11.5 Polynomial and Rational Inequalities

So far this chapter we have solved equations in the format $f(x) = 0$. What if instead we had an inequality that involved a quadratic such as $x^2 - 4 > 0$? We no longer have one or two values of x that satisfy the equation – we have entire intervals that satisfy it, much like in chapter 9. Unfortunately, solving quadratic inequalities is not as straightforward as linear inequalities, but if you follow the process below, it should make some sense.

Procedure

1. Solve $f(x) = 0$ to get the boundary points.
2. Draw a number line and include the boundary points.
3. Choose a test value in each interval.
   (a) If $f(x)$ becomes positive, it is positive for all values in the interval.
   (b) If $f(x)$ becomes negative, it is negative for all values in the interval.
4. Write the solution set choosing the intervals that satisfy the inequality.

Example 11.5.1
Solve and graph the solution set of $x^2 - 4 > 0$.

Example 11.5.2
Solve and graph the solution set of $x^2 - x > 20$.

Example 11.5.3
Solve and graph the solution set of $2x^2 \leqslant -6x - 1$.

Rational Inequalities

Rational inequalities are solved using the same methods as quadratic inequalities, but we find the boundaries using a different method. Boundaries are found by setting both the numerator and denominator equal to zero.

Example 11.5.4
Solve and graph the solution set of $\dfrac{x-5}{x+2} < 0$.

Example 11.5.5

Solve and graph the solution set of $\dfrac{2x}{x+1} \geqslant 1$.

Example 11.5.6
An object is propelled straight up from ground level with an initial velocity of 80 feet per second. The height of the object is given by $h(t) = -16t^2 + 80t$. During which time interval is the object more than 64 feet above the ground?