MATH1210: PRACTICE FINAL ANSWERS

1. a) $\sum_{i=3}^{8} (i+1)^2 = 4^2+5^2+6^2+7^2+8^2+9^2$

$= 16+25+36+49+64+81 = 271$

b) $\sum_{k=3}^{7} \frac{(-1)^k 2^k}{k+1} = \frac{-8}{4} + \frac{16}{5} - \frac{32}{6} + \frac{64}{7} - \frac{128}{8} = \frac{-1154}{105}$

c) $\sum_{i=1}^{10} (i-1)(4i+3) = \sum_{i=1}^{10} 4i^2 - i - 3 = 4\sum_{i=1}^{10} i^2 - \sum_{i=1}^{10} i - \sum_{i=1}^{10} 3$

$= 4\,\frac{10\cdot 11\cdot 21}{6} - \frac{10(11)}{2} - 30 = 1455$

d) $\sum_{j=1}^{n} (2j-3)^2 = \sum_{j=1}^{n} 4j^2 - 12j + 9 = 4\,\frac{n\cdot(n+1)(2n+1)}{6} - 12\,\frac{n(n+1)}{2} + 9n$

2. a) $\int_0^2 x^2+1\,dx \qquad R_n = \sum_{i=1}^{n} \frac{2}{n} f\left(\frac{2i}{n}\right) = \sum_{i=1}^{n} \frac{2}{n}\left[\left(\frac{2i}{n}\right)^2 + 1\right]$

$= \frac{8}{n^3} \sum_{i=1}^{n} i^2 + \frac{2}{n}\sum_{i=1}^{n} 1 = \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6} + 2$

$= \frac{8}{6}\cdot\frac{2n^2+3n+1}{n^2} + 2. \qquad \lim_{n\to\infty} R_n = \frac{8\cdot 2}{6} + 2 = 2 + \frac{8}{3}$

b) $\int_{-1}^{2} 3x^2+2\,dx. \quad R_n = \sum_{i=1}^{n} \frac{3}{n}\left[3\left(-1+\frac{3i}{n}\right)^2 + 2\right]$

$= \frac{3}{n}\sum_{i=1}^{n} 3\left(1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right) + 2 = \frac{3}{n}\sum_{i=1}^{n}\left(5 - \frac{6i}{n} + \frac{9i^2}{n^2}\right)$

$= \frac{3}{n}\left[5n - \frac{6}{n}\cdot\frac{n(n+1)}{2} + \frac{9}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$

$= 15 - 9 - \frac{9}{n} + \frac{9(2n^2+3n+1)}{2n^2} = 6 - \frac{9}{n} + 9 + \frac{27}{2n} + \frac{9}{2n^2} \to 15$

as $n\to\infty$

3. a) $\frac{d}{dx}\left[\int_1^x 3t^2\,dt\right] = 3x^2$

b) $\frac{d}{dx}\left[\int_{-x}^{x} \sin t\,dt\right] = \frac{d}{dx}\left[\int_{-x}^{0} \sin t\,dt + \int_0^x \sin t\,dt\right]$

$= -\sin(-x) + \sin x = 0.$

c) $\frac{d}{dx}\left[\int_1^{x^2+x} \sqrt{2z+\sin z}\,dz\right] = \sqrt{2(x^2+x)+\sin(x^2+x)}\cdot(2x+1)$

4. a) $\int_1^4 \frac{s^4-8}{s^2}\,ds = \int_1^4 s^2 - \frac{8}{s^2}\,ds = \left[\frac{s^3}{3}+\frac{8}{s}\right]_1^4 = \frac{64}{3}+2-\frac{1}{3}-8 = 15$

b) $\int_1^8 w^{1/3}\,dw = \left[\frac{3}{4}w^{4/3}\right]_1^8 = \frac{3}{4}(15) = \frac{45}{4}$

c) $\int x(\sqrt{3}\,x^2+\pi)^{7/8}\,dx = \frac{1}{2\sqrt{3}}\frac{8}{15}(\sqrt{3}\,x^2+\pi)^{15/8} = \frac{4}{15\sqrt{3}}(\sqrt{3}\,x^2+\pi)^{15/8}$

d) $\int s^2\cos(s^3+5)\,ds = \frac{1}{3}\sin(s^3+5)$

e) $\int_0^{1/2} \sin(2\pi x)\,dx = \left[\frac{-1}{2\pi}\cos(2\pi x)\right]_0^{1/2} = \frac{-1}{2\pi}(-1-1) = \frac{1}{\pi}$

