Proof of optimal values of SSError

We start by obtaining an expression for SSError in terms of α, β and a measured $\|Cx - p^*\|_2^2$ where $x = (\alpha, \beta)$:

$$\|Cx - p^*\|_2^2 = (-P(\alpha+\beta) + R\alpha + R\beta - p_1^*)^2 + (-P(\alpha+\beta) + S\alpha + T\beta - p_2^*)^2 + (-P(\alpha+\beta) + S\beta + T\alpha - p_3^*)^2 \tag{1}$$

In order to find SSError we need to optimise the above in terms of α, β.
Differentiating (1), with respect to α and equating to 0 we obtain following value for α:

$$\alpha = \frac{-3P^2\beta + 2PR\beta + 2PS\beta + 2PT\beta - Pp_1^* - Pp_2^* - Pp_3^* - R^2\beta + Rp_1^* - 2ST\beta + Sp_2^* + Tp_3^*}{3P^2 - 2PR - 2PS - 2PT + R^2 + S^2 + T^2} \tag{2}$$

Differentiating (1), with respect to β and equating to 0, and substituting (2) we obtain the following value for β:

$$\beta = \frac{-3P^2p_2^* + 3P^2p_3^* + 2PRp_2^* - 2PRp_3^* - PSp_1^* + PSp_2^* - 3PSp_3^* + PTp_1^* + 3PTp_2^* - PTp_3^* - R^2p_2^* + R^2p_3^* + RSp_1^* - RTp_1^* + S^2p_3^* - STp_2^* + STp_3^* - T^2p_2^*}{(S-T)(6P^2 - 4PR - 4PS - 4PT + 2R^2 + S^2 + 2ST + T^2)} \tag{3}$$

Substituting this back in to (2) we obtain:

$$\alpha = \frac{3P^2p_2^* - 3P^2p_3^* - 2PRp_2^* + 2PRp_3^* - PSp_1^* - 3PSp_2^* + PSp_3^* + PTp_1^* - PTp_2^* + 3PTp_3^* + R^2p_2^* - R^2p_3^* + RSp_1^* - RTp_1^* + S^2p_2^* + STp_2^* - STp_3^* - T2p_3^*}{6P^2S - 6P^2T - 4PRS + 4PRT - 4PS^2 + 4PT^2 + 2R^2S - 2R^2T + S^3 + S^2T - ST^2 - T^3} \tag{4}$$

Substituting (3-4) in to (1) gives the required expression for SSError.
The condition $\chi > 1$ corresponds to $-\beta > \alpha$:

$$-\beta - \alpha = \frac{2p_1^*(P-R) + (p_2^* + p_3^*)(2P - S - T)}{6P^2 - 4PR - 4PS - 4PT + 2R^2 + S^2 + 2ST + T^2} \tag{5}$$

However, the denominator of (5) is positive as:

$$(6P^2 - 4PR - 4PS - 4PT + 2R^2 + S^2 + 2ST + T^2) = (T - 2P - S - \sqrt{2}i(P-R))(T - 2P - S + \sqrt{2}i(P-R)) \tag{6}$$

so it is a quadratic in T with no real roots and leading coefficient 1. So $\chi > 1$ corresponds to:

$$(2p_1^*(P-R) + (p_2^* + p_3^*)(2P - S - T)) > 0 \tag{7}$$

which gives the required expression because $2(P-R) < 0$ by the definition of the Prisoner's Dilemma.