MATH1220: Final Exam Practice Problems

The following are practice problems for the final exam. For review of any earlier material, refer to Midterms 1,2, & 3 along with the corresponding practice problems.

1. Find the convergence set for the following power series:

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{5^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty}2n(x+\pi)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{(2n+1)!}$

2. Find the power series representation for $f(x)=(\sin x)(\cos x)$ by multiplying the power series for the functions $\sin x = x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\ldots$ and $\cos x = 1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\ldots$.

3. Find the power series for $\frac{x}{1+x^2}$ by using polynomial long division (this is the same as "equating coefficients").

4. Find the power series for $\frac{1}{2}\ln(1+x^2)\int\frac{x}{1+x^2}dx$ by integrating the result of the last problem term-by-term.

5. Find the first 5 terms of the Taylor polynomial about $x=1$ for $g(x)=\frac{x}{1+x^2}$ and compare it to the result of 3.

6. Approximate $\cos(0.1)$ by using the Taylor polynomial of order five for $\cos x$ about $a=0$. Estimate the error using Taylor's remainder formula. $(5!=120)$

7. Find the Taylor polynomial of order 3 for $f(x)=\sqrt{x}$ based at $a=1$.

8. Convert the equation $\theta=\frac{\pi}{4}$ into rectangular coordinates (If you think about it, you can write down the answer without doing any work at all, but show work anyways).

9. Convert $x^2+y^2=8+2x$ into a polar equation.

10. Compute the area of the region bounded by the graph of $r=5+4\cos\theta$.

11. Compute the area of the region bounded by the graph of $r=5\sin(3\theta)$.