MATH1210-002

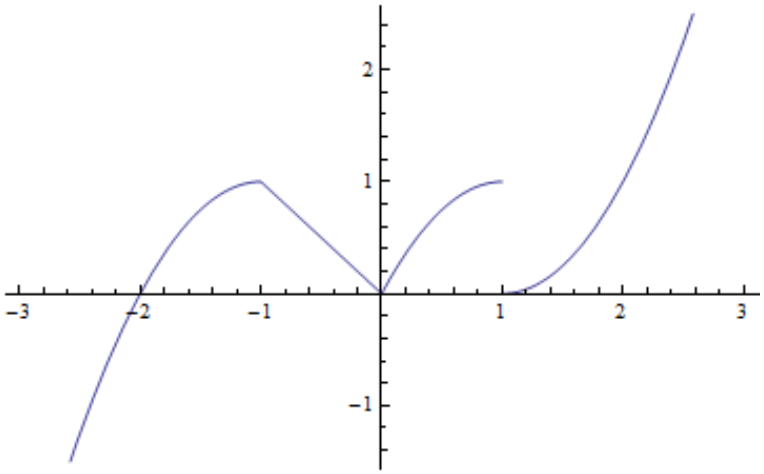
NAME:

Midterm 2

July 8, 2016

Instructions: Do all the problems on **both sides** of each page. Show all your work and box your answers. If you get stuck on a problem, skip it and come back to it at the end.

1. **[6 points]** Let f be the function shown on the graph below. Sketch the graph of the derivative of f in the space below the graph.



2. A ball is thrown upwards from the edge of the roof of a building that is 64 feet tall. The height of the ball *off the ground* as a function of time is $h(t) = -16t^2 + 48t + 64$. Answer the following:

   (a) **[3 points]** When does the ball have zero velocity?

   (b) **[2 points]** What is the acceleration of the ball?

   (c) **[3 points]** When does the ball hit the ground?

3. **[4 points each]** Let f and g be functions and suppose

$$g(0) = 3 \qquad g(3) = 2 \qquad g'(0) = -2 \qquad g'(3) = 1 \qquad f(3) = -2 \qquad f'(3) = 5$$

   (a) $(f \cdot g)'(3) =$

   (b) $(f \circ g)'(0) =$

4. **[10 points]** Find the equation of the tangent line to the function $f(x) = 2\cos x$ at $x = \pi/6$. Then sketch both $f(x)$ and the tangent line you just calculated.

5. **[8 points]** Assume the following equation determines a differentiable function of x. Find dy/dx using implicit differentiation.

$$xy - 12 = 2x^2 + 3y^3$$

6. **[5 points each]** Find the derivative of the following functions. Do not simplify your answers

(a) $g(x) = x^{12} + 5x^{-2} - 8\sqrt{x}$

(b) $f(x) = x^3 \cos x$

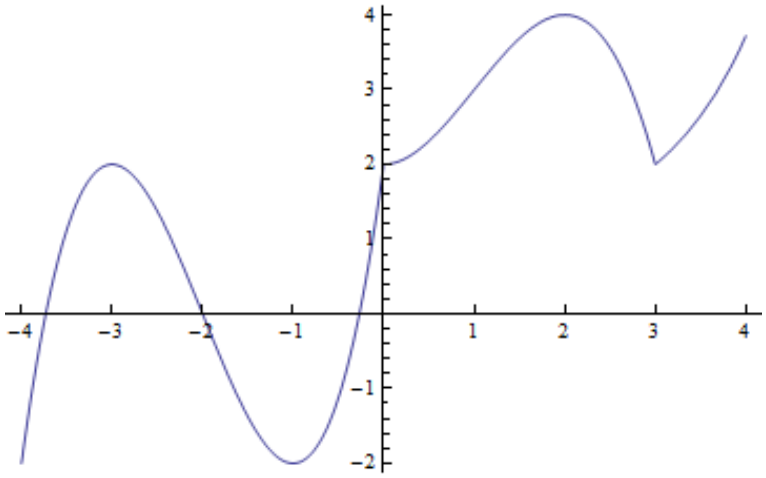
(c) $g(x) = \tan\left(\dfrac{2x + 5\sin x}{3x^2}\right)$

7. **[5 points]** Find $f^{(100)}(x)$ if $f(x) = 23x^{94} - 8x^{61} + x^{37} - 1 + \sin(x)$

8. **[8 points]** Compute the derivative of $f(x) = x^2 + 1$ directly from the definition. *IF YOU DO NOT USE THE LIMIT DEFINITION OF THE DERIVATIVE, YOU WILL NOT RECEIVE CREDIT*

9. [**8 points**] A cone-shaped hour glass has a height of 20 inches and its base has a radius of 10 inches. Sand drains from the hour glass at a constant rate of 10 cubic inches per minute. How fast is the height of the sand changing when the height is 10 inches?

10. Shown here is the graph of a function $f(x)$:



(a) **[5 points]** Identify the critical points of f. Write each point as a pair $(x, f(x))$.

(b) **[3 points]** Identify the inflection points of f. Write each point as a pair $(x, f(x))$.

(c) **[4 points]** On the two lines above, draw sign graphs for f' and f''.

11. **[6 points]** Use differentials to approximate $\sqrt{100.5}$.

12. **[6 points]** Does the function $f(x) = x^3 - 3x$ necessarily have a maximum and minimum on the interval $[-2, 3]$? Explain why or why not. If so, find the maximum and minimum of f on this interval.