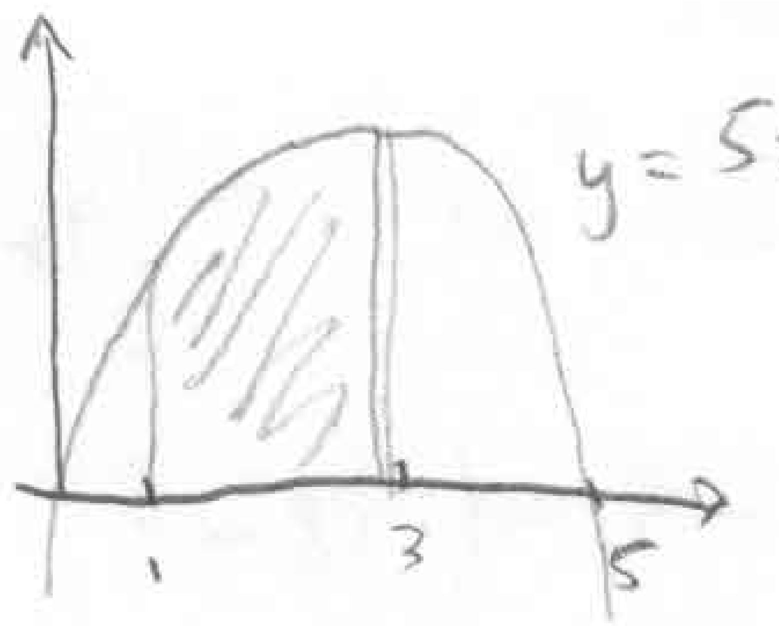
f) $\int_1^4 \frac{(\sqrt{t}-1)^3}{\sqrt{t}}\,dt = \left[\frac{(\sqrt{t}-1)^4}{2}\right]_1^4 = \frac{1}{2}-0 = \frac{1}{2}$

5. $\text{avg.} = \frac{1}{\pi/4}\int_0^{\pi/4} \tan x\sec^2 x\,dx = \frac{4}{\pi}\left[\frac{\tan^2 x}{2}\right]_0^{\pi/4} = \frac{2}{\pi}$

6. $\text{avg.} = \frac{1}{2}\int_0^2 x^3\,dx = 2 = f(c)$ for some c. $c^3 = 2$

$\Rightarrow c = \sqrt[3]{2}.$

7. a)

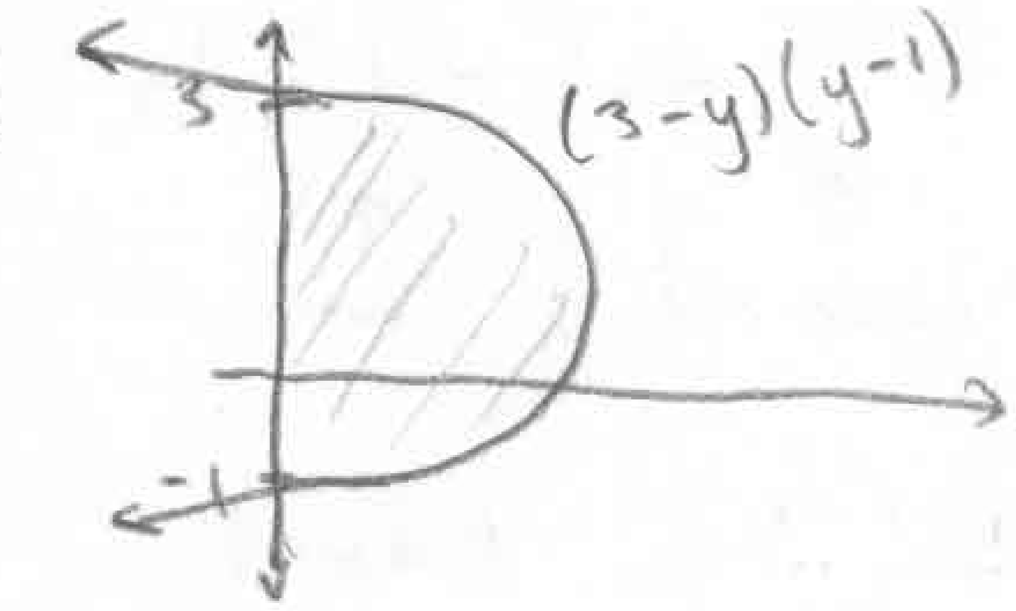


$$\int_1^3 5x - x^2\,dx = \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_1^3$$

$$= \frac{45}{2} - 9 - \frac{5}{2} + \frac{1}{3}$$

$$= \frac{34}{3}$$

b)

