Suspicion: Recognising and evaluating the effectiveness of extortion in the Iterated Prisoner's Dilemma

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Abstract

Since the introduction of zero-determinant strategies extortionate strategies have received considerable interest. While interesting theoretically, the definitions of extortionate strategies are algebraically rigid, apply only to memory-one strategies, and require full knowledge of a strategy (memory-one cooperation probabilities). We describe a method to detect extortionate behaviour from the history of play of a strategy. When applied to a corpus of 204 strategies this method detects extortionate behaviour in well-known extortionate strategies as well others that do not fit the algebraic definition. The highest performing strategies in this corpus are able to exhibit selectively extortionate behavior, cooperating with strong strategies while exploiting weaker strategies, which no memory one strategy can do. These strategies emerged from an evolutionary selection process and contradict widely-repeated folklore in the evolutionary game theory literature: complex strategies can be extraordinarily effective, zero-determinant strategies can be outperformed by non-zero determinant strategies, and longer memory strategies are able to outperform short memory strategies. Moreover, while resistance to extortion is critical for the evolution of cooperation, the extortion of weak opponents does not prevent cooperation between stronger opponents.

1 Introduction

The Iterated Prisoner's Dilemma is a model for rational and evolutionary interactive behaviour, having applications in biology, the study of human social behaviour, and many other domains.

Since the introduction of zero-determinant strategies in [**press2012**], extortionate strategies have received considerable interest in the literature [] []. These strategies "enforce" a linear difference in stationary payouts between themselves and their opponents. The definition requires a precise algebraic relationship between the probabilities of cooperation given the outcome of the previous round of play and a slight alterations to these probabilities can cause a strategy to no longer satisfy the necessary equations.

These algebraic relationships define a subspace of $\mathbb{R}^4$ and we can broaden the definition of an extortionate strategy by requiring only that the defining cooperation probabilities of a strategy are close to an algebraically extortionate strategy, by the usual technique of orthogonal projection onto the subspace. Moreover, given the history of play of a strategy, we can empirically compute its four cooperation probabilities, measure the distance to the subspace of extortionate strategies, and use this distance as a measure of the extortionality of a strategy. This method can be applied to any strategy regardless of the memory depth and avoids the algebraic rigidity issues. We apply this method to the largest known corpus of strategies for the iterated prisoner's dilemma and show empirically that the method in fact detects extortionate strategies.

Several theoretical insights that emerge from this work. Infamously, extortionate strategies do not play well with themselves. In cite Press and Dyson claim that a player with a "theory of mind" would rationally chose to cooperate against an opponent that also has knowledge of zero determinant strategies to avoid sustained mutual defection. This is not possible for a strategy of memory depth of one. We show that this behavior is exhibited by relatively simple strategies of memory depth two or more from strategies that emerged from an evolutionary selection process. Similarly, in [**adami2013evolutionary**], Adami and Hintze suggest that the may exist strategies that are able to selectively behave extortionately to some opponents and to others. We show that this is indeed the case for the same evolved strategies. It seems that humans have trouble defining such strategies but evolution is able to simply by optimizing for total payoff in IPD interactions. Accordingly, while resistance to extortionate behavior appears critical to the evolution of cooperation, there is no prohibition on extorting weaker opponents, even in population dynamics.

Much attention has been given to a class of strategies called Zero Determinant strategies. It has been theoretically shown that these strategies can "extort" large classes of other strategies.

This work will conceptually demonstrate that it is possible to recognise this extortion using a linear algebraic approach. Further to this, a large tournament with 204 strategies is considered with which it will be demonstrated that sophisticated strategies exist that do in fact recognise this behaviour and adapt to their opponents. Statistical analysis of these strategies will demonstrated that one of the biggest factors correlated to evolutionary stability is in fact adaptability.

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Agent-based game-theoretic models have become a stalwart of the underpinning mathematics of interactive behaviours. One of the major pieces of work in this area is the pair of computer tournaments run by Robert Axelrod [**Axelrod1980**, **Axelrod1980a**]. These tournaments pitted submitted computer strategies against each other in plays of the Iterated Prisoner's Dilemma, which is a common game where agents can choose to pay a slight cost to their immediate utility in the hope of building a reputation. This has been used in economic and evolutionary game theory to understand the evolution of cooperative behaviour.

In [**Press2012**] a class of strategies that can provably extort given opponents was described. These so called Zero-Determinant (ZD) strategies caused a substantial reaction in the game theoretic community. As stated in [**hilbe2015partners**], the American Mathematical Society's news section stated that 'the world of game theory is currently on fire'. In [**adami2013evolutionary**, **Hilbe2013**, **hilbe2013adaptive**, **hilbe2015partners**, **ichinose2018zero**, **Moran1707**] some questions have already been asked about the true effectiveness of these strategies in an evolutionary setting. For example [**adami2013evolutionary**] showed that ZD strategies were not evolutionarily stable. Furthermore, in that work it was also postulated that 'evolutionarily successful ZD strategies can be designed that use longer memory to distinguish self from non-self'.

ZD strategies are defined by a constrained set of transition probabilities $p \in \mathbb{R}^4$ (so called memory-one strategies). In this work, a procedure is offered that will quantify if any strategy is acting in an extortionate way. A $p \in \mathbb{R}^4$ can be measured from actual interactions, regardless of whether or not the strategy is defined in such a way. In Section 2 a measure is used to capture how close a given $p \in \mathbb{R}^4$ is to being ZD. This allows for the recognition of extortionate behaviour even in the cases of numeric inexactitude that would otherwise imply a strategy was not extortionate. It is also possible to recognise sophisticated strategies that act extortionately against some opponents but not others. This was the concept suggested in [**adami2013evolutionary**] when speaking about being able to distinguish self from non-self.

This work makes use of the Axelrod Python library [**Knight2016**, **Knight2018**] which contains a large number of Prisoner Dilemma strategies available for extensive numerical examples. All of the code and data discussed in Section 3 is open sourced, archived and written according to best scientific principles [**Wilson2014**]. The data archive can be found at [**vincent˙knight˙2018˙1297075**]. In Section 4, this large tournament is complemented with an evolutionary consideration that offers some insight in to the effectiveness of extortionate strategies.

2 Recognising Extortion

Zero Determinant (ZD) strategies are introduced in [**Press2012**] in the context of matches between two memory-one strategies. Memory-one strategies are represented as elements of $\mathbb{R}^4$ mapping a state of $\{C, D\}^2$ to a probability of cooperating. A match between two such strategies creates a Markov chain with transient states $\{C, D\}^2$. The main result of [**Press2012**] is that given two memory-one players $p, q \in \mathbb{R}^4$, a linear relationship between the players' scores can be forced by one of the players.

Using the notation of [**Press2012**], the utilities for player p are given by $S_x = (R, S, T, P)$ and for player q by $S_y = (R, T, S, P)$ and the stationary scores of each player are given by S_X and S_Y respectively. The main result of [**Press2012**] is that if

$$\tilde{p} = \alpha S_x + \beta S_y + \gamma \tag{1}$$

or

$$\tilde{q} = \alpha S_x + \beta S_y + \gamma \tag{2}$$

where $\tilde{p} = (1 - p_1, 1 - p_2, p_3, p_4)$ and $\tilde{q} = (1 - q_1, 1 - q_2, q_3, q_4)$ then:

$$\alpha S_X + \beta S_Y + \gamma = 0 \tag{3}$$

In [**Press2012**] a particular type of ZD strategy is defined: extortionate strategies. If:

$$\gamma = -P(\alpha + \beta) \tag{4}$$

then the player can ensure they get a score χ times larger than the opponent. This extortion coefficient is given by:

$$\chi = \frac{-\beta}{\alpha} \tag{5}$$

Thus, if (4) holds and $\chi > 1$ a player is said to extort their opponent. Here, the reverse problem is considered: given a $p \in \mathbb{R}^4$ how does one identify α, β if they exist and is the strategy in fact acting in an extortionate way?

