5.7 Negative Exponents & Scientific Notation

Suppose that we have $\frac{b^3}{b^5}$. By our definitions so far, we can determine the following:

$$\frac{b^3}{b^5} = \frac{b \cdot b \cdot b}{b \cdot b \cdot b \cdot b \cdot b} = \frac{1}{b \cdot b} = \frac{1}{b^2}$$

However, by the quotient rule, we have $\frac{b^3}{b^5} = b^{3-5} = b^{-2}$. Since we assume that both methods are correct, we can safely say that $\frac{1}{b^2} = b^{-2}$ by the *transitive property*.

Definition 5.7.1 (Negative Exponent Rule)

- $b^{-n} = \frac{1}{b^n}$ for $b \neq 0$
- $\frac{1}{b^{-n}} = b^n$ for $b \neq 0$

Example 5.7.1
Rewrite each of the following with positive exponents.

1. 6^{-2}
2. $(-3)^{-4}$
3. -3^{-4}
4. 7^{-1}
5. $\frac{2^{-3}}{7^{-2}}$
6. $\left(\frac{4}{5}\right)^{-2}$
7. $\frac{1}{7y^{-2}}$
8. $\frac{x^{-1}}{y^{-8}}$

Simplifying Exponential Expressions

Example 5.7.2
Simplify each of the following:

1. $x^{-12} \cdot x^2 =$

2. $\dfrac{x^2}{x^{10}} =$

3. $\dfrac{75x^3}{5x^9} =$

4. $\dfrac{50y^8}{-25y^{-14}} =$

5. $\dfrac{(5x^3)^2}{x^{10}} =$

6. $\left(\dfrac{x^8}{x^4}\right)^{-5} =$

Scientific Notation

Scientific notation is used as a shorthand method of writing *very* large numbers. For those of you who have taken Chemistry, you may remember Avogadro's number: $6.02214076 \times 10^{23}$. If, for some unknown reason, we wanted to write this without using scientific notation, we would have:

$$602,214,076,000,000,000,000,000$$

Definition 5.7.2 (Scientific Notation)
A number written in scientific notation has the form $a \times 10^n$ where $1 \leqslant |a| < 10$ and n is some integer.

Procedure: Convert from Scientific Notation

- If $n > 0$, move the decimal to the *right* by n places, adding in 0s as necessary. This should give you a *large* number.
- If $n < 0$, move the decimal the the *left* by n places, adding in 0s as necessary. This should give you a *small* number.

Example 5.7.3
Convert 7.4×10^9 to standard decimal notation.

Example 5.7.4
Convert 3.017×10^{-6} to standard decimal notation.

Procedure: Convert to Scientific Notation

- Determine a – move the decimal around until $1 \leqslant |a| < 10$.
- Determine n – n is the number of places that the decimal point was moved. $n > 0$ if the original number is larger than 10 (or smaller than -10) and $n < 0$ if the original is between -1 and 1.

Example 5.7.5
Convert $7,410,000,000$ to scientific notation.

Example 5.7.6
Convert $-4,120,000$ to scientific notation.

Example 5.7.7
Convert 0.000023 to scientific notation.

Operations on Scientific Notations

- Multiplication: $(a \times 10^m)(b \times 10^n) = ab \times 10^{m+n}$
- Division: $\dfrac{a \times 10^m}{b \times 10^n} = \left(\dfrac{a}{b}\right) \times 10^{m-n}$
- Exponentiation: $(a \times 10^m)^n = a^n \times 10^{mn}$

Example 5.7.8
Simplify and write in scientific notation:

$$(3 \times 10^8)(2 \times 10^2)$$

Example 5.7.9
Simplify and write in scientific notation:

$$\frac{8.4 \times 10^7}{4 \times 10^{-4}}$$

Example 5.7.10
Simplify and write in scientific notation:

$$(4 \times 10^{-2})^3$$

Example 5.7.11
Simplify and write in scientific notation:

$$\frac{(4 \times 10^5)(9 \times 10^{-4})}{2 \times 10^{-3}}$$