Food for Thought 6

Due Monday, October 16

Spend the rest of today's class period working through these problems. I encourage you to work with your classmates and discuss the problems. If you are finished with the assignment at the end of class today, then you can turn it in today. If you would like to work on the assignment more, take it home over the weekend and turn it in on Monday. This assignment will be graded for ***effort*** *(which means you have written down thoughtful, complete solutions to each problem), not correctness. Solutions to these problems will be posted on Canvas on Monday for future reference.*

1. Consider the collection $M_{n\times m}$ of $n \times m$ matrices, with addition and scalar multiplication as defined in chapter 2. Is $M_{n\times m}$ a vector space?

2. $\mathbb{P}_n$ is defined to be the collection of polynomials of degree **at most** n. Why didn't we define it to be the collection of polynomials of degree **exactly** n? Which axiom(s) of a vector space fail in this alternate definition?

3. For each of the following vector spaces V, determine whether the subset H is a subspace. Justify your choice.

   (a) $V = \mathbb{R}^2$, and $H = \operatorname{span}\left\{\begin{bmatrix}3\\4\end{bmatrix}\right\}$.

   (b) $V = \mathbb{P}_3$, and H is the collection of polynomials $\mathbf{p}(t)$ in $\mathbb{P}_3$ such that $\mathbf{p}(2) = 0$.

   (c) $V = \mathbb{P}_3$, and H is the collection of polynomials $\mathbf{p}(t)$ in $\mathbb{P}_3$ such that $\mathbf{p}(2) = 3$.

(d) $V = \mathbb{R}^2$, and $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}$. (In other words, H is the union of the first and third quadrants in $\mathbb{R}^2$.)

(e) $V = M_{2\times3}$, and H is the set of matrices in $M_{2\times3}$ of the form $\begin{bmatrix} a & 0 & b \\ 0 & a & c \end{bmatrix}$.

4. Define a linear transformation $T : \mathbb{P}_2 \longrightarrow \mathbb{R}^2$ by $\mathbf{p}(t) \mapsto \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(0) \end{bmatrix}$. Find polynomials $\mathbf{p}_1(t), \mathbf{p}_2(t) \in \mathbb{P}_2$ that span the kernel of T, and describe the range of T.

5. $C^\infty(\mathbb{R})$ (or just C^∞) is defined as the set of all functions $f\colon \mathbb{R} \to \mathbb{R}$ that are "infinitely differentiable." This very scary sounding term is not that bad at all; it means that f is differentiable everywhere. If you differentiate f, then you get another function that is differentiable everywhere. If you differentiate f', you still get a differentiable function. If you then differentiate f'', you again get a differentiable function, and so on... As it happens, $C^\infty([a, b])$ is a vector space! Give two examples of vectors in C^∞.