MATH2270: Midterm 2 Study Guide

The following is an overview of the material that will be covered on the second exam.

§2.8 Subspaces of $\mathbb{R}^n$ This section is subsumed by material in Ch. 4.

§2.9 Dimension and Rank This section is subsumed by material in Ch. 4.

§3.1 & §3.2 Determinants

- Be able to compute determinants using cofactor expansions. You should choose the row/column to expand in strategically.
- Be able to compute the determinant of a matrix, A, by row reducing A to an upper triangular matrix, and keeping track of how the row operations change the determinant.
- Know the defining properties of the determinant:
  1. The determinant is linear in each row
  2. The determinant is alternating (if you swap two rows, the determinant changes sign).
  3. $\det I_n = 1$.
- Know the other basic properties of the determinant:
  1. $\det(AB) = \det(A)\det(B)$
  2. $\det A^T = \det A$
  3. A is invertible if and only if $\det A \neq 0$.

§3.3 Cramer's Rule, Volume, and Linear Transformations

- Know Cramer's rule for computing the inverse of a matrix.
- Understand how the volume of a region in $\mathbb{R}^n$ changes under the application of a linear transformation $T\colon \mathbb{R}^n \to \mathbb{R}^n$. Specifically, $\mathrm{vol}(T(R)) = \mathrm{vol}(R) \cdot |\det(T)|$.
- Understand how the determinant of a matrix relates to the volume of the parallelepiped determined by the columns of the matrix.

§4.1 Vector Spaces and Subspaces

- You should have some familiarity with abstract vector spaces, including but not limited to: $\mathbb{P}_n$, $\mathbb{P}$, $C(R)$, $C([a,b])$, $C^\infty(R)$, $C^\infty([a,b])$, and the solution set to certain differential equations (for an example, see §4.1 exercise 19).
- You should be comfortable with subspaces of these vector spaces.
- If $H, K \leq V$ (i.e., H and K are subspaces of V), you should know the definitions of the subspaces $H + K$ and $H \cap K$.

§4.2 Null Spaces, Column Spaces and Linear Transformations

- You should know the definition of a linear transformation and be able to check whether a function $f\colon V \to W$ is one.
- Understand linear transformations on arbitrary vector spaces.

- Know the definition of the kernel (or null space) of a linear transformation and be able to do computations involving kernels.
- You should be able to do all these things in more generality than just linear transformations on $\mathbb{R}^n$.

§4.3 Linearly Independent Sets: Bases

- Know what a basis is and be able to find a basis for a subspace (not just of $\mathbb{R}^n$, but of a general vector space).
- Be able to find a basis for the kernel of a linear transformation $T\colon V \to V$.
- This is one of the most important fundamental concepts about a vector space.

§4.4 Coordinate Systems/ §4.7 Change of Coordinates

- Know how a basis for a vector space (or subspace) is related to a coordinate system on the vector space (or subspace).
- Be able to find the coordinates of $v \in V$ given a basis $\mathcal{B}$ for V.
- Given a linear transformation, $T\colon V \to V$, and a basis $\mathcal{B}$ for V, you should be able to compute the matrix for T with respect to the basis $\mathcal{B}$.
- Be able to compute the change of coordinates matrix between two bases, $\mathcal{B}$ and $\mathcal{C}$.
- Given a linear transformation $T\colon V \to V$, and two bases, $\mathcal{B}$ and $\mathcal{C}$ for V, you should be able to compute the matrix for T in the basis $\mathcal{B}$ and the basis $\mathcal{C}$. (This isn't covered in §4.7, but we talked about it in class. The book covers this in §5.4.)

§4.5 The Dimension of a Vector Space/ §4.6 Rank

- Know the definition of the dimension of a vector space (or subspace).
- Know the definition of rank of a matrix.
- Be able to compute a basis for $\operatorname{col} A$, $\ker A$, and $\operatorname{row} A$ for a matrix A.
- Be able to do basic calculations/deductions using dimension (i.e., relating the dimensions the kernel and image of a linear transformation to the dimension of the domain).

§5.1 Eigenvectors and Eigenvalues

- Know the definition of eigenvectors and eigenvalues.
- Be able to determine if a given vector is an eigenvector for a linear transformation.
- Be able to determine if a given number is an eigenvalue for a linear transformation.
- Be able to compute a basis for the eigenspace of a linear transformation associated to a specified eigenvalue.