In this case constraints (1) and (4) correspond to:

$$\tilde{p}_1 = \alpha R + \beta R - P(\alpha + \beta) \tag{6}$$
$$\tilde{p}_2 = \alpha S + \beta T - P(\alpha + \beta) \tag{7}$$
$$\tilde{p}_3 = \alpha T + \beta S - P(\alpha + \beta) \tag{8}$$
$$\tilde{p}_4 = \alpha P + \beta P - P(\alpha + \beta) = 0 \tag{9}$$

Equation (9) ensures that $p_4 = \tilde{p}_4 = 0$. Equations (6-8) can be used to eliminate α, β, giving:

$$\tilde{p}_1 = \frac{(R-P)(\tilde{p}_2 + \tilde{p}_3)}{S + T - 2P} \tag{10}$$

with:

$$\chi = \frac{\tilde{p}_2(P-T) + \tilde{p}_3(S-P)}{\tilde{p}_2(P-S) + \tilde{p}_3(T-P)} \tag{11}$$

Given a strategy $p \in \mathbb{R}^4$ equations (9-11) can be used to check if a strategy is extortionate. The conditions correspond to:

$$p_1 = \frac{(R-P)(p_2 + p_3) - R + T + S - P}{S + T - 2P} \tag{12}$$
$$p_4 = 0 \tag{13}$$
$$1 > p_2 + p_3 \tag{14}$$

The algebraic steps necessary to prove these results are available in the supporting materials, and note that an equivalent formulation was obtained in [**adami2013evolutionary**].

All extortionate strategies reside on a triangular (14) plane (12) in 3 dimensions (13). Using this formulation it can be seen that a necessary (but not sufficient) condition for an extortionate strategy is that it cooperates on average less than 50% of the time when in a state of disagreement with the opponent (14).

As an example, consider the known extortionate strategy $p = (8/9, 1/2, 1/3, 0)$ from [**Stewart2012**] which is referred to as `Extort-2`. In this case, for the standard values of $(R, S, T, P) = (3, 0, 5, 1)$ constraint (12) corresponds to:

$$p_1 = \frac{2(p_2 + p_3) + 1}{3} = \frac{2(1/2 + 1/3) + 1}{3} = \frac{8}{9} \tag{15}$$

It is clear that in this case all constraints hold. As a counterexample, consider the strategy that cooperates 25% of the time: $p = (1/4, 1/4, 1/4, 1/4)$ obeys (14) but is not extortionate as:

$$p_1 \neq \frac{2(p_2 + p_3) + 1}{3} = \frac{2(1/4 + 1/4) + 1}{3} = \frac{2}{3} \tag{16}$$

Not all strategies are memory-one strategies but it is possible to measure a given p from any set of interactions between two strategies. This approach can then be used to confirm that a given strategy is acting in an extortionate manner even if it is not a memory-one strategy. However, in practice, if an exact form for p is not known but measured from observed plays of the game then measurement and/or numerical error might lead to an extortionate strategy not being confirmed as such.

As an example consider Table 1 which shows some actual plays of $p = (8/9, 1/2, 1/3, 0)$ against an alternating strategy ($p = (0, 0, 1, 1)$):

Turn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
(8/9, 1/2, 1/3, 0)	C	C	D	D	D	C	D	D	D	D	D	C	C	C	D	D	D	C	D	D
Alternator	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D	C	D

Table 1: A seeded play of 20 turns of two strategies.

In this particular instance the measured value of p for the known extortionate strategy would be: $(2/2, 1/5, 3/8, 0/4)$ which does not fit the definition of a ZD strategy.

Note that measurement of behaviour might in some cases lead to missing values. For example the strategy $p = (8/9, 1/2, 1/3, 0)$ when playing against a cooperator will in fact never visit any state which would allow to measure p_3 and p_4. To overcome this, it is proposed that if s is a state that is not visited than p_s is approximated using a sensible prior. In Section 3 the overall cooperation rate is used. Another approach to overcoming this measurement error would be to measure our strategies in a minimalistically noisy environment.

Identifying if a strategy is ZD strategy can be written in the following linear algebraic form where $x = (\alpha, \beta)$ and $p^* = (\tilde{p}_1 - 1, \tilde{p}_2 - 1, \tilde{p}_3, \tilde{p}_4)$:

$$Cx = p^* \tag{17}$$

C corresponds to equations (6-8) and is given by:

$$C = \begin{bmatrix} R-P & R-P \\ S-P & T-P \\ T-P & S-P \\ 0 & 0 \end{bmatrix} \tag{18}$$

Note that in general, equation (17) will not necessarily have a solution. From the Rouché-Capelli theorem if there is a solution it is unique as $\text{rank}(C) = 2$ which is the dimension of the variable x. The best fitting x^* is defined by:

$$x^* = \text{argmin}_{x \in \mathbb{R}^2} \|Cx - p^*\|_2^2 \tag{19}$$

Known results [**kutner2004applied**, **rao1973linear**, **wakefield2013bayesian**] can now be applied:

$$x^* = \left(C^T C\right)^{-1} C^T p^* \tag{20}$$

This x^* corresponds to the nearest ZD strategy to the measured p. It is in fact a normal projection of p on to the plane defined by (12).

The squared norm of the remaining error is referred to as sum of squared errors of prediction (SSE):

$$\text{SSE} = \|Cx^* - p^*\|_2^2 \tag{21}$$

This gives expressions for α, β as $\alpha = x_1^*$ and $\beta = x_2^*$ thus the conditions for a strategy to be acting extortionately becomes:

$$-x_2^* < x_1^* \tag{22}$$

A further known result [**kutner2004applied**, **rao1973linear**, **wakefield2013bayesian**] gives an expression for SSE:

$$\text{SSE} = {p^*}^T p^* - p^* C \left(C^T C\right)^{-1} C^T p^* = {p^*}^T p^* - p^* C x^* \tag{23}$$

Using this approach, the memory-one representation $p \in \mathbb{R}^4$ of any strategy against any other can can be measured and if (22) holds then (23) can be used to identify if a strategy is acting extortionately.

For a measured p, SSE corresponds to the best fitting α, β. Suspicion of extortion then corresponds to a threshold on SSE and a comparison of the measured $\chi = \frac{-\beta}{\alpha}$.

Comparing theoretic and actual plays of the IPD is not novel, see for example [**Rand2013**].

In the next section, this idea will be illustrated by observing the interactions that take place in a large computer based tournament of the IPD.

3 Numerical experiments

In [**Stewart2012**] results from a tournament with 19 strategies, were presented with specific consideration given to ZD strategies. This tournament is reproduced here using the Axelrod-Python library [**Knight2016**]. To obtain a good measure of the corresponding transition rates for each strategy all matches have been run for 2000 turns and every match has been repeated 60 times. All of this interaction data is available at [**vincent˙knight˙2018˙1297075**]. Note that in the interest of open scientific practice, [**vincent˙knight˙2018˙1297075**] also contains interaction data for noisy and probabilistic ending interactions which are not investigated here.

Figure 1 shows the SSE values for all the strategies in the tournament, as reported in [**Stewart2012**] the extortionate strategy (which has an expected SSE approximately 0) gains a large number of wins.

Here, the work of [**Stewart2012**] is extended by investigating a tournament with 204 strategies. The results of this analysis are shown in Figure 2. The top ranking strategies by number of wins act in an extortionate way (but not against all strategies) and it can be seen that a small subgroup of strategies achieve mutual defection. All the top ranking strategies according to score achieve mutual cooperation and do not extort each other, however they **do** exhibit extortionate behaviour towards a number of the lower ranking strategies.

Figure 3 shows the relationship between χ and SSE, whilst Figure 2 seems to indicate that if a strategy has a low SSE then it is acting extortionately $\chi > 1$ this is not necessarily the case.

