MATH1210: Final Practice Exam

The following are practice problems for the final exam from Chapters 4 and 5. For practice problems from the first three chapters, refer to the previous study guides.

1. Evaluate the following sums (either directly or using the formulas on page 218 of the text.

   (a) $\displaystyle\sum_{i=3}^{8}(i+1)^2$

   (b) $\displaystyle\sum_{k=3}^{7}\frac{(-1)^k 2^k}{(k+1)}$

   (c) $\displaystyle\sum_{i=1}^{10}[(i-1)(4i+3)]$

   (d) $\displaystyle\sum_{j=1}^{n}(2j-3)^2$

   Ignore this question.

2. Evaluate the following integrals using a Riemann sum

   (a) $\displaystyle\int_0^2 (x^2+1)dx$

   (b) $\displaystyle\int_{-2}^{1} (3x^2+2)dx$

   Ignore this question

3. Use the first fundamental theorem of calculus to find

   (a) $\displaystyle\frac{d}{dx}\left[\int_1^x 3t^2 dt\right]$ Answer: $3x^2$

   (b) $\displaystyle\frac{d}{dx}\left[\int_{-x}^x \sin t dt\right]$ Answer: 0 or $\sin(x)+\sin(-x)$

   (c) $\displaystyle\frac{d}{dx}\left[\int_1^{x^2+x} \sqrt{2z+\sin z}dz\right]$ Answer: $\sqrt{2(x^2+x)+\sin(x^2+x)}(2x+1)$

4. Evaluate the following definite and indefinite integrals using any means at your disposal:

   (a) $\displaystyle\int_1^4 \frac{s^4-8}{s^2}ds$ Answer: 15

   (b) $\displaystyle\int_1^8 \sqrt[3]{w}dw$ Answer: 45/4

   (c) $\displaystyle\int x\left(\sqrt{3}x^2+\pi\right)^{7/8}dx$ Answer: $\displaystyle\frac{4\sqrt{3}}{45}(\sqrt{3}x^2+\pi)^{15/8}$

(d) $\int s^2 \cos(s^3+5)ds$Answer: $\frac{\sin(x^3+5)}{3}$

(e) $\int_0^{1/2} \sin(2\pi x)dx$Answer: $\frac{1}{\pi}$

(f) $\int_1^4 \frac{(\sqrt{t}-1)^3}{\sqrt{t}}dt$Answer: $\frac{1}{2}$

5. Find the average value of $g(x) = \tan x \sec^2 x$ on the interval $[0, \pi/4]$.Answer: $\frac{2}{\pi}$

6. Find all values of c that satisfy the mean value theorem for integrals for the function $f(x) = x^3$ on the interval $[0, 2]$.

7. For each part, sketch the region bounded by the given functions and then find its area using a definite integral.Answer: $c = \sqrt[3]{2}$

   (a) $y = 5x - x^2$, $y = 0$, $x = 1$ and $x = 3$.Answer: $\frac{34}{3}$

   (b) $x = (3-y)(y+1)$, $x = 0$.Answer: $\frac{32}{3}$

   (c) $y = x^2 - 9$, $y = (2x-1)(x+3)$Answer: $1/6$

8. For each part, sketch the region R bounded by the given equations. Then find the volume of the solid generated by revolving R about the given axis.

   (a) $y = x^3$, $x = 3$, and $y = 0$ revolved around the y-axis.Answer: $\frac{486\pi}{5}$

   (b) The same region revolved around the x-axis.Answer: $\frac{3^7\pi}{7}$

   (c) $y = \sqrt{9-x^2}$, $y - 0$, between $x = -2$ and $x = 3$ revolved around the x-axis.Answer: $\frac{100\pi}{3}$

   (d) Same region revolved around the line $x = -1$. Answer: Ignore this problem

   (e) $y = x^2$ $y = 3x$ about the y-axis.Answer: $\frac{27\pi}{2}$

   (f) $x = y^2$, $y = 2$, $x = 0$ about the line $y = 2$.Answer: $\frac{8\pi}{3}$

9. Set up (but do not evaluate) an integral to compute the length of the given plane curve.

   (a) $x(t) = t\cos t$, $y(t) = 2t\sin t$, for $t \in [0, 5\pi]$Answer:

$$\int_0^{5\pi} \sqrt{(\cos t - t\sin t)^2 + (2\sin t + 2t\cos t)^2}\, dt$$

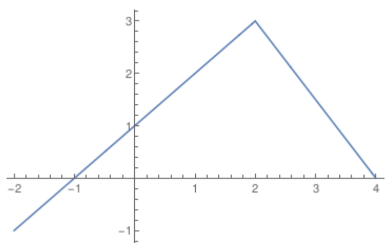
(b) $x(t) = t^2$, $y(t) = \sqrt{t}$ for $t \in [1, 4]$Answer:

$$\int_1^4 \sqrt{4t^2 + \left(\frac{1}{2\sqrt{t}}\right)^2}\, dt$$

(c) $y = \tan x$ for $x \in [0, \pi/4]$Answer:

$$\int_0^{\pi/4} \sqrt{1 + \sec^4 x}\, dx$$

10. §5.5 Exercises 4 and 18Answers to book problems are on the internet
11. §5.6 Exercises 2, 8, and 14Answers to book problems are on the internet
12. The graph of g' is shown here. Answer the following questions.

