MATH1210-002

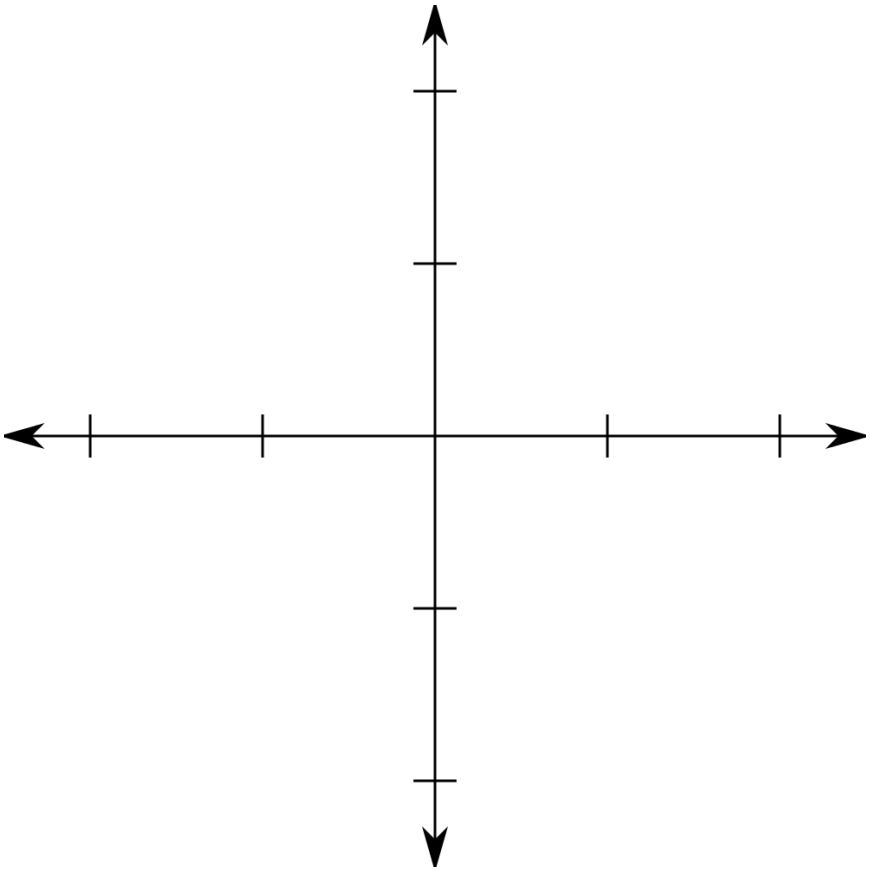
NAME:

Midterm 1

June 23, 2016

Instructions: Do all the problems on **both sides** of each page. Show all your work and box your answers. If you get stuck on a problem, skip it and come back to it at the end.

1. **[10 points]** Invent a function f satisfying the following conditions:
   - The domain of f is $[-2, 2]$
   - $f(-2) = f(2) = 1$
   - $f(0) = f(1) = -1$
   - $f(-1) = 2$
   - $\lim_{x \to 1^+} f(x) = 0$
   - $\lim_{x \to 1^-} f(x) = 2$
   - f is continuous at $x = 0$
   - $\lim_{x \to -1} f(x) = \infty$



2. **[8 points]** Find $\lim_{x \to -2} f(x)$ from the following table of functional values

x	f(x)
-2.25	9.047
-2.1	9.069
-2.01	9.010
-2.001	9.001
-2	-7
-1.999	8.999
-1.99	8.990
-1.9	8.871
-1.75	8.578

$\lim_{x \to -2} f(x) =$_____

Is f continuous at $x = -2$?
(Circle one)

Yes or No

3. **[8 points]** Show that the polynomial $x^4 + 3x^3 - 2x^2 - x + 1$ has a root on the interval $[-1, 0]$. (*Hint:* Intermediate Value Theorem)

4. **[5 points each]** Compute the following limits:

