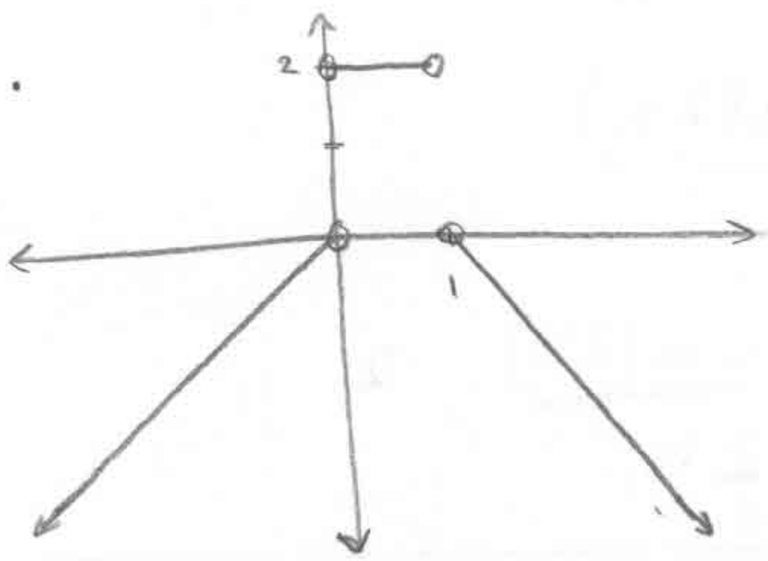
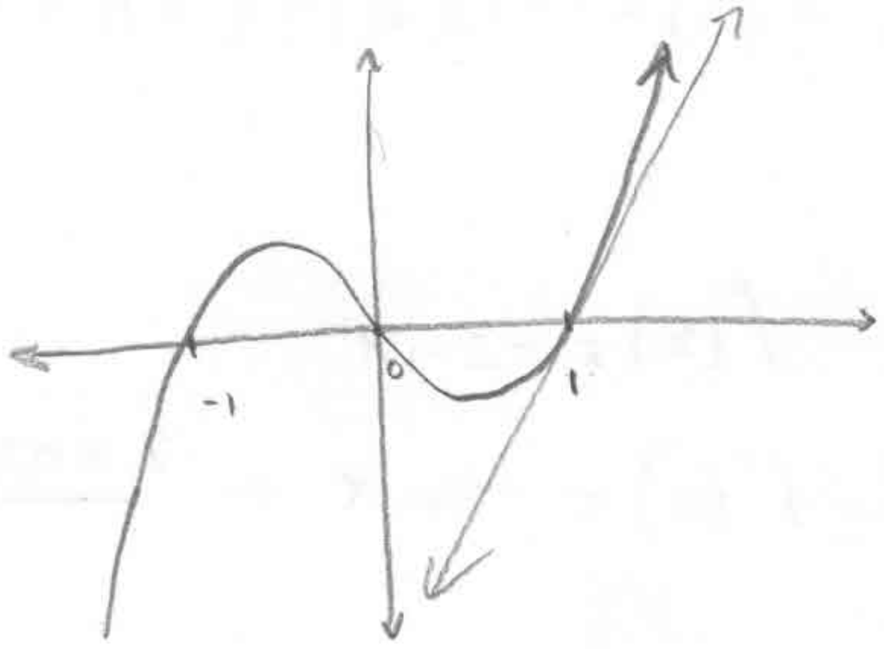
MATH1210: MIDTERM 2 PRACTICE ANSWERS

1.



2. $f'(x) = 3x^2 - 1$. $f'(1) = 2$ $\quad y - 0 = 2(x-1)$

$y = 2x - 2$



3. a) $\lim_{h\to 0} \dfrac{2(x+h)^2 + 3(x+h) - 2x^2 - 3x}{h} = \lim_{h\to 0} \dfrac{4xh + 2h^2 + 3h}{h} = 4x + 3$

b) $\lim_{h\to 0} \dfrac{3(2+h) - 4 - 3(2) + 4}{h} = \lim_{h\to 0} \dfrac{3h}{h} = 3$

c) $\lim_{h\to 0} \dfrac{1}{h}\left[\dfrac{x+h+1}{x+h-1} - \dfrac{x+1}{x-1}\right] = \lim_{h\to 0} \dfrac{1}{h}\left[\dfrac{(x+h+1)(x-1) - (x+h-1)(x+1)}{(x+h-1)(x-1)}\right]$

$= \lim_{h\to 0} \dfrac{1}{h}\left[\dfrac{x^2 + xh - h - 1 - (x^2 + hx + h - 1)}{(x+h-1)(x-1)}\right] = \lim_{h\to 0} \dfrac{-2}{(x+h-1)(x-1)} = \dfrac{-2}{(x-1)^2}$

d) $\lim_{h\to 0} \dfrac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h}\left(\dfrac{\sqrt{2(x+h)-1} + \sqrt{2x-1}}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}\right)$

