10.4 Adding, Subtracting, and Dividing Radical Expressions

Adding and Subtracting Rational Expressions

We know that we can combine like terms to simplify expressions. For example,

$$2x + 3x = (2 + 3)x = 5x$$

We can add and subtract radicals in the same manner, however, we have to have the same radicand for it to work.

$$2\sqrt{3} + 3\sqrt{3} = (2 + 3)\sqrt{3} = 5\sqrt{3}$$

If instead, we had unlike radicands, we would treat them as unlike terms and would be unable to combine them:

$$-4\sqrt{7} + 8\sqrt{7} + 2\sqrt{3} = (-4 + 8)\sqrt{7} + 2\sqrt{3} = 4\sqrt{7} + 2\sqrt{3}$$

Example 10.4.1. Simplify by combining like terms:

1. $8\sqrt{13} + 2\sqrt{13}$

2. $9\sqrt[3]{7} - 6x\sqrt[3]{7} + 12\sqrt[3]{7}$

3. $7\sqrt[4]{3x} - 2\sqrt[4]{3x} + 2\sqrt[3]{3x}$

Example 10.4.2. Simplify by combining like terms, but reduce the radicals if necessary.

1. $3\sqrt{20} + 5\sqrt{45}$

2. $3\sqrt{12x} - 6\sqrt{27x}$

3. $8\sqrt{5} - 6\sqrt{2}$

Example 10.4.3. Simplify by combining like terms, but reduce the radicals if necessary.

1. $3\sqrt[3]{24} - 5\sqrt[3]{81}$

2. $5\sqrt[3]{x^2y} + \sqrt[3]{27x^5y^4}$

Quotient Rule for Radicals

Let $\sqrt[n]{a}$ and $\sqrt[n]{b}$ be two real numbers with $b \neq 0$, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Example 10.4.4. Simplify using the quotient rule:

1. $\sqrt[3]{\dfrac{24}{125}}$

2. $\sqrt{\dfrac{9x^3}{y^{10}}}$

3. $\sqrt[3]{\dfrac{8y^7}{x^{12}}}$

4. $\sqrt{\dfrac{x^2}{25y^6}}$

Example 10.4.5. Divide and simplify as appropriate.

1. $\dfrac{\sqrt{40x^5}}{\sqrt{2x}}$

2. $\dfrac{\sqrt{50xy}}{2\sqrt{2}}$

3. $\dfrac{\sqrt[3]{48xy}}{\sqrt[3]{6xy^{-2}}}$

4. $\dfrac{\sqrt[3]{16x^5y^2}}{\sqrt[3]{2xy^{-1}}}$