MATH1210: Final Practice Exam

The following are practice problems for the final exam from Chapters 4 and 5. For practice problems from the first three chapters, refer to the previous study guides.

1. Evaluate the following sums (either directly or using the formulas on page 218 of the text.
   (a) $\sum_{i=3}^{8}(i+1)^2$
   (b) $\sum_{k=3}^{7}\frac{(-1)^k 2^k}{(k+1)}$
   (c) $\sum_{i=1}^{10}[(i-1)(4i+3)]$
   (d) $\sum_{j=1}^{n}(2j-3)^2$
2. Evaluate the following integrals using a Riemann sum
   (a) $\int_0^2 (x^2+1)dx$
   (b) $\int_{-2}^1 (3x^2+2)dx$
3. Use the first fundamental theorem of calculus to find
   (a) $\frac{d}{dx}\left[\int_1^x 3t^2 dt\right]$
   (b) $\frac{d}{dx}\left[\int_{-x}^x \sin t dt\right]$
   (c) $\frac{d}{dx}\left[\int_1^{x^2+x} \sqrt{2z+\sin z}dz\right]$
4. Evaluate the following definite and indefinite integrals using any means at your disposal:
   (a) $\int_1^4 \frac{s^4-8}{s^2}ds$
   (b) $\int_1^8 \sqrt[3]{w}dw$
   (c) $\int x\left(\sqrt{3}x^2+\pi\right)^{7/8}dx$
   (d) $\int s^2\cos(s^3+5)ds$

(e) $\displaystyle\int_0^{1/2} \sin(2\pi x)dx$

(f) $\displaystyle\int_1^4 \frac{(\sqrt{t}-1)^3}{\sqrt{t}}dt$

5. Find the average value of $g(x) = \tan x \sec^2 x$ on the interval $[0, \pi/4]$.

6. Find all values of c that satisfy the mean value theorem for integrals for the function $f(x) = x^3$ on the interval $[0, 2]$.

7. For each part, sketch the region bounded by the given functions and then find its area using a definite integral.

   (a) $y = 5x - x^2$, $y = 0$, $x = 1$ and $x = 3$.

   (b) $x = (3 - y)(y + 1)$, $x = 0$.

   (c) $y = x^2 - 9$, $y = (2x - 1)(x + 3)$

8. For each part, sketch the region R bounded by the given equations. Then find the volume of the solid generated by revolving R about the given axis.

   (a) $y = x^3$, $x = 3$, and $y = 0$ revolved around the y-axis.

   (b) The same region revolved around the x-axis.

   (c) $y = \sqrt{9 - x^2}$, $y - 0$, between $x = -2$ and $x = 3$ revolved around the x-axis.

   (d) Same region revolved around the line $x = -1$.

   (e) $y = x^2$ $y = 3x$ about the y-axis.

   (f) $x = y^2$, $y = 2$, $x = 0$ about the line $y = 2$.

9. Set up (but do not evaluate) an integral to compute the length of the given plane curve.

   (a) $x(t) = t \cos t$, $y(t) = 2t \sin t$, for $t \in [0, 5\pi]$

   (b) $x(t) = t^2$, $y(t) = \sqrt{t}$ for $t \in [1, 4]$

   (c) $y = \tan x$ for $x \in [0, \pi/4]$

10. §5.5 Exercises 4 and 18

11. §5.6 Exercises 2, 8, and 14