

NAME:

Final Exam

August 4, 2016

Instructions: Do all the problems on **both sides** of each page. Show all your work and box your answers. If you get stuck on a problem, skip it and come back to it at the end.

1. **[8 points each]** Evaluate the following definite and indefinite integrals.

(a) $\int (x^3 + 5x + 9)^8 (12x^2 + 20) dx$

(b) $\int_1^{27} \sqrt[3]{x} dx$

(c) $\int_{\pi/4}^{2\pi/3} \sin(2t) \, dt$

2. **[12 points]** The radius of a circle is increasing at a rate of 3 inches per second. How fast is the area of the circle increasing when the radius is 9 inches?

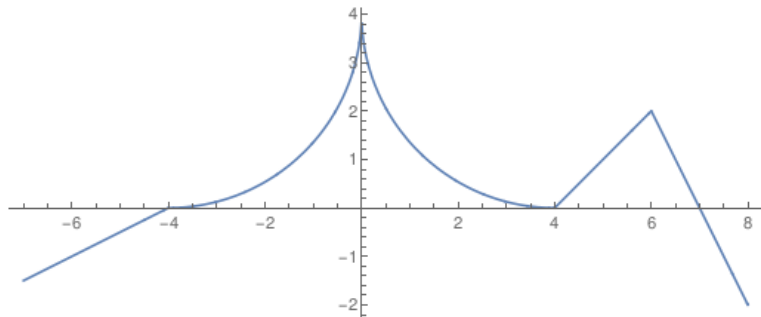
3. Consider the ellipse with equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

(a) [**7 points**] Find $\frac{dy}{dx}$ for any point on the ellipse in terms of x and y .

(b) [**8 points**] Write an equation for the tangent line to the ellipse at the point $\left(\frac{12}{5}, \frac{12}{5}\right)$.

- (c) **[10 points]** *This is is a continuation of the problem from the previous page.* Consider the region R bounded by the ellipse and *above* the x -axis. Set up an integral to find the volume of the solid obtained when the region R is rotated about the x -axis.

4. The graph of $y = f(x)$, shown below, consists of three line segments and two quarter circles. The function $g(x)$ is defined by $g(x) = \int_4^x f(t) dt$ for all x in the closed interval $[-7, 8]$.



- (a) **[4 points]** Is $g(x)$ continuous on $[-7, 8]$? Give a reason for your answer.
- (b) **[7 points]** Which is larger, $g(-4)$ or $g(-3)$? Give a reason for your answer.
- (c) **[8 points]** What is the absolute maximum of g on the closed interval $[-7, 8]$? *I am asking for the maximum value of g !*
- (d) **[6 points]** Find the x -coordinate of each point of inflection on the graph of $y = g(x)$.

5. **[12 points]** Let $F(x) = \int_{-x}^{3x} 2t^2 dt$. Find $F'(x)$.

6. **[15 points]** Find two numbers x and y such that $x + 2y = 2$ and such that $x^2 + y^2$ is minimal.

7. [7 points each] Evaluate the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{2x^9 + x^5 + 8x^2}{10x^9}$

(b) Let $f(x) = \begin{cases} x^2 + 10 & x < 3 \\ 6x & x \geq 3 \end{cases}$. Find $\lim_{x \rightarrow 3} f(x)$

(c) $\lim_{x \rightarrow -2} \frac{x + 2}{x^2 - 2x - 8}$

(d) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}$

8. **[7 points each]** Find the derivative of each function. *Do not simplify your answers.*

(a) $f(x) = x^2 \sin x$

(b) $g(x) = (x^2 - 3\sqrt{x})^{100}$

9. A differentiable function f has the values shown in the table:

x	0	1	2	3	4	5	6
$f(x)$	0	3	1	0.5	-1	-4	-6

(a) **[5 points]** Estimate $f'(4.5)$.

(b) **[7 points]** Find the average rate of change of $f(x)$ on the interval $0 \leq x \leq 6$. What does the Mean Value Theorem for Derivatives tell you?

(c) **[8 points]** Estimate $\int_0^6 f(x) dx$ using a midpoint Riemann sum with three equal sized subintervals.

10. **[15 points]** Find the volume of the solid obtained by revolving about the y -axis the region in the plane bounded by the y -axis and the curves $y = x$ and $y = 3 - 2x^2$.

11. **[10 points]** Set up an integral to find the circumference of the ellipse with parametric equations $x(t) = 3 \cos t$, $y(t) = 4 \sin t$ with $0 \leq t \leq 2\pi$. *Do not evaluate the integral!!!*

12. **Extra Credit:** [?? points] Some question.