## 9.1 A Review of Solving Linear Inequalities

**Solving a Linear Inequality**

1. Simplify both sides of the inequality
2. Collect all variable terms on one side and all constant terms on the other
3. Isolate the variable and solve; be sure to change the direction of the inequality if you multiply or divide by a negative number
4. State the solution in interval notation and graph on a number line

**Example 9.1.1.** Solve the following:

$$3x + 1 > 7x - 15$$

**Example 9.1.2.** Solve the following:

$$\frac{x-4}{2} \geqslant \frac{x-2}{3} + \frac{5}{6}$$

## 9.2 Compound Inequalities

## Introductory Set Theory

**Definition 9.2.1** (Set)**.** a set is a collection of *distinct* objects; each object in the set is called an *element*

**Definition 9.2.2** (Intersection of Sets)**.** the intersection of sets $A$ and $B$ is given as $A \cap B$ and is the set of elements that are found in *both* sets

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

**Definition 9.2.3** (Union of Sets)**.** the union of sets $A$ and $B$ is given as $A \cup B$ and is the set of elements that are found *either* set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

**Definition 9.2.4** (Set Substraction)**.** the subtraction of two sets $A$ and $B$ is given as $A \backslash B$ and represents what remains after all elements that occur in $B$ are removed from $A$

$$A \backslash B = \{x \mid x \in A \text{ and } x \notin B\}$$

**Definition 9.2.5** (Set Cardinality)**.** the cardinality (size) of a set is the number of distinct elements in the set and is given by $\|A\|$

**Example 9.2.1.** Given $A = \{a, b, c, d, e, f\}$ and $B = \{b, d, f, h, j, l\}$, find each of the following:

1. $A \cap B =$
2. $A \cup B =$
3. $A \backslash B =$
4. $B \backslash A =$
5. $\|A \cap B\| =$
6. $\|A \cup B\| =$
7. $\|A \backslash B\| =$
8. $\|B \backslash A\| =$

**Example 9.2.2.** Given $A = \{1, 2, 3, \ldots, 10\}$ and $B = \{2, 4, 6, \ldots, 20\}$, find each of the following:

1. $A \cap B =$
2. $A \cup B =$
3. $A \backslash B =$
4. $B \backslash A =$
5. $\|A \cap B\| =$
6. $\|A \cup B\| =$
7. $\|A \backslash B\| =$
8. $\|B \backslash A\| =$

## Compound Inequalities with "And"

A number is a solution of a compound inequality involving "and" if and only if it satisfies both of the given inequalities. In other words, the solution set is the *intersection* of the solution to each individual inequality.

**Example 9.2.3.** Solve the compound inequality:

$$x + 2 < 5 \text{ and } 2x - 4 < -2$$

**Example 9.2.4.** Solve the compound inequality:

$$4x - 5 > 7 \text{ and } 5x - 2 < 3$$

**Example 9.2.5.** Solve the compound inequality:

$$1 \leqslant 2x + 3 < 11$$

## Compound Inequalities with "Or"

A number is a solution of a compound inequality with the word "or" if it is a solution of either inequality. In other words, the solution set is the *union* of the solution to each individual inequality.

**Example 9.2.6.** Solve the compound inequality:

$$3x - 5 \leqslant -2 \text{ or } 10 - 2x < 4$$

**Example 9.2.7.** Solve the compound inequality:

$$2x + 5 \geqslant 3 \text{ or } 2x + 3 < 3$$

## 9.3 Equations and Inequalities with Absolute Values

## Equations with Absolute Values

Recall that the absolute value of a number, $|x|$, represents the distance from 0 for that number. Essentially, it takes any number and turns it positive. If we have an equation involving absolute values, we typically end up with 2 values as answers - why?

**Rewriting Absolute Value Equations**
If $c$ is some positive number and $u$ is an algebraic expression, then we can rewrite $|u| = c$ as $u = c$ or $u = -c$.

**Example 9.3.1.** Solve:

$$|2x - 1| = 5$$

**Example 9.3.2.** Solve:

$$2|1 - 3x| - 28 = 0$$

**Rewriting Equations with 2 Absolute Values**

If $u$ and $v$ are both some algebraic expression, then we can rewrite $|u| = |v|$ as $u = v$ or $u = -v$.

**Example 9.3.3.** Solve:

$$|2x - 7| = |x + 3|$$

## Inequalities with Absolute Values

**Inequalities of the form $|u| < c$**

If $c$ is a positive number and $u$ is an algebraic expression, then $|u| < c$ can be rewritten as the compound inequality $-c < u < c$.

**Example 9.3.4.** Solve:

$$|x - 2| < 5$$

**Example 9.3.5.** Solve:

$$-3|5x-2|+20 \geqslant -19$$

**Inequalities of the form $|u| > c$**

If $c$ is a positive number and $u$ is an algebraic expression, then $|u| > c$ can be rewritten as $u < -c$ or $u > c$.

