Food for Thought 1

Due Friday, August 25

Spend the rest of today's class period working through these problems. I encourage you to work with your classmates and discuss the problems. If you are finished with the assignment at the end of class today, then you can turn it in today. If you would like to work on the assignment more, take it home and turn it in on Friday. This assignment will be graded for ***effort*** *(which means you have written down thoughtful, complete solutions to each problem), not correctness. Solutions to these problems will be posted on Canvas on Friday evening for future reference.*

1. Give examples of the following:

   (a) A linear system of two equations in two variables that is inconsistent.

   (b) A linear system of two equations in two variables that has infinitely many solutions.

   (c) A linear system of two equations in two variables that has exactly one solution.

2. Suppose a 4×7 coefficient matrix for a system of equations has 4 pivots. Is the system consistent? If the system is consistent, how many solutions are there?

3. Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?

4. What would you have to know about the pivot columns in an augmented matrix in order to know that the linear system is consistent and has a unique solution?

5. Suppose a, b, c, and d are constants such that a is not zero and the system below is consistent for all possible values of f and g. What can you say about the numbers a, b, c, and d? Justify your answer.

$$ax_1 + bx_2 = f$$
$$cx_1 + dx_2 = g$$

6. Let A be a 6×4 matrix viewed as the augmented matrix of a linear system. Assume the linear system is consistent and that A is in reduced row echelon form. Determine how many entries of A are equal to 0 in each of the following cases:

   (a) A has as many entries equal to 0 as possible.

   (b) A has as few entries equal to 0 as possible.

7. A system of linear equations with more equations than unknowns is sometimes called an *overdetermined* system. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.