10.3 Multiplying and Simplifying Radical Expressions

Product Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then we have

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

which follows from the rule for multiplying like-bases with exponents.

Example 10.3.1. Find each of the following products:

1. $\sqrt{5}\sqrt{11}$

2. $\sqrt[3]{6}\,\sqrt[3]{10}$

3. $\sqrt{x+4}\sqrt{x-4}$

4. $\sqrt[7]{2x}\,\sqrt[7]{6x^3}$

Simplifying Radical Expressions by Factoring

We say that a radical expression (with index n) is simplified if the radicand has no factors that are perfect n powers. Use the following method to simplify:

1. Write the radicand as the product of two factors where one of the factors has an exponent that is a multiple of n
2. Use the product rule to take the n^{th} root of each factor
3. Find the n^{th} root of the perfect n^{th} power.

Example 10.3.2. Simplify by factoring:

1. $\sqrt{80}$
2. $\sqrt[4]{32}$
3. $\sqrt[3]{40}$
4. $\sqrt{200x^2y}$

Example 10.3.3. Simplify the following radical function:

$$f(x) = \sqrt{3x^2 - 12x + 12}$$

Example 10.3.4. Simplify the following expression:

$$\sqrt{x^9 y^{11} z^3}$$

Example 10.3.5. Simplify the following expression:

$$\sqrt[3]{40x^{10} y^{14}}$$

Example 10.3.6. Simplify the following expression:

$$\sqrt[5]{32x^{12} y^2 z^8}$$

Example 10.3.7. Multiply then simplify:

$$\sqrt{6} \cdot \sqrt{2}$$

Example 10.3.8. Multiply then simplify:

$$10\sqrt[3]{16} \cdot 5\sqrt[3]{2}$$

Example 10.3.9. Multiply then simplify:

$$\sqrt[4]{4x^2y} \cdot \sqrt[4]{8x^6y^3}$$