MATH1210: Final Practice Exam

The following are practice problems for the final exam from Chapters 4 and 5. For practice problems from the first three chapters, refer to the previous study guides.

1. Evaluate the following sums (either directly or using the formulas on page 218 of the text.

   (a) $\displaystyle\sum_{i=3}^{8}(i+1)^2$

   (b) $\displaystyle\sum_{k=3}^{7}\frac{(-1)^k 2^k}{(k+1)}$

   (c) $\displaystyle\sum_{i=1}^{10}[(i-1)(4i+3)]$

   (d) $\displaystyle\sum_{j=1}^{n}(2j-3)^2$

2. Evaluate the following integrals using a Riemann sum

   (a) $\displaystyle\int_0^2 (x^2+1)dx$

   (b) $\displaystyle\int_{-2}^1 (3x^2+2)dx$

3. Use the first fundamental theorem of calculus to find

   (a) $\displaystyle\frac{d}{dx}\left[\int_1^x 3t^2 dt\right]$

   (b) $\displaystyle\frac{d}{dx}\left[\int_{-x}^x \sin t dt\right]$

   (c) $\displaystyle\frac{d}{dx}\left[\int_1^{x^2+x} \sqrt{2z+\sin z} dz\right]$

4. Evaluate the following definite and indefinite integrals using any means at your disposal:

   (a) $\displaystyle\int_1^4 \frac{s^4-8}{s^2} ds$

   (b) $\displaystyle\int_1^8 \sqrt[3]{w} dw$

   (c) $\displaystyle\int x\left(\sqrt{3}x^2+\pi\right)^{7/8} dx$

   (d) $\displaystyle\int s^2\cos(s^3+5)ds$

(e) $\int_0^{1/2} \sin(2\pi x)dx$

(f) $\int_1^4 \frac{(\sqrt{t}-1)^3}{\sqrt{t}}dt$

5. Find the average value of $g(x) = \tan x \sec^2 x$ on the interval $[0, \pi/4]$.

6. Find all values of c that satisfy the mean value theorem for integrals for the function $f(x) = x^3$ on the interval $[0, 2]$.

7. For each part, sketch the region bounded by the given functions and then find its area using a definite integral.

   (a) $y = 5x - x^2$, $y = 0$, $x = 1$ and $x = 3$.

   (b) $x = (3 - y)(y + 1)$, $x = 0$.

   (c) $y = x^2 - 9$, $y = (2x - 1)(x + 3)$

8. For each part, sketch the region R bounded by the given equations. Then find the volume of the solid generated by revolving R about the given axis.

   (a) $y = x^3$, $x = 3$, and $y = 0$ revolved around the y-axis.

   (b) The same region revolved around the x-axis.

   (c) $y = \sqrt{9 - x^2}$, $y - 0$, between $x = -2$ and $x = 3$ revolved around the x-axis.

   (d) Same region revolved around the line $x = -1$.

   (e) $y = x^2$ $y = 3x$ about the y-axis.

   (f) $x = y^2$, $y = 2$, $x = 0$ about the line $y = 2$.

9. Set up (but do not evaluate) an integral to compute the length of the given plane curve.

   (a) $x(t) = t\cos t$, $y(t) = 2t\sin t$, for $t \in [0, 5\pi]$

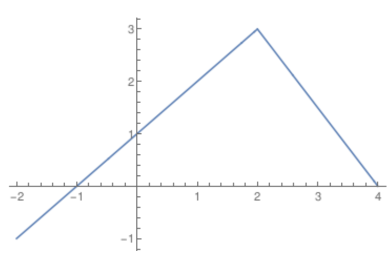
   (b) $x(t) = t^2$, $y(t) = \sqrt{t}$ for $t \in [1, 4]$

   (c) $y = \tan x$ for $x \in [0, \pi/4]$

10. §5.5 Exercises 4 and 18

11. §5.6 Exercises 2, 8, and 14

12. The graph of g' is shown here. Answer the following questions.

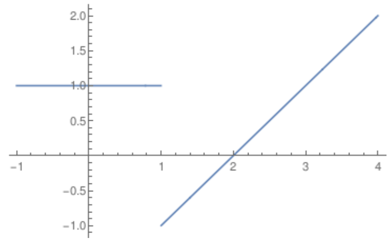


(a) Is g continuous at $x = 2$?

