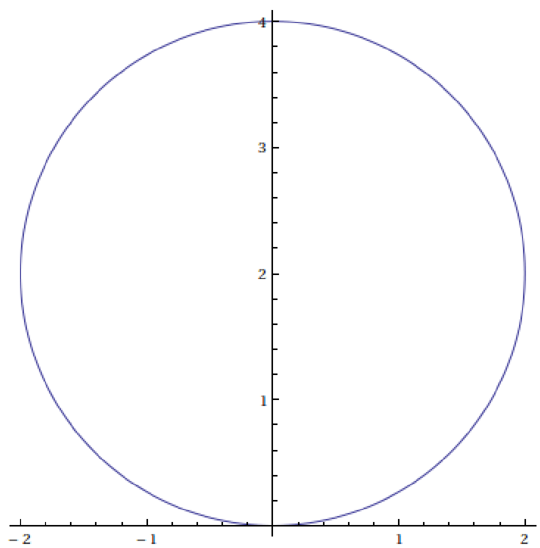
# MATH1060: §10.8 Handout

Derrick Wigglesworth

December 9, 2013

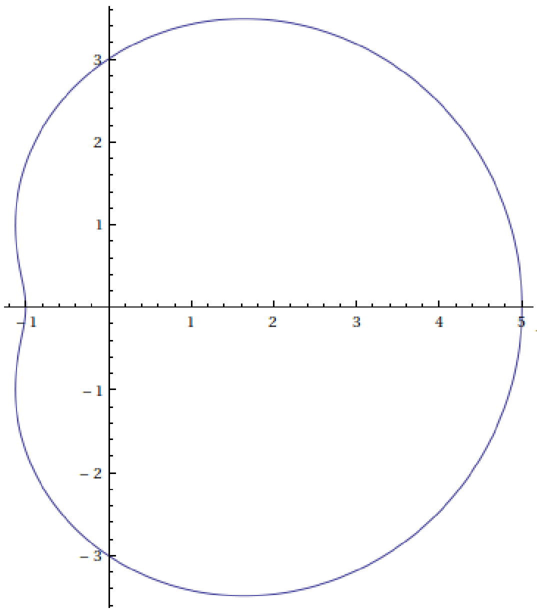
## 1 Circles

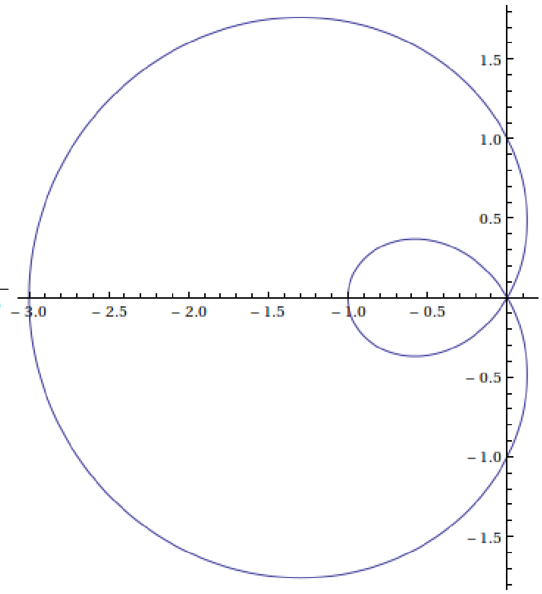
We saw in class that circles centered at the origin have polar equations of the form $r = c$ where $c$ is a constant. The equation of a circle which is not centered at the origin is not quite so simple in polar coordinates. The circle shown here is the graph of $r = 4\sin\theta$.




