Food for Thought 2

Due Friday, September 1

*Spend the rest of today's class period working through these problems. I encourage you to work with your classmates and discuss the problems. If you are finished with the assignment at the end of class today, then you can turn it in today. If you would like to work on the assignment more, take it home and turn it in on Friday. This assignment will be graded for **effort** (which means you have written down thoughtful, complete solutions to each problem), not correctness. Solutions to these problems will be posted on Canvas on Friday evening for future reference.*

1. Give examples of the following:

   (a) A vector $\mathbf{v}$ with no zero entries that is a linear combination of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

   (b) A 2×4 matrix A whose columns do not span $\mathbb{R}^2$.

   (c) A vector $\mathbf{b} \in \mathbb{R}^3$ so that $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{b}$ has no solution.

2. Classify each of the following statements as **true** or **false** and justify your choice.

   (a) If $\mathbf{x}$ is a non-trivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is non-zero.

   (b) The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution.

   (c) The set $\text{span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.

   (d) When $\mathbf{u}$ and $\mathbf{v}$ are non-zero vectors, $\text{span}\{\mathbf{u}, \mathbf{v}\}$ contains the line through $\mathbf{u}$ and the origin.

3. Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$, where $n < m$?

4. If $\mathbf{b} \neq \mathbf{0}$, can the solution set of $A\mathbf{x} = \mathbf{b}$ be a plane through the origin?

5. Suppose A is a 3×3 matrix and $\mathbf{b}$ is a vector in $\mathbb{R}^3$ with the property that $A\mathbf{x} = \mathbf{b}$ has a unique solution. Must the columns of A span $\mathbb{R}^3$? Why or why not? Explain.