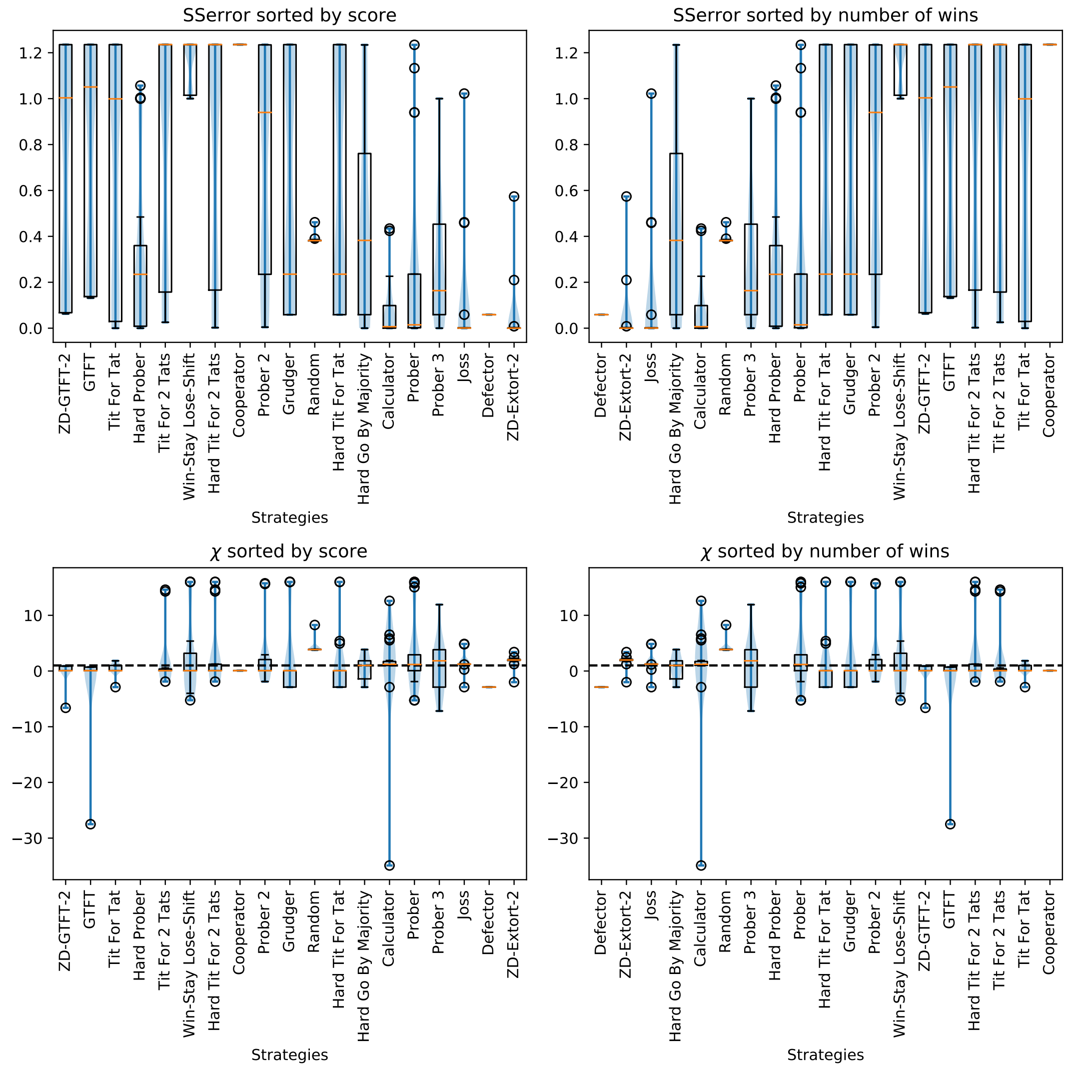

Figure 1: SSE and χ for [**Stewart2012**], ordered both by number of wins and overall score. The dashed line shows the $\chi = 1$ boundary highlighting which strategies act in an extortionate manner. The strategies which a low variation in SSE and high χ win the most matches, although even the known extortionate strategy does not act in a perfectly extortionate manner in all matches. The strategies with a high score have a large variation in SSE.

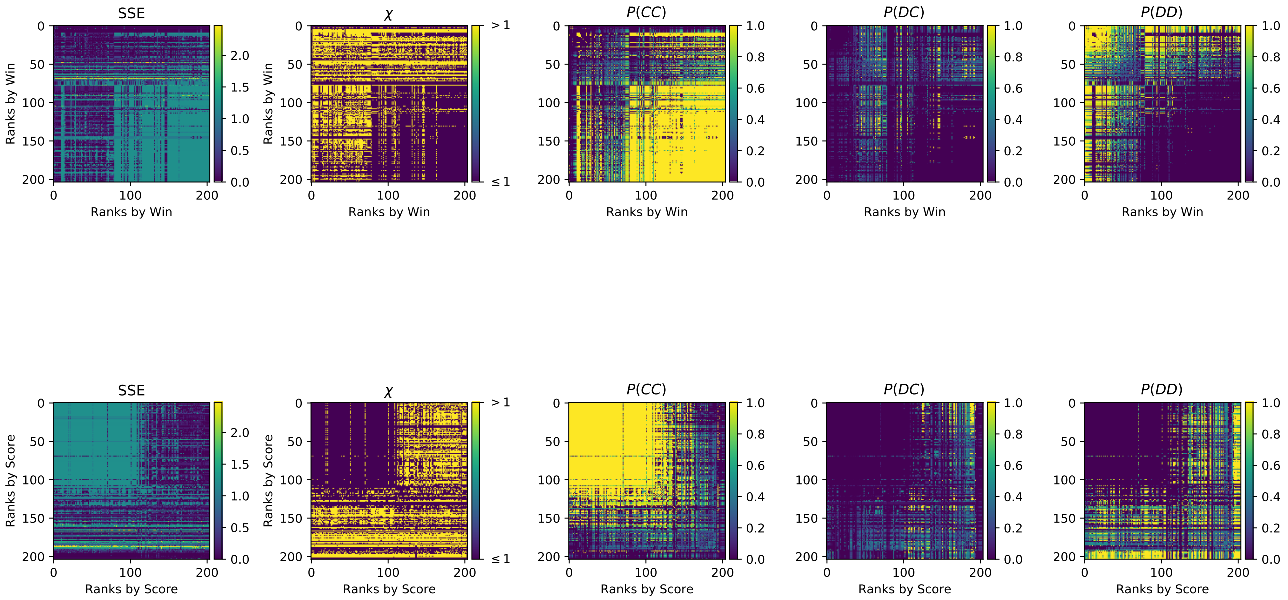

Figure 2: SSE, χ and state probabilities for the strategies for the full tournament. Note that $P(CD)$ is not shown as it corresponds to the transpose of $P(DC)$. The strategies with high number of wins have a low SSE and $\chi > 1$. The strategies with a high score have a high SSE against the other high scoring strategies but a low SSE and high χ against the low scoring strategies.

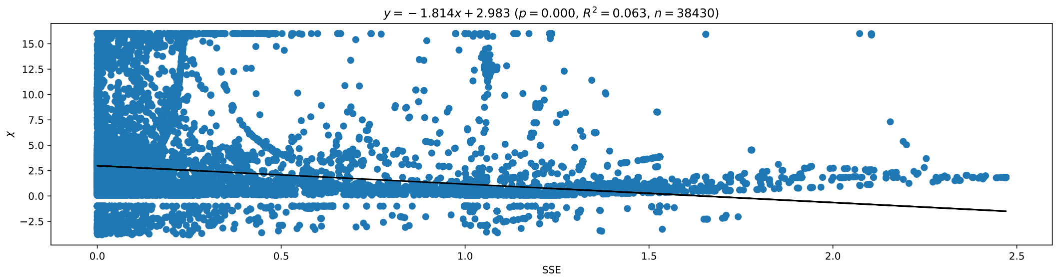

Figure 3: Linear relationship between SSE and χ, note that 5% and 95% confidence levels are considered to ignore extreme values of χ. Whilst a significant relationship between SSE and χ exists it is clear that having a low SSE does not guarantee extortionate behaviour.

