MATH1220: Midterm 3 Study Guide

The following is an overview of the material that will be covered on the first exam.

§6.1 The Natural Logarithm Function

- The definition of the natural logarithm, including its derivative.
- Computing integrals of the form $\int du/u$.
- Using properties of logarithms to simplify the computation of derivatives (this is called logarithmic differentiation).
- Computing certain trig integrals (e.g., $\int \tan x \, dx$).

§6.2 Inverse Functions and Their Derivatives

- Finding the inverse of a function.
- Show a function has an inverse (without actually finding it). Our standard method for doing this is using Theorem A from §6.2.
- Checking that two functions are inverses of each other (we just show that $f \circ f^{-1}(y) = y$ and $f^{-1} \circ f(x) = x$).
- Using the Inverse Function Theorem.

§6.3 The Natural Exponential Function

- The definition of the natural exponential function, including its derivative.
- Computing derivatives of the form $D_x(e^u)$ and integrals of the form $\int e^u \, du$.

§6.4 General Exponential and Logarithmic Functions

- Derivatives and integrals involving general exponential functions (i.e., a^x for arbitrary a) and general logarithms ($\log_a x$).
- Differentiating (or integrating) using the definition of a^x (e.g.,$D_x(x^x) = D_x(e^{x \ln x})$).

§6.5 Exponential Growth and Decay

- Solving word problems involving exponential growth/decay.
- Know that $\lim_{h \to 0}(1+h)^{1/h} = e$.
- Solving separable differential equations by integration.

§6.6 First Order Linear Differential Equations

- Solving linear first-order differential equations using the integrating factor technique (*I guarantee you will be asked to do this on the exam*).
- Finding the general solution to such a differential equation.
- Finding a specific solution using given initial conditions.

§6.7 Approximations for Differential Equations

- Sketch a specific solution to a differential equation when given the slope field and an initial condition.
- Use Euler's Method to approximate a solution to a differential equation.

§6.8 Inverse Trig Functions and Their Derivatives

- Deriving the identities from Theorem A (these are the ones that look like $\sin(\cos^{-1} x) = \sqrt{1-x^2}$).
- The derivatives of the six standard trig functions.
- Integrals involving inverse trig functions (e.g., $\int \frac{3}{\sqrt{5-9x^2}} dx$).
- You will be given the derivatives of the inverse trig functions (see formula sheet).

§6.9 The Hyperbolic Functions and Their Inverses

- The definitions of the hyperbolic functions.
- The derivatives of the hyperbolic functions.
- Integrals involving inverse hyperbolic functions (e.g., $\int \frac{dx}{\sqrt{x^2+1}}$). There are multiple ways to do this integral. If you do a trig substitution (as in §7.4) you will get the algebraic expression for $\sinh^{-1} x$.
- You will be given the derivatives of the inverse trig functions.

§7.1 Basic Integration Rules

- You should be able to integrate anything resembling 1-12, or 16,17 on p$\dot{3}$84 in the text.
- You will be given 13-15 on the formula sheet.
- You should be (very) comfortable with u-substitution.

§7.2 Integration By Parts

- Using integration by parts in definite and indefinite integrals.
- Recognizing when it is appropriate to try integration by parts.
- Repeated integration by parts.

§7.3 Some Trigonometric Integrals

- Integrals like $\int \sin^n x \, dx$.
- Integrals like $\int \sin^n x \cos^m x \, dx$.
- Integrals like $\int \sin(mx) \cos(nx) \, dx$.
- Integrals like $\int \tan^n x \, dx$.
- You will be given the half-angle formulas and the product identities.

§7.4 Rationalizing Substitutions

- Rationalizing substitutions for integrands involving $\sqrt[n]{ax+b}$.
- Trig substitutions for integrands involving $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, or $\sqrt{a^2+x^2}$.

§7.5 Partial Fraction Decompotisions

- Integrating rational functions using partial fractions.
- Distinct or repeated linear factors.
- Distinct or repeated quadratic factors.
- The logistic differential equation will NOT be covered.

§7.6 Strategies for Integration

- Determining which technique(s) you should use to evaluate an integral.

§8.1 Indeterminate Forms of Type 0/0

- L'Hôpital's Rule for forms of type 0/0
- Repeated L'Hôpital's Rule for forms of type 0/0

§8.2 Other Indeterminate Forms

- L'Hôpital's Rule for forms of type ∞/∞
- Indeterminate forms of type $0 \cdot \infty$ and $\infty - \infty$
- Indeterminate forms of type 0^0, ∞^0, and 1^∞

§8.3 Improper Integrals: Infinite Limits of Integration

- Integrals of the form $\int_a^\infty f(x)\,dx$
- Integrals of the form $\int_{-\infty}^\infty f(x)\,dx$

§8.4 Improper Integrals: Infinite Integrands

- Integrals where the integrand is infinite at a limit of integration.
- Integrals of the form $\int_a^b f(x)\,dx$ where the $f(x)$ is infinite at some point in (a, b).

§9.1 Infinite Sequences

- The definition of a sequence.
- Writing a general formula when given a list of terms or a recursive formula.
- Writing a recursive formula when given a general formula or a list of terms.
- Writing a list of the first few terms when given a general or recursive formula.
- Computing the limit of a sequence by computing the limit of a function (e.g., Example 3 in §9.1).
- Applying the Squeeze Theorem to show a sequence converges.
- Using the Monotone Sequence Theorem to show a sequence converges.

§9.2 Infinite Series

- Deriving a formula for the n-th partial sum of a series.

- Conditions under which a geometric series converges, and computing the sum of a geometric series.
- Finding the sum of a collapsing series.
- The n-th term test for divergence.
- The harmonic series.

§9.3 Positive Series: The Integral Test

- The Integral Test (make sure the hypotheses are satisfied).
- The p-series test.
- Using the integral test to bound the error on the n-th partial sum.

§9.4 Positive Series: Other Tests

- The Ordinary Comparison Test.
- The Limit Comparison Test.
- The Ratio Test.
- When each test is appropriate to try and what the hypotheses of the tests are.

§9.5 Alternating Series, Absolute Convergence, and Conditional Convergence

- The Alternating Series test (make sure the hypotheses are satisfied).
- The Absolute Convergence Test.
- The Absolute Ratio Test.
- Conditional Convergence.

§9.6 Power Series

- Finding the convergence set (or radius of convergence) of a power series in x or $x - a$.

§9.7 Operations on Power Series

- Integrating and differentiating power series term by term.
- ‘ Adding and subtracting power series term by term.
- Multiplying and dividing power series.

§9.8 Taylor and Maclaurin Series

- Computing a Taylor series based at $x = a$.
- Taylor's remainder formula.
- Note that a Maclauring series is just a special case of a Taylor series (where $a = 0$).

§9.9 Taylor's Approximation to a Function

- Approximating a function using the first few terms of the taylor polynomial based at $x = a$.

- Finding an error for the remainder in such an approximation.

§10.5 The Polar Coordinate System

- Converting points and equations between polar and rectangular coordinates.
- Polar equations for lines, circles, and conics.

§10.6 Graphs of Polar Equations

- Limaçons, cardioids and spirals.

§10.7 Calculus in Polar Coordinates

- Computing areas via integrals in polar coordinates.