MATH 1210 : MIDTERM 3 PRACTICE SOLUTIONS

1. $48.7 = 49 - 0.3$

$y = \sqrt{x}$

$dy = \frac{1}{2\sqrt{x}}\,dx$

$$\sqrt{48.7} \approx \sqrt{49} + dy$$

$$= 7 + \frac{1}{2\sqrt{49}}(-0.3)$$

$$= 7 - \frac{.3}{14} \approx 7 - .021 = \boxed{6.979}$$

$$\begin{array}{r} .021 \\ 14\,\overline{)\,.300} \\ \underline{28} \\ 20 \end{array}$$

2. a) $f(x) = \frac{1}{x}$. critical points are $x = 3$.

f has min of $\frac{1}{3}$ at $x = 3$. No max.

b) $x^3 - 3x^2 - x + 3$ on $[-1, 4]$

$g'(x) = 3x^2 - 6x - 1$

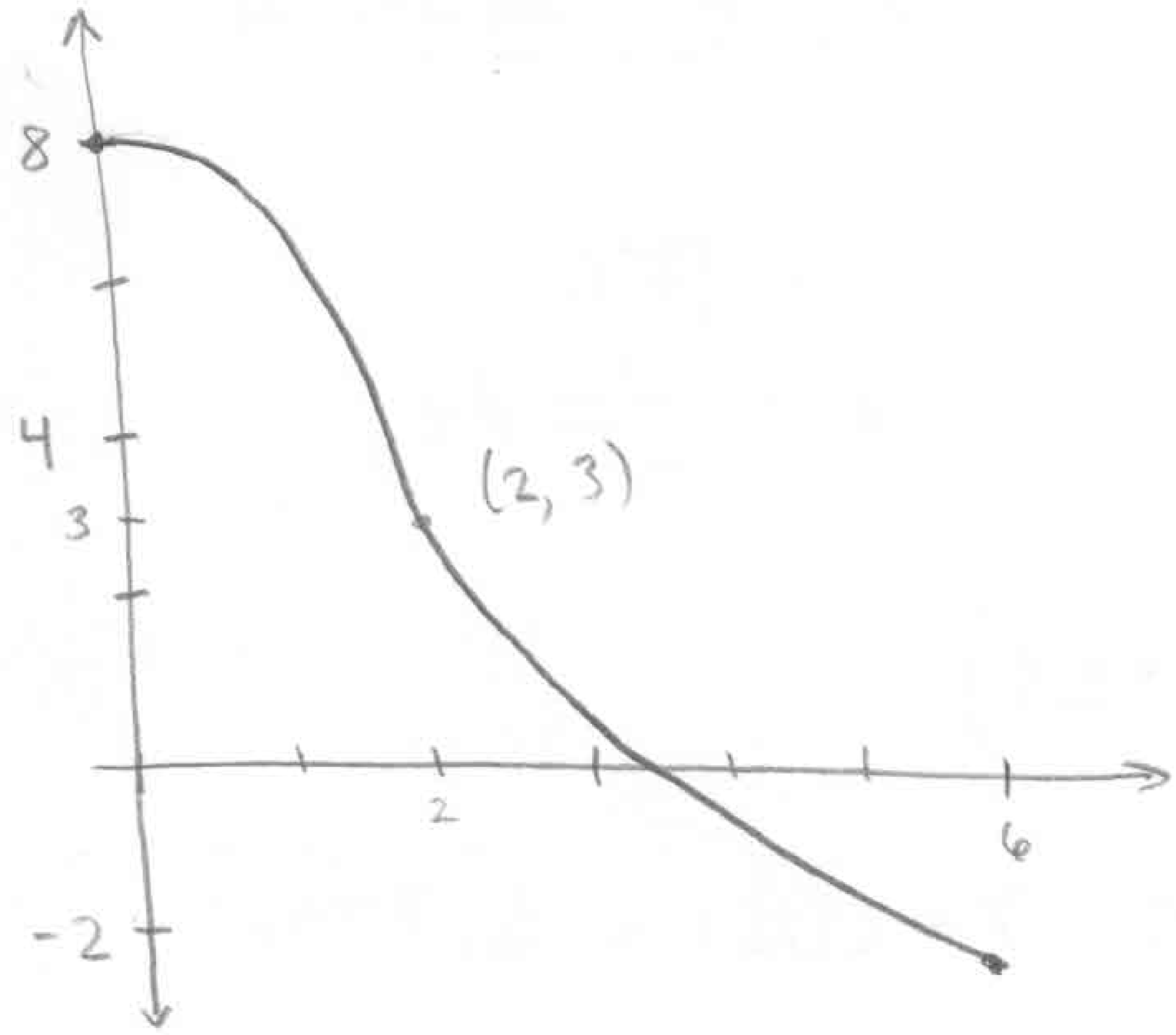
$g'(x) = 0 \iff 3x^2 - 6x - 1 = 0$

$\iff x = \frac{6 \pm \sqrt{36 + 4(3)}}{6} = 1 \pm \frac{\sqrt{48}}{6}$

This is a bad problem & would not be on the test. Try

$g(x) = x^3 - 3x^2 + 3x + 3$ instead.

3.



4. a) critical pts. (end pts., disc. deriv. & deriv. = 0)

$x = -3, -2, -1, 1, 2, 3$

b) inflection pts

$x = -1, 0$

5. $\underline{\S 3.4 \# 12}$ $A = xy$, $2x + 2y - 100 = 180$

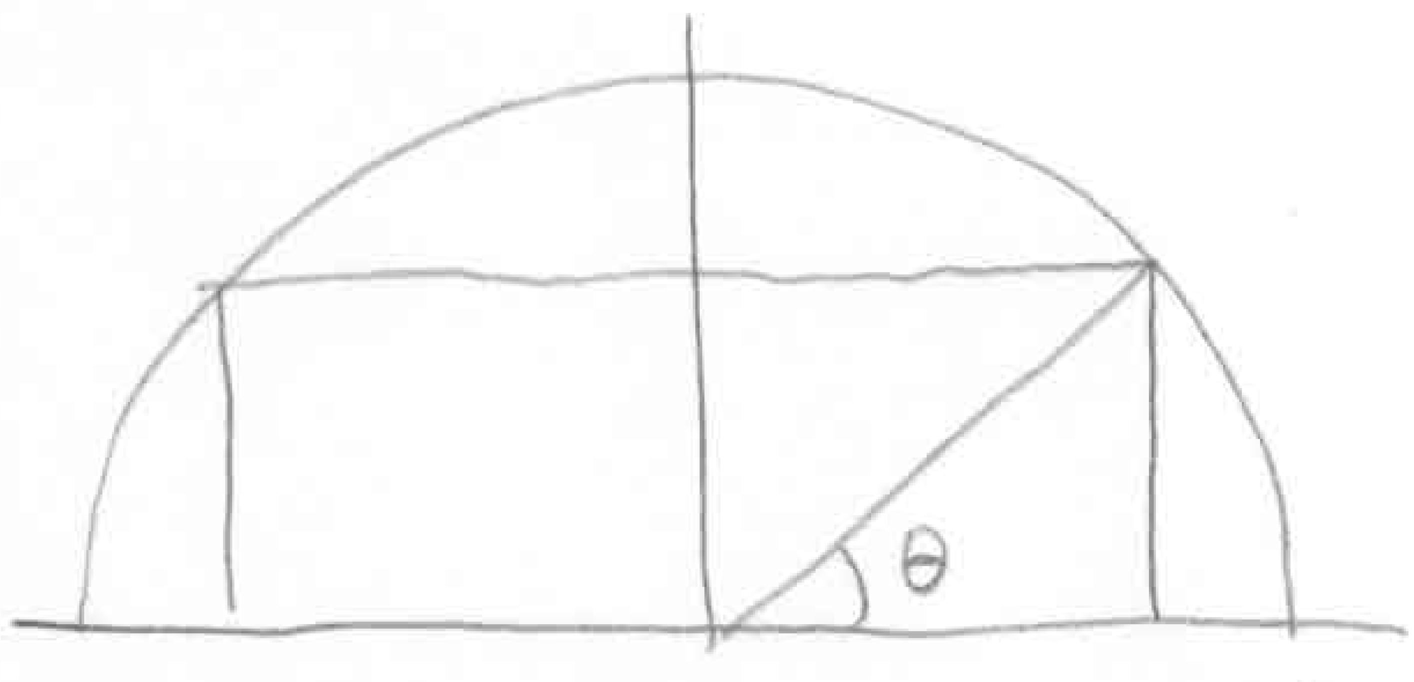
$\Rightarrow y = 140 - x \Rightarrow A = 140x - x^2$

