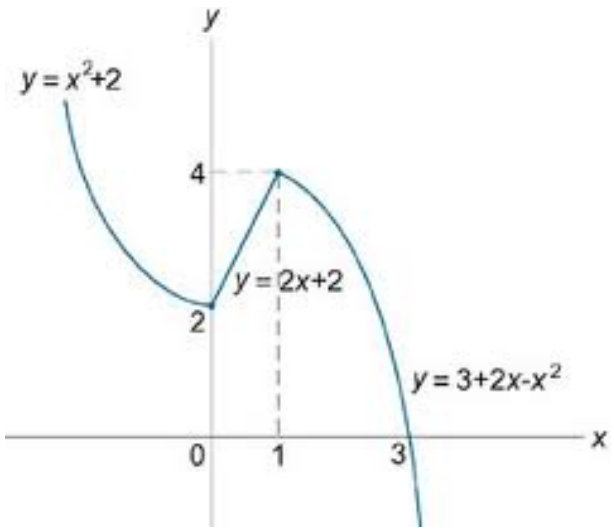
MATH1210: Midterm 2 Practice Exam

The following are practice problems for the second exam. This is not meant to mimic the length of the actual exam.

1. Sketch the graph of the derivative of the following function:



2. Find the equation of the tangent line to $f(x) = x^3 - x$ at $x = 1$. Then sketch the graph of $f(x)$ and the tangent line just calculated. (*Hint:* To sketch the graph, try factoring the polynomial to find its roots)

3. Compute the derivatives of the following functions (at the specified point if asked) directly from the definition:

   (a) $f(x) = 2x^2 + 3x$

   (b) $g(x) = 3x - 4$ at $x = 2$

   (c) $h(x) = \dfrac{x+1}{x-1}$

   (d) $i(x) = \sqrt{2x-1}$

4. Directly from the definition, find the derivative of $f(x) = \cos x$.

5. Suppose $f(3) = 7$, $f'(3) = -1$, $g(3) = 2$, and $g'(3) = 2$. Find $(f \cdot g)'(3)$.

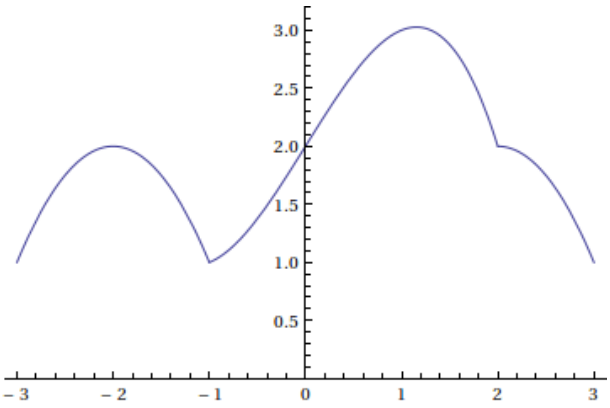
6. Using the rules for trig derivatives, find $\dfrac{df}{dx}$

   (a) $f(x) = \tan^3 x$

   (b) $f(x) = (\sin x)(\cot x)$

   (c) $f(x) = \dfrac{\sin x + \cos x}{\tan x}$

7. Find $f'(x)$ if
   (a) $f(x) = (4 + 2x^2)^7$
   (b) $f(x) = \dfrac{x^2 - 9}{x + 4}$
   (c) $f(x) = (2 - 3x^2)^4(x^7 + 3)^3$
   (d) $\cos^2(\cos(\cos x))$
   (e) $\cos^2(\cos(\cos t))$ (not a typo)
8. Suppose $f'(4) = 2$, $g(0) = 4$, and $g'(0) = 3$. Find $(f \circ g)'(0)$.
9. Find f', f'', and f''' if
   (a) $f(x) = \sin(3x)$
   (b) $f(x) = x^4 + 2x^3 + 3x^2 + 4x + 20$
10. Exercise 31 from §2.6
11. Find $D_x y$ using implicit differentiation.
   (a) $x^2 + 2x^2y + 3xy^3 = 0$
   (b) $x\sqrt{y + 1} = xy + 1$
   (c) $\cos^2(xy) = y^2 + x^2$
12. Exercises 18, 20, and 22 from §2.8
13. Use differentials to approximate $\sqrt{48.7}$.
14. Find the maxima and minima (if they exist) of the following functions on the specified intervals
   (a) $f(x) = \dfrac{1}{x}$ on $(1, 3]$
   (b) $g(x) = x^3 - 3x^2 - x + 3$ on $[-1, 4]$
15. Sketch the graph of a function, f, defined on $[0, 6]$ and satisfying
   - $f(0) = 8$
   - $f(6) = -2$
   - f is decreasing on the interval $(0, 6)$
   - f has an inflection point at the ordered pair $(2, 3)$
   - f is concave up on $(2, 6)$
16. Let $f(x)$ be the function whose graph is shown here:



(a) Identify all the critical points of $f(x)$.

(b) Identify all the inflection points of $f(x)$.