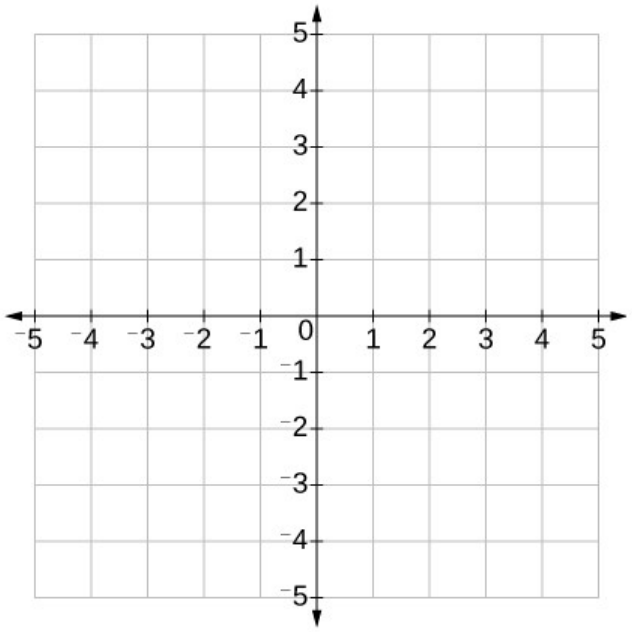
3.1 Graphing Linear Equations in 2 Variables

So far our equations have typically only involved one variable, whether it is x, y, or any other that may be used. In this chapter, we add a *second dimension*, meaning that we use two variables instead of one.

Equations of the form $ax + b = c$ have *one* solution for x and we can plot/graph this solution on a number line. However, when we increase this to two variables, our solutions jump in quantity - there are now an *infinite* number of solutions. Instead of graphing this on a number line, we introduce the concept of a *Cartesian plane* that is constructed from both an x and a y-axis.

Cartesian Planes



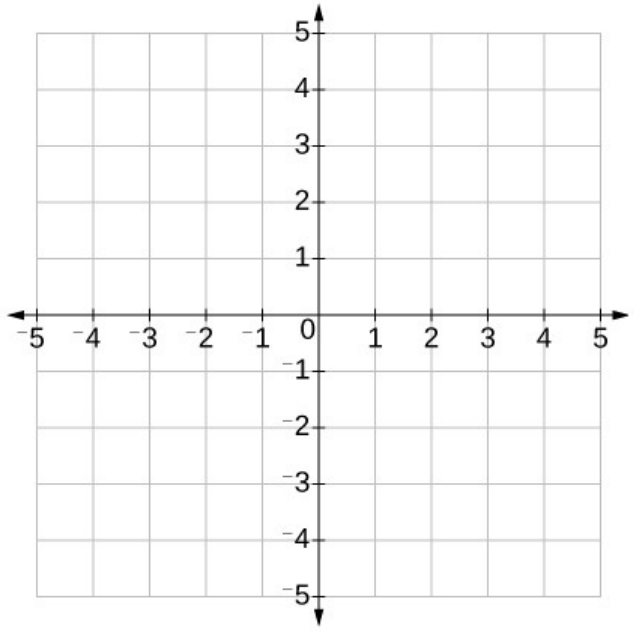
This plane is also referred to as an "xy-plane"; however, the axes are not required to be x and y. For example, if we were working on a problem in a physics class, we may refer to the x axis as t (for time) and the y axis as h (for height).
Positions or placements on the xy-plane are referred to as *points* or *ordered-pairs*. All of this together – the plane, points, axes, etc. – are collectively referred to as a "*rectangular coordinate system*".

Definition 3.1.1 (Point)
written as (x, y) where x is the x-coordinate (left/right of the center) and y is the y-coordinate (above/below the center)

Regarding individual points, we can make some inferences from our plane.

- In Q1, $x > 0$, $y > 0$ so our points are $(+, +)$.
- In Q2, $x < 0$, $y > 0$ so our points are $(-, +)$.
- In Q3, $x < 0$, $y < 0$ so our points are $(-, -)$.
- In Q4, $x > 0$, $y < 0$ so our points are $(+, -)$.

Example 3.1.1
Graph the following points on the coordinate plane below.



- A$(-2, 4)$
- B$(4, -2)$
- C$(-3, 0)$
- D$(0, -3)$

Solutions of Equations in 2-Variables

A *solution* is an ordered pair (x, y) such that (x, y) satisfies the given equation. Each equation in two variables has an infinite number of solutions – an infinite number of ordered pairs that satisfy the equation.

Example 3.1.2
Is the point $(3, -2)$ a solution to $x - 3y = 9$?

Example 3.1.3
Is the point $(-2, 3)$ a solution to $x - 3y = 9$?

Finding Solutions

1. Choose your x values – (pick some negative, 0, and some positive)
2. Plug the values of x into the equation and solve for y.
3. Write each pair of x and y as an ordered pair.

Example 3.1.4
Find 3 solutions of $y = 3x + 2$.

Graphing Equations in 2-Variables

There are a few methods available to graph these type of equations. The easiest, of course, is to use a graphing calculator. Throughout this chapter we will see a few of the various methods.

Point-Plotting Method

1. Find several ordered pairs that are solutions to the equation
   - Aim for 3 or more ordered pairs.
2. Plot each of these points on the coordinate plane.
3. Connect the dots - preferably with a straightedge of some sort.

Example 3.1.5
Graph the equation $y = 3x$.

Example 3.1.6

Graph the equation $y = \frac{1}{2}x - 2$.