Suspicion: Recognising and evaluating the effectiveness of extortion in the Iterated Prisoner's Dilemma

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Abstract

The Iterated Prisoner's Dilemma is a model for rational and evolutionary interactive behaviour, having applications in biology, the study of human social behaviour, and many other domains. It is often used to understand when and how a rational individual might accept an immediate cost to their own utility for the direct benefit of another.

Much attention has been given to a class of strategies called Zero Determinant strategies. It has been theoretically shown that these strategies can "extort" large classes of other strategies.

In this work, we describe an approach to identify if observed strategies are playing in an extortionate manner. This corresponds to a mathematical model of suspicion. Furthermore, experimental analysis of a large tournament with 204 strategies is considered. In this setting the most highly performing strategies do not play in an extortionate manner against each other but do against lower performing strategies. This suggests that whilst the theory of Zero Determinant strategies suggests that memory is not of fundamental importance to the evolution of cooperative behaviour, this narrative is incomplete.

1 Introduction

Agent-based game-theoretic models have become a stalwart of the underpinning mathematics of interactive behaviours. One of the major pieces of work in this area is the pair of computer tournaments run by Robert Axelrod [1, 2]. These tournaments pitted submitted computer strategies against each other in plays of the Iterated Prisoner's Dilemma, which is a common game where agents can choose to pay a slight cost to their immediate utility in the hope of building a reputation. This has been used in economic and evolutionary game theory to understand the evolution of cooperative behaviour.

In [16] a class of strategies that can provably extort given opponents was described. These so called Zero-Determinant (ZD) strategies ... In [6, 10] some questions have already been asked about the true effectiveness of these strategies in an evolutionary setting. Here another question is asked: is it possible to recognise this extortionate behaviour? A mathematical procedure for suspicion is presented, reflecting the way that the continued actions of an extortionate individual might raise suspicion.

ZD strategies are defined by a constrained set of transition probabilities $p \in \mathbb{R}^4$ (so called memory-one strategies). In this work, a procedure is offered that will quantify if any strategy is acting in an extortionate way. A $p \in \mathbb{R}^4$ can be measured from actual interactions, regardless of whether or not the strategy is defined in such a way. In Section 2 a κ -ZD strategy is defined as a strategy for which the measured p is sufficiently close (this will be done in a rigorous manner) to a ZD strategy. The value of κ is a measure of how close a given strategy is to being ZD. This allows for the recognition of extortionate behaviour even in the cases of numeric inexactitude that would otherwise imply a strategy was not extortionate. It is also possible to recognise sophisticated strategies that act extortionately against some opponents but not others. This was a concept suggested in [].

This work makes use of the Axelrod Python library [9, 11] which contains a large number of Prisoner Dilemma strategies available for extensive numerical examples. The approach is presented in Section 2. All of the code and data discussed in Section 3 is open sourced, archived and written according to best scientific principles [22]. The data archive can be found at [8].

This large tournament is complemented with an evolutionary consideration that offers some insight in to the effectiveness of extortionate strategies.

2 Recognising Extortion

Zero Determinant (ZD) strategies are introduced in [16] in the context of matches between two memory-one strategies. Memory-one strategies are represented as elements of \mathbb{R}^4 mapping a state of $\{C, D\}^2$ to a probability of cooperating. A match between two such strategies creates a Markov chain with transient states $\{C, D\}^2$. The main result of [16]

is that given two memory-one players $p, q \in \mathbb{R}^4$, a linear relationship between the players' scores can be forced by one of the players.

Using the notation of [16], the utilities for player p are given by $S_x = (R, S, T, P)$ and for player q by $S_y = (R, T, S, P)$ and the stationary scores of each player are given by S_X and S_Y respectively. The main result of [16] is that if

$$\tilde{p} = \alpha S_x + \beta S_y + \gamma \tag{1}$$

or

$$\tilde{q} = \alpha S_x + \beta S_y + \gamma \tag{2}$$

where $\tilde{p} = (1 - p_1, 1 - p_2, p_3, p_4)$ and $\tilde{q} = (1 - q_1, 1 - q_2, q_3, q_4)$ then:

$$\alpha S_X + \beta S_Y + \gamma = 0 \tag{3}$$

In [16] a particular type of ZD strategy is defined: extortionate strategies. If:

$$\gamma = -P(\alpha + \beta) \tag{4}$$

then the player can ensure they get a score χ times larger than the opponent. This extortion coefficient is given by:

$$\chi = \frac{-\beta}{\alpha} \tag{5}$$

Thus, if (4) holds and $\chi > 1$ a player is said to extort their opponent. Here, the reverse problem is considered: given a $p \in \mathbb{R}^4$ how does one identify α, β if they exist and is the strategy in fact acting in an extortionate way? In this case constraints (1) and (4) correspond to:

