MATH1210: Midterm 1 Practice Exam

The following are practice problems for the first exam.

1. Sketch a function f satisfying the following conditions
   - The domain of f is $[0, 4]$
   - $f(0) = f(1) = f(2) = f(3) = f(4) = 1$
   - $\lim_{x \to 1} f(x) = 2$
   - $\lim_{x \to 2} f(x) = 1$
   - $\lim_{x \to 3^-} f(x) = 2$
   - $\lim_{x \to 3^+} f(x) = 1$
2. Estimate $\lim_{x \to 5} f(x)$ from the following table of functional values

x	f(x)
4.75	-.5
4.9	-.2
4.99	-.02
4.999	-.002
5	17
5.001	.002
5.01	.02
5.1	.2
5.25	.5

3. Compute the following limits
   (a) $\lim_{x \to -3} \sqrt{5x^2 + 2x}$
   (b) $\lim_{w \to 2} \dfrac{(w-2)(w^2 - w - 6)}{w^2 - 4w + 4}$
   (c) $\lim_{x \to -1} \dfrac{x^2 + x}{x^2 + 1}$
4. Assume that $\lim_{x \to a} f(x) = 1$ and that $\lim_{x \to a} g(x) = \pi$. Compute
   (a) $\lim_{x \to a} [\sqrt{g(x)} + 2f(x)]$
   (b) $\lim_{x \to a} \dfrac{g^2(x) - f(x)}{g(x) + f(x)}$
   (c) $\lim_{x \to a} [f(x) + g(x)]^3$
5. Show that $\lim_{x \to 0} x^2 \cos(1/x) = 0$. (*Hint:* Use the squeeze theorem.)

6. Compute the following limits

   (a) $\lim_{t\to 0} \dfrac{\tan 2t}{\sin 2t - 1}$

   (b) $\lim_{x\to 0} \dfrac{\sin 6x}{\tan 9x}$

   (c) $\lim_{\theta\to 0} \dfrac{\sin^2 2t}{3t}$

   (d) $\lim_{t\to 0^+} \dfrac{\cos 2t}{5t}$

7. Compute the following limits AT INFINITY AND BEYOND

   (a) $\lim_{x\to\infty} \dfrac{3x^2}{x^2 - 8x + 100}$

   (b) $\lim_{\theta\to -\infty} \dfrac{\pi\theta^5}{\theta^5 - 5\theta^4}$

   (c) $\lim_{n\to -\infty} \dfrac{n}{n^2 + 1}$

8. Compute the following (possibly infinite) limits

   (a) $\lim_{t\to \pi^+} \dfrac{t^2}{\sin t}$

   (b) $\lim_{x\to 2^+} \dfrac{x^2 + 2x - 8}{x^2 - 4}$

9. Let $f(x) = \dfrac{(x-3)(x^2+2x+2)}{x^2+2x-15}$. Compute the following

   (a) $\lim_{x\to 3} f(x)$

   (b) $\lim_{x\to 5^+} f(x)$

   (c) $\lim_{x\to 5^-} f(x)$

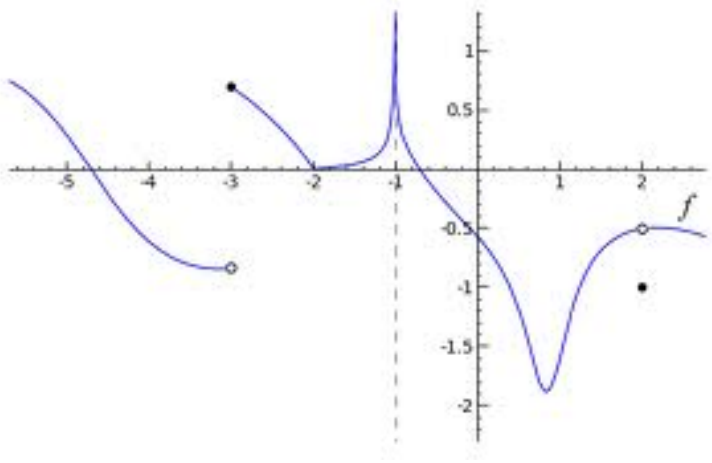
10. Compute the following limits

    (a) $\lim_{x\to 0^-} \dfrac{2x}{|x|}$

    (b) $\lim_{x\to\infty} \dfrac{\sin x}{x}$ (*Hint:* The squeeze theorem works for limits at infinity.)

    (c) $\lim_{x\to\infty} x \sin(1/x)$ (*Hint:* $\lim_{x\to\infty} f(x) = \lim_{x\to 0} f(1/x)$)

11. Let $f(x)$ be determined by the graph shown here:



At each of the following points, determine if f is continuous. If so, is the discontinuity removable or non-removable?

(a) $x = -3$

(b) $x = -2$

(c) $x = -1$

(d) $x = -2$

12. At what points (if any) is the function

$$f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \le x \le 1 \\ 2 - x & \text{if } x > 1 \end{cases}$$

discontinuous?

13. Show that $x^3 + 3x = 2$ has a solution between 0 and 1. (*Hint:* Intermediate value theorem)