1. $x \cdot y = -16 \Rightarrow y = \frac{-16}{x}$

$s(x) = x^2 + \left(\frac{-16}{x}\right)^2 = x^2 + \frac{256}{x^2}$

- want to minimize s, so

$s'(x) = 2x - \frac{128}{x^3}$

$s'(x) = 0 \Leftrightarrow 2x = \frac{128}{x^3} \Leftrightarrow x^4 = 64 \Leftrightarrow \boxed{x = 2\sqrt[4]{4}}$

$\boxed{y = \frac{-8}{\sqrt[4]{4}}}$

3. Maximize $f(x) = \sqrt[4]{x} - 2x$

$f'(x) = \frac{1}{4} x^{-3/4} - 2$

$f'(x) = 0 \Leftrightarrow \frac{1}{x^{3/4}} = 8 \Leftrightarrow x^{3/4} = \frac{1}{8} \Leftrightarrow x^{1/4} = 2$

$\Leftrightarrow \boxed{x = 16}$

5. - dist^2 to $(0,5)$ is $(x-0)^2 + (y-5)^2$

- for pts on $y = x^2$, the dist^2 is $(x)^2 + (x^2 - 5)^2$

- we want to minimize $D(x) = x^2 + (x^2-5)^2$

$= x^4 - 9x^2 + 25$

- $D'(x) = 4x^3 - 18x$

$D'(x) = 0 \Leftrightarrow x = 0$, or $x^2 = \frac{18}{4} \Leftrightarrow x = \pm \frac{3\sqrt{2}}{2}$

- $x = 0$ is a local maximum. (Draw a picture or use 2nd deriv. test)

- $x = \pm \frac{3\sqrt{2}}{2}$ are both minima.

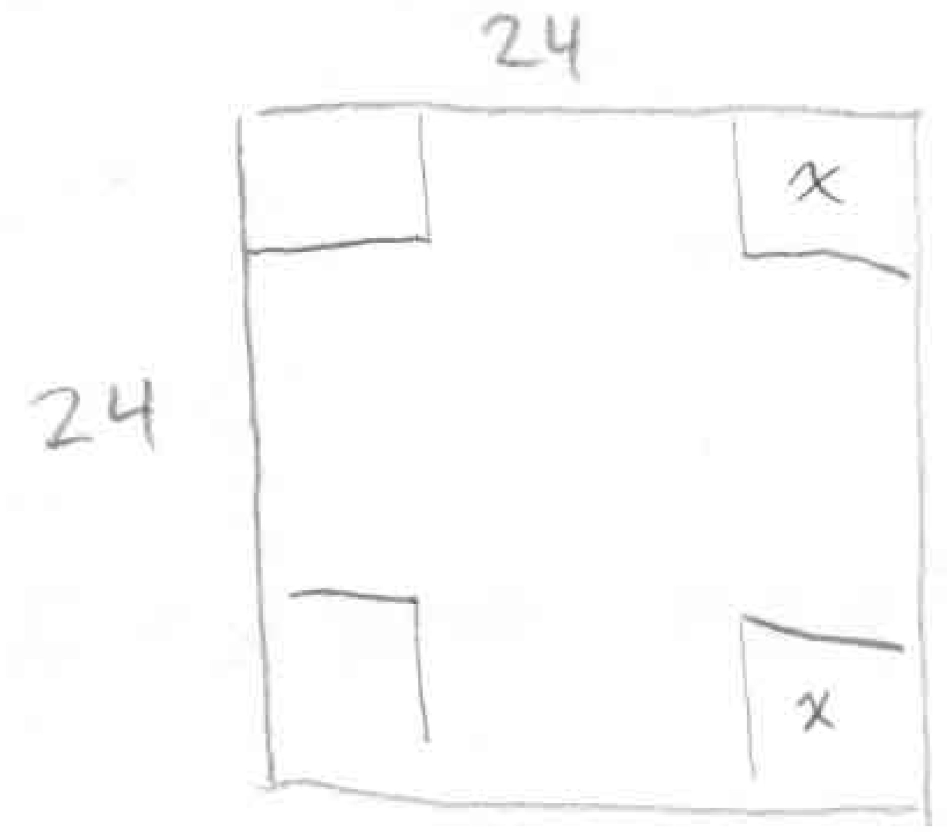
- points are $\left(\pm\frac{3\sqrt{2}}{2}, \frac{18}{4}\right)$

7. Maximize $f(x) = x - x^2$

$f'(x) = 1 - 2x$

$f'(x) = 0 \iff \boxed{x = \frac{1}{2}}$

9.