A detailed look at selected strategies is given in Table 2 and Figure 4. The high scoring strategies presented have a large variation in SSE whilst the ZD strategies have a low score but high probability of winning. A version of Figure 4 with all the strategies considered is available in the appendix and gives the same conclusion. This evidences an idea proposed in [**adami2013evolutionary**]: sophisticated strategies are able to recognise their opponent and defend themselves against extortion. The high ranking strategies were in fact trained to maximise score [**Harper2017**] which seems to have created strategies able to extort weaker strategies whilst cooperating with stronger ones. Indeed unconditional extortion is self defeating.

Rank	Name	Score per turn	P(Win)	$P(CC)$	Median χ	5% CI SSerror	Mean SSerror	Std SSerror	95% CI SSerror
1	EvolvedLookerUp2_2_2	2.944	0.230	0.673	0.063	0.235	1.057	0.400	1.235
2	Evolved HMM 5	2.944	0.205	0.718	0.063	0.078	0.796	0.542	1.235
3	PSO Gambler 2_2_2	2.913	0.204	0.624	0.063	0.084	0.899	0.505	1.235
4	PSO Gambler Mem1	2.908	0.211	0.715	0.063	0.065	0.705	0.577	1.235
5	PSO Gambler 1_1_1	2.906	0.221	0.696	0.063	0.055	0.737	0.544	1.235
7	Evolved ANN 5	2.893	0.225	0.682	0.063	0.001	0.804	0.578	1.235
31	ZD-GTFT-2	2.721	0.000	0.806	0.063	0.064	0.786	0.537	1.235
45	ZD-GEN-2	2.689	0.016	0.801	0.063	0.015	0.694	0.599	1.235
47	Eatherley	2.682	0.000	0.828	0.063	0.007	0.895	0.474	1.235
69	Tit For Tat	2.638	0.000	0.723	0.063	0.000	0.773	0.549	1.235
75	Grumpy	2.630	0.075	0.793	0.063	0.000	0.978	0.495	1.235
88	Win-Stay Lose-Shift	2.616	0.099	0.649	0.063	1.000	1.172	0.164	1.235
103	Eventual Cycle Hunter	2.565	0.067	0.770	0.063	0.001	0.728	0.597	1.235
107	Tricky Level Punisher	2.537	0.062	0.828	0.063	0.000	0.710	0.595	1.235
127	Adaptive	2.272	0.500	0.363	-1.000	0.000	0.084	0.098	0.274
169	Bully	1.970	0.381	0.141	-1.000	0.200	1.373	0.375	1.529
179	Alternator	1.945	0.392	0.157	3.857	0.779	1.332	0.347	1.529
181	Negation	1.941	0.356	0.141	-1.000	1.130	1.470	0.288	1.529
183	Cycler DC	1.931	0.324	0.149	3.857	0.382	1.279	0.374	1.529
188	Hopeless	1.908	0.352	0.261	1.833	1.235	2.247	0.372	2.471
194	Gradual Killer	1.892	0.354	0.439	0.063	0.000	0.254	0.326	1.004
196	Aggravater	1.879	0.930	0.087	-2.889	0.059	0.163	0.256	1.023
200	ZD-Extort-2	1.821	0.851	0.179	2.005	0.000	0.019	0.094	0.010
201	ZD-Extort-4	1.820	0.865	0.106	4.003	0.000	0.021	0.069	0.204
202	ZD-Extort3	1.810	0.862	0.133	3.028	0.000	0.015	0.070	0.017
203	Defector	1.808	0.929	0.000	-2.889	0.059	0.059	0.000	0.059
204	Handshake	1.806	0.870	0.046	-2.888	0.000	0.126	0.288	1.200

Table 2: Summary of results for a selected list of strategies. Similarly to Figure 1, the high scoring strategies have a higher standard deviation of SSE. The strategies with a large number of wins have a low SSE and low variation of SSE. Note that a value of $\chi = 0.063$ and SSE $= 1.235$ corresponds to a vector $p = (1, 1, 1, 1)$ which highlights that the high scoring strategies, adapt and in fact cooperate often. A graphical representation of this table is given in Figure 4).

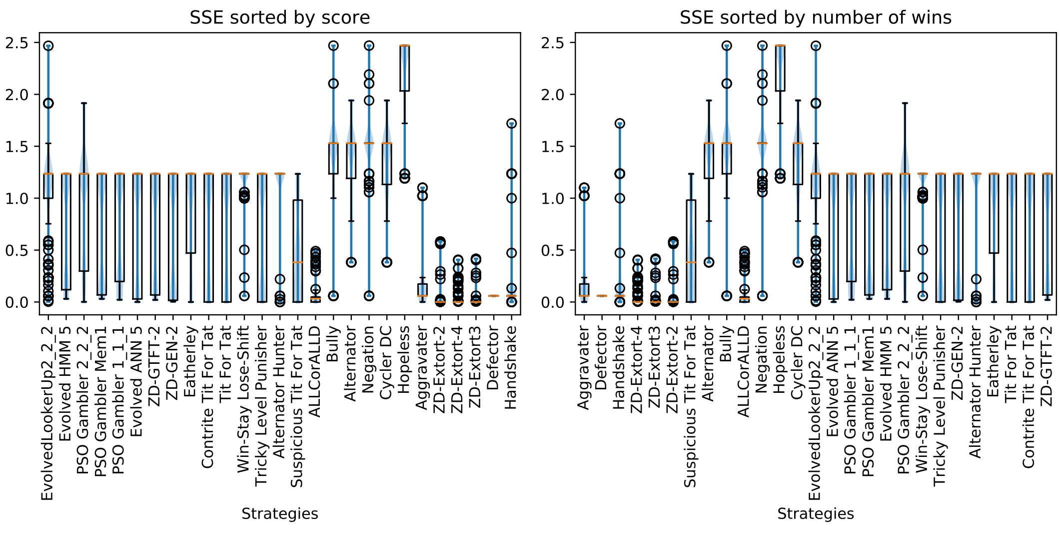

Figure 4: SSE and χ for the same selection of strategies as Table 2. A similar conclusion to that of Figure 1 can be made: the strategies that win often do not have a large variation of SSE whilst the strategies that score highly do.

4 Evolutionary dynamics

From the large number of interactions a payoff matrix S can be measured where S_{ij} denotes the score (using standard values of $(R, S, T, P) = (3, 0, 5, 1)$) of the ith strategy against the jth strategy. Using this, the replicator equation describes the evolution of the system based on a population density fitness function:

$$\frac{dx_i}{dt} = x_i((Sx)_i - x^T Sx) \tag{24}$$

Equation (24) is solved numerically through an integration technique described in [**Petzold1983**] and Figure 5 shows the evolution of the distribution of the system: the various strategies are ranked by scores. It is clear to see that only the high ranking strategies survive the evolutionary process (in fact, only 25 have a final long run probability value greater than 10^{-2}).

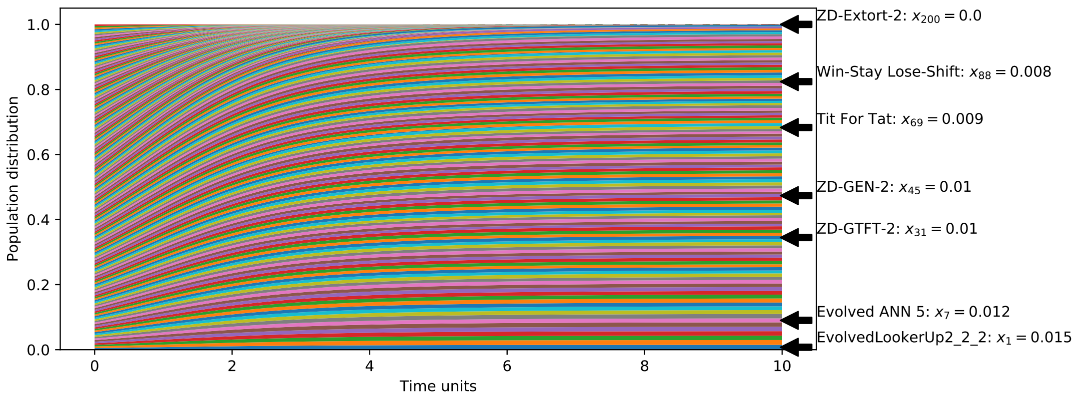

Figure 5: Numerical simulation of the replicator equation (24): strategies are ordered by score. Some selected strategies are highlighted with their long run population distribution.