## 2 Limaçons

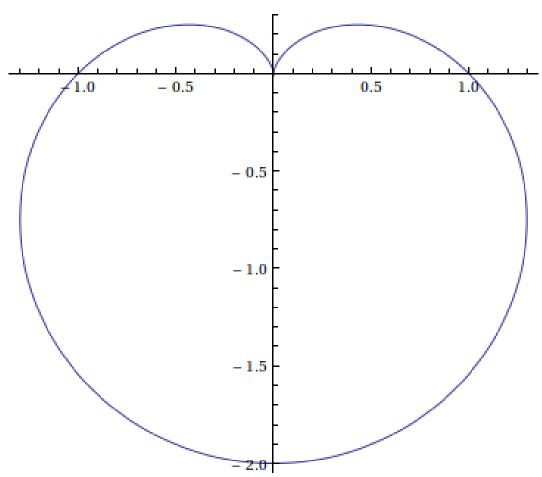
The first graph shown below is the graph of $r = 3 + 2\cos\theta$. You can convince yourself that this is reasonably accurate by plugging in $\theta = 0, \pi/2, \pi, 3\pi$ to the given equation. This shape is called a limaçon.




The figure on the right is a limaçon with a loop. The equation of this graph is $r = 1 - 2\cos\theta$. In general, a limaçon is the graph of an equation of the form $r = a + b\cos\theta$. The ratio $\frac{a}{b}$ determines the exact type. Try plugging in different values of $\theta$ to see where the loop comes from.

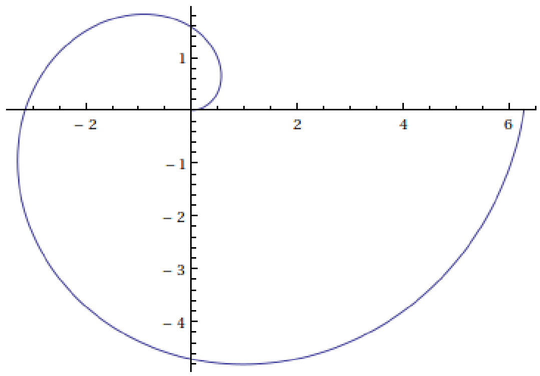
When the ratio $\frac{a}{b}$ is equal to 1, we don't quite get a loop. The limaçon will be in between the first one (with no loop) and the second one (with a loop). This shape (pictured below) is called a cardioid. The figure below is the graph of $r = 1 - \sin\theta$.

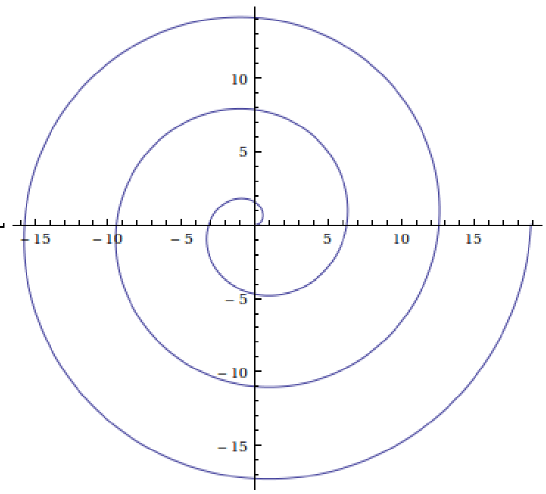



# 3 Spiral of Archimedes

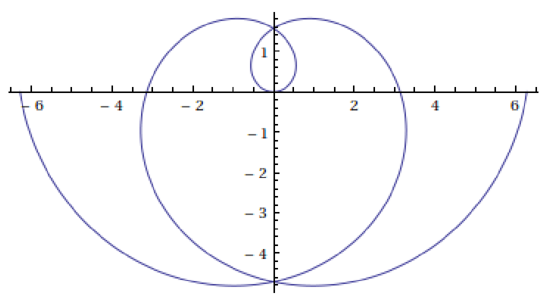
In rectangular coordinates, the graph of $y = x$ is simply a line. In polar coordinates, the graph of $r = \theta$ is not just a line. When $\theta = 0$, $r = 0$, so we have a point at the origin. As $\theta$ starts increasing,

$r$ increases proportionally. This curve is called the Spiral of Archimedes. It's graph is shown below for various ranges of $\theta$. The first is $\theta \in [0, 2\pi]$. The second is the graph for $\theta \in [0, 6\pi]$.



