University of Arkansas
Department of Mathematics

Exam 2

Calculus III

Printed Name (Last, First): ____________________ Student ID: ____________________

TA Name: ____________________ Drill Time: ____________________

Instructions: This exam has a total of 160 points. You have 50 minutes. **CLEARLY SHOW ALL YOUR WORK** to receive full credit. Put a $\boxed{\textbf{box}}$ around your final answer. You may use any result covered in class. The points attached to each problem are indicated beside the problem.

Question	Score	Question	Score
1		5	
2		6	
3		7	
4		8	

Final Score: ____________________________

v1

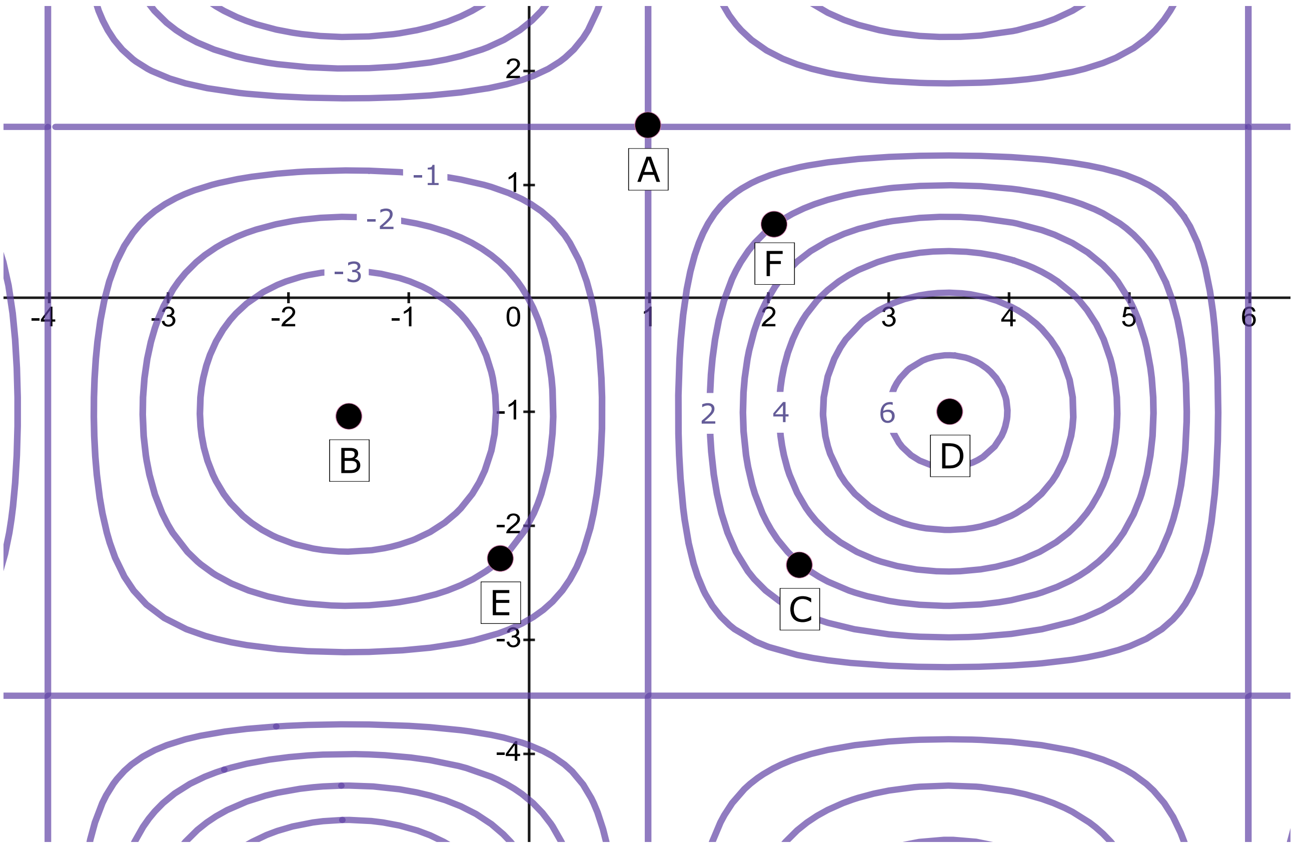
1. **[20 points]** Find the arc length of the curve with parametrization given by $\overrightarrow{r}(t)$ from $t = 1$ to $t = 3$.

$$\overrightarrow{r}(t) = \left\langle \frac{1}{2}t^2 - t\ ,\ \frac{4}{3}t^{3/2} \right\rangle$$

2. **[20 points]** Show the following limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{x+2y}{x-2y}.$$

3. **[20 points total]** Shown here are several level curves of a function $f\colon \mathbb{R}^2 \to \mathbb{R}$. Values of f are shown on some of the level curves. Six points on the plot are labeled A thru F.



(a) What is the approximate value of f at the point labeled C?

(b) Which point(s) is/are local maxima of f?

(c) Which point(s) is/are saddle points of f?

(d) At which point(s) does the gradient have the largest length?

(e) At which labeled point(s) does the gradient point southeast?

4. **[10 points each]** Let $f(x, y) = \ln(4 + x^2 + y^2)$ and $\vec{u} = \langle 2, 1 \rangle$.

   (a) Compute $\nabla f(-1, 2)$.

   (b) Find the directional derivative of f at $(-1, 2)$ in the direction of $\vec{u}$.

5. **[20 points]** Find an equation of the plane tangent to the surface defined by $z = \sin(xy) + 2$ at the point $(1, 0, 2)$.

6. **[10 points each]** Assume w is a function of x and y, and that each of x, y is a functions of s and t.

(a) Draw a labeled tree diagram showing the relationships among the variables. Use this to write the Chain Rule formula for $\frac{\partial w}{\partial s}$.

(b) Suppose that $w(x, y) = xy$ and $x = 2s + t$ and $y = s + t$. Use your answer to part 6a to find $\frac{\partial w}{\partial s}$ at the point $s = 2, t = 4$.

7. **[20 points]** Find all critical points of the function

$$f(x, y) = y^4 - 2y^2 + x^2 - 4x + 5.$$

For each critical point, use the second derivative test to classify it as either a local minimum, a local maximum, or a saddle point.

8. **[20 points]** Find the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 - 2x + 4y + 5$$

subject to the constraint $x^2 + y^2 = 1$.