MATH2270: Midterm 3 Study Guide

The following is an overview of the material that will be covered on the third exam.

§4.1 Vector Spaces and Subspaces

- Know and have familiarity with vector spaces besides $\mathbb{R}^n$. Some examples: the vector space of polynomials of degree less than or equal to n, $\mathbb{P}_n$; set of all polynomials, $\mathbb{P}$; the vector space of $m \times n$ matrices, $M_{m\times n}$; the vector space of infinite sequences, $\mathbb{R}^\infty$; the vector space of all functions $\mathcal{F}([a,b]) = \{f\colon [a,b] \to \mathbb{R}\}$; the vector space of continuous functions $\mathcal{C}([a,b]) = \{f\colon [a,b] \to \mathbb{R} \mid f \text{ is cont.}\}$; the vector space of k-times differentiable functions $\mathcal{C}^k([a,b]) = \{f\colon [a,b] \to \mathbb{R} \mid f \text{ is differentiable } k \text{ times}\}$.
- You should be able to use your intuition from $\mathbb{R}^n$ to determine whether or not a given set is a subspace of a vector space V. You should be able to check, using the definition, if a given set is a subspace.

§4.2 Linear Transformations, Kernels & Ranges

- Find the kernel of a linear transformation
- Describe the range of a linear transformation explicitly.
- Use your intuition from $\mathbb{R}^n$ to understand linear transformations.

§4.3 Linear (In)Dependence & Bases

- Be able to answer linear algebra questions in general vector spaces
  - Is this set linearly independent?
  - Is this set a basis?
  - Find a basis for this subspace.
- Know the definitions (as always) of linear (in)dependence, basis, and other relevant terms.

§4.4 Coordinate Systems

- Understand coordinate mappings.
- Be able to use them to translate questions/computations in general vector spaces into questions/computations in $\mathbb{R}^n$.

§4.5/4.6 Dimension & the Rank-Nullity Theorem

- As always, know the definition of dimension of a vector space or subspace.
- Know the rank-nullity theorem: If $T\colon V \to W$ is a linear transformation, then $\dim(\ker T) + \dim(\Im T) = \dim(V)$.
- Understand the rank-nullity theorem using your intuition from $\mathbb{R}^n$. Can you draw a picture?

§4.7 Change of Basis

- If V is a vector space, and $\mathcal{B}, \mathcal{C}$ are bases for V, in what sense are the associated coordinate mappings the same? In what sense are they different?
- Be able to compute change of coordinate matrices.
- Understand what changing coordinates does!

§5.1 Eigenvectors & Eigenvalues

- Know the definitions... eigenvector, eigenvalue
- Be able to answer questions like:
  - Is $\lambda = 7$ an eigenvalue of A?
  - Is $\vec{v}$ an eigenvector for A?
  - Write down a 3×3 matrix that has $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ as an eigenvector.
  - Write down a 3×3 matrix that has $\lambda = 2$ as an eigenvalue.
- Compute a basis for an eigenspace of A.

§5.2 The Characteristic Equation

- Compute the characteristic polynomial of a matrix.
- Find the eigenvalues of a matrix (or linear transformation).

§5.3/5.4 Eigenvectors in relation to Linear Transformations/Diagonalization

- Be able to diagonalize a matrix. That is, write $A = PDP^{-1}$. If the matrix is not diagonalizable, you should be able to determine that.
- Given a diagonalizable matrix, A, be able to find a basis of $\mathbb{R}^n$ consisting of eigenvectors of A.
- Understand the connection between diagonalizing a matrix, and viewing a linear transformation in a "well chosen" basis.
- Given a linear transformation, $T\colon V \to V$, be able to find a basis of V (if one exists) for which the matrix of T with respect to $\mathcal{B}$ is diagonal.
- Given a linear transformation, $T\colon V \to V$, know the relationship between $[T]_{\mathcal{B}}$ and $[T]_{\mathcal{C}}$ for bases $\mathcal{B}, \mathcal{C}$ of V.

§4.9/5.6 Markov Chains & Discrete Dynamical Systems

- Apply the theory of eigenvectors and eigenvalues to determine the long term or steady-state behavior of a Markov chain or discrete dynamical system.
- Plot trajectories for discrete dynamical systems.
- Determine if the origin is an attractor/repeller/saddle point for a discrete dynamical system.
- Use the above analysis to describe the long-term behavior of such a system.

§5.7 Applications to Differential Equations

- Use the process of diagonalization to decouple a dynamical system determined by a system of first order differential equations.
- Determine if the origin is an attractor/repeller/saddle point for a system of differential equations.