$$\tilde{p}_1 = \alpha R + \beta R - P(\alpha + \beta) \tag{6}$$

$$\tilde{p}_2 = \alpha S + \beta T - P(\alpha + \beta) \tag{7}$$

$$\tilde{p}_3 = \alpha T + \beta S - P(\alpha + \beta) \tag{8}$$

$$\tilde{p}_4 = \alpha P + \beta P - P(\alpha + \beta) \tag{9}$$

Equation (9) ensures that $p_4 = \tilde{p}_4 = 0$. Equations (6-8) can be used to eliminate α, β , giving:

$$\tilde{p}_1 = \frac{(R-P)(\tilde{p}_2 + \tilde{p}_3)}{S+T-2P} \tag{10}$$

with:

$$\chi = \frac{\tilde{p}_2(P-T) + \tilde{p}_3(S-P)}{\tilde{p}_2(P-S) + \tilde{p}_3(T-P)}$$
(11)

Given a strategy $p \in \mathbb{R}^4$ equations (9-11) can be used to check if a strategy is extortionate. The conditions correspond to:

$$p_1 = \frac{(R-P)(p_2+p_3) - R + T + S - P}{S + T - 2P}$$
(12)

$$p_4 = 0 (13)$$

$$1 > p_2 + p_3 \tag{14}$$

The algebraic steps necessary to prove these results are available in the supporting materials.

All extortionate strategies reside on a triangular (14) plane (12) in 3 dimensions (13). Using this formulation it can be seen that a necessary (but not sufficient) condition for an extortionate strategy is that it cooperates on average less than 50% of the time when in a state of disagreement with the opponent (14).

As an example, consider the known extortionate strategy p = (8/9, 1/2, 1/3, 0) from [19] which is referred to as Extort-2. In this case, for the standard values of (R, S, T, P) = (3, 0, 5, 1) constraint (12) corresponds to:

$$p_1 = \frac{2(p_2 + p_3) + 1}{3} = \frac{2(1/2 + 1/3) + 1}{3} = \frac{8}{9}$$
 (15)

It is clear that in this case all constraints hold. As a counterexample, consider the strategy that cooperates 25% of the time: p = (1/4, 1/4, 1/4, 1/4) obeys (14) but is not extortionate as:

$$p_1 \neq \frac{2(p_2 + p_3) + 1}{3} = \frac{2(1/4 + 1/4) + 1}{3} = \frac{2}{3}$$
(16)

Not all strategies are memory-one strategies but it is possible to measure a given p from any set of interactions between two strategies. This approach can then be used to confirm that a given strategy is acting in an extortionate manner even if it is not a memory-one strategy. However, in practice, if an exact form for p is not known but measured from observed plays of the game then measurement and/or numerical error might lead to an extortionate strategy not being confirmed as such.

As an example consider Table 1 which shows some actual plays of p = (8/9, 1/2, 1/3, 0) against an alternating strategy (p = (0, 0, 1, 1)):

Turn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
(8/9, 1/2, 1/3, 0)	С	С	D	D	D	С	D	D	D	D	D	С	С	С	D	D	D	С	D	D
Alternator										D										

Table 1: A seeded play of 20 turns of two strategies.

In this particular instance the measured value of p for the known extortionate strategy would be: (2/2, 1/5, 3/8, 4/4) which is does not fit the definition of an extortionate strategy.

Inverting the definition of a ZD strategy can be written in the following linear algebraic form where $x = (\alpha, \beta)$ and $p^* = (\tilde{p}_1 - 1, \tilde{p}_2 - 1, \tilde{p}_3)$:

$$Cx = p^* (17)$$

C corresponds to equations (6-8) and is given by:

$$C = \begin{bmatrix} R - P & R - P \\ S - P & T - P \\ T - P & S - P \end{bmatrix}$$
 (18)

Note that in general, equation (17) will not necessarily have a solution. From the Rouché-Capelli theorem if there is a solution it is unique as rank(C) = 2 which is the dimension of the variable x. The best fitting x^* is defined by:

$$x^* = \operatorname{argmin}_{x \in \mathbb{R}^2} \|Cx - p^*\|_2^2 \tag{19}$$

Known results [12, 18, 21] can now be applied:

$$x^* = (C^T C)^{-1} C^T p^* (20)$$

This x^* corresponds to the nearest ZD strategy that corresponds to the measured p. It is in fact a normal projection of p on to the plane defined by (12).

