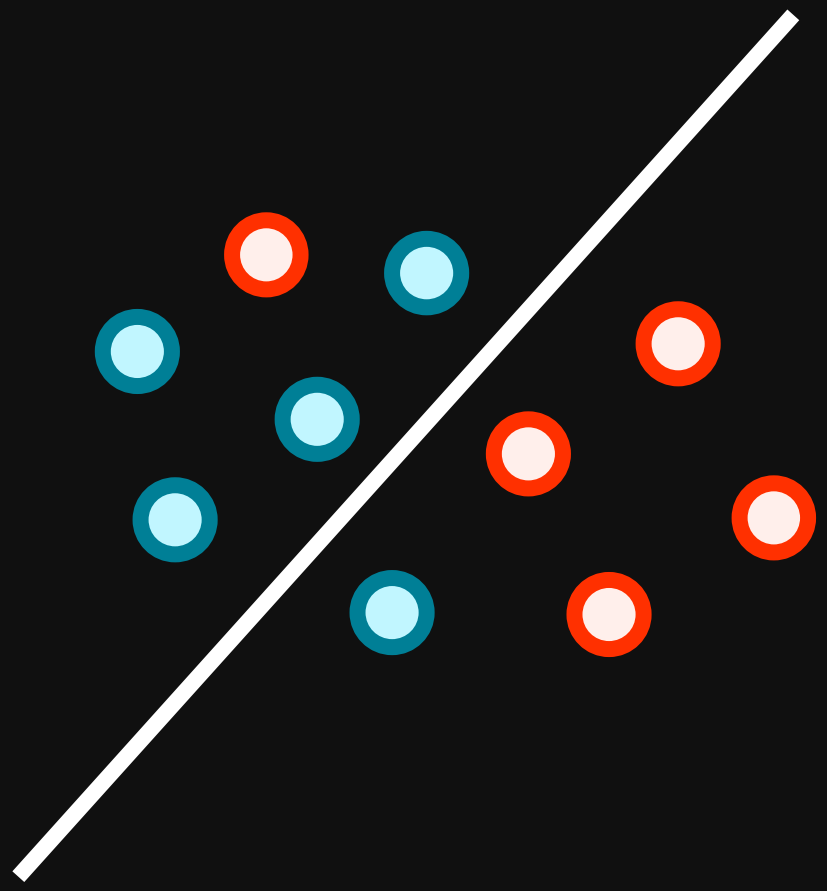
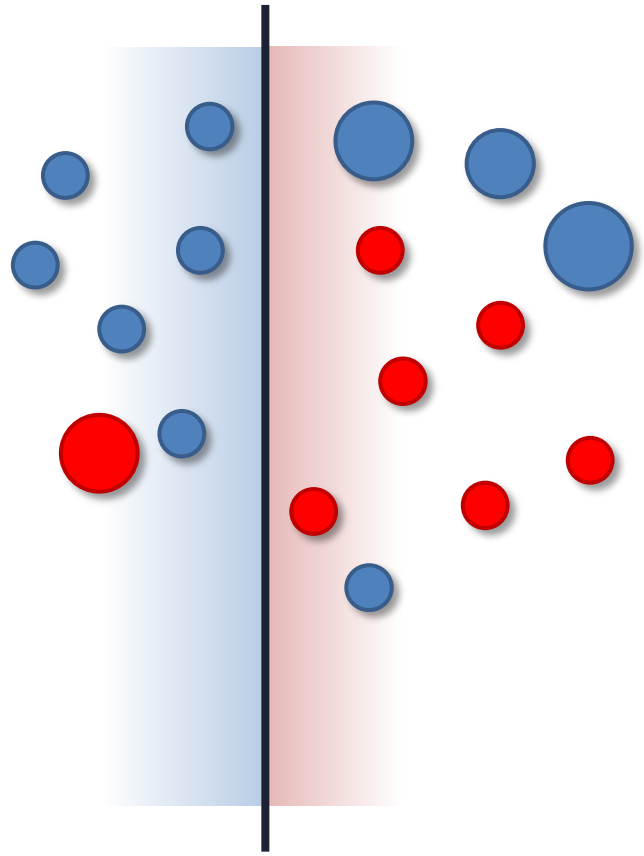
Basics of Machine Learning