In Figure 6 linear models are fitted to the mean and variance of SSE against the long run probabilities s of (24). In both cases the predictive power of these models is small (a low R^2) however, we note a significant positive correlation between the variance of SSE and s. Indeed: strategies that perform strongly according to equation (24) seem to be strategies that are able to modify their memory one representation depending on the opponent. Whilst the specific memory one representation might not be one that acts extortionately, a high SSE does imply that a strategy is not extortionate. A general linear model is shown in Table 3 with stronger predictive power and giving similar conclusions.

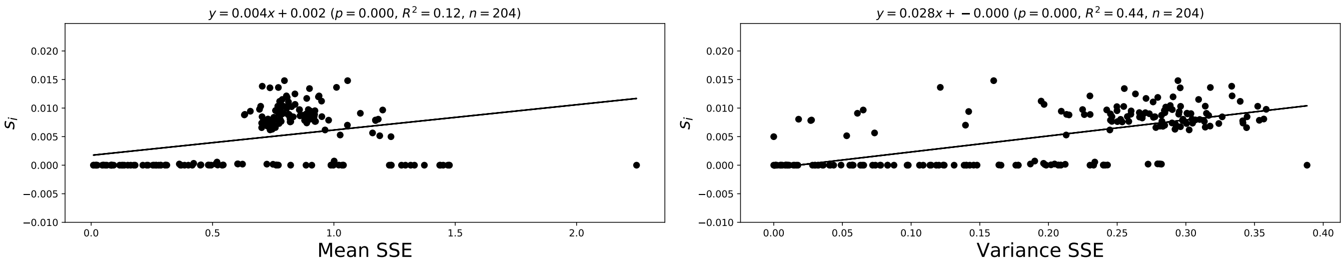

Figure 6: Linear regression analysis of long run probabilities of (24) against the mean, and variance of SSE (for a given strategy).

Dep. Variable:	s_i	**R-squared:**	0.667
Model:	OLS	**Adj. R-squared:**	0.662
Method:	Least Squares	**F-statistic:**	133.3
Date:	Mon, 19 Nov 2018	**Prob (F-statistic):**	1.82e-47
Time:	16:09:10	**Log-Likelihood:**	916.43
No. Observations:	204	**AIC:**	-1825.
Df Residuals:	200	**BIC:**	-1812.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	0.0007	0.001	1.351	0.178	-0.000	0.002
('residual', 'mean')	-0.0131	0.001	-9.054	0.000	-0.016	-0.010
('residual', 'median')	0.0136	0.001	11.172	0.000	0.011	0.016
('residual', 'var')	0.0056	0.003	2.163	0.032	0.000	0.011

Omnibus:	13.613	**Durbin-Watson:**	1.697
Prob(Omnibus):	0.001	**Jarque-Bera (JB):**	21.684
Skew:	0.383	**Prob(JB):**	1.96e-05
Kurtosis:	4.401	**Cond. No.**	24.9

Table 3: General linear model. This shows that strategies with a high mean and low median are more likely to survive the evolutionary dynamics. This corresponds to negatively skewed distributions of SSE which again highlights the importance of adaptability.

In [**Moran1707**] a large data set of pairwise fixation probabilities in the Moran process is made available at [**vincent˙knight˙2**

Figure 7 shows linear models fitted to three summary measures of SSE and the mean (over population size N and opponents) value of $x_1 \cdot N$. This specific measure of fixation is chosen as x_1 is usually compared to the neutral fixation probability of $1/N$. As was noted in [**Moran1707**], the specific case of $N = 2$ differs from all other population sizes which is why it is presented in isolation. Similarly to the conclusions from Figure 6 we note that there is a significant relationship between the variability of SSE and the ability for a strategy to become fixed.

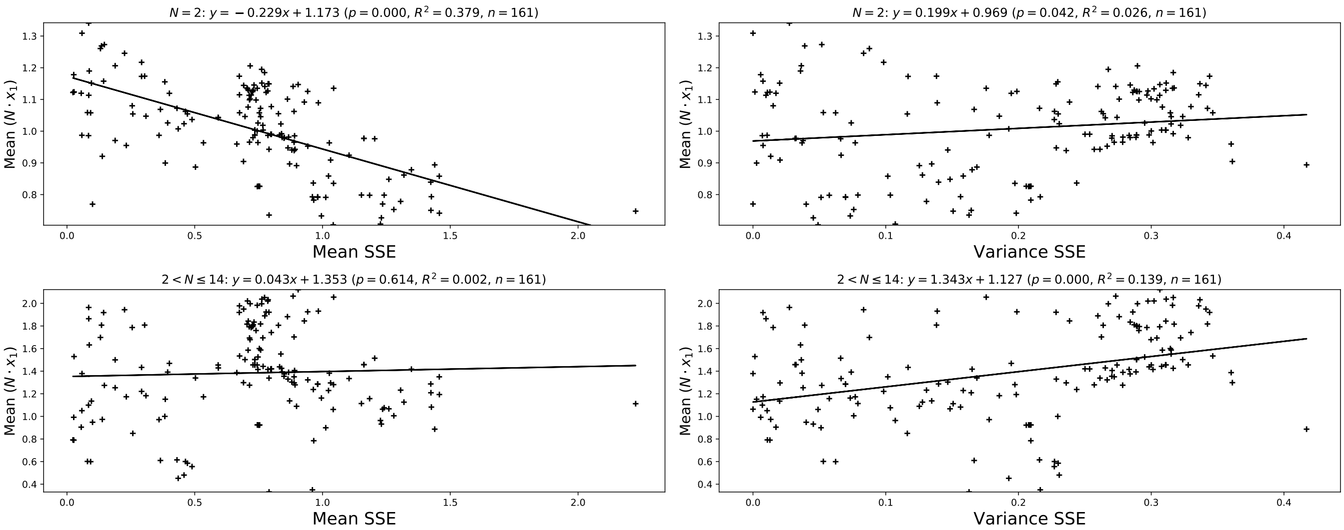

Figure 7: Linear regression analysis of pairwise fixation probabilities from [**Moran1707**] against the mean and variance of SSE (for a given strategy averaged over all opponents and population sizes).

Using recursive feature elimination a better linear model can be found that compliments these results. This is shown in Table 4.

Dep. Variable:	mean	**R-squared:**	0.319
Model:	OLS	**Adj. R-squared:**	0.310
Method:	Least Squares	**F-statistic:**	36.53
Date:	Mon, 19 Nov 2018	**Prob (F-statistic):**	9.74e-14
Time:	16:08:50	**Log-Likelihood:**	-42.272
No. Observations:	159	**AIC:**	90.54
Df Residuals:	156	**BIC:**	99.75
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	1.2815	0.056	22.993	0.000	1.171	1.392
('residual', 'mean')	-1.0620	0.145	-7.323	0.000	-1.348	-0.776
('residual', 'median')	0.9037	0.106	8.535	0.000	0.695	1.113

Omnibus:	2.302	**Durbin-Watson:**	1.716
Prob(Omnibus):	0.316	**Jarque-Bera (JB):**	1.850
Skew:	-0.199	**Prob(JB):**	0.397
Kurtosis:	3.348	**Cond. No.**	11.2

Table 4: General linear model. This shows that strategies with a high mean and low median are likely to be evolutionarily stable. This corresponds to negatively skewed distributions of SSE which again highlights the importance of adaptability.

These findings confirm the work of [**Moran1707**] in which sophisticated strategies resist evolutionary invasion of shorter memory strategies. This also confirms the work of [**adami2013evolutionary**, **hilbe2015partners**] which proved that ZD strategies where not evolutionarily stable due to the fact that they score poorly against themselves.

The work also provides strong evidence to the importance of adaptability: strategies that offer a variety of behaviours corresponding to a higher standard deviation of SSE are significantly more likely to survive the evolutionary process. This corresponds to the following quote of [**darwin1869origin**]:

> "It is not the most intellectual of the species that survives; it is not the strongest that survives; but the species that survives is the one that is able to adapt to and to adjust best to the changing environment in which it finds itself."