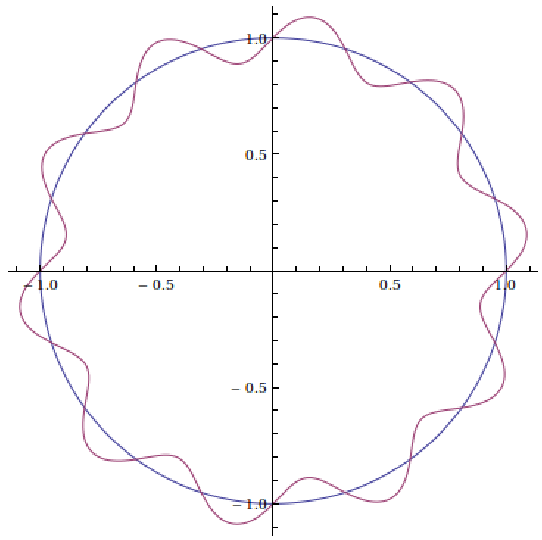




Now if we allow negative values of $\theta$, we see another copy of the graph but reflect across the $y$-axis. The graph below is the spiral of Archimedes for $\theta \in [-2\pi, 2\pi]$.




## 4 A Wiggly Shape

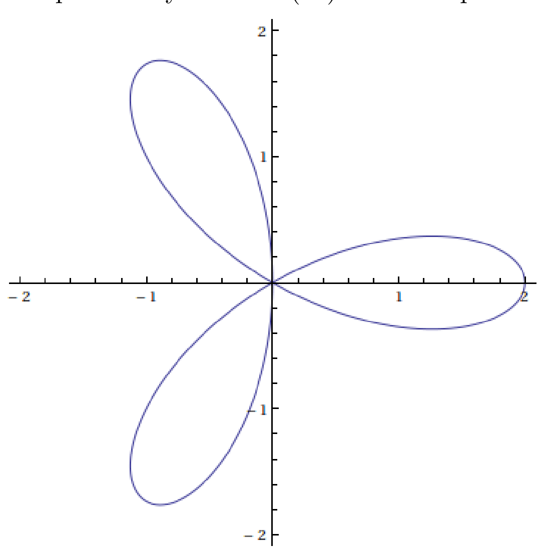
This is just a cool picture. It is the graph of $r = 1 + \frac{1}{10}\sin(10\theta)$. For reference, I also graphed the circle $r = 1$.

