

Basics of Machine Learning

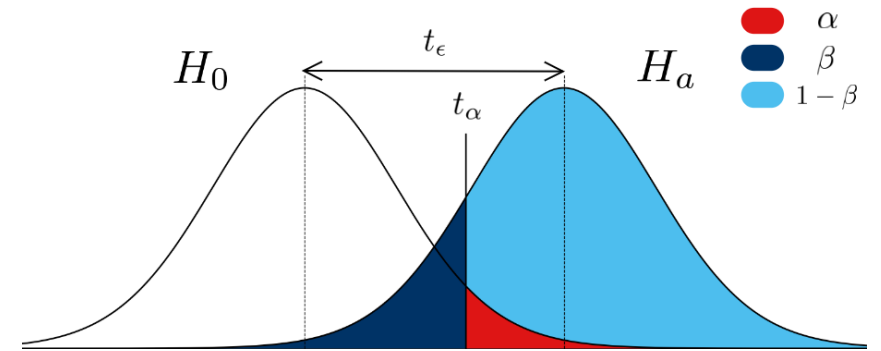
Dmitry Ryabokon, github.com/dryabokon





Lesson 13

Non Bayesian Approach



Non Bayesian Approach

Summary

- Limitations of the Bayesian approach
- Type I error
- Type II error
- Neyman-Pearson approach
- Minimax approach

Neyman-Pearson approach

Limitations of the Bayesian approach

$x \in X$ feature

$k \in K$ hidden state

$p(x|k)$ conditional probability to expose x *by* state k

$p(k)$ priory probability of the state

$W : K \times K \rightarrow R$ penalty function

Neyman-Pearson approach

Limitations of the Bayesian approach

$x \in X$ feature

$k \in K$ hidden state

$p(x|k)$ conditional probability to expose x *by* state k

~~$p(k)$ priory probability of the state~~

~~$W: K \times K \Rightarrow R$ penalty function~~

Neyman-Pearson approach

Limitations of the Bayesian approach: example

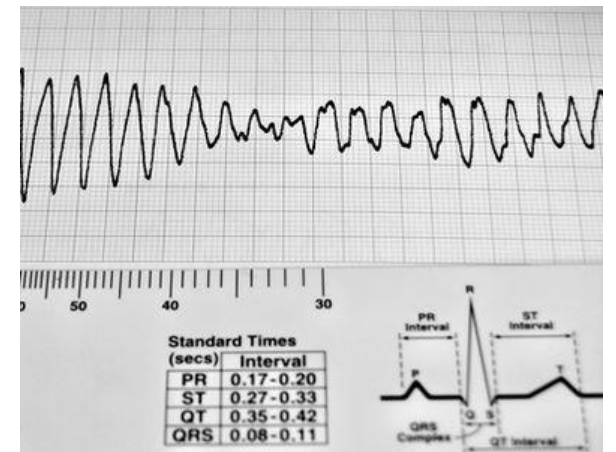
$x \in X$ heartbeat

$k \in K$ state: normal, sick

$p(x|k)$ distribution of the heartbeat for
normal and sick

$p(k)$???

W(normal, sick) ???
W(sick, normal)



Neyman-Pearson approach

The problem

$$x \in X \quad k \in K = \{normal, risky\}$$

$$p(x|k = norm) \quad X_{norm} \cup X_{risk} = X$$

$$p(x|k = risk) \quad X_{norm} \cap X_{risk} = \emptyset$$

input

$$error\ type\ 1 = P(false\ alarm) = \sum_{x \in X_{risk}} p(x|norm) \rightarrow min$$

$$error\ type\ 2 = P(miss\ the\ risk) = \sum_{x \in X_{norm}} p(x|risk) \leq \varepsilon$$

$$X_{norm}, X_{risk} = ?$$

output

Neyman-Pearson approach

The problem

$$x \in X \quad k \in K = \{normal, risky\}$$

$$\alpha_{norm}(x) = \begin{cases} 1 & \text{when } x \in X_{norm} \\ 0 & \text{when } x \notin X_{norm} \end{cases}$$

$$\alpha_{risk}(x) = \begin{cases} 1 & \text{when } x \in X_{risk} \\ 0 & \text{when } x \notin X_{risk} \end{cases}$$

Neyman-Pearson approach

The problem

$$\begin{aligned} \sum_{x \in X_{risk}} p(x|norm) &\rightarrow \min \\ \sum_{x \in X_{norm}} p(x|risk) &\leq \varepsilon \end{aligned}$$

$$\left\{ \begin{array}{l} \sum_{x \in X} \alpha_{risk}(x) \cdot p(x|norm) \rightarrow \min \\ - \sum_{x \in X} \alpha_{norm}(x) \cdot p(x|norm) \geq -\varepsilