Food for Thought 5

Due Friday, September 29

Spend the rest of today's class period working through these problems. I encourage you to work with your classmates and discuss the problems. If you are finished with the assignment at the end of class today, then you can turn it in today. If you would like to work on the assignment more, take it home over the weekend and turn it in on Tuesday. This assignment will be graded for ***effort*** *(which means you have written down thoughtful, complete solutions to each problem), not correctness. Solutions to these problems will be posted on Canvas on Tuesday for future reference.*

1. Give examples of the following:

   (a) A matrix A such that $\det A = -4$ and $\det(3A) = -36$.

   (b) A 3×3 matrix A such that the volume of the parallelepiped in $\mathbb{R}^3$ given by the columns of A is 18.

2. Classify each of the following statements as **true** or **false** and justify your choice.

(a) If two rows of a 3×3 matrix A are the same, then $\det A = 0$.

(b) If A is a square matrix, then $\det(-A) = -\det A$.

(c) If A is a square matrix, then $\det A^T A \geq 0$.

(d) If A is a square matrix and $A^3 = 0$, then $\det A = 0$.

(e) If A is a square matrix and $A^3 = 0$, then $A = 0$.

(f) The determinent of a square matrix A is the product of the diagonal entries of A.

(g) If A and B are $n \times n$ matrices, then $\det(A + B) = \det A + \det B$.

3. Determine whether or not the set $H \subset \mathbb{R}^3$ described below is a subspace of $\mathbb{R}^3$.

$$H = \left\{ \vec{x} = \left[\begin{smallmatrix} x_1 \\ x_2 \\ x_3 \end{smallmatrix}\right] \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 0 \quad \text{or} \quad x_1 + x_2 + x_3 = 0 \right\}$$

4. Let A be a square matrix such that $\det(A^3) = 0$. Can A be invertible?

5. Use the concept of volume to explain why the determinant of a 3×3 matrix A is zero if and only if A is not invertible. (Try to do so *without* using the fact that $\det A \neq 0$ if and only if A is invertible.)

6. Find a formula for the area of a triangle whose verticies are $\mathbf{0}$, $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ in $\mathbb{R}^2$