# MATH1210: HOMEWORK SOLUTIONS §1.3

1. $\lim_{x\to 1}(2x+1) \overset{3,4}{=} 2\left[\lim_{x\to 1} x\right] + \lim_{x\to 1} 1$

$\overset{1,2}{=} 2(1)+1 = 3$

3. $\lim_{x\to 0}\left[(2x+1)(x-3)\right] \overset{6}{=} \left[\lim_{x\to 0} 2x+1\right]\left[\lim_{x\to 0} x-3\right]$

$\overset{3,4,5}{=} \left[2\lim_{x\to 0} x + \lim_{x\to 0} 1\right]\left[\lim_{x\to 0} x - \lim_{x\to 0} 3\right]$

$\overset{1,2}{=} \left[2(0)+1\right]\left[0-3\right] = -3$

5. $\lim_{x\to 2}\frac{2x+1}{5-3x} \overset{7}{=} \frac{\lim_{x\to 2}(2x+1)}{\lim_{x\to 2}(5-3x)}$

$\overset{3,4,5}{=} \frac{2\lim_{x\to 2} x + \lim_{x\to 2} 1}{\lim_{x\to 2} 5 - 3\lim_{x\to 2} x} \overset{1,2}{=} \frac{2\cdot 2+1}{5-3\cdot 2} = -5$

7. $\lim_{x\to 3}\sqrt{3x-5} \overset{9}{=} \sqrt{\lim_{x\to 3}(3x-5)}$

$\overset{3,5}{=} \sqrt{3\lim_{x\to 3} x - \lim_{x\to 3} 5} \overset{1,2}{=} \sqrt{3\cdot 3-5} = \sqrt{4} = 2$

9. $\lim_{t\to -2}(2t^3+15)^{13} \overset{8}{=} \left(\lim_{t\to -2}(2t^3+15)\right)^{13}$

$\overset{3,4,8}{=} \left(2\left[\lim_{t\to -2} t\right]^3 + \lim_{t\to -2} 15\right)^{13} \overset{1,2}{=} \left(2(-2)^3+15\right)^{13} = (-1)^{13}$

$= -1$

11. $$\lim_{y\to 2}\left(\frac{4y^3+8y}{y+4}\right)^{1/3} \overset{9,7}{=} \left[\frac{\lim_{y\to 2}\left(4y^3+8y\right)}{\lim_{y\to 2}\left(y+4\right)}\right]^{1/3}$$

$$\overset{3,4,8}{=} \left[\frac{4\left(\lim_{y\to 2} y\right)^3 + 8\lim_{y\to 2} y}{\lim_{y\to 2} y + \lim_{y\to 2} 4}\right]^{1/3} \overset{1,2}{=} \left[\frac{4(2)^3+8(2)}{2+4}\right]^{1/3}$$

$$= \left(\frac{48}{6}\right)^{1/3} = 8^{1/3} = 2$$

13. $$\lim_{x\to 2}\frac{x^2-4}{x^2+4} = \frac{2^2-4}{2^2+4} = \frac{0}{8} = \boxed{0}$$

15. $$\lim_{x\to -1}\frac{x^2-2x-3}{x+1} = \lim_{x\to -1}\frac{(x+1)(x-3)}{x+1} = \lim_{x\to -1} x-3 = -1-3 = \boxed{-4}$$

17. $$\lim_{x\to -1}\frac{x^3-6x^2+11x-6}{x^3+4x^2-19x+14} = \frac{(-1)^3-6(-1)^2+11(-1)-6}{(-1)^3+4(-1)^2-19(-1)+14}$$

$$= \frac{-1-6-11-6}{-1+4+19+14} = \boxed{\frac{-24}{35}}$$

19. $$\lim_{x\to 1}\frac{x^2+x-2}{x^2-1} = \lim_{x\to 1}\frac{(x+2)(x-1)}{(x+1)(x-1)} = \lim_{x\to 1}\frac{x+2}{x+1} = \frac{2+1}{1+1} = \boxed{\frac{3}{2}}$$

21. $$\lim_{u\to -2}\frac{u^2-ux+2u-2x}{u^2-u-6} = \lim_{u\to -2}\frac{(u+2)(u-x)}{(u+2)(u-3)} = \lim_{u\to -2}\frac{u-x}{u-3}$$

$$= \frac{-2-x}{-5} = \boxed{\frac{2+x}{5}}$$

23. $$\lim_{x\to\pi}\frac{2x^2-6x\pi+4\pi^2}{x^2-\pi^2} = \lim_{x\to\pi}\frac{2(x-\pi)(x-2\pi)}{(x-\pi)(x+\pi)}$$

$$= \lim_{x\to\pi}\frac{2(x-2\pi)}{x+\pi} = \frac{-2\pi}{2\pi} = -1$$

25. $$\lim_{x\to a}\sqrt{f^2(x)+g^2(x)} = \sqrt{\left(\lim_{x\to a} f(x)\right)^2+\left(\lim_{x\to a} g(x)\right)^2}$$
$$= \sqrt{3^2+(-1)^2} = \boxed{\sqrt{10}}$$

