MATH1210: HOMEWORK SOLUTIONS §2.6

1. $y = x^3 + 3x^2 + 6x$

$\frac{dy}{dx} = 3x^2 + 6x + 6$

$\frac{d^2y}{dx^2} = 6x + 6$

$\frac{d^3y}{dx^3} = 6$

3. $y = (3x+5)^3$

$\frac{dy}{dx} = 3(3x+5)^2 \cdot 3$

$\frac{d^2y}{dx^2} = 2 \cdot 9(3x+5) \cdot 3$

$\frac{d^3y}{dx^3} = 2 \cdot 9 \cdot 3 \cdot 3$

5. $y = \sin 7x$

$y' = 7\cos 7x$

$y'' = -49 \sin 7x$

$y''' = -343 \cos 7x$

7. $y = \frac{1}{x-1}$

$y' = -(x-1)^{-2}$

$y'' = +2(x-1)^{-3}$

$y''' = -6(x-1)^{-4}$

9. $f(x) = x^2 + 1$

$f'(x) = 2x$

$f''(x) = 2$

$f''(2) = 2$

11. $f(t) = 2t^{-1}$

$f'(t) = -2t^{-2}$

$f''(t) = 4t^{-3}$

$f''(2) = \frac{1}{2}$

13. $f(\theta) = (\cos(\pi\theta))^{-2}$

$f'(\theta) = -2(\cos(\pi\theta))^{-3} \cdot (-\sin\pi\theta) \, \pi$

$f''(\theta) = -6(\cos(\pi\theta))^{-4} \cdot (\sin\pi\theta)\pi$

$\quad + 2\pi^2 \cos(\pi\theta)^{-3} \cdot \cos(\pi\theta)$

$f''(2) = 0 + 2\pi^2$

15. $f(s) = s(1-s^2)^3$

$= s(1 - 2s^2 + s^4)(1-s^2)$

$= s - 3s^3 + 3s^5 - s^7$

$f'(s) = -7s^6 + 15s^4 - 9s^2 + 1$

$f''(s) = -42s^5 + 60s^3 - 18s$

$f''(2) = -42(32) + 60(8) - 36$

$= -900$

17. We proceed by induction.

<u>Base</u>: $D_x^1(x^1) = 1 = 1!$ ✓

<u>Ind Step</u>: Suppose $D_x^n(x^n) = n!$

Then $D_x^{n+1}(x^{n+1}) = D_x^n((n+1)x^n)$

$= (n+1) D_x^n(x^n) = (n+1)\,n!$

by inductive hypothesis. This is $(n+1)!$ and we are done.

19. a) $D_x^4(3x^2+2x-19)=0$

b) 0

c) 0

21. $f(x)= x^3+3x^2-45x-6$

$f'(x)=3x^2+6x-45$

$=3(x^2+2x-15)$

$f''(x)= 6x+6$

$f'(x)=0 \iff x=3,-5$

$f''(3)=24 \quad f''(-5)=-24$

23. a) $v(t)=12-4t$

$a(t)=-4$

b) $v(t)>0 \iff 12>4t$

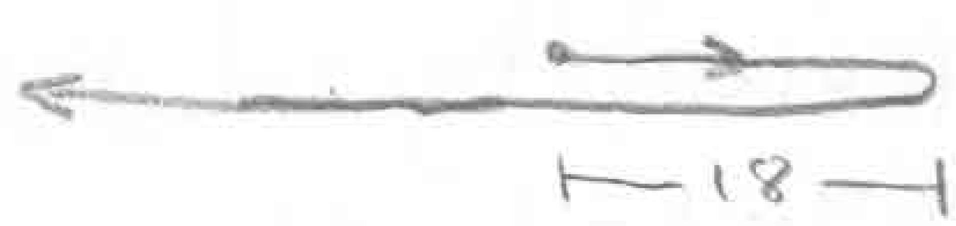
$\iff t<3$

moving right on $(-\infty,3)$

c) moving left on $(3,\infty)$

d) acceleration is always neg.

e)



25. a) $v(t)=3t^2-18t+24$

$a(t)=6t-18$

b) $v(t)=0 \iff$

$t^2-6t+8=0$

$\iff t=2,4$

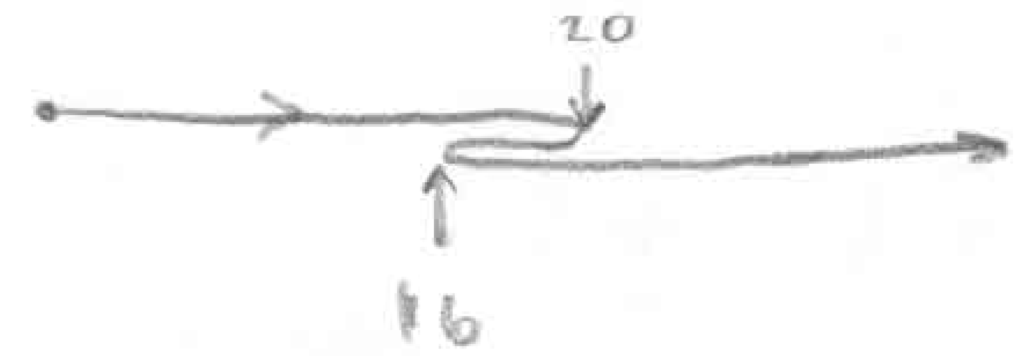
$\Rightarrow$ object is moving right on $(-\infty,2)$ and $(4,\infty)$