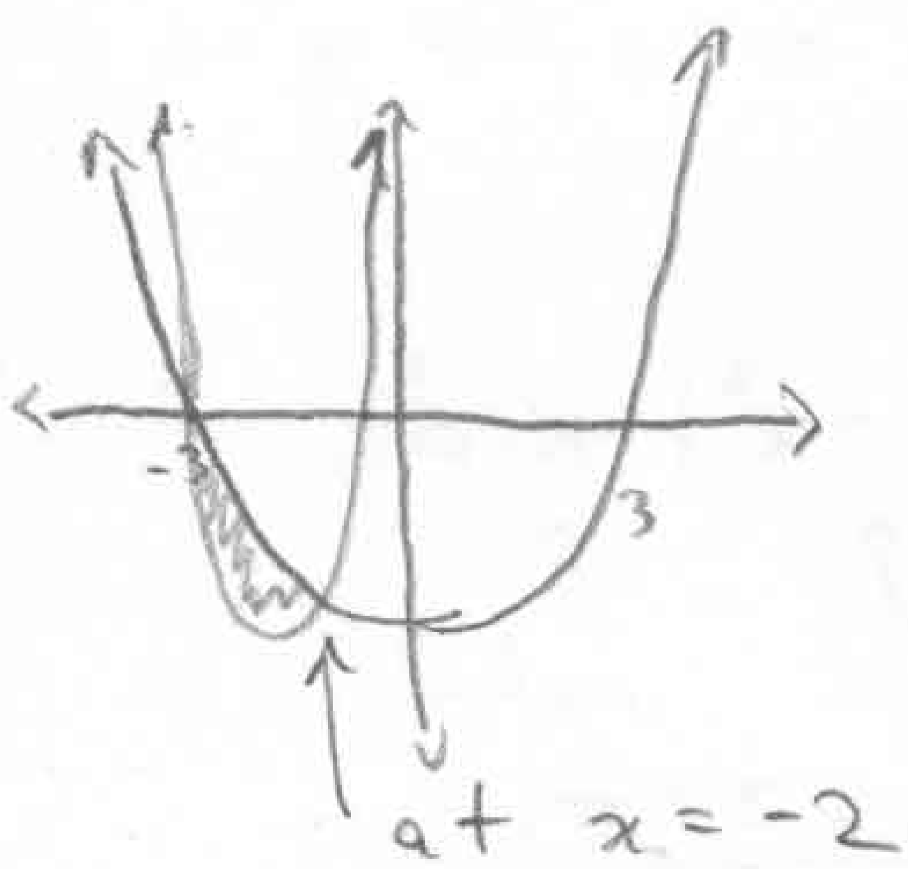


$$\int_{-1}^3 (3-y)(y-1)\,dy = \int_{-1}^3 -y^2 + 4y - 3\,dy$$

$$= \left[-\frac{y^3}{3} + 2y^2 - 3y\right]_{-1}^3 = -9 + 18 - 9 - \left(\frac{1}{3} + 2 + 3\right)$$

$$= -\frac{14}{3}$$

c)



$$x^2 - 9 = 2x^2 + 5x - 3$$

$$0 = x^2 + 5x + 6$$

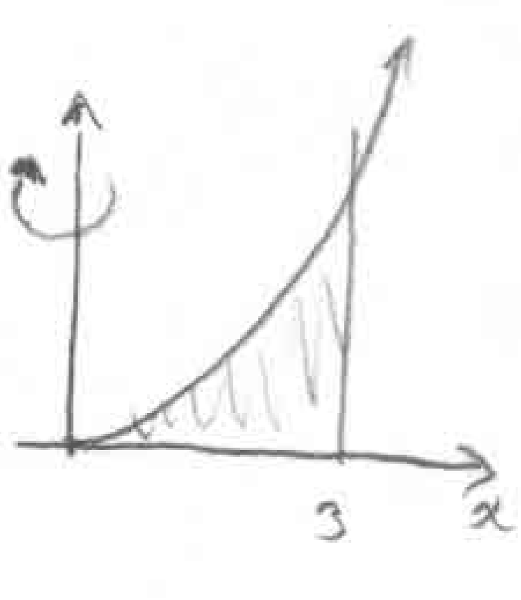
$$= (x+3)(x+2)$$

$$A = \int_{-3}^{-2} x^2 - 9 - (2x^2 + 5x - 3)\,dx$$

$$= \int_{-3}^{-2} -x^2 - 5x - 6\,dx = \left[-\frac{x^3}{3} - \frac{5x^2}{2} - 6x\right]_{-3}^{-2}$$

