

MATH 1030: Homework 1 Solutions

due January 10, 2014

Problem 1

Come by my office in the LCB Loft before Friday afternoon. Find my desk in the maze of cubicles and sign the sheet of paper so I know you came.

Problem 2

Get an appropriate scientific calculator for the quiz on Friday.

Problem 3

Five red, five green, and five blue marbles are placed in a jar.

- a) How many marbles must you remove (without looking) from the jar to be sure that you have at least one of each color?

The clearest way to think about this is to ask “What’s the worst possible scenario?” First, I could remove all the red marbles. Then I might remove all the green marbles, and I still wouldn’t have one of each color. But the 11-th marble would have to be blue, which would give me one of each. So the answer is 11.

- b) How many marbles must you remove (still without looking) from the jar to be sure that you have two marbles of the same color?

Again we ask “What’s the worst possible scenario?” First, I draw a red marble. Then I draw a green marble, and I still don’t have two of the same color. Next I might draw a blue marble. Now I have one of each (three marbles), and I still don’t have two of the same color. But no matter which marble I choose next, I’ll have 2 of the same color. So the answer is 4.

Problem 4

Solve for x in each of the equations below. Be sure to check your answer.

- a) $2x + 17 = 9x - 4$

$$2x + 17 = 9x - 4$$

$$17 = 7x - 4$$

$$21 = 7x$$

$$x = 3$$

- b) $x^2 + 4x - 12 = 0$

$$x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0$$

So either $x + 6 = 0$ in which case $x = -6$ or $x - 2 = 0$ so that $x = 2$.

c) $|x - 1| + 4 = 9$

$$|x - 1| + 4 = 9$$

$$|x - 1| = 5$$

Recall the definition of the absolute value function

$$|y| = \begin{cases} y & \text{if } y \geq 0 \\ -y & \text{if } y < 0 \end{cases}$$

So if $x - 1 \geq 0$, then $|x - 1| = 5$ implies $x - 1 = 5$ so $x = 6$. On the other hand, if $x - 1 < 0$, then $|x - 1| = -(x - 1)$. Then we have $-(x - 1) = 5$, so $-x + 1 = 5$, which gives that $x = -4$.

Problem 5

Simplify the following expressions

a) $\frac{36p^5q^4}{9p^3q}$

We have

$$\frac{36p^5q^4}{9p^3q} = \frac{4p^5q^4}{p^3q} = \frac{4p^2q^4}{q} = 4p^2q^3$$

b) $(2y^3 + 4y^3)^2 \cdot y^{-3}$

We first combine the terms in the parentheses, giving

$$(6y^3)^2 \cdot y^{-3} = 36y^6 \cdot y^{-3} = 36y^3$$

c) $(a^2b^{-5})^3 \cdot (ab^9)^0$

We have

$$(a^2b^{-5})^3 \cdot (ab^9)^0 = a^{2 \cdot 3}b^{-5 \cdot 3}(a^0b^{9 \cdot 0}) = a^6b^{-15}a^0b^0 = \frac{a^6}{b^{15}}$$

Problem 6

Solve the following word problems.

- a) The length of a rectangle is 7 inches more than its width. The area of the rectangle is 78 square inches. Find the length and width of the rectangle.

If we call the length of the rectangle l and the width w , then we are given $l = w + 7$. The area of a rectangle is $78 = l \cdot w$, so $78 = l \cdot w = (w + 7)w$. Distributing the w , and moving everything to the right hand side, we get $0 = w^2 + 7w - 78$, which factors as $0 = (w + 13)(w - 6)$. So $w = 6$ or $w = -13$, but w is supposed to be the length of the side of a triangle, and hence cannot be negative. Then $w = 6$ which means that $l = 7$.

- b) Nick Andopolis has four times as many LPs as Ken Miller. Ken has 33 less LPs than Nick. How many LPs do each of them have?

Call the number of LPs Nick has n and the number that Ken has k . Then Nick has $4 \cdot k$ LPs, so $n = 4 \cdot k$. The difference between the number of LPs Nick has and the number that Ken has is 33, so $n - k = 33$. Using the first equation, we can plug in the value of $4k$ for n in the second, yeilding $4n - n = 33$. So $3n = 33$ and therefore $n = 11$. Thus, Cate has 11 LPs and Kathy has $4 \cdot 11 = 44$ LPs.