5 Conclusion

This work defines an approach to measure whether or not a player is playing a strategy that corresponds to an extortionate strategy as defined in [**Press2012**]: a mathematical model for suspicion. All extortionate strategies have been classified as lying on a triangular plane. This rigorous classification fails to be robust to small measurement error, thus a statistical approach is proposed approximating the solution of a linear system. Using this, a large number of pairwise interactions is simulated.

The work of [**Press2012**], whilst showing that a clever approach to taking advantage of another memory-one strategy exists: this is not the full story. Though the elegance of this result is very attractive, just as the simplicity of the victory of Tit For Tat in Axelrod's original tournaments was, it is incomplete. Extortionate strategies achieve a high number of wins but they do not achieve a high score and fail to be evolutionarily stable.

Instead, it is in fact the more sophisticated strategies that are able to adapt to their opponent and act extortionately against weaker strategies and cooperate with like minded strategies that perform well.

Following Axelrod's seminal work [**Axelrod1980**, **Axelrod1980a**], it was commonly thought that evolutionary cooperation required strategies that followed a simple set of rules. The discovery/definition of extortionate strategies [**Press2012**] seemingly showed that complex strategies could be taken advantage of. In this manuscript it has been shown that not only is it possible to detect and prevent extortionate behaviour but that more complex strategies can be evolutionary stable. The complex strategies in question were obtained through reinforcement learning approaches [**Harper2017**, **Moran1707**]. Thus, this demonstrates that it is possible to recognise extortion, both theoretically using SSE but also that this ability can develop through reinforcement learning. This work shows the possibility for the evolution of cooperation through suspicion.

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- The Axelrod [**Knight2016**, **Knight2018**] library (IPD strategies and tournaments).
- The sympy library [**Meurer2017**] (verification of all symbolic calculations).
- The matplotlib [**Droettboom2018**] library (visualisation).
- The pandas [**Structures2010**], dask [**Dask2016**] and NumPy [**Oliphant2015**] libraries (data manipulation).
- The SciPy [**Jones2001**] library (numerical integration of the replicator equation).

This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

Proof of algebraic condition for extortionate strategies

The defining equations for an extortionate strategy are:

$$\tilde{p}_1 = \alpha(R-P) + \beta(R-P) \tag{1}$$
$$\tilde{p}_2 = \alpha(S-P) + \beta(T-P) \tag{2}$$
$$\tilde{p}_3 = \alpha(T-P) + \beta(S-P) \tag{3}$$
$$\tilde{p}_4 = 0 \tag{4}$$

Using equation (2), α is isolated

$$\alpha = \frac{-\beta(P-T) - \tilde{p}_2}{P-S} \tag{5}$$

Substituting this value in to equation (3), β is isolated:

$$\beta = -\frac{P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1}{(S-T)(2P-S-T)} \tag{6}$$

Substituting this back in to (5) gives:

$$\alpha = \frac{-\tilde{p}_2 + (P-T)(P\tilde{p}_1 - P\tilde{p}_2 + S\tilde{p}_2 - T\tilde{p}_1)}{(S-T)(2P-S-T)(P-S)} \tag{7}$$

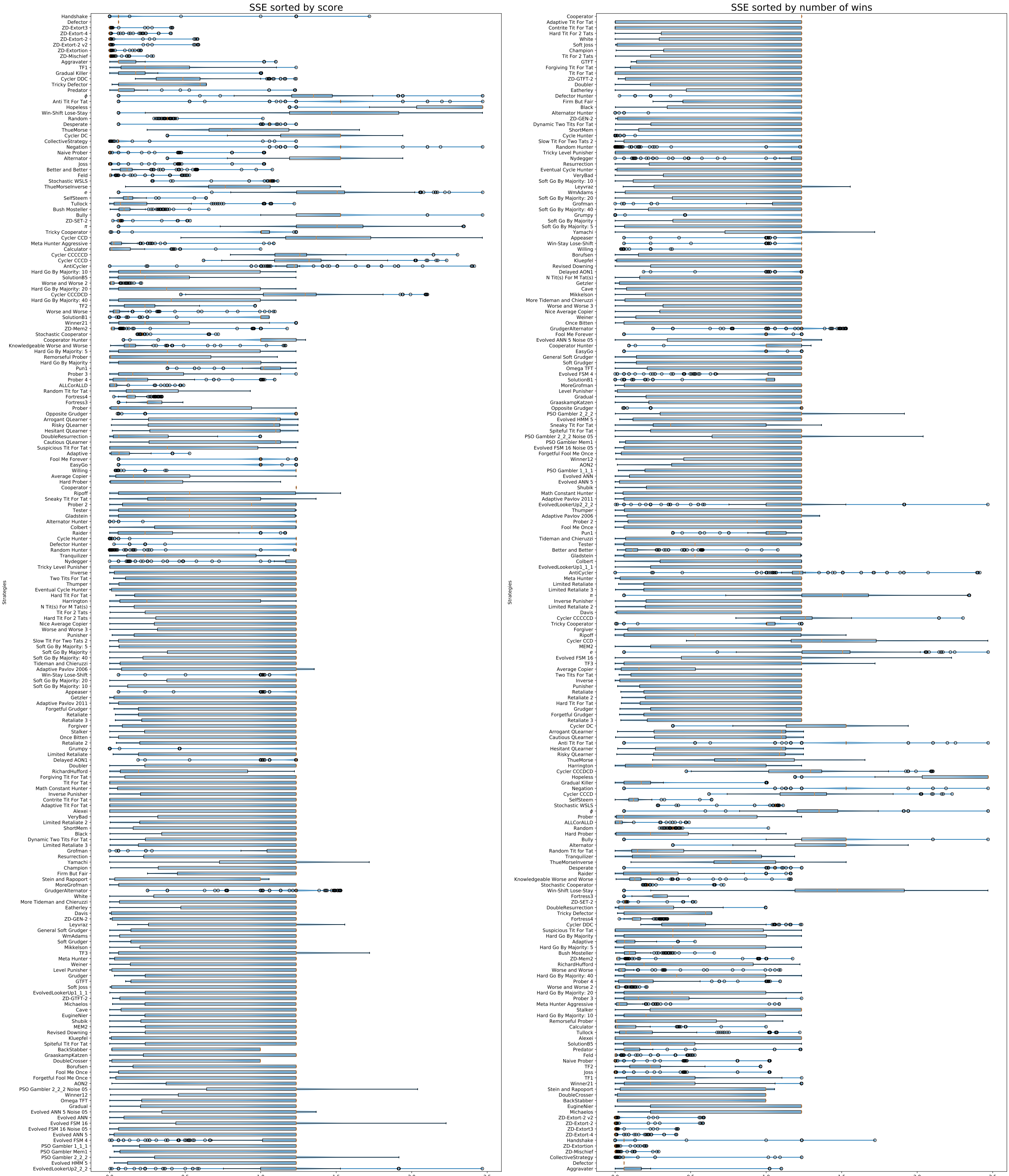
Substituting equations (6-7) in to equation (1) gives the required expression for p_1.
Taking the ratio of equations (6-7) gives the required expression for χ.
Finally, the condition $\chi > 1$ corresponds to:

$$\tilde{p}_2(P-T) + \tilde{p}_3(S-P) > \tilde{p}_2(P-S) + \tilde{p}_3(T-P) \tag{8}$$

which can be simplified to:

$$\tilde{p}_2 < -\tilde{p}_3 \tag{9}$$

recalling that $\tilde{p}_2 = p_2 - 1$ and $\tilde{p}_3 = p_3$ gives the required result.