Dmitry Ryabokon, github.com/dryabokon



Lesson 16
Adaboost



Adaboost

Definitions

$x_1, x_2, \ x_i, \ \dots \ , x_N$ Training features

$k_1, k_2, \ k_i, \ \dots \ , k_N$ = {-1 ,1} Training targets

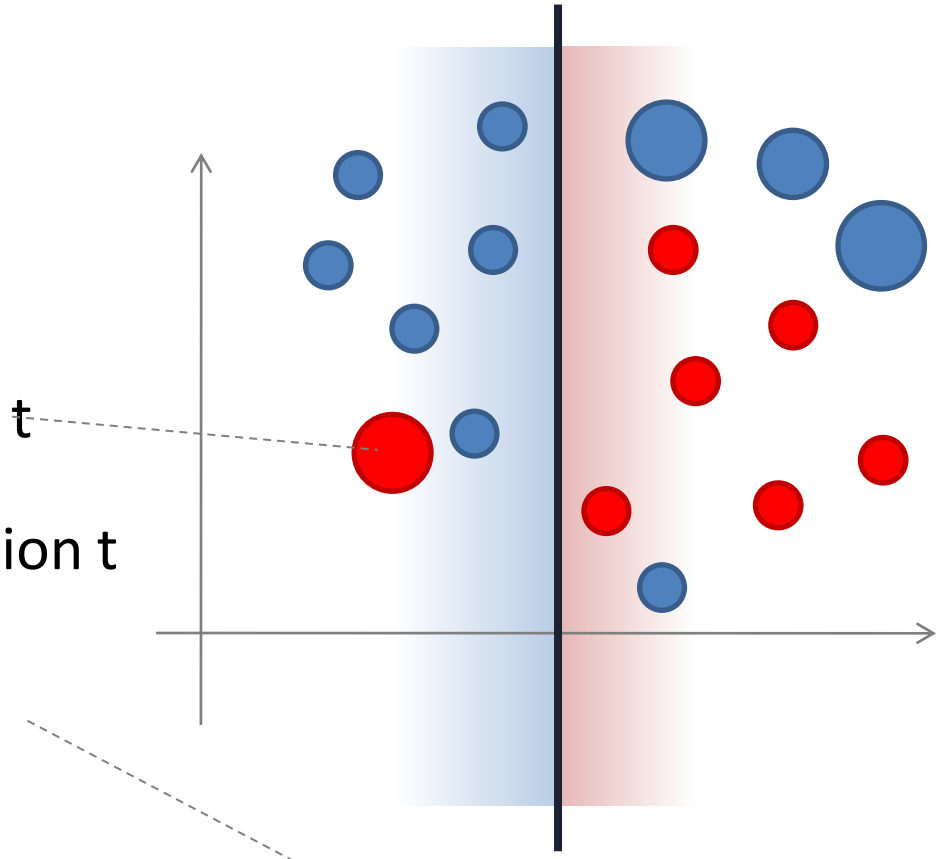
$t = 1,2,3\dots$ Iteration number

$W_t(x_i)$ Weight of the element on iteration t

α_t Weight if a weak classifier on iteration t

$h_t(x)$ Weak classifier on iteration t

$$k(x) \cong \sum_t \alpha_t \cdot h_t(x)$$



$$h(x) = \{x > \theta \,?\, 0 : 1\}$$

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Algorithm

Weak classifier to minimize the weighted error

$$h_t(x) = \arg\min_h \sum_i W_t(i) \cdot [k_i \neq h(x_i)]$$

Update weights

$$W_{t+1}(i) = W_t(i) \cdot \exp\{-\alpha_t \cdot k_i \cdot h_t(x_i)\} = W_t(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & if\ h_t(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & if\ h_t(x) \neq k \end{cases}$$

Calculate the weight od classifier

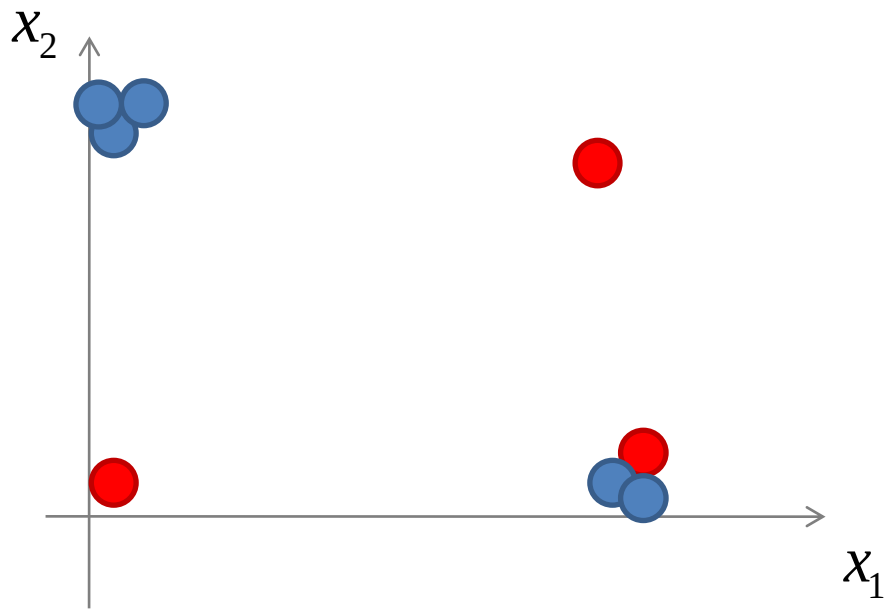
$$\alpha_t = \frac{1}{2}\log\left(\frac{1+r_t}{1-r_t}\right)$$