$= \lim_{h\to 0} \dfrac{1}{h}\left[\dfrac{2h}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}\right] = \dfrac{1}{\sqrt{2x-1}}$

4. $\lim_{h\to 0} \frac{\cos(2(x+h)) - \cos(2x)}{h} = \lim_{h\to 0} \frac{\cos(2x+2h) - \cos(2x)}{h}$

$= \lim_{h\to 0} \frac{\cos(2x)\cdot\cos(2h) - \sin(2x)\sin(2h) - \cos(2x)}{h}$

$= \lim_{h\to 0} \cos(2x)\frac{(\cos(2h)-1)}{h} - \lim_{h\to 0} \sin(2x)\frac{\sin(2h)}{2h}\cdot 2$

$= 0 - 2\sin(2x) = -2\sin(2x)$

5. $(f\cdot g)'(3) = f'(3)\cdot g(3) + f(3)\cdot g'(3) = (-1)(2) + (7)(2) = 12$

6. a) $f'(x) = 3\tan^2 x \cdot \sec^2 x$

b) $f(x) = \sin x \cdot \frac{\cos x}{\sin x} = \cos x \quad f'(x) = -\sin x$

c) $f(x) = \cos x + \frac{\cos^2 x}{\sin x} \quad f'(x) = -\sin x + \frac{2\sin x\cos x - \cos^3 x}{\sin^2 x}$

7. a) $f'(x) = 7(4+2x^2)^6(4x)$

b) $f'(x) = \frac{(x+4)(2x) - (x^2-9)}{(x+4)^2}$

c) $4(2-3x^2)^3\cdot(-6x)(x^7+3)^3 + (2-3x^2)^4(3)(x^7+3)^2\cdot 7x^6$

d) $2\cos(\cos(\cos x))\{-\sin(\cos(\cos x))\}(-\sin(\cos x))(-\sin x)$

e) 0

8. $(f\circ g)'(0) = f'(g(0))\cdot g'(0) = 2\cdot 3 = 6$

9. a) $f'(x) = 3\cos(3x)$
$f''(x) = -9\sin(3x)$
$f'''(x) = -27\cos(3x)$

b) $f'(x) = 4x^3 + 6x^2 + 6x + 4$
$f''(x) = 12x^2 + 12x + 6$
$f'''(x) = 24x + 12$

10. a) $s_1(t) = 4t - 3t^2$ $\quad$ $s_2(t) = t^2 - 2t$

$s_1'(t) = 4 - 6t$ $\quad$ $s_2'(t) = 2t - 2$

same velocity when $4 - 6t = 2t - 2$

$\Rightarrow 8t = 6 \Rightarrow t = 3/4$

b) same speed if $|s_1'| = |s_2'|$

so $8t - 6 = \pm(2t - 2)$ + gives $t = 3/4$

$8t - 6 = -2t + 2 \Rightarrow 10t = 8 \Rightarrow t = 4/5$

c) $s_1(t) = s_2(t)$

$4t - 3t^2 = t^2 - 2t \Rightarrow 4t^2 - 6t = 0$

$t = 0, \frac{3}{2}$

11. a) $2x + 4xy + 2x^2 \frac{dy}{dx} + 3y^3 + 9xy^2 \frac{dy}{dx} = 0$

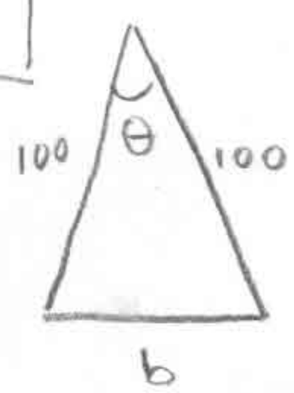
$\Rightarrow \frac{dy}{dx} = \frac{-(2x + 4xy + 3y^3)}{2x^2 + 9xy^2}$

b) $\sqrt{y+1} + \frac{x}{2\sqrt{y+1}} \frac{dy}{dx} = x\frac{dy}{dx} + y \Rightarrow \frac{dy}{dx} = \frac{\sqrt{y+1} - y}{x - \frac{x}{2\sqrt{y+1}}}$

c) $2\cos(xy)(-\sin(xy))\left(y + x\frac{dy}{dx}\right) = 2y\frac{dy}{dx} + 2x$

$\Rightarrow \frac{dy}{dx} = \frac{2x + 2y\cos(xy)\sin(xy)}{-2y - 2x\cos(xy)\sin(xy)}$

