MATH1210: HOMEWORK SOLUTIONS §1.4

1. $\lim_{x\to 0} \dfrac{\cos x}{x+1} = \dfrac{\cos 0}{0+1} = \dfrac{1}{1} = \boxed{1}$

3. $\lim_{t\to 0} \dfrac{\cos^2 t}{1+\sin t} = \dfrac{\cos^2 0}{1+\sin 0} = \dfrac{1^2}{1+0} = \boxed{1}$

5. $\lim_{x\to 0} \dfrac{\sin x}{2x} = \dfrac{1}{2}\lim_{x\to 0} \dfrac{\sin x}{x} = \dfrac{1}{2}\cdot 1 = \boxed{\dfrac{1}{2}}$

7. $\lim_{\theta\to 0} \dfrac{\sin 3\theta}{\tan\theta} = \lim_{\theta\to 0} \dfrac{\sin 3\theta}{\left(\frac{\sin\theta}{\cos\theta}\right)}$

$= \lim_{\theta\to 0} \dfrac{\sin 3\theta}{\sin\theta}\cos\theta\left(\dfrac{3\theta}{3\theta}\right)$

$= \lim_{\theta\to 0} \left(\dfrac{\sin 3\theta}{3\theta}\right)\cdot\left(\dfrac{\theta}{\sin\theta}\right)\cdot 3\cos\theta$

$= \left[\lim_{\theta\to 0} \dfrac{\sin 3\theta}{3\theta}\right]\left[\lim_{\theta\to 0} \dfrac{1}{\left(\frac{\sin\theta}{\theta}\right)}\right]\left[\lim_{\theta\to 0} 3\cos\theta\right]$

$= 1\cdot\left(\dfrac{1}{1}\right)\cdot(3) = \boxed{3}$

9. $\lim_{\theta\to 0} \dfrac{\cot(\pi\theta)\sin\theta}{2\sec\theta} = \lim_{\theta\to 0} \dfrac{\cos(\pi\theta)\sin(\theta)}{2\sin(\pi\theta)}\cdot(\cos\theta)$

$= \lim_{\theta\to 0} \dfrac{\cos(\pi\theta)\cos(\theta)}{2\pi}\left(\dfrac{\sin\theta}{\theta}\right)\left(\dfrac{\pi\theta}{\sin(\pi\theta)}\right)$

$= \dfrac{1}{2\pi}\cdot 1\cdot 1 = \boxed{\dfrac{1}{2\pi}}$

11. $\lim_{t\to 0} \dfrac{\tan^2(3t)}{2t} = \lim_{t\to 0} \dfrac{\sin(3t)\cdot\sin(3t)}{2t\cdot\cos(3t)\cos(3t)}$

$= \lim_{t\to 0} \left(\dfrac{\sin(3t)}{3t}\right)\left(\dfrac{\sin(3t)}{3t}\right)\left(\dfrac{9t}{2\cos^2(3t)}\right)$

$= 1\cdot 1\cdot \dfrac{0}{1} = \boxed{0}$

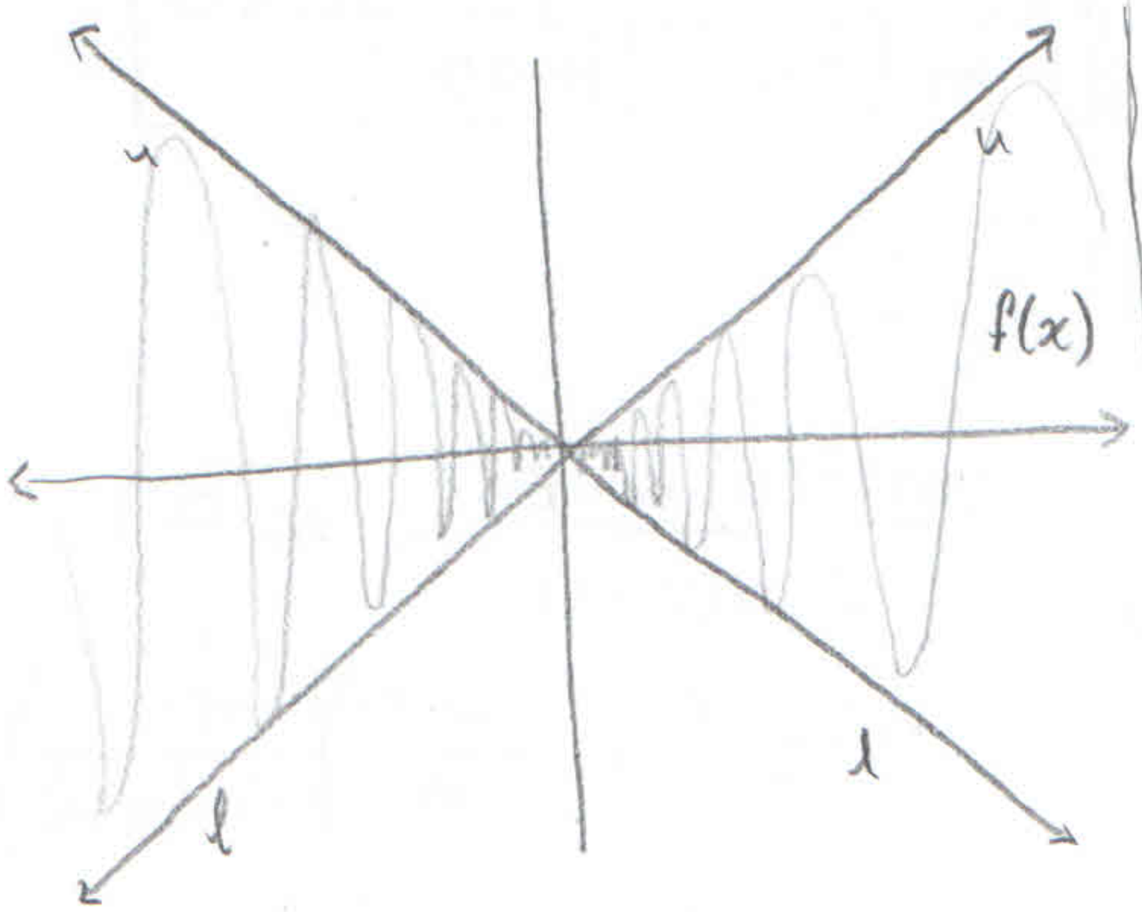
13. $\lim_{t\to 0} \dfrac{\sin 3t + 4t}{t\sec t} = \lim_{t\to 0} \dfrac{\sin(3t)+4t}{t}(\cos t)$

$= \lim_{t\to 0}\left(\dfrac{\sin 3t}{t}\cdot\cos t\right) + \lim_{t\to 0}\left(\dfrac{4t}{t}\cos t\right)$

$= 3\left[\lim_{t\to 0} \dfrac{\sin 3t}{3t}\cos t\right] + \lim_{t\to 0} 4\cos t$

$= 3\cdot 1\cdot 1 + 4 = \boxed{7}$

15.



$\lim_{x\to 0} f(x) = 0$

23. $\lim_{t\to c} \sec t = \lim_{t\to c} \dfrac{1}{\cos t}$

$= \dfrac{\lim_{t\to c} 1}{\lim_{t\to c} \cos t} = \dfrac{1}{\cos c}$

$= \sec c$

The other proof is similar.