12.1 Exponential Functions

Definition 12.1.1 (Exponential Function)

- $f(x) = b^x,\ b > 0,\ b \neq 1$
- Domain: $\mathbb{R} = (-\infty, \infty)$
- Range: $\mathbb{Z}^+ = (0, \infty)$
- Exponential functions always have two guaranteed points: $(0, 1)$ and $(1, b)$.
- The x-axis is a horizontal asymptote.

Graphs

- If $0 < b < 1$, then the function exhibits *exponential decay*.
- If $b > 1$, then the function exhibits *exponential growth*.

Example 12.1.1
Identify the base, make a table of values, and graph the function.

$$f(x) = 2^x$$

Example 12.1.2
Identify the base, make a table of values, and graph the function.

$$f(x) = 3^{-x}$$

Base "e"

Definition 12.1.2 (Euler's Number)
$e \approx 2.718281827\ldots$
$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Example 12.1.3
The exponential function

$$f(x) = 1145e^{0.0325x}$$

gives the population of gray wolves x years after 1978.

1. What is the approximate gray wolf population in 1979?

2. What about in 1990?

3. What about in 2020?

Compound Interest

There are two main types of compounding interest that we'll cover in this course.

Periodically Compounding

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- A = balance/amount
- P = principal/initial amount
- r = interest rate (as a decimal)
- n = number of times compounded in *one* time period
- t = the number of time periods

Continuously Compounding

$$A = Pe^{rt}$$

- A = balance/amount
- P = principal/initial amount
- r = interest rate (as a decimal)
- t = the number of time periods

Example 12.1.4
Say that I invested \$10,000 at 8% interest for 5 years. Compare the ending balances if compounded quarterly, twice monthly, and continuously.

1. Quarterly:

2. Twice monthly:

3. Continuously: