Food for Thought 4

Due Friday, September 22

Spend the rest of today's class period working through these problems. I encourage you to work with your classmates and discuss the problems. If you are finished with the assignment at the end of class today, then you can turn it in today. If you would like to work on the assignment more, take it home over the weekend and turn it in on Friday. This assignment will be graded for ***effort*** *(which means you have written down thoughtful, complete solutions to each problem), not correctness. Solutions to these problems will be posted to Canvas on Friday for future reference.*

1. Give examples of the following:

   (a) A 3×3 matrix A that is not invertible.

   (b) Two 2×2 matrices A and B such that $AB \neq BA$.

   (c) Two 2×2 matrices A and B (where $A \neq I_2$ and $B \neq I_2$) such that $AB = BA$.

   (d) Two matrices A and B such that $AB = 0$, but $A \neq 0$ and $B \neq 0$.

2. Suppose that A and B are square and A is invertible. Show that if $AB = 0$, then $B = 0$. Why does the example you gave in 1(d) not contradict this? If **both** A and B are invertible, is it possible that $AB = 0$?

3. Suppose $A^n = 0$ for some $n > 1$. Find an inverse for $I - A$. (Hint: Factor $I - A^n$.)

4. Suppose A is invertible. Explain why $A^T A$ is also invertible. Find a formula for A^{-1} in terms of A^T and $(A^T A)^{-1}$.

5. Let A be a 6×4 matrix and B a 4×6 matrix. Show that the 6×6 matrix AB cannot be invertible.

6. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$ for a 3×3 matrix A. One possible reduced row echelon form of A is

$$\begin{bmatrix} 1 & * & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \qquad T \text{ is not one-to-one, } T \text{ is onto.}$$

Compute the other seven possible reduced row echelon forms of A (using $*$ to denote entries that could be any real number). In each case, state whether T is one-to-one and whether T is onto.