

MATH1220: Midterm 1 Practice Problems

The following are practice problems for the first exam.

1. Compute the following derivatives:

$$\begin{aligned} \text{(a)} \quad D_x \left(\ln \sqrt{3x^2 + 2} \right) &= \frac{3x}{3x^2 + 2} \\ \text{(b)} \quad D_x [e^{\sin x}] &= e^{\sin x} \cos x \\ \text{(c)} \quad \frac{d}{dx} [\log_a(2x^2) \cos x] &= \frac{2 \cos x}{x \ln a} - \sin x \log_a 2x^2 \\ \text{(d)} \quad \frac{d}{dx} [4^{3x^2+x+1}] &= 4^{3x^2+x+1} (6x + 1) \ln 4 \end{aligned}$$

2. Compute the following integrals:

$$\begin{aligned} \text{(a)} \quad \int \frac{6x^2 + 16x}{x^3 + 4x^2 - 3} dx &= 2 \ln(x^3 + 4x^2 - 3) + C \\ \text{(b)} \quad \int_0^5 5xe^{x^2} dx &= \frac{5}{2} (e^{25} - 1) \\ \text{(c)} \quad \int 3^x dx &= \frac{3^x}{\ln 3} + C \\ \text{(d)} \quad \int \tan x dx &= -\ln |\cos x| + C \end{aligned}$$

3. Find the inverse of the function $f(x) = \frac{5x-3}{2x-1}$ and verify that it is actually the inverse by showing that $f \circ f^{-1}(y) = y$ and $f^{-1} \circ f(x) = x$. **Answer:** $f^{-1}(x) = \frac{-x+3}{-2x+5}$

4. Show that $f(x) = x^5 + 2x^3 + 4x + \sin(\pi x)$ has an inverse (don't try to find the inverse) and compute $(f^{-1})'(7)$. (*Hint:* You can find an x such that $f(x) = 7$ by inspection) **Answer:** $(f^{-1})'(7) = \frac{1}{15-\pi}$

5. Compute $\frac{d}{dx} [(1+x^2)^{\cos x}] = (1+x^2)^{\cos x} \left[-\sin x \ln(1+x^2) + \frac{2x \cos x}{1+x^2} \right]$

6. A radioactive substance loses 15% of its radioactivity in 2 days. What is its half-life? **Answer:** $\tau_{1/2} = \frac{2 \ln 0.5}{\ln 0.85}$

7. Find the general solution to the following differential equation:

$$\frac{dy}{dx} + \frac{2y}{x+1} = (x+1)^3 \quad \text{Answer:} \quad y = \frac{(x+1)^4}{6} + \frac{C}{(x+1)^2}$$

8. Use Euler's Method with $h = 0.5$ to approximate the solution to

$$y' = 2y - 2x \quad y(0) = 1$$

over the interval $[0, 1]$. **Answer:** $y(1) \approx 3.5$

9. Sketch the solution to $y' = x^2 - y$, whose slope field is shown below, satisfying the initial conditions $y(-2) = 1$.