$$r_t = \sum_i W_t(i) \cdot h(x_i) \cdot k_i \Big/ \sum_i W_t(i)$$

Adaboost

Example: step 1

$$h_{t=1}(\bar{x}) =$$



Adaboost

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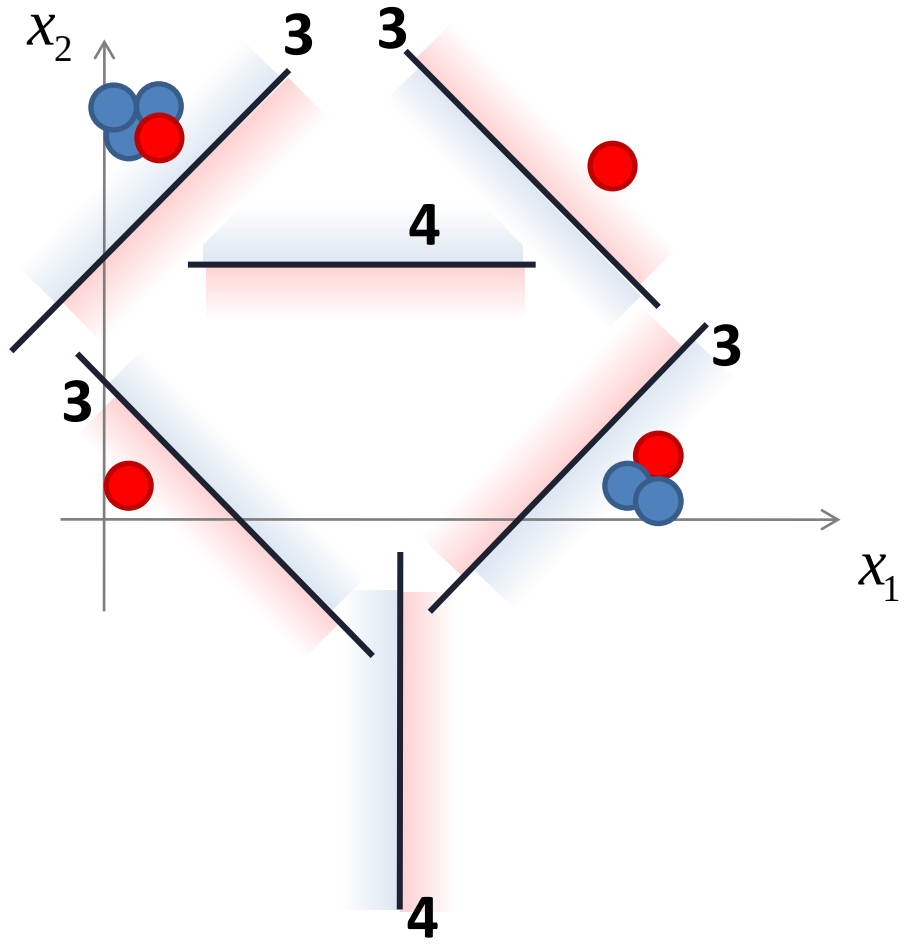
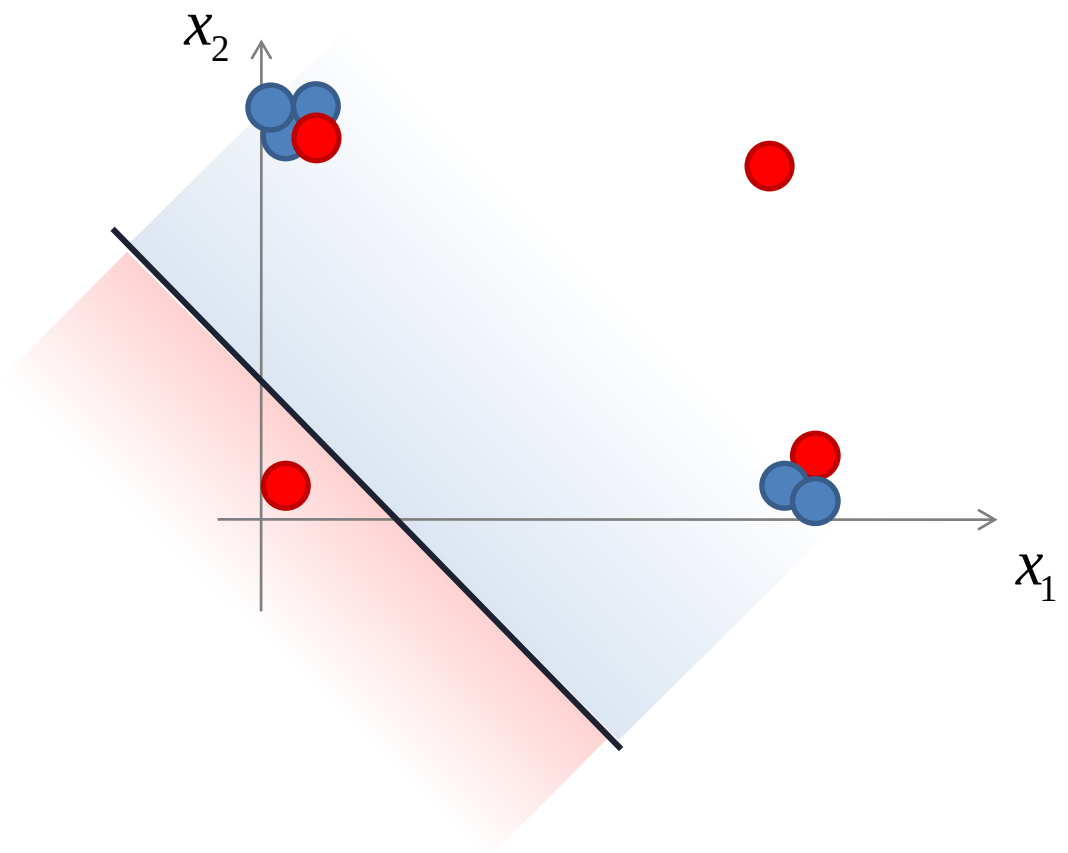


Adaboost

Example: step 1

$$h_{t=1}(\bar{x}) = \{x_1 + x_2 < 0.5? \text{ ●} : \text{ ●}\}$$

$$r_{t=1} = 0.33 \quad \alpha_{t=1} = 0.34$$



Adaboost

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$$h_{t=1}(\bar{x}) = \{x_1 + x_2 < 0.5? \text{ ● } : \text{ ● }\}$$