The squared norm of the remaining error is referred to as SS_{error} :

$$SS_{error} = ||Cx^* - p^*||_2^2 \tag{21}$$

This gives expressions for α, β as $\alpha = x_1^*$ and $\beta = x_2^*$ thus the conditions for a strategy to be acting extortionately becomes:

$$-x_2^* < x_1^* \tag{22}$$

A further known result [12, 18, 21] gives an expression for SS_{error}:

$$SS_{error} = p^{*T}p^* - p^*C (C^T C)^{-1} C^T p^* = p^{*T}p^* - p^*Cx^*$$
(23)

By observing interactions (human or otherwise), their memory-one representation can be inferred and if (22) hold then (23) can be used to identify if a strategy is acting extortionately.

For a measured p, SS_{error} corresponds to the best fitting α, β , which omits the value of p_4 which is 0 if the strategy is extortionate (??). Suspicion of extortion then corresponds to a threshold on the following value κ :

$$\kappa = SS_{error} + p_4^2 \tag{24}$$

Comparing theoretic and actual plays of the IPD is not novel, see for example [17].

In the next section, this idea will be illustrated by observing the interactions that take place in a large computer based tournament of the IPD.

3 Numerical experiments

In [19] results from a tournament with 19 strategies, was presented with specific consideration given to ZD strategies. This tournament is reproduced here using the Axelrod-Python library [9]. To obtain a good measure of the corresponding transition rates for each strategy all matches have been run for 2000 turns and every match has been repeated 60 times. All of this interaction data is available at [8]. Note that in the interest of open scientific practice, [8] also contains interaction data for noisy and probabilistic ending interactions which are not investigated here. A good match between the inferred Markov chain and the state distribution of the actual interactions has been verified. Data for this is presented in the supplementary materials.

Figure 1 shows the κ values for all the strategies in the tournament, as reported in [19] the extortionate strategy (which has an expected κ approximately 0) gains a large number of wins.

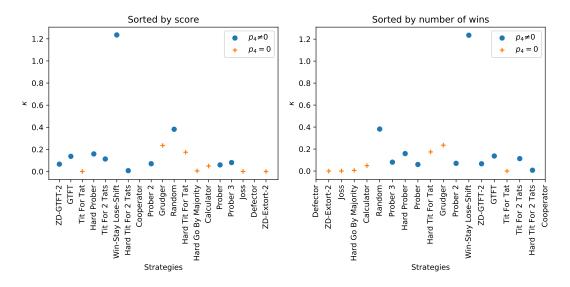


Figure 1: κ and state probabilities for the strategies of [19], ordered both by number of wins and overall score. Note that Cooperator and Defector do not have measures of p in all states and so no κ can be computed.

Here, the work of [19] is extended by investigating a tournament with 204 strategies. The results of this analysis are shown in Figure 2. The top ranking strategies by number of wins seem to be extortionate (but not against all strategies) and it can be seen that a small subgroup of strategies achieve mutual defection. All the top ranking strategies according to score achieve mutual cooperation and do not extort each other, however they **do** exhibit extortionate behaviour towards a number of the lower ranking strategies.

A detailed look at selected strategies is given in Table 2. It can be seen that Extort-2 wins many matches but does not achieve a high mean score or a high mutual cooperation rate (P(CC)).

Rank	Name	Score per turn	P(Win)	P(CC)	P(C CC)	P(C CD)	P(C DC)	P(C DD)	SSError	α	β	χ
1	EvolvedLookerUp2_2_2	2.944	0.230	0.673	0.938	0.498	0.207	0.470	0.0095	0.038	-0.104	2.765
7	Evolved ANN 5	2.893	0.225	0.682	1.000	0.930	0.001	0.000	0.0011	0.001	-0.013	12.225
31	ZD-GTFT-2	2.721	0.000	0.806	1.000	0.125	1.000	0.250	0.0037	0.199	-0.176	0.889
45	ZD-GEN-2	2.689	0.016	0.801	1.000	0.562	0.500	0.125	0.0009	0.099	-0.088	0.889
69	Tit For Tat	2.638	0.000	0.723	1.000	0.000	1.000	0.000	0.0000	0.200	-0.200	1.000
88	Win-Stay Lose-Shift	2.616	0.099	0.649	1.000	0.000	0.000	1.000	0.2353	0.012	-0.188	16.000
200	ZD-Extort-2	1.821	0.851	0.179	0.889	0.500	0.334	0.000	0.0000	0.056	-0.111	1.998

Table 2: Summary of overall results for a selected list of strategies. The transition rates are computed as an average over all matches.

4 Conclusion

This work defines an approach to measure whether or not a player is playing a strategy that corresponds to an extortionate strategy as defined in [16]: a mathematical model for suspicion. All extortionate strategies have been classified as lying on a triangular plane. This rigorous classification fails to be robust to small measurement error, thus a statistical approach is proposed approximating the solution of a linear system. Using this, a large number of pairwise interactions is simulated and in fact very few strategies are found to act extortionately.

