## 10.1 Radical Expressions and Functions

**Definition 10.1.1** (Principal Square Root). If a is a non-negative real number, then the non-negative number b such that  $b^2 = a$ , denoted by  $b = \sqrt{a}$ , is the principal square root of a.

Example 10.1.1. Evaluate each of the following square roots.

1. 
$$\sqrt{64}$$

3. 
$$\sqrt{\frac{16}{25}} = \frac{4}{5}$$

5. 
$$\sqrt{9+16} = \sqrt{25} = 5$$

$$2. -\sqrt{49}$$

$$= -7$$

4. 
$$\sqrt{0.0081}$$

6. 
$$\sqrt{9} + \sqrt{16}$$

## Functions with Square Roots

We can define the square root as a function with  $f(x) = \sqrt{x}$ . Both the domain and range of this function are the non-negative numbers -  $[0, \infty)$ .

To evaluate square root functions, we treat them the same as anything - make the substitution for the independent variable and simplify.

Example 10.1.2. Find the indicated value for each given function:

1. 
$$f(3)$$
 when  $f(x) = \sqrt{12x - 20}$ 

$$= \sqrt{12 \cdot 3 - 20}$$

$$= \sqrt{36 - 20}$$

$$= \sqrt{16} = 4$$

2. 
$$g(-5)$$
 when  $g(x) = -\sqrt{9-3x}$ 

$$= -\sqrt{4-3(-5)}$$

$$= -\sqrt{4+15}$$

$$= -\sqrt{24} = -2\sqrt{6}$$

The square root function is only defined when the radicand - the portion under the radical - is non-negative. If we need to find the domain of a square root function, set the radicand greater than or equal to 0 and solve for x. State the domain using whichever method is specified.

Example 10.1.3. Find the domain of each of the following functions:

1. 
$$f(x) = \sqrt{9x - 27}$$

$$9x - 27 > 0$$

$$9x > 27$$

$$\times 7.3$$

2. 
$$g(x) = -3\sqrt{2(3x-4)} + 4$$

$$2(3x-4) \ge 0$$

$$6x-8 \ge 0$$

$$6x \ge 8$$

$$x \ge 4/3$$

Simplifying  $\sqrt{a^2}$ 

For any real value of  $\mathfrak{a}$ , we have

$$\sqrt{a^2} = |a|$$

Example 10.1.4. Simplify:

1. 
$$\sqrt{(-7)^2}$$

2. 
$$\sqrt{(x+8)^2}$$

$$= \left| x+8 \right|$$

3. 
$$\sqrt{49x^{10}}$$
=  $\sqrt{49x^{10}}$ 
=  $\sqrt{49x^{10}}$ 
=  $\sqrt{49x^{10}}$ 

$$4. \sqrt{x^2 - 6x + 9}$$

$$= \sqrt{(x - 3)^2}$$

$$= \sqrt{x - 3}$$

#### **Cube Roots**

Similar to a square root, a cube root is given as  $\sqrt[3]{a} = b$  where  $b^3 = a$ . The 3 is the *index* of the radical. Unlike the square root, however, the cube root has negative numbers in its domain. Both the domain and range of  $f(x) = \sqrt[3]{x}$  are  $(-\infty, \infty)$ .

Example 10.1.5. Find the indicated value for each given function:

1. 
$$f(127)$$
 when  $f(x) = \sqrt[3]{x-2}$ 

$$= \sqrt[3]{127-2}$$

$$= \sqrt[3]{125}$$

$$= 5$$

2. 
$$g(-7)$$
 when  $g(x) = \sqrt[3]{8x - 8}$ 

$$y(-7) = \sqrt[3]{8(-7)-8}$$

$$= \sqrt[3]{-54-8}$$

$$= \sqrt[3]{-64} = -4$$

$$= \sqrt[3]{-64} = -4$$

$$= \sqrt[3]{-64} = -4$$

$$= \sqrt[3]{-64} = -4$$

# Simplifying $\sqrt[3]{\mathfrak{a}^3}$

For any real number a, we have

$$\sqrt[3]{a^3} = a$$

Example 10.1.6. Simplify the following:

$$\sqrt[3]{-27x^3} = \sqrt[3]{-27} \sqrt[3]{x^3}$$

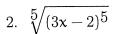
## Simplifying Odd or Even Roots

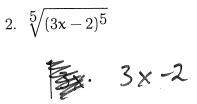
For any real number a:

- If n is even,  $\sqrt[n]{a^n} = |a|$ .
- If n is odd,  $\sqrt[n]{a^n} = a$ .

Example 10.1.7. Find each of the following:

1. 
$$\sqrt[4]{(x+6)^4}$$





3. 
$$\sqrt[6]{(-8)^6}$$
$$|-8| = 8$$