \beta R - P(\alpha + \beta) \tag{6}$$
$$\tilde{p}_2 = \alpha S + \beta T - P(\alpha + \beta) \tag{7}$$
$$\tilde{p}_3 = \alpha T + \beta S - P(\alpha + \beta) \tag{8}$$
$$\tilde{p}_4 = \alpha P + \beta P - P(\alpha + \beta) = 0 \tag{9}$$

Equation (9) ensures that $p_4 = \tilde{p}_4 = 0$. Equations (6-8) can be used to eliminate α, β, giving:

$$\tilde{p}_1 = \frac{(R - P)(\tilde{p}_2 + \tilde{p}_3)}{S + T - 2P} \tag{10}$$

with:

$$\chi = \frac{\tilde{p}_2(P - T) + \tilde{p}_3(S - P)}{\tilde{p}_2(P - S) + \tilde{p}_3(T - P)} \tag{11}$$

Given a strategy $p \in \mathbb{R}^4$ equations (9-11) can be used to check if a strategy is extortionate. The conditions correspond to:

$$p_1 = \frac{(R-P)(p_2+p_3) - R + T + S - P}{S + T - 2P} \tag{12}$$

$$p_4 = 0 \tag{13}$$

$$1 > p_2 + p_3 \tag{14}$$

The algebraic steps necessary to prove these results are available in the supporting materials, and note that an equivalent formulation was obtained in [**adami2013evolutionary**].

All extortionate strategies reside on a triangular (14) plane (12) in 3 dimensions (13). Using this formulation it can be seen that a necessary (but not sufficient) condition for an extortionate strategy is that it cooperates on average less than 50% of the time when in a state of disagreement with the opponent (14).

As an example, consider the known extortionate strategy $p = (8/9, 1/2, 1/3, 0)$ from [**Stewart2012**] which is referred to as `Extort-2`. In this case, for the standard values of $(R, S, T, P) = (3, 0, 5, 1)$ constraint (12) corresponds to:

$$p_1 = \frac{2(p_2+p_3)+1}{3} = \frac{2(1/2+1/3)+1}{3} = \frac{8}{9} \tag{15}$$

It is clear that in this case all constraints hold. As a counterexample, consider the strategy that cooperates 25% of the time: $p = (1/4, 1/4, 1/4, 1/4)$ obeys (14) but is not extortionate as:

$$p_1 \neq \frac{2(p_2+p_3)+1}{3} = \frac{2(1/4+1/4)+1}{3} = \frac{2}{3} \tag{16}$$

2.2 Measuring Extortion from the History of Play

Not all strategies are memory-one strategies but it is possible to measure a given p from any set of interactions between two strategies. This approach can then be used to confirm that a given strategy is acting in an extortionate manner even if it is not a memory-one strategy. However, in practice, if an exact form for p is not known but measured from observed plays of the game then measurement and/or numerical error might lead to an extortionate strategy not being confirmed as such. [1]

As an example consider Table 1 which shows some actual plays of Extort-2 ($p = (8/9, 1/2, 1/3, 0)$) against an alternating strategy ($p = (0, 0, 1, 1)$). In this particular instance the measured value of p for the known extortionate strategy would be: $(2/2, 1/5, 3/8, 0/4)$ which does not fit the definition of a ZD strategy.

Turn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
(8/9, 1/2, 1/3, 0)	C	C	D	D	D	C	D	D	D	D	D	C	C	C	D	D	D	C	D	D
Alternator	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D

Table 1: A seeded play of 20 turns of two strategies.

Note that measurement of behaviour might in some cases lead to missing values. For example the strategy $p = (8/9, 1/2, 1/3, 0)$ when playing against an opponent that always cooperates will in fact never visit any state which would allow measurement of p_3 and p_4. To overcome this, it is proposed that if s is a state that is not visited than p_s is approximated using a sensible prior or imputation. In Section 3 the overall cooperation rate is used. Another approach to overcoming this measurement error would be to measure our strategies in a sufficiently noisy environment.

We can measure how close a strategy is to being extortionate with a bit of linear algebra. Essentially we attempt to find $x = (\alpha, \beta)$ and $p^* = (\tilde{p}_1 - 1, \tilde{p}_2 - 1, \tilde{p}_3, \tilde{p}_4)$ such that

$$Cx = p^* \tag{17}$$

where C corresponds to equations (6-8) and is given by:

$$C = \begin{bmatrix} R-P & R-P \\ S-P & T-P \\ T-P & S-P \\ 0 & 0 \end{bmatrix} \tag{18}$$

Note that in general, equation (17) will not necessarily have a solution. From the Rouché-Capelli theorem if there is a solution it is unique since $\text{rank}(C) = 2$ which is the dimension of the variable x. The best fitting x^* is defined by:

[1] Comparing theoretic and actual plays of the IPD is not novel, see for example [**Rand2013**].

$$x^* = \text{argmin}_{x\in\mathbb{R}^2} \|Cx - p^*\|_2^2 \tag{19}$$

Known results [**kutner2004applied**, **rao1973linear**, **wakefield2013bayesian**] yield x^*, corresponding to the nearest extortionate strategy to the measured p. It is in fact an orthogonal projection of p on to the plane defined by (12).

$$x^* = \left(C^T C\right)^{-1} C^T p^* \tag{20}$$

The squared norm of the remaining error is referred to as sum of squared errors of prediction (SSE):

$$\text{SSE} = \|Cx^* - p^*\|_2^2 \tag{21}$$

This gives expressions for α, β as $\alpha = x_1^*$ and $\beta = x_2^*$ thus the conditions for a strategy to be acting extortionately becomes:

$$-x_2^* < x_1^* \tag{22}$$

A further known result [**kutner2004applied**, **rao1973linear**, **wakefield2013bayesian**] gives an expression for SSE:

$$\text{SSE} = {p^*}^T p^* - p^* C \left(C^T C\right)^{-1} C^T p^* = {p^*}^T p^* - p^* C x^* \tag{23}$$

Using this approach, the memory-one representation $p \in \mathbb{R}^4$ of any strategy against any other can can be measured and if (22) holds then (23) can be used to identify if a strategy is acting extortionately. For a measured p, SSE corresponds to the best fitting α, β. Suspicion of extortion then corresponds to a threshold on SSE and a comparison of the measured $\chi = \frac{-\beta}{\alpha}$.

3 Numerical experiments

[**Stewart2012**] presents results from a tournament with 19 strategies with specific consideration given to ZD strategies. This tournament is reproduced here using the Axelrod-Python library [**Knight2016**]. To obtain a good measure of the corresponding transition rates for each strategy all matches have been run for 2000 turns and every match has been repeated 60 times. All of this interaction data is available at [**vincent˙knight˙2018˙1297075**]. Note that in the interest of open scientific practice, [**vincent˙knight˙2018˙1297075**] also contains interaction data for noisy and probabilistic ending interactions which are not investigated here.

Figure 1 shows the SSE values for all the strategies in the tournament, as reported in [**Stewart2012**] the extortionate strategy Extort-2 gains a large number of wins. Notice that the mean SSE for Extort-2 is approximately zero, while for the always cooperating strategy Cooperator the SSE is far from zero.

Next we investigate a tournament with 204 strategies. The results of this analysis are shown in Figure 2. The top ranking strategies by number of wins act in an extortionate way (but not against all opponents) and it can be seen that a small subgroup of strategies achieve mutual defection. All the top ranking strategies according to score achieve mutual cooperation and do not extort each other, however they **do** exhibit extortionate behaviour towards a number of the lower ranking strategies.

Figure 3 shows the relationship between χ and SSE. Note that while a strategy may attempt to act extortionately, not all opponents can be extorted. For example, a strategy that always defects never receives a lower score than its opponent, and therefore $\chi \leq 1$ for the would-be extortionate opponent. Accordingly, a low SSE indicates extortionate behavior rather than successful extortion. This is why there is not a strong correlation between SSE and χ in Figure 2.

