MATH1060: Midterm 3 Practice Problems

The following are practice problems for the first exam.

1. Find all solutions to the following trigonometric equations:

   (a) $4\cos^2\phi - 1 = 0$ $\phi = \dfrac{\pi}{3} + n\pi, \dfrac{2\pi}{3} + n\pi$

   (b) $\csc\nu + \cot\nu = 1$ $\nu = \dfrac{\pi}{2} + 2\pi n$

   (c) $\tan(3\eta) - 1 = 0$ $\eta = \dfrac{\pi}{12} + \dfrac{n\pi}{3}$

   (d) This question was impossible to solve analytically.

2. Find the exact value of each of the following expressions:

   (a) $\sin(\pi/12) = \dfrac{\sqrt{6} - \sqrt{2}}{4}$

   (b) $\tan(165^\circ) = \dfrac{\sqrt{3} - 3}{\sqrt{3} + 3}$

   (c) $\cos(18^\circ)\cos(12^\circ) - \sin(18^\circ)\sin(12^\circ) = \dfrac{\sqrt{3}}{2}$

   (d) $\sin\dfrac{\pi}{12}\cos\dfrac{\pi}{4} + \sin\dfrac{\pi}{4}\cos\dfrac{\pi}{12} = \dfrac{\sqrt{3}}{2}$

   (e) $\cos 75^\circ + \cos 15^\circ = \dfrac{\sqrt{6}}{2}$

3. Find the exact solutions to the following trigonometric equations in the interval $[0, 2\pi)$

   (a) $\cos 2\chi - \cos\chi = 0$. $\chi = 0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

   (b) $\tan 2\nu - 2\cos\nu = 0$. This one is tough. $\nu = \dfrac{\pi}{6}, \dfrac{3\pi}{2}, \dfrac{5\pi}{6}$

   (c) $4 - 8\sin^2\mu = 0$. $\mu = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

   (d) $\sin\dfrac{\rho}{2} + \cos\rho = 0$. $\rho = \pi$. There is a subtlety in this problem (and problems like it) that we did not discuss in class. This subtlety will not come up on the exam, however there may be a problem similar to this one. Briefly, the issue is that since the original equation has the angle $\dfrac{\rho}{2}$, if we solve for ρ, and add an integer multiple of 2π to get a coterminal angle, then $\dfrac{\rho + 2\pi n}{2}$ will not necessarily be coterminal to $\dfrac{\rho}{2}$. In class, we solved this problem and determined $\dfrac{-\pi}{3}$ is a solution. Then we hastily concluded that $\dfrac{5\pi}{3}$ is also a solution. A quick check shows that $\dfrac{5\pi}{3}$ is not a solution, so the only solution in $[0, 2\pi)$ is $\rho = \pi$. Again, this will not be an issue you need to worry about on the exam. It is easily dealt with, though.

(e) $\sin\frac{\beta}{2} + \cos\beta = 1$. $\beta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$

4. Use the power reducing formula to rewrite the expression $\sin^4 x \cos^2 x$ in terms of the first power of cosine. $\frac{1}{16}\left[1 - \cos(2x) - \cos(4x) - \cos(2x)\cos(4x)\right]$

5. Use the sum-to-product or product-to-sum formula to rewrite each expression:

   (a) $\sin 5\gamma \sin 3\gamma = \frac{1}{2}\left[\cos(2\gamma) - \cos(8\gamma)\right]$

   (b) $\cos 6\delta + \cos 2\delta = 2\cos(4\delta)\cos(2\delta)$

6. Use the half-angle formula to simplify $\sqrt{\frac{1 - \cos 14x}{2}}$. $\sin(7x)$

7. If $\sin u = \frac{7}{25}$ and u lies in the second quadrant, find $\cos(u/2)$. $\sqrt{1/50}$

8. Use any means you like to solve the triangle with $\alpha = 24.3°$, $\gamma = 54.6°$, and $c = 10.3$. $\beta = 101.1°$, $a = 5.20$, $b = 12.40$

9. Use any means you like to solve the triangle with $\alpha = 120°$, $\beta = 45°$, and $c = 16$. $\gamma = 15°$, $a = 53.54$, $b = 43.71$

10. Use any means you like to solve the triangle with $\beta = 63.2°$, $\gamma = 47.6°$ and $b = 12.2$. $\alpha = 69.2°$, $a = 12.78$, $c = 10.09$

11. Use any means you like to solve the triangle with $\alpha = 110°$, $a = 125$, and $b = 100$. $\beta_1 = 48.74°$, $\gamma_1 = 21.26°$, $c_1 = 48.23$. Only one solution.

12. Use any means you like to solve the triangle with $\beta = 100°$, $b = 14$, and $c = 19$. Impossible.

13. Use any means you like to solve the triangle with $\alpha = 28°$, $b = 12.8$, and $a = 8$. $\beta_1 = 48.69°$, $\gamma_1 = 103.31°$, $c_1 = 16.58$, and $\beta_2 = 131.31°$, $\gamma_2 = 20.69°$, $c_2 = 6.42$

14. Use Heron's Formula to find the area of a triangle with side lengths $7, 8$, and 9. Answer: $12\sqrt{5}$

15. Use any means you like to solve the triangle with $\alpha = 50°$, $b = 15$ and $c = 30$. $a = 23.38$, $\beta = 29.44°$, $\gamma = 100.56°$

16. Use any means you like to solve the triangle with side lengths $7, 8$, and 9. $\beta = 48.19°$, $\alpha = 73.40°$, $\gamma = 58.41°$. Depending on how you labelled the angles, your answer may differ, but the numbers should be the same.

17. Write the vector, $\vec{v}$, with initial point $(-4, 5)$ and terminal point $(3, -1)$ in standard form. Answer: $\vec{v} = \langle 7, -6\rangle$

18. Write the vector from the last question as a linear combination of the standard unit vectors $\hat{\imath}$ and $\hat{\jmath}$. Answer: $\vec{v} = 7\hat{\imath} - 6\hat{\jmath}$

19. Compute the magnitude of the vector, $\vec{v}$, from the last question. Answer: $\|\vec{v}\| = \sqrt{85}$

20. Let $\vec{v} = \langle 3, -1\rangle$ and $\vec{w} = \langle -1, 4\rangle$. Write $3\vec{v} - 2\vec{w}$ in standard form. Answer: $\langle 11, -11\rangle$