2.3 Solving Linear Equations

Method - Solving Linear Equations

1. simplify the algebraic expressions on both sides
2. collect all variable terms on one side, constants to the other
3. isolate the variable
4. check your solution

Example 2.3.1
Solve for x and check:

$$-7x + 25 + 3x = 16 - 2x - 3$$

Example 2.3.2
Solve for x and check:

$$8x = 2(x + 6)$$

Example 2.3.3
Solve for x and check:

$$4(2x+1)-29=3(2x-5)$$

Example 2.3.4
Solve for x and check:

$$\frac{x}{4}=\frac{2x}{3}+\frac{5}{6}$$

Equations with No or Infinite Solutions

Consider the equation $x = x + 4$. If we solve it by subtracting x from both sides, we determine that $0 = 4$ which we know is not true - i.e., a *false statement*. This tells us that there are **no** values of x that satisfy the original equation. We write this as either $\varnothing$ or $\{\}$.

Now consider a similar statement, $x + 3 = 5 + x - 2$. Solving this yields the statement that $3 = 3$, which we should recognize as being true - i.e., a *true statement*. This is called an *identity* and tells us that *any* value of x is a solution to the original equation. This is typically notated as one of the following ways: "all real numbers", $\mathbb{R}$, $\{x \mid x \text{ is a real number}\}$, or $\{x \mid x \in \mathbb{R}\}$.

Example 2.3.5
Solve for x and verify:

$$3x + 7 = 3(x + 1)$$

Example 2.3.6
Solve for x and verify:

$$3(x - 1) + 9 = 8x + 6 - 5x$$