A detailed look at selected strategies is given in Table 2 and Figure 4. The high scoring strategies presented have a large variation in SSE whilst the ZD strategies have a low score but high probability of winning. A version of Figure 4 with all the strategies considered is available in the appendix and gives the same conclusion. This evidences an idea proposed in [**adami2013evolutionary**]: sophisticated strategies are able to recognise their opponent and defend themselves against extortion. The high ranking strategies were in fact trained to maximise score [**Harper2017**] which seems to have created strategies able to extort weaker strategies whilst cooperating with stronger ones. Indeed unconditional extortion is self defeating.

4 Evolutionary dynamics

4.1 Replicator Dynamics

From the large number of interactions a payoff matrix S can be measured where S_{ij} denotes the score (using standard values of $(R, S, T, P) = (3, 0, 5, 1)$) of the ith strategy against the jth strategy. This defines a fitness landscape for which the replicator equation describes the evolution of a population of strategies:

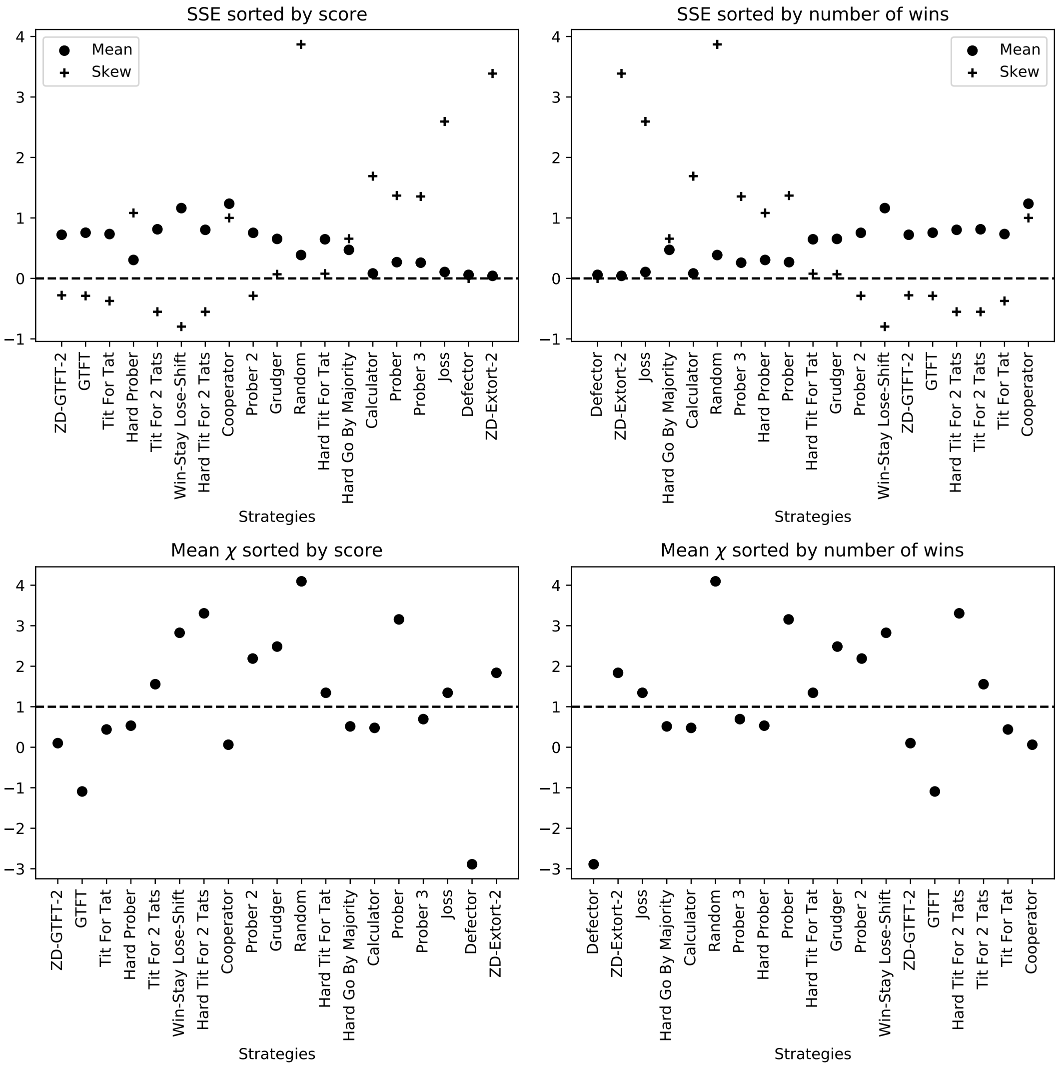


Figure 1: SSE and χ for [**Stewart2012**], ordered both by number of wins and overall score. The dashed line shows the $\chi = 1$ boundary highlighting which strategies act in an extortionate manner. The strategies which a low variation in SSE and high χ win the most matches, although even the known extortionate strategy does not act in a perfectly extortionate manner in all matches. The strategies with a high score have a large variation in SSE.

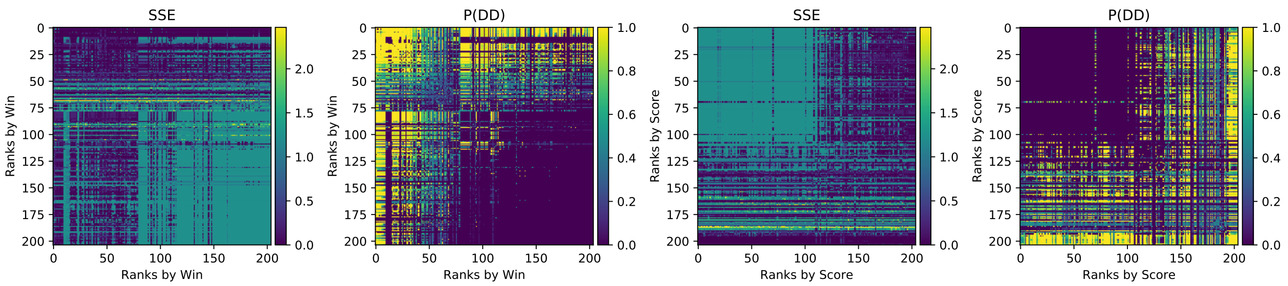


Figure 2: SSE, χ and state probabilities for the strategies for the full tournament. Note that $P(CD)$ is not shown as it corresponds to the transpose of $P(DC)$. The strategies with high number of wins have a low SSE and $\chi > 1$. The strategies with a high score have a high SSE against the other high scoring strategies but a low SSE and high χ against the low scoring strategies.

./assets/img/relationship_between_sse_and_chi/main.pdf

Figure 3: Linear relationship between SSE and χ, note that 5% and 95% confidence levels are considered to ignore extreme values of χ. Whilst a significant relationship between SSE and χ exists it is clear that having a low SSE does not guarantee extortionate behaviour.