SSE sorted by score
SSE sorted by number of wins
Strategies
Strategies
0.0
0.5
1.0
1.5
2.0
2.5

List of all strategies used from [21]

1. Adaptive - Deterministic - Memory length: ∞ - [25]
2. Adaptive Tit For Tat: 0.5 - Deterministic - Memory length: ∞ - [39]
3. Aggravater - Deterministic - Memory length: ∞ - [21]
4. Alexei: (D,) - Deterministic - Memory length: ∞ - [43]
5. ALLCorALLD - Stochastic - Memory length: 1 - [2]
6. Alternator - Deterministic - Memory length: 1 - [31, 11]
7. Alternator Hunter - Deterministic - Memory length: ∞ - [21]
8. AntiCycler - Deterministic - Memory length: ∞ - [21]
9. Anti Tit For Tat - Deterministic - Memory length: 1 - [18]
10. AON2 - Deterministic - Memory length: 2 - [19]
11. Adaptive Pavlov 2006 - Deterministic - Memory length: ∞ - [24]
12. Adaptive Pavlov 2011 - Deterministic - Memory length: ∞ - [25]
13. Appeaser - Deterministic - Memory length: ∞ - [21]
14. Arrogant QLearner - Stochastic - Memory length: ∞ - [21]
15. Average Copier - Stochastic - Memory length: ∞ - [21]
16. BackStabber: (D, D) - Deterministic - Memory length: ∞ - [21]
17. Better and Better - Stochastic - Memory length: ∞ - [29]
18. Black - Stochastic - Memory length: 5 - [10]
19. Borufsen - Deterministic - Memory length: ∞ - [10]
20. Bully - Deterministic - Memory length: 1 - [32]
21. Bush Mosteller: 0.5, 0.5, 3.0, 0.5 - Stochastic - Memory length: ∞ - [20]
22. Calculator - Stochastic - Memory length: ∞ - [29]
23. Cautious QLearner - Stochastic - Memory length: ∞ - [21]
24. Cave - Stochastic - Memory length: ∞ - [10]
25. Champion - Stochastic - Memory length: ∞ - [10]
26. Colbert - Deterministic - Memory length: 4 - [10]
27. CollectiveStrategy - Deterministic - Memory length: ∞ - [26]
28. Contrite Tit For Tat - Deterministic - Memory length: 3 - [42]
29. Cooperator - Deterministic - Memory length: 0 - [31, 11, 34]
30. Cooperator Hunter - Deterministic - Memory length: ∞ - [21]
31. Cycle Hunter - Deterministic - Memory length: ∞ - [21]
32. Cycler CCCCCD - Deterministic - Memory length: 5 - [21]

33. Cycler CCCD - Deterministic - Memory length: 3 - [21]
34. Cycler CCD - Deterministic - Memory length: 2 - [31]
35. Cycler DC - Deterministic - Memory length: 1 - [21]
36. Cycler DDC - Deterministic - Memory length: 2 - [31]
37. Cycler CCCDCD - Deterministic - Memory length: 5 - [21]
38. Davis: 10 - Deterministic - Memory length: ∞ - [9]
39. Defector - Deterministic - Memory length: 0 - [31, 11, 34]
40. Defector Hunter - Deterministic - Memory length: ∞ - [21]
41. Desperate - Stochastic - Memory length: 1 - [41]
42. Delayed AON1 - Deterministic - Memory length: 2 - [19]
43. DoubleCrosser: (D, D) - Deterministic - Memory length: ∞ - [21]
44. Doubler - Deterministic - Memory length: ∞ - [29]
45. DoubleResurrection - Deterministic - Memory length: 5 - [15]
46. EasyGo - Deterministic - Memory length: ∞ - [25, 29]
47. Eatherley - Stochastic - Memory length: ∞ - [10]
48. EugineNier: (D,) - Deterministic - Memory length: ∞ - [43]
49. Eventual Cycle Hunter - Deterministic - Memory length: ∞ - [21]
50. Evolved ANN - Deterministic - Memory length: ∞ - [21]
51. Evolved ANN 5 - Deterministic - Memory length: ∞ - [21]
52. Evolved ANN 5 Noise 05 - Deterministic - Memory length: ∞ - [21]
53. Evolved FSM 4 - Deterministic - Memory length: 4 - [21]
54. Evolved FSM 16 - Deterministic - Memory length: 16 - [21]
55. Evolved FSM 16 Noise 05 - Deterministic - Memory length: 16 - [21]
56. EvolvedLookerUp1_1_1 - Deterministic - Memory length: ∞ - [21]
57. EvolvedLookerUp2_2_2 - Deterministic - Memory length: ∞ - [21]
58. Evolved HMM 5 - Stochastic - Memory length: 5 - [21]
59. Feld: 1.0, 0.5, 200 - Stochastic - Memory length: 200 - [9]
60. Firm But Fair - Stochastic - Memory length: 1 - [16]
61. Fool Me Forever - Deterministic - Memory length: ∞ - [21]
62. Fool Me Once - Deterministic - Memory length: ∞ - [21]
63. Forgetful Fool Me Once: 0.05 - Stochastic - Memory length: ∞ - [21]
64. Forgetful Grudger - Deterministic - Memory length: 10 - [21]
65. Forgiver - Deterministic - Memory length: ∞ - [21]
66. Forgiving Tit For Tat - Deterministic - Memory length: ∞ - [21]
67. Fortress3 - Deterministic - Memory length: 3 - [7]
68. Fortress4 - Deterministic - Memory length: 4 - [7]
69. GTFT: 0.33 - Stochastic - Memory length: 1 - [17, 33]

70. General Soft Grudger: n=1,d=4,c=2 - Deterministic - Memory length: ∞ - [21]
71. Getzler - Stochastic - Memory length: ∞ - [10]
72. Gladstein - Deterministic - Memory length: ∞ - [10]
73. Soft Go By Majority - Deterministic - Memory length: ∞ - [31, 11, 10]
74. Soft Go By Majority: 10 - Deterministic - Memory length: 10 - [21]
75. Soft Go By Majority: 20 - Deterministic - Memory length: 20 - [21]
76. Soft Go By Majority: 40 - Deterministic - Memory length: 40 - [21]
77. Soft Go By Majority: 5 - Deterministic - Memory length: 5 - [21]
78. ϕ - Deterministic - Memory length: ∞ - [21]
79. GraaskampKatzen - Deterministic - Memory length: ∞ - [10]
80. Gradual - Deterministic - Memory length: ∞ - [13]
81. Gradual Killer: (D, D, D, D, D, C, C) - Deterministic - Memory length: ∞ - [29]
82. Grofman - Stochastic - Memory length: ∞ - [9]
83. Grudger - Deterministic - Memory length: 1 - [12, 25, 13, 41, 9]
84. GrudgerAlternator - Deterministic - Memory length: ∞ - [29]
85. Grumpy: Nice, 10, -10 - Deterministic - Memory length: ∞ - [21]
86. Handshake - Deterministic - Memory length: ∞ - [35]
87. Hard Go By Majority - Deterministic - Memory length: ∞ - [31]
88. Hard Go By Majority: 10 - Deterministic - Memory length: 10 - [21]
89. Hard Go By Majority: 20 - Deterministic - Memory length: 20 - [21]
90. Hard Go By Majority: 40 - Deterministic - Memory length: 40 - [21]
91. Hard Go By Majority: 5 - Deterministic - Memory length: 5 - [21]
92. Hard Prober - Deterministic - Memory length: ∞ - [29]
93. Hard Tit For 2 Tats - Deterministic - Memory length: 3 - [38]
94. Hard Tit For Tat - Deterministic - Memory length: 3 - [40]
95. Harrington - Stochastic - Memory length: ∞ - [10]
96. Hesitant QLearner - Stochastic - Memory length: ∞ - [21]
97. Hopeless - Stochastic - Memory length: 1 - [41]
98. Inverse - Stochastic - Memory length: ∞ - [21]
99. Inverse Punisher - Deterministic - Memory length: ∞ - [21]
100. Joss: 0.9 - Stochastic - Memory length: 1 - [38, 9]
101. Kluepfel - Stochastic - Memory length: ∞ - [10]
102. Knowledgeable Worse and Worse - Stochastic - Memory length: ∞ - [21]
103. Level Punisher - Deterministic - Memory length: ∞ - [15]
104. Leyvraz - Stochastic - Memory length: 3 - [10]
105. Limited Retaliate: 0.1, 20 - Deterministic - Memory length: ∞ - [21]
106. Limited Retaliate 2: 0.08, 15 - Deterministic - Memory length: ∞ - [21]