$\frac{dA}{dx} = 140 - 2x$ $\quad \frac{dA}{dx} = 0 \Leftrightarrow x = 70.$ ← This is outside the parameters of the problem, so x should be $\boxed{40.}$

$\underline{\S 3.4 \# 14}$ $xy = 300$, $P = 4y + 6x$

$\Rightarrow x = \frac{300}{y} \Rightarrow P = 4y + \frac{1800}{y}$, $\frac{dP}{dy} = 0 \Leftrightarrow y = \sqrt{450}$

§3.4 #36 Can write area as a fn of θ, where θ is



$$h = r\sin\theta$$
$$w = 2r\cos\theta \Rightarrow A = 2r^2\sin\theta\cos\theta$$

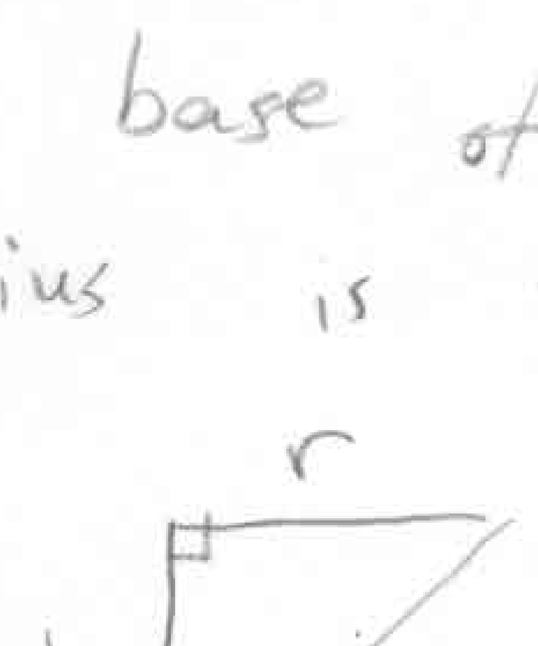
$$\frac{dA}{d\theta} = 0 \Leftrightarrow 4r\sin\theta\cos\theta + 2r^2(\cos^2\theta - \sin^2\theta) = 0$$

$$\Leftrightarrow 2\sin\theta\cos\theta + r(1-2\sin^2\theta) = 0$$
$$2y\sqrt{1-y^2} + r - 2ry^2 = 0$$

$$2y\sqrt{1-y^2} = 2ry^2 - r$$
$$4y^2(1-y^2) = 4r^2y^4 - 4r^2y^2 + r^2$$
$$0 = (4r^2+4)y^4 + (-4r^2-4)y^2 + r^2$$

now use quadratic formula to get y^2, then take square root.

