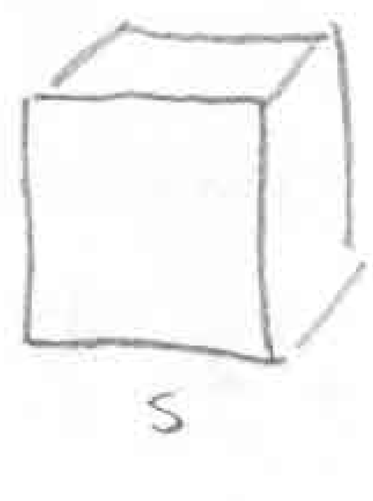
# MATH1210: HOMEWORK SOLUTIONS §2.8

1.

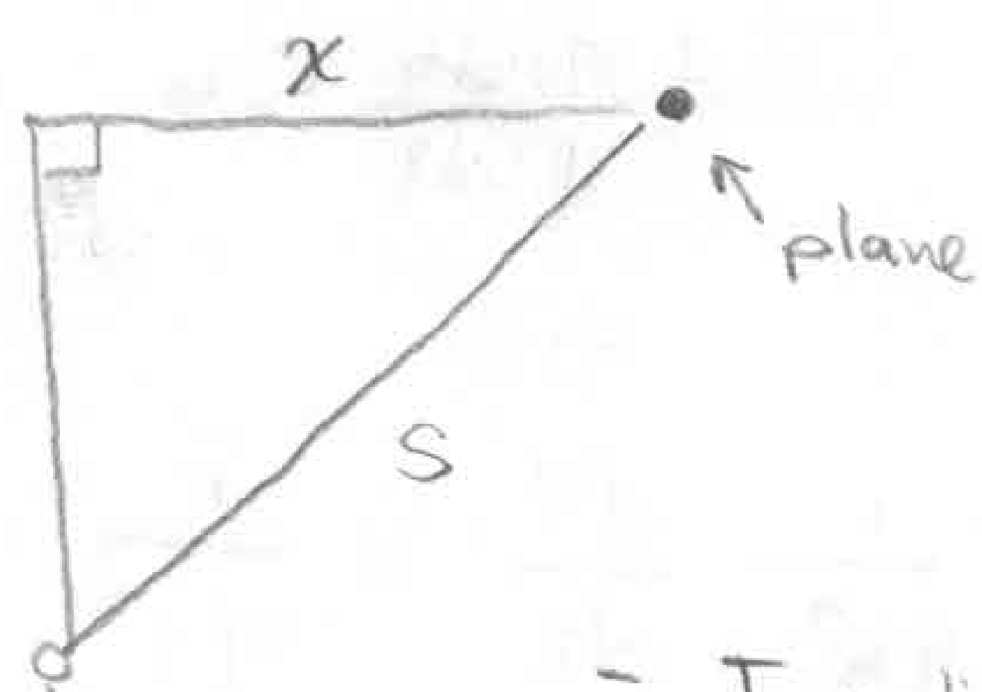



$V = s^3$ Given $\frac{ds}{dt} = 3$ in/s. Want $\frac{dV}{dt}$ when $s = 12$ in.

- $\frac{dV}{dt} = \frac{dV}{ds}\cdot\frac{ds}{dt}$  $\frac{dV}{ds} = 3s^2 \Rightarrow \frac{dV}{dt} = 3s^2\frac{ds}{dt}$

when $s = 12$ in, $\frac{dV}{dt} = 3(12\text{ in})^2\left(3\text{ in/s}\right) = \boxed{1296\text{ in}^3/\text{s}}$

3.




$1^2 + x^2 = s^2$. Given $\frac{dx}{dt} = 400$ mi/hr.

- Want $\frac{ds}{dt} = \frac{ds}{dx}\cdot\frac{dx}{dt}$

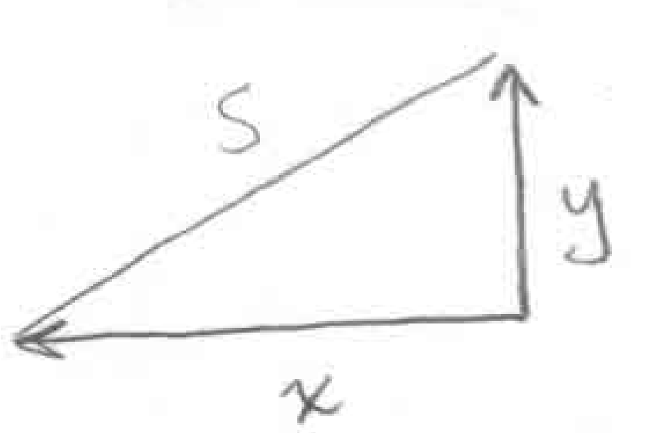
- Implicit differentiation yeilds $2x = 2s\cdot\frac{ds}{dx}$

so $\frac{ds}{dx} = \frac{x}{s}$

- when $x = 5$ mi, $\frac{ds}{dt} = \frac{x}{s}\cdot\left(400\text{ mi/hr}\right) = \frac{5}{\sqrt{26}}\cdot 400$ mi/hr

$\boxed{\approx 392\text{ mi/hr}}$

5.




