MATH1210-002

NAME:

Midterm 3

July 22, 2016

Instructions: Do all the problems on **both sides** of each page. Show all your work and box your answers. If you get stuck on a problem, skip it and come back to it at the end.

1. **[10 points]** Consider the function $f(s) = \dfrac{s+5}{s+1}$ on the interval $[0, 3]$. Does the Mean Value Theorem for Derivatives apply to $f(s)$? If so, find all points $c \in [0, 3]$ that satisfy the theorem. If not, explain why.

2. **[10 points]** Approximate $\displaystyle\int_1^5 \frac{1}{x}\, dx$ by using a right Riemann sum with four equal sized subintervals.

3. **[10 points]** Kathy is trying to compute the definite integral $\int_0^2 (x^2 + x)\, dx$ directly from the definition. She found the following formula for the right Riemann sum with n equal sized subintervals. Help her finish by eliminating the summations from the formula and taking a limit to find a value for the integral.

$$S_n = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{2i}{n} \right)^2 + \frac{2i}{n}$$

4. **[10 points]** A rectangular piece of sheet metal with perimeter 36 meters is to be rolled into a cylinder. What are the dimensions of the rectangle that give the greatest volume?

5. This problem is worth 30 points. The purpose of this problem is to graph the function $f(x) = x^4 - 2x^2 - 3$.

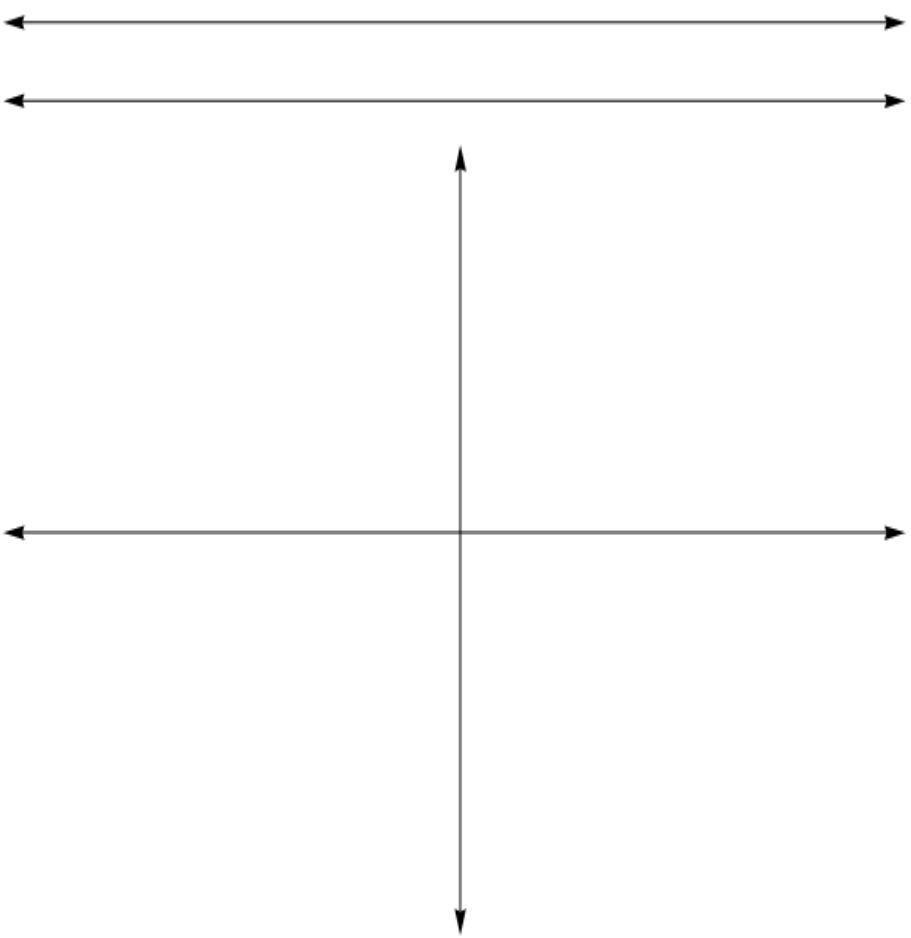
   (a) **[5 points]** Find the x and y intercepts of $f(x)$. (*Hint: Notice that $f(x)$ is a quadratic polynomial in x^2*)

   (b) **[5 points]** Find the critical points of $f(x)$. Then identify the regions where $f(x)$ is increasing and decreasing. Fill in the sign line for f' on the next page

   (c) **[5 points]** Find the inflection points of $f(x)$. Then identify the regions where $f(x)$ is concave up and where $f(x)$ is concave down. Fill in the sign line for f''.

(d) **[5 points]** Find the values of f at the critical points.

(e) **[10 points]** Graph $f(x)$.



6. **[5 points each]** Evaluate the following indefinite integrals.

(a) $\int \left(x^4 + \sqrt[3]{x} - 2x^{7/6}\right)\, dx$

(b) $\int \left(z + \sqrt{2}\right)^2\, dz$

(c) $\int 3\cos^4(x)\sin(x)\, dx$

(d) $\int x^2\sqrt{x^3+4}\,dx$

7. **[10 points]** Use Newton's Method to approximate the root of $f(x) = x^2 - x - 1$ that lies between 1 and 2 to an accuracy of 0.1 by starting at $x_0 = 1$. This number is called the *golden ratio*.