MATH2270: Midterm 1 Practice Problems

The following are practice problems for the first exam.

1. For what values of h and k is the following system consistent?

$$\begin{aligned} 2x_1 - x_2 &= h \\ -6x_1 + 3x_2 &= k \end{aligned}$$

   Answer: $k = -3h$, $h \in \mathbb{R}$

2. Give a parametric description of the solutions to the equation $A\vec{x} = \vec{0}$ where A is the matrix shown below:

$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   Answer:$\{x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -29 \\ 0 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix} \mid x_2, x_3, x_5 \in \mathbb{R}\}$

3. Determine if the vector $\vec{b} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ is in Span$\{v_1, v_2\}$ where

$$\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \qquad \vec{v_2} = \begin{bmatrix} 5 \\ 9 \\ 7 \end{bmatrix}$$

   If the answer is yes, then write $\vec{b}$ as a linear combination of $\vec{v_1}$ and $\vec{v_2}$. Answer: Yes. $\vec{b} = -3\vec{v_1} + \vec{v_2}$

4. If $\vec{b}$ is in the span of the vectors $\vec{v_1}, \dots, \vec{v_k}$, what can you say about solutions to the matrix equation $A\vec{x} = \vec{b}$ where A is the matrix whose columns are $\vec{v_1}, \dots, \vec{v_k}$ (i.e., $A = [\vec{v_1}\ \vec{v_2}\ \dots\ \vec{v_k}]$)? Answer: The equation $A\vec{x} = \vec{b}$ is consistent.

5. Is the set of vectors $\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\}$ linearly independent? Why or why not? Answer: No

6. Is the set of vectors $\{\text{Span}\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\}\}$ linearly independent? Why or why not? Answer: No

7. Determine if the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$, whose standard matrix is A, is 1-1. Is it onto?

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

   Answer: Yes

8. Suppose $S\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation such that $S\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ and $S\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$.

   (a) Find $S\left(\begin{bmatrix} -3 \\ 3 \end{bmatrix}\right)$. Answer: $\begin{bmatrix} -12 \\ 12 \\ 9 \end{bmatrix}$

(b) Find the standard matrix for S. Answer: $\begin{bmatrix} 3 & -1 \\ -2 & 2 \\ -1 & 2 \end{bmatrix}$

9. Be able to multiply matrices...

10. Write down the inverse of the following matrix:

$$\begin{bmatrix} 3 & 2 \\ 8 & 5 \end{bmatrix}$$

Answer: $\begin{bmatrix} -5 & 2 \\ 8 & -3 \end{bmatrix}$

11. Compute the inverse of the following matrix:

$$\begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 3 \end{bmatrix}$$

Answer: $\begin{bmatrix} 3 & 0 & 1 \\ -6 & -1 & -1 \\ -10 & -2 & -1 \end{bmatrix}$. This question is probably more computationally intensive than anything you will see on the exam.

12. Suppose a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^n$ has the property that $T(\vec{u}) = T(\vec{v})$ for some pair of distinct vectors $\vec{u}$ and $\vec{v}$. Can T be onto? Why or why not? No. Something about invertible matrices.