1.3 The Real Numbers

We like to classify numbers and groups of numbers by similar properties. A few of these groupings may seem familiar or intuitive, while others may not.

Definition 1.3.1 (set)**.** one of the most fundamental objects in math; a collection of *distinct* items; often denoted using curly brackets - $\{\}$

Example 1.3.1. Which of the following are sets and which are not?

1. $A = \{a, b, c\}$
2. $B = \{97, 98, 110, 133\}$
3. $C = \{20, 24, 26, 20\}$
4. $D = \{1, 2, 3, \dots\}$
5. $E = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Definition 1.3.2 (Real Numbers)**.** abbreviated as $\mathbb{R}$; the numbers that we use in day-to-day life; the largest set of numbers that we'll use in this class

Definition 1.3.3 (Natural Numbers)**.** abbreviated as $\mathbb{N}$; also known as the counting numbers; $\mathbb{N} = \{1, 2, 3, \dots\}$

Definition 1.3.4 (Whole Numbers)**.** the natural numbers with 0 included;
$\{0, 1, 2, 3, \dots\}$

Definition 1.3.5 (Integers)**.** abbreviated as $\mathbb{Z}$; all natural numbers along with their negatives and 0;
$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
$\mathbb{Z}^+ = \{1, 2, 3, \dots\} = \mathbb{N}$
$\mathbb{Z}^- = \{\dots, -3, -2, -1\}$

The Number Line

The number line is a graphical representation of the real numbers. It is a one-dimensional object that displays the real numbers from $-\infty$ to ∞ going from left to right.

Example 1.3.2. Draw a number line and graph the following:

1. -2
2. 0
3. 3
4. $\frac{1}{2}$

Rational Numbers

Definition 1.3.6 (Rational Numbers)**.** root word: ratio
abbreviated as $\mathbb{Q}$; all numbers that can be expressed as the quotient of two integers where the denominator is not 0; essentially, all numbers that can be written as fractions

We can easily see that integers are rational numbers. How could we show that the integers 10 and -15 are rational?

What about mixed numbers/mixed fractions?

What happens if we add or subtract two rational numbers? Is that new number rational?

How about decimals? Are they rational numbers?

Graphing Rational Numbers

Example 1.3.3. Draw a number line and plot the following rational numbers:

1. $\dfrac{9}{2}$
2. -1.2
3. $3\dfrac{1}{3}$

Converting Rationals to Decimals

There are two easy ways to convert rational numbers to a decimal format - either long division or use a calculator.

Example 1.3.4. Convert the rational numbers $\dfrac{3}{8}$ and $\dfrac{2}{5}$ to a decimal.

Irrational Numbers

Most things in math have an inverse or a complement (opposite). The complement of the rational numbers are the *irrationals*.

Definition 1.3.7 (Irrational Numbers)**.** numbers that are not rational; numbers that can't be written as the quotient of two integers

A famous irrational number is π. Most people can name the first few digits of π as 3.14 and if you've got a good memory, you might be able to get to 3.14159; however, π continues on and *does not terminate*, meaning that there are an infinite number of decimals places. These decimal values do not follow a pattern and they do not repeat. Therefor, π is irrational. We can, however, approximate π as $\pi \approx \frac{22}{7} = 3.142857\ldots$.

We could also consider another famous example, $\sqrt{2}$. In general, radicals are irrational if they cannot be simplified. We can find that $\sqrt{2} = 1.41414\ldots$. There is a proof that is often taught in introductory discrete math classes that proves the irrationality of $\sqrt{2}$, but we will leave that be for now.

What is an example of a radical that isn't irrational?

Example 1.3.5. Consider the following set of numbers. Put each number into the correct category. Note that numbers may belong to more than one set.

$$\{-9, -1.3, 0, 0.\bar{3}, \frac{\pi}{2}, \sqrt{9}, \sqrt{10}\}$$

- Natural?
- Whole?
- Integers?
- Rational?
- Irrational?
- Real?

Ordering the Reals

Reals are considered *well ordered* meaning that if we have two real numbers, we can determine which is larger and which is smaller. We typically denote this with inequality symbols $(<, >, \leqslant, \geqslant)$. On a number line, numbers are ordered from smallest to largest going from left to right.

Example 1.3.6.

- $14 > 5$ because 14 is right of 5 on the number line
- $-19 < -6$ because -19 is left of -6 on the number line
- $\frac{1}{4} < \frac{1}{2}$ because $\frac{1}{4}$ is left of $\frac{1}{2}$ on the number line

We can modify the inequality symbols to include a number as well.

- $a \leqslant b$ reads as "a is less than or equal to b"
- $a \geqslant b$ reads as "a is greater than or equal to b"

Example 1.3.7. True or false?

1. $-2 \leqslant 3$
2. $-2 \geqslant -2$
3. $-4 \geqslant 1$

Definition 1.3.8 (Absolute Value)**.** represents the distance from zero on a number line; makes a number positive; denoted as $|a|$

Example 1.3.8. Find each of the following:

1. $|-4| =$
2. $|6| =$
3. $\left|-\sqrt{2}\right| =$
4. $|3 - 5| =$
5. $-2|-4 + 3| =$