The work of [16], whilst showing that a clever approach to taking advantage of another memory-one strategy exists: this is not the full story. Though the elegance of this result is very attractive, just as the simplicity of the victory of

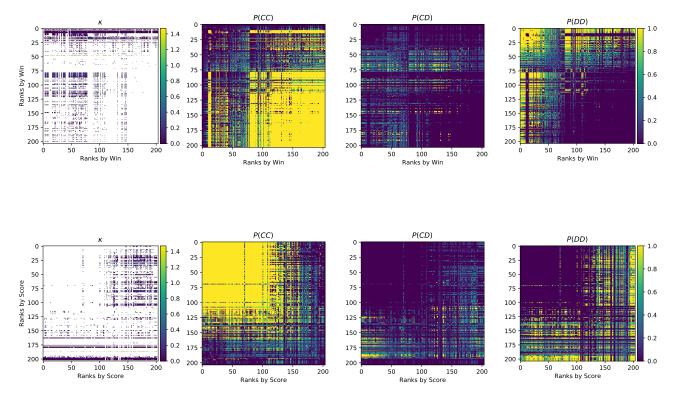


Figure 2: κ for the strategies for the full tournament. For the κ plot only strategy interactions for which $p_4 = 0$ and $\chi > 1$ are displayed. Note that P(DC) is not shown as it corresponds to the transpose of P(CD).

Tit For Tat in Axelrod's original tournaments was, it is incomplete. Extortionate strategies achieve a high number of wins but they do not achieve a high score. From the large number of interactions a payoff matrix S can be measured where S_{ij} denotes the score (using standard values of (R, S, T, P) = (3, 0, 5, 1)) of the ith strategy against the jth strategy. Using this, the replicator equation describes the evolution of the system based on a population density fitness function:

$$\frac{dx_i}{dt} = x_i(S_i - x^T S x) \tag{25}$$

Equation (25) is solved numerically through an integration technique described in [15] and Figure 3 shows the evolution of the distribution of the system: the various strategies are ranked by scores. It is clear to see that only the high ranking strategies survive the evolutionary process (in fact, only 25 have a final distribution value greater than 10^{-2}). This confirms the findings of [10] in which sophisticated strategies resist evolutionary invasion of shorter memory strategies. Recalling Figure 2 this demonstrates that:

- Cooperation emerges through the evolutionary process: the high scoring strategies do not exhibit extortionate behaviour towards each other.
- Purely extortionate strategies do not survive the evolutionary process.

Following Axelrod's seminal work [1, 2], it was commonly thought that evolutionary cooperation required strategies that followed a simple set of rules. The discovery/definition of extortionate strategies [16] seemingly showed that complex strategies could be taken advantage of. In this manuscript it has been shown that not only is it possible to be detect and prevent extortionate behaviour but that more complex strategies can be evolutionary stable. The complex strategies in question were obtained through reinforcement learning approaches [5, 10]. Thus, this work demonstrates the possibility for the evolution of cooperation through suspicion.

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The following open source software libraries were used in this research:

- The Axelrod [9, 11] library (IPD strategies and tournaments).
- The sympy library [13] (verification of all symbolic calculations).

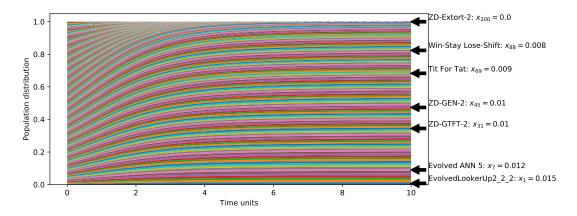


Figure 3: Numerical simulation of the replicator equation (25): strategies are ordered by score. Some selected strategies are highlighted with their long run population distribution.

- The matplotlib [4] library (visualisation).
- The pandas [20], dask [3] and NumPy [14] libraries (data manipulation).
- The SciPy [7] library (numerical integration of the replicator equation).

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Supplementary materials

Proof of algebraic condition for extortionate strategies

The defining equations for an extortionate strategy are:

$$\tilde{p}_1 = \alpha(R - P) + \beta(R - P) \tag{1}$$

$$\tilde{p}_2 = \alpha(S - P) + \beta(T - P) \tag{2}$$

$$\tilde{p}_3 = \alpha(T - P) + \beta(S - P) \tag{3}$$

$$\tilde{p}_4 = 0 \tag{4}$$

Using equation (2), α is isolated

$$\alpha = \frac{-\beta(P-T) - \tilde{p}_2}{P-S} \tag{5}$$

Substituting this value in to equation (3), β is isolated:

$$\beta = -\frac{P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1}{(S - T)(2P - S - T)} \tag{6}$$

