6.1 GCF and Factoring by Grouping

Definition 6.1.1 (Factor)
Factoring a polynomial means to express it as the product of prime (or irreducible) polynomials. Essentially, un-distribute.

For example, we could rewrite the following polynomial in factored form as:

$$12x^3 - 4x^2 = 4x^2(3x - 1)$$

If we were to work backward, we would distribute the $4x^2$ to the $3x - 1$ and would end up with $12x^3 - 4x^2$.

There are many methods of factoring which will be covered in this chapter. The specific method that we use depends on exactly what we are trying to factor. Some methods will always work, some will work for certain polynomials, but not for others.

The most fundamental method of factoring is the **GCF** method - **G**reatest **C**ommon **F**actor.

Example 6.1.1
Find the GCF of $18x^3$ and $15x^2$.

Example 6.1.2
Find the GCF of $-20x^2$, $12x^4$, and $40x^3$.

Example 6.1.3
Find the GCF of x^4y, x^3y^2, and x^2y.

Factoring with the GCF

1. Identify the GCF of all terms.
2. Divide all terms by the GCF.
3. Write the GCF in front of the remaining polynomial.

Example 6.1.4
Factor $6x^2 + 18$

Example 6.1.5
Factor $25x^2 + 35x^3$

Example 6.1.6
Factor $15x^5 + 12x^4 - 27x^3$

Example 6.1.7
Factor $8x^3y^2 - 14x^2y + 2xy$

Example 6.1.8
Factor $-16a^4b^5 + 24a^3b^4 - 20ab^2$

Factoring by Grouping

We can factor out *any* polynomial factor, not just a monomial GCF. For example, say we have $x^2(x+3)+5(x+3)$. We have a common factor between each - $(x+3)$. This can be treated the same as any GCF and can be factored out to the front, giving us $(x+3)(x^2+5)$. This method is essentially the inverse of the FOIL method that we learned in chapter 5.

Factoring by Grouping

1. Group terms that have a common monomial factor - typically 2 groups.
2. Factor the GCF from each group.
3. Factor out the remaining binomial factor.

Example 6.1.9
Factor $x^3 + 5x^2 + 2x + 10$

Example 6.1.10
Factor $xy + 3x - 5y - 15$

Example 6.1.11
Factor $10x^2 - 12xy + 35xy - 42y^2$