

Basics of Machine Learning

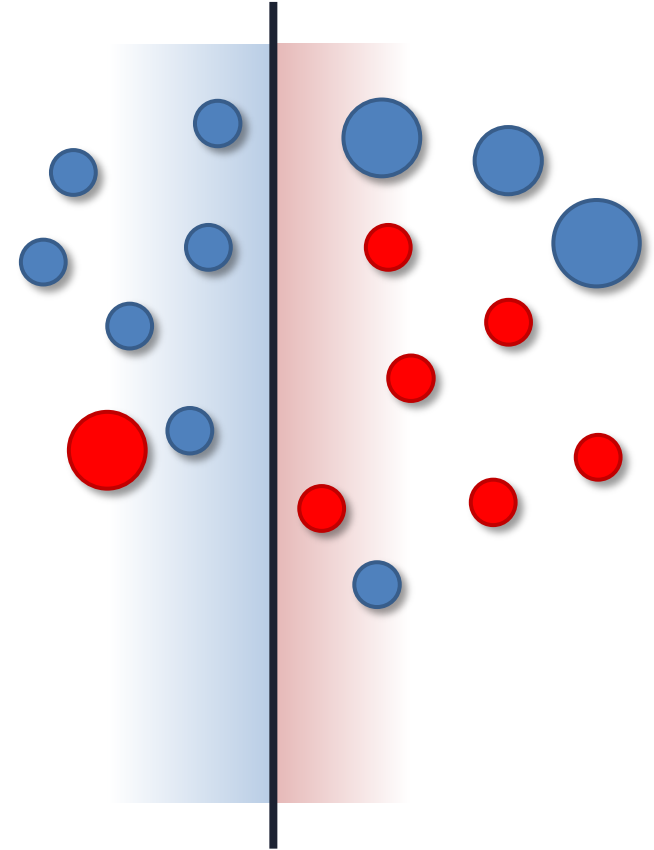
Dmitry Ryabokon, github.com/dryabokon





Lesson 16

Adaboost



Adaboost

Definitions

$x_1, x_2, x_i, \dots, x_N$ Training features
 $k_1, k_2, k_i, \dots, k_N$ = $\{-1, 1\}$ Training targets

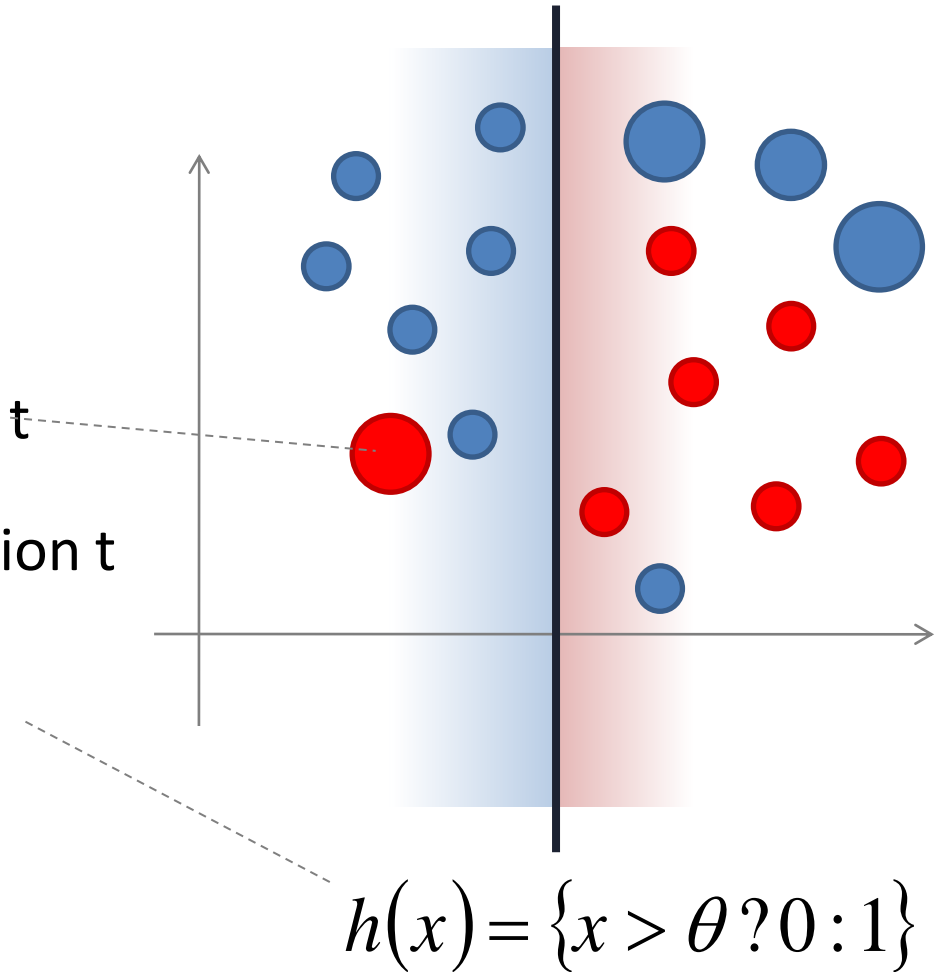
$t = 1, 2, 3, \dots$ Iteration number

$W_t(x_i)$ Weight of the element on iteration t

α_t Weight of a weak classifier on iteration t

$h_t(x)$ Weak classifier on iteration t

$$k(x) \cong \sum_t \alpha_t \cdot h_t(x)$$



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Algorithm

Weak classifier to minimize the weighted error

$$h_t(x) = \arg \min_h \sum_i W_t(i) \cdot [k_i \neq h(x_i)]$$

Update weights

$$W_{t+1}(i) = W_t(i) \cdot \exp\{-\alpha_t \cdot k_i \cdot h_t(x_i)\} = W_t(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & \text{if } h_t(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & \text{if } h_t(x) \neq k \end{cases}$$