$\frac{dx}{dt} = 300$ mi/hr  $\frac{dy}{dt} = 400$ mi/hr  $x^2 + y^2 = s^2$

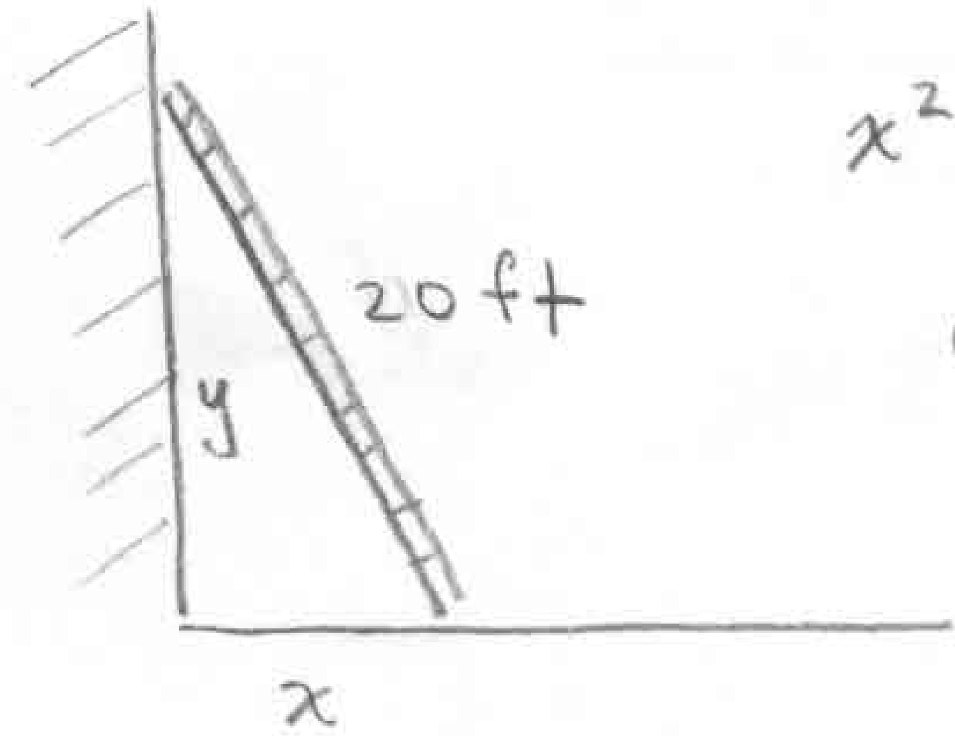
$\Rightarrow 2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

At 2:00, $x = 600$
$y = 400$
$s = \sqrt{520000}$

so $2(\sqrt{520000})\frac{ds}{dt} = 2(600)(300) + 2(400)(400)$

$\Rightarrow \frac{ds}{dt} \approx \boxed{471\text{ mi/hr}}$

7.



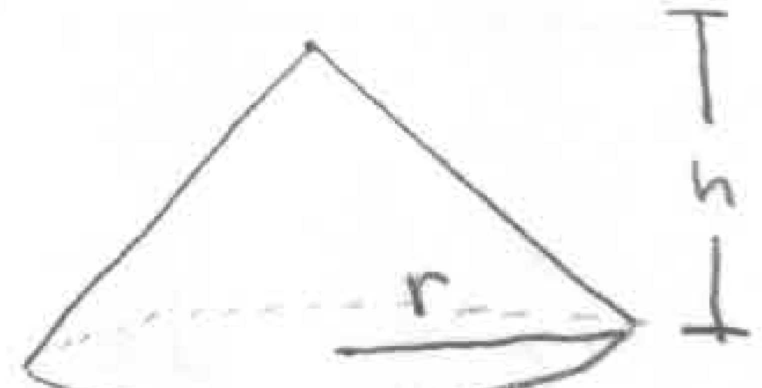

$x^2 + y^2 = 400$ Want $\frac{dy}{dt}$ when $x = 5$ ft.

