MATH1220: Midterm 1 Practice Problems

The following are practice problems for the first exam.

1. Compute the following derivatives:

   (a) $D_x\left(\ln\sqrt{3x^2+2}\right)$

   (b) $D_x\left[e^{\sin x}\right]$

   (c) $\dfrac{d}{dx}\left[\log_a(2x^2)\cos x\right]$

   (d) $\dfrac{d}{dx}\left[4^{3x^2+x+1}\right]$

2. Compute the following integrals:

   (a) $\displaystyle\int \frac{6x^2+16x}{x^3+4x^2-3}dx$

   (b) $\displaystyle\int_0^5 5xe^{x^2}\,dx$

   (c) $\displaystyle\int 3^x\,dx$

   (d) $\displaystyle\int \tan x\,dx$

3. Find the inverse of the function $f(x) = \dfrac{5x-3}{2x-1}$ and verify that it is actually the inverse by showing that $f\circ f^{-1}(y) = y$ and $f^{-1}\circ f(x) = x$.

4. Show that $f(x) = x^5 + 2x^3 + 4x + \sin(\pi x)$ has an inverse (don't try to find the inverse) and compute $(f^{-1})'(7)$. (*Hint:* You can find an x such that $f(x) = 7$ by inspection)

5. Compute $\dfrac{d}{dx}\left[(1+x^2)^{\cos x}\right]$

6. A radioactive substance loses 15% of its radioactivity in 2 days. What is its half-life?

7. Find the general solution to the following differential equation:

$$\frac{dy}{dx} + \frac{2y}{x+1} = (x+1)^3$$

8. Use Euler's Method with $h = 0.5$ to approximate the solution to

$$y' = 2y - 2x \qquad\qquad y(0) = 1$$

   over the interval $[0,1]$.

9. Sketch the solution to $y' = x^2 - y$, whose slope field is shown below, satisfying the initial conditions $y(-2) = 1$.