5.2 Multiplying Polynomials

Exponent Rules

We know that $a^n = a \cdot a \cdot a \cdots a$. For example, we know that $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$.

What about $2^4 \cdot 2^3$ though?

$$2^4 \cdot 2^3 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$$

This gives us our first rule for exponents, the product rule:

Product Rule
$b^m \cdot b^n = b^{m+n}$

Example 5.2.1
Find each of the following:

1. $2^2 \cdot 2^3 =$

2. $x^6 \cdot x^4 =$

3. $y \cdot y^7 =$

4. $y^4 \cdot y^3 \cdot y^2 =$

Now, what if we raised an exponent to an exponent, such as $(2^3)^2$?

$$\begin{aligned} (2^3)^2 &= (2^3)(2^3) \\ &= (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= 2^6 = 64 \end{aligned}$$

This can be generalized into our second rule, the power rule:

Power Rule
$(b^m)^n = b^{mn}$

Example 5.2.2
Find each of the following:

1. $(3^4)^5 =$

2. $(x^9)^{10} =$

3. $[(-5)^7]^3 =$

Let's generalize this further - what if we had a product raised to a power? For example, $(2x^2)^3$?

$$\begin{aligned}(2x^2)^3 &= (2x^2)\cdot(2x^2)\cdot(2x^2)\\ &= (2\cdot 2\cdot 2)\cdot(x^2\cdot x^2\cdot x^2)\\ &= 2^3\cdot(x^2)^3\\ &= 2^3\cdot x^{2\cdot 3} = 8x^6\end{aligned}$$

General Power Rule
$(ab)^n = a^n b^n$

Example 5.2.3
Find each of the following:

1. $(2x)^4 =$

2. $(-4y^2)^3 =$

Multiplying Polynomials

Example 5.2.4
Find the following:

$$(7x^2)(10x)$$

Example 5.2.5
Find the following:

$$(-5x^4)(4x^5)$$

Example 5.2.6
Find the following:

$$3x(x+5)$$

Example 5.2.7
Find the following:

$$6x^2(5x^3-2x+3)$$

What if both factors have more than one term? **FOIL**

FOIL – **F**irst, **O**utside, **I**nside, **L**ast

Example 5.2.8
Find the following:

$$(x+4)(x+5)$$

Example 5.2.9
Find the following:

$$(5x+3)(2x-7)$$

Example 5.2.10
Find the following:

$$(5x+2)(x^2-4x+3)$$

Example 5.2.11
Find the following:

$$(3x^2-2x)(2x^3-5x^2+4x)$$