Calculate the weight of classifier

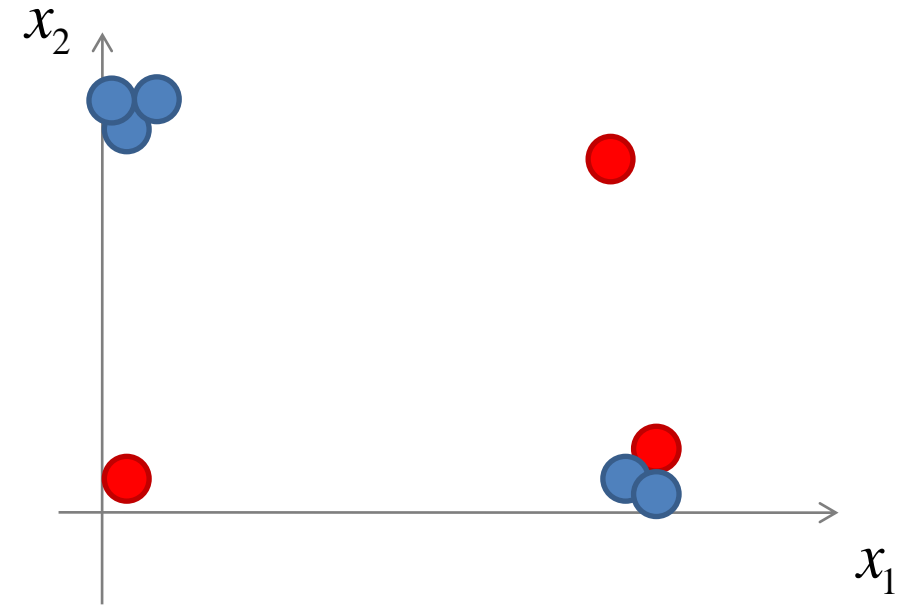
$$\alpha_t = \frac{1}{2} \log \left(\frac{1+r_t}{1-r_t} \right)$$

