Global alignment VS Local alignment

Group 1 October 9, 2013

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Background

Why alignment?

"Nothing in Biology Makes Sense Except in the Light of Evolution"

——T.Dobzhansky,1973

Why computer arithmetic?

Why dynamic programming?

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α													
J													
C													
J													
N													
R													
C													
K													
С													
R													
В													
P													

(modified from fig.1 of Needleman&Wunsch,1970)

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1	0	0	0	0	0	0	0	0	0	0	0	0
J	0	0	0	0	1	0	0	0	0	0	0	0	0
С	0	0	1	0	0	0	0	1	0	1	0	0	0
J	0	0	0	0	1	0	0	0	0	0	0	0	0
N	0	0	0	1	0	0	0	0	0	0	0	0	0
R	0	0	0	0	0	1	0	0	0	0	1	0	0
С	0	0	1	0	0	0	0	1	0	1	0	0	0
K	0	0	0	0	0	0	0	0	0	0	0	0	0
С	0	0	1	0	0	0	0	1	0	1	0	0	0
R	0	0	0	0	0	1	0	0	0	0	1	0	0
В	0	1	0	0	0	0	0	0	0	0	0	0	0
Р	0	0	0	0	0	0	0	0	0	0	0	1	0

(modified from fig.1 of Needleman&Wunsch,1970)

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1												
J					1								
C			1					1		1			
J					1								
N				1									
R						1					1		
С			1					1		1			
K													
С			1					1		1			
R						1					1		
В		1											
P												1	

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1												
J					1								
С			1					1		1			
J					1								
N				1									
R						1					1		
С			1					1		1			
K													
С			1					1		1			
R						1					1		
В		1											
Р	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1												
J					1								
С			1					1		1			
J					1								
N				1									
R						1					1		
С			1					1		1			
K													
С			1					1		1			
R						1					1		
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1												
J					1								
С			1					1		1			
J					1								
N				1									
R						1	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1												
J					1								
С			1					1		1			
J					1								
N				1									
R						1	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1												
J					1								
С			1					1		1			
J					1								
N				1									
R						1	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	1												
J					1								
С			1					1		1			
J					1								
N				1									
R						5	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	8	7	6	6	5	4	4	3	3	2	1	0	0
J	7	7	6	6	6	4	4	3	3	2	1	0	0
С	6	6	7	6	5	4	4	4	3	3	1	0	0
J	6	6	6	5	6	4	4	3	3	2	1	0	0
N	5	5	5	6	5	4	4	3	3	2	1	0	0
R	4	4	4	4	4	5	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	8	7	6	6	5	4	4	3	3	2	1	0	0
J	7	7	6	6	6	4	4	3	3	2	1	0	0
С	6	6	7	6	5	4	4	4	3	3	1	0	0
J	6	6	6	5	6	4	4	3	3	2	1	0	0
N	5	5	5	6	5	4	4	3	3	2	1	0	0
R	4	4	4	4	4	5	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
A	8	7	6	6	5	4	4	3	3	2	1	0	0
J	7	7	6	6	6	4	4	3	3	2	1	0	0
С	6	6	7	6	5	4	4	4	3	3	1	0	0
J	6	6	6	5	6	4	4	3	3	2	1	0	0
N	5	5	5	6	5	4	4	3	3	2	1	0	0
R	4	4	4	4	4	5	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	8	7	6	6	5	4	4	3	3	2	1	0	0
J	7	7	6	6	6	4	4	3	3	2	1	0	0
С	6	6	7	6	5	4	4	4	3	3	1	0	0
J	6	6	6	5	6	4	4	3	3	2	1	0	0
N	5	5	5	6	5	4	4	3	3	2	1	0	0
R	4	4	4	4	4	5	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

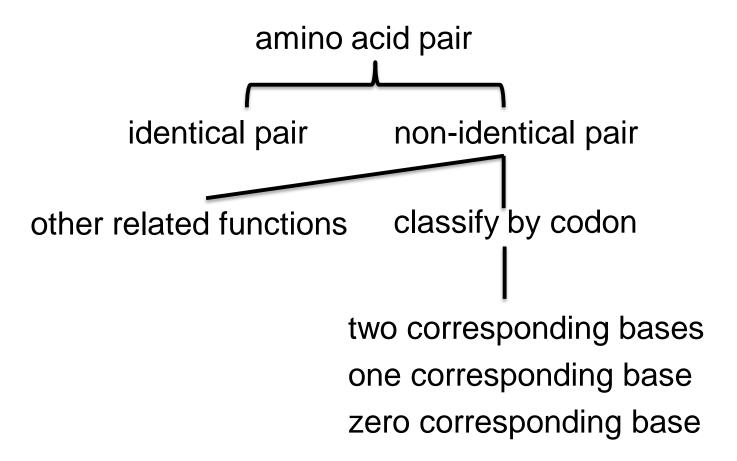
	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	8	7	6	6	5	4	4	3	3	2	1	0	0
J	7	7	6	6	6	4	4	3	3	2	1	0	0
С	6	6	7	6	5	4	4	4	3	3	1	0	0
J	6	6	6	5	6	4	4	3	3	2	1	0	0
N	5	5	5	6	5	4	4	3	3	2	1	0	0
R	4	4	4	4	4	5	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$

