MATH1210: HOMEWORK SOLUTIONS §2.7

1. $\frac{d}{dx}\left[y^2 - x^2 = 1 \right]$

$$2y\left(\frac{dy}{dx}\right) - 2x = 0$$

$$\frac{dy}{dx} = \frac{x}{y}$$

3. $1 \cdot y + x \cdot \left(\frac{dy}{dx}\right) = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

5. $1 \cdot y^2 + x \cdot 2y \cdot \left(\frac{dy}{dx}\right) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - y^2}{2xy}$$

7. $12x^2 + 7\left[1 \cdot y^2 + x(2y)\left(\frac{dy}{dx}\right)\right] = 6y^2\left(\frac{dy}{dx}\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{12x^2 + 7y^2}{6y^2 - 14xy}$$

9. $\frac{1}{2}(5xy)^{-1/2}\left[5y + 5x\frac{dy}{dx}\right] + 2\frac{dy}{dx} = 2y\left(\frac{dy}{dx}\right) + y^3 + 3xy^2\left(\frac{dy}{dx}\right)$

$$\left(\frac{dy}{dx}\right)\left[\frac{5x}{2\sqrt{5xy}} + 2 - 2y - 3xy^2\right] = y^3 - \frac{5y}{2\sqrt{5xy}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^3 - \frac{5y}{2\sqrt{xy}}}{\frac{5x}{2\sqrt{xy}} + 2 - 2y - 3xy^2}$$

11. $x\frac{dy}{dx} + y + \cos(xy)\left[x\frac{dy}{dx} + y\right] = 0$

$$\frac{dy}{dx} = \frac{-y}{x}$$

13. $3x^2y + x^3\frac{dy}{dx} + 3y^2\frac{dy}{dx}x + y^3 = 0$

$\frac{dy}{dx} = \frac{-y^3 - 3x^2y}{3y^2x + x^3}$ at $(1,3)$ $\frac{dy}{dx} = \frac{-27 - 3\cdot 3}{3\cdot 9 + 1} = \frac{-36}{28} = -9/7$

$y - 3 = \frac{-9}{7}(x-1)$

15. $\cos(xy)\left[y + x\frac{dy}{dx}\right] = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$ at $(\pi/2, 1)$ $\frac{dy}{dx} = \frac{\cos(\pi/2)}{1 - \pi/2\cos(\pi/2)} = \frac{0}{1} = 0$

so tangent is $y = 1$

17. $\frac{2}{3}x^{-1/3} - \frac{2}{3}y^{-1/3}\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{3\sqrt[3]{x}}}{1 + \frac{1}{3\sqrt[3]{y}}}$ at $(1,-1)$ $\frac{dy}{dx} = \frac{\frac{1}{3}}{1 + \frac{1}{-3}} = \frac{1}{2}$

tangent line is $y + 1 = \frac{1}{2}(x-1)$

19. $\frac{dy}{dx} = 5x^{2/3} + \frac{1}{2}x^{-1/2}$

21. $\frac{dy}{dx} = \frac{1}{3}x^{-2/3} - \frac{1}{3}x^{-4/3}$

23. $\frac{dy}{dx} = \frac{1}{4}(3x^2 - 4x)^{-3/4}\cdot(6x-4)$

25. $\frac{dy}{dx} = \frac{-2}{3}(x^3 + 2x)^{-5/3}(3x^2 + 2)$

27. $\frac{dy}{dx} = \frac{1}{2}(x^2 + \sin x)^{-1/2}(2x + \cos x)$

29. $\frac{-(2x\sin x + x^2\cos x)}{3(x^2\sin x)^{4/3}}$

31. $$\frac{dy}{dx} = \frac{-\sin(x^2+2x)(2x+2)}{4\left(1+\cos(x^2+2x)\right)^{3/4}}$$

