12.2 Logarithmic Functions

Definition 12.2.1 (Logarithm)
For $x > 0,\ b > 0,\ b \neq 1,$ $y = \log_b(x) \equiv b^y = x$

Circle Trick

$$y = \log_b(x)$$

Example 12.2.1
Write each of the following in exponential form.

1. $3 = \log_7(x)$

2. $2 = \log_b(25)$

3. $y = \log_4(26)$

Example 12.2.2
Write each exponential as a logarithm.

1. $2^5 = x$

2. $b^3 = 27$

3. $e^y = 33$

Example 12.2.3
Evaluate.

1. $\log_{10}(100) =$

2. $\log_3(3) =$

3. $\log_{36}(6) =$

Basic Log Properties

1. $\log_b(b) = 1$
2. $\log_b(1) = 0$
3. $\log_b\left(b^x\right) = x$
4. $b^{\log_b(x)} = x$

Example 12.2.4
Evaluate.

1. $\log_7(7) =$

2. $\log_8(1) =$

3. $\log_7\left(7^8\right) =$

4. $3^{\log_3(15)} =$

Graphs of $\log_b(x)$

- Domain: $\mathbb{Z}^+ = (0, \infty)$
- Range: $\mathbb{R} = (-\infty, \infty)$
- Logarithmic functions always have two guaranteed points: $(1, 0)$ and $(b, 1)$.
- The y-axis is a horizontal asymptote.
- If $b > 1$, it is an *increasing* function.
- If $0 < b < 1$, it is a *decreasing* function.

Example 12.2.5
Find the domain of each log function.

1. $h(x) = \log_4 (x - 5)$

2. $g(x) = \log_6 (2x - 4)$

3. $f(x) = \log_3 (6 - 4x) + 3$

Special Logs

1. Common Log
   - Base 10
   - $\log(x) = \log_{10}(x)$
2. Natural Log
   - Base e
   - $\ln(x) = \log_e(x)$

Example 12.2.6
Find each of the following values:

1. $\log(1000) =$

2. $\ln(50 =$