9.2 Compound Inequalities

Introductory Set Theory

Definition 9.2.1 (Set)**.** a set is a collection of *distinct* objects; each object in the set is called an *element*

Definition 9.2.2 (Intersection of Sets)**.** the intersection of sets A and B is given as $A \cap B$ and is the set of elements that are found in *both* sets

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Definition 9.2.3 (Union of Sets)**.** the union of sets A and B is given as $A \cup B$ and is the set of elements that are found *either* set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Definition 9.2.4 (Set Substraction)**.** the subtraction of two sets A and B is given as $A \backslash B$ and represents what remains after all elements that occur in B are removed from A

$$A \backslash B = \{x \mid x \in A \text{ and } x \notin B\}$$

Definition 9.2.5 (Set Cardinality)**.** the cardinality (size) of a set is the number of distinct elements in the set and is given by $\|A\|$

Example 9.2.1. Given $A = \{a, b, c, d, e, f\}$ and $B = \{b, d, f, h, j, l\}$, find each of the following:

1. $A \cap B =$
2. $A \cup B =$
3. $A \backslash B =$
4. $B \backslash A =$
5. $\|A \cap B\| =$
6. $\|A \cup B\| =$
7. $\|A \backslash B\| =$
8. $\|B \backslash A\| =$

Example 9.2.2. Given $A = \{1, 2, 3, \ldots, 10\}$ and $B = \{2, 4, 6, \ldots, 20\}$, find each of the following:

1. $A \cap B =$
2. $A \cup B =$
3. $A \backslash B =$
4. $B \backslash A =$
5. $\|A \cap B\| =$
6. $\|A \cup B\| =$
7. $\|A \backslash B\| =$
8. $\|B \backslash A\| =$

Compound Inequalities with "And"

A number is a solution of a compound inequality involving "and" if and only if it satisfies both of the given inequalities. In other words, the solution set is the *intersection* of the solution to each individual inequality.

Example 9.2.3. Solve the compound inequality:

$$x + 2 < 5 \text{ and } 2x - 4 < -2$$

Example 9.2.4. Solve the compound inequality:

$$4x - 5 > 7 \text{ and } 5x - 2 < 3$$

Example 9.2.5. Solve the compound inequality:

$$1 \leqslant 2x + 3 < 11$$

Compound Inequalities with "Or"

A number is a solution of a compound inequality with the word "or" if it is a solution of either inequality. In other words, the solution set is the *union* of the solution to each individual inequality.

Example 9.2.6. Solve the compound inequality:

$$3x - 5 \leqslant -2 \text{ or } 10 - 2x < 4$$

Example 9.2.7. Solve the compound inequality:

$$2x + 5 \geqslant 3 \text{ or } 2x + 3 < 3$$