MATH1210: Final Study Guide

The following is a list of the material we have covered in this course. All of this is valid material for the final exam. Approximately two thirds of the final exam will be on material from Chapters 4 and 5. The final third will test material from the first 3 chapters.

§1.1 Introduction to Limits

- The intuitive meaning of a limit.
- Calculating limits from the graph of a function.
- Calculating limits from a table of functional values.
- One sided limits.
- Example of a limit that doesn't exist.
- Example of a one-sided limit that doesn't exist.
- $\lim_{x\to c} f(x) = L$ if and only if $\lim_{x\to c^+} f(x) = L$ and $\lim_{x\to c^-} f(x) = L$

§1.3 Limit Theorems

- Know the Main Limit Theorem. If given one side of any equality in the theorem, you should be able to fill in the other side.
- The substitution theorem (this is the one that says you can just plug in $x = c$ for polynomials and rational functions).
- Theorem C (this is the one that tells us we are justified in cancelling out like factors in numerator and denominator).
- *Know the squeeze theorem and how to use it.*

§1.4 Limits Involving Trig Functions

- Know the Trig Limit Theorem (this is the substitution theorem for trig functions).
- Know $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ and $\lim_{x\to 0} \dfrac{1-\cos t}{t} = 0$.
- You should be able to evaluate trig limits like the ones we did in class and for homework. For example $\lim_{\theta\to 0} \dfrac{\tan 5\theta}{\sin 2\theta}$.
- Know the pythagorean identity ($\sin^2 x + \cos^2 x = 1$).
- Know the sum and difference of angles formulas (note that the double angle formulas are special cases of these).

§1.5 Limits at Infinity/Infinite Limits

- How to calculate limits at $\pm\infty$.
- How to calculate infinite limits.
- Graphing rational functions with asymptotes.

§1.6 Continuity of Functions

- Definition of continuity at a point.
- Definition of continuity on an interval (either open or closed).
- Restatement of previous limit and substitution theorems in term of continuity.
- The composition limit theorem.
- *The Intermediate Value Theorem and how to use it.*

§2.1 The Idea Behind The Derivative

- The intuitive meaning of the derivative (as the slope of the tangent to the graph of $f(x)$)
- Calculating derivatives from the graph of a function.
- Calculating the equation of the tangent line to a graph at a specified point.

§2.2 The Derivative

- *KNOW THE LIMIT DEFINITION OF THE DERIVATIVE AND BE ABLE TO USE IT.*
- Calculating the derivative of functions using the limit definition.
- Recognizing all the different notations for the derivative (i.e., $f'(x)$, df/dx, $D_x(f)$, etc.).
- Using the graph of $f(x)$ to graph the derivative, $f'(x)$.
- Identifying places where the derivative of a function is not defined using the graph.

§2.3 Rules for Finding Derivatives

- Know the power rule, the sum/difference rule, the constant multiple rule, and the product/quotient rules for differentiating functions.
- Taking derivatives of polynomials and rational functions.
- Problems similar to $45 - 53$ from §2.1.

§2.4 Derivatives of Trigonometric Functions

- Know the pythagorean identity ($\sin^2 x + \cos^2 x = 1$).
- Know the sum and difference of angles formulas (note that the double angle formulas are special cases of these).
- Know the derivatives of the standard trigonometric functions (you can always get away with just knowing sine and cosine, and then deriving the others using the product and quotient rules).
- Prove that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cos x) = -\sin x$.
- Find points where the tangent line to the graph of a function is horizontal.

§2.5 The Chain Rule

- *KNOW THE CHAIN RULE AND BE ABLE TO USE IT IN YOUR SLEEP.*
- Computing derivatives using the chain rule.

§2.6 Higher Order Derivatives

- Know how to find higher order derivatives.
- Know the different notations for higher order derivatives.
- The physical interpretation of the first and second derivatives of position with respect to time.

§2.7 Implicit Differentiation

- *KNOW HOW TO DO IMPLICIT DIFFERENTIATION.*

§2.8 Related Rates

- Be able to do related rate problems. This almost always involves using a combination of the chain rule and implicit differentiation.