$$= +\frac{8}{3} - 10 + 12 - \left(9 - \frac{45}{2} + 18\right) = -25 + \frac{8}{3} - \frac{45}{2}$$

8. a)

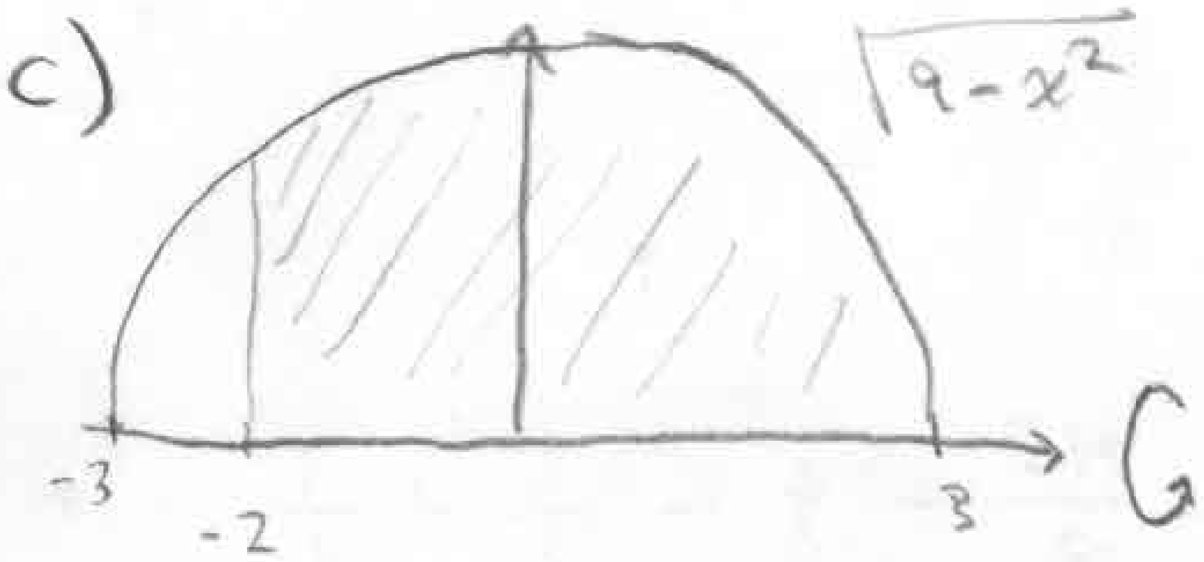


Washer or shell works. Shell:

$$V = 2\pi \int_0^3 \underbrace{x^3}_{\text{height}} \cdot \underbrace{x}_{\text{radius}}\,dx = 2\pi\left[\frac{x^5}{5}\right]_0^3 = \frac{486\pi}{5}$$

b) Washer method (Can use shells too):

$$V = \int_0^3 \pi(\underbrace{x^3}_{r})^2\,dx = \pi\int_0^3 x^6\,dx = \pi\left[\frac{x^7}{7}\right]_0^3 = \boxed{\frac{3^7\pi}{7}}$$

c) 

Shell method won't work.

washers: $V = \pi \int_{-2}^{3} \left(\sqrt{9-x^2}\right)^2 dx = \pi \left[9x - \frac{x^3}{3} \right]_{-2}^{3}$