$V(x) = x \cdot (24 - 2x)^2$

- Maximize $V(x)$

$V'(x) = (24 - 2x)^2 - 4x(24 - 2x)$

$V'(x) = 0 \iff$

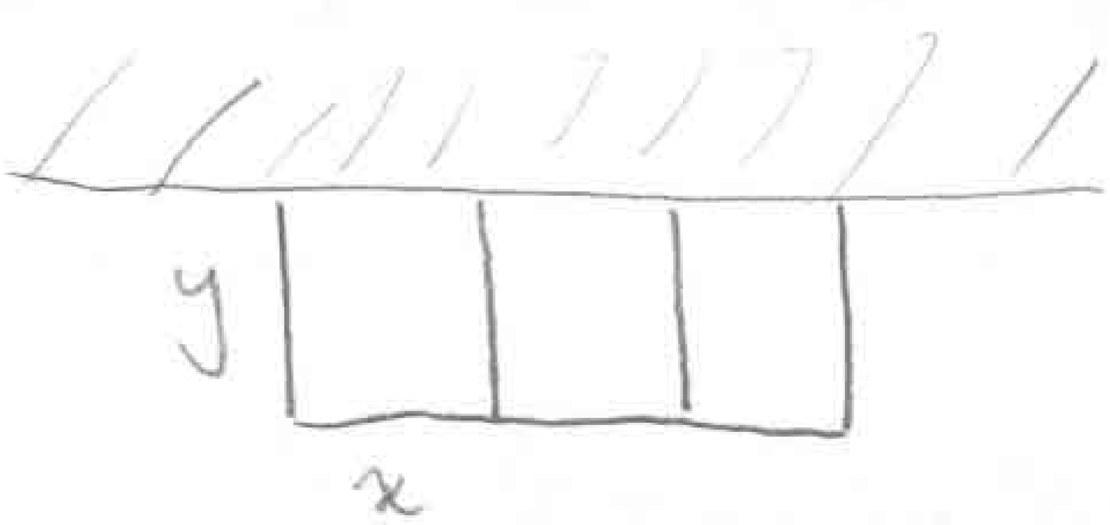
$24 - 2x = 0$ or $24 - 2x = 4x$

$\iff x = 12$ or $\boxed{x = 4}$

degenerate case

Dimensions $4 \times 16 \times 16$

11.



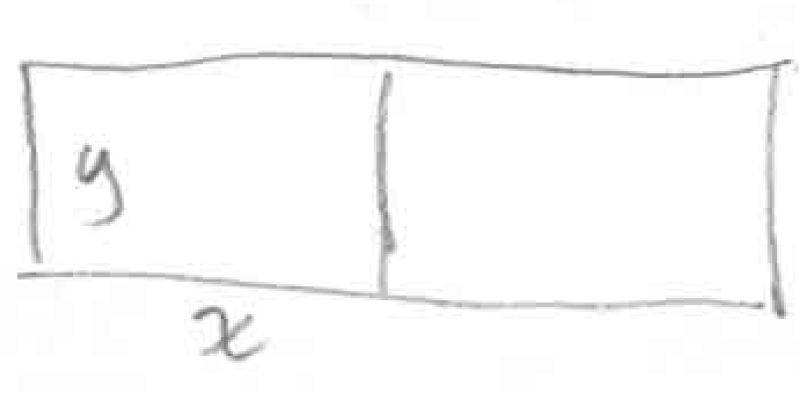
$3x + 4y = 80$

$y = 20 - \frac{3x}{4}$

$A = 3xy = 3x\left(20 - \frac{3x}{4}\right) = \left(60x - \frac{9x^2}{4}\right)$

$\frac{dA}{dx} = 60 - \frac{9x}{2} = 0 \Rightarrow 120 = 9x \boxed{\Rightarrow x = \frac{40}{3} \quad y = 10}$

13.



