10.1 Radical Expressions and Functions

Definition 10.1.1 (Principal Square Root). If a is a non-negative real number, then the non-negative number b such that $b^2 = a$, denoted by $b = \sqrt{a}$, is the principal square root of a.

Example 10.1.1. Evaluate each of the following square roots.

1. $\sqrt{64}$

   8

2. $-\sqrt{49}$

   $= -7$

3. $\sqrt{\frac{16}{25}} = \frac{4}{5}$

4. $\sqrt{0.0081}$

   $= .09$

5. $\sqrt{9+16} = \sqrt{25} = 5$

6. $\sqrt{9} + \sqrt{16}$

   $= 3 + 4 = 7$

Functions with Square Roots

We can define the square root as a function with $f(x) = \sqrt{x}$. Both the domain and range of this function are the non-negative numbers - $[0, \infty)$.

To evaluate square root functions, we treat them the same as anything - make the substitution for the independent variable and simplify.

Example 10.1.2. Find the indicated value for each given function:

1. $f(3)$ when $f(x) = \sqrt{12x - 20}$

   $= \sqrt{12 \cdot 3 - 20}$

   $= \sqrt{36 - 20}$

   $= \sqrt{16} = 4$

2. $g(-5)$ when $g(x) = -\sqrt{9 - 3x}$

   $= -\sqrt{9 - 3(-5)}$

   $= -\sqrt{9 + 15}$

   $= -\sqrt{24} = -2\sqrt{6}$

The square root function is only defined when the *radicand* - the portion under the radical - is non-negative. If we need to find the domain of a square root function, set the radicand greater than or equal to 0 and solve for x. State the domain using whichever method is specified.

Example 10.1.3. Find the domain of each of the following functions:

1. $f(x) = \sqrt{9x - 27}$

$9x - 27 \geq 0$

$9x \geq 27$

$x \geq 3$

D: $(-\infty, 3]$

2. $g(x) = -3\sqrt{2(3x - 4)} + 4$

$2(3x-4) \geq 0$

$6x - 8 \geq 0$

$6x \geq 8$

$x \geq 4/3$

D: $[4/3, \infty)$

Simplifying $\sqrt{a^2}$

For any real value of a, we have

$$\sqrt{a^2} = |a|$$

Example 10.1.4. Simplify:

1. $\sqrt{(-7)^2}$

$|-7| = 7$

2. $\sqrt{(x+8)^2}$

$= |x+8|$

3. $\sqrt{49x^{10}}$

$= \sqrt{49} \cdot \sqrt{x^{10}}$

$= 7 \cdot |x^5|$

4. $\sqrt{x^2 - 6x + 9}$

$= \sqrt{(x-3)^2}$

$= |x-3|$

Cube Roots

Similar to a square root, a cube root is given as $\sqrt[3]{a} = b$ where $b^3 = a$. The 3 is the *index* of the radical. Unlike the square root, however, the cube root has negative numbers in its domain. Both the domain and range of $f(x) = \sqrt[3]{x}$ are $(-\infty, \infty)$.

Example 10.1.5. Find the indicated value for each given function:

1. $f(127)$ when $f(x) = \sqrt[3]{x-2}$

$= \sqrt[3]{127-2}$

$= \sqrt[3]{125}$

$= 5$

2. $g(-7)$ when $g(x) = \sqrt[3]{8x - 8}$

$g(-7) = \sqrt[3]{8(-7)-8}$
$= \sqrt[3]{-56-8}$
$= \sqrt[3]{-64} = -4$

or

$g(-7) = \sqrt[3]{8(-7-1)}$
$= 2\sqrt[3]{-8}$
$= 2 \cdot (-2)$
$= -4$

Simplifying $\sqrt[3]{a^3}$

For any real number a, we have

$$\sqrt[3]{a^3} = a$$

.

Example 10.1.6. Simplify the following:

$$\sqrt[3]{-27x^3} = \sqrt[3]{-27}\sqrt[3]{x^3}$$
$= -3x$

Simplifying Odd or Even Roots

For any real number a:

- If n is even, $\sqrt[n]{a^n} = |a|$.
- If n is odd, $\sqrt[n]{a^n} = a$.

Example 10.1.7. Find each of the following:

1. $\sqrt[4]{(x+6)^4}$

$|x+6|$

2. $\sqrt[5]{(3x-2)^5}$

$3x - 2$

3. $\sqrt[6]{(-8)^6}$

$|-8| = 8$