# Trig Cheat Sheet

#### Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$$
$$\cos(\gamma \pm \delta) = \cos \gamma \cos \delta \mp \sin \gamma \sin \delta$$
$$\tan(\epsilon \pm \zeta) = \frac{\tan \epsilon \pm \tan \zeta}{1 \mp \tan \epsilon \tan \zeta}$$

### Double-Angle Formulas

$$\sin(2\eta) = 2\sin\eta\cos\eta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan(2\iota) = \frac{2\tan\iota}{1 - \tan^2\iota}$$

### Power-Reducing Formulas

$$\sin^2 \kappa = \frac{1 - \cos(2\kappa)}{2}$$
$$\cos^2 \lambda = \frac{1 + \cos(2\lambda)}{2}$$
$$\tan^2 \mu = \frac{1 - \cos(2\mu)}{1 + \cos(2\mu)}$$

# Half-Angle Formulas

$$\sin(\nu/2) = \pm \sqrt{\frac{1 - \cos \nu}{2}}$$
$$\cos(\xi/2) = \pm \sqrt{\frac{1 + \cos \xi}{2}}$$
$$\tan(\rho/2) = \frac{1 - \cos \rho}{\sin \rho} = \frac{\sin \rho}{1 + \cos \rho}$$

#### Product-to-Sum Formulas

$$\sin \sigma \sin \tau = \frac{1}{2} \left[ \cos(\sigma - \tau) - \cos(\sigma + \tau) \right]$$

$$\cos \upsilon \cos \phi = \frac{1}{2} \left[ \cos(\upsilon - \phi) + \cos(\upsilon + \phi) \right]$$

$$\sin \chi \cos \psi = \frac{1}{2} \left[ \sin(\chi + \psi) + \sin(\chi - \psi) \right]$$

### **Sum-to-Product Formulas**

$$\sin \omega + \sin \alpha = 2 \sin \left(\frac{\omega + \alpha}{2}\right) \cos \left(\frac{\omega - \alpha}{2}\right)$$

$$\sin \omega - \sin \alpha = 2 \cos \left(\frac{\omega + \alpha}{2}\right) \sin \left(\frac{\omega - \alpha}{2}\right)$$

$$\cos \omega + \cos \alpha = 2 \cos \left(\frac{\omega + \alpha}{2}\right) \cos \left(\frac{\omega - \alpha}{2}\right)$$

$$\cos \omega - \cos \alpha = -2 \sin \left(\frac{\omega + \alpha}{2}\right) \sin \left(\frac{\omega - \alpha}{2}\right)$$

### Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

#### Law of Cosines

$$c^{2} = a^{2} + b^{2} - 2ab\cos\gamma$$
  
 $b^{2} = a^{2} + c^{2} - 2ac\cos\beta$   
 $a^{2} = b^{2} + c^{2} - 2bc\cos\alpha$ 

## Area of a Triangle

#### From Law of Sines

$$A = \frac{1}{2}bc\sin\alpha$$

#### Heron's Formula

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
  
$$s = \frac{a+b+c}{2}$$

### **Vector Projection**

$$\operatorname{proj}_{\vec{u}} \vec{v} = \frac{\vec{v} \cdot \vec{u}}{\|\vec{u}\|} \vec{u}$$

### **Complex Roots**

The *n*-th roots of  $re^{i\theta}$  are

$$z_k = \sqrt[n]{r}e^{(\theta + 2\pi k)i/n}$$

for 
$$k = 0, 1, \dots, n - 1$$