Needleman-Wunsch Algorithm

	Α	В	С	N	J	R	0	С	L	С	R	Р	M
Α	8	7	6	6	5	4	4	3	3	2	1	0	0
J	7	7	6	6	6	4	4	3	3	2	1	0	0
С	6	6	7	6	5	4	4	4	3	3	1	0	0
J	6	6	6	5	6	4	4	3	3	2	1	0	0
N	5	5	5	6	5	4	4	3	3	2	1	0	0
R	4	4	4	4	4	5	4	3	3	2	2	0	0
С	3	3	4	3	3	3	3	4	3	3	1	0	0
K	3	3	3	3	3	3	3	3	3	2	1	0	0
С	2	2	3	2	2	2	2	3	2	3	1	0	0
R	2	1	1	1	1	2	1	1	1	1	2	0	0
В	1	2	1	1	1	1	1	1	1	1	1	0	0
P	0	0	0	0	0	0	0	0	0	0	0	1	0

$$F_{ij} = \max_{h < i, k < j} \{ F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j) \}$$



Gap penalty: avoid too many gaps

Needleman-Wunsch Algorithm

$$F_{i0} = d * i$$
 $F_{0j} = d * j$ $F_{ij} = \max(F_{i-1,j-1} + S(A_i,B_j),\; F_{i,j-1} + d,\; F_{i-1,j} + d)$

$$F_{ij} = \max_{h < i, k < j} \{F_{h,j-1} + S(A_i, B_j), F_{i-1,k} + S(A_i, B_j)\}$$

	С	Α	G	С	С	U	С	G	С	U	U	Α	G
Α													
Α													
U													
U G C C													
С													
С													
U													
U													
U U G													
C G													
G													
G													

	С	Α	G	С	С	U	С	G	С	U	U	Α	G
Α	8	8	7	7	7	6	6	5	4	3	2	2	0
Α	7	87-	-7	_7	7	6	6	5	4	3	2	2	0
U	7	7	6	6	6	7	6	5	4	4	3	1	0
G	6	6	7	-6	6	6	5	6	4	3	2	1	1
С	6	5	5	6	6-		8	5	*	3	2	1	0
С	6	4	4	5	3	4	3	4	3//	3	2	1	0
Α	4	5	4	4	4	4	4	34	4	3	2	2	0
U	4	4	4	4	4	4.	3 -	3		4	3	1	0
U	4	4	3	3	3	4	3	2	2			1	0
G	4	3	4	3	3	3	2	3 -	2	2	2	1	1
Α	3	4	3	3	3	3	2	2	1	1	1	2	0
С	3	2	2	3	3	2	3	1	2	1	1	1	4 0
G	1	1	2	1	1	1	1	4	1	I	4.	1	1
G	0	0	1	0	0	0	0	1	0	0	0	0	1

Pairwise Alignment Result

LENGTH	SCORE	IDENTITY	SIMILARITY	GAPS	
19	40.0	8/19 (42.1%)	8/19 (42.1%)	11/19 (57.9%)	a

Pairwise Alignment Result

LENGTH	SCORE	IDENTITY	SIMILARITY	GAPS	h
19	40.0	8/19 (42.1%)	8/19 (42.1%)	11/19 (57.9%)	D

Needleman algorithm (simplified)

Pairwise Alignment Result

LENGTH	SCORE	IDENTITY	SIMILARITY	GAPS	
19	34.0	8/19 (42.1%)	8/19 (42.1%)	11/19 (57.9%)	

Pairwise Alignment Result

LENGTH	SCORE	IDENTITY	SIMILARITY	GAPS	
19	34.0	8/19 (42.1%)	8/19 (42.1%)	11/19 (57.9%)	

Needleman algorithm

Alignment significance evaluation

• Question:

Whether a particular result differs significance from a fortuitous match between two random sequences?

