MATH 1210: HOMEWORK SOLUTIONS §2.2

1. $f'(1) = \lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \dfrac{(1+h)^2 - 1}{h} = \lim_{h\to 0} \dfrac{2h+h^2}{h} = 2$

3. $f'(3) = \lim_{h\to 0} \dfrac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \dfrac{(3+h)^2-(3+h)-(3^2-3)}{h}$

$= \lim_{h\to 0} \dfrac{9+6h+h^2-3-h-9+3}{h} = 5$

5. $s'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{2(x+h)+1-2x-1}{h} = 2$

7. $r'(x) = \lim_{h\to 0} \dfrac{3(x+h)^2+4-3x^2-4}{h} = \lim_{h\to 0} \dfrac{3x^2+6xh+3h^2-3x^2}{h} = 6x$

9. $f'(x) = \lim_{h\to 0} \dfrac{a(x+h)^2+b(x+h)+c-ax^2-bx-c}{h}$

$= \lim_{h\to 0} \dfrac{ax^2+2axh+ah^2+bx+bh-ax^2-bx}{h} = 2ax+b$

11. $f'(x) = \lim_{h\to 0} \dfrac{(x+h)^3+2(x+h)^2+1-x^3-2x^2-1}{h}$

$= \lim_{h\to 0} \dfrac{x^3+3x^2h+3xh^2+h^3+2x^2+4xh+h^2-x^3-2x^2}{h}$

$= \lim_{h\to 0} \dfrac{3x^2h+3xh^2+h^3+4xh+h^2}{h} = 3x^2+4x$

13. $$h'(x) = \lim_{h\to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{2x}{x(x+h)} - \frac{2(x+h)}{x(x+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\cdot\left[\frac{-h}{x(x+h)}\right] = \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

15. $$F'(x) = \lim_{h\to 0} \left[\frac{6}{(x+h)^2+1} - \frac{6}{x^2+1}\right]\cdot\frac{1}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{6(x^2+1) - 6((x+h)^2+1)}{(x^2+1)((x+h)^2+1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{6x^2+6-6x^2-12xh-6h^2-6}{(x^2+1)((x+h)^2+1)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{-12xh-6h^2}{(x^2+1)((x+h)^2+1)}\right] = \frac{-12x}{(x^2+1)^2}$$

17. $$G'(x) = \lim_{h\to 0} \frac{1}{h}\left[\frac{2(x+h)-1}{x+h-4} - \frac{2x-1}{x-4}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{(2x+2h-1)(x-4) - (2x-1)(x+h-4)}{(x+h-4)(x-4)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{2x^2} - \cancel{8x} + \cancel{2xh} - 8h - \cancel{x} + \cancel{4} - \cancel{2x^2} + \cancel{x} - \cancel{2xh} + h + \cancel{8x} - \cancel{4}}{(x+h-4)(x-4)}\right]$$

$$= \lim_{h\to 0} \left[\frac{2x-8-2x+1}{(x+h-4)(x-4)}\right] = \frac{-7}{(x+4)^2}$$

19. $$g'(x) = \lim_{h\to 0} \frac{1}{h}\left[\sqrt{3(x+h)} - \sqrt{3x}\right] = \lim_{h\to 0} \frac{3(x+h)-3x}{h\left(\sqrt{3(x+h)} + \sqrt{3x}\right)}$$

$$= \lim_{h\to 0} \frac{+3}{\sqrt{3(x+h)} - \sqrt{3x}} = \frac{+3}{2\sqrt{3x}}$$

21. $$H'(x) = \lim_{h\to 0} \frac{1}{h}\left[\frac{3}{\sqrt{x+h-2}} - \frac{3}{\sqrt{x-2}}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{3\sqrt{x-2} - 3\sqrt{x+h-2}}{\sqrt{(x+h-2)(x-2)}}\right]$$

$$= \lim_{h\to 0} \frac{3}{h}\left[\frac{x-2-(x+h-2)}{\left(\sqrt{x-2}+\sqrt{x+h-2}\right)\sqrt{(x+h-2)(x-2)}}\right]$$

$$= \lim_{h\to 0} \frac{-3}{\left(\sqrt{x-2}+\sqrt{x+h-2}\right)\sqrt{(x+h-2)(x-2)}} = \frac{-3}{2\sqrt{x-2}\cdot(x-2)}$$

$$= \frac{-3}{2(x-2)^{3/2}}$$

23. $$f'(x) = \lim_{t\to x} \frac{t^2-3t-(x^2-3x)}{t-x} = \lim_{t\to x} \frac{(t-x)(t+x-3)}{t-x} = 2x-3$$

