10.1 Radical Expressions and Functions

Definition 10.1.1 (Principal Square Root). If a is a non-negative real number, then the non-negative number b such that $b^2 = a$, denoted by $b = \sqrt{a}$, is the principal square root of a.

Example 10.1.1. Evaluate each of the following square roots.

1. $\sqrt{64}$
2. $-\sqrt{49}$
3. $\sqrt{\dfrac{16}{25}}$
4. $\sqrt{0.0081}$
5. $\sqrt{9+16}$
6. $\sqrt{9}+\sqrt{16}$

Functions with Square Roots

We can define the square root as a function with $f(x) = \sqrt{x}$. Both the domain and range of this function are the non-negative numbers - $[0, \infty)$.

To evaluate square root functions, we treat them the same as anything - make the substitution for the independent variable and simplify.

Example 10.1.2. Find the indicated value for each given function:

1. $f(3)$ when $f(x) = \sqrt{12x - 20}$
2. $g(-5)$ when $g(x) = -\sqrt{9 - 3x}$

The square root function is only defined when the *radicand* - the portion under the radical - is non-negative. If we need to find the domain of a square root function, set the radicand greater than or equal to 0 and solve for x. State the domain using whichever method is specified.

Example 10.1.3. Find the domain of each of the following functions:

1. $f(x) = \sqrt{9x - 27}$

2. $g(x) = -3\sqrt{2(3x-4)} + 4$

Simplifying $\sqrt{a^2}$

For any real value of a, we have

$$\sqrt{a^2} = |a|$$

Example 10.1.4. Simplify:

1. $\sqrt{(-7)^2}$

2. $\sqrt{(x+8)^2}$

3. $\sqrt{49x^{10}}$

4. $\sqrt{x^2 - 6x + 9}$

Cube Roots

Similar to a square root, a cube root is given as $\sqrt[3]{a} = b$ where $b^3 = a$. The 3 is the *index* of the radical. Unlike the square root, however, the cube root has negative numbers in its domain. Both the domain and range of $f(x) = \sqrt[3]{x}$ are $(-\infty, \infty)$.

Example 10.1.5. Find the indicated value for each given function:

1. $f(127)$ when $f(x) = \sqrt[3]{x-2}$

2. $g(-7)$ when $g(x) = \sqrt[3]{8x-8}$

Simplifying $\sqrt[3]{a^3}$

For any real number a, we have

$$\sqrt[3]{a^3} = a$$

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Example 10.1.6. Simplify the following:

$$\sqrt[3]{-27x^3}$$

Simplifying Odd or Even Roots

For any real number a:

- If n is even, $\sqrt[n]{a^n} = |a|$.
- If n is odd, $\sqrt[n]{a^n} = a$.

Example 10.1.7. Find each of the following:

1. $\sqrt[4]{(x+6)^4}$

2. $\sqrt[5]{(3x-2)^5}$

3. $\sqrt[6]{(-8)^6}$