MATH1060: Final Exam Practice Problems

The following are practice problems for the first exam.

1. Convert the following angles to radians:
   (a) $175°$
   (b) $10\pi°$
   (c) $36°$

2. Convert the following angles to degrees:
   (a) $\frac{\pi}{20}$
   (b) 3
   (c) $\frac{11\pi}{12}$

3. Find an angle coterminal with $\frac{107\pi}{7}$ such that $0 \leq \theta < 2\pi$:

4. Find the area of a sector of a circle of radius 7 with central angle $\frac{5\pi}{6}$.

5. Find the length of an arc cut out by a central angle of $\frac{5\pi}{6}$ in a circle of radius 3.

6. Be able to fill out a unit circle without thinking.

7. Take a right triangle with side lengths $7, 24$, and 25, and let θ be the angle opposite the side of length 24. Find:
   (a) $\sec\theta$
   (b) $\cos(90° - \theta)$
   (c) $\sin\theta$
   (d) $\cot(90° - \theta)$

8. You are skiing down a mountain with a vertical height of 1500 feet. The distance from the top of the mountain to the base is 3000 feet. What is the angle of elevation from the base to the top of the mountain.

9. If $\sin\theta = \sqrt{2/3}$, find $\cos\theta$ using one of the Pythagorean identities.

10. Show that $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \csc\theta\sec\theta$.

11. Show that $\sec^2\theta + \csc^2\theta - \tan^2\theta - \cot^2\theta = 2$.

12. Suppose $\cos\theta = \frac{\sqrt{2}}{2}$ and θ is in quadrant IV. Find θ, then find the value of all six standard trig functions at θ.

13. Suppose $\tan\theta = -\sqrt{3}$ and θ is in quadrant II. Find θ, then find the value of all six standard trig functions at θ.

14. Graph $y = 5\sin(2x)$.

15. Graph $y = -\frac{1}{2}\cos(3x+6) - 1$.

16. Sketch the graph of the following functions:

    (a) $y = \csc(x)$

    (b) $y = \arctan(x)$

17. Evaluate the following expressions:

    (a) $\arcsin \frac{\sqrt{3}}{2}$

    (b) $\cos^{-1}(-1)$

    (c) $\sin^{-1} \frac{\sqrt{3}}{2}$

    (d) $\arctan -\sqrt{3}$

    (e) $\arccos \frac{\sqrt{2}}{2}$

    (f) $\tan^{-1}(1)$

18. A motorcycle is moving at 110 miles per hour. The radius of its wheel is 10 inches. What is the angular velocity of the wheel in revolutions per second?

19. The earth makes one complete revolution around the sun every 365 days. The average distance from the earth to the sun is 92.96 million miles. What is the linear speed of the earth hurtling through space?

20. The sun is 25° above the horizon. If Michelle is 5 feet tall, how long is her shadow?

21. An airplane is 150 miles north and 100 miles east of the airport. The pilot wants to fly directly to the airport. What bearing should be taken?

22. Identify each of the following expressions as one of the standard trigonometric functions:

    (a) $\sec\beta\csc\beta - \cot\beta$

    (b) $\dfrac{\cos\gamma}{1-\sin^2\gamma}$

23. Simplify the following expressions:

    (a) $\csc^4\alpha - \cot^4\alpha$

    (b) $-1 + 2\sin^2\delta + \sin^4\delta$

    (c) $\sin^2\zeta + 3\cos\zeta + 3$

    (d) $\sin\epsilon\tan\epsilon + \cos\epsilon$

(e) $\tan\iota - \dfrac{\sec^2\iota}{\tan\iota}$

24. Use the trigonometric substitution $x = 5\tan\lambda$ to simplify the expression $\sqrt{x^2+25}$

25. Verify the following identities:

    (a) $\cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$

    (b) $\dfrac{1}{\tan\gamma} + \dfrac{1}{\cot\gamma} = \tan\gamma + \cot\gamma$

    (c) $\cos^2\theta + \cos^2(\frac{\pi}{2} - \theta) = 1$

    (d) $\tan(\sin^{-1}x) = \dfrac{x}{\sqrt{1-x^2}}$

26. Find all solutions to the following trigonometric equations:

    (a) $4\cos^2\phi - 1 = 0$

    (b) $\csc\nu + \cot\nu = 1.$

    (c) $\tan(3\eta) - 1 = 0$

27. Find the exact value of each of the following expressions:

    (a) $\cos(5\pi/12)$

    (b) $\tan(165°)$

    (c) $\cos(17°)\cos(13°) - \sin(17°)\sin(13°)$

    (d) $\sin\dfrac{\pi}{12}\cos\dfrac{\pi}{4} + \sin\dfrac{\pi}{4}\cos\dfrac{\pi}{12}$

