MATH 1030: Homework 5

due February 10, 2014

Instructions: Do the following problems on a separate sheet of paper. Show all of your work.

Problem 1: Pillaging Pirates

(a) The fearsome pirate Captain Bartolomeo needs to buy a new ship. He knows that a good ship these days (in the 16th century, that is) will cost him 125 Pirate Doubloons. Unfortunately, most of his money is in swindled Spanish Reales. Using the chart below, determine how many Spanish Reales a good ship will cost.

Currency	US Dollars per Foreign	Foreign per US Dollar
Pirate Doubloons	20	0.05
Spanish Reales	0.25	4

Bartolomeo has already pillaged 7,258 Spanish Reales. Does he have enough to buy his ship? If not, how much more does he need?

We have to convert the price of a ship from 125 Doubloons into Spanish Reales. We have

$$125\,\mathrm{D}\left(\frac{20\,\mathrm{USD}}{1\,\mathrm{D}}\right)\left(\frac{4\,\mathrm{Reales}}{1\,\mathrm{USD}}\right) = 10,000\,\mathrm{Reales}$$

So Captain Bartolomeo needs $2,742$ more Spanish Reales to buy his ship.

(b) Bartolomeo has obtained his ship, *Converter of the Seas.* The dimensions of the living quarters are 22 meters in length and 9 meters in width. Bartolomeo's crew is 77 pirates. What is the population density of the living quarters in pirates per square foot? How much space (in feet) is available per pirate?

The living quarters has an area of

$$22\,\mathrm{m}\cdot 9\,\mathrm{m} = 190\,\mathrm{m}^2\left(\frac{3.28\,\mathrm{ft}}{1\,\mathrm{m}}\right)^2 = 2130\,\mathrm{ft}^2$$

So the population density is

$$\frac{77\,\mathrm{pirates}}{2130\,\mathrm{ft}^2} = .036\,\mathrm{pirates}/\mathrm{ft}^2$$

There are $2130\,\mathrm{ft}^2$ to share between 77 pirates so each pirate has

$$\frac{2130\,\mathrm{ft}^2}{77\,\mathrm{pirates}} = 27.6\,\mathrm{ft}^2$$

of living space

(c) Salty Sam quickly drinks 6 ounces of rum, which contains 57 grams of alcohol. Given that Salty Sam weighs 200 pounds and has 6 liters of blood, what is his blood alcohol content in

grams per 100 milliliters? Should Bartolomeo let Salty Sam drive *Converter of the Seas*?
Salty Sam's blood alcohol content is

$$\frac{57\,\text{grams}}{6\,\text{L}}\left(\frac{1\,\text{L}}{1000\,\text{mL}}\right) = .0095\,\text{grams/mL} = 0.95\,\text{grams/100mL}$$

(d) Salty Sam, still reeling from the rum, challenges Bartolomeo to a sword fight, which incidentally burns 408 calories per hour. How many watts of power is generated from an hour of sword fighting? Considering that Salty Sam just had 388 calories of rum, how long does he need to sword fight to burn off those calories?
We must convert 408 Calories per hour into Watts (recall that $1\,\text{W} = 1\,\text{J/s}$).

$$408\frac{\text{Cal}}{\text{hr}}\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)\left(\frac{4184\,\text{J}}{1\,\text{Cal}}\right) = 474\,\text{W}$$

In order to burn 388 Cal Sam must sword fight for

$$\frac{388\,\text{Cal}}{408\,\text{Cal/hr}} = 0.95\,\text{hr} = 57\,\text{min}$$

(e) Bartolomeo orders Salty Sam to walk the plank. Salty Sam knows that it will take him 2 hours to swim to the closest shore, and that to survive for two hours the water temperature needs to be at least 60° F. "Arr, you'll never make it," says Bartolomeo, "the water is only 17° C." Salty Sam says, "I'll take my chances." Does Salty Sam survive?
We can convert $17°C$ to Fahrenheit.

$$1.8 \cdot 17 + 32 = 62.6°\,\text{F}$$

So the water temperature is greater than $60°$ F and Sam will survive.

Problem 2

Use scientific notation to compute:

(a) $(.0000000000000038234) \cdot (239,700,000,000,000,000,000,000)$
Converting into scientific notation, we have

$$(3.8234 \times 10^{-15}) \cdot (2.397 \times 10^{23}) = 9.165 \times 10^{8} = 916,500,000$$

(b) $\dfrac{13,463,000,000,000,000}{.000000000000000000000000334}$

$$\frac{1.3463 \times 10^{16}}{3.34 \times 10^{-25}} = 4.03 \times 10^{40}$$

Problem 3

The number of violent crimes in Twin Peaks has been on the rise. This year there were 16 violent crimes, 23% more than last year. How many violent crimes were there last year?
The problem says that 16 is 23% more than some number (the number of crimes last year). Symbolically, this translates to $16 = 1.23x$. Then $x = \frac{16}{1.23} = 13$, so there were 13 crimes last year.

