Problem Statement

In the following problem we are trying to visualize the corresponding lattice in this application for each n:

Consider the allowed lattice symmetries for n-fold rotational axes, i.e. with a rotation angle ϕ = 2π/n. What are the possible values for n? To do this, consider four lattice points A, B, C, and D in the plane orthogonal to an n-fold rotation axis. Let A and B be nearest neighbor points in the Bravais lattice with distance a, through each of which one of the n-fold rotation axes shall pass. Let rotation about B bring A to C and rotation about A bring B to D. Show that the geometric properties of the lattice allow only n = 1, 2, 3, 4, and 6. Hint: Make a sketch. What conditions must the sides of the resulting trapezoid satisfy if A, B, C, and D are lattice points?

Basic Math stuff

Points

- A: at the origin $(0, 0)$
- B: at $(a, 0)$

Rotation Angles:

- $\varphi = \frac{2\pi}{n}$

Rotations:

- Rotate A around B to get C
- Rotate B around A to get D

Rotation Matrix for rotating P around O by θ:

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

The new point is $P' = O + R \times (P - O)$.