

MATH 1030: Homework 7

due March 6, 2013

Instructions: Do the following problems on a separate sheet of paper. Show all of your work.

§4C Exercise 63

- (a) The mutual fund currently costs \$10.93 per share. If you invest \$5,000 in this mutual fund today, you will buy $\$5,000 \div (\$10.93/\text{share}) = 457.46$ shares.
- (b) The annualized 5-year return is 3.20%, so the fund has earned 3.20% APY per year for the last 5 years. The value today is then

$$A = \$5,000(1 + .032)^5 = \$5,852.86$$

- (c) The 10-year annualized return is 3.69%, so today we would have

$$A = \$5,000(1 + .0369)^{10} = \$7,183.54$$

had we invested \$5,000 ten years ago.

§4C Exercise 76

On average, each month has 30 days. If the person spends \$4.50 on cigarettes per day, then he will save $\$4.50 \times 30 = \135 each month. Making deposits into a savings account that earns 9% APR for 30 years, he would have approximately

$$A = PMT \frac{\left(1 + \frac{APR}{n}\right)^{nY} - 1}{\frac{APR}{n}} = \$135 \frac{\left(1 + \frac{.09}{12}\right)^{12 \cdot 30} - 1}{\frac{.09}{12}} = \$247,150$$

so the claim is accurate. Good luck finding an investment that earns 9% APR.

§4C Exercise 77

- (a) We can use the savings plan formula for the first 10 years of Mitch's investment because he is making regular contributions. After that, since he is no longer making contributions, we need to use the compound interest formula. After 10 years he will have

$$A = PMT \times \frac{\left(1 + \frac{APR}{n}\right)^{nY} - 1}{\frac{APR}{n}} = \$1,000 \times \frac{\left(1 + \frac{.07}{1}\right)^{1 \cdot 10} - 1}{\frac{.07}{1}} = \$13,816.40$$

At this point his money just sits there and earns interest via the compound interest formula for the next 40 years (he is 35 when he stops contributing and we are asked to calculate how much he has when he is 75). He will have

$$A = P \left(1 + \frac{APR}{n}\right)^{nY} = \$13,816.40 \times (1 + .07)^{40} = \$206,894$$

- (b) Bill doesn't start saving until he is 35, but then he makes regular contributions for the next 40 years. We can use the savings plan formula to find out how much he has when he is 75.

$$A = PMT \times \frac{\left(1 + \frac{APR}{n}\right)^{nY} - 1}{\frac{APR}{n}} = \$1,000 \times \frac{\left(1 + \frac{.07}{1}\right)^{1 \cdot 40} - 1}{\frac{.07}{1}} = \$199,635$$

- (c) Mitch only made contributions for 10 years, so he contributed a total of \$10,000. Bill, on the other hand, made contributions for 40 years totalling \$40,000.
- (d) If you take one thing away from this problem, this section, or even this entire class, it's this "START SAVING NOW!!!"

Problem 1

Agent Cooper is interested in purchasing some retirement property at what he thinks "will be a very reasonable price." Suppose his savings account compounds continuously with an APR of 4.65%.

- (a) Find the APY of his account.
Recall the formula for APY when the interest is continuously compounded.

$$APY = e^{APR} - 1 \quad \implies \quad APR = e^{.0465} - 1 = 4.76\%$$

- (b) Cooper wants to deposit some money into his account and leave it until he retires 25 years from now. Suppose he wants to have a total of \$150,000 in his account when he retires. How much should he deposit right now?

We want the account balance after 25 years to be \$150,000. How much should we put in today?

$$\$150,000 = Pe^{.0465 \cdot 25} \quad \implies \quad P = \frac{\$150,000}{e^{.0465 \cdot 25}} = \$46,905.50$$

Problem 2

Her competition with Arthur has gotten Joanne thinking about her future. Her bank offers a savings account that compounds monthly at an APR of 3.8%. Suppose that she decides to deposit \$200 into her account each month until she retires in 35 years.

- (a) What will Joanne's savings account balance be after the 35 years?
We can use the savings plan formula to find out how much Joanne will have in her account 35 years from now

$$A = PMT \times \frac{\left(1 + \frac{APR}{n}\right)^{nY} - 1}{\frac{APR}{n}} = \$200 \times \frac{\left(1 + \frac{.038}{12}\right)^{12 \cdot 35} - 1}{\frac{.038}{12}} = \$175,144$$

- (b) How much of the total was deposited by Joanne and how much did she earn in interest?
 She deposited a total of $\$200 \times 12 \times 35 = \$84,000$. So the rest must have been earned in interest. She earned $\$175,144 - \$84,000 = \$91,144$.
- (c) What percentage of the balance was earned in interest?
 This is asking what percent of the final balance after 35 years was earned interest. It is $\frac{\$91,144}{\$175,144} = .52 = 52\%$.