$$r_t = \sum_i W_t(i) \cdot h(x_i) \cdot k_i / \sum_i W_t(i)$$

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Example: step 1

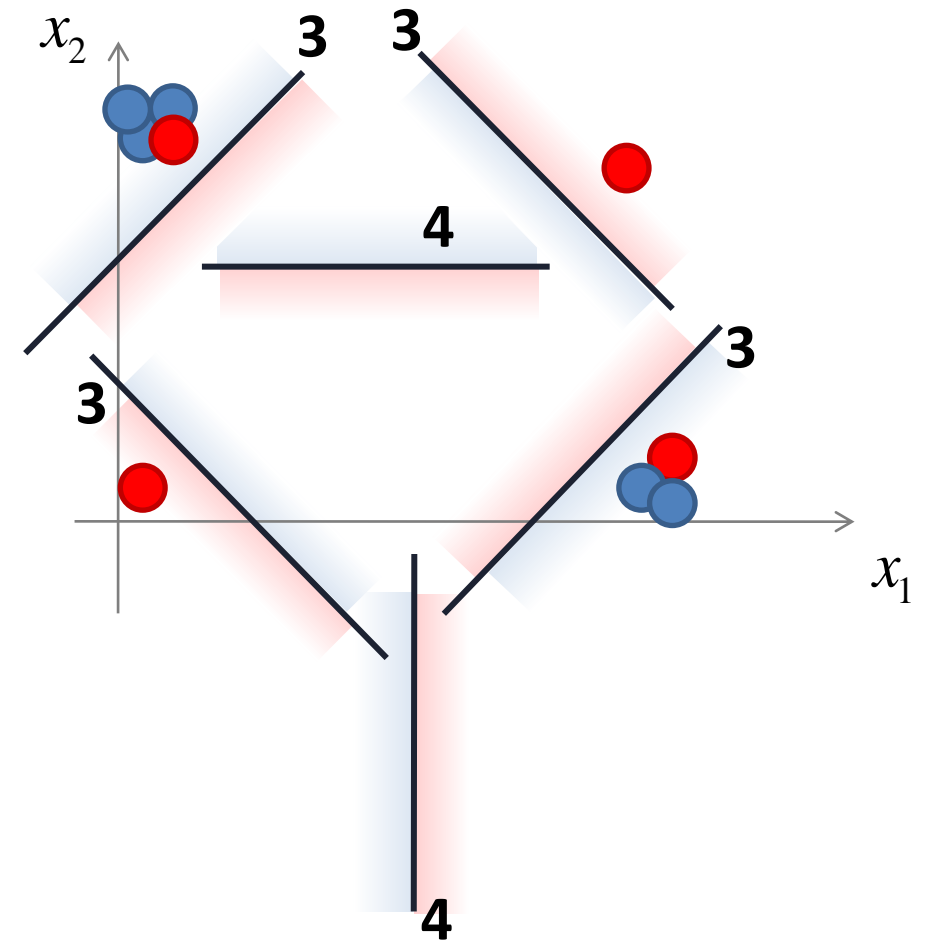
$$h_{t=1}(\bar{x}) =$$



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Example: step 1

$$h_{t=1}(\bar{x}) =$$

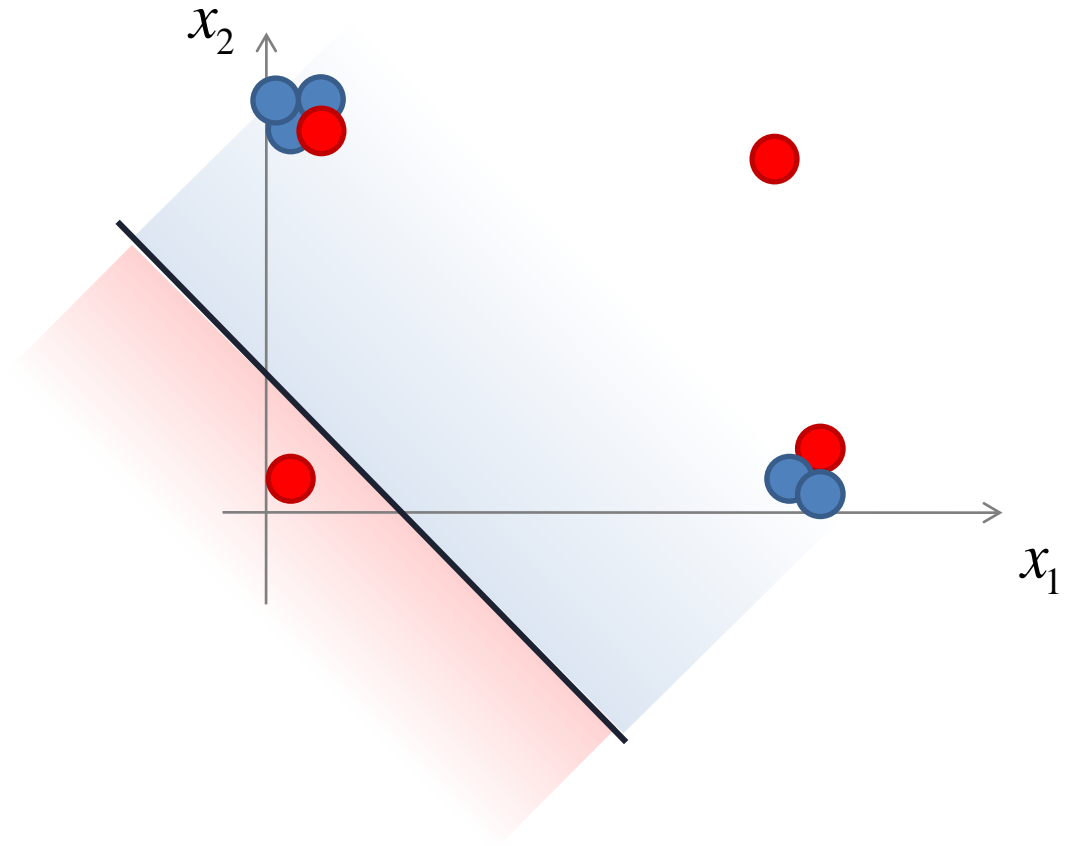


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Example: step 1

$$h_{t=1}(\bar{x}) = \{x_1 + x_2 < 0.5? \text{ red} : \text{blue}\}$$

$$r_{t=1} = 0.33 \quad \alpha_{t=1} = 0.34$$



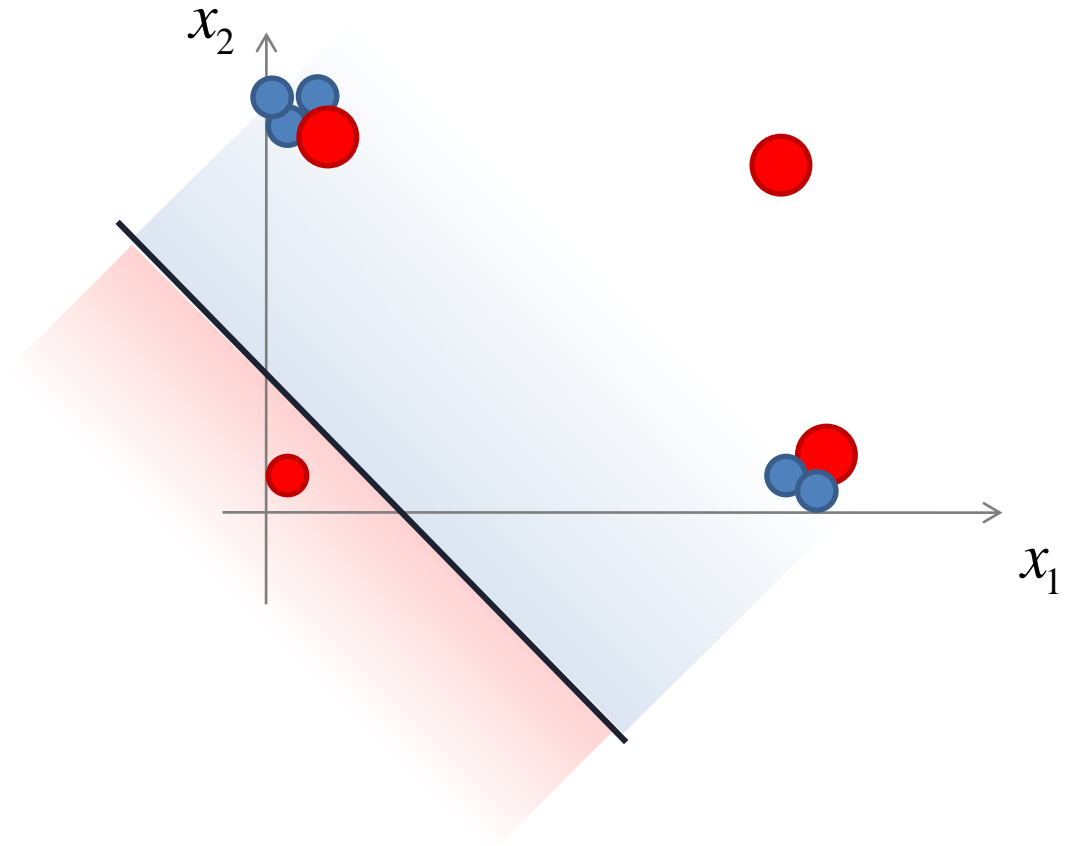
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$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\}$$