Substituting this back in to (5) gives:

$$\alpha = \frac{-\tilde{p}_2 + (P - T)(P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1)}{(S - T)(2P - S - T)(P - S)}$$
(7)

Substituting equations (6-7) in to equation (1) gives the required expression for p_1 . Taking the ratio of equations (6-7) gives the required expression for χ . Finally, the condition $\chi > 1$ corresponds to:

$$\tilde{p}_2(P-T) + \tilde{p}_3(S-P) > \tilde{p}_2(P-S) + \tilde{p}_3(T-P)$$
 (8)

which can be simplified to:

$$\tilde{p}_2 < -\tilde{p}_3 \tag{9}$$

recalling that $\tilde{p}_2 = p_2 - 1$ and $\tilde{p}_3 = p_3$ gives the required result.

Comparison of steady states

Using the pair wise interactions the transition rates p, q can be measured and the steady state probabilities inferred and compared to the actual probabilities of each state. This is done numerically by computing the singular eigenvector of the matrix A [1]:

$$A = \begin{bmatrix} p_1q_1 & p_1(1-q_1) & (1-p_1)q_1 & (1-p_1)(1-q_1) \\ p_2q_2 & p_2(1-q_2) & (1-p_2)q_2 & (1-p_2)(1-q_2) \\ p_3q_3 & p_3(1-q_3) & (1-p_3)q_3 & (1-p_3)(1-q_3) \\ p_4q_4 & p_4(1-q_4) & (1-p_4)q_4 & (1-p_4)(1-q_4) \end{bmatrix}$$

Figure 1 shows a regression line fitted to every pairwise interaction with a reported SSError value (pairwise interactions with missing states were omitted). This serves to validate the approach: a part from some edge cases the relationship is consistent.

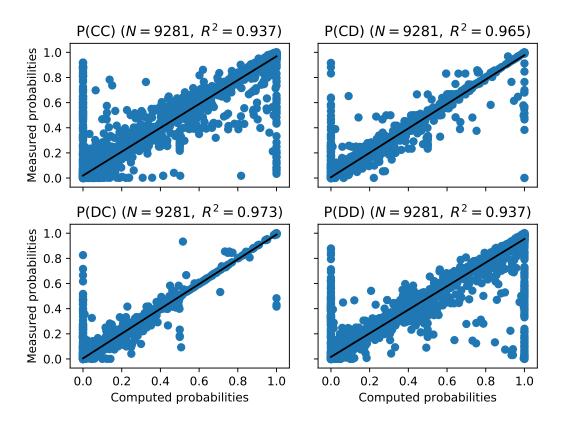


Figure 1: The relationship between the steady state probabilities inferred from the measured transitions and the actual steady state probabilities. A linear regression line is included validating the approach.

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List of all strategies used from [21]

- 1. Adaptive Deterministic Memory length: ∞ [25]
- 2. Adaptive Tit For Tat: 0.5 Deterministic Memory length: ∞ [39]
- 3. Aggravater Deterministic Memory length: ∞ [21]
- 4. Alexei: (D,) Deterministic Memory length: ∞ [43]
- 5. ALLCorALLD Stochastic Memory length: 1 [2]
- 6. Alternator Deterministic Memory length: 1 [31, 11]
- 7. Alternator Hunter Deterministic Memory length: ∞ [21]
- 8. AntiCycler Deterministic Memory length: ∞ [21]
- 9. Anti Tit For Tat Deterministic Memory length: 1 [18]
- 10. AON2 Deterministic Memory length: 2 [19]
- 11. Adaptive Pavlov 2006 Deterministic Memory length: ∞ [24]
- 12. Adaptive Pavlov 2011 Deterministic Memory length: ∞ [25]
- 13. Appeaser Deterministic Memory length: ∞ [21]
- 14. Arrogant QLearner Stochastic Memory length: ∞ [21]
- 15. Average Copier Stochastic Memory length: ∞ [21]
- 16. BackStabber: (D, D) Deterministic Memory length: ∞ [21]
- 17. Better and Better Stochastic Memory length: ∞ [29]
- 18. Black Stochastic Memory length: 5 [10]
- 19. Borufsen Deterministic Memory length: ∞ [10]
- 20. Bully Deterministic Memory length: 1 [32]
- 21. Bush Mosteller: 0.5, 0.5, 3.0, 0.5 Stochastic Memory length: ∞ [20]
- 22. Calculator Stochastic Memory length: ∞ [29]
- 23. Cautious QLearner Stochastic Memory length: ∞ [21]
- 24. Cave Stochastic Memory length: ∞ [10]
- 25. Champion Stochastic Memory length: ∞ [10]
- 26. Colbert Deterministic Memory length: 4 [10]
- 27. CollectiveStrategy Deterministic Memory length: ∞ [26]
- 28. Contrite Tit For Tat Deterministic Memory length: 3 [42]
- 29. Cooperator Deterministic Memory length: 0 [34, 31, 11]
- 30. Cooperator Hunter Deterministic Memory length: ∞ [21]
- 31. Cycle Hunter Deterministic Memory length: ∞ [21]
- 32. Cycler CCCCCD Deterministic Memory length: 5 [21]

