10.2 Rational Exponents

Rational Exponent Definitions

For some integers m and n, we have the following:

1. $a^{1/n} = \sqrt[n]{a}$
2. $a^{m/n} = \sqrt[n]{a^m} = \sqrt[n]{a}^m$

Example 10.2.1. Rewrite each of the following using radical notations. Simplify if possible.

1. $25^{1/2}$
2. $(-8)^{1/3}$
3. $(5xy^2)^{1/4}$
4. $8^{4/3}$
5. $25^{3/2}$
6. $-81^{3/4}$

Example 10.2.2. Rewrite each of the following using rational exponents. Simplify if possible.

1. $\sqrt[4]{5xy}$

2. $\sqrt[5]{\dfrac{a^3b}{2}}$

3. $\sqrt[3]{6^4}$

4. $\sqrt[5]{2xy}^7$

Properties of Rational Exponents

1) Product Rule:
$a^m \cdot a^n = a^{m+n}$

2) Quotient Rule:
$\dfrac{a^m}{a^n} = a^{m-n}$

3) Power-to-Power Rule:
$\left(a^m\right)^n = a^{mn}$

4) Product-to-Power Rule:
$(ab)^m = a^m b^m$

5) Quotient-to-Power Rule:
$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

6) $a^m = a^n$ iff $m = n$

7) $a^m = b^m$ iff $a = b, m \neq 0$

8) Negative Exponents:
   (a) $a^{-m} = \dfrac{1}{a^m}$
   (b) $ba^{-m} = \dfrac{b}{a^m}$
   (c) $\dfrac{1}{a^{-m}} = a^m$
   (d) $\dfrac{b}{a^{-m}} = ba^m$

9) Zero Exponent Rule:
$a^0 = 1$

Example 10.2.3. Use the properties of rational exponents to simplify the following statements:

1. $7^{1/2} \cdot 7^{1/3}$

2. $\dfrac{50x^{1/3}}{10x^{4/3}}$

3. $\left(9.1^{2/5}\right)^{3/4}$

4. $\left(x^{-3/5}y^{1/4}\right)^{1/3}$

Example 10.2.4. Use the properties of rational exponents to simplify the following radical statements:

1. $\sqrt[6]{x^3}$

2. $\sqrt[3]{8x^{12}}$

3. $\dfrac{\sqrt{x}}{\sqrt[3]{x}}$

4. $\sqrt{\sqrt[3]{x}}$