Problem 3

Joanne's rival Arthur is still angry about not saving as much money as Joanne. He hears about Joanne's plan to retire and he devises a plan to have a better retirement than her. His plan is to save enough money so that when he retires, he can live solely on the interest earned in the account. When he retires, he wants to have enough money in the bank so that he can earn a modest \$25,000 per year in interest alone. It turns out that Arthur banks at the same place as Joanne and has the same savings account that compounds monthly at an APR of 3.8%

- (a) What should the account balance be when Arthur retires so that he earns \$25,000 in interest each year?
 If his account compounds monthly at an APR of 3.8%, we have to find out how much he needs to have so that he earns \$25,000 in interest in one year. There are several ways to do this. First, we suppose he has P dollars in his account. We then want him to have $P + \$25,000$ in his account after one year. So

$$P + \$25,000 = P \left(1 + \frac{.038}{12}\right)^{12}$$

If we now solve for P , we get $P = \$646,515$. Alternatively, we could compute the APY of the account and then require that $P \cdot APY = \$25,000$, so that $P = \frac{\$25,000}{APY} = \frac{\$25,000}{.038669}$.

- (b) Supposing Arthur also plans to retire 35 years from now, what monthly payment does he need to make in order to reach his goal?
 How much does he need to save every month if he wants to have \$646,515 when he retires 35 years from now. Using the savings plan formula, we have

$$\$646,515 = PMT \frac{\left(1 + \frac{.038}{12}\right)^{12 \cdot 35} - 1}{\frac{.038}{12}} \implies PMT = \frac{\$646,515}{875.718} = \$738,269$$

Problem 4

Bonnie and Dexter are looking into buying a house. They secure a 30-year mortgage that compounds monthly at an APR of 7.1% for a house that costs \$230,000.

- (a) What will Bonnie's and Dexter's monthly payments be?
 Using the loan payment formula, we have

$$PMT = P \frac{\frac{APR}{n}}{1 - \left(1 + \frac{APR}{n}\right)^{-nY}} = \$230,000 \frac{\frac{.071}{12}}{1 - \left(1 + \frac{.071}{12}\right)^{-12 \cdot 30}} = \$1,545.67$$

- (b) How much will they actually pay for the house?

Making payments of \$1,545.67 every month for 30 years, they will pay

$$\$1,545.67 \times 12 \times 30 = \$556,442$$

- (c) Of the total amount paid, what percentage was paid in interest?

The original principal was \$230,000, so they paid a total of $\$556,442 - \$230,000 = \$326,442$ in interest. As a fraction this is

$$\frac{\$326,442}{\$556,442} = 58.6\%$$

- (d) If Bonnie and Dexter decide to instead pay off the loan in 20 years, how much will their monthly payment be and how much (in dollars) will they save in interest?

This amounts to doing parts (a) and (b) over with a loan term of 20 years, then finding how much they save. Indeed, their monthly payments will be

$$PMT = P \frac{\frac{APR}{n}}{1 - \left(1 + \frac{APR}{n}\right)^{-nY}} = \$230,000 \frac{\frac{.071}{12}}{1 - \left(1 + \frac{.071}{12}\right)^{-12 \cdot 20}} = \$1,797.02$$

So they pay a total of $\$1,797.02 \times 12 \times 20 = \$431,285$. Since the loan principal was the same, they saved $\$556,442 - \$431,285 = \$125,157$ in interest.

Problem 5

Chester wants to buy a new car. His bank offers car loans that compound monthly at an APR of 6.2%. Suppose that he can only afford to pay \$350 per month towards a car loan.

- (a) If Chester takes out a 3-year loan, what is the largest loan amount that he can afford?

Using the loan payment formula, we have

$$\$350 = P \frac{\frac{.062}{12}}{1 - \left(1 + \frac{.062}{12}\right)^{-12 \cdot 3}} \implies P = \$11,470.70$$

- (b) If Chester takes out a 4-year loan, what is the largest loan amount that he can afford?

Doing the same calculation as before except with a loan term of 4 years, we get

$$\$350 = P \frac{\frac{.062}{12}}{1 - \left(1 + \frac{.062}{12}\right)^{-12 \cdot 4}} \implies P = \$14,845.10$$

- (c) If Chester takes out a 5-year loan, what is the largest loan amount that he can afford?

This time, we get

$$\$350 = P \frac{\frac{.062}{12}}{1 - \left(1 + \frac{.062}{12}\right)^{-12 \cdot 5}} \implies P = \$18,017.20$$