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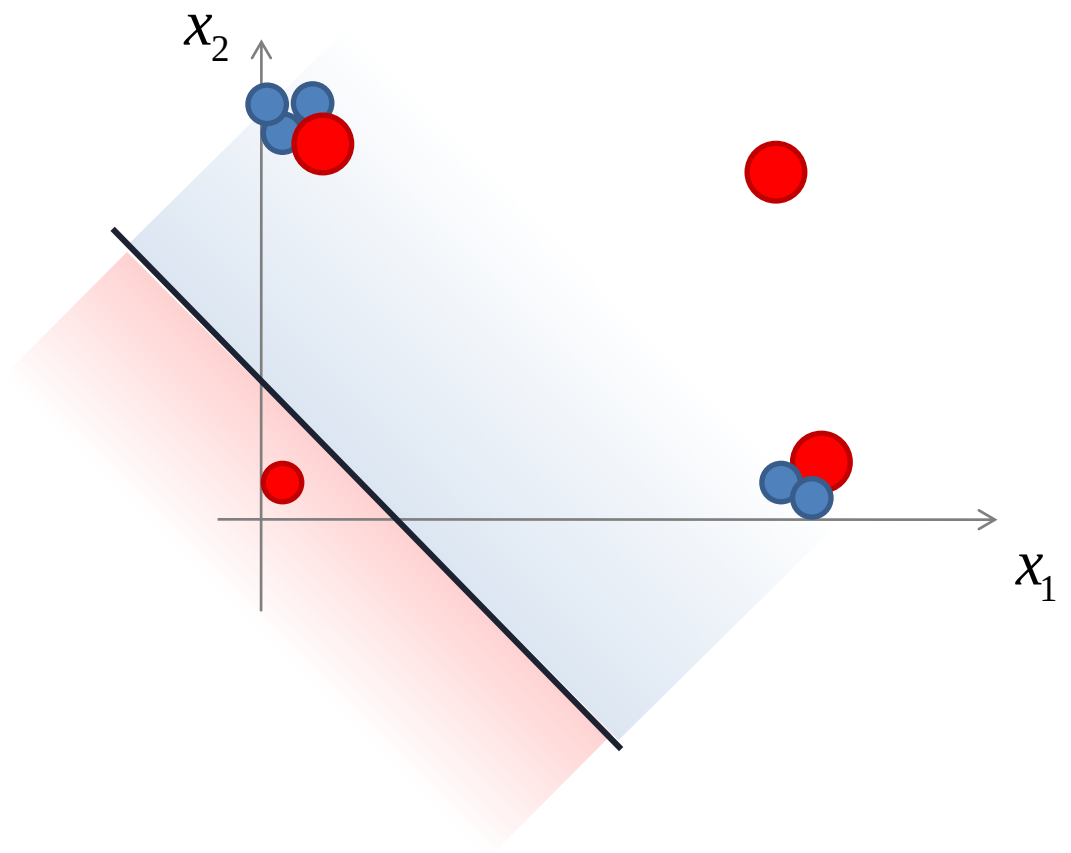
$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\}$$



Adaboost

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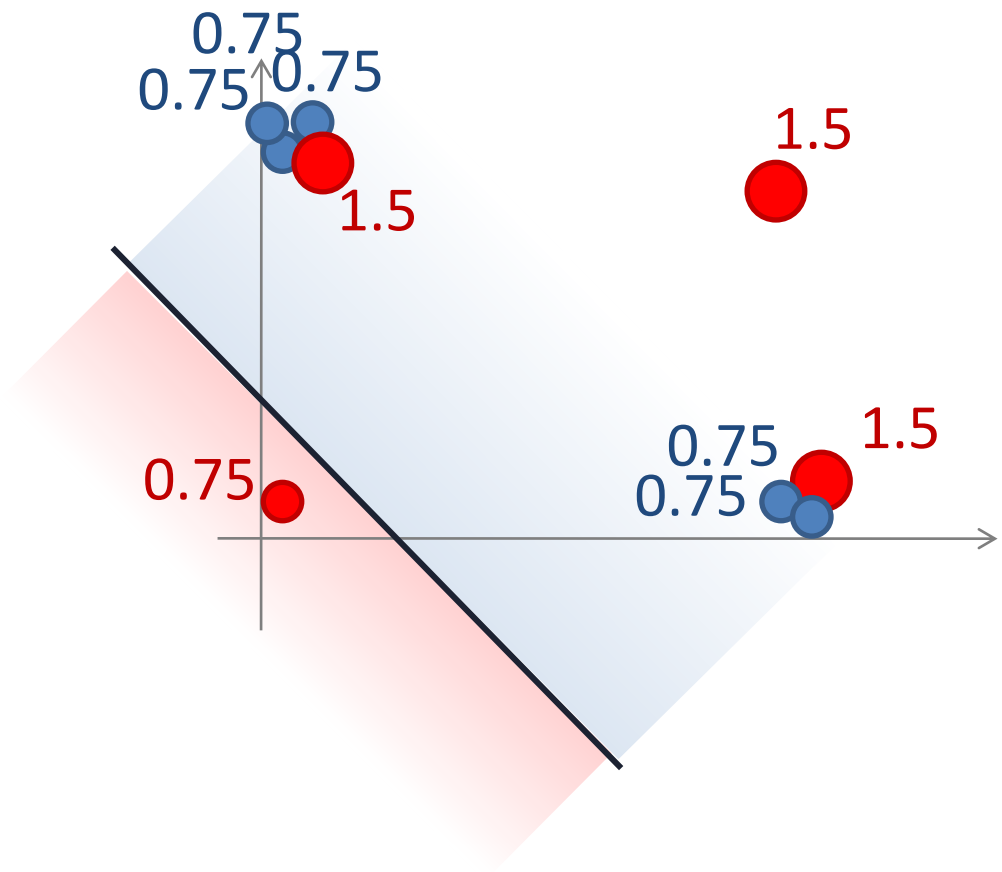