To answer this question we can do:
 Sequence alignment between two sets of random sequences

Or:

Sequence alignment between one set of random sequences and a real sequence

$$H_{k0} = H_{0l} = 0$$
 for $0 \le k \le n$ and $0 \le l \le m$.

$$H_{ij} = \max\{H_{i-1,j-1} + s(\mathbf{a}_i, \mathbf{b}_j), \max_{k \ge 1} \{H_{i-k,j} - W_k\}, \max_{i \ge 1} \{H_{i,j-1} - W_i\}, 0\}.$$
 (1)

 $1 \le i \le n$ and $1 \le j \le m$.

$$W_k = 1.0 + 1/3 * k$$
.

(source: Smith & Waterman, 1981)

	gap	a1	a2	a3	a4
gap	0	1 gap	2 gaps	3 gaps	4 gaps
b1	1 gap	A11	A21	A31	A41
b2	2 gaps	A12	A22	A32	A42
b3	3 gaps	A13	A23	A33	A43

$$H_{k0} = H_{0l} = 0$$
 for $0 \le k \le n$ and $0 \le l \le m$.

$$H_{ij} = \max\{H_{i-1,j-1} + s(a_i,b_j), \max_{k\geq 1} \{H_{i-k,j} - W_k\}, \max_{l\geq i} \{H_{i,j-l} - W_l\}, 0\}.$$
 (1)

 $1 \le i \le n$ and $1 \le j \le m$.

$$W_k = 1.0 + 1/3 * k$$
.

(source: Smith & Waterman, 1981)

	gap	a1	a2	a3	a4
gap	0	1 gap	2 gaps	3 gaps	4 gaps
b1	1 gap	A11	A21	A31	A41
b2	2 gaps	A12	A22	A32	A42
b3	3 gaps	A13	A23	A33	A43

$$H_{k0} = H_{0l} = 0$$
 for $0 \le k \le n$ and $0 \le l \le m$.

$$H_{ij} = \max\{H_{i-1,j-1} + s(\mathbf{a}_i, \mathbf{b}_j), \max_{k \ge 1} \{H_{i-k,j} - W_k\}, \max_{i \ge 1} \{H_{i,j-1} - W_i\}, 0\}. \tag{1}$$

 $1 \le i \le n$ and $1 \le j \le m$.

$$W_k = 1.0 + 1/3 * k$$
.

(source: Smith & Waterman, 1981)

	gap	a1	a2	a3	a4
gap	0	1 gap	2 gaps	3 gaps	4 gaps
b1	1 gap	A11	A21	A31	A41
b2	2 gaps	A12	A22	A32	A42
b3	3 gaps	A13	A23	A33	A43



$$H(i,0) = 0, \ 0 \le i \le m$$

$$H(0,j) = 0, \ 0 \le j \le n$$
if $a_i = b_j$ then $w(a_i,b_j) = w(\text{match})$ or if $a_i \ne b_j$ then $w(a_i,b_j) = w(\text{mismatch})$

$$H(i,j) = \max \begin{cases} 0 \\ H(i-1,j-1) + w(a_i,b_j) & \text{Match/Mismatch} \\ H(i-1,j) + w(a_i,-) & \text{Deletion} \\ H(i,j-1) + w(-,b_j) & \text{Insertion} \end{cases}, \ 1 \le i \le m, 1 \le j \le n$$
(source: http://or

	gap	a1	a2	a3	a4
gap	0	1 gap	2 gaps	3 gaps	4 gaps
b1	1 gap	A11	A21	A31	A41
b2	2 gaps	A12	A22	A32	A42
b3	3 gaps	A13	A23	A33	A43

(source: http://en.wikipedia.org/wiki/Smith-Waterman_algorithm)

O

- Compared with global alignment:
 - Zero could terminate the current local alignment
 - Mismatch must be negative scored
- Other properties:
 - Suitable to identify conserved local sequence (substring)
 - Guaranteed to find the best local alignment
 - Perform poorly when dealing with separated regions within a long sequence

Pairwise Alignment Result

LENGTH	SCORE	IDENTITY	SIMILARITY	GAPS
17	34.333	8/17 (47.1%)	8/17 (47.1%)	9/17 (52.9%)

Pairwise Alignment Result

LENGTH	SCORE	IDENTITY	SIMILARITY	GAPS
17	34.333	8/17 (47.1%)	8/17 (47.1%)	9/17 (52.9%)

Comparision

Match	1
Mismatch	0
Gap	0

	Α	K	С	A	С	K
С	2	2	3	1	1	0
A	1	1	1	2	0	0
С	0	0	1	0	1	0

Needleman-Wunsch (Simplified)

Match	1
Mismatch	-1
Gap	-1

		A	K	С	Α	С	K
	0	0	0	0	0	0	0
С	0	0	0	1	0	1	0
A	0	1	0	0	2	0	0
С	0	0	0	1	0	3	1

Neardielm\a/a-ta/rumach

References

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