25. $$f'(x) = \lim_{t\to x} \left(\frac{1}{t-x}\right)\left[\frac{t}{t-5} - \frac{x}{x-5}\right]$$

$$= \lim_{t\to x} \left(\frac{1}{t-x}\right)\left[\frac{t(x-5)-x(t-5)}{(t-5)(x-5)}\right]$$

$$= \lim_{t\to x} \frac{1}{t-x}\left[\frac{(t-x)(-5)}{(t-5)(x-5)}\right] = \frac{-5}{(x-5)^2}$$

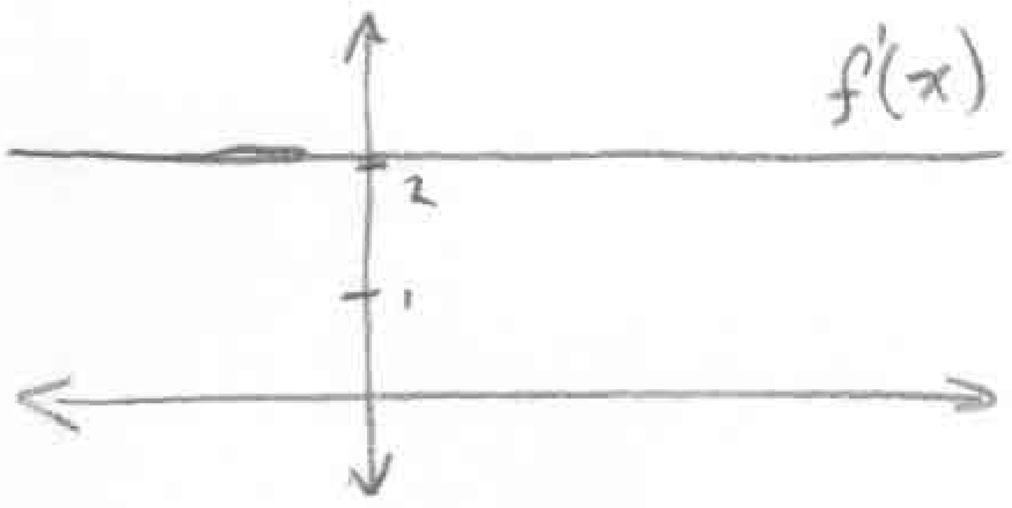
27. $f(x) = 2x^3$ at $x=5$

29. $f(x) = x^2$ at $x=2$

31. $f(x) = x^2$ at x

33. $f(x) = \frac{2}{t}$ at t

37.



