10.5 Multiplying with More than One Term & Rationalizing Denominators

Multiplying Radical Expressions with Multiple Terms

The same methods of FOILing, distributing, etc., that we've been using for polynomials still apply for radicals.

Example 10.5.1. Multiply each of the following. Be sure to simplify if possible.

1. $\sqrt{6}\left(x+\sqrt{10}\right)$
2. $\sqrt[3]{y}\left(\sqrt[3]{y^2}-\sqrt[3]{7}\right)$
3. $\left(6\sqrt{5}+3\sqrt{2}\right)\left(2\sqrt{5}-4\sqrt{2}\right)$
4. $\left(5\sqrt{2}+2\sqrt{3}\right)\left(4\sqrt{2}-3\sqrt{3}\right)$

In Algebra 1, we were introduced to *special forms* for multiplication – binomial sums, binomial differences and differences of squares. These rules still hold for radicals.

Special Forms

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a-b)(a+b) = a^2 - b^2$

Example 10.5.2. Multiply each of the following special forms. Be sure to simplify if possible.

1. $\left(\sqrt{5}+\sqrt{6}\right)^2$
2. $\left(\sqrt{6}-\sqrt{5}\right)^2$
3. $\left(\sqrt{a}-\sqrt{7}\right)\left(\sqrt{a}+\sqrt{7}\right)$
4. $\left(\sqrt{2x}-\sqrt{3b}\right)\left(\sqrt{2x}+\sqrt{3b}\right)$

Rationalizing Denominators with One Term

Rationalizing a denominator is the process of rewriting an expression so that there are no radicals in the denominator. To rationalize denominators, we need to use the trick of multiplying by 1 (written in a specific way). If we have squareroots, we use whatever the denominator. If we have a higher index radical, we need to choose what to multiply by carefully.

Example 10.5.3. Rationalize each denominator.

1. $\dfrac{\sqrt{7}}{\sqrt{3}}$
2. $\dfrac{\sqrt{5}}{\sqrt{12}}$

If, however, we have an index greater than 2, we need to choose the radicand in such a way that we have exponents that are multiples of the index – allowing us to simplify the radical in the denominator.

Example 10.5.4. Rationalize each denominator.

1. $\dfrac{\sqrt[3]{2}}{\sqrt[3]{9}}$

2. $\dfrac{\sqrt[4]{2}}{\sqrt[4]{3}}$

If the radicand contains multiple factors (constants and variables), we need to make sure that each factor in the radicand ends with an exponent that is a multiple of the index. This is true regardless of the index.

Example 10.5.5. Rationalize each denominator.

1. $\sqrt{\dfrac{2x}{7x}}$

2. $\dfrac{\sqrt[3]{x}}{\sqrt[3]{9y}}$

3. $\dfrac{6x}{\sqrt[5]{8x^2y^4}}$

Rationalizing Denominators with Multiple Terms

If the denominator contains two or more terms where at least one of them is a radical, we need to multiply by the *conjugate* of the denominator. The conjugate is an idea that will get used a few times in various forms throughout this class. To find the conjugate of a radical

expression, change the operation to the opposite - addition to subtraction, subtraction to addition.

Why use the conjugate? The conjugate allows us to use the difference of squares method to simplify the denominator. If the denominator is $a + b$, then the conjugate is $a - b$. Multiplying them together gives us

$$(a+b)(a-b) = a^2 - b^2$$

Example 10.5.6. Rationalize the following:

$$\frac{18}{2\sqrt{3}+3}$$

Example 10.5.7. Rationalize the following:

$$\frac{3+\sqrt{7}}{\sqrt{5}-\sqrt{2}}$$