    (e) $\cos 75° + \cos 15°$

28. Find the exact solutions to the following trigonometric equations in the interval $[0, 2\pi)$

    (a) $\cos 2\chi - \cos\chi = 0$

    (b) $4 - 8\sin^2\mu = 0$

    (c) $\sin\dfrac{\beta}{2} + \cos\beta = 1$

29. Use the power reducing formula to rewrite the expression $\sin^2 x\cos^2 x$ in terms of the first power of cosine.

30. Use the sum-to-product or product-to-sum formula to rewrite each expression:

    (a) $\sin 5\gamma\sin 3\gamma$

    (b) $\cos 6\delta + \cos 2\delta$

31. Use the half-angle formula to simplify $\sqrt{\dfrac{1-\cos 14x}{2}}$.

32. If $\sin u = \dfrac{7}{25}$ and u lies in the second quadrant, find $\cos(u/2)$.

33. Use any means you like to solve the triangle with $\alpha = 24.3^\circ$, $\gamma = 54.6^\circ$, and $c = 10.3$.

34. Use any means you like to solve the triangle with $\beta = 63.2^\circ$, $\gamma = 47.6^\circ$ and $b = 12.2$.

35. Use any means you like to solve the triangle with $\beta = 100^\circ$, $b = 14$, and $c = 19$.

36. Use Heron's Formula to find the area of a triangle with side lengths $7, 8$, and 9.

37. Use any means you like to solve the triangle with side lengths $7, 8$, and 9.

38. Write the vector, $\vec{v}$, with initial point $(4, 5)$ and terminal point $(5, -1)$ in standard form.

39. Write the vector from the last question as a linear combination of the standard unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

40. Compute the magnitude of the vector, $\vec{v}$, from the last question.

41. Let $\vec{v} = \langle 2, -7 \rangle$ and $\vec{w} = \langle -2, 2 \rangle$. Write $2\vec{v} + 3\vec{w}$ in standard form.

42. Let $\vec{u} = \langle 3, 7 \rangle$ and $\vec{v} = \langle 1, 0 \rangle$. Find $\vec{u} \cdot \vec{v}$.

43. Find the angle between $\vec{u}$ and $\vec{v}$ from the previous problem.

44. Find $\mathrm{proj}_{\vec{v}}\, \vec{u}$ and $\mathrm{proj}_{\vec{u}}\, \vec{v}$ where $\vec{u}$ and $\vec{v}$ are the same vectors used in the previous problem.

45. Are $3\hat{\imath} + 17\hat{\jmath}$ and $-10\hat{\imath} + 3\hat{\jmath}$ orthogonal.

46. Decompose $7\hat{\imath} - 5\hat{\jmath}$ into two orthogonal vectors, one of which is a scalar multiple of $2\hat{\imath} + 4\hat{\jmath}$.

47. Convert $4 - 2i$ into trig form.

48. Convert $7e^{3\pi i/4}$ into standard form.

49. Let $\zeta_1 = 8e^{3\pi i/4}$ and $\zeta_2 = 2e^{2\pi i/3}$. Find $\zeta_1\zeta_2$.

50. Find $\dfrac{\zeta_1}{\zeta_2}$.

51. Compute ζ_2^3, and write your answer in standard form.

52. Find all the 4-th roots of ζ_1. Write your answers in trigonometric form.

53. Convert the point $(-5, 5)$ into polar form.

54. Convert the point $\left(7, \dfrac{7\pi}{6}\right)$ into rectangular form.

55. Convert the equation $r = 4\sin\theta$ into rectangular form. What shape is the graph of this equation?

56. Convert the equation $y = 7x$ into polar form.