Basics of Machine Learning

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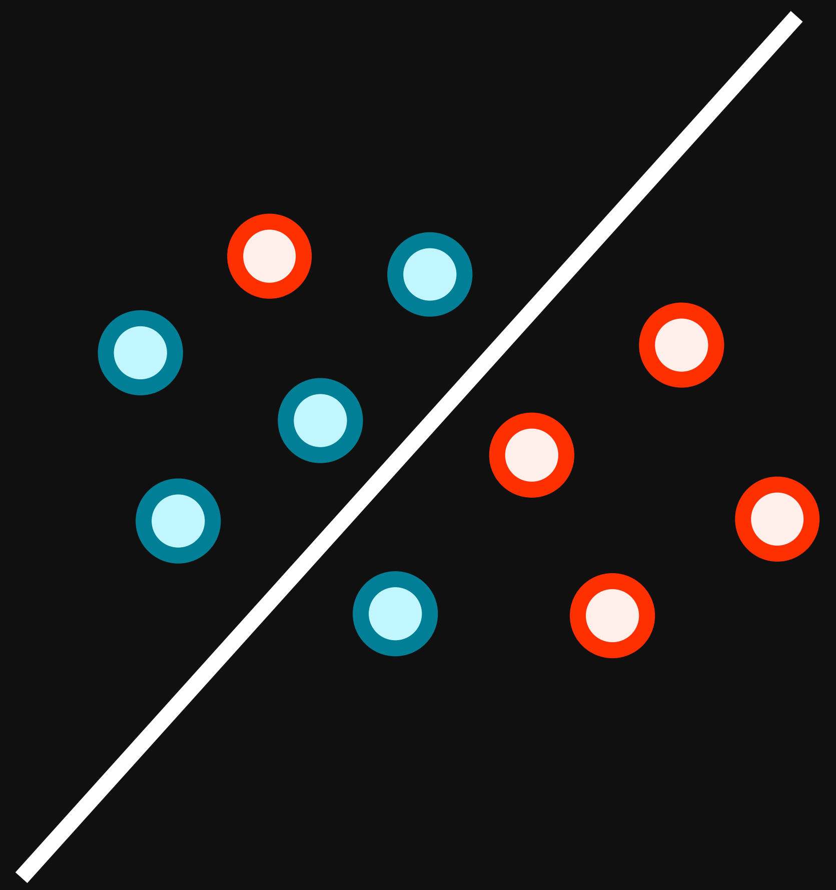
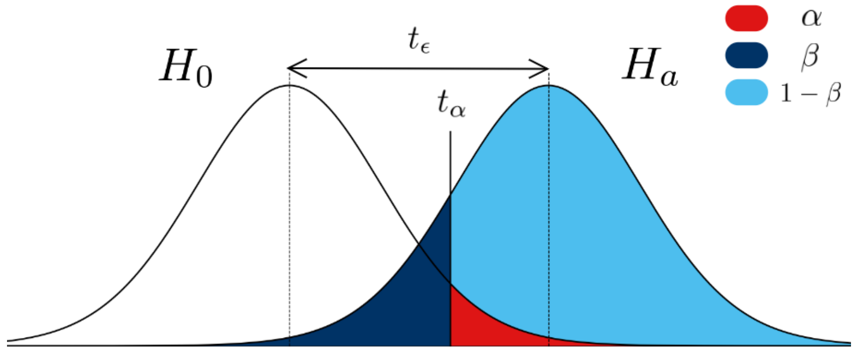


Lesson 13
Non Bayesian Approach



Non Bayesian Approach

Summary

- Limitations of the Bayesian approach
- Type I error
- Type II error
- Neyman-Pearson approach
- Minimax approach