- 33. Cycler CCCD Deterministic Memory length: 3 [21]
- 34. Cycler CCD Deterministic Memory length: 2 [31]
- 35. Cycler DC Deterministic Memory length: 1 [21]
- 36. Cycler DDC Deterministic Memory length: 2 [31]
- 37. Cycler CCCDCD Deterministic Memory length: 5 [21]
- 38. Davis: 10 Deterministic Memory length: ∞ [9]
- 39. Defector Deterministic Memory length: 0 [34, 31, 11]
- 40. Defector Hunter Deterministic Memory length: ∞ [21]
- 41. Desperate Stochastic Memory length: 1 [41]
- 42. Delayed AON1 Deterministic Memory length: 2 [19]
- 43. Double Crosser: (D, D) Deterministic Memory length: ∞ [21]
- 44. Doubler Deterministic Memory length: ∞ [29]
- 45. DoubleResurrection Deterministic Memory length: 5 [15]
- 46. Easy
Go Deterministic Memory length: ∞ [25, 29]
- 47. Eatherley Stochastic Memory length: ∞ [10]
- 48. EugineNier: (D,) Deterministic Memory length: ∞ [43]
- 49. Eventual Cycle Hunter Deterministic Memory length: ∞ [21]
- 50. Evolved ANN Deterministic Memory length: ∞ [21]
- 51. Evolved ANN 5 Deterministic Memory length: ∞ [21]
- 52. Evolved ANN 5 Noise 05 Deterministic Memory length: ∞ [21]
- 53. Evolved FSM 4 Deterministic Memory length: 4 [21]
- 54. Evolved FSM 16 Deterministic Memory length: 16 [21]
- 55. Evolved FSM 16 Noise 05 Deterministic Memory length: 16 [21]
- 56. Evolved Looker Up
1_1_1 - Deterministic - Memory length: ∞ - [21]
- 57. Evolved Looker Up
2_2_2 - Deterministic - Memory length: ∞ - [21]
- 58. Evolved HMM 5 Stochastic Memory length: 5 [21]
- 59. Feld: 1.0, 0.5, 200 Stochastic Memory length: 200 [9]
- 60. Firm But Fair Stochastic Memory length: 1 [16]
- 61. Fool Me Forever Deterministic Memory length: ∞ [21]
- 62. Fool Me Once Deterministic Memory length: ∞ [21]
- 63. Forgetful Fool Me Once: 0.05 Stochastic Memory length: ∞ [21]
- 64. Forgetful Grudger Deterministic Memory length: 10 [21]
- 65. Forgiver Deterministic Memory length: ∞ [21]
- 66. Forgiving Tit For Tat Deterministic Memory length: ∞ [21]
- 67. Fortress3 Deterministic Memory length: 3 [7]
- 68. Fortress4 Deterministic Memory length: 4 [7]
- 69. GTFT: 0.33 Stochastic Memory length: 1 [33, 17]

