Name:

Midterm 1 Answers
August 25, 2015

Instructions: Do all the problems on **both sides** of each page. Show all your work and box your answers. If you get stuck on a problem, skip it and come back to it at the end.

1. (a) $\vec{b}$ is not a linear combination of $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$.

   (b) The set is not linearly independent. There are many valid justifications. For example, 3 linearly independent vectors in $\mathbb{R}^3$ must span $\mathbb{R}^3$, but $b \notin \mathrm{Span}\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$.

   (c) $\{\vec{a}_1, \vec{a}_2, \vec{b}\}$ is a linearly independent set. Using a theorem, we just have to check that each vector is not in the span of the ones coming before it. $\vec{a}_2 \notin \mathrm{Span}\{\vec{a}_1\}$ since it's not a multiple and $\vec{b} \notin \mathrm{Span}\{\vec{a}_1, \vec{a}_2\}$ by part (a).

2. Yes.

3.
$$\vec{x} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

4. (a) $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 0 \end{bmatrix}$

   (b) $AB = \begin{bmatrix} 5 & 3 & 7 \\ 7 & 4 & 10 \\ 0 & 0 & 0 \end{bmatrix}$

   (c) $S \circ T$ is not invertible. There are valid justifications. For example, the matrix isn't invertible (its rows are clearly linearly dependent).

   (d) $BA = \begin{bmatrix} 4 & 3 \\ 7 & 5 \end{bmatrix}$

   (e) Yes, it is invertible. The matrix for $(T \circ S)^{-1}$ is $\begin{bmatrix} -5 & 3 \\ 7 & -4 \end{bmatrix}$

5. No. Four vector in $\mathbb{R}^3$ will always form a linearly dependent set.

6. $A^{-1} = \begin{bmatrix} 4 & -3 & -3 \\ 2 & -1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$

7. No. It contains the zero vector.

8. (a) No. Many possible justifications.

   (b) Many possible correct answers.

9. $A^8 = \begin{bmatrix} 1 & -255 & -255 \\ 255 & 511 & 255 \\ -255 & -255 & 1 \end{bmatrix}$