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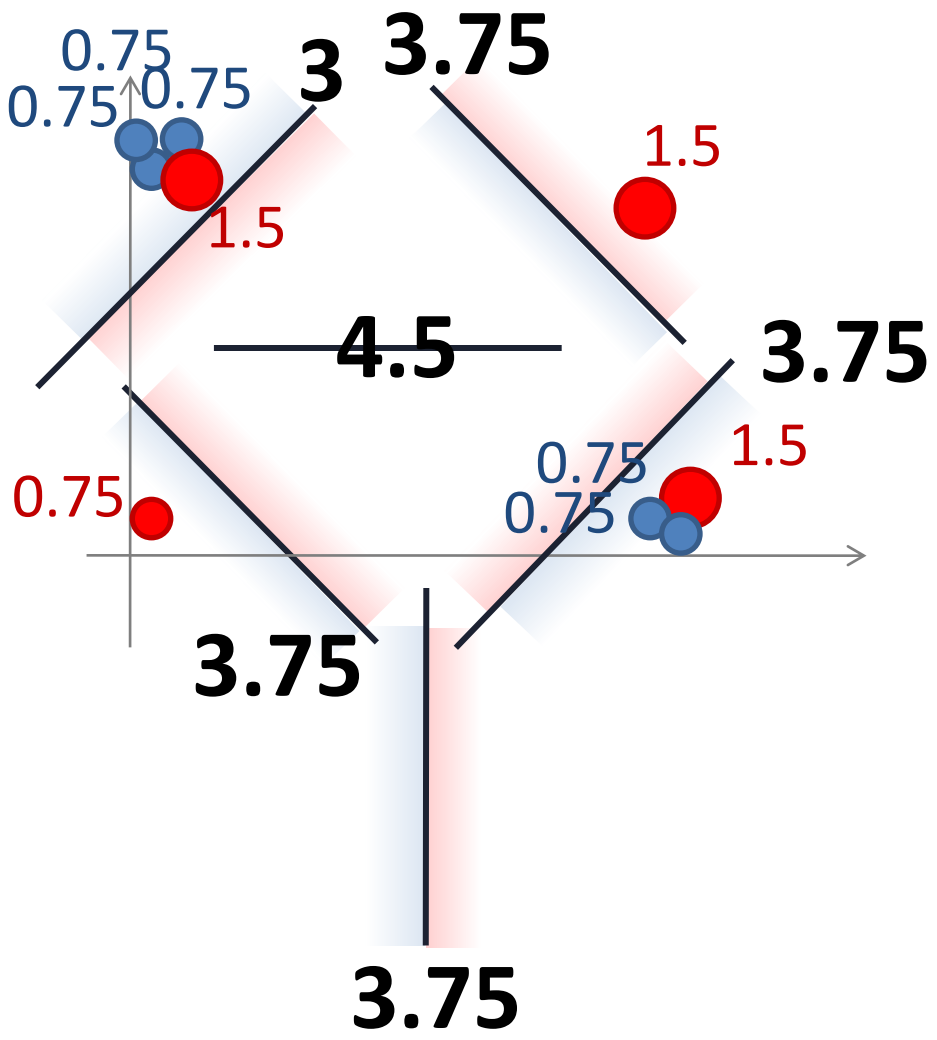
Example: step 2

$$h_{t=2}(\bar{x}) =$$

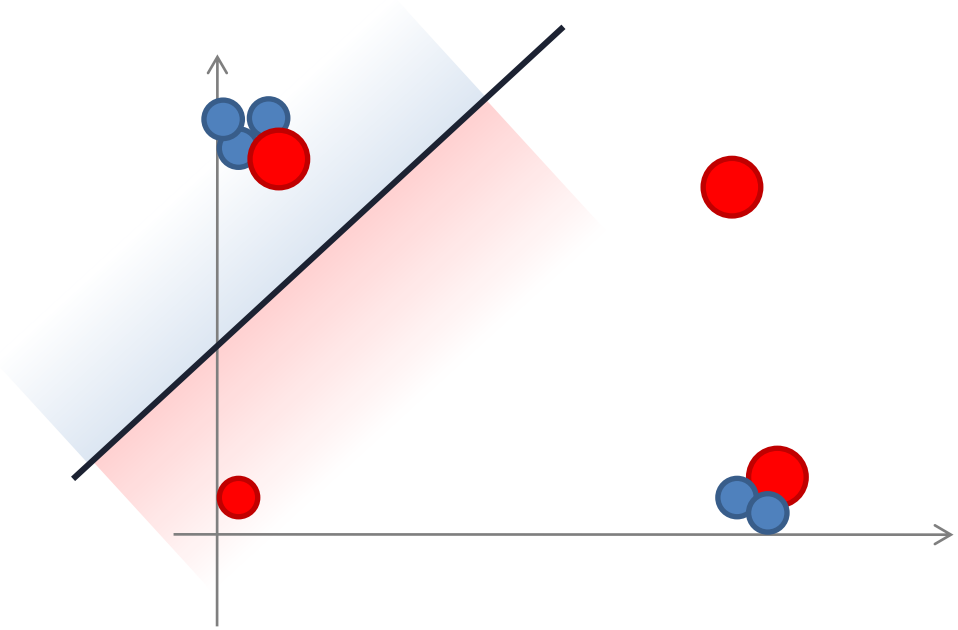


Adaboost

Example: step 2

$$h_{t=2}(\bar{x}) = \{x_1 - x_2 < -0.5? \text{ ● } : \text{ ● }\}$$

$$r_{t=2} = 0.33 \quad \alpha_{t=2} = 0.34$$



Adaboost

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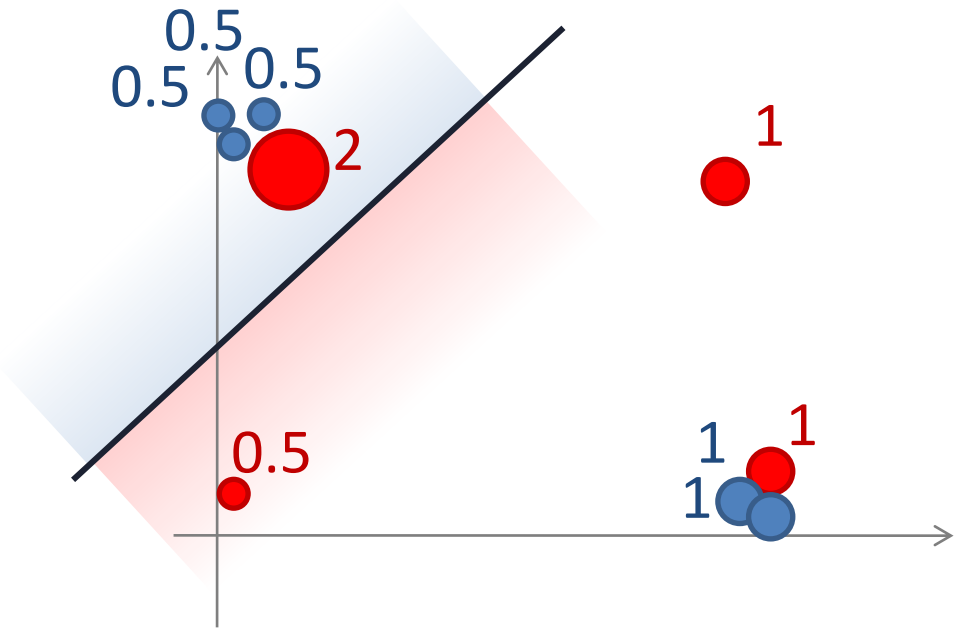