(b) Is g differentiable at $x = 2$?

(c) Is g increasing at $x = 2$?

13. The graph of f' is shown here. Which of the following statements about f must be false:



- f is continuous at $x = 1$.
- $f(1) = 0$.
- f has a vertical asymptote at $x = 1$.
- f has a jump discontinuity at $x = 1$.
- f has a removable discontinuity at $x = 1$.

14. A differentiable function f has the values shown.

x	2.5	3.2	3.5	4.0	4.6	5.0
$f(x)$	7.6	5.7	4.2	3.1	2.2	1.5

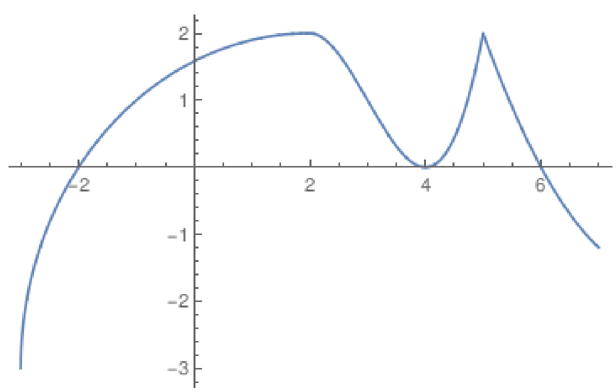
(a) Estimate $f'(4.0)$ and $f'(4.8)$.

(b) Find the average rate of change of f on the interval $2.5 \le x \le 5.0$. What does the Mean Value Theorem for derivatives tell you?

(c) Estimate $\int_{2.5}^{5} f(x)\,dx$ with a Riemann sum using left endpoints.

15. Given the graph of a function $f(x)$, sketch the graph of $f'(x)$.

16. The figure below shows the graph of f', the derivative of f, with domain $-3 \le x \le 7$. The graph of f' has horizontal tangents at $x = 2$ and $x = 4$ as well as corner at $x = 5$.



(a) Is f continuous? Why or why not?

(b) Find all values of x at which f attains a relative minimum.

(c) Find all values of x at which f attains a relative maximum.

(d) At what value of x does f attain its absolute maximum?

(e) Find all values of x at which the graph of f has an inflection point.

17. The graph of a function $y = f(x)$ passes through the point $(2, 5)$ and satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{6x^2 - 4}{y}$.

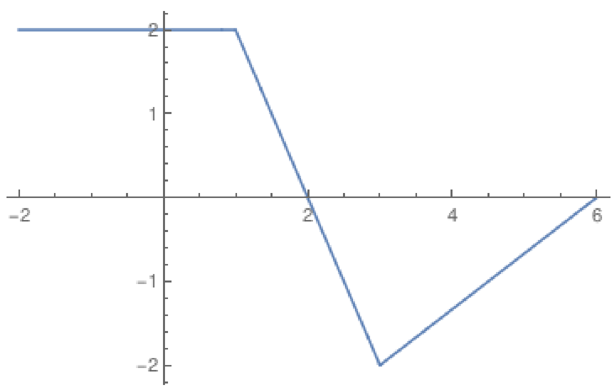
(a) Write an equation of the tangent line to f at $(2, 5)$.

(b) Using this tangent line, estimate $f(2.1)$.

(c) Solve the differential equation, expressing f as a function of x.

(d) Using part (c), find $f(2.1)$.

18. The figure below shows the graph of f, whose domain is the interval $[-2, 6]$. Let $F(x) = \int_1^x f(t)\,dt$.



(a) Find $F(-2)$ and $F(6)$.

(b) For what value(s) of x does $F(x) = 0$?

(c) For what value(s) of x is F increasing?

(d) Find the maximum value and the minimum value of F.

(e) At what value(s) of x does the graph of F have points of inflection?