Rank	Name	Score per turn	P(Win)	P(DD)	Median χ	Mean SSE	Skew SSE	Var SSE
1	EvolvedLookerUp2_2_2	2.944	0.230	0.092	0.063	1.057	-0.857	0.160
2	Evolved HMM 5	2.944	0.205	0.110	0.063	0.796	-0.448	0.294
3	PSO Gambler 2_2_2	2.913	0.204	0.128	0.063	0.899	-0.508	0.255
4	PSO Gambler Mem1	2.908	0.211	0.128	0.063	0.705	-0.186	0.333
5	PSO Gambler 1_1_1	2.906	0.221	0.145	0.063	0.737	-0.209	0.296
7	Evolved ANN 5	2.893	0.225	0.185	0.063	0.804	-0.608	0.334
31	ZD-GTFT-2	2.721	0.000	0.081	0.063	0.786	-0.502	0.289
45	ZD-GEN-2	2.689	0.016	0.096	0.063	0.694	-0.227	0.358
69	Tit For Tat	2.638	0.000	0.157	0.063	0.773	-0.507	0.301
75	Grumpy	2.630	0.075	0.100	0.063	0.978	-1.438	0.245
88	Win-Stay Lose-Shift	2.616	0.099	0.122	0.063	1.172	-4.501	0.027
103	Eventual Cycle Hunter	2.565	0.067	0.052	0.063	0.728	-0.338	0.357
127	Adaptive	2.272	0.500	0.314	-1.000	0.084	2.171	0.010
169	Bully	1.970	0.381	0.141	-1.000	1.373	-2.221	0.140
179	Alternator	1.945	0.392	0.259	3.857	1.332	-1.021	0.120
181	Negation	1.941	0.356	0.141	-1.000	1.470	-3.204	0.083
182	CollectiveStrategy	1.931	0.915	0.762	-2.888	0.085	6.082	0.028
183	Cycler DC	1.931	0.324	0.256	3.857	1.279	-0.900	0.140
188	Hopeless	1.908	0.352	0.048	1.833	2.247	-1.694	0.139
194	Gradual Killer	1.892	0.354	0.367	0.063	0.254	1.669	0.106
196	Aggravater	1.879	0.930	0.739	-2.889	0.163	2.951	0.066
200	ZD-Extort-2	1.821	0.851	0.652	2.005	0.019	5.435	0.009
201	ZD-Extort-4	1.820	0.865	0.697	4.003	0.021	3.677	0.005
202	ZD-Extort3	1.810	0.862	0.687	3.028	0.015	5.066	0.005
203	Defector	1.808	0.929	0.800	-2.889	0.059	0.000	0.000
204	Handshake	1.806	0.870	0.737	-2.888	0.126	3.825	0.083

Table 2: Summary of results for a selected list of strategies. Similarly to Figure 1, the high scoring strategies have a higher standard deviation of SSE. The strategies with a large number of wins have a low SSE and low variation of SSE. Note that a value of $\chi = 0.063$ and SSE $= 1.235$ corresponds to a vector $p = (1, 1, 1, 1)$ which highlights that the high scoring strategies, adapt and in fact cooperate often. A graphical representation of this table is given in Figure 4).

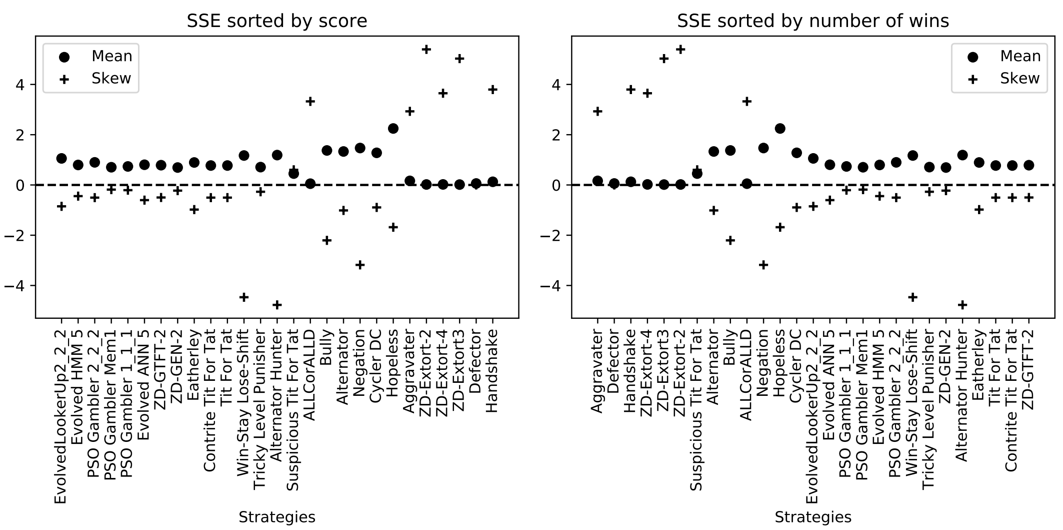


Figure 4: SSE and χ for the same selection of strategies as Table 2. A similar conclusion to that of Figure 1 can be made: the strategies that win often do not have a large variation of SSE whilst the strategies that score highly do.

$$\frac{dx_i}{dt} = x_i((Sx)_i - x^T Sx) \tag{24}$$

Equation (24) is solved numerically through an integration technique described in [**Petzold1983**] and Figure 5 shows the evolution of the distribution of the system with strategies ranked by score. It is clear to see that only the high ranking strategies survive the evolutionary process (in fact, only 39 have a final long run probability value greater than 10^{-2}, these strategies are on the bottom right.)

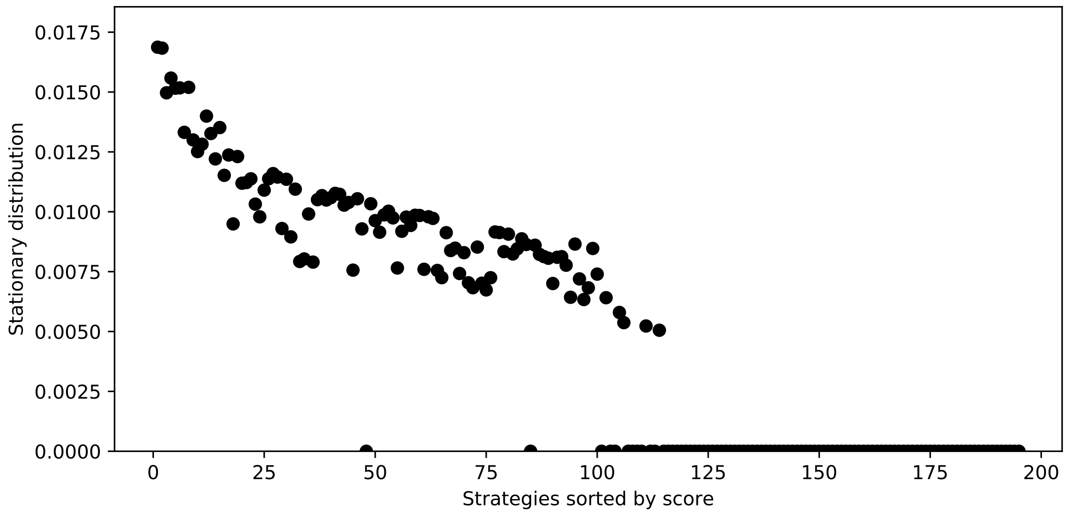


Figure 5: Numerical simulation of the replicator equation (24): strategies are ordered by score. Some selected strategies are highlighted with their long run population distribution.

Figure 6 plots the mean and variance of SSE against the long run probabilities s of (24). Strategies that perform strongly according to equation (24) seem to be strategies that are able to modify their memory one representation depending on the opponent. While the specific memory-one representation might not be one that acts extortionately, a high SSE does imply that a strategy is not extortionate. A general linear model is shown in Table 3 with stronger predictive power and giving similar conclusions.

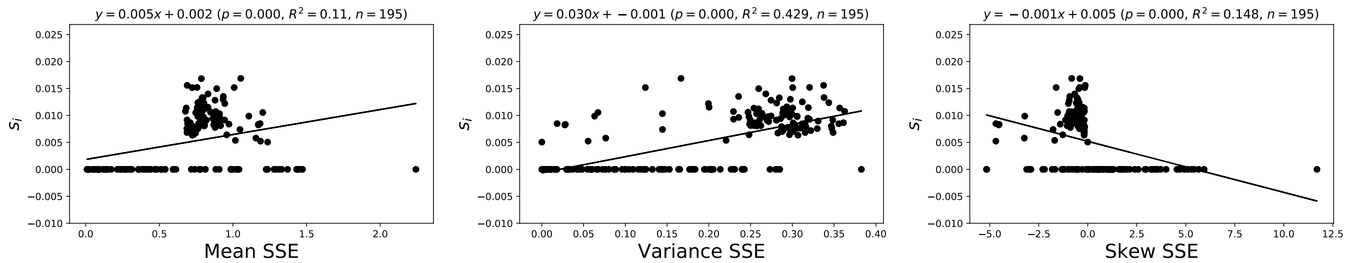


Figure 6: Linear regression analysis of long run probabilities of (24) against the mean, and variance of SSE (for a given strategy). The clusters in each plot consist of the same strategies.