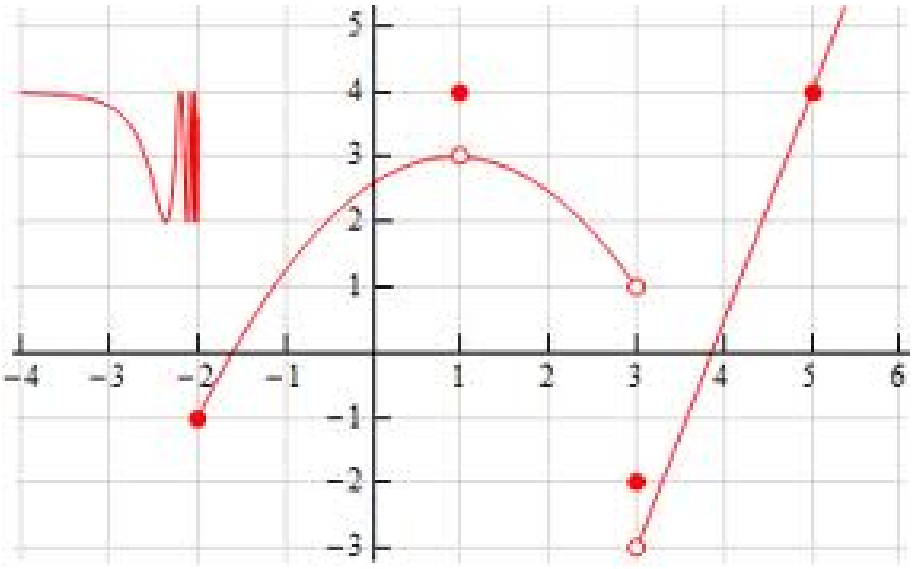
   (a) $\lim_{x \to 4} \dfrac{x^2 - 8x + 16}{x^2 - 2x - 8}$

(b) $\lim_{x \to -2} \dfrac{x+2}{x^2 - x - 6}$

(c) $\lim_{x \to \pi/2^-} \dfrac{\cos x}{x \sin x}$

(d) $\lim_{\theta \to 0} \dfrac{\tan 5\theta}{\sin 6\theta}$

5. **[12 points]** Shown below is the graph of the function $f(x)$. Find the following:



(a) $\lim_{x\to 1} f(x) =$_____

(b) $\lim_{x\to -2^+} f(x) =$_____

(c) $\lim_{x\to 3^+} f(x) =$_____

(d) $\lim_{x\to 3^-} f(x) =$_____

(e) $f(-2) =$_____

(f) $f(3) =$_____

6. **[5 points each]** Compute each of the following limits.

(a) $\lim_{x\to 3^-} \dfrac{x}{[[x]]}$

(b) $\lim_{x \to 0^-} \dfrac{|x|}{x}$

7. **[10 points]** State whether this function is continuous or not. If the function is discontinuous, give the x-value(s) where the discontinuities occur and also state what type of discontinuities occur at each of those x-values. Then "patch" the hole(s), if possible.

   (a) $f(x) = \dfrac{(x+1)(x-3)}{x^2+x-12}$ Continuous everywhere? Yes or No (circle one)

   If no, it's discontinuous at $x =$_____

   (b) For each of the x-values you listed above, why is it discontinuous? (You may not need all of the lines provided). Show work to support your answer.

   At $x =$_____, there is a hole or jump discontinuity or vertical asymptote

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   (c) If $f(x)$ has any discontinuities, how can we re-define $f(x)$ so that its holes are "patched"? (*Recall that we cannot patch vertical asymptotes or jumps. For this problem, just patch any holes that exist.*)

8. **[5 points each]** Let $f(x) = \dfrac{2x^2 - 6x + 4}{x^2 - 6x + 9}$ and let $g(x) = \dfrac{x}{x^2 - 9}$. Compute:

(a) $\lim_{x \to 3^+} f(x) =$

(b) $\lim_{x \to \infty} f(x) =$

(c) $\lim_{x \to -\infty} g(x) =$

9. **[7 points]** Find $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$ and be sure to justify your steps. (*Hint:* Squeeze Theorem)

10. **Extra Credit: [5 points]** Let f be a continuous function defined on $[0, 1]$ and suppose that $0 \leq f(x) \leq 1$. Prove that f has a fixed point. That is, prove there is a point $c \in [0, 1]$ such that $f(c) = c$.