27. $$\lim_{x\to a}\sqrt[3]{g(x)}\left[f(x)+3\right] = \sqrt[3]{\lim_{x\to a} g(x)}\left(\lim_{x\to a} f(x)+3\right)$$
$$= \sqrt{-1}\,(3+3) = -1(6) = \boxed{-6}$$

29. $$\lim_{t\to a}\left[|f(t)|+|3g(t)|\right] = \left[\left|\lim_{t\to a} f(t)\right|+\left|3\lim_{t\to a} g(t)\right|\right]$$
$$= |3|+|3\cdot -1| = \boxed{6}$$

31. $$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{3x^2-3(2)^2}{x-2} = \lim_{x\to 2}\frac{3(x+2)(x-2)}{x-2}$$
$$= \lim_{x\to 2} 3(x+2) = 3\cdot 4 = \boxed{12}$$

33. $$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2}\frac{2-x}{2x}\cdot\left(\frac{1}{x-2}\right)$$
$$= \lim_{x\to 2}\frac{-(x-2)}{2x(x-2)} = \lim_{x\to 2}\frac{-1}{2x} = \frac{-1}{4}$$

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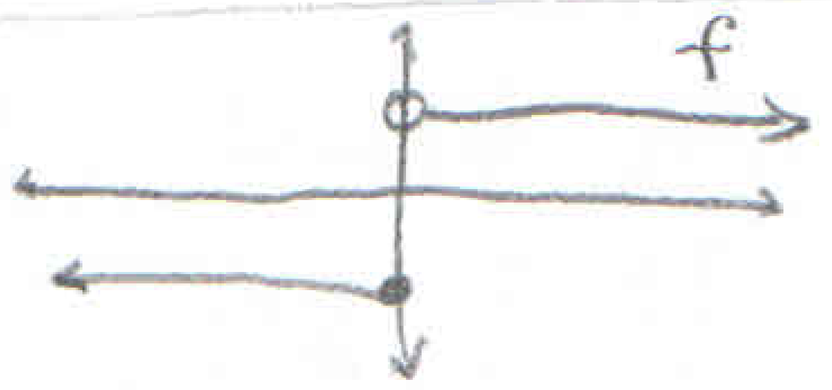
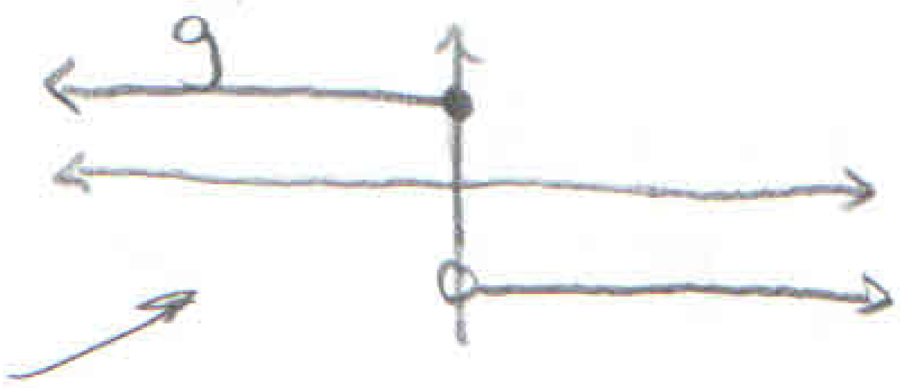
40. a) $f(x) =$




$g(x) =$




$-f(x)$

Then $f+g \equiv 0$

b) Use $f$ from part a) for both $f$ and $g$.

Then $f\cdot g$ is constant.

41. $\lim_{x \to -3^+} \frac{\sqrt{3+x}}{x}$. For $x$ close to but $\neq -3$, and to the right of $-3$, $\sqrt{3+x}$ is defined because the argument is positive. Since all functions involved are cont., we can evaluate by plugging in $x = -3$ and the limit is $0$. If we were asked to evaluate $\lim_{x \to -3^-} \frac{\sqrt{3+x}}{x}$, the limit would not exist.

43. $\lim_{x \to 3^+} \frac{x-3}{\sqrt{x^2-9}}$. If we try substitution, we get $\frac{0}{0}$, so we should try to simplify.

$$= \lim_{x \to 3^+} \frac{x-3}{(x+3)^{1/2}(x-3)^{1/2}} = \lim_{x \to 3^+} \frac{(x-3)^{1/2}}{(x+3)^{1/2}}$$

$= \lim_{x \to 3^+} \sqrt{\frac{x-3}{x+3}}$. Now we are in the same position as #41. The argument of the $\sqrt{\phantom{x}}$ is approaching $0$. As long as it's positive when $x$ is near but to the right of 3, the limit is defined. When $x$ is say 3.01, we have $\frac{3.01-3}{3.01+3} > 0$, so the limit is defined. $\lim_{x \to 3^+} \frac{x-3}{\sqrt{x^2-9}} = 0$.