- 70. General Soft Grudger: n=1, d=4, c=2 Deterministic Memory length: ∞ [21]
- 71. Getzler Stochastic Memory length: ∞ [10]
- 72. Gladstein Deterministic Memory length: ∞ [10]
- 73. Soft Go By Majority Deterministic Memory length: ∞ [10, 31, 11]
- 74. Soft Go By Majority: 10 Deterministic Memory length: 10 [21]
- 75. Soft Go By Majority: 20 Deterministic Memory length: 20 [21]
- 76. Soft Go By Majority: 40 Deterministic Memory length: 40 [21]
- 77. Soft Go By Majority: 5 Deterministic Memory length: 5 [21]
- 78. ϕ Deterministic Memory length: ∞ [21]
- 79. GraaskampKatzen Deterministic Memory length: ∞ [10]
- 80. Gradual Deterministic Memory length: ∞ [13]
- 81. Gradual Killer: (D, D, D, D, D, C, C) Deterministic Memory length: ∞ [29]
- 82. Grofman Stochastic Memory length: ∞ [9]
- 83. Grudger Deterministic Memory length: 1 [9, 12, 41, 13, 25]
- 84. Grudger Alternator - Deterministic - Memory length:
 ∞ - [29]
- 85. Grumpy: Nice, 10, -10 Deterministic Memory length: ∞ [21]
- 86. Handshake Deterministic Memory length: ∞ [35]
- 87. Hard Go By Majority Deterministic Memory length: ∞ [31]
- 88. Hard Go By Majority: 10 Deterministic Memory length: 10 [21]
- 89. Hard Go By Majority: 20 Deterministic Memory length: 20 [21]
- 90. Hard Go By Majority: 40 Deterministic Memory length: 40 [21]
- 91. Hard Go By Majority: 5 Deterministic Memory length: 5 [21]
- 92. Hard Prober Deterministic Memory length: ∞ [29]
- 93. Hard Tit For 2 Tats Deterministic Memory length: 3 [38]
- 94. Hard Tit For Tat Deterministic Memory length: 3 [40]
- 95. Harrington Stochastic Memory length: ∞ [10]
- 96. Hesitant Q
Learner Stochastic Memory length: ∞
 [21]
- 97. Hopeless Stochastic Memory length: 1 [41]
- 98. Inverse Stochastic Memory length: ∞ [21]
- 99. Inverse Punisher Deterministic Memory length: ∞ [21]
- 100. Joss: 0.9 Stochastic Memory length: 1 [9, 38]
- 101. Kluepfel Stochastic Memory length: ∞ [10]
- 102. Knowledgeable Worse and Worse Stochastic Memory length: ∞ [21]
- 103. Level Punisher Deterministic Memory length: ∞ [15]
- 104. Leyvraz Stochastic Memory length: 3 [10]
- 105. Limited Retaliate: 0.1, 20 Deterministic Memory length: ∞ [21]
- 106. Limited Retaliate 2: 0.08, 15 Deterministic Memory length: ∞ [21]

- 107. Limited Retaliate 3: 0.05, 20 Deterministic Memory length: ∞ [21]
- 108. Math Constant Hunter Deterministic Memory length: ∞ [21]
- 109. Naive Prober: 0.1 Stochastic Memory length: 1 [25]
- 110. MEM2 Deterministic Memory length: ∞ [27]
- 111. Michaelos: (D,) Stochastic Memory length: ∞ [43]
- 112. Mikkelson Deterministic Memory length: ∞ [10]
- 113. MoreGrofman Deterministic Memory length: 8 [10]
- 114. More Tideman and Chieruzzi Deterministic Memory length: ∞ [10]
- 115. Negation Stochastic Memory length: 1 [40]
- 116. Nice Average Copier Stochastic Memory length: ∞ [21]
- 117. N Tit(s) For M Tat(s): 3, 2 Deterministic Memory length: 3 [21]
- 118. Nydegger Deterministic Memory length: 3 [9]
- 119. Omega TFT: 3, 8 Deterministic Memory length: ∞ [37]
- 120. Once Bitten Deterministic Memory length: 12 [21]
- 121. Opposite Grudger Deterministic Memory length: ∞ [21]
- 122. π Deterministic Memory length: ∞ [21]
- 123. Predator Deterministic Memory length: 9 [7]
- 124. Prober Deterministic Memory length: ∞ [25]
- 125. Prober 2 Deterministic Memory length: ∞ [29]
- 126. Prober 3 Deterministic Memory length: ∞ [29]
- 127. Prober 4 Deterministic Memory length: ∞ [29]
- 128. Pun1 Deterministic Memory length: 2 [6]
- 129. PSO Gambler 1_1_1 Stochastic Memory length: ∞ [21]
- 130. PSO Gambler 2_2_2 Stochastic Memory length: ∞ [21]
- 131. PSO Gambler 2_2_2 Noise 05 Stochastic Memory length: ∞ [21]
- 132. PSO Gambler Mem1 Stochastic Memory length: 1 [21]
- 133. Punisher Deterministic Memory length: ∞ [21]
- 134. Raider Deterministic Memory length: 3 [8]
- 135. Random: 0.5 Stochastic Memory length: 0 [9, 39]
- 136. Random Hunter Deterministic Memory length: ∞ [21]
- 137. Random Tit for Tat: 0.5 Stochastic Memory length: 1 [21]
- 138. Remorseful Prober: 0.1 Stochastic Memory length: 2 [25]
- 139. Resurrection Deterministic Memory length: 5 [15]
- 140. Retaliate: 0.1 Deterministic Memory length: ∞ [21]
- 141. Retaliate 2: 0.08 Deterministic Memory length: ∞ [21]
- 142. Retaliate 3: 0.05 Deterministic Memory length: ∞ [21]
- 143. Revised Downing: True Deterministic Memory length: ∞ [9]