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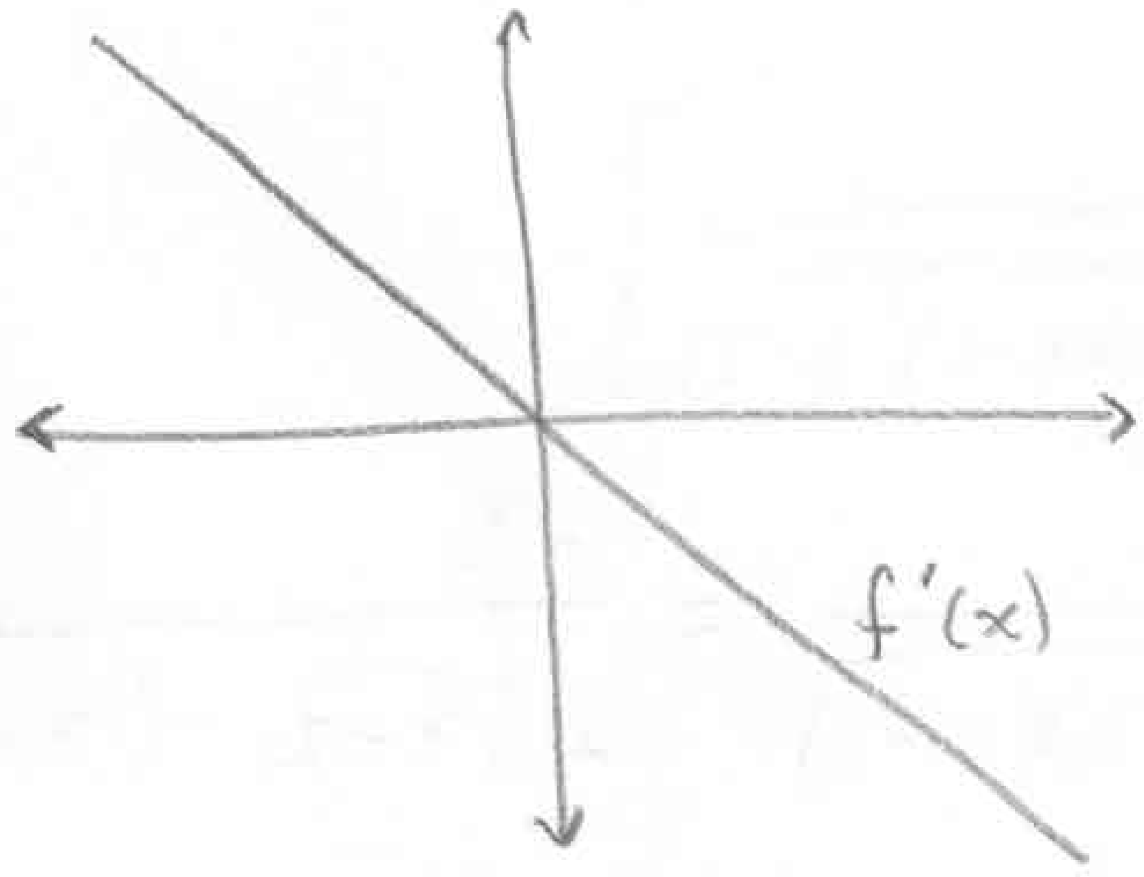

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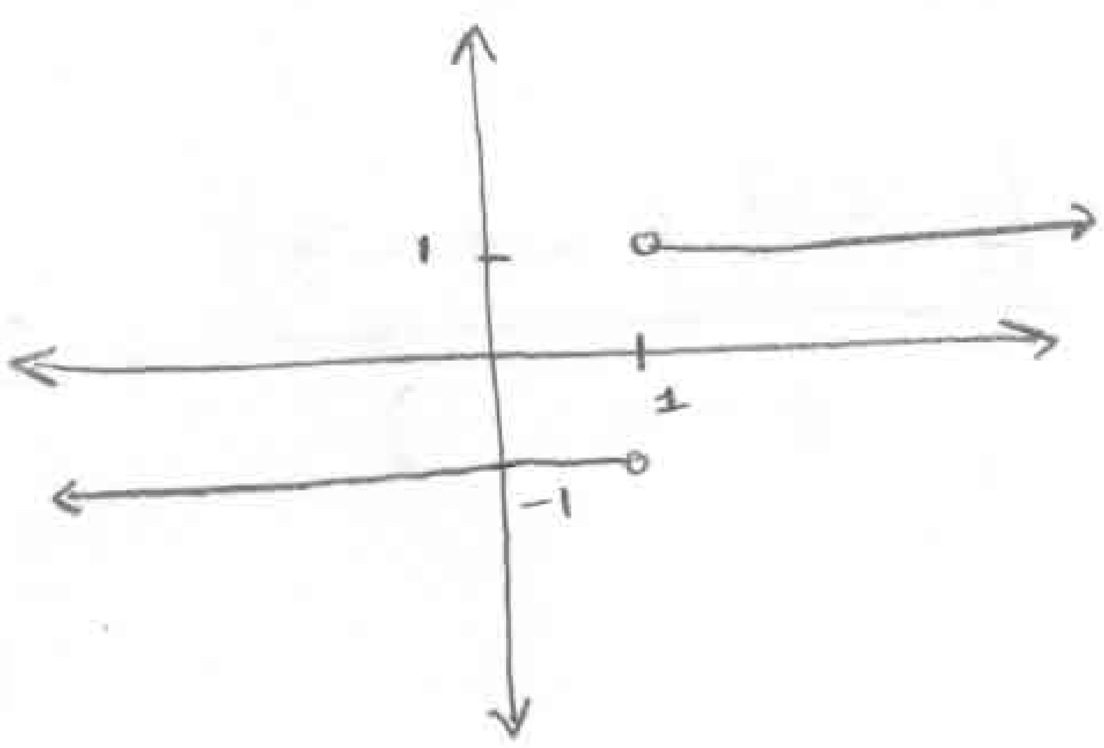

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