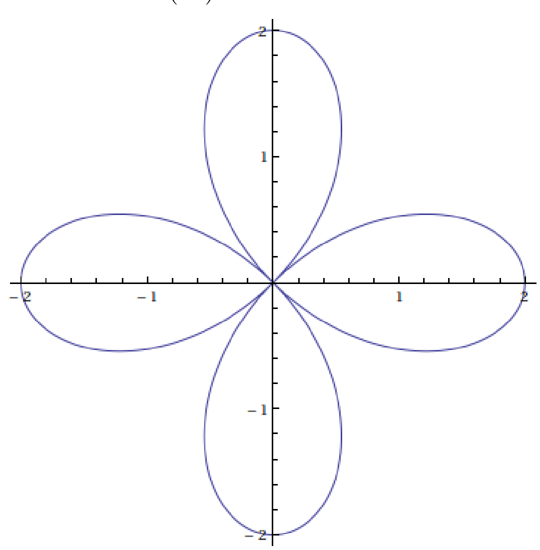



# 5 Roses (or Daisies)

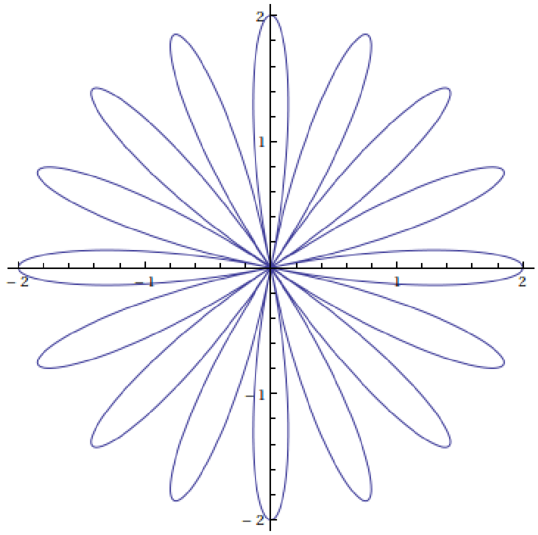
## 5.1 Closed Roses

Shown below are the graphs of $r = 2\cos(3\theta)$ and $r = 2\cos(2\theta)$. As $\theta$ varies from 0 to $2\pi$, the argument of $\cos(3\theta)$ varies from 0 to $6\pi$, so we see 3 periods of the cosine function. This gives us a "rose" with 3 pedals (we would see 6 pedals, but the graph repeats itself from $\pi$ to $2\pi$). For an even number, $n$, the graph of $r = \cos(n\theta$ has $2n$ pedals (because this graph doesn't repeat in $[0, 2\pi]$. This explains why $r = 2\cos(2\theta)$ has more pedals than $r = 2\cos(3\theta)$.



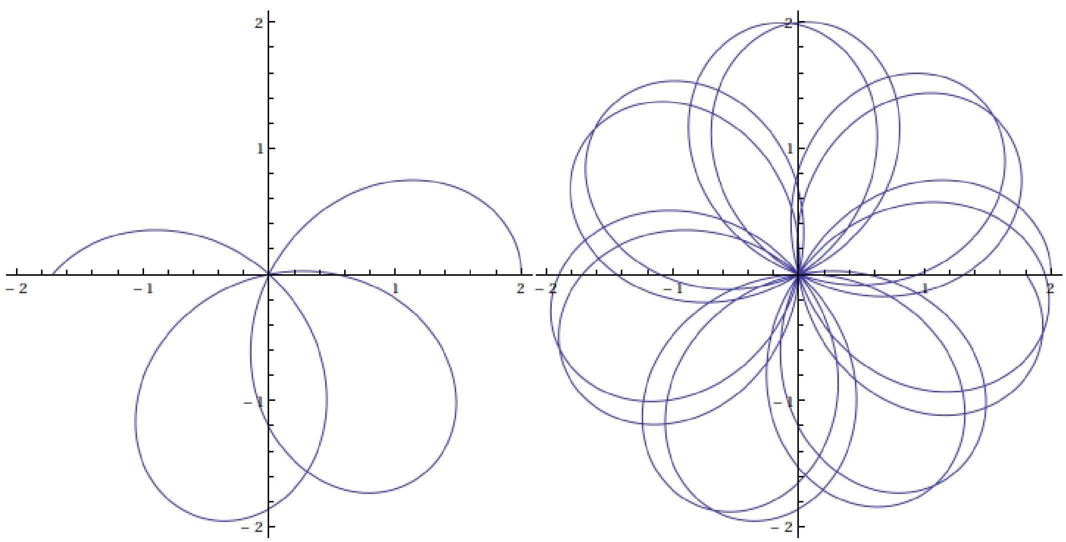




Shown below is the graph of $r = 2\cos(8\theta)$, which, by the preceding discussion, should have 16 pedals.

