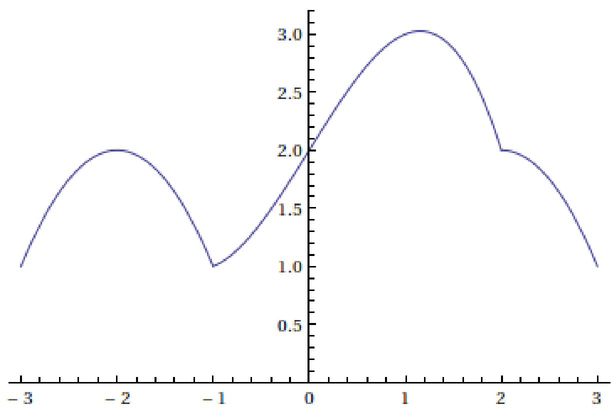
MATH1210: Midterm 3 Practice Problems

The following are practice problems for the third exam. This is not meant to mimic the length of the actual exam.

1. Sketch the graph of a function, f, defined on $[0, 6]$ and satisfying
   - $f(0) = 8$
   - $f(6) = -2$
   - f is decreasing on the interval $(0, 6)$
   - f has an inflection point at the ordered pair $(2, 3)$
   - f is concave up on $(2, 6)$

2. Let $f(x)$ be the function whose graph is shown here:



   (a) Identify all the critical points of $f(x)$.

   (b) Identify all the inflection points of $f(x)$.

3. §3.4 Exercise 12, 14, 36, 42

4. Let $f(x) = \dfrac{x^2 + 1}{x}$

   (a) Find the x and y intercepts of f.

   (b) Find the critical points of f.

   (c) Identify the regions where f is increasing and where f is decreasing.

   (d) Find the inflection points of f.

   (e) Identify the regions where f is concave up and where f is concave down.

   (f) Find the values of f at the critical and inflection points.

   (g) Graph f.

5. Consider the function $f(s) = s^2 + 3s - 1$ on $[-3, 1]$. Does the Mean Value Theorem for derivatives apply to $f(s)$? If so, find all points $c \in [-3, 1]$ that satisfy the mean value theorem. If not, explain why.

6. The function $g(x) = x^2 - 2$ has a root in between $x = 1$ and $x = 2$. Use the Bisection Method to approximate the root of $g(x)$ to an accuracy of 0.25.

7. The function $f(x) = 3x^3 - 3x + 2$ has a root between $x = -2$ and $x = -1$. Use Newton's method to approximate the root to an accuracy of 0.01. *(You can use a calculator when doing this problem. If there is a Newton's method problem on the test, the numbers will be nice enough that you won't need a calculator.)*

8. Compute the following indefinite integrals:

   (a) $\displaystyle\int 3x^2 + \sqrt{3}\,\mathrm{d}x$

   (b) $\displaystyle\int \frac{s(s+1)^2}{\sqrt{s}}\,\mathrm{d}s$

9. Approximate the area under the graph of $f(x) = 2x^2 + x$ between $x = 0$ and $x = 3$ using a midpoint Riemann sum with three subintervals of equal size.

10. Compute the definite integral $\displaystyle\int_{-2}^{3} 2x - 2\,\mathrm{d}x$ without using the Fundamental Theorem of Calculus or a Riemann sum. *Hint: Draw a picture*

11. The right Riemann sum for $f(x) = \dfrac{1}{2}x^2 + 1$ between $a = 0$ and $b = 0$ with n equal size subintervals is given by

$$\frac{1}{n}\sum_{i=1}^{n} f(i/n) = \frac{1}{n}\left[\frac{1}{2n^2}\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} 1\right]$$

Use the sum formulas provided on page 218 of the textbook to obtain an equation (without any summations) for the n-th Riemann sum. Then take a limit to evaluate the definite integral $\displaystyle\int_0^1 \frac{1}{2}x^2 + 1\,\mathrm{d}x$. *Do not use the fundamental theorem of calculus.*