Problem 4

The Norwegians' investment offer for the Ghostwood Development Project was 12.5% less than the Icelanders' offer. If the Icelanders offered $27.2 million, how much did the Norwegians offer?
The Norwegians' offer is

$$27.2\,\text{million} - (12.5\%\text{ of }27.2\,\text{million}) = 27,200,000 - (.125 \cdot 27,200,000) = 23,800,000$$

or 23.8 million.

Problem 5

Special Agent Dale Cooper just finished two pieces of Norma's incredible huckleberry pie and a cup of coffee. Suppose the pie costs $3.49 per slice and the coffee costs $1.69. If Agent Cooper leaves $11.00, what percent tip did he leave?
The bill was $2 \cdot 3.49 + 1.69 = 8.67$. The difference between the amount he left and the bill is $T = 11 - 8.67 = 2.33$. What percent of 8.67 is 2.33? It's $\frac{2.33}{8.67} = .268 = 26.8\%$

Problem 6

Profits at the Great Northern Hotel have fluctuated greatly over the last few years. Three years ago profits increased by 6%, two years ago profits decreased by 13%, and last year profits increased by 9%. By what percent did profits increase over the last three years combined?(*Tip:* Don't just add/subtract these numbers.)
Suppose the profits were initially x dollars. Then three years ago, profits would have been $1.06x$. Two years ago, profits decreased by 13% from the previous year, so the profits were $.87 \cdot (1.06x)$. Last year, the profits increased 9% from two years ago so the profits were $1.09 \cdot (.87 \cdot (1.06x)) = 1.0052x$. This means the profits are up 0.52% over the last 3 years.

§3B Exercise 21

a. 10^{35} is $1,000,000,000$ times as large as 10^{26}, or 1 billion times as large.

b. 10^{17} is 10^{-10} or 10 billion times smaller than 10^{27}.

c. 1 billion is $1,000$ times as large as 1 million.

d. 7 trillion is $1,000,000,000$ (or 1 billion) times as large as 7 thousand.

e. 2×10^{-6} is $1,000$ times as large as 10^{-9}.

f. 10^{12} is 10^{21} (or one sextillion) times as large as 10^{-9}.

§3B Exercise 29

a. We can just multiply 3×1 then add the appropriate number of zeros. We have $300,000 \times 100 = 30,000,000$. This is exact.

b. 5.1×1.9 is about 5.0×2.0 and one million times one thousand is one billion so the answer should be approximately 10 billion or 10^9. The actual answer is 9.69×10^8 which is pretty close.

c. Approximate $4/2.1$ by $4/2$, then we get

$$4 \times 10^9 \div 2.1 \times 10^6 \approx 4 \times 10^9 \div 2 \times 10^6 = 2 \times 10^3 = 2,000$$

The actual value is $1,904$, so we are off by about 5%.

d. We want to approximate $(33 \times 10^6) \cdot (3.1 \times 10^3)$. We have

$$(33 \times 10^6) \cdot (3.1 \times 10^3) \approx (33 \times 10^6) \cdot (3 \times 10^3) = 99 \times 10^9 = 99,000,000,000$$

The actual answer is $102,000,000,000$, so we are off by about 3%.

e. This one is a little harder. We could say $4,288,364 \approx 4,200,000$ and $2,132 \approx 2,100$. The reason I chose to make these specific approximations is because $4.2 = 2 \times 2.1$, so the division will be easy even without a calculator. In scientific notation we have

$$4,288,364 \div 2,132 \approx 4,200,000 \div 2,100 = (4.2 \times 10^6) \div (2.1 \times 10^3) = 2 \times 10^3 = 2,000$$

The actual answer is 2011.43, so we are very close. In fact we are about 0.5% off.

f. We can estimate $6.129845 \times 10^6 \approx 6 \times 10^6$ and $2.198 \times 10^4 \approx 2 \times 10^4$. Then we have

$$(6.129845 \times 10^6)(2.198 \times 10^4) \approx (6 \times 10^6)(2 \times 10^4) = 12 \times 10^{10} = 1.2 \times 10^{11}$$

The actual answer is 1.34×10^{11}, so we are off by about 12%.

§3B Exercise 32

It is about 3,000 miles from New York to California. The average person walks about 3.5 miles per hour. Assuming you walk 12 hours each day, not including breaks. You could walk $365 \times 12 \times 3.5 \approx 15,000$ miles in one year. So the answer is yes.