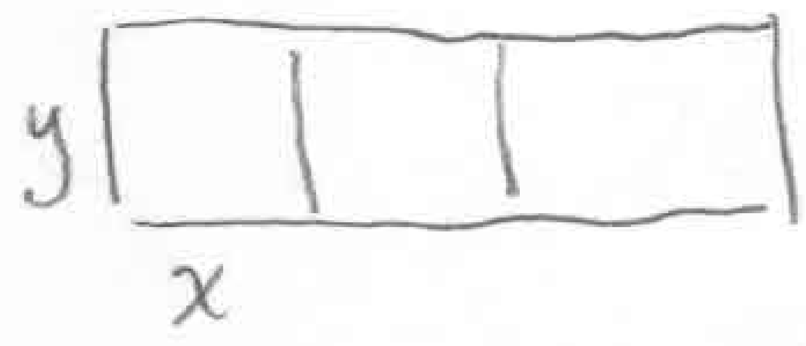
$xy = 900$, $P = 3y + 4x$

$y = \frac{900}{x}$ $\quad = \frac{2700}{x} + 4x$

$\frac{dP}{dx} = -\frac{2700}{x^2} + 4 = 0 \iff 4x^2 = 2700 \iff x^2 = 675$

$x = \boxed{\sqrt{675}}$

15.