4.2 Finite Population Dynamics: Moran Process

In [**Moran1707**] a large data set of pairwise fixation probabilities in the Moran process is made available at [**vincent˙knight˙201** Figure 7 shows linear models fitted to three summary measures of SSE and the mean (over population size N and opponents) value of $x_1 \cdot N$. This specific measure of fixation is chosen as x_1 is usually compared to the neutral fixation probability of $1/N$. As was noted in [**Moran1707**], the specific case of $N = 2$ differs from all other population sizes which is why it is presented in isolation. Similarly to the conclusions from Figure 6 we note that there is a significant relationship between the variability of SSE and the ability for a strategy to become fixed.

Using recursive feature elimination a better linear model can be found that compliments these results. This is shown in Table 4.

These findings confirm the work of [**Moran1707**] in which sophisticated strategies resist evolutionary invasion of shorter memory strategies. This also confirms the work of [**adami2013evolutionary**, **hilbe2015partners**] which proved that ZD strategies where not evolutionarily stable due to the fact that they score poorly against themselves.

Dep. Variable:	s_i	**R-squared:**	0.648
Model:	OLS	**Adj. R-squared:**	0.642
Method:	Least Squares	**F-statistic:**	117.0
Date:	Fri, 21 Dec 2018	**Prob (F-statistic):**	5.00e-43
Time:	11:01:35	**Log-Likelihood:**	851.41
No. Observations:	195	**AIC:**	-1695.
Df Residuals:	191	**BIC:**	-1682.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	0.0007	0.001	1.137	0.257	-0.000	0.002
('SSE', 'mean')	-0.0134	0.002	-8.369	0.000	-0.017	-0.010
('SSE', 'median')	0.0139	0.001	10.433	0.000	0.011	0.017
('SSE', 'var')	0.0069	0.003	2.402	0.017	0.001	0.013

Omnibus:	17.190	**Durbin-Watson:**	1.664
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	25.453
Skew:	0.530	**Prob(JB):**	2.97e-06
Kurtosis:	4.418	**Cond. No.**	23.7

Table 3: General linear model for the stationary probabilities of the replicator dynamics. This shows that strategies with a high mean and low median are more likely to survive the evolutionary dynamics. This corresponds to negatively skewed distributions of SSE which again highlights the importance of adaptability.

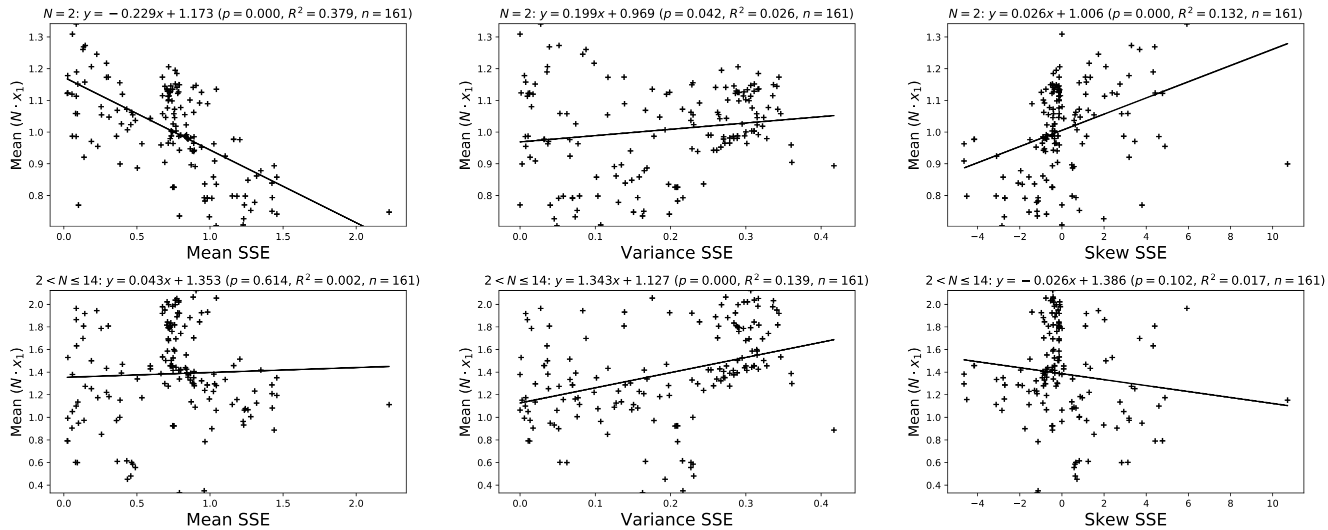


Figure 7: Linear regression analysis of pairwise fixation probabilities from [**Moran1707**] against the mean and variance of SSE (for a given strategy averaged over all opponents and population sizes).

Dep. Variable:	mean	**R-squared:**	0.319
Model:	OLS	**Adj. R-squared:**	0.310
Method:	Least Squares	**F-statistic:**	36.53
Date:	Fri, 21 Dec 2018	**Prob (F-statistic):**	9.74e-14
Time:	10:42:28	**Log-Likelihood:**	-42.272
No. Observations:	159	**AIC:**	90.54
Df Residuals:	156	**BIC:**	99.75
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	1.2815	0.056	22.993	0.000	1.171	1.392
('SSE', 'mean')	-1.0620	0.145	-7.323	0.000	-1.348	-0.776
('SSE', 'median')	0.9037	0.106	8.535	0.000	0.695	1.113

Omnibus:	2.302	**Durbin-Watson:**	1.716
Prob(Omnibus):	0.316	**Jarque-Bera (JB):**	1.850
Skew:	-0.199	**Prob(JB):**	0.397
Kurtosis:	3.348	**Cond. No.**	11.2

Table 4: General linear model for fixation probabilities of the Moran process. This shows that strategies with a high mean and low median are likely to be evolutionarily stable. This corresponds to negatively skewed distributions of SSE which again highlights the importance of adaptability.

The work also provides strong evidence to the importance of adaptability: strategies that offer a variety of behaviours corresponding to a higher standard deviation of SSE are significantly more likely to survive the evolutionary process. This corresponds to the following quote of [**darwin1869origin**]:

> "It is not the most intellectual of the species that survives; it is not the strongest that survives; but the species that survives is the one that is able to adapt to and to adjust best to the changing environment in which it finds itself."

5 Conclusion

This work defines an approach to measure whether or not a player is using an extortionate strategy as defined in [**Press2012**], or a strategy that behaves similarly, broadening the definition of extortionate behavior. All extortionate strategies have been classified as lying on a triangular plane. This rigorous classification fails to be robust to small measurement error, thus a statistical approach is proposed approximating the solution of a linear system. This method was applied to a large number of pairwise interactions.

The work of [**Press2012**], while showing that a clever approach to taking advantage of another memory-one strategy exists, is not the full story. Though the elegance of this result is very attractive, just as the simplicity of the victory of Tit For Tat in Axelrod's original tournaments was, it is incomplete and in the author's opinions, has been oversimplified and overgeneralized in subsequent work. Extortionate strategies achieve a high number of wins but they do generally not achieve a high score and fail to be evolutionarily stable.

Rather more sophisticated strategies are able to adapt to a variety of opponents and act extortionately only against weaker strategies while cooperating with like-minded strategies that are not susceptible to extortion. This adaptability may be key to maintaining sustained cooperation, as some of these strategies emerged naturally from evolutionary processes trained to maximize payoff in IPD tournaments and fixation in population dynamics.

Following Axelrod's seminal work [**Axelrod1980**, **Axelrod1980a**], it was commonly thought that evolutionary cooperation required strategies that followed a simple set of rules. The discovery/definition of extortionate strategies [**Press2012**] seemingly showed that complex strategies could be taken advantage of. In this manuscript it has been shown that not only is it possible to detect and prevent extortionate behaviour but that more complex strategies can be evolutionary stable. The complex strategies in question were obtained through reinforcement learning approaches [**Harper2017**, **Moran1707**]. Thus, this demonstrates that it is possible to recognise extortion, both theoretically using SSE but also that this ability can develop through reinforcement learning. It seems human difficulty in directly developing effective complex strategies has been incorrectly generalized to a weakness in complex strategies themselves, which is demonstrable not the case. In fact, complex strategies can be the most effective against a diverse set of opponents.