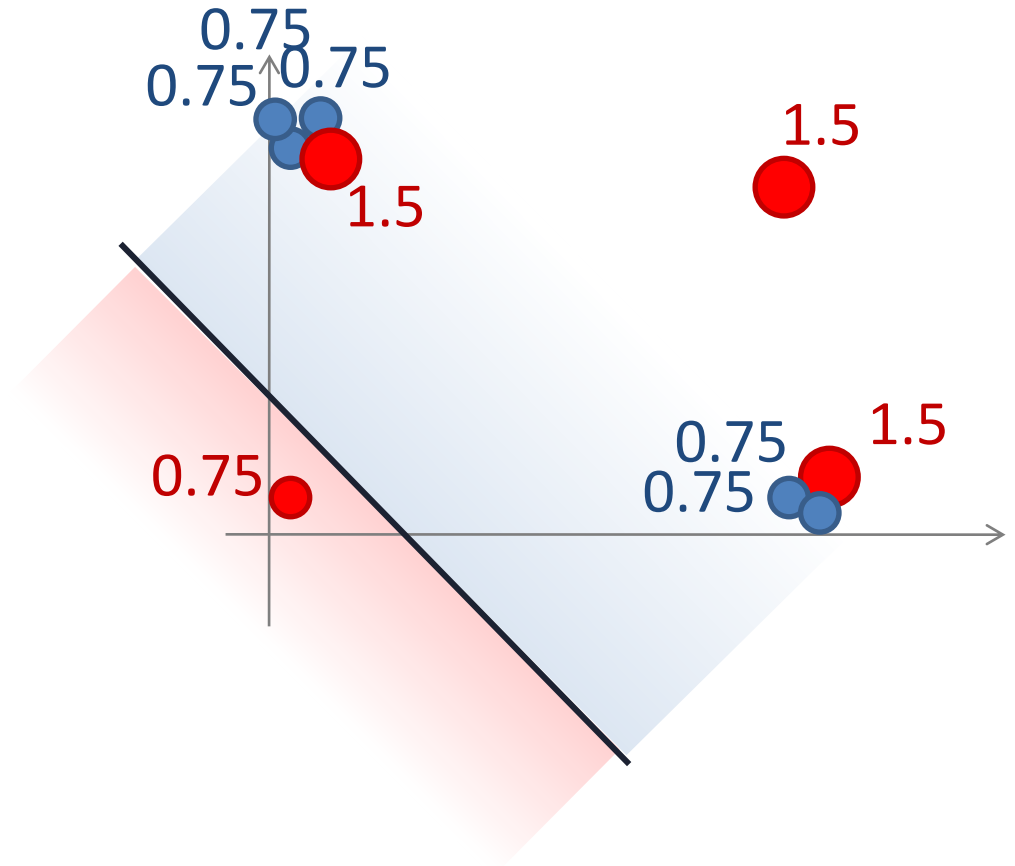
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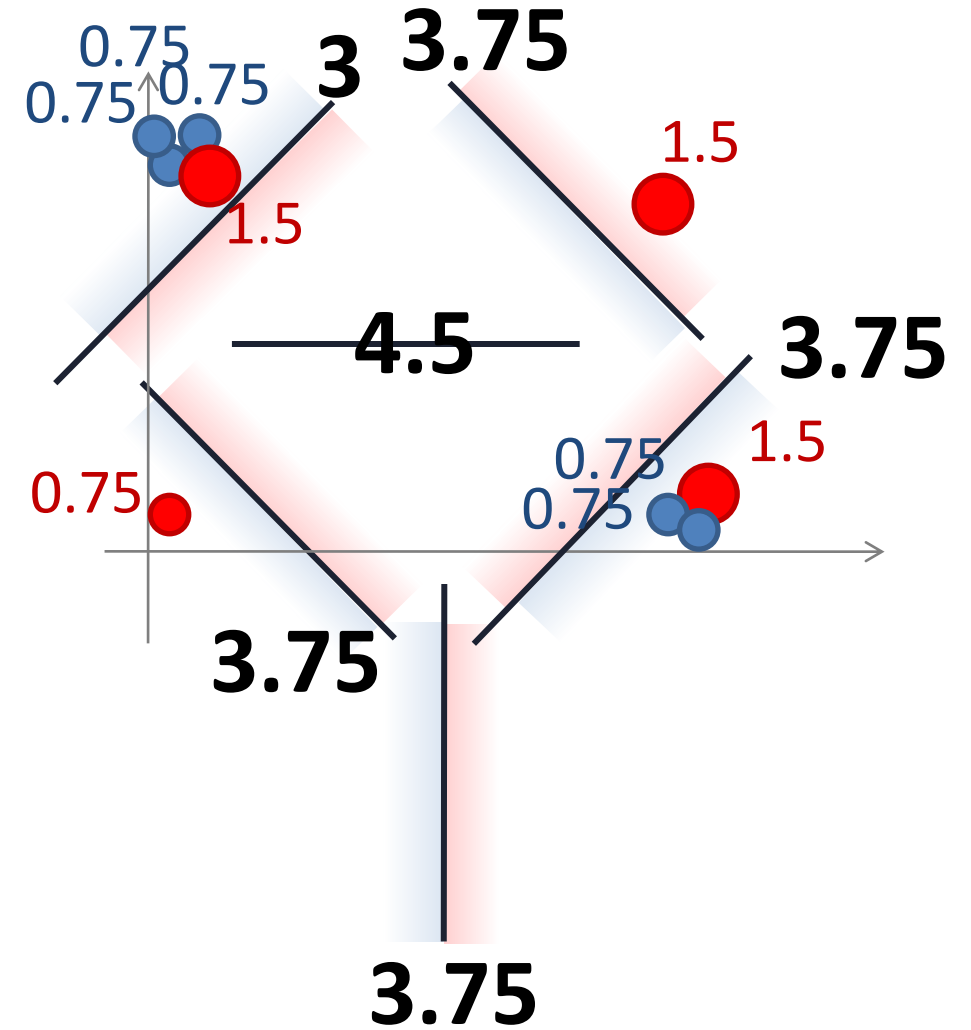
$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\}$$