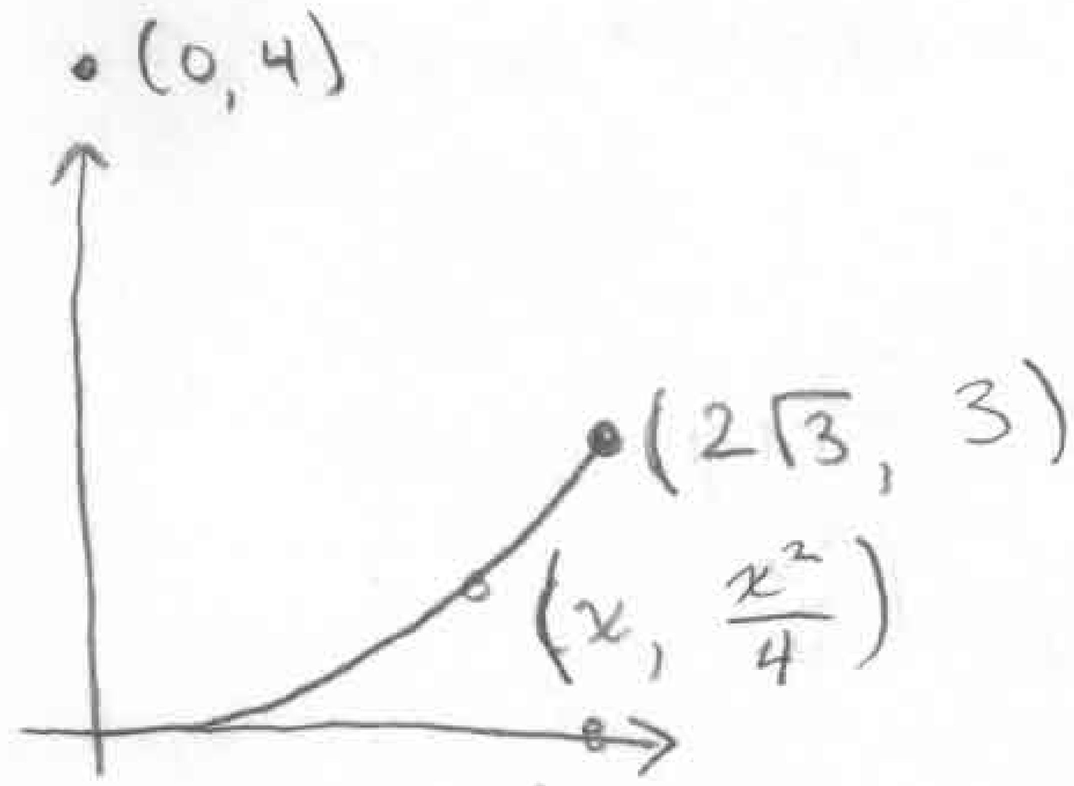
$$xy = 300 \qquad y = \frac{300}{x}$$

$$C = 3\cdot(2y+6x) + 2\cdot(2y) = 10y + 18x = \frac{3000}{x} + 18x$$

$$\frac{dC}{dx} = -\frac{3000}{x^2} + 18 = 0$$

$$\Leftrightarrow 18x^2 = 3000 \Leftrightarrow x = \sqrt{\frac{500}{3}} = \boxed{\frac{10\sqrt{15}}{3}}$$

17.



$$D^2 = (0-x)^2 + \left(4 - \frac{x^2}{4}\right)^2 = x^2 + 16 - 2x^2 + \frac{x^4}{16} = 16 - x^2 + \frac{x^4}{16}$$

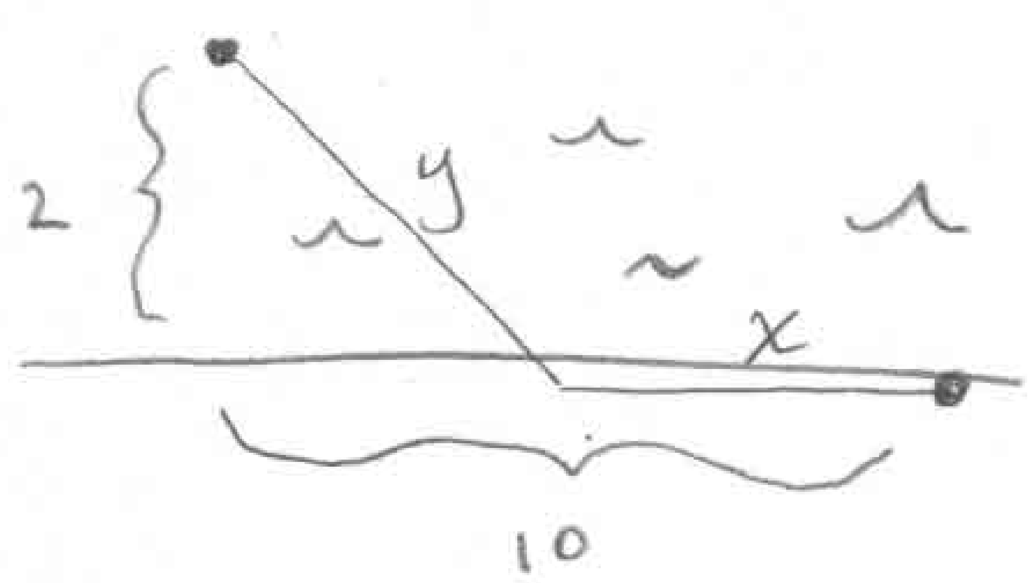
$$\frac{d(D^2)}{dx} = \frac{x^3}{4} - 2x = 0 \Leftrightarrow x^3 - 8x = 0 \Leftrightarrow x(x^2-8) = 0$$

$x = 0, 2\sqrt{2}$ ← need to check both

$$D^2(0) = 16 \qquad D^2(2\sqrt{2}) = 16 - 4\cdot 2 + \frac{64}{16} = 16$$

Both $(0,0)$ and $(2\sqrt{2}, 2)$

19.



$$y^2 = (10-x)^2 + 2^2 = 100 - 2x + x^2 + 4$$

$$T = 3y + 4x = 312 - 2x + x^2$$

$$\frac{dT}{dx} = 2x - 2 = 0 \Leftrightarrow \boxed{x = 1}$$

23.