Given $\frac{dx}{dt} = 1$ ft/s. Differentiate implicitly with respect to $t$. $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$$\Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt} = \frac{-5}{\sqrt{375}} \cdot (1 \text{ ft/s}) \approx -.258 \text{ ft/s}.$$

The ladder is sliding down the wall at $\boxed{.258 \text{ ft/s}}$ when $x = 5$ ft.

9.




$r = 2h$

$V = \frac{1}{3}r^2 h$

$= \frac{4}{3}h^3$

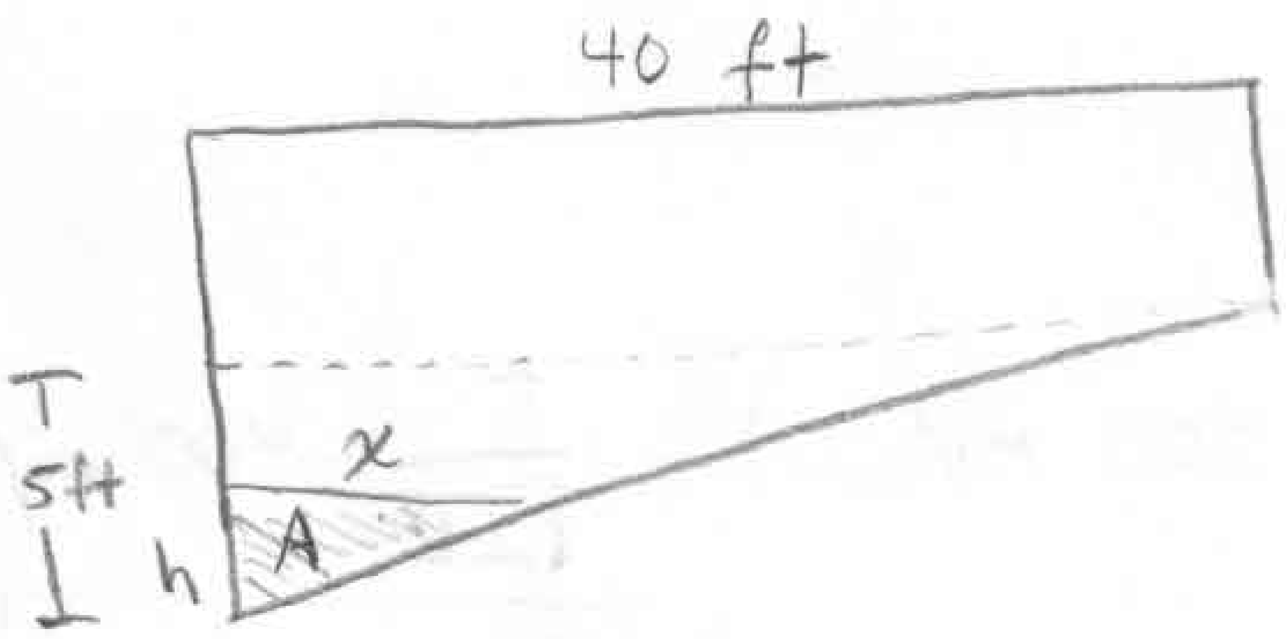
Given $\frac{dV}{dt} = 16$ ft³/s

Want $\frac{dh}{dt}$ when $h = 4$ ft.

Differentiate w.r.t. $t$

$$\Rightarrow \frac{dV}{dt} = 4h^2\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{4h^2}\frac{dV}{dt} = \frac{1}{4(4^2)} \cdot 16 = \boxed{.25 \text{ ft/s}}$$

11.




$\frac{dV}{dt} = 40$ ft³/min

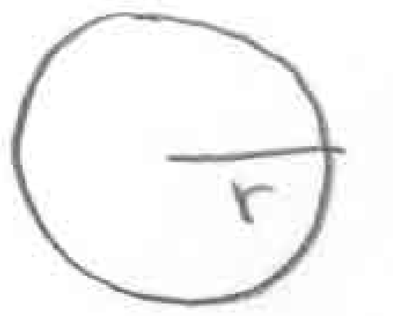
$\frac{x}{h} = \frac{40}{5} \Rightarrow x = 8h$

$$V = A \cdot 20\text{ft} = \frac{1}{2}h \cdot x \cdot 20 = 10h(8h) = 80h^2$$

$$\frac{dV}{dt} = 160h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{160 \cdot h} \cdot \frac{dV}{dt}$$

so at $h = 3$, $\frac{dh}{dt} = \frac{1}{160 \cdot 3} \cdot 40 = \boxed{\frac{1}{120} \text{ ft/min}}$