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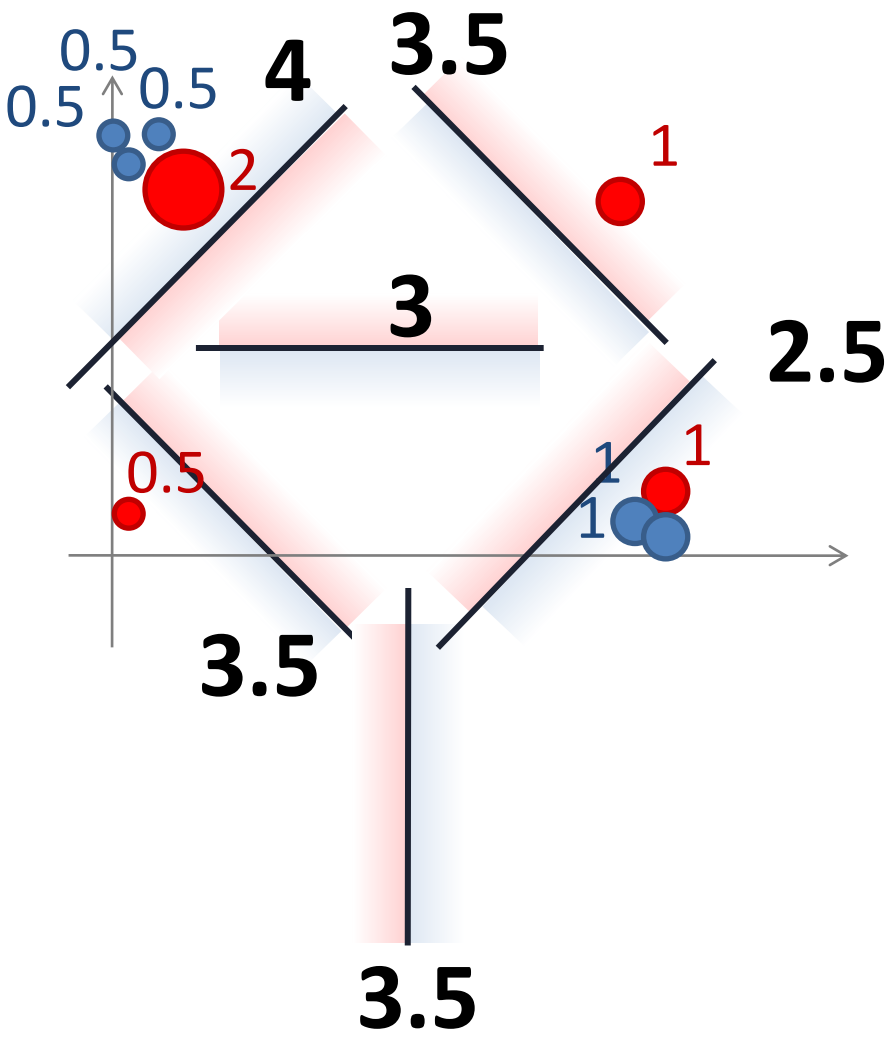
Example: step 3