$$D^2 = \left((60-20t) - 30\cos\tfrac{\pi}{4}t\right)^2 + \left(0 - 30\sin(\tfrac{\pi}{4})\,t\right)^2$$

$$= \left(60 - (20 + 15\sqrt{2})t\right)^2 + \frac{900\,t^2}{2}$$

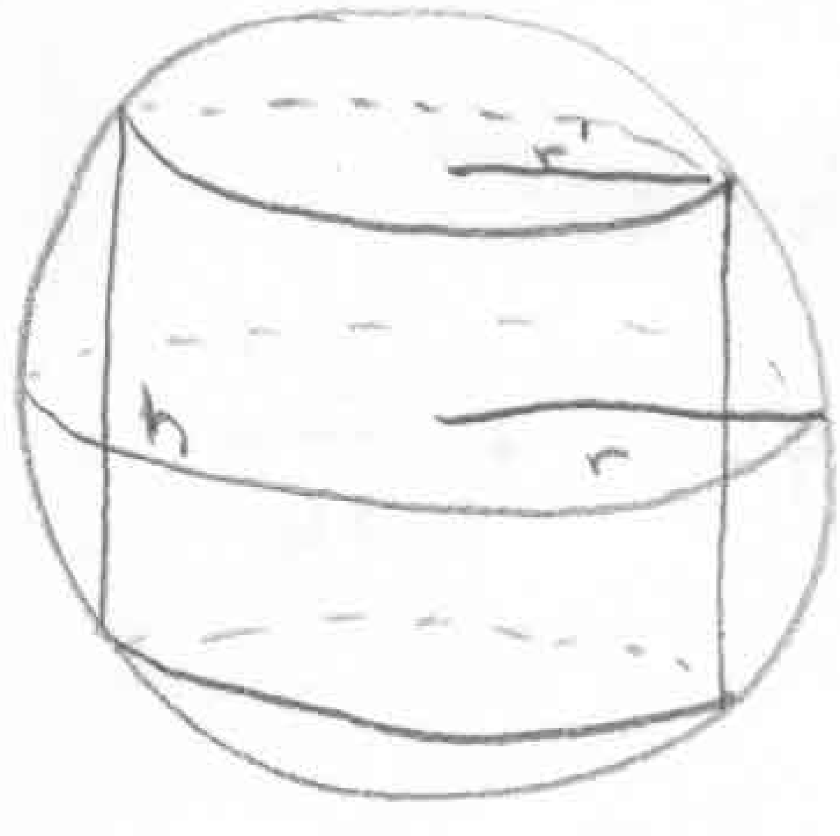
$$= 3600 - (1200 + 900\sqrt{2})t + (400 + 600\sqrt{2} + 450)t^2 + 450t^2$$

$$\frac{d(D^2)}{dt} = 2(1300 + 600\sqrt{2})t - (1200 + 900\sqrt{2}) = 0$$

$$\Longleftrightarrow \quad (2600 + 1200\sqrt{2})t = 1200 + 900\sqrt{2}$$

$$t = \frac{1200 + 900\sqrt{2}}{2600 + 1200\sqrt{2}} = \boxed{0.575 \text{ hrs or } 34.5 \text{ min}}$$

25.



$$\left(\frac{h}{2}\right)^2 + (r')^2 = r^2 \implies r'^2 = r^2 - \left(\frac{h}{2}\right)^2$$

$$V = \pi r'^2 h = \pi h\left(r^2 - \frac{h^2}{4}\right) = \pi r^2 h - \frac{\pi h^3}{4}$$

$$\frac{dV}{dh} = \pi r^2 - \frac{3\pi h^2}{4} = 0 \Longleftrightarrow h^2 = \frac{4\pi r^2}{3\pi} = \frac{4}{3}r^2$$

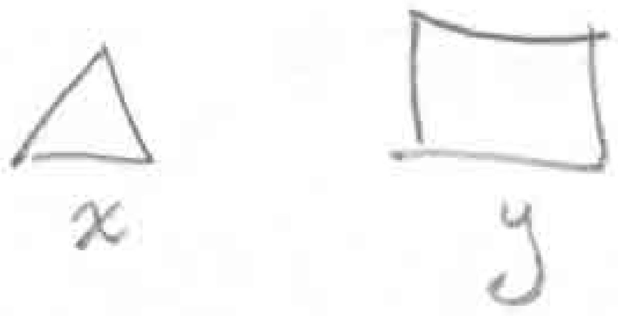
$$\boxed{h = \frac{2r}{\sqrt{3}}}, \quad r' = \sqrt{r^2 - \frac{r^2}{3}} = \boxed{r\sqrt{\frac{2}{3}}}$$

27.