In closing, the authors wish to emphasize the the role of comprehensive simulations to temper theoretical results from overgeneralization, and perhaps more importantly, the ability of simulations to provide insights that are difficult to obtain from theory.

Acknowledgements

The following open source software libraries were used in this research:

- The Axelrod [**Knight2016**, **Knight2018**] library (IPD strategies and tournaments).
- The sympy library [**Meurer2017**] (verification of all symbolic calculations).
- The matplotlib [**Droettboom2018**] library (visualisation).
- The pandas [**Structures2010**], dask [**Dask2016**] and NumPy [**Oliphant2015**] libraries (data manipulation).
- The SciPy [**Jones2001**] library (numerical integration of the replicator equation).

This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

Proof of algebraic condition for extortionate strategies

The defining equations for an extortionate strategy are:

$$\tilde{p}_1 = \alpha(R-P) + \beta(R-P) \tag{1}$$
$$\tilde{p}_2 = \alpha(S-P) + \beta(T-P) \tag{2}$$
$$\tilde{p}_3 = \alpha(T-P) + \beta(S-P) \tag{3}$$
$$\tilde{p}_4 = 0 \tag{4}$$

Using equation (2), α is isolated

$$\alpha = \frac{-\beta(P-T) - \tilde{p}_2}{P-S} \tag{5}$$

Substituting this value in to equation (3), β is isolated:

$$\beta = -\frac{P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1}{(S-T)(2P-S-T)} \tag{6}$$

Substituting this back in to (5) gives:

$$\alpha = \frac{-\tilde{p}_2 + (P-T)(P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1)}{(S-T)(2P-S-T)(P-S)} \tag{7}$$

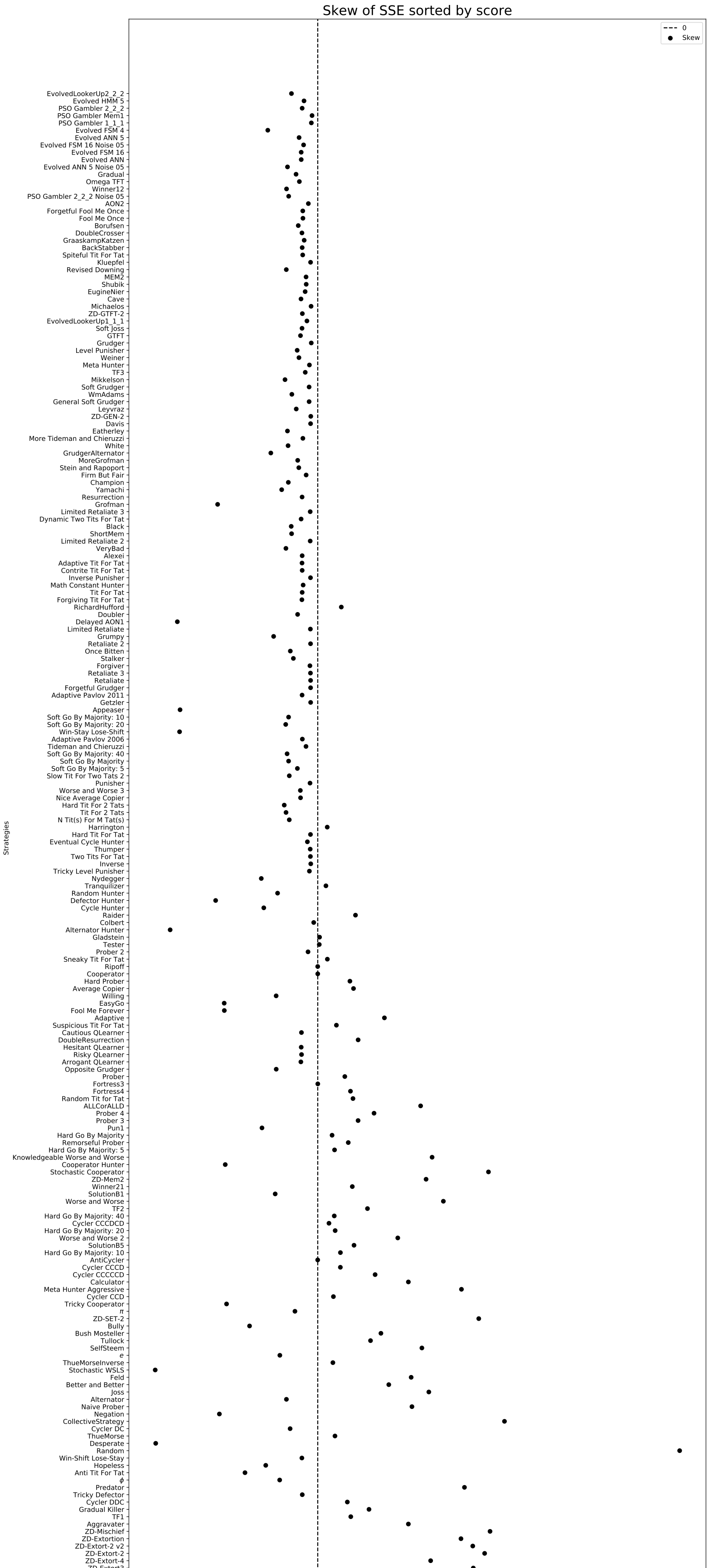
Substituting equations (6-7) in to equation (1) gives the required expression for p_1.
Taking the ratio of equations (6-7) gives the required expression for χ.
Finally, the condition $\chi > 1$ corresponds to:

$$\tilde{p}_2(P-T) + \tilde{p}_3(S-P) > \tilde{p}_2(P-S) + \tilde{p}_3(T-P) \tag{8}$$

which can be simplified to:

$$\tilde{p}_2 < -\tilde{p}_3 \tag{9}$$

recalling that $\tilde{p}_2 = p_2 - 1$ and $\tilde{p}_3 = p_3$ gives the required result.


