Practice and Challenge Problems from §4.6, 5.1, 5.2, & 5.3

Not collected

I will post solutions to these problems approximately one week before the third midterm.

1. Is it possible that all solutions of a homogeneous system of 10 equations in 12 variables are multiples of one non-zero solution? Explain.

2. Let A be an $m \times n$ matrix. What is the relationship between $\dim(\text{Row}A)$, $\dim(\text{Nul}A)$, and the number of columns of A? What is the relationship between $\dim(\text{Col}A)$, $\dim(\text{Nul}A^T)$, and the number of rows of A? Write down two equations describing the relationships.

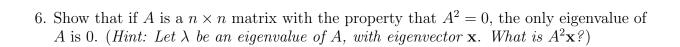
3.	Using the previous exercise, explain why the equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} in \mathbb{R}^m if and only if the equation $A^T\mathbf{x} = 0$ has only the trivial solution.
4.	Give examples of the following:
	(a) A 2×2 matrix A with only one distinct eigenvalue.
	(b) A matrix A with a two-dimensional eigenspace.
	(c) A matrix A with all non-zero entries that has 0 as an eigenvalue.

5. Prove the following theorem which we will discuss on Friday.

Theorem: If $n \times n$ matrices A and B are similar, then they have the same characteristic polynomial. (And therefore the same eigenvalues.)

HINTS:

- Start by analyzing the matrix $B \lambda I$.
- Use the facts that $B = P^{-1}AP$ and $I = P^{-1}P$ to write $B \lambda I$ in terms of A, λ, P, P^{-1} , and I.
- Compute the characteristic polynomial of B.
- Remember that the determinant of the product is the product of the determinants. (In other words, det(AB) = detAdetB.)



7. Show that A and A^T have the same characteristic polynomial.

8. Let A be an $n \times n$ matrix with the property that each of its rows sums to the same number s. (For example, if s = 4, the matrix $A = \begin{pmatrix} 4 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$ has this property.) Show that s is an eigenvalue of A. (*Hint: Find an eigenvector.*)