§3.4 #42 cutout section reduces perimeter by 10θ, so the base of the cone has perimeter $(2\pi-\theta)10$. The radius is therefore $r = \frac{(2\pi-\theta)10}{2\pi}$. For the height,



$h^2 + r^2 = 10^2$, so $V = \frac{1}{3}\pi\cdot 100\left(1-\frac{\theta}{2\pi}\right)^2 h$

$$\frac{dV}{d\theta} = \frac{100\pi}{3}\cdot 2\left(1-\frac{\theta}{2\pi}\right)\left(\frac{-1}{2\pi}\right)\frac{dh}{d\theta}$$

$$2h\cdot\frac{dh}{dr} + 2r = 0, \quad \frac{dh}{dr} = \frac{-r}{h}, \quad \frac{dh}{d\theta} = \frac{dh}{dr}\cdot\frac{dr}{d\theta}$$

$$\frac{dV}{d\theta} = \frac{-100}{3}\left(1-\frac{\theta}{2\pi}\right)\left(\frac{10\left(1-\frac{\theta}{2\pi}\right)}{\sqrt{100-10^2\left(1-\frac{\theta}{2\pi}\right)^2}}\right)\left(\frac{-10}{2\pi}\right)$$

$$= \frac{10000}{6\pi}\left(1-\frac{\theta}{2\pi}\right)^2\frac{1}{10\sqrt{1-\left(1-\frac{\theta}{2\pi}\right)^2}} = 0$$

Then solve for θ

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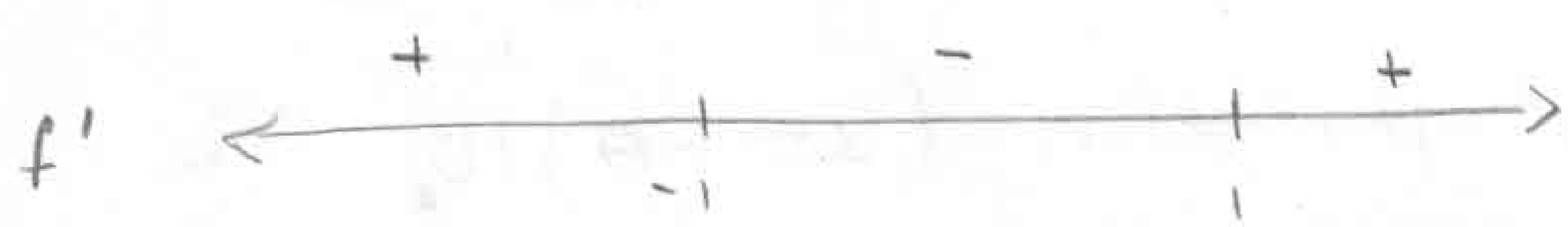
5. a) $f(x) = \frac{x^2+1}{x}$, no y-int. $f(x)=0 \iff x^2+1=0$ $\emptyset$

no x-int.

b) $f'(x) = \frac{2x^2 - x^2 - 1}{x^2} = 0 \iff x^2 - 1 = 0 \iff x = \pm 1$.