Skew of SSE sorted by score
0
Skew
Strategies
EvolvedLookerUp2_2_2
Evolved HMM 5
PSO Gambler 2_2_2
PSO Gambler Mem1
PSO Gambler 1_1_1
Evolved FSM 4
Evolved ANN 5
Evolved FSM 16 Noise 05
Evolved FSM 16
Evolved ANN
Evolved ANN 5 Noise 05
Gradual
Omega TFT
Winner12
PSO Gambler 2_2_2 Noise 05
AON2
Forgetful Fool Me Once
Fool Me Once
Borufsen
DoubleCrosser
GraaskampKatzen
BackStabber
Spiteful Tit For Tat
Kluepfel
Revised Downing
MEM2
Shubik
EugineNier
Cave
Michaelos
ZD-GTFT-2
EvolvedLookerUp1_1_1
Soft Joss
GTFT
Grudger
Level Punisher
Weiner
Meta Hunter
TF3
Mikkelson
Soft Grudger
WmAdams
General Soft Grudger
Leyvraz
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GrudgerAlternator
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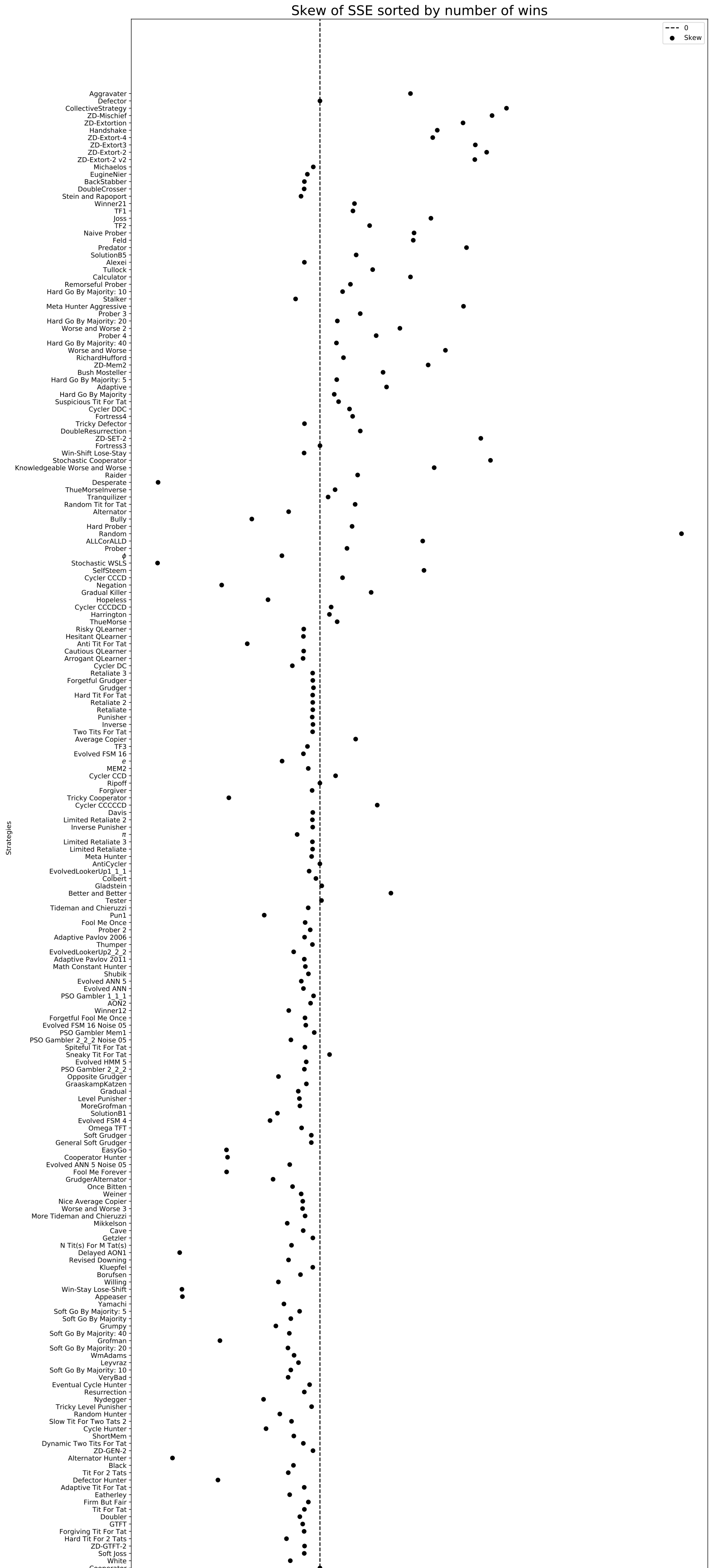
Stein and Rapoport
Firm But Fair
Champion
Yamachi
Resurrection
Grofman
Limited Retaliate 3
Dynamic Two Tits For Tat
Black
ShortMem
Limited Retaliate 2
VeryBad
Alexei
Adaptive Tit For Tat
Contrite Tit For Tat
Inverse Punisher
Math Constant Hunter
Tit For Tat
Forgiving Tit For Tat
RichardHufford
Doubler
Delayed AON1
Limited Retaliate
Grumpy
Retaliate 2
Once Bitten
Stalker
Forgiver
Retaliate 3
Retaliate
Forgetful Grudger
Adaptive Pavlov 2011
Getzler
Appeaser
Soft Go By Majority: 10
Soft Go By Majority: 20
Win-Stay Lose-Shift
Adaptive Pavlov 2006
Tideman and Chieruzzi
Soft Go By Majority: 40
Soft Go By Majority
Soft Go By Majority: 5
Slow Tit For Two Tats 2
Punisher
Worse and Worse 3
Nice Average Copier
Hard Tit For 2 Tats
Tit For 2 Tats
N Tit(s) For M Tat(s)
Harrington
Hard Tit For Tat
Eventual Cycle Hunter
Thumper
Two Tits For Tat
Inverse
Tricky Level Punisher
Nydegger
Tranquilizer
Random Hunter
Defector Hunter
Cycle Hunter
Raider
Colbert
Alternator Hunter
Gladstein
Tester
Prober 2
Sneaky Tit For Tat
Ripoff
Cooperator
Hard Prober
Average Copier
Willing
EasyGo
Fool Me Forever
Adaptive
Suspicious Tit For Tat
Cautious QLearner
DoubleResurrection
Hesitant QLearner
Risky QLearner
Arrogant QLearner
Opposite Grudger
Prober
Fortress3
Fortress4
Random Tit for Tat
ALLCorALLD
Prober 4
Prober 3
Pun1
Hard Go By Majority
Remorseful Prober
Hard Go By Majority: 5
Knowledgeable Worse and Worse
Cooperator Hunter
Stochastic Cooperator
ZD-Mem2
Winner21
SolutionB1
Worse and Worse
TF2
Hard Go By Majority: 40
Cycler CCCDCD
Hard Go By Majority: 20
Worse and Worse 2
SolutionB5
Hard Go By Majority: 10
AntiCycler
Cycler CCCD
Cycler CCCCCD
Calculator
Meta Hunter Aggressive
Cycler CCD
Tricky Cooperator
π
ZD-SET-2
Bully
Bush Mosteller
Tullock
SelfSteem
e
ThueMorseInverse
Stochastic WSLS
Feld
Better and Better
Joss
Alternator
Naive Prober
Negation
CollectiveStrategy
Cycler DC
ThueMorse
Desperate
Random
Win-Shift Lose-Stay
Hopeless
Anti Tit For Tat
ϕ
Predator
Tricky Defector
Cycler DDC
Gradual Killer
TF1
Aggravater
ZD-Mischief
ZD-Extortion
ZD-Extort-2 v2
ZD-Extort-2
ZD-Extort-4
ZD-Extort3
Defector
Handshake
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5


Skew of SSE sorted by number of wins
0
Skew
Strategies
Aggravater
Defector
CollectiveStrategy
ZD-Mischief
ZD-Extortion
Handshake
ZD-Extort-4
ZD-Extort3
ZD-Extort-2
ZD-Extort-2 v2
Michaelos
EugineNier
BackStabber
DoubleCrosser
Stein and Rapoport
Winner21
TF1
Joss
TF2
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Feld
Predator
SolutionB5
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Hard Go By Majority: 40
Worse and Worse
RichardHufford
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Bush Mosteller
Hard Go By Majority: 5
Adaptive
Hard Go By Majority
Suspicious Tit For Tat
Cycler DDC
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Tricky Defector
DoubleResurrection
ZD-SET-2
Fortress3
Win-Shift Lose-Stay
Stochastic Cooperator
Knowledgeable Worse and Worse
Raider
Desperate
ThueMorseInverse
Tranquilizer
Random Tit for Tat
Alternator
Bully
Hard Prober
Random
ALLCorALLD
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ϕ
Stochastic WSLS
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Cycler CCCD
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Retaliate 2
Retaliate
Punisher
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Two Tits For Tat
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π
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Evolved ANN
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Forgetful Fool Me Once
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PSO Gambler Mem1
PSO Gambler 2_2_2 Noise 05
Spiteful Tit For Tat
Sneaky Tit For Tat
Evolved HMM 5
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Gradual
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Nice Average Copier
Worse and Worse 3
More Tideman and Chieruzzi
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Eventual Cycle Hunter
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Tit For 2 Tats
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Firm But Fair
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Doubler
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Forgiving Tit For Tat
Hard Tit For 2 Tats
ZD-GTFT-2
Soft Joss
White
Cooperator
Contrite Tit For Tat
Champion
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5

List of all strategies used from [21]