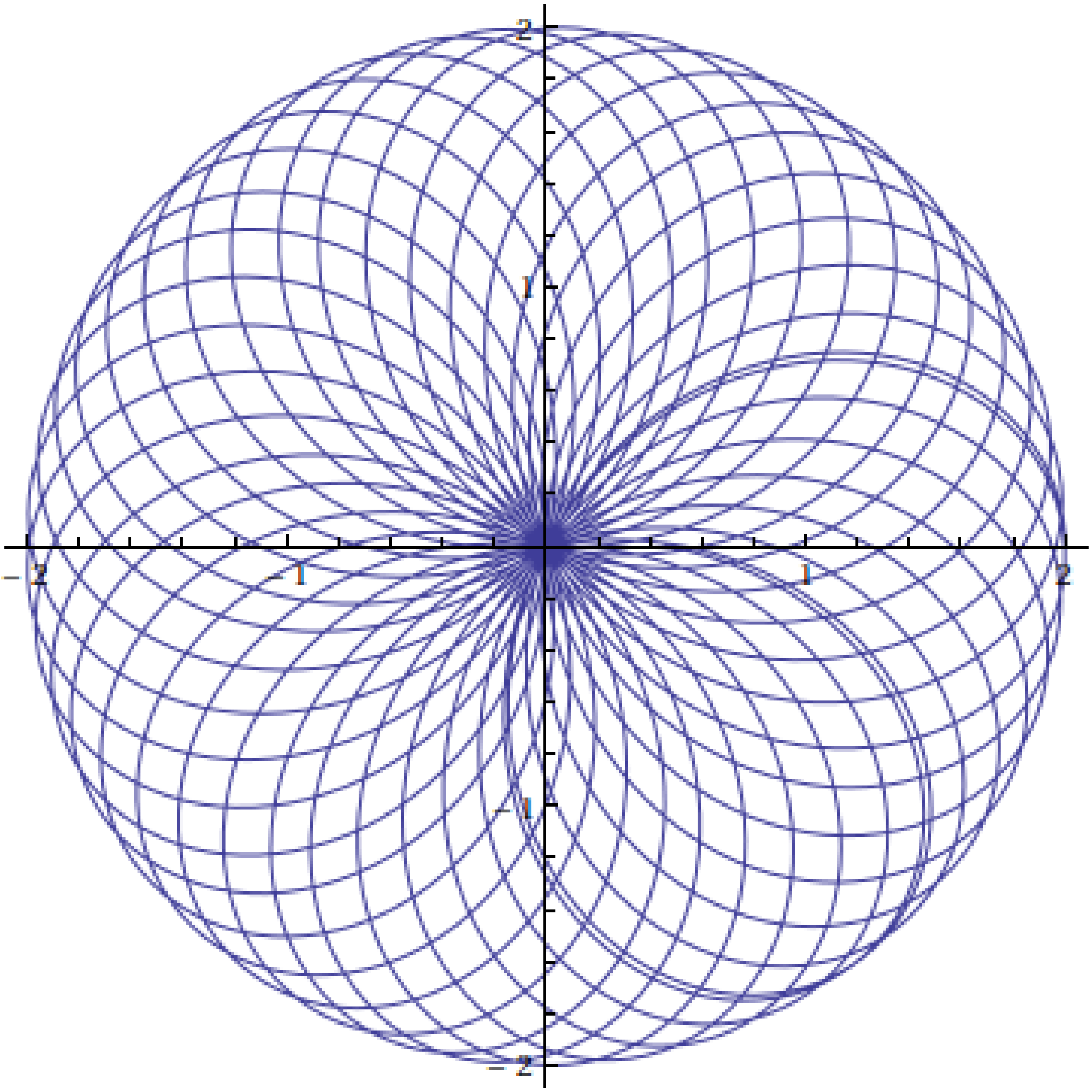


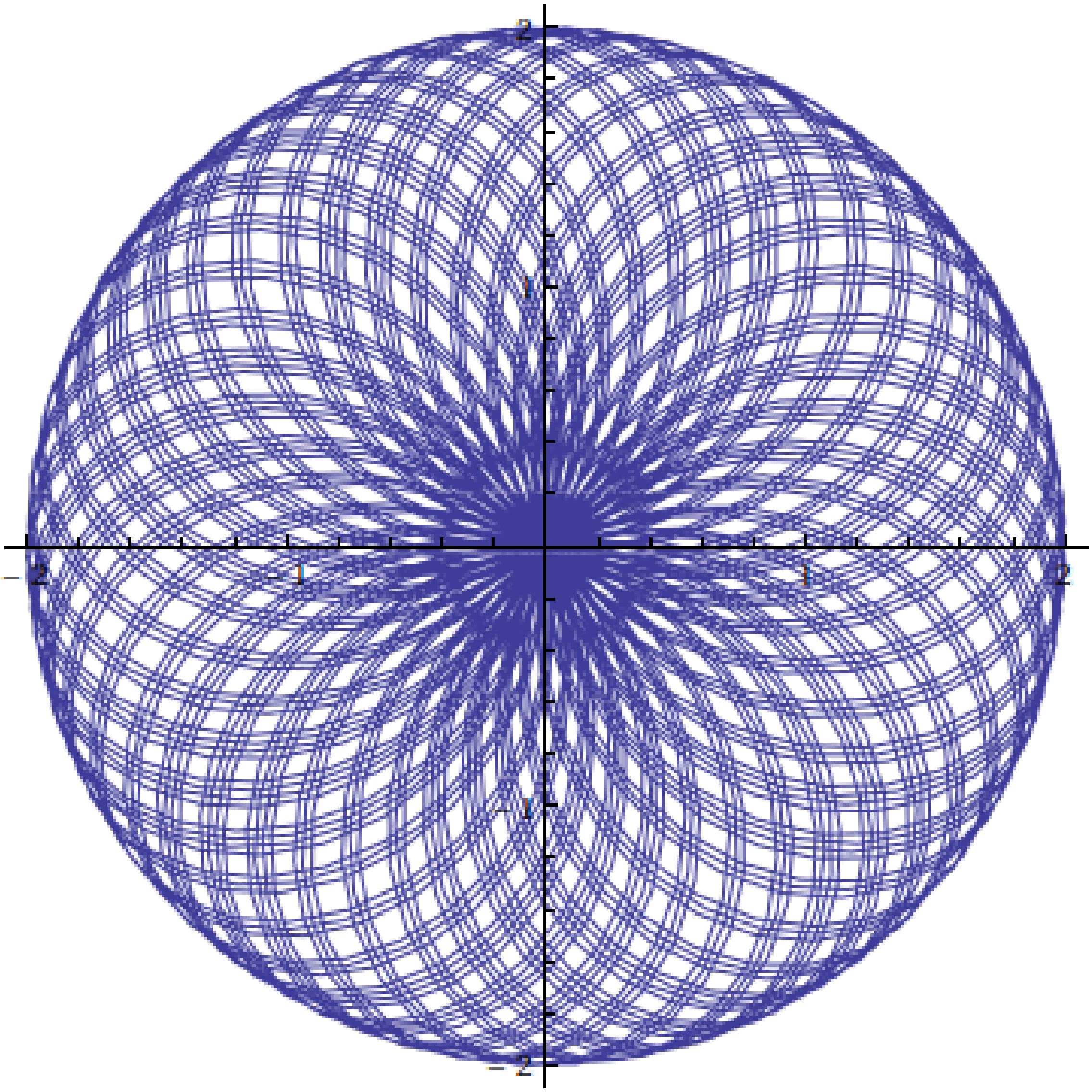

## 5.2 Non-closed Roses

If we alter the period to be an irrational number ($\sqrt{2}$ for example), then graph will never close up on itself. Shown here is the graph of $r = 2\cos(\sqrt{2}\theta)$ for $\theta \in [0, 2\pi]$ and then for $\theta \in [0, 10\pi]$.



If you look closely at the positive $x$-axis, you can see that the graph has not closed up on itself. Pictured below are graphs of the same function with $\theta \in [0, 30, \pi]$ and $\theta \in [0, 100\pi]$ respectively.


2
1
-2
-1
1
2
-1
-2



2
1
−2
−1
1
2
−1
−2