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Example: step 2

$$h_{t=2}(\bar{x}) =$$

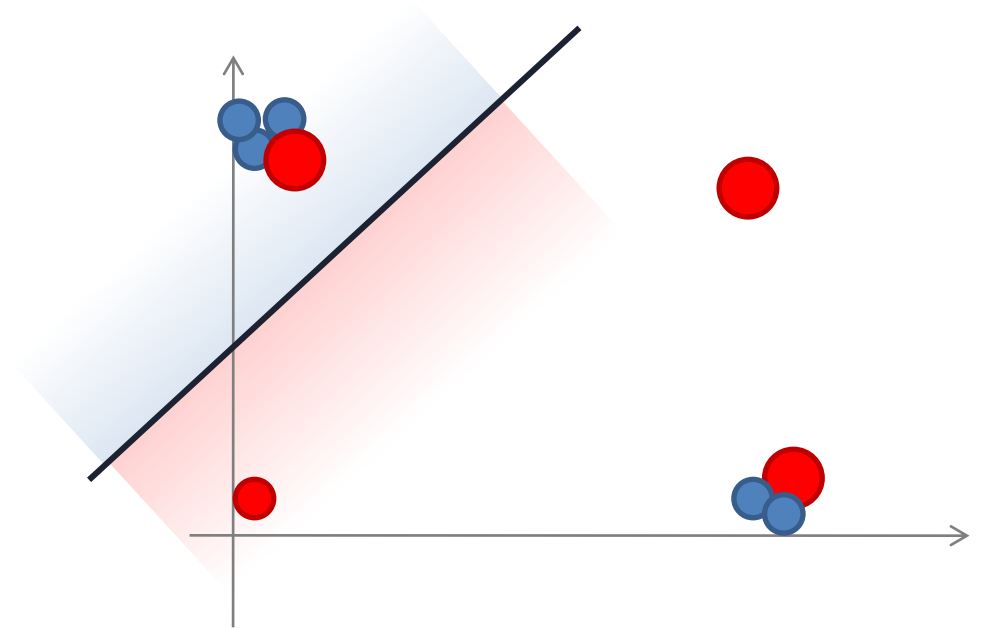


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Example: step 2

$$h_{t=2}(\bar{x}) = \{x_1 - x_2 < -0.5? \text{ red} : \text{blue}\}$$

$$r_{t=2} = 0.33 \quad \alpha_{t=2} = 0.34$$



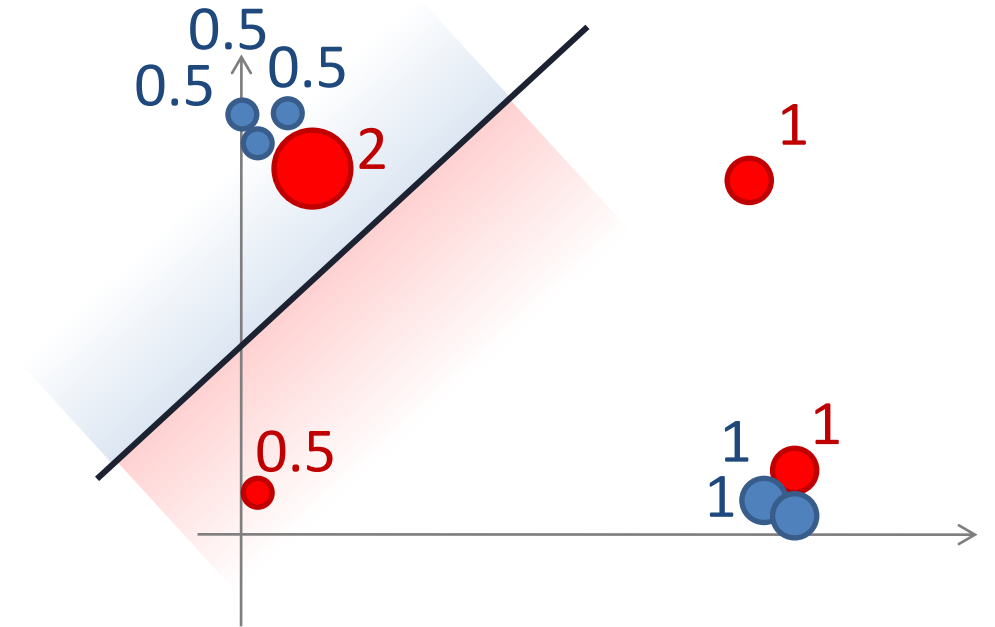
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Example: step 2

