11.3 Quadratics and their Graphs

Quadratics can be written in two typical formats, given below. Each format has pros and cons which will be discussed and shown throughout this section.

Definition 11.3.1 (Standard Form)
Standard form of a quadratic is given as $f(x) = ax^2 + bx + c$ where $a \neq 0$.

Definition 11.3.2 (Vertex Form)
Vertex form of a quadratic is given as $f(x) = a(x - h)^2 + k$ where $a \neq 0$. The point (h, k) gives the *vertex* of the quadratic.

Definition 11.3.3 (Vertex)
A *vertex* is otherwise known as a *turning point* in a graph. It represents the maximum or minimum value and is the point at which the function changes between increasing and decreasing.

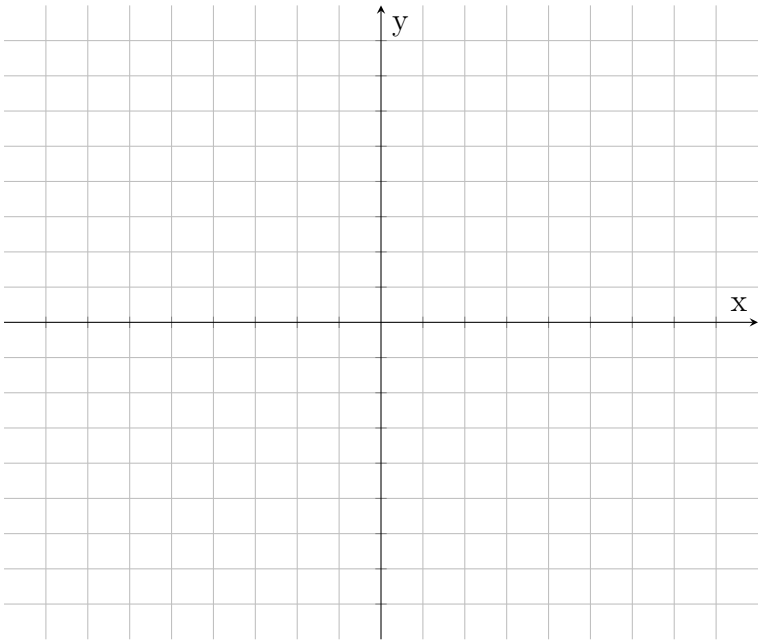
We can consider two cases for a for either form of a quadratic. Depending on the value of a, we are able to determine a few facts about the graph.

- $a > 0$
  - $f(x)$ opens up
  - $f(x)$ has a *minimum* value
- $a < 0$
  - $f(x)$ opens down
  - $f(x)$ has a *maximum* value

Graphing Quadratics

We can graph quadratics in a variety of ways – some easier than others. For example, we could construct a table of points on the graph and plot the points. However, we have new tools at our disposal that we did not have when we first learned how to graph functions in Algebra 1.

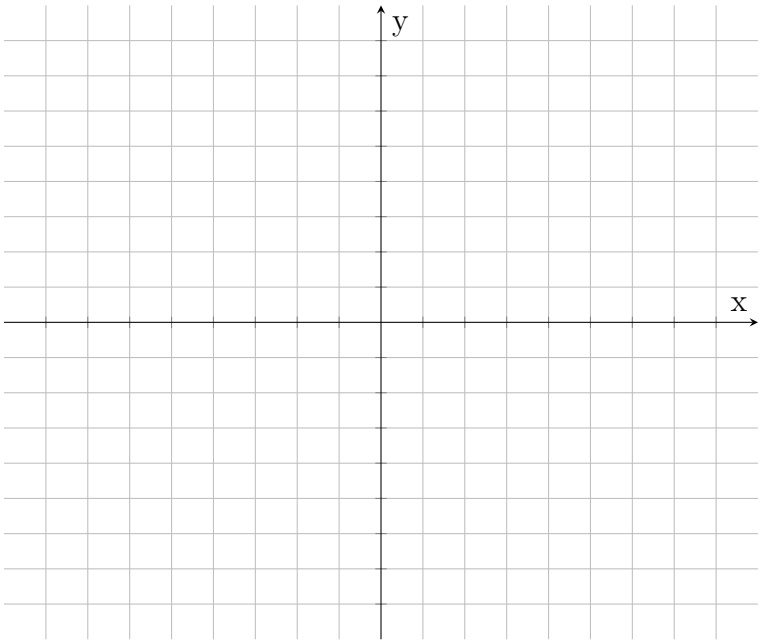
Example 11.3.1
Graph the quadratic $f(x) = -2(x-3)^2 + 8$.



Quadratics are easiest to graph when given in vertex form since we can easily pick out the vertex. When given in standard form however, we have to do some extra work in the form of *completing the square*.

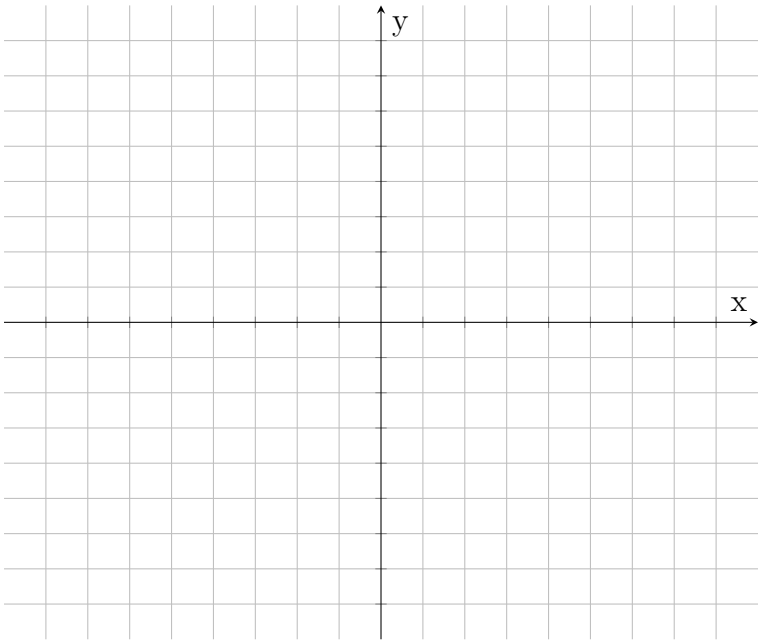
Example 11.3.2
Convert the following standard form equation into vertex form and identify the vertex. Then graph the *parabola*.

$$f(x) = 2x^2 + 8x - 1$$



Example 11.3.3
Identify the vertex of the following quadratic. Then graph the *parabola*.

$$f(x) = -x^2 - 2x + 1$$



Applications of Quadratics

As stated on the first page of this section, the function $f(x)$ has a minimum if $a > 0$ and has a maximum if $a < 0$. Use this information to solve the following examples.

Example 11.3.4
You have 100 yards of fencing to enclose a rectangular field. What are the dimensions and area of the largest field possible?

Example 11.3.5
Find the dimensions and area of the largest possible field (referring to the previous example) if you only fence in 3 of the 4 sides.

Example 11.3.6
Among all pairs of numbers whose sum is 16, find a pair with the largest possible *product*. What is the product?

Example 11.3.7
Among all pairs of numbers whose difference is 24, find the pair whose product is smallest.