§2.9 Differentials and Approximations

- The definition of differentials.
- Approximating increments by differentials.

$$f(x + \Delta x) \approx f(x) + dy = f(x) = f'(x)\Delta x$$

§3.1 Maxima and Minima

- The definition of a global maximum or minimum and an extreme value.
- The Max-Min Existence Theorem. This is the one that says a continuous function on a closed interval always has a max and a min.
- The Critical Point Theorem. This says that maxima and minima always happen at critical points.

§2.2 Monotonicity and Concavity

- The (intuitive) definition of an increasing or decreasing function.
- The Monotonicity Theorem.
- The definition of concavity.
- The Concavity Theorem.
- The definition of an inflection point. Know how to identify inflection points from the graph of a function (See pg159 in textbook).

§3.3 Local Extrema and Extrema on Open Intervals

- The definition of a local maximum or minimim as well as the definition of a local extreme value.
- The First Derivative Test.
- The Second Derivative Test.

§3.4 Practical Problems

- You should be able to apply the theorems of the previous 3 sections to "real world" problems.

§3.5 Graphing Functions Using Calculus

- I guarantee that there will be a graphing problem on the exam. It will be worth 20-30 points. Know how to apply the methods of calculus to draw a *detailed* graph of a function. See example 1 in §3.5.

§3.6 The Mean Value Theorem for Derivatives

- The Mean Value Theorem for Derivatives.
- Using the Mean Value Theorem for Derivatives.

§3.7 Solving Equations Numerically

- The Bisection Method for finding solutions to $f(x) = 0$.
- Newton's Method for finding solutions to $f(x) = 0$.
- The Fixed Point Method for solving $g(x) = x$.

§3.8 Antiderivatives

- Know how to compute indefinite integrals, including using the power rule and the "Generalized Power Rule" (in class I called this "the chain rule backwards").
- Don't forget the integration constant, C.

§4.1 Introduction to Area

- Summation notation
- Evaluating sums using the special sum formulas on page 218 of the text. These formulas will be given on the exam.

§4.2 The Definite Integral

- Computing definite integrals by taking the limit of a Riemann sum as $\|P\| \to 0$.
- The additivity of the definite integral. That is

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

§4.3 The First Fundamental Theorem of Calculus

- KNOW THE FIRST FUNDAMENTAL THEOREM OF CALCULUS.
- Using the first fundamental theorem to evaluate derivatives of integrals

§4.4 The Second Fundamental Theorem of Calculus and u-substitution

- KNOW THE SECOND FUNDAMENTAL THEOREM OF CALCULUS.
- Use the SFT to evaluate definite integrals

- Use u-substitution to evaluate definite and indefinite integrals

§4.5 The Mean Value Theorem for Integrals and Symmetry

- Know the Mean Value Theorem for Integrals.
- Be able to evaluate definite integrals using symmetry.

§5.1 The Area of a Plane Region

- Find the area of a region in the plane by setting up and evaluating a definite integral.
- Recognize when it is appropriate to integrate over "horizontal" versus "vertical" slices.

§5.2 Volumes of Solids: Slabs, Disks, and Washers

- Find the volume of solids using a definite integral.
- Be able to determine whether horizontal or vertical slicing is more appropriate for a given solid.
- Be able to set up "disk," "washer," and "slab" integrals.

§5.3 Volumes of Solids of Revolution: Shells

- Compute volumes of solids of revolution using the "shell" method.
- Determine the most appropriate method of "slicing" for a given solid.

§5.4 The Length of a Plane Curve

- Set up an integral to compute the length of a plane curve when given:
  - parametric equations describing the curve
  - y as a function of x
  - x as a function of y
- Set up an integral to find the surface area of a surface of revolution.

§5.5 Work and Fluid Force

- Use the definite integral to compute the work done by a force.
- Use the definite integral to compute the force exerted by a fluid.

§5.6 Moments and Center of Mass

- Use the definite integral to compute the moment of inertia of an object about a given axis of rotation.
- Use the definite integral to compute the center of mass of an object.