Also $x=0$ since deriv. is disc. there

c) Denom of f' always >0, so we want where $x^2-1 \lessgtr 0$



d) $f''(x) = \frac{2x^3 - 2x(x^2-1)}{x^4} = \frac{2x}{x^4}$

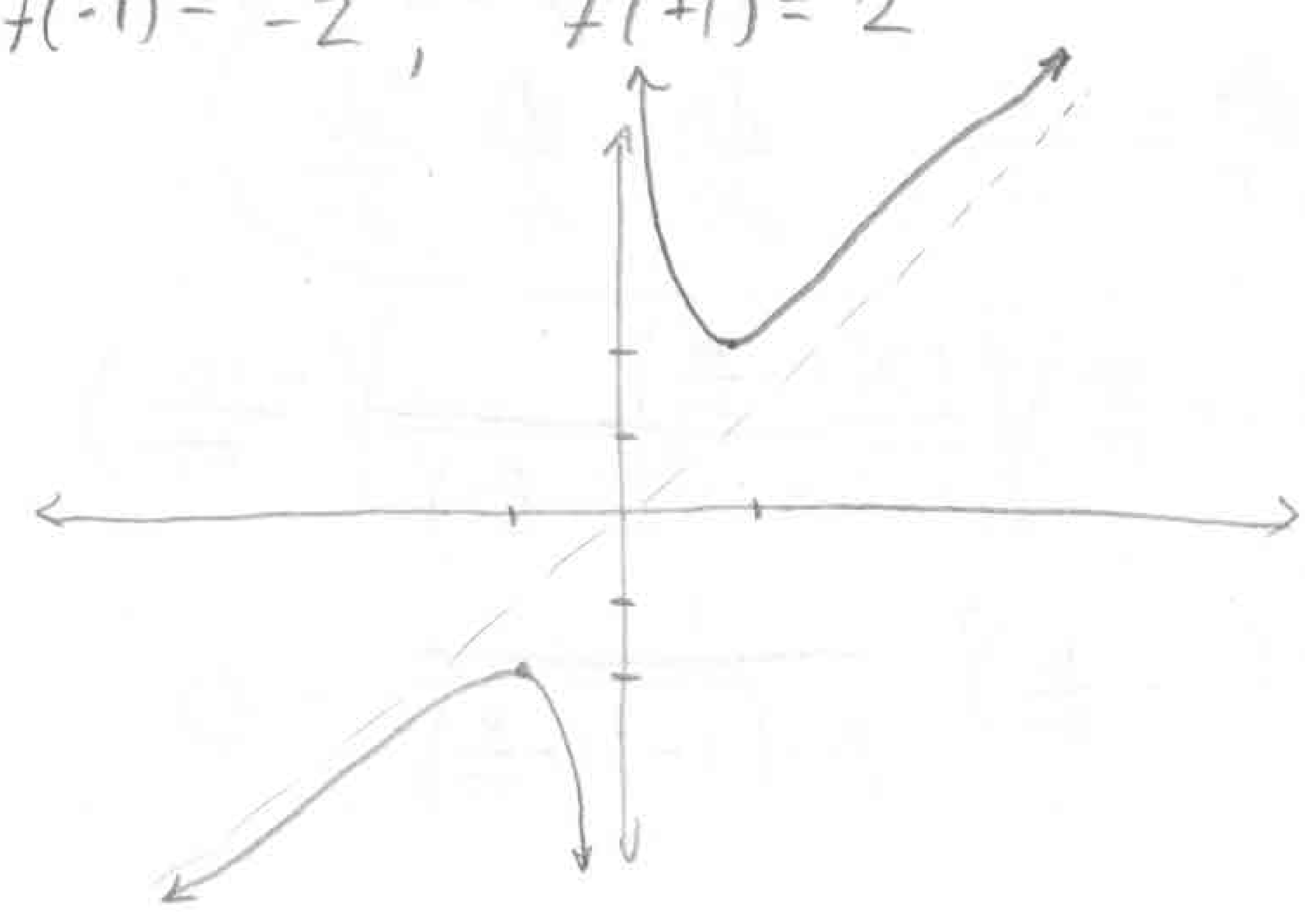
f'' changes sign at $x=0$.

e)

$$x \overline{)x^2+1}$$ quotient $\boxed{x + \frac{1}{x}}$

f) $f(-1) = -2$, $f(+1) = 2$

g)



7. $f(s)$ is diff. in $[-3, 1]$ so MVT applies

$\exists\ c$ st. $f'(c) = \frac{f(1) - f(-3)}{1 - -3} = \frac{3-2}{4} = \frac{1}{4}$

$f'(s) = 2s + 3$

$f'(c) = 2c + 3 = \frac{1}{4} \Rightarrow \boxed{c = \frac{-11}{8}}$

8.

$g(1) = -1$

$g(2) = 2$

$g(1.5) = 2.25 - 2 = 0.25$ so root in $[1, 1.5]$

$\boxed{x = 1.25}$ is within 0.25 of root.

9. a) $\int 3x^2 + \sqrt{3}\, dx = x^3 + \sqrt{3}\, x + c$

b) $\int \frac{s(s+1)^2}{\sqrt{s}}\, ds = \int \sqrt{s}\,(s^2 + 2s + 1)\, ds$

$= \int s^{5/2} + 2s^{3/2} + \sqrt{s}\, ds = \boxed{\frac{2}{7} s^{7/2} + \frac{4}{5} s^{5/2} + \frac{2}{3} s^{3/2} + C.}$