$$h_{t=2}(\bar{x}) = \{x_1 - x_2 < -0.5? \text{ red} : \text{blue}\}$$

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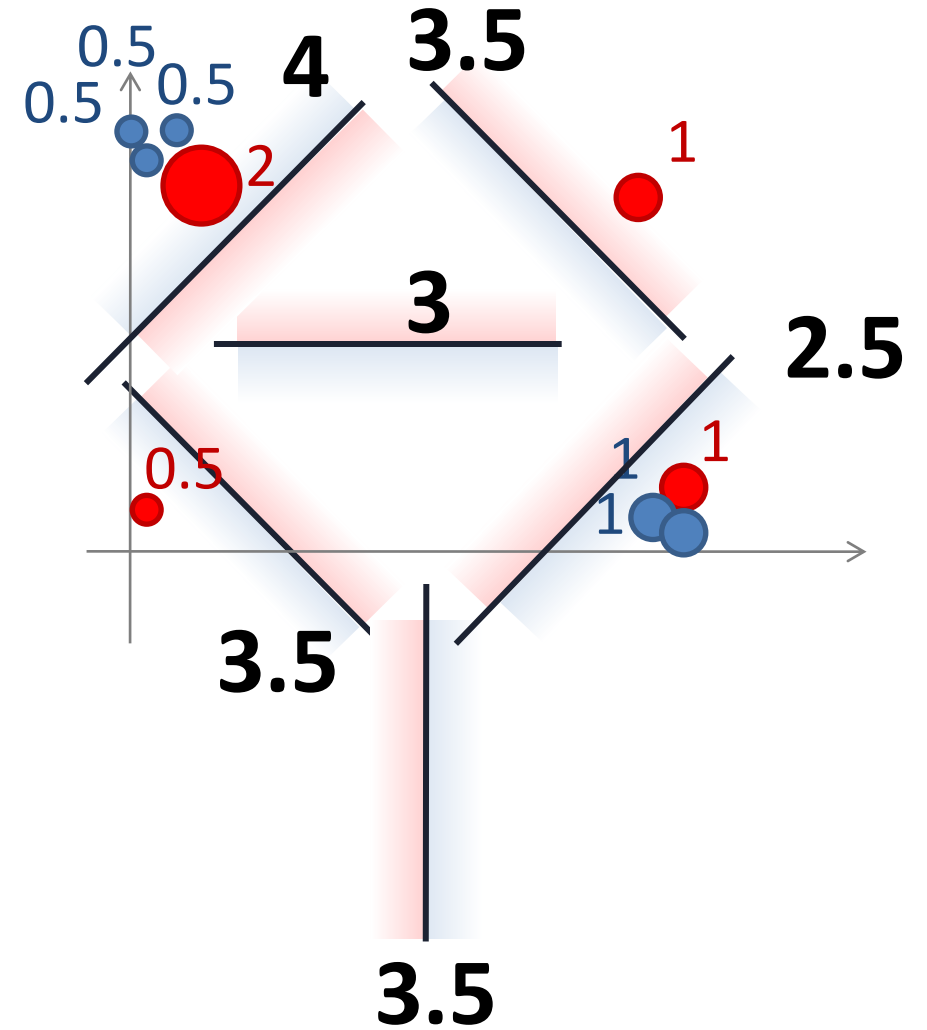
$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\}$$



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Example: step 3

$$h_{t=3}(\bar{x}) =$$

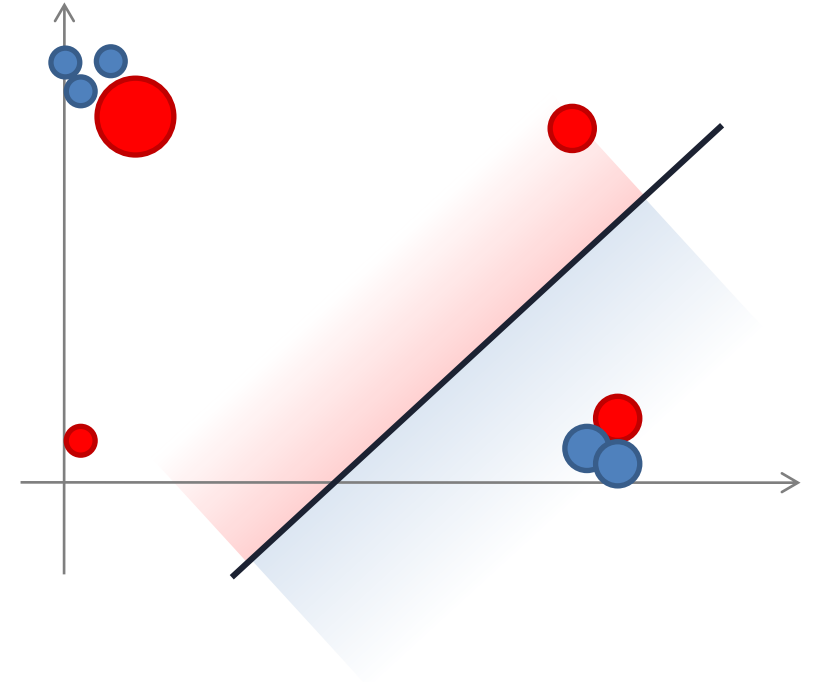


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Example: step 3

$$h_{t=3}(\bar{x}) = \{x_1 - x_2 > 0.5? \text{ red} : \text{blue}\}$$

$$r_{t=3} = 0.38 \quad \alpha_{t=3} = 0.39$$



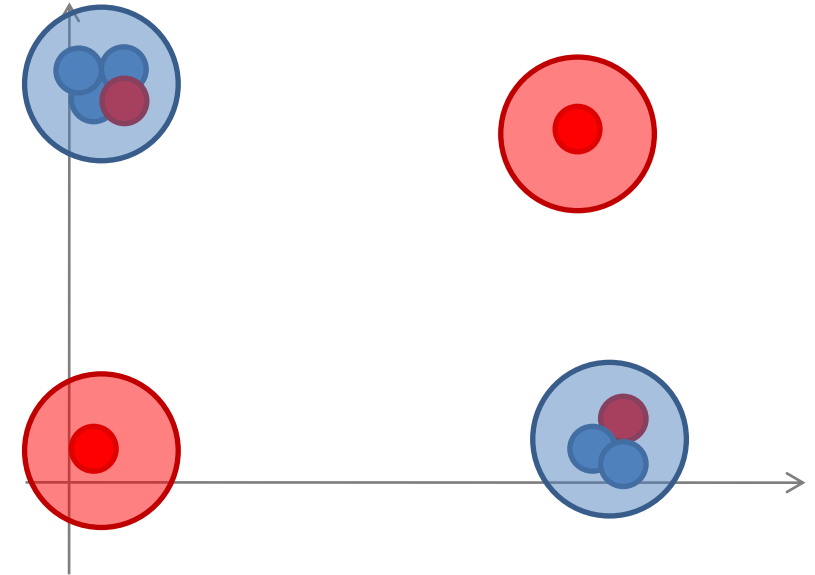
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Example: result

