Food for Thought 3

Due Friday, September 8

*Spend the rest of today's class period working through these problems. I encourage you to work with your classmates and discuss the problems. If you are finished with the assignment at the end of class today, then you can turn it in today. If you would like to work on the assignment more, take it home over the weekend and turn it in on Friday. This assignment will be graded for **effort** (which means you have written down thoughtful, complete solutions to each problem), not correctness. Solutions to these problems will be posted on Canvas on Friday for future reference.*

1. Give examples of the following:

   (a) A 2×2 matrix A such that the solution set of the equation $A\mathbf{x} = \mathbf{0}$ is the line in $\mathbb{R}^2$ through $(4, 1)$ and the origin.

   (b) A matrix A so that $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a solution of $A\mathbf{x} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$.

   (c) A 4×3 matrix $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$ such that $\{\mathbf{a}_1, \mathbf{a}_2\}$ is linearly independent.

2. Classify each of the following statements as **true** or **false** and justify your choice.

   (a) Assume $A\mathbf{x} = \mathbf{b}$ has a solution. The solution is unique precisely when $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

   (b) Two vectors are linearly dependent if and only if they line on a line through the origin.

   (c) If a set contains fewer vectors than entries in the vectors, then the set is linearly independent.

   (d) Every matrix transformation is a linear transformation.

3. Suppse $T : \mathbb{R}^5 \longrightarrow \mathbb{R}^2$ is a linear transformation and $T(\mathbf{x}) = A\mathbf{x}$ for some matrix A and for each $\mathbf{x} \in \mathbb{R}^5$. How many rows and columns does A have?

4. Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.