Neyman-Pearson approach

Limitations of the Bayesian approach

$x \in X$ feature

$k \in \mathrm{K}$ hidden state

$p(x|k)$ conditional probability to expose *x by* state *k*

$p(k)$ priory probability of the state

$W : \mathrm{K} \times \mathrm{K} \rightarrow R$ penalty function

Limitations of the Bayesian approach

$x \in X$ feature

$k \in K$ hidden state

$p(x|k)$ conditional probability to expose *x by* state *k*

~~$p(k)$ priory probability of the state~~

~~$W : K \times K \to R$ penalty function~~

Neyman-Pearson approach

Limitations of the Bayesian approach: example

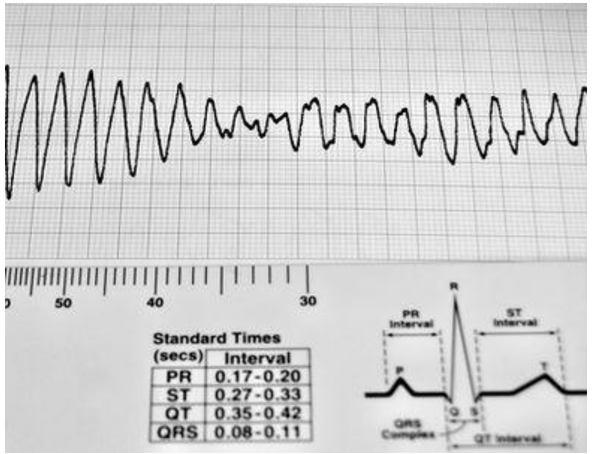
$x \in X$ heartbeat

$k \in \mathrm{K}$ state: normal, sick

$p(x|k)$ distribution of the heartbeat for normal and sick

$p(k)$ **???**

W(normal, sick)
W(sick, normal) **???**



Neyman-Pearson approach

The problem

$x \in X \quad k \in K = \{nornal, risky\}$

$p(x|k = norm) \quad X_{norm} \cup X_{risk} = X$

$p(x|k = risk) \quad X_{norm} \cap X_{risk} = 0$

input

$$error\ type\ 1 = P(false\ alarm) = \sum_{x \in X_{risk}} p(x|norm) \to min$$

$$error\ type\ 2 = P(miss\ the\ risk) = \sum_{x \in X_{norm}} p(x|risk) \leq \varepsilon$$

$X_{norm}, X_{risk} = ?$

output

Neyman-Pearson approach

The problem

$x \in X \qquad k \in K = \{nornal, risky\}$

$$\alpha_{norm}(x) = \begin{cases} 1 \; when \; x \in X_{norm} \\ 0 \; when \; x \notin X_{norm} \end{cases}$$

$$\alpha_{risk}(x) = \begin{cases} 1 \; when \; x \in X_{risk} \\ 0 \; when \; x \notin X_{risk} \end{cases}$$

Neyman-Pearson approach

The problem

$$\sum_{x \in X_{risk}} p(x|norm) \to min$$

$$\sum_{x \in X_{norm}} p(x|risk) \le \varepsilon$$

$$\begin{cases} \sum_{x \in X} \alpha_{risk}(x) \cdot p(x|norm) \to min \\ -\sum_{x \in X} \alpha_{norm}(x) \cdot p(x|norm) \ge -\varepsilon \\ \alpha_{norm}(x) + \alpha_{risk}(x) = 1 \;\; \forall\, x \in X \\ \alpha_{norm}(x) \;\ge 0 \;\; \forall\, x \in X \\ \alpha_{risk}(x) \;\ge 0 \;\; \forall\, x \in X \end{cases}$$

Neyman-Pearson approach

$$\begin{cases} p_1 \cdot x_1 + \ \ldots \ + p_n \cdot x_n \to max \\ a_{11} \cdot x_1 + \ \ldots \ + a_{1n} \cdot x_n \le b_1 \\ a_{k1} \cdot x_1 + \ \ldots \ + a_{kn} \cdot x_n \le b_k \\ a_{k+1,1} \cdot x_1 + \ \ldots \ + a_{k+1,n} \cdot x_n = b_{k+1} \\ a_{m1} \cdot x_1 + \ \ldots \ + a_{mn} \cdot x_n = b_m \\ x_1 \ge 0 \\ x_s \ge 0 \\ x_{s+1} - св. \ \ перем. \\ x_n - св. \ \ перем. \end{cases}
\qquad
\begin{cases} b_1 \cdot y_1 + \ \ldots \ + b_m \cdot y_m \to min \\ y_1 \ge 0 \\ y_k \ge 0 \\ y_{k+1} - св. \ \ перем. \\ y_m - св. \ \ перем. \\ a_{11} \cdot y_1 + \ \ldots \ + a_{m1} \cdot y_m \ge p_1 \\ a_{1s} \cdot y_1 + \ \ldots \ + a_{ms} \cdot y_m \ge p_s \\ a_{1,s+1} \cdot y_1 + \ \ldots \ + a_{m,s+1} \cdot y_m = p_{s+1} \\ a_{1n} \cdot y_1 + \ \ldots \ + a_{mn} \cdot y_m = p_n \end{cases}$$