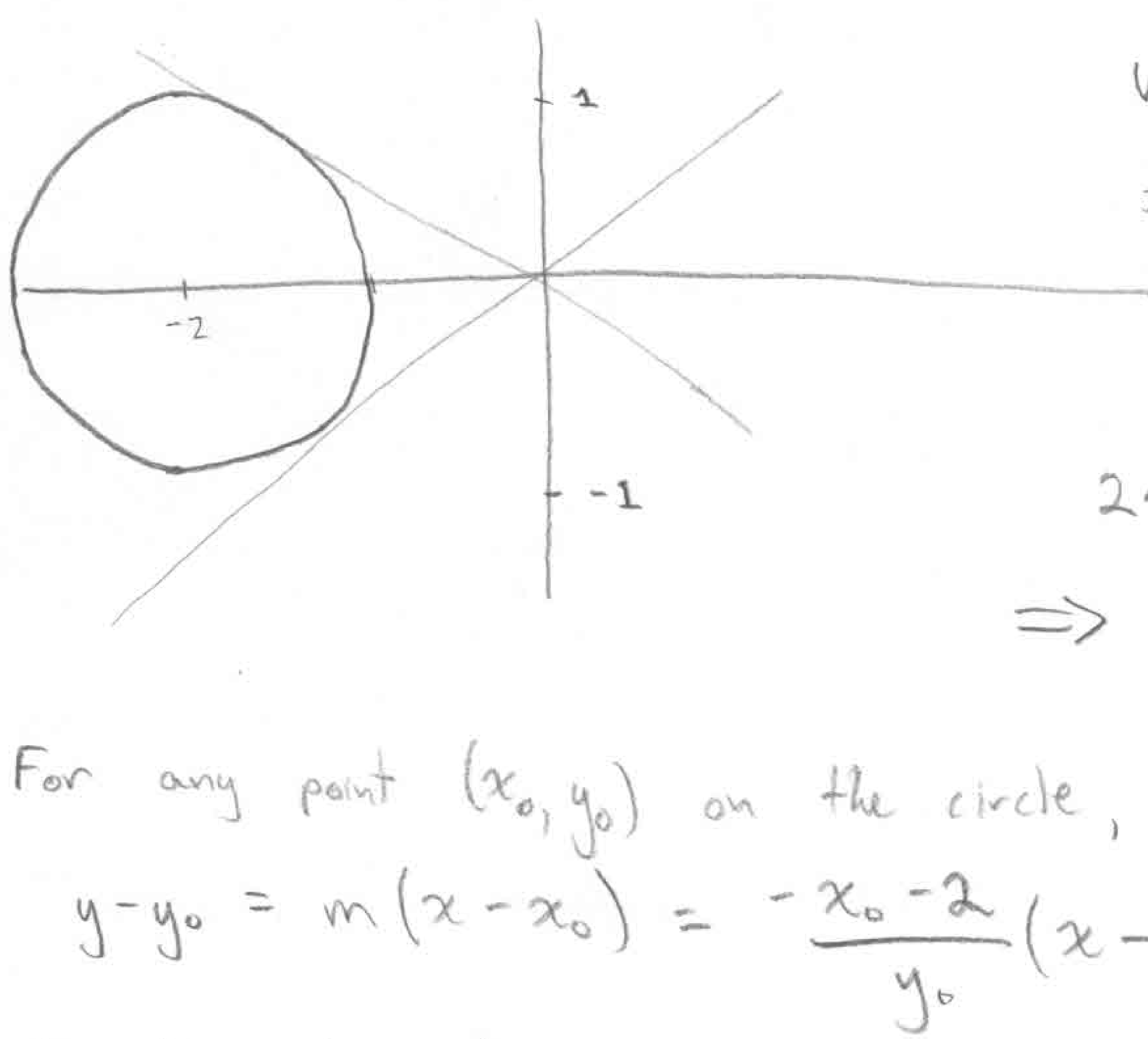
33. $$s^2 t + t^3 = 1$$

$$2s\left(\frac{ds}{dt}\right)t + s^2 + 3t^2 = 0 \Rightarrow \frac{ds}{dt} = \frac{-3t^2 - s^2}{2st}$$

$$2st + s^2\frac{dt}{ds} + 3t^2\frac{dt}{ds} = 0 \Rightarrow \frac{dt}{ds} = \frac{-2st}{s^2+3t^2}$$

35. $$x^2 + 4x + y^2 + 3 = 0 \qquad \text{center} = (-2, 0)$$

$$\Leftrightarrow (x+2)^2 + y^2 = 1 \qquad \text{rad} = 1$$



We diff implicitly to find the slope of the tangent at an arbitrary point on the circle.

$$2x + 4 + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x-2}{y}$$

- For any point (x_0, y_0) on the circle, the tangent line is

$$y - y_0 = m(x - x_0) = \frac{-x_0 - 2}{y_0}(x - x_0)$$

- If the tangent line passes through origin, then $(0,0)$ is on the line, so

$$0 - y_0 = \frac{-x_0 - 2}{y_0}(0 - x_0) \Rightarrow y_0^2 = -x_0^2 - 2x_0$$

- This relation is about points on the circle whose tangent goes through the origin.

$\Rightarrow$ (over)

- We can substitute $-x_0^2 - 2x_0$ for y_0^2 to get

$$(x_0+2)^2 - x_0^2 - 2x_0 = 1 \Rightarrow 2x_0 = -3 \Rightarrow x_0 = \frac{-3}{2}$$

- Now we solve for $y_0 = \pm\sqrt{-x_0^2 - 2x_0} = \pm\sqrt{\frac{-9}{4} + 3} = \pm\sqrt{\frac{3}{4}}$

and the tangent lines are

$$y - \sqrt{\tfrac{3}{4}} = \frac{-1/2}{\sqrt{3/4}}\left(x + 3/2\right) \quad \text{and} \quad \boxed{y + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}\left(x + \frac{3}{2}\right)}$$

$$\boxed{y - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{3}\left(x + 3/2\right)}$$