MATH1210: HOMEWORK SOLUTIONS §1.5

1. $\lim_{x\to\infty} \frac{x}{x-5} = \lim_{x\to\infty} \frac{1}{1-\frac{5}{x}} = \frac{\lim_{x\to\infty} 1}{\lim_{x\to\infty} 1 - \lim_{x\to\infty} \frac{5}{x}} = \frac{1}{1-0} = \boxed{1}$

3. $\lim_{t\to-\infty} \frac{t^2}{7-t^2}\left(\frac{\frac{1}{t^2}}{\frac{1}{t^2}}\right) = \lim_{t\to-\infty} \frac{1}{\frac{7}{t^2}-1} \boxed{= -1}$

5. $\lim_{x\to\infty} \frac{x^2}{(x-5)(3-x)} = \lim_{x\to\infty} \frac{x^2}{-x^2+8x-15} = \lim_{x\to\infty} \frac{1}{-1+\frac{8}{x}-\frac{15}{x^2}}$

$= \frac{1}{-1+0-0} \boxed{= -1}$

7. $\lim_{x\to\infty} \frac{x^3}{2x^3-100x^2} = \lim_{x\to\infty} \frac{1}{2-\frac{100}{x}} = \frac{1}{2-0} = \boxed{\frac{1}{2}}$

9. $\lim_{x\to\infty} \frac{3x^3-x^2}{\pi x^3-5x^2} = \lim_{x\to\infty} \frac{3-\frac{1}{x}}{\pi-\frac{5}{x}} = \frac{3-0}{\pi-0} = \boxed{\frac{3}{\pi}}$

11. $\lim_{x\to\infty} \frac{3\sqrt{x^3}+3x}{\sqrt{2x^3}} = \lim_{x\to\infty} \frac{3x^{3/2}+3x}{\sqrt{2}\,x^{3/2}} \cdot \left(\frac{x^{-3/2}}{x^{-3/2}}\right)$

$= \lim_{x\to\infty} \frac{3+\frac{3}{x^{1/2}}}{\sqrt{2}} = \frac{3+0}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \boxed{\frac{3\sqrt{2}}{4}}$

13. $\lim_{x\to\infty} \sqrt[3]{\frac{1+8x^2}{x^2+4}} = \sqrt[3]{\lim_{x\to\infty} \frac{1+8x^2}{x^2+4}} = \left(\lim_{x\to\infty} \frac{\frac{1}{x^2}+8}{1+\frac{4}{x^2}}\right)^{1/3}$

$= \left(\frac{8}{1}\right)^{1/3} \boxed{= 2}$

15. $\lim_{n\to\infty} \frac{n}{2n+1} = \lim_{n\to\infty} \frac{1}{2+\frac{1}{n}} = \frac{1}{2+0} = \boxed{1/2}$

17. $\lim_{n\to\infty} \frac{n^2}{n+1} = \lim_{n\to\infty} \frac{1}{\frac{1}{n}+\frac{1}{n^2}} = \boxed{\infty}$

19. $\lim_{n\to\infty} \frac{2x+1}{\sqrt{x^2+3}} = \lim_{n\to\infty} \frac{2+\frac{1}{x}}{\frac{1}{x}\sqrt{x^2+3}} = \lim_{n\to\infty} \frac{2+\frac{1}{x}}{\sqrt{1+\frac{3}{x^2}}}$

$= \frac{2+0}{\sqrt{1+0}} = \boxed{2}$

21. $\lim_{x\to\infty} \left(\sqrt{2x^2+3} - \sqrt{2x^2-5}\right)\left(\frac{\sqrt{2x^2+3}+\sqrt{2x^2-5}}{\sqrt{2x^2+3}+\sqrt{2x^2-5}}\right)$

$= \lim_{x\to\infty} \frac{2x^2+3-2x^2+5}{\sqrt{2x^2+3}+\sqrt{2x^2-5}}$

$= \lim_{x\to\infty} \frac{8}{\sqrt{2x^2+3}+\sqrt{2x^2-5}} = 0$

since the denominator can be made arbitrarily large by choosing x large enough and the numerator is a constant.

23. $\lim_{y\to-\infty} \frac{9y^3+1}{y^2-2y+2} = \lim_{y\to-\infty} \frac{9y+\frac{1}{y^2}}{1-\frac{2}{y}+\frac{2}{y^2}}$

$= \frac{\left(\lim_{y\to-\infty} 9y\right)+0}{1-0+0} = \lim_{y\to-\infty} 9y = \boxed{-\infty}$

25. $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\frac{1}{n}\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}}$

$= \frac{1}{\sqrt{1+0}} = \boxed{1}$

27. $\lim_{x \to 4^+} \frac{x}{x-4}$: If we try substitution, we get $\frac{4}{0}$, so the limit should be $\pm\infty$. To find out which, we test with a value of x slightly larger than 4. Both numerator and denominator will be positive so the limit is $+\infty$.

29. $\lim_{t \to 3^-} \frac{t^2}{9-t^2} = \lim_{t \to 3^-} \frac{t^2}{(3+t)(3-t)}$. Again, substitution gives $\frac{9}{0}$, so the limit is $\pm\infty$. When x is slightly less than 3, we get $\frac{(+)}{(+)(+)}$ which is positive, so the limit is $+\infty$.

31. $\lim_{x \to 5^-} \frac{x^2}{(x-5)(3-x)}$. Substitution gives $\frac{25}{0}$, so the limit is $\pm\infty$. For x slightly less than 5, $\frac{x^2}{(x-5)(3-x)}$ is $\frac{(+)}{(-)(-)}$ which is positive, so the limit is $+\infty$.

33. $\lim_{x \to 3^-} \frac{x^3}{x-3}$. Same as before, we get $\frac{(+)}{(-)}$ so the limit is $-\infty$.

35. $\lim_{x \to 3^-} \frac{x^2-x-6}{x-3} = \lim_{x \to 3^-} \frac{(x+2)(x-3)}{(x-3)} = \lim_{x \to 3^-} x+2 = \boxed{5}$

This problem is here to remind us that if substitution gives $\frac{0}{0}$, we can't expect the limit to be infinite (though it still could be).

37. $\lim_{x\to 0^+} \frac{[\![x]\!]}{x}$. substitution gives $\frac{0}{0}$. When x is slightly larger than 0, substitution gives $\frac{0}{x} = 0$, so the limit is 0.

39. $\lim_{x\to 0^-} \frac{|x|}{x}$. Substitution gives $\frac{0}{0}$. When $x<0$, $|x| = -x$, so $\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} \frac{-x}{x} = \boxed{-1}$

49. $f(x) = \frac{2x^4+3x^3-2x-4}{x^3-1}$. We do polynomial long division:

$$\begin{array}{r}
2x+3 \\
x^3-1 \overline{\big) 2x^4+3x^3 \qquad -2x-4} \\
-(2x^4 \qquad\qquad -2x) \\
\hline
3x^3 \qquad\qquad -4 \\
-(3x^3 \qquad\qquad -3) \\
\hline
-1
\end{array}$$

remainder is -1 or $\frac{-1}{x^3-1}$

so $f(x) = 2x+3+\frac{-1}{x^3-1}$

Then $\lim_{x\to\infty} f(x) - [2x+3] = \lim_{x\to\infty} \frac{-1}{x^3-1}$

$$= \lim_{x\to\infty} \frac{\frac{-1}{x^3}}{1-\frac{1}{x^3}} = \frac{0}{1-0} = 0$$

so $\boxed{y = 2x+3}$ must be the oblique asymptote for $f(x)$.