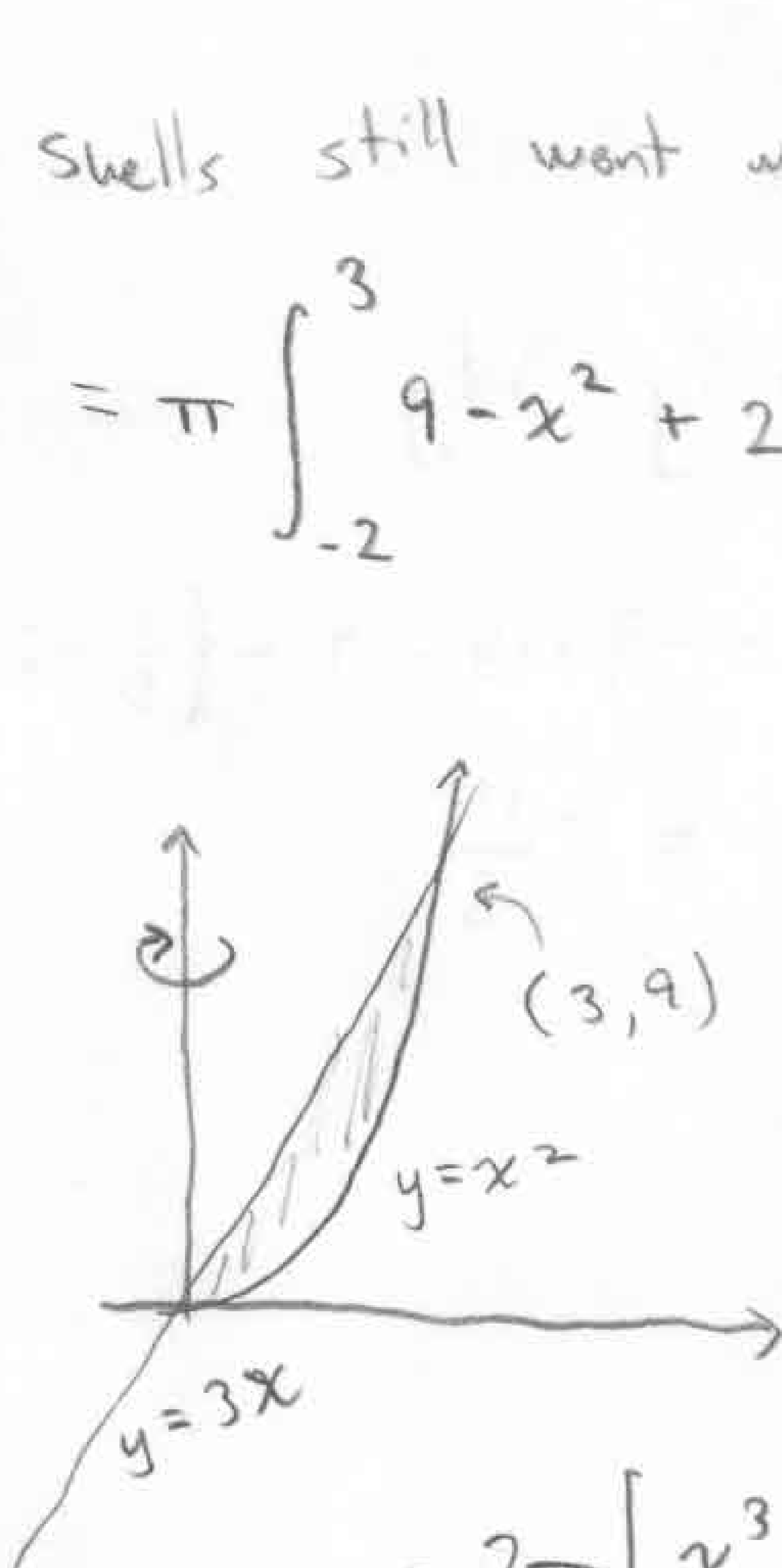
$= \pi \left(27 - 9 - \left(-18 + \frac{8}{3} \right) \right) = \left(36 - \frac{8}{3} \right) \pi$

d) Shells still won't work. Washers: $V = \pi \int_{-2}^{3} \left[\left(\sqrt{9-x^2} + 1 \right)^2 - 1^2 \right] dx$

outer rad. inner rad.

$= \pi \int_{-2}^{3} 9 - x^2 + 2\sqrt{9-x^2} \, dx$

can't integrate this term, so leave it as is.

e) 