$$k(x) \cong \sum_t \alpha_t \cdot h_t(x)$$

$h_{t=3}$

$h_{t=2}$



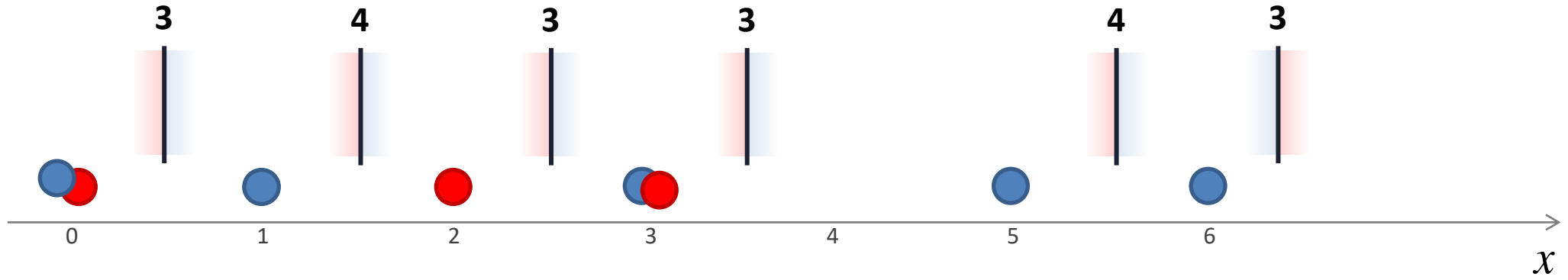
	x	(0,0)	(1,0)	(1,0)	(1,0)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(1,1)
$\alpha_1 = 0.35$	h_1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1
$\alpha_2 = 0.35$	h_2	1	1	1	1	-1	-1	-1	-1	-1	1
$\alpha_3 = 0.39$	h_3	1	-1	-1	-1	1	1	1	1	1	1
$k(x) \cong \sum_t \alpha_t \cdot h_t(x)$		1	-1	-1	-1	-1	-1	-1	-1	-1	1

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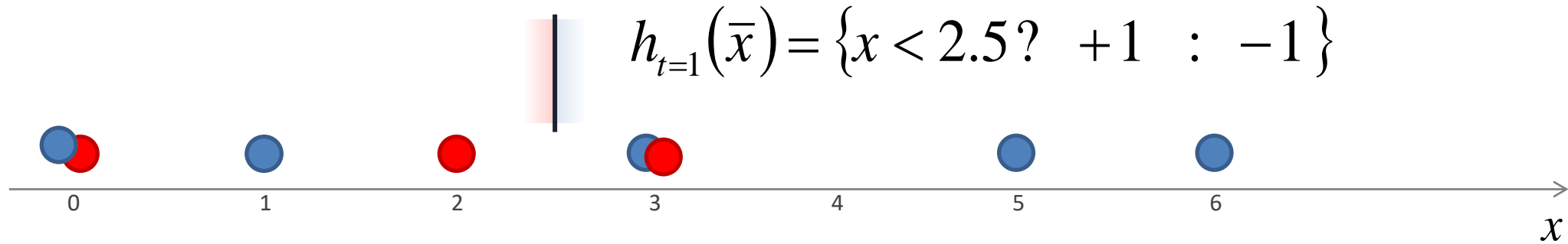
Another example



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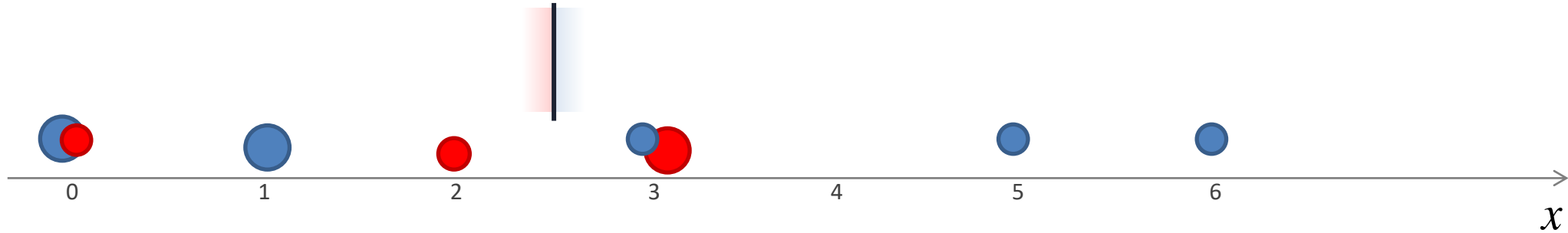


$$r_{t=1} = \frac{1}{\sum_i W_1(i)} \sum_i W_1(i) \cdot h_1(x_i) \cdot k_i = 2/8$$

$$\alpha_t = \frac{1}{2} \log \left(\frac{1+r_t}{1-r_t} \right)$$

$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\} = W_1(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & \text{if } h_1(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & \text{if } h_1(x) \neq k \end{cases}$$

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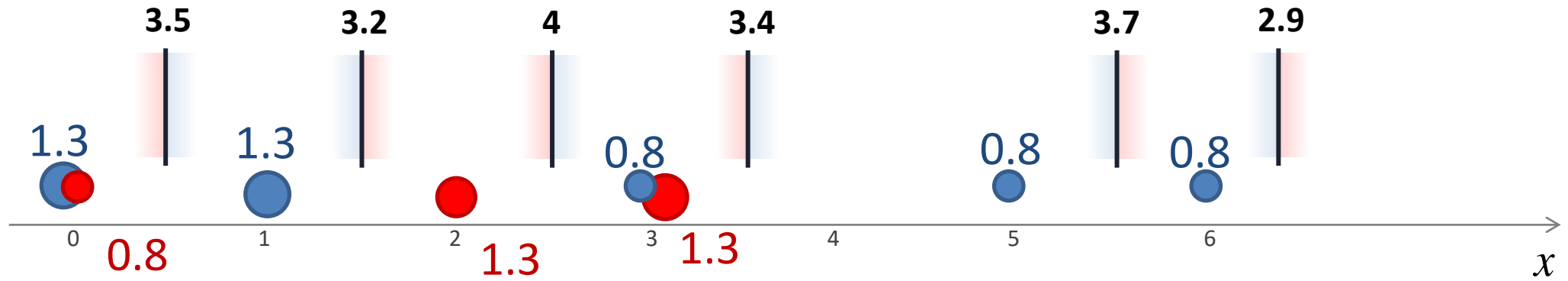


$$r_{t=1} = \frac{1}{\sum_i W_1(i)} \sum_i W_1(i) \cdot h_1(x_i) \cdot k_i = 2/8$$

