8.4 Composite and Inverse Functions

Composite Functions

Composite functions are *composed* of two other functions.

Definition 8.4.1 (Function Composition)**.** Function composition uses the following notation:

$$(f \circ g)(x) = f(g(x))$$

which reads as "f composed with g at/of x". The domain of $f \circ g$ is the set of all values of x such that x is in the domain of g and $g(x)$ is in the domain of f.

Example 8.4.1. Given $f(x) = 5x + 6$ and $g(x) = x^2 - 1$, find the following:

1. $(f \circ g)(x) =$

2. $(g \circ f)(x) =$

Example 8.4.2. Given $f(x) = -2x + 3$ and $g(x) = 2x^2 - 4x$, find the following:

1. $(f \circ g)(x) =$

2. $(g \circ f)(x) =$

Inverse Functions

Two functions are said to be inverses if they undo each other; that is, if $(f \circ g)(x) = f(g(x)) = x$. We denote the inverse function of $f(x)$ as $f^{-1}(x)$. The -1 in this notation is not an exponent.

Definition 8.4.2 (Function Inverse)**.** Let f and g be two functions such that $f(g(x)) = x$ for all x in the domain of g and $g(f(x)) = x$ for all x in the domain of f. The functions f and g are then *inverses.*

If we notate the inverse of f as f^{-1}, we then have $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is the range of f^{-1} and the domain of f^{-1} is the range of f.

Example 8.4.3. Show that the functions $f(x) = 3x$ and $g(x) = \frac{x}{3}$ are inverses.

Example 8.4.4. Show that the functions $f(x) = 4x - 7$ and $g(x) = \frac{x+7}{4}$ are inverses.

Finding the Inverse of a Function

1. Replace $f(x)$ with y
2. Swap x with y
3. Solve the new equation for y
4. Replace y with f^{-1}
5. Verify that $(f \circ f^{-1})(x) = x$

Example 8.4.5. Find the inverse of $f(x) = 4x + 5$.

Example 8.4.6. Find the inverse of $f(x) = -3x - 7$.

Example 8.4.7. Find the inverse of $f(x) = \dfrac{2}{x-1}$.

Example 8.4.8. Find the inverse of $f(x) = \dfrac{2x-1}{x+3}$.