NAME:

Midterm 3 Answers
November 20, 2015

Instructions: Do all the problems on **both sides** of each page. Show all your work and box your answers. If you get stuck on a problem, skip it and come back to it at the end.

1. (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

    (b) $\lambda = 1, 2, 3$

2. $\begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$

3. (a) Yes

    (b) $\begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix}$

    (c) $\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$

    (d) $\left\{ \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$

4. $\vec{x}(t) = c_1 e^{2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

5. (a) $x_k = 1.1^k \begin{bmatrix} 1 \\ 3 \end{bmatrix} + .5^k \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

    (b) Origin is a saddle. Draw a picture to show trajectories.

6. (a) $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

    (b) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

    (c) No. There is no basis of eigenvectors.

7. $\frac{2}{7} \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$

8. No. There is no basis of eigenvectors since the sum of the dimensions of the eigenspaces of A is less than 6.

9. $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ Any matrix with linearly dependent columns will do (as long as no entries are 0).

10. Expanding in the first row, we get

$$(2 - \lambda)\left[(4 - \lambda)(2 - \lambda) + 1\right] + 1\left[(2 - \lambda) + 1\right] - 1\left[-1 + (4 - \lambda)\right]$$