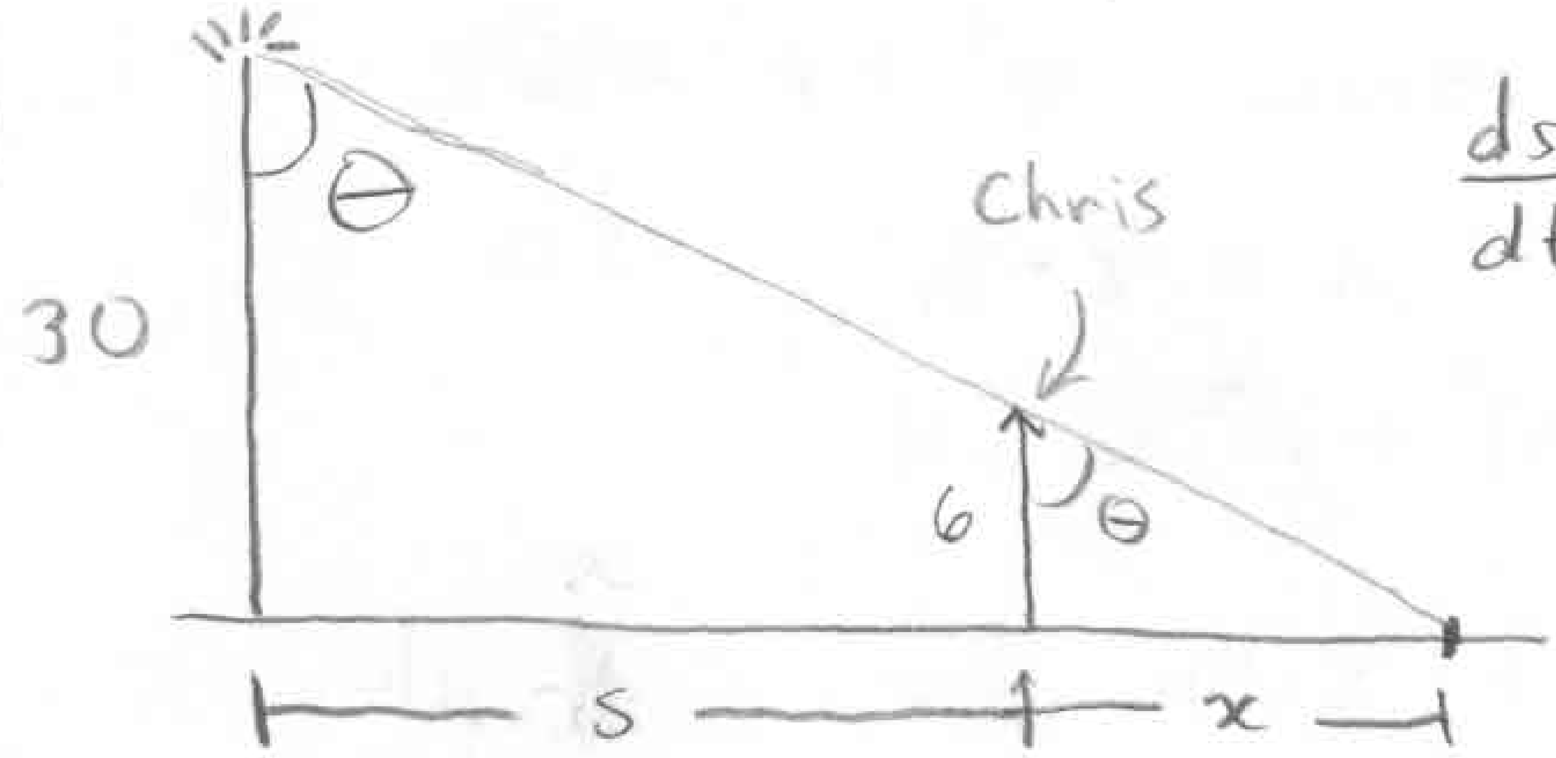
13.




