MATH2270: Midterm 3 Study Guide

The following is an overview of the material that will be covered on the third exam.

§5.2 The Characteristic Equation

- Compute the characteristic polynomial of a matrix.
- Find the eigenvalues of a matrix (or linear transformation).

§5.3 Diagonalization

- Be able to diagonalize a matrix. That is, write $A = PDP^{-1}$. If the matrix is not diagonalizable, you should be able to determine that.
- Given a diagonalizable matrix, A, be able to find a basis of $\mathbb{R}^n$ consisting of eigenvectors of A.

§5.4 Eigenvectors and Linear Transformation

- Understand the connection between diagonalizing a matrix, and viewing a linear transformation in a "well chosen" basis.
- Given a linear transformation, $T\colon V \to V$, be able to find a basis of V (if one exists) for which the matrix of T with respect to $\mathcal{B}$ is diagonal.

§4.9 Application to Markov Chains/ §5.6 Discrete Dynamical Systems

- Know the following definitions: Markov chain, discrete dynamical system, stochastic matrix.
- Be able to compute the state of a dynamical system after a given amount of time for given initial conditions.
- Be able to find steady-state vectors for Markov chains and discrete dynamical systems.
- You should be able to find trajectories of a discrete dynamical system under iteration and classify the origin as an attractor/repeller/saddle.
- Be able to "diagonalize" a discrete dynamical system.

§5.7 Applications to Differential Equations

- Solve systems of first order linear differential equations by "diagonalizing"

§6.1 Inner Product, Length, and Orthogonality

- Know the basic properties of the dot product and be able to do computations.
- Know the definitions of orthogonal vectors and the orthogonal complement of a subspace.
- Know theorem 3 on page 335 of the text book and be able to apply it.

§6.2 Orthogonal Sets/§6.3 Orthogonal Projections

- Know the definitions of orthogonal sets, orthonormal sets, and orthogonal projections onto subspaces. Be able to do computations.
- Is the set orthogonal? Orthonormal? You should be able to answer this.
- Find $\text{proj}_W v$ for a vector $v \in \mathbb{R}^n$ and a subspace W.
- Write $v \in \mathbb{R}^n$ as $v = w + z$ where $w \in W$ and $z \in W^{\perp}$.