University of Arkansas
Department of Mathematics

Exam 1

Calculus III

Printed Name (Last, First): _____ Student ID: _____

TA Name: _____ Drill Time: _____

Instructions: This exam has a total of 85 points. You have 50 minutes. **CLEARLY SHOW ALL YOUR WORK** to receive full credit. Put a **box** around your final answer. You may use any result covered in class. The points attached to each problem are indicated beside the problem.

Question	Score	Question	Score
1		5	
2		6	
3		7	
4			

Final Score: _____

v1

1. **[5 points each]** Consider the vectors $\mathbf{u} = \langle 1, 3, -2 \rangle$ and $\mathbf{v} = \langle 1, 1, -1 \rangle$.

   (a) Find the projection of $\mathbf{u}$ onto $\mathbf{v}$ and call this vector $\mathbf{w}$.

   (b) Find the vector $\mathbf{u} - \mathbf{w}$.

   (c) Show that $\mathbf{u} - \mathbf{w}$ is orthogonal to $\mathbf{v}$.

2. **[5 points each]** Consider the points $P(1,1,3)$, $Q(4,0,1)$, and $R(0,1,6)$.

(a) Find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(b) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

(c) Find the area of the parallelogram whose sides are $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

3. **[10 points]** Find an equation of the plane that contains the point $(1, 2, -3)$ and is perpendicular to the line with vector equation $\mathbf{r}(t) = \langle 2 - 3t, 3 + t, -1 \rangle$.

4. **[5 points each]** Suppose the position of a particle in space is given by

$$\mathbf{r}(t) = \langle 2\sin(t), e^t - 2t \rangle.$$

(a) Find the vector valued function that describes the velocity of the particle as a function of time.

(b) What is the speed of the particle at $t = 0$?

5. **[5 points each]** Consider the point $P(1, 2, 3)$ and the plane defined by the equation $x + y - 2z = 0$. In this problem, you will find the point on the plane that is closest to P.

   (a) Starting at P, in which direction should you travel to get to the plane as quickly as possible? Your answer should be a vector.

   (b) Write parametric equations for the line through P in the direction of the vector from part (5a). If you did not get an answer for part (5a), you may use the direction vector $\mathbf{v} = \langle a, b, c \rangle$ instead.

   (c) **[Extra Credit]** Find the point Q on the plane that is closest to P. Then calculate the distance between these two points.

6. **[10 points]** A particle is moving in space with acceleration described by the function

$$\mathbf{a}(t) = \langle 2\sin t, 2\cos t, 0\rangle.$$

At time $t = 0$ the particle has velocity $\overrightarrow{0}$ and is located at $\mathbf{r}(0) = \langle -1, 1, 1\rangle$. Find the vector equation of the position function $\mathbf{r}(t)$.

7. **[15 points] (3 for each blank)** Each of the two surfaces below is defined by one of the five equations shown. Match each surface to its corresponding equation.

I. $\frac{x^2}{4} + y^2 + z^2 = 1$

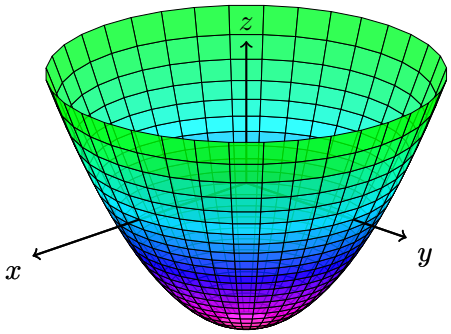
II. $z = x^2 - y^2$

III. $2 = x^2 + y^2$

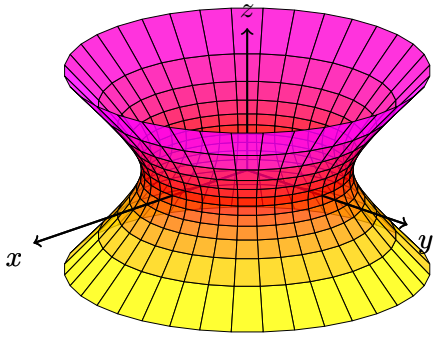
IV. $z = (x-1)^2 + y^2$

V. $x^2 + y^2 - z^2 = 1$

(a)



(b)



Equation: _____

Equation: _____

(c) Consider the surface defined by $y = z^2 - x^2$. Describe the traces of this surface in each of the three coordinate planes:

xy-plane: _____ xz-plane: _____ yz-plane: _____

SCRATCH WORK