107. Limited Retaliate 3: 0.05, 20 - Deterministic - Memory length: ∞ - [21]
108. Math Constant Hunter - Deterministic - Memory length: ∞ - [21]
109. Naive Prober: 0.1 - Stochastic - Memory length: 1 - [25]
110. MEM2 - Deterministic - Memory length: ∞ - [27]
111. Michaelos: (D,) - Stochastic - Memory length: ∞ - [43]
112. Mikkelson - Deterministic - Memory length: ∞ - [10]
113. MoreGrofman - Deterministic - Memory length: 8 - [10]
114. More Tideman and Chieruzzi - Deterministic - Memory length: ∞ - [10]
115. Negation - Stochastic - Memory length: 1 - [40]
116. Nice Average Copier - Stochastic - Memory length: ∞ - [21]
117. N Tit(s) For M Tat(s): 3, 2 - Deterministic - Memory length: 3 - [21]
118. Nydegger - Deterministic - Memory length: 3 - [9]
119. Omega TFT: 3, 8 - Deterministic - Memory length: ∞ - [37]
120. Once Bitten - Deterministic - Memory length: 12 - [21]
121. Opposite Grudger - Deterministic - Memory length: ∞ - [21]
122. π - Deterministic - Memory length: ∞ - [21]
123. Predator - Deterministic - Memory length: 9 - [7]
124. Prober - Deterministic - Memory length: ∞ - [25]
125. Prober 2 - Deterministic - Memory length: ∞ - [29]
126. Prober 3 - Deterministic - Memory length: ∞ - [29]
127. Prober 4 - Deterministic - Memory length: ∞ - [29]
128. Pun1 - Deterministic - Memory length: 2 - [6]
129. PSO Gambler 1_1_1 - Stochastic - Memory length: ∞ - [21]
130. PSO Gambler 2_2_2 - Stochastic - Memory length: ∞ - [21]
131. PSO Gambler 2_2_2 Noise 05 - Stochastic - Memory length: ∞ - [21]
132. PSO Gambler Mem1 - Stochastic - Memory length: 1 - [21]
133. Punisher - Deterministic - Memory length: ∞ - [21]
134. Raider - Deterministic - Memory length: 3 - [8]
135. Random: 0.5 - Stochastic - Memory length: 0 - [39, 9]
136. Random Hunter - Deterministic - Memory length: ∞ - [21]
137. Random Tit for Tat: 0.5 - Stochastic - Memory length: 1 - [21]
138. Remorseful Prober: 0.1 - Stochastic - Memory length: 2 - [25]
139. Resurrection - Deterministic - Memory length: 5 - [15]
140. Retaliate: 0.1 - Deterministic - Memory length: ∞ - [21]
141. Retaliate 2: 0.08 - Deterministic - Memory length: ∞ - [21]
142. Retaliate 3: 0.05 - Deterministic - Memory length: ∞ - [21]
143. Revised Downing: True - Deterministic - Memory length: ∞ - [9]

144. RichardHufford - Deterministic - Memory length: ∞ - [10]
145. Ripoff - Deterministic - Memory length: 2 - [5]
146. Risky QLearner - Stochastic - Memory length: ∞ - [21]
147. SelfSteem - Stochastic - Memory length: ∞ - [14]
148. ShortMem - Deterministic - Memory length: 10 - [14]
149. Shubik - Deterministic - Memory length: ∞ - [9]
150. Slow Tit For Two Tats 2 - Deterministic - Memory length: 2 - [29]
151. Sneaky Tit For Tat - Deterministic - Memory length: ∞ - [21]
152. Soft Grudger - Deterministic - Memory length: 6 - [25]
153. Soft Joss: 0.9 - Stochastic - Memory length: 1 - [29]
154. SolutionB1 - Deterministic - Memory length: 3 - [4]
155. SolutionB5 - Deterministic - Memory length: 5 - [4]
156. Spiteful Tit For Tat - Deterministic - Memory length: ∞ - [29]
157. Stalker: (D,) - Stochastic - Memory length: ∞ - [14]
158. Stein and Rapoport: 0.05: (D, D) - Deterministic - Memory length: ∞ - [9]
159. Stochastic Cooperator - Stochastic - Memory length: 1 - [1]
160. Stochastic WSLS: 0.05 - Stochastic - Memory length: 1 - [3]
161. Suspicious Tit For Tat - Deterministic - Memory length: 1 - [13, 18]
162. Tester - Deterministic - Memory length: ∞ - [10]
163. TF1 - Deterministic - Memory length: ∞ - [21]
164. TF2 - Deterministic - Memory length: ∞ - [21]
165. TF3 - Deterministic - Memory length: ∞ - [21]
166. ThueMorse - Deterministic - Memory length: ∞ - [21]
167. ThueMorseInverse - Deterministic - Memory length: ∞ - [21]
168. Thumper - Deterministic - Memory length: 2 - [5]
169. Tideman and Chieruzzi - Deterministic - Memory length: ∞ - [9]
170. Tit For Tat - Deterministic - Memory length: 1 - [9]
171. Tit For 2 Tats - Deterministic - Memory length: 2 - [11]
172. Tranquilizer - Stochastic - Memory length: ∞ - [9]
173. Tricky Cooperator - Deterministic - Memory length: 10 - [21]
174. Tricky Defector - Deterministic - Memory length: ∞ - [21]
175. Tricky Level Punisher - Deterministic - Memory length: ∞ - [15]
176. Tullock: 11 - Stochastic - Memory length: 11 - [9]
177. Two Tits For Tat - Deterministic - Memory length: 2 - [11]
178. VeryBad - Deterministic - Memory length: ∞ - [14]
179. Weiner - Deterministic - Memory length: ∞ - [10]
180. White - Deterministic - Memory length: ∞ - [10]

181. Willing - Stochastic - Memory length: 1 - [41]
182. Winner12 - Deterministic - Memory length: 2 - [30]
183. Winner21 - Deterministic - Memory length: 2 - [30]
184. Win-Shift Lose-Stay: D - Deterministic - Memory length: 1 - [25]
185. Win-Stay Lose-Shift: C - Deterministic - Memory length: 1 - [38, 33, 22]
186. WmAdams - Stochastic - Memory length: ∞ - [10]
187. Worse and Worse - Stochastic - Memory length: ∞ - [29]
188. Worse and Worse 2 - Stochastic - Memory length: ∞ - [29]
189. Worse and Worse 3 - Stochastic - Memory length: ∞ - [29]
190. Yamachi - Deterministic - Memory length: ∞ - [10]
191. ZD-Extortion: 0.2, 0.1, 1 - Stochastic - Memory length: 1 - [36]
192. ZD-Extort-2: 0.1111111111111111, 0.5 - Stochastic - Memory length: 1 - [38]
193. ZD-Extort3: 0.11538461538461539, 0.3333333333333333, 1 - Stochastic - Memory length: 1 - [34]
194. ZD-Extort-2 v2: 0.125, 0.5, 1 - Stochastic - Memory length: 1 - [23]
195. ZD-Extort-4: 0.23529411764705882, 0.25, 1 - Stochastic - Memory length: 1 - [21]
196. ZD-GTFT-2: 0.25, 0.5 - Stochastic - Memory length: 1 - [38]
197. ZD-GEN-2: 0.125, 0.5, 3 - Stochastic - Memory length: 1 - [23]
198. ZD-Mem2 - Stochastic - Memory length: 2 - [28]
199. ZD-Mischief: 0.1, 0.0, 1 - Stochastic - Memory length: 1 - [36]
200. ZD-SET-2: 0.25, 0.0, 2 - Stochastic - Memory length: 1 - [23]
201. e - Deterministic - Memory length: ∞ - [21]
202. Dynamic Two Tits For Tat - Stochastic - Memory length: ∞ - [21]
203. Meta Hunter: 6 players - Deterministic - Memory length: ∞ - [21]
204. Meta Hunter Aggressive: 7 players - Deterministic - Memory length: ∞ - [21]

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