University of Arkansas
Department of Mathematics

Final Exam

Calculus III

Printed Name (Last, First): _____ Student ID: _____

TA Name: _____ Drill Time: _____

Instructions, Rules, Submission, etc.

This cover page should answer any questions you may have about the exam. Failure to follow these policies may result in a score of **zero**.

	Am I allowed to ...	**Answer**
...	look something up in my notes? the book? from the homework?	Yes
...	graph something with Geogebra? Desmos?	Yes
...	plug an integral into MatLab? Mathematica? Google?	No
...	discuss the exam with a classmate?	No
...	ask for help from my tutor?	No

What if I have a question? I will be available in our chat room (`Dr. Wigglesworth's Room`) at these times: Monday 8:35-9:25 am, and 12:55-1:45 pm; Tuesday 1:00 pm. I will also answer any emails you send **before 2pm** on Tuesday. Any questions sent by email after that time will not be answered.

How will I submit my exam? You will submit this exam through Gradescope. If you have not submitted an assignment on Gradescope yet, that means you have received zeros on several assignments. You absolutely must get Gradescope set up in order to submit your exam. There are several ways to submit your exam. All of them end with you uploading a pdf to Gradescope.

1. Print this pdf, fill out the exam, scan it, then upload via Gradescope.
2. Save this pdf to a tablet, fill it out. Save the filled out version to a pdf, then upload it in Gradescope.
3. Use blank sheets of paper. One cover page with your name, student ID, TA Name, etc. Then use **one sheet of paper for each question**. Write your solutions on looseleaf paper, scan them then upload the pdf to Gradescope.

What if I don't have a scanner? Please DO NOT take photos of your work with your phone. Instead, there are several apps that will allow you to use the camera on your phone as a scanner: AdobeScan (this one is free!), Scannable, Genius Scan, TurboScan, among others.

What if I don't know how to use Gradescope? Can I email you my exam? No! Exams submitted via email will not be accepted. You will receive a **zero**.

What if I don't have a printer? Read the above more carefully.

SUBMISSION DEADLINE – 7:00PM on Tuesday, May 5.

1. **[20 points]** Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = x\ln(x+2y)$.

   (a) Find the gradient of $f(x,y)$ at the point $P(e/3, e/3)$.

   (b) Use the gradient to find the directional derivative of f at $P(e/3, e/3)$ in the direction of the vector $\vec{u} = \langle -4, 3\rangle$.

   (c) Find a unit vector (based at P) pointing in the direction in which f increases most rapidly at P.

2. **[20 points]**[1] Use the method of Lagrange multipliers to find the absolute maximum and minimum values of the function $f(x, y, z) = x^2yz^2$ subject to the constraint $2x^2 + 3y^2 + 6z^2 = 33$.

[1] The following things will earn you points:

- Drawing an appropriate picture,
- correctly making any preliminary computations
- setting up an appropriate equation/system of equations for Lagrange multipliers
- solving correctly, using clear work
- clearly stating your conclusions in complete sentences.

3. **[20 points]**[2] Let D be the solid region in $\mathbb{R}^3$ bounded by the cone $z = 2\sqrt{x^2 + y^2}$ and the plane $z = 4$. Let $\vec{F}$ be the vector field in $\mathbb{R}^3$ defined by $\vec{F}(x, y, z) = \langle -x^3/3, -y^3/3, 9z \rangle$. Evaluate the outward flux integral $\iint_S \vec{F} \cdot \vec{n}\, dS$, where S is the closed surface that is boundary of the region D.

[2]The following things will earn you points:

- Drawing an appropriate picture,
- clearly stating theorems you use (if any)
- setting up your integral by choosing appropriate parameterization and/or coordinate system
- correct limits of integration
- evaluating the integral correctly, with clear work

4. **[20 points]**[3] Evaluate the line integral $\int_C (x^2 + y^2)\, ds$ where C is the line segment from $(1, 1)$ to $(2, 5)$.

[3]The following things will earn you points:

- Drawing an appropriate picture,
- using a relevant formula,
- parameterizing the curve correctly
- computing the integrand correctly
- evaluating the integral correctly

5. **[20 points]**[4] Calculate the (outward) flux of the vector field $\mathbf{F} = \langle 2x^2, y - \cos x \rangle$ through the boundary of the region in the xy-plane, enclosed by curves $y = 2x - x^2$ and $y = 0$ (oriented counterclockwise).

[4]The following things will earn you points:

- Drawing an appropriate picture,
- clearly stating any formulas/theorems you
- setting up your integral by choosing appropriate parameterization and/or coordinate system
- correctly performing any preliminary computations
- correct limits of integration
- evaluating your integral correctly

6. **[20 points]** Let C denote a curve joining point $P(0, 3\pi/2, \pi)$ to point $Q(2, \pi/2, \pi/2)$. Let $\vec{F}$ be the vector field in $\mathbb{R}^3$ defined by the equation

$$\vec{F} = \langle e^z \cos y + 2xy - 3x^2, -xe^z \sin y + x^2 - 3z \cos y, xe^z \cos y - 3 \sin y + \cos z \rangle.$$

(a) Evaluate the following circulation integral by finding an appropriate potential function.

$$\int_C \vec{F} \cdot \vec{T} \, ds$$

(b) Explain in a sentence or two why your work above shows that the value of the integral is independent of the path C.

7. **[20 points]**[5] Evaluate the surface integral $\iint_S y \, dS$ where S is the cylinder with parametrization

$$\mathbf{r}(u, v) = \langle 3\cos(u), v, 3\sin(u) \rangle \qquad 0 \leq u \leq 2\pi \quad \text{and} \quad 0 \leq v \leq 4.$$

[5] The following things will earn you points:

- Drawing an appropriate picture,
- clearly stating any formulas/theorems you
- correctly performing any preliminary computations
- correct limits of integration
- evaluating your integral correctly

8. **[20 points]**[6] Evaluate the flux integral $\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$ where $\vec{F}(x, y, z) = \langle x^2 + z, -2z, e^z \rangle$ and S is the upper part of the sphere $x^2 + y^2 + z^2 = 25$ with $3 \leq z \leq 5$.

[6] The following things will earn you points:

- Drawing an appropriate picture,
- clearly stating any formulas/theorems you
- setting up your integral by choosing appropriate parameterization and/or coordinate system
- correctly performing any preliminary computations
- correct limits of integration
- evaluating your integral correctly

9. **[20 points]**[7] Evaluate $\iiint_D z\, dV$ where D is the solid region in $\mathbb{R}^3$ bounded by the paraboloid $z = 16 - x^2 - 4y^2$ and the xy-plane. Use the change of variables

$$x = 4u\cos v \quad y = 2u\sin v \quad z = w.$$

[7] The following things will earn you points:

- Drawing an appropriate picture,
- clearly stating any formulas/theorems you
- correctly setting up limits of integration
- correctly performing any preliminary computations
- evaluating your integral correctly