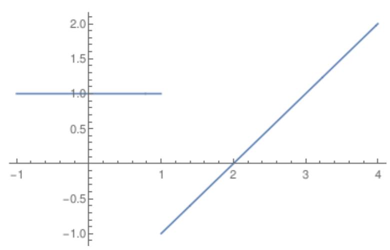


(a) Is g continuous at $x = 2$? Answer: Yes. Differentiability implies continuity

(b) Is g differentiable at $x = 2$? Answer: Yes. $g'(2) = 3$

(c) Is g increasing at $x = 2$? Answer: Yes. $g'(2) = 3$

13. The graph of f' is shown here. Which of the following statements about f must be false:



- f is continuous at $x = 1$.
- $f(1) = 0$.
- f has a vertical asymptote at $x = 1$.
- f has a jump discontinuity at $x = 1$.
- f has a removable discontinuity at $x = 1$.

Answer: f cannot have a vertical asymptote at $x = 1$

14. A differentiable function f has the values shown.

x	2.5	3.2	3.5	4.0	4.6	5.0
$f(x)$	7.6	5.7	4.2	3.1	2.2	1.5

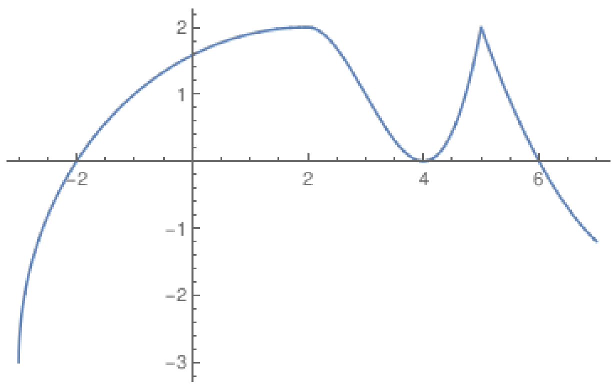
   (a) Estimate $f'(4.0)$ and $f'(4.8)$. Answer: $f'(4.0) \approx -1.818$ and $f'(4.8) \approx -1.75$

   (b) Find the average rate of change of f on the interval $2.5 \le x \le 5.0$. What does the Mean Value Theorem for derivatives tell you? Answer: average rate of change is -2.44. The Mean Value Theorem guarantees that there is some c in $[2.5, 5]$ such that $f'(c) = -2.44$

   (c) Estimate $\int_{2.5}^{5} f(x)\,dx$ with a Riemann sum using left endpoints. Answer: 11.87

15. Given the graph of a function $f(x)$, sketch the graph of $f'(x)$.

16. The figure below shows the graph of f', the derivative of f, with domain $-3 \le x \le 7$. The graph of f' has horizontal tangents at $x = 2$ and $x = 4$ as well as corner at $x = 5$.



   (a) Is f continuous? Why or why not? Answer: Yes. Differentiability implies continuity

   (b) Find all values of x at which f attains a relative minimum. Answer: $x = -2, 7$ by the first derivative test

   (c) Find all values of x at which f attains a relative maximum. Answer: $x = -3, 6$ by the first derivative test

   (d) At what value of x does f attain its absolute maximum? Answer: $x = 6$. We know $f(6) > f(-3)$ because $f(6) - f(3) = \int_{-3}^{6} f'(t)\,dt$ which is positive.

   (e) Find all values of x at which the graph of f has an inflection point. Answer: $x = 2, 5$

17. The graph of a function $y = f(x)$ passes through the point $(2, 5)$ and satisfies the differential equation $\dfrac{dy}{dx} = \dfrac{6x^2 - 4}{y}$.

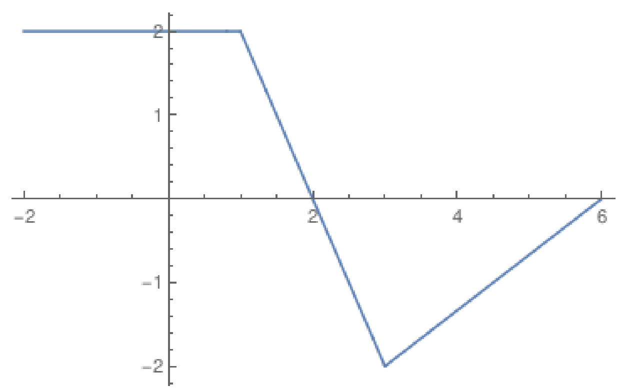
(a) Write an equation of the tangent line to f at $(2, 5)$. Answer: $y - 5 = 4(x - 2)$

(b) Using this tangent line, estimate $f(2.1)$. Answer: $f(2.1) \approx 5.4$

(c) Solve the differential equation, expressing f as a function of x. Answer: $y = \sqrt{4x^3 - 8x + 9}$

(d) Using part (c), find $f(2.1)$. Answer: 5.408

18. The figure below shows the graph of f, whose domain is the interval $[-2, 6]$. Let $F(x) = \int_1^x f(t)\,dt$.



(a) Find $F(-2)$ and $F(6)$. Answer: $F(-2) = -6$ and $F(6) = -3$

(b) For what value(s) of x does $F(x) = 0$? Answer: $x = 1, 3$

(c) For what value(s) of x is F increasing? Answer: $-2 \leq x \leq 2$ because F is increasing when $F' = f$ is positive

(d) Find the maximum value and the minimum value of F. Answer: Maximum value is 1 at $x = 2$. Minimum value is -6 at $x = -2$

(e) At what value(s) of x does the graph of F have points of inflection? Answer: $x = 3$ (maybe 1 also, but F is neither concave up nor concave down when $x < 1$)