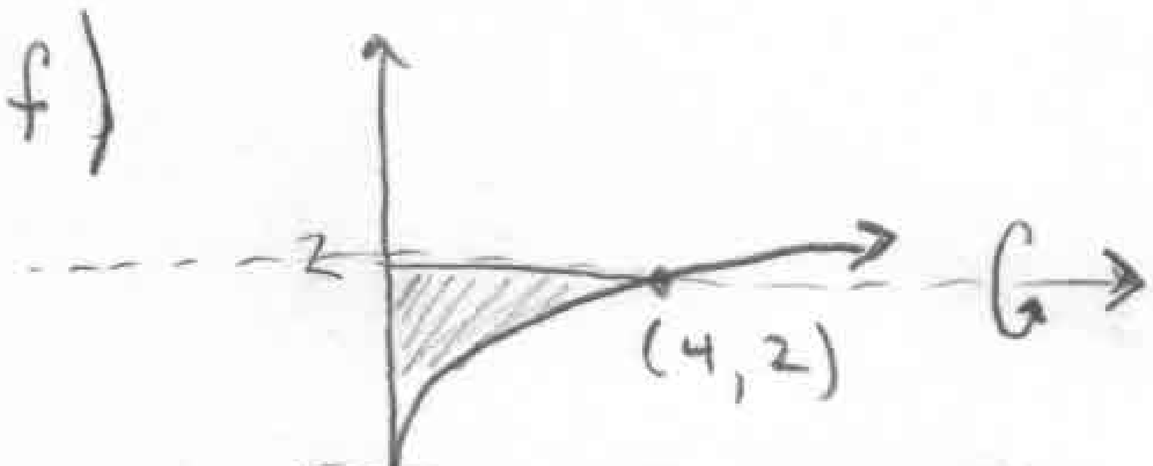
Find intersection $\Rightarrow x^2 = 3x \Rightarrow x = 0, 3$

shell or washers ok.

<u>shells</u>: $V = 2\pi \int_0^3 (3x - x^2) \cdot x \, dx$

height radius

$= 2\pi \left[x^3 - \frac{x^4}{4} \right]_0^3 = 2\pi \left(27 - \frac{81}{4} \right) = \frac{2 \cdot 27 \pi}{4}$

f) 

Shells or washers ok.

<u>washers</u>: $V = \pi \int_0^4 (2 - \sqrt{x})^2 dx = \pi \left[4x + \frac{x^2}{2} - \frac{8}{3} x^{3/2} \right]_0^4$

rad.

$= \pi \left(16 + 8 - \frac{64}{3} \right)$

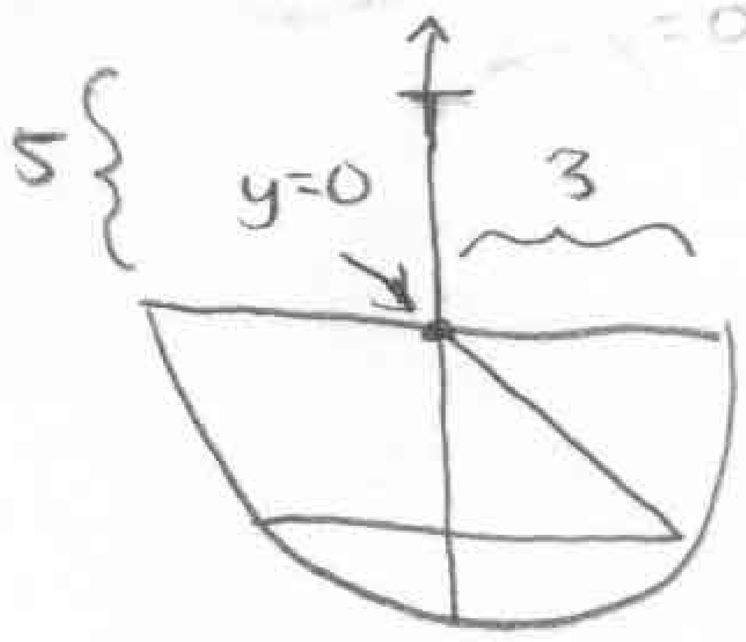
9. a) $L = \int_0^{5\pi} \sqrt{(\cos t - t \sin t)^2 + (2 \sin t + 2t \cos t)^2} \, dt$

b) $L = \int_1^4 \sqrt{(2t)^2 + \left(\frac{1}{2\sqrt{t}} \right)^2} \, dt$

c) $L = \int_0^{\pi/4} \sqrt{1 + \sec^4 x} \, dx$

§5.5 #41 Did this problem in class.

§5.5 #12 (The 18 was a typo.)



$$W = \underset{\text{density}}{\delta} \int_{-3}^{0} \underset{\text{length}}{10} \cdot \underset{\text{width}}{\left(2\sqrt{3^2 - y^2}\right)} \cdot \underset{\text{dist moved}}{(5-y)}\, dy$$