c) moving left on $(2,4)$

d) acceleration is neg. when

$6t-18<0 \iff t<3$

e)



27. a) $v(t)=2t-\frac{16}{t^2}$

$a(t)=2+\frac{32}{t^3}$

b) $v(t)=0 \iff 2t-\frac{16}{t^2}=0$

$\iff 2t^3=16 \Rightarrow t=2$

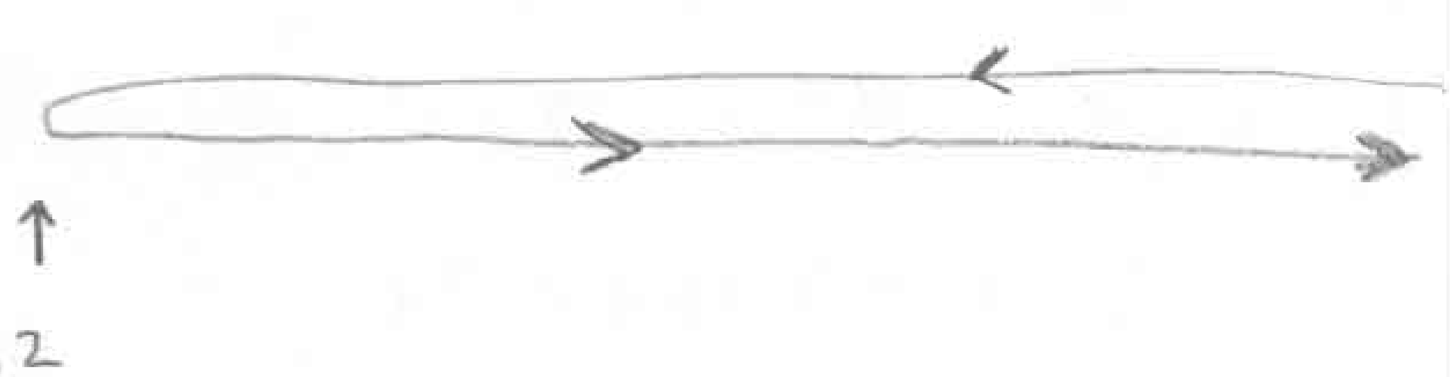
$v>0$ when $t>2$ so

moving right on $(2,\infty)$

c) moving left on $(0,2)$

d) acceleration is always pos.

e)



29. $v(t)=2t^3-15t^2+24t$

$a(t)=6t^2-30t+24$

$=6(t-4)(t-1)$

$a=0$ at $t=1,4$

at $t=1$, $v=11$

at $t=4$, $v=-16$

33. a) $s(t) = -16t^2 + 48t + 256$

$v(t) = -32t + 48$

$a(t) = -32$

initial velocity $= v(0) = 48$ ft/s.

b) max height occurs when $v(t) = 0$

$\Leftrightarrow \quad 32t = 48 \Rightarrow t = 1.5$ sec.

c) max height $= s(1.5) = 292$ ft.

d) hits ground when $s(t) = 0$

$\Leftrightarrow \quad 16(t^2 - 3t - 16) = 0$

$\Leftrightarrow \quad t = \frac{+3 \pm \sqrt{9 - 4(1)(-16)}}{2} = \frac{3 \pm \sqrt{73}}{2} \approx 5.77$ s

we only care about pos. soln so

e) $v(5.77) \approx -136.70$ ft/s

35. $s(t) = v_0 t - 16t^2$ - max height occurs when $v(t) = 0$

$v(t) = v_0 - 32t$ or at time $t = \frac{v_0}{32}$

- at $t = \frac{v_0}{32}$, the height is $s\left(\frac{v_0}{32}\right) = v_0\left(\frac{v_0}{32}\right) - 16\left(\frac{v_0}{32}\right)^2$.

- If the max height is 1 mile, then

$s\left(\frac{v_0}{32}\right) = 5280$ ft $\Rightarrow \quad \frac{v_0^2}{32} - \frac{v_0^2}{64} = 5280$

$\frac{v_0^2}{64} = 5280$

$\Rightarrow v_0 = \sqrt{5280 \cdot 64}$

$\boxed{\approx 581 \text{ ft/s}}$