1. Adaptive - Deterministic - Memory length: ∞ - [25]
2. Adaptive Tit For Tat: 0.5 - Deterministic - Memory length: ∞ - [39]
3. Aggravater - Deterministic - Memory length: ∞ - [21]
4. Alexei: (D,) - Deterministic - Memory length: ∞ - [43]
5. ALLCorALLD - Stochastic - Memory length: 1 - [2]
6. Alternator - Deterministic - Memory length: 1 - [31, 11]
7. Alternator Hunter - Deterministic - Memory length: ∞ - [21]
8. AntiCycler - Deterministic - Memory length: ∞ - [21]
9. Anti Tit For Tat - Deterministic - Memory length: 1 - [18]
10. AON2 - Deterministic - Memory length: 2 - [19]
11. Adaptive Pavlov 2006 - Deterministic - Memory length: ∞ - [24]
12. Adaptive Pavlov 2011 - Deterministic - Memory length: ∞ - [25]
13. Appeaser - Deterministic - Memory length: ∞ - [21]
14. Arrogant QLearner - Stochastic - Memory length: ∞ - [21]
15. Average Copier - Stochastic - Memory length: ∞ - [21]
16. BackStabber: (D, D) - Deterministic - Memory length: ∞ - [21]
17. Better and Better - Stochastic - Memory length: ∞ - [29]
18. Black - Stochastic - Memory length: 5 - [10]
19. Borufsen - Deterministic - Memory length: ∞ - [10]
20. Bully - Deterministic - Memory length: 1 - [32]
21. Bush Mosteller: 0.5, 0.5, 3.0, 0.5 - Stochastic - Memory length: ∞ - [20]
22. Calculator - Stochastic - Memory length: ∞ - [29]
23. Cautious QLearner - Stochastic - Memory length: ∞ - [21]
24. Cave - Stochastic - Memory length: ∞ - [10]
25. Champion - Stochastic - Memory length: ∞ - [10]
26. Colbert - Deterministic - Memory length: 4 - [10]
27. CollectiveStrategy - Deterministic - Memory length: ∞ - [26]
28. Contrite Tit For Tat - Deterministic - Memory length: 3 - [42]
29. Cooperator - Deterministic - Memory length: 0 - [31, 11, 34]
30. Cooperator Hunter - Deterministic - Memory length: ∞ - [21]
31. Cycle Hunter - Deterministic - Memory length: ∞ - [21]
32. Cycler CCCCCD - Deterministic - Memory length: 5 - [21]

33. Cycler CCCD - Deterministic - Memory length: 3 - [21]
34. Cycler CCD - Deterministic - Memory length: 2 - [31]
35. Cycler DC - Deterministic - Memory length: 1 - [21]
36. Cycler DDC - Deterministic - Memory length: 2 - [31]
37. Cycler CCCDCD - Deterministic - Memory length: 5 - [21]
38. Davis: 10 - Deterministic - Memory length: ∞ - [9]
39. Defector - Deterministic - Memory length: 0 - [31, 11, 34]
40. Defector Hunter - Deterministic - Memory length: ∞ - [21]
41. Desperate - Stochastic - Memory length: 1 - [41]
42. Delayed AON1 - Deterministic - Memory length: 2 - [19]
43. DoubleCrosser: (D, D) - Deterministic - Memory length: ∞ - [21]
44. Doubler - Deterministic - Memory length: ∞ - [29]
45. DoubleResurrection - Deterministic - Memory length: 5 - [15]
46. EasyGo - Deterministic - Memory length: ∞ - [25, 29]
47. Eatherley - Stochastic - Memory length: ∞ - [10]
48. EugineNier: (D,) - Deterministic - Memory length: ∞ - [43]
49. Eventual Cycle Hunter - Deterministic - Memory length: ∞ - [21]
50. Evolved ANN - Deterministic - Memory length: ∞ - [21]
51. Evolved ANN 5 - Deterministic - Memory length: ∞ - [21]
52. Evolved ANN 5 Noise 05 - Deterministic - Memory length: ∞ - [21]
53. Evolved FSM 4 - Deterministic - Memory length: 4 - [21]
54. Evolved FSM 16 - Deterministic - Memory length: 16 - [21]
55. Evolved FSM 16 Noise 05 - Deterministic - Memory length: 16 - [21]
56. EvolvedLookerUp1_1_1 - Deterministic - Memory length: ∞ - [21]
57. EvolvedLookerUp2_2_2 - Deterministic - Memory length: ∞ - [21]
58. Evolved HMM 5 - Stochastic - Memory length: 5 - [21]
59. Feld: 1.0, 0.5, 200 - Stochastic - Memory length: 200 - [9]
60. Firm But Fair - Stochastic - Memory length: 1 - [16]
61. Fool Me Forever - Deterministic - Memory length: ∞ - [21]
62. Fool Me Once - Deterministic - Memory length: ∞ - [21]
63. Forgetful Fool Me Once: 0.05 - Stochastic - Memory length: ∞ - [21]
64. Forgetful Grudger - Deterministic - Memory length: 10 - [21]
65. Forgiver - Deterministic - Memory length: ∞ - [21]
66. Forgiving Tit For Tat - Deterministic - Memory length: ∞ - [21]
67. Fortress3 - Deterministic - Memory length: 3 - [7]
68. Fortress4 - Deterministic - Memory length: 4 - [7]
69. GTFT: 0.33 - Stochastic - Memory length: 1 - [17, 33]

70. General Soft Grudger: n=1,d=4,c=2 - Deterministic - Memory length: ∞ - [21]
71. Getzler - Stochastic - Memory length: ∞ - [10]
72. Gladstein - Deterministic - Memory length: ∞ - [10]
73. Soft Go By Majority - Deterministic - Memory length: ∞ - [31, 11, 10]
74. Soft Go By Majority: 10 - Deterministic - Memory length: 10 - [21]
75. Soft Go By Majority: 20 - Deterministic - Memory length: 20 - [21]
76. Soft Go By Majority: 40 - Deterministic - Memory length: 40 - [21]
77. Soft Go By Majority: 5 - Deterministic - Memory length: 5 - [21]
78. ϕ - Deterministic - Memory length: ∞ - [21]
79. GraaskampKatzen - Deterministic - Memory length: ∞ - [10]
80. Gradual - Deterministic - Memory length: ∞ - [13]
81. Gradual Killer: (D, D, D, D, D, C, C) - Deterministic - Memory length: ∞ - [29]
82. Grofman - Stochastic - Memory length: ∞ - [9]
83. Grudger - Deterministic - Memory length: 1 - [12, 25, 13, 41, 9]
84. GrudgerAlternator - Deterministic - Memory length: ∞ - [29]
85. Grumpy: Nice, 10, -10 - Deterministic - Memory length: ∞ - [21]
86. Handshake - Deterministic - Memory length: ∞ - [35]
87. Hard Go By Majority - Deterministic - Memory length: ∞ - [31]
88. Hard Go By Majority: 10 - Deterministic - Memory length: 10 - [21]
89. Hard Go By Majority: 20 - Deterministic - Memory length: 20 - [21]
90. Hard Go By Majority: 40 - Deterministic - Memory length: 40 - [21]
91. Hard Go By Majority: 5 - Deterministic - Memory length: 5 - [21]
92. Hard Prober - Deterministic - Memory length: ∞ - [29]
93. Hard Tit For 2 Tats - Deterministic - Memory length: 3 - [38]
94. Hard Tit For Tat - Deterministic - Memory length: 3 - [40]
95. Harrington - Stochastic - Memory length: ∞ - [10]
96. Hesitant QLearner - Stochastic - Memory length: ∞ - [21]
97. Hopeless - Stochastic - Memory length: 1 - [41]
98. Inverse - Stochastic - Memory length: ∞ - [21]
99. Inverse Punisher - Deterministic - Memory length: ∞ - [21]
100. Joss: 0.9 - Stochastic - Memory length: 1 - [38, 9]
101. Kluepfel - Stochastic - Memory length: ∞ - [10]
102. Knowledgeable Worse and Worse - Stochastic - Memory length: ∞ - [21]
103. Level Punisher - Deterministic - Memory length: ∞ - [15]
104. Leyvraz - Stochastic - Memory length: 3 - [10]
105. Limited Retaliate: 0.1, 20 - Deterministic - Memory length: ∞ - [21]
106. Limited Retaliate 2: 0.08, 15 - Deterministic - Memory length: ∞ - [21]