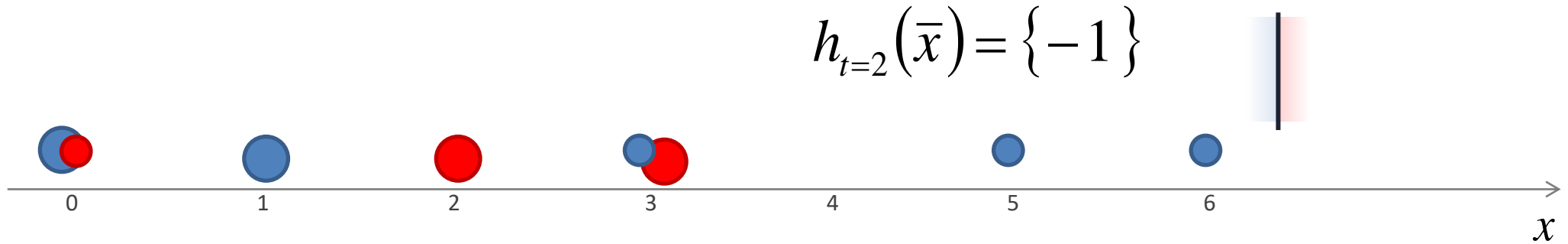
$$\alpha_t = \frac{1}{2} \log \left(\frac{1 + r_t}{1 - r_t} \right)$$

$$W_{t=2}(i) = W_1(i) \cdot \exp\{-\alpha_1 \cdot k_i \cdot h_1(x_i)\} = W_1(i) \times \begin{cases} \sqrt{\frac{3}{5}} & \text{if } h_1(x) = k \\ \sqrt{\frac{5}{3}} & \text{if } h_1(x) \neq k \end{cases}$$

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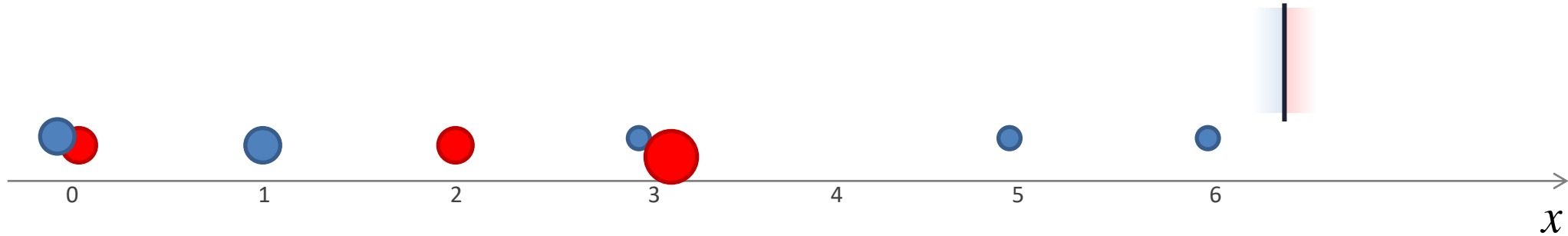


$$W_{t=3}(i) = W_2(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & \text{if } h_2(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & \text{if } h_2(x) \neq k \end{cases}$$

$$r_{t=2} = \frac{1}{\sum_i W_2(i)} \sum_i W_2(i) \cdot h_2(x_i) \cdot k_i = 2.9$$

$$\alpha_t = \frac{1}{2} \log \left(\frac{1+r_t}{1-r_t} \right)$$

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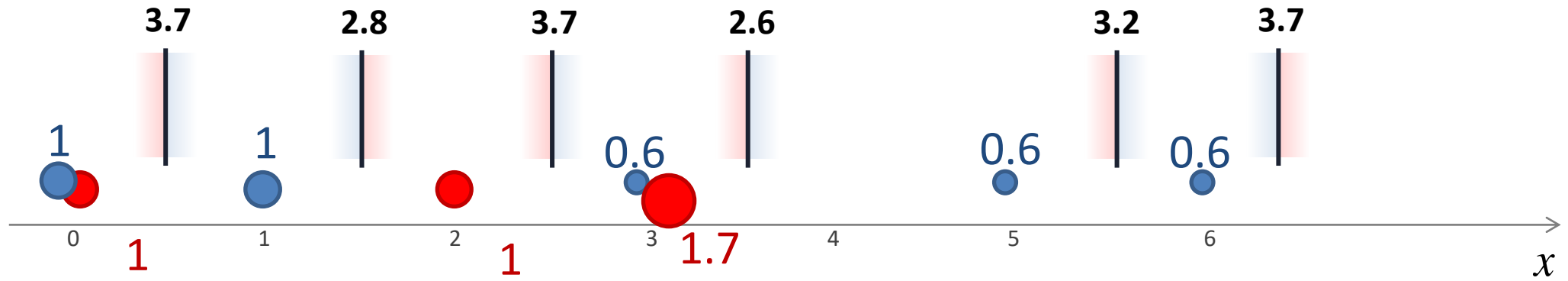


$$W_{t=3}(i) = W_2(i) \times \begin{cases} \sqrt{\frac{1-r_t}{1+r_t}} & \text{if } h_2(x) = k \\ \sqrt{\frac{1+r_t}{1-r_t}} & \text{if } h_2(x) \neq k \end{cases}$$

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