$$h_{t=3}(\bar{x}) =$$

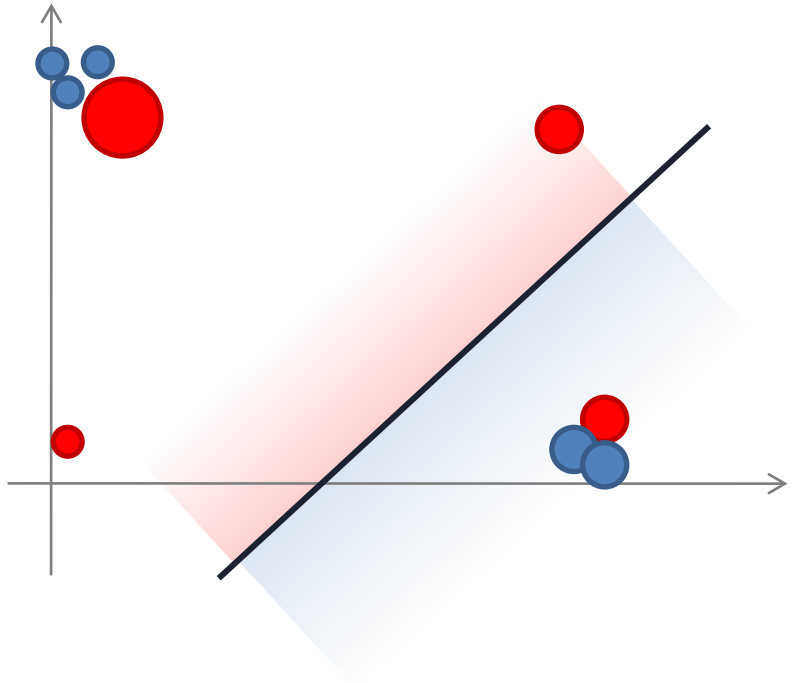


Adaboost

Example: step 3

$$h_{t=3}(\bar{x}) = \{x_1 - x_2 > 0.5? \ \bullet : \bullet\}$$

$$r_{t=3} = 0.38 \quad \alpha_{t=3} = 0.39$$



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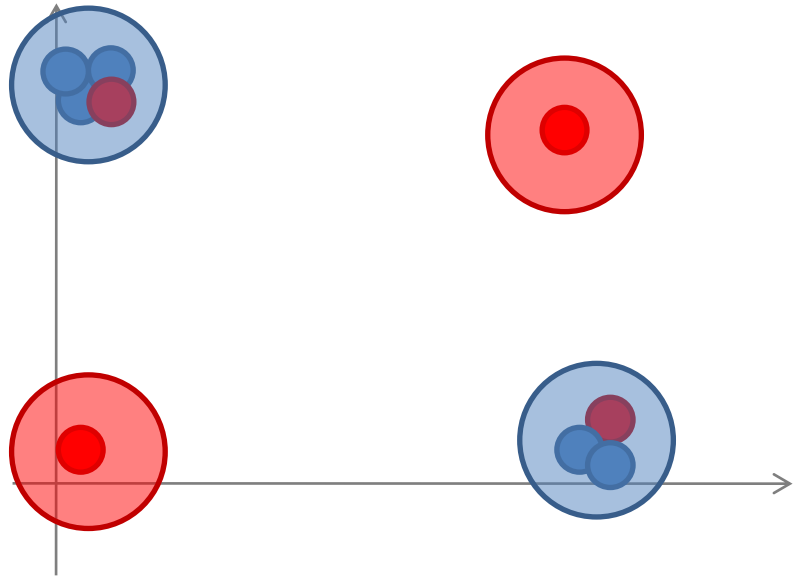
Example: result

$$k(x) \cong \sum_t \alpha_t \cdot h_t(x)$$

$h_{t=3}$ $h_{t=2}$



	x	(0,0)	(1,0)	(1,0)	(1,0)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
$\alpha_1 = 0.35$	h_1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1
$\alpha_2 = 0.35$	h_2	1	1	1	1	-1	-1	-1	-1	-1	1
$\alpha_3 = 0.39$	h_3	1	-1	-1	-1	1	1	1	1	1	1
$k(x) \cong \sum_t \alpha_t \cdot h_t(x)$		1	-1	-1	-1	-1	-1	-1	-1	-1	1

Adaboost

Another example

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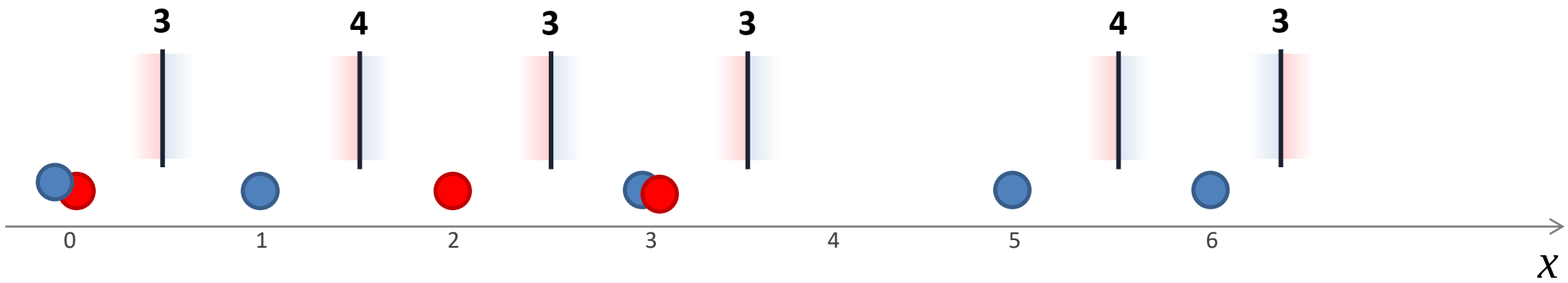

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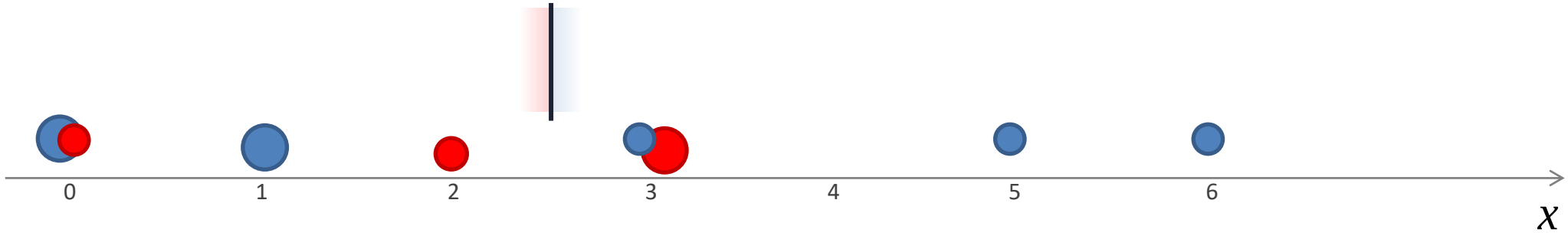
$$h_{t=1}(\bar{x}) = \{x < 2.5?\ \ +1\ \ :\ \ -1\}$$

$$r_{t=1} = \frac{1}{\sum_i W_1(i)} \sum_i W_1(i) \cdot h_1(x_i) \cdot k_i = 2/8$$

$$\alpha_t = \frac{1}{2} \log\left(\frac{1+r_t}{1-r_t}\right)$$

$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\} = W_1(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & if\ h_1(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & if\ h_1(x) \neq k \end{cases}$$

