7.2 Multiplying & Dividing Rational Expressions

Multiplying and dividing rational expressions works exactly the same as with fractions. If

P, Q, R, and S are polynomials and $Q \neq 0$, $S \neq 0$, then:

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$$

Since rational expressions are essentially fractions, treat them the same way. When you

multiply (or divide) fractions, you should *always* make sure that you reduce the fractions before performing the operation(s).

Multiplication

Method - Multiplication

1. factor *all* numerators and denominators
2. divide and cancel common factors
3. multiply the remaining numerators together and multiply the remaining denominators together

Example 7.2.1. Find the following:

$$\frac{x-5}{x-2} \cdot \frac{x^2-4}{9x-45}$$

Example 7.2.2. Find the following:

$$\frac{5x+5}{7x-7x^2} \cdot \frac{2x^2+x-3}{4x^2-9}$$

Division

When we divide fractions, we follow the mnemonic: *keep* the first, *change* the operation, *flip* the second. We use the same rule with rational expressions. Keep the first fraction as is (but reduced), change the operation from division to multiplication and flip the second fraction (use the reciprocal).

Method - Division

1. factor *all* numerators and denominators
2. change the operation to multiplication
3. flip the second fraction (turn it into the reciprocal)
4. divide and cancel common factors
5. multiply the remaining numerators together and multiply the remaining denominators together

Example 7.2.3. Find the following quotient:

$$\frac{7}{4} \div \frac{21}{8}$$

Example 7.2.4. Find the following quotient:

$$(x+3) \div \frac{x-4}{x+7}$$

Example 7.2.5. Find the following quotient:

$$\frac{x^2+5x+6}{x^2-25} \div \frac{x+2}{x+5}$$

Example 7.2.6. Find the following quotient:

$$\frac{y^2+3y+2}{y^2+1} \div (5y^2+10y)$$