10.6 Radical Equations

Solving Radical Equations

Solving a *radical equation* involves removing the radical from the equation and turning the equation into a form that we can otherwise solve. When doing so, however, we need to be careful of *extraneous solutions* – solutions that we can arrive at alegebraically, but are not actually solutions of the original equation. These typically occur when there is a contradiction of some sort during the process.

Method

1. Isolate the radical

2. Raise both sides of the equation to the nth exponent (removing the radical)

3. Solve the new equation using an appropriate technique

4. Verify possible solutions

Example 10.6.1. Solve and verify:

$$\sqrt{3x+4} = 8$$

$$\sqrt{3x+4}^2 = 8^2$$
$$3x+4 = 64$$
$$\frac{3x}{3} = \frac{60}{3}$$
$$x = 20$$

Verify: $\sqrt{3 \cdot 20 + 4} = 8$
$$\sqrt{64} = 8$$
$$8 = 8 \checkmark$$

Example 10.6.2. Solve and verify:

$$\sqrt{x-1} + 7 = 2$$
$$\underline{\quad -7 \quad -7}$$
$$\sqrt{x-1} = -5$$
$$x-1 = (-5)^2$$
$$x-1 = 25$$
$$x = 26$$

Verify: $\sqrt{26-1} + 7 = 2$
$$\sqrt{25} + 7 = 2$$
$$5 + 7 = 2$$
$$12 = 2$$
$$\times$$

No Solutions!

Example 10.6.3. Solve and verify:

$$\sqrt{6x+7} - x = 2$$

$$\sqrt{6x+7}^2 = (x+2)^2$$

$$6x+7 = (x+2)(x+2)$$

$$6x+7 = x^2+4x+4$$

$$0 = x^2-2x-3$$

$$(x-3)(x+1) = 0$$

$$x = -1, 3$$

Verify:

$x = -1$

$$\sqrt{6(-1)+7} - (-1) = 2$$

$$\sqrt{-6+7} + 1 = 2$$

$$\sqrt{1} + 1 = 2$$

$$2 = 2 \checkmark$$

$x = 3$

$$\sqrt{6(3)+7} - 3 = 2$$

$$\sqrt{18+7} - 3 = 2$$

$$\sqrt{25} - 3 = 2$$

$$5 - 3 = 2$$

$$2 = 2 \checkmark$$

Example 10.6.4. Solve and verify:

$$\sqrt{x+5} - \sqrt{x-3} = 2$$

$$\sqrt{x+5}^2 = (\sqrt{x-3}+2)^2$$

$$x+5 = (\sqrt{x-3}+2)(\sqrt{x-3}+2)$$

$$x+5 = \sqrt{x-3}^2 + 2\sqrt{x-3} + 2\sqrt{x-3} + 4$$

$$x+5 = x-3 + 4\sqrt{x-3} + 4$$

$$5 = 1 + 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3}$$

$$1 = \sqrt{x-3}$$

$$1 = x-3$$

$$\boxed{x = 4}$$

Check

$$\sqrt{4+5} - \sqrt{4-3} = 2$$

$$\sqrt{9} - \sqrt{1} = 2$$

$$3 - 1 = 2$$

$$2 = 2 \checkmark$$

Example 10.6.5. Solve and verify:

$$(2x - 3)^{1/3} + 3 = 0$$

$$\left[(2x-3)^{1/3}\right]^3 = (-3)^3$$

$$2x - 3 = -27$$

$$2x = -24$$

$$x = -12$$

Verify:

$$\left(2(-12)-3\right)^{1/3} + 3 = 0$$

$$(-24-3)^{1/3} + 3 = 0$$

$$(-27)^{1/3} + 3 = 0$$

$$-3 + 3 = 0$$

$$0 = 0 \checkmark$$

Example 10.6.6. Solve and verify:

$$(9x + 2)^{1/4} - (5x + 18)^{1/4} = 0$$

$$\left[(9x+2)^{1/4}\right]^4 = \left[(5x+18)^{1/4}\right]^4$$

$$9x + 2 = 5x + 18$$

$$4x = 16$$

$$x = 4$$

Check

$$\left(9\cdot4+2\right)^{1/4} - \left(5\cdot4+18\right)^{1/4} = 0$$

$$(36+2)^{1/4} - (20+18)^{1/4} = 0$$

$$(38)^{1/4} - (38)^{1/4} = 0$$

$$\checkmark$$