$\frac{dr}{dt} = .02$ in/sec $\qquad A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

When $r = 8.1$, $\frac{dA}{dt} = 2\pi(8.1)(.02) \approx \boxed{1.02 \text{ in}^2/\text{s}}$

17. a)




$\frac{ds}{dt} = 2$ ft/s $\qquad$ Using similar $\Delta$s,

$$\frac{x}{6} = \frac{s+x}{30} \quad \text{so}$$

$$24x = 6s$$

- Diff. wrt $t$ to get $\quad 24\frac{dx}{dt} = 6\frac{ds}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{4}\frac{ds}{dt}$

- This eqn doesn't depend on $s$, so

$$\frac{dx}{dt} = \frac{1}{4}(2 \text{ ft/s}) = \boxed{0.5 \text{ ft/s}}$$ regardless of how far he is.

b) Tip of shadow is moving at $\frac{d}{dt}(s+x) = \frac{ds}{dt} + \frac{dx}{dt} = 2. + 0.5 = \boxed{2.5 \text{ ft/s.}}$

c)

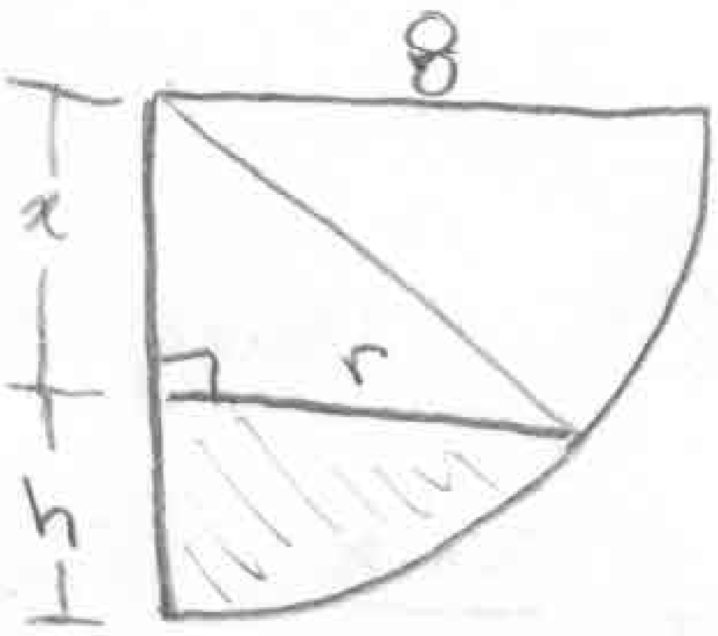
$$\tan\theta = \frac{x}{6} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{6}\frac{dx}{dt}$$

$\Rightarrow \frac{d\theta}{dt} = \frac{1}{6}(0.5) \cdot \frac{1}{\sec^2\theta} = \frac{1}{12} \cdot \cos^2\theta$ and when $x = 6$,

$$\cos\theta = \frac{6}{\sqrt{6^2+6^2}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{d\theta}{dt} = \boxed{\frac{1}{24} \text{ rad/sec}}$$

21. $\frac{dV}{dt} = 2\ ft^3/hr$ $\qquad V = \pi h^2[r - h/3]$

- Need an eqn relating $r$ & $h$.
- Look at a cross-section




we know $x^2 + r^2 = 64$
and $x = 8 - h$. so
$(8-h)^2 + r^2 = 64$

- At this point, we could solve for $r$ & plug it in to formula for $V$, but it's easier if we implicitly differentiate both equations with respect to $t$. The volume eqn will have $\frac{dV}{dt}$, $\frac{dh}{dt}$ and $\frac{dr}{dt}$. We know $\frac{dV}{dt}$; we want $\frac{dh}{dt}$; and the second eqn will give a relationship between $\frac{dh}{dt}$ and $\frac{dr}{dt}$.

- $$\frac{dV}{dt} = 2\pi h \cdot \frac{dh}{dt}\left[r - h/3\right] + \pi h^2\left[\frac{dr}{dt} - \frac{1}{3}\frac{dh}{dt}\right] \qquad (*)$$

- $$2(8-h)\left(-\frac{dh}{dt}\right) + 2r\frac{dr}{dt} = 0 \implies \frac{dr}{dt} = \left(\frac{8-h}{r}\right)\left(\frac{dh}{dt}\right)$$

- When $h = 3$, $r = \sqrt{64-25} = \sqrt{39}$

so using $(*)$, we get

$$2 = 2\pi(3)\cdot\frac{dh}{dt}\left[\sqrt{39} - 1\right] + \pi(3)^2\left[\left(\frac{8-3}{\sqrt{39}}\right)\frac{dh}{dt} - \frac{1}{3}\frac{dh}{dt}\right]$$

$$\implies \frac{dh}{dt} = \frac{2}{6\pi(\sqrt{39}-1) + 9\pi\left(\frac{5}{\sqrt{39}} - \frac{1}{3}\right)} \approx .0178$$