- 144. Richard Hufford - Deterministic - Memory length: ∞ - [10]
- 145. Ripoff Deterministic Memory length: 2 [5]
- 146. Risky Q
Learner Stochastic Memory length: ∞ [21]
- 147. SelfSteem Stochastic Memory length: ∞ [14]
- 148. ShortMem Deterministic Memory length: 10 [14]
- 149. Shubik Deterministic Memory length: ∞ [9]
- 150. Slow Tit For Two Tats 2 Deterministic Memory length: 2 [29]
- 151. Sneaky Tit For Tat Deterministic Memory length: ∞ [21]
- 152. Soft Grudger Deterministic Memory length: 6 [25]
- 153. Soft Joss: 0.9 Stochastic Memory length: 1 [29]
- 154. SolutionB1 Deterministic Memory length: 3 [4]
- 155. SolutionB5 Deterministic Memory length: 5 [4]
- 156. Spiteful Tit For Tat Deterministic Memory length: ∞ [29]
- 157. Stalker: (D,) Stochastic Memory length: ∞ [14]
- 158. Stein and Rapoport: 0.05: (D, D) Deterministic Memory length: ∞ [9]
- 159. Stochastic Cooperator Stochastic Memory length: 1 [1]
- 160. Stochastic WSLS: 0.05 Stochastic Memory length: 1 [3]
- 161. Suspicious Tit For Tat Deterministic Memory length: 1 [13, 18]
- 162. Tester Deterministic Memory length: ∞ [10]
- 163. TF1 Deterministic Memory length: ∞ [21]
- 164. TF2 Deterministic Memory length: ∞ [21]
- 165. TF3 Deterministic Memory length: ∞ [21]
- 166. ThueMorse Deterministic Memory length: ∞ [21]
- 167. ThueMorseInverse Deterministic Memory length: ∞ [21]
- 168. Thumper Deterministic Memory length: 2 [5]
- 169. Tideman and Chieruzzi Deterministic Memory length: ∞ [9]
- 170. Tit For Tat Deterministic Memory length: 1 [9]
- 171. Tit For 2 Tats Deterministic Memory length: 2 [11]
- 172. Tranquilizer Stochastic Memory length: ∞ [9]
- 173. Tricky Cooperator Deterministic Memory length: 10 [21]
- 174. Tricky Defector Deterministic Memory length: ∞ [21]
- 175. Tricky Level Punisher Deterministic Memory length: ∞ [15]
- 176. Tullock: 11 Stochastic Memory length: 11 [9]
- 177. Two Tits For Tat Deterministic Memory length: 2 [11]
- 178. VeryBad Deterministic Memory length: ∞ [14]
- 179. Weiner Deterministic Memory length: ∞ [10]
- 180. White Deterministic Memory length: ∞ [10]

- 181. Willing Stochastic Memory length: 1 [41]
- 182. Winner12 Deterministic Memory length: 2 [30]
- 183. Winner21 Deterministic Memory length: 2 [30]
- 184. Win-Shift Lose-Stay: D Deterministic Memory length: 1 [25]
- 185. Win-Stay Lose-Shift: C Deterministic Memory length: 1 [33, 38, 22]
- 186. WmAdams Stochastic Memory length: ∞ [10]
- 187. Worse and Worse Stochastic Memory length: ∞ [29]
- 188. Worse and Worse 2 Stochastic Memory length: ∞ [29]
- 189. Worse and Worse 3 Stochastic Memory length: ∞ [29]
- 190. Yamachi Deterministic Memory length: ∞ [10]
- 191. ZD-Extortion: 0.2, 0.1, 1 Stochastic Memory length: 1 [36]
- 192. ZD-Extort-2: 0.11111111111111111, 0.5 Stochastic Memory length: 1 [38]
- 193. ZD-Extort3: 0.11538461538461539, 0.333333333333333333333, 1 Stochastic Memory length: 1 [34]
- 194. ZD-Extort-2 v2: 0.125, 0.5, 1 Stochastic Memory length: 1 [23]
- 195. ZD-Extort-4: 0.23529411764705882, 0.25, 1 Stochastic Memory length: 1 [21]
- 196. ZD-GTFT-2: 0.25, 0.5 Stochastic Memory length: 1 [38]
- 197. ZD-GEN-2: 0.125, 0.5, 3 Stochastic Memory length: 1 [23]
- 198. ZD-Mem2 Stochastic Memory length: 2 [28]
- 199. ZD-Mischief: 0.1, 0.0, 1 Stochastic Memory length: 1 [36]
- 200. ZD-SET-2: 0.25, 0.0, 2 Stochastic Memory length: 1 [23]
- 201. e Deterministic Memory length: ∞ [21]
- 202. Dynamic Two Tits For Tat Stochastic Memory length: ∞ [21]
- 203. Meta Hunter: 6 players Deterministic Memory length: ∞ [21]
- 204. Meta Hunter Aggressive: 7 players Deterministic Memory length: ∞ [21]

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