Adaboost

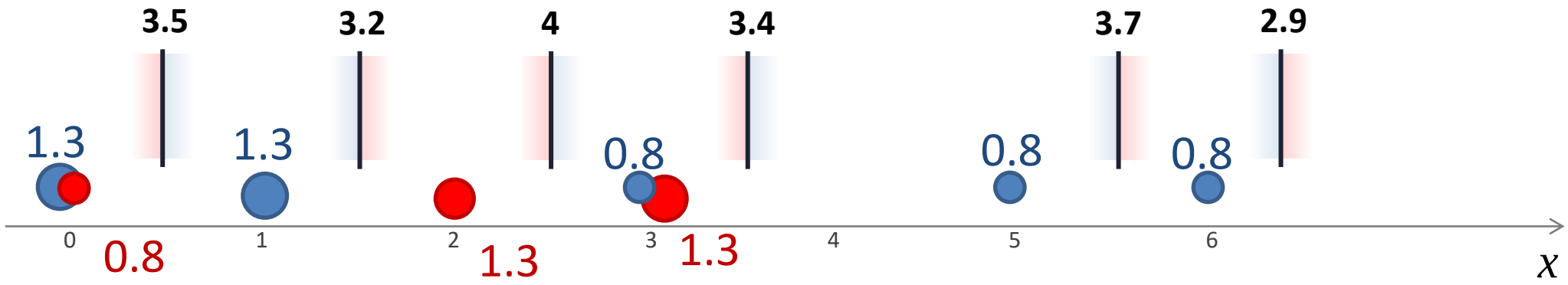


$$r_{t=1} = \frac{1}{\sum_i W_1(i)} \sum_i W_1(i) \cdot h_1(x_i) \cdot k_i = 2/8$$

$$\alpha_t = \frac{1}{2} \log\left(\frac{1+r_t}{1-r_t}\right)$$

$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\} = W_1(i) \times \begin{cases} \sqrt{\frac{3}{5}} & if\ h_1(x) = k \\ \sqrt{\frac{5}{3}} & if\ h_1(x) \neq k \end{cases}$$

Adaboost



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$$h_{t=2}(\bar{x}) = \{-1\}$$

$$W_{t=3}(i) = W_2(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & if\ h_2(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & if\ h_2(x) \neq k \end{cases}$$

$$r_{t=2} = \frac{1}{\sum_i W_2(i)} \sum_i W_2(i) \cdot h_2(x_i) \cdot k_i = 2.9$$

$$\alpha_t = \frac{1}{2} \log\left(\frac{1+r_t}{1-r_t}\right)$$

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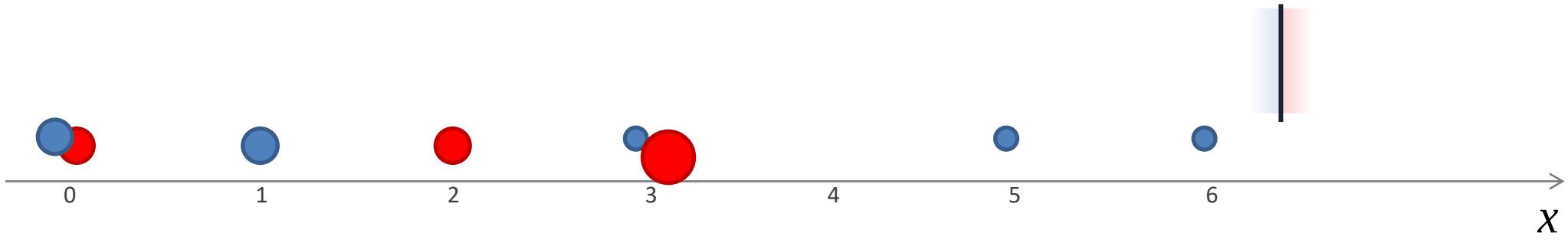

$$W_{t=3}(i) = W_2(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & if\ h_2(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & if\ h_2(x) \neq k \end{cases}$$

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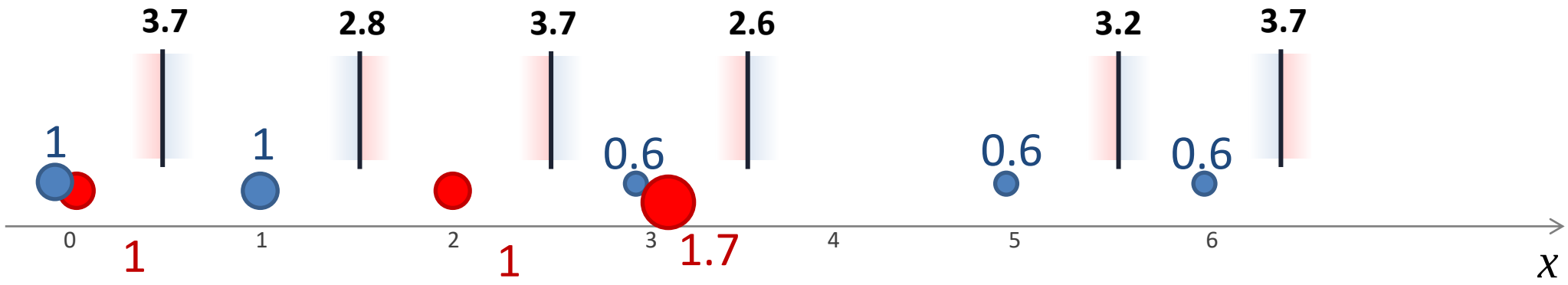

Adaboost

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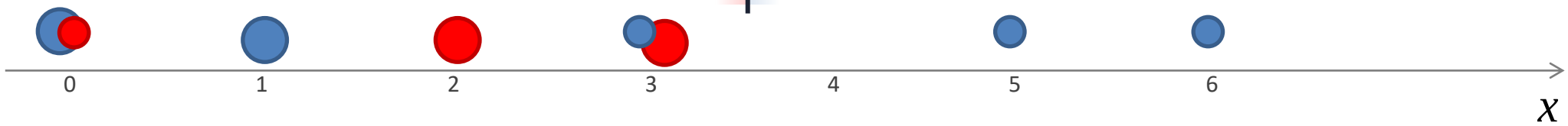

$\alpha_1 = 0.25$	1	1	1	-1	-1	-1
$\alpha_2 = 0.27$	-1	-1	-1	-1	-1	-1
$\alpha_3 = 0.33$	1	1	1	1	-1	-1
	1	1	1	-1	-1	-1