§2.8 #18



$\frac{d\theta}{dt} = .1$ rad/min Want $\frac{dA}{dt}$

$A = \frac{1}{2}ab\sin\theta$

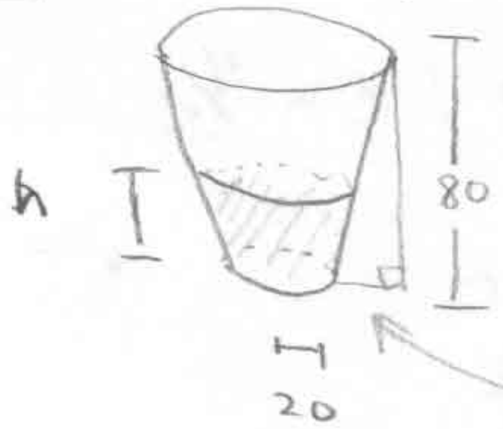
$= \frac{1}{2}(100)^2 \sin\theta$

$\frac{dA}{dt} = \frac{1}{2}(100)^2 \cos\theta \frac{d\theta}{dt}$

When $\theta = \pi/6$, $\frac{dA}{dt} = \frac{1}{2}(10{,}000)\left(\frac{1}{10}\right)\cos\left(\frac{\pi}{6}\right)$

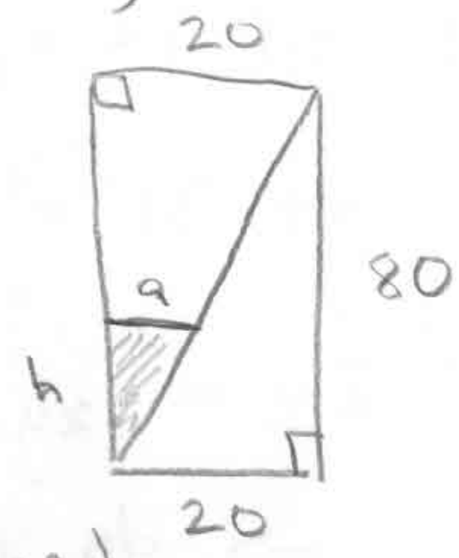
$= \frac{1000\sqrt{3}}{4} = \boxed{250\sqrt{3} \text{ cm}^2/\text{min}}$

§2.8 #20



$V = \frac{1}{3}\pi h\left(20^2 + 20a + a^2\right)$

Look at this △



Triangles are similar so $\frac{h}{a} = \frac{80}{20} = 4$

$\Rightarrow h = 4a \Rightarrow V = \frac{1}{3}\pi h\left(20^2 + 5h + \frac{h^2}{16}\right)$

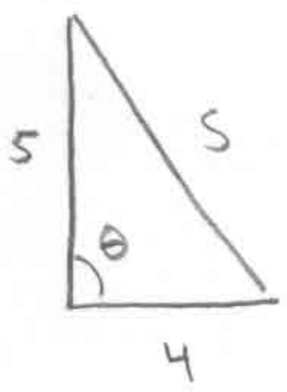
- Diff W.R.T. $t \Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi \frac{dh}{dt}\left(20^2 + 5h + \frac{h^2}{16}\right)$

$+ \frac{1}{3}\pi h\left(5\frac{dh}{dt} + \frac{h}{8}\frac{dh}{dt}\right)$

- When $h = 30$, we get $\left(2000 \text{ cm}^3/\text{min}\right) = \frac{1}{3}\pi\left[\frac{dh}{dt}\left(\frac{2425}{4} + \frac{525}{2}\right)\right]$

$\Rightarrow \frac{dh}{dt} = 2.20$ cm/min

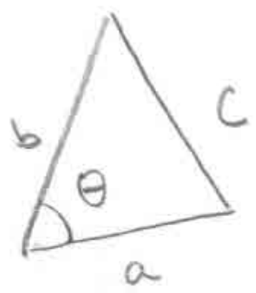
§2.8 #22



BOTH hands are moving. We'll call the ∠ made by min. hand with vertical Θ_m and the ∠ made by hour hand w/ vertical Θ_h. Then

$$\frac{d\Theta_m}{dt} = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/min} \qquad \frac{d\Theta_h}{dt} = \frac{2\pi}{12\cdot 60} = \frac{\pi}{360} \text{ rad/min}$$

- Law of cosines says



$$c^2 = a^2 + b^2 - 2ab\cos\Theta$$

$$\Rightarrow s^2 = 4^2 + 5^2 - 2(5)(4)\cos(\Theta_h - \Theta_m)$$

- Differentiate implicitly w.r.t. time

$$2s\frac{ds}{dt} = 0 + 40\sin(\Theta_h - \Theta_m)\left[\frac{d\Theta_h}{dt} - \frac{d\Theta_m}{dt}\right]$$

- At 3:00, we get

$$2(\sqrt{41})\frac{ds}{dt} = 40\sin\left(\frac{\pi}{2}\right)\left[\frac{\pi}{360} - \frac{\pi}{30}\right]$$

$$\Rightarrow \frac{ds}{dt} = \frac{-40\cdot 11\cdot\pi}{720\sqrt{41}} = \boxed{-.299 \text{ in/min.}}$$