$\left(\frac{h}{2}\right)^2 + r'^2 = r^2$ $\qquad A = 2\pi r' h$. The algebra will be easier if wee look at $A^2 = 4\pi^2 r'^2 h^2 = 4\pi^2 h^2\left(r^2 - \frac{h^2}{4}\right)$

$$\frac{dA}{dh} = 8\pi^2 r^2 h - 4\pi^2 h^3 = 0 \Longleftrightarrow 2r^2h - h^3 = 0 \qquad h = 0, \boxed{\sqrt{2}\,r}$$

29. a)



$$3x + 4y = 100 \Rightarrow y = 25 - \frac{3x}{4}$$

$$A = y^2 + \frac{1}{2}x \cdot \frac{\sqrt{3}}{2}x$$

$$A = 625 - \frac{3x}{2} + \frac{9x^2}{16} + \frac{\sqrt{3}}{4}x^2$$

$$\frac{dA}{dx} = \frac{9x}{8} + \frac{\sqrt{3}\,x}{2} - \frac{3}{2} = 0 \Leftrightarrow (9+4\sqrt{3})x = 12$$

$$\Leftrightarrow \quad x = \frac{12}{9+4\sqrt{3}} \approx 1.118$$

$$\frac{d^2A}{dx^2} > 0 \text{ at } \boxed{x = 1.118 \Rightarrow} \text{ this is a local min.}$$

- The max occurs at an endpoint. Either $x = 0,\ y = 25$

or $x = \frac{100}{3},\ y = 0$

b) A quick check tells us the max is at $\boxed{x = 0,\ y = 25.}$

31.

Say we want an observatory with volume V. then $V = \pi r^2 h + \frac{2}{3}\pi r^3$

$$\Rightarrow \quad h = \frac{V - \frac{2}{3}\pi r^3}{\pi r^2} \leftarrow (*)$$

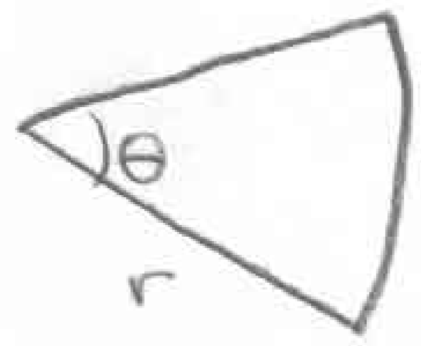
- If cost is a units/ft^2, then $C = 2\pi r h a + 2\pi r^2(2a)$

$$= 2\pi a(rh + 2r^2)$$

- Substituting h with $(*)$, $C = 2\pi a\left(\frac{V - \frac{2}{3}\pi r^3}{\pi r^2}\cdot r + 2r^2\right)$

$$= 2\pi a\left[\frac{V}{\pi r} - \frac{2r^2}{3} + 2r^2\right]$$

$$\frac{dC}{dr} = 0 \Leftrightarrow 2\pi a\left[\frac{-V}{\pi r^2} - \frac{4r}{3} + 4r\right] = 0 \Leftrightarrow r = \frac{3V}{8\pi r^2} \Leftrightarrow r = \left(\frac{3V}{8\pi}\right)^{1/3}$$

33 

A is fixed. $A = \pi r^2 \left(\frac{\theta}{2\pi}\right) = \pi r^2 \theta$

$$P = 2r + r\theta = 2r + r\left(\frac{A}{\pi r^2}\right) = 2r + \frac{A}{\pi r}$$

$$\frac{dP}{dr} = 2 - \frac{A}{\pi r^2} = 0 \iff 2 = \frac{A}{\pi r^2} \iff \boxed{r = \sqrt{\frac{A}{2\pi}}}$$