Neyman-Pearson approach

$p_1 \cdot x_1 + \quad \dots \quad + p_n \cdot x_n \to max$	$b_1 \cdot y_1 + \quad \dots \quad + b_m \cdot y_m \to min$
$a_{11} \cdot x_1 + \quad \dots \quad + a_{1n} \cdot x_n \le b_1$	$y_1 \ge 0$
$a_{k1} \cdot x_1 + \quad \dots \quad + a_{kn} \cdot x_n \le b_k$	$y_k \ge 0$
$a_{k+1,1} \cdot x_1 + \quad \dots \quad + a_{k+1,n} \cdot x_n = b_{k+1}$	$y_{k+1} - св. \quad перем.$
$a_{m1} \cdot x_1 + \quad \dots \quad + a_{mn} \cdot x_n = b_m$	$y_m - св. \quad перем.$
$x_1 \ge 0$	$a_{11} \cdot y_1 + \quad \dots \quad + a_{m1} \cdot y_m \ge p_1$
$x_s \ge 0$	$a_{1s} \cdot y_1 + \quad \dots \quad + a_{ms} \cdot y_m \ge p_s$
$x_{s+1} - св. \quad перем.$	$a_{1,s+1} \cdot y_1 + \quad \dots \quad + a_{m,s+1} \cdot y_m = p_{s+1}$
$x_n - св. \quad перем.$	$a_{1n} \cdot y_1 + \quad \dots \quad + a_{mn} \cdot y_m = p_n$

$$\left(a_{11} \cdot x_1 + \quad \dots \quad + a_{1n} \cdot x_n - b_1\right) \cdot y_1 = 0$$

$$\left(a_{11} \cdot y_1 + \quad \dots \quad + a_{m1} \cdot y_m - p_1\right) \cdot x_1 = 0$$

Neyman-Pearson approach

Solution

$$\begin{cases} \sum_{x \in X} \alpha_{risk}(x) \cdot p(x|norm) \rightarrow min \\ -\sum_{x \in X} \alpha_{norm}(x) \cdot p(x|norm) \geq -\varepsilon \\ \alpha_{norm}(x) + \alpha_{risk}(x) = 1 \quad \forall\, x \in X \\ \alpha_{norm}(x) \;\geq 0 \;\; \forall\, x \in X \\ \alpha_{risk}(x) \;\geq 0 \;\; \forall\, x \in X \end{cases}$$

$$\begin{cases} -\varepsilon \cdot \tau + 1 \cdot t(x_1) + \cdots + 1 \cdot t(x_i) \rightarrow max \\ \tau \geq 0 \\ t(x)\ free\ var \\ -p(x|risk) \cdot \tau + 1 \cdot t(x) \leq 0 \\ 1 \cdot t(x) \leq p(x|norm) \end{cases}$$

Neyman-Pearson approach

Solution

$$x \in X_{norm} \;\Rightarrow\; \alpha_{norm}(x) = 1, \alpha_{risk}(x) = 0 \quad \Rightarrow \quad \begin{cases} -p(x|risk) \cdot \tau + 1 \cdot t(x) = 0 \\ 1 \cdot t(x) < p(x|norm) \end{cases} \quad \Rightarrow$$

$$\Rightarrow \quad p(x|risk) \cdot \tau \le p(x|norm)$$

$$x \in X_{risk} \;\Rightarrow\; \alpha_{norm}(x) = 0,\; \alpha_{risk}(x) = 1 \quad \Rightarrow \quad \begin{cases} -p(x|risk) \cdot \tau + 1 \cdot t(x) \le 0 \\ 1 \cdot t(x) = p(x|norm) \end{cases} \quad \Rightarrow$$

$$\Rightarrow \quad p(x|risk) \cdot \tau \ge p(x|norm)$$

Neyman-Pearson approach

Solution

$$\frac{p(x|norm)}{p(x|risk)} \underset{risk}{\overset{norm}{\gtrless}} \tau$$

Neyman-Pearson approach

Some Variations: two risky states

$$\sum_{x \in X_{risk}} p(x|norm) \to min$$

$$\sum_{x \in X_{norm}} p(x|risk1) \le \varepsilon \qquad \sum_{x \in X_{norm}} p(x|risk2) \le \varepsilon$$

Neyman-Pearson approach

Some Variations: two normal states

$$\sum_{x \in X_{risk}} p(x|norm1) + \sum_{x \in X_{risk}} p(x|norm2) \to min$$

$$\sum_{x \in X_{norm}} p(x|risk) \leq \varepsilon$$

Neyman-Pearson approach

Minimax problem

$$max\left\{\sum_{x\notin X_1} p(x|1)\ ,\sum_{x\notin X_2} p(x|2)\ ..,\sum_{x\notin X_K} p(x|K)\ \right\}\rightarrow min$$