107. Limited Retaliate 3: 0.05, 20 - Deterministic - Memory length: ∞ - [21]
108. Math Constant Hunter - Deterministic - Memory length: ∞ - [21]
109. Naive Prober: 0.1 - Stochastic - Memory length: 1 - [25]
110. MEM2 - Deterministic - Memory length: ∞ - [27]
111. Michaelos: (D,) - Stochastic - Memory length: ∞ - [43]
112. Mikkelson - Deterministic - Memory length: ∞ - [10]
113. MoreGrofman - Deterministic - Memory length: 8 - [10]
114. More Tideman and Chieruzzi - Deterministic - Memory length: ∞ - [10]
115. Negation - Stochastic - Memory length: 1 - [40]
116. Nice Average Copier - Stochastic - Memory length: ∞ - [21]
117. N Tit(s) For M Tat(s): 3, 2 - Deterministic - Memory length: 3 - [21]
118. Nydegger - Deterministic - Memory length: 3 - [9]
119. Omega TFT: 3, 8 - Deterministic - Memory length: ∞ - [37]
120. Once Bitten - Deterministic - Memory length: 12 - [21]
121. Opposite Grudger - Deterministic - Memory length: ∞ - [21]
122. π - Deterministic - Memory length: ∞ - [21]
123. Predator - Deterministic - Memory length: 9 - [7]
124. Prober - Deterministic - Memory length: ∞ - [25]
125. Prober 2 - Deterministic - Memory length: ∞ - [29]
126. Prober 3 - Deterministic - Memory length: ∞ - [29]
127. Prober 4 - Deterministic - Memory length: ∞ - [29]
128. Pun1 - Deterministic - Memory length: 2 - [6]
129. PSO Gambler 1_1_1 - Stochastic - Memory length: ∞ - [21]
130. PSO Gambler 2_2_2 - Stochastic - Memory length: ∞ - [21]
131. PSO Gambler 2_2_2 Noise 05 - Stochastic - Memory length: ∞ - [21]
132. PSO Gambler Mem1 - Stochastic - Memory length: 1 - [21]
133. Punisher - Deterministic - Memory length: ∞ - [21]
134. Raider - Deterministic - Memory length: 3 - [8]
135. Random: 0.5 - Stochastic - Memory length: 0 - [39, 9]
136. Random Hunter - Deterministic - Memory length: ∞ - [21]
137. Random Tit for Tat: 0.5 - Stochastic - Memory length: 1 - [21]
138. Remorseful Prober: 0.1 - Stochastic - Memory length: 2 - [25]
139. Resurrection - Deterministic - Memory length: 5 - [15]
140. Retaliate: 0.1 - Deterministic - Memory length: ∞ - [21]
141. Retaliate 2: 0.08 - Deterministic - Memory length: ∞ - [21]
142. Retaliate 3: 0.05 - Deterministic - Memory length: ∞ - [21]
143. Revised Downing: True - Deterministic - Memory length: ∞ - [9]

144. RichardHufford - Deterministic - Memory length: ∞ - [10]
145. Ripoff - Deterministic - Memory length: 2 - [5]
146. Risky QLearner - Stochastic - Memory length: ∞ - [21]
147. SelfSteem - Stochastic - Memory length: ∞ - [14]
148. ShortMem - Deterministic - Memory length: 10 - [14]
149. Shubik - Deterministic - Memory length: ∞ - [9]
150. Slow Tit For Two Tats 2 - Deterministic - Memory length: 2 - [29]
151. Sneaky Tit For Tat - Deterministic - Memory length: ∞ - [21]
152. Soft Grudger - Deterministic - Memory length: 6 - [25]
153. Soft Joss: 0.9 - Stochastic - Memory length: 1 - [29]
154. SolutionB1 - Deterministic - Memory length: 3 - [4]
155. SolutionB5 - Deterministic - Memory length: 5 - [4]
156. Spiteful Tit For Tat - Deterministic - Memory length: ∞ - [29]
157. Stalker: (D,) - Stochastic - Memory length: ∞ - [14]
158. Stein and Rapoport: 0.05: (D, D) - Deterministic - Memory length: ∞ - [9]
159. Stochastic Cooperator - Stochastic - Memory length: 1 - [1]
160. Stochastic WSLS: 0.05 - Stochastic - Memory length: 1 - [3]
161. Suspicious Tit For Tat - Deterministic - Memory length: 1 - [13, 18]
162. Tester - Deterministic - Memory length: ∞ - [10]
163. TF1 - Deterministic - Memory length: ∞ - [21]
164. TF2 - Deterministic - Memory length: ∞ - [21]
165. TF3 - Deterministic - Memory length: ∞ - [21]
166. ThueMorse - Deterministic - Memory length: ∞ - [21]
167. ThueMorseInverse - Deterministic - Memory length: ∞ - [21]
168. Thumper - Deterministic - Memory length: 2 - [5]
169. Tideman and Chieruzzi - Deterministic - Memory length: ∞ - [9]
170. Tit For Tat - Deterministic - Memory length: 1 - [9]
171. Tit For 2 Tats - Deterministic - Memory length: 2 - [11]
172. Tranquilizer - Stochastic - Memory length: ∞ - [9]
173. Tricky Cooperator - Deterministic - Memory length: 10 - [21]
174. Tricky Defector - Deterministic - Memory length: ∞ - [21]
175. Tricky Level Punisher - Deterministic - Memory length: ∞ - [15]
176. Tullock: 11 - Stochastic - Memory length: 11 - [9]
177. Two Tits For Tat - Deterministic - Memory length: 2 - [11]
178. VeryBad - Deterministic - Memory length: ∞ - [14]
179. Weiner - Deterministic - Memory length: ∞ - [10]
180. White - Deterministic - Memory length: ∞ - [10]

181. Willing - Stochastic - Memory length: 1 - [41]

182. Winner12 - Deterministic - Memory length: 2 - [30]

183. Winner21 - Deterministic - Memory length: 2 - [30]

184. Win-Shift Lose-Stay: D - Deterministic - Memory length: 1 - [25]

185. Win-Stay Lose-Shift: C - Deterministic - Memory length: 1 - [38, 33, 22]

186. WmAdams - Stochastic - Memory length: ∞ - [10]

187. Worse and Worse - Stochastic - Memory length: ∞ - [29]

188. Worse and Worse 2 - Stochastic - Memory length: ∞ - [29]

189. Worse and Worse 3 - Stochastic - Memory length: ∞ - [29]

190. Yamachi - Deterministic - Memory length: ∞ - [10]

191. ZD-Extortion: 0.2, 0.1, 1 - Stochastic - Memory length: 1 - [36]

192. ZD-Extort-2: 0.1111111111111111, 0.5 - Stochastic - Memory length: 1 - [38]

193. ZD-Extort3: 0.11538461538461539, 0.3333333333333333, 1 - Stochastic - Memory length: 1 - [34]

194. ZD-Extort-2 v2: 0.125, 0.5, 1 - Stochastic - Memory length: 1 - [23]

195. ZD-Extort-4: 0.23529411764705882, 0.25, 1 - Stochastic - Memory length: 1 - [21]

196. ZD-GTFT-2: 0.25, 0.5 - Stochastic - Memory length: 1 - [38]

197. ZD-GEN-2: 0.125, 0.5, 3 - Stochastic - Memory length: 1 - [23]

198. ZD-Mem2 - Stochastic - Memory length: 2 - [28]

199. ZD-Mischief: 0.1, 0.0, 1 - Stochastic - Memory length: 1 - [36]

200. ZD-SET-2: 0.25, 0.0, 2 - Stochastic - Memory length: 1 - [23]

201. e - Deterministic - Memory length: ∞ - [21]

202. Dynamic Two Tits For Tat - Stochastic - Memory length: ∞ - [21]

203. Meta Hunter: 6 players - Deterministic - Memory length: ∞ - [21]

204. Meta Hunter Aggressive: 7 players - Deterministic - Memory length: ∞ - [21]

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