**Example 9.3.6.** Solve:

$$|2x-5| \geqslant 3$$

## 9.4 Linear Inequalities in Two Variables

**Method:**

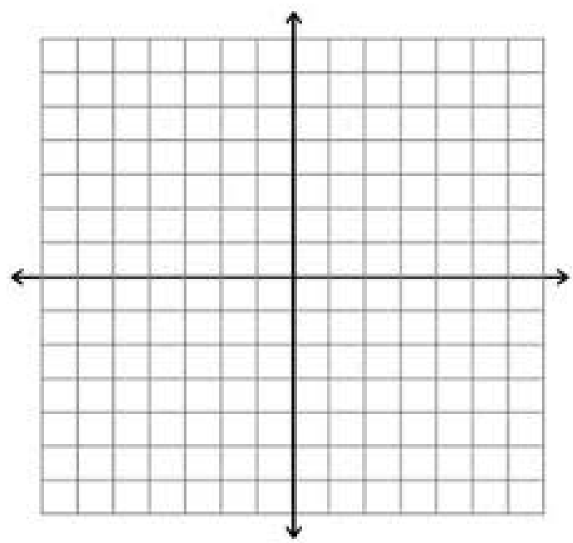
1. Replace the inequality symbol with an equal sign and graph the equation. Use a dashed line if the symbol is $<$ or $>$ and a solid line otherwise.

2. Decide on which side of the line to shade.

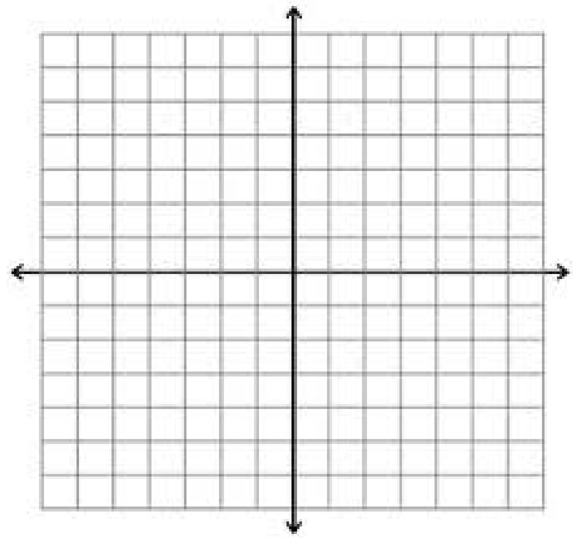
   (a) Choose a test point. If the inequality evaluated at the point is true, graph on the side that contains the test point; otherwise, graph the other side.

   (b) If the inequality is solved for $y$, shade based on the inequality symbol. Shade below the line if you have $y < \ldots$ and shade above the line if you have $y > \ldots$.

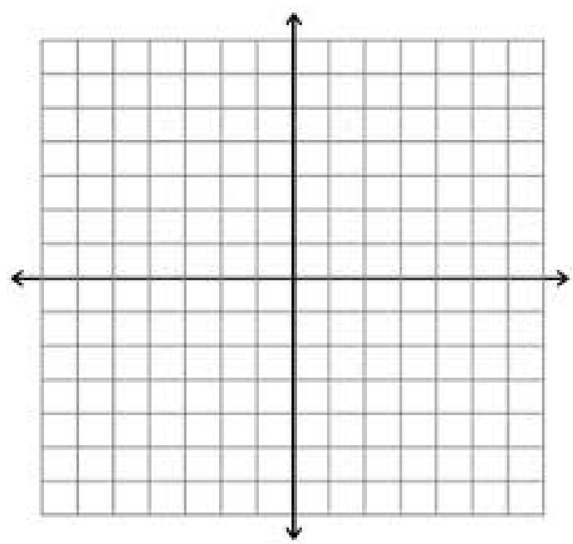
**Example 9.4.1.** Graph: $4x - 2y \geqslant 8$



**Example 9.4.2.** Graph: $y > \dfrac{-3}{4}x$



**Example 9.4.3.** Graph: $x \leqslant -2$

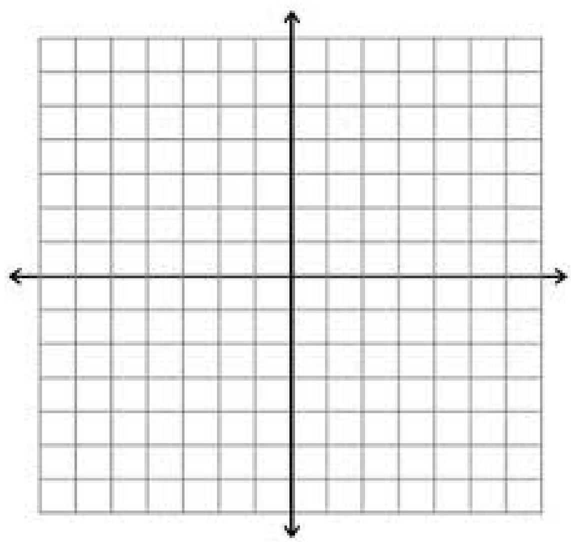


**Graphing Systems of Inequalities**

Systems of linear inequalities have a *solution set* that is a portion of the plane, not just a point. To find this solution set, graph each of the inequalities individually and look for the overlap (intersection) of their solutions.

**Example 9.4.4.** Graph the solution set of the following system:

$$\begin{cases} x - 3y < 6 \\ 2x + 3y \geqslant -6 \end{cases}$$



**Example 9.4.5.** Graph the solution set of the following system:

$$\begin{cases} x + y < 2 \\ -2 \leqslant x < 1 \\ y > -3 \end{cases}$$