MATH1220: Midterm 3 Practice Problems

The following are practice problems for the second exam.

1. Compute the following limits:

   (a) $\lim\limits_{x\to 1} \dfrac{x^2-2x+1}{\sin(\pi x)}$

   (b) $\lim\limits_{x\to 0} \dfrac{e^x-e^{-x}}{2\sin x}$

   (c) $\lim\limits_{x\to\infty} \dfrac{(\ln x)^2}{2^x}$

   (d) $\lim\limits_{x\to\infty} \left(1+\dfrac{1}{x}\right)^x$

2. Evaluate the following integrals:

   (a) $\displaystyle\int_{-\infty}^{1} e^{4x}dx$

   (b) $\displaystyle\int_{5}^{\infty} \frac{x}{1+x^2}dx$

   (c) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2+9)^2}$

   (d) $\displaystyle\int_{0}^{3} \frac{dx}{x^2-2x-3}dx$

   (e) $\displaystyle\int_{-4}^{0} \frac{dx}{(x+3)^2}$

3. Write an explicit formula for the n-th term of the sequence. Then determine whether the sequence converges or diverges. If it converges, find what number it converges to:

   (a) $a_1 = 7$, $a_{n+1} = a_n\left(\dfrac{2}{3}\right)$

   (b) $-1, 2, 5, 8, 11, \ldots$

   (c) $0, \dfrac{1}{4}, \dfrac{2}{6}, \dfrac{3}{8}, \dfrac{4}{10}, \dfrac{5}{12}, \dfrac{6}{14}, \ldots$

4. Find the limit of the sequence $a_n = \dfrac{2n^3}{5n^3-2n+2}$.

5. Show that the sequence $a_n = \dfrac{n}{n+1}\left(2-\dfrac{1}{n^2}\right)$ converges using the monotone sequence theorem.

6. Determine the convergence/divergence of the following series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$

(b) $\displaystyle\sum_{k=1}^{\infty}\left[5\left(\frac{1}{2}\right)^k - 3\left(\frac{1}{7}\right)^k\right]$

(c) $\displaystyle\sum_{k=1}^{\infty}\frac{2}{(k+2)k}$

(d) $\displaystyle\sum_{k=1}^{\infty}\ln(k/(k+1))$

(e) $\displaystyle\sum_{k=1}^{\infty}\frac{3}{2k^2+1}$

(f) $\displaystyle\sum_{k=1}^{\infty}\frac{1000k^2}{1+k^3}$

(g) $\displaystyle\sum_{k=1}^{\infty}k\sin(1/k)$

(h) $\displaystyle\sum_{k=1}^{\infty}\frac{\sqrt[5]{3n^4+3}}{n^2}$

(i) $\displaystyle\sum_{k=1}^{\infty}\frac{3^k+k}{k!}$

(j) $\displaystyle\sum_{k=1}^{\infty}\frac{\ln k}{2^k}$

(k) $\displaystyle\sum_{k=1}^{\infty}\frac{4^{2n}}{n!}$

(l) $\displaystyle\frac{\ln 2}{2^2}+\frac{\ln 3}{3^3}+\frac{\ln 4}{4^4}+\frac{\ln 5}{5^5}+\dots$

7. Determine whether each of the following is absolutely convergent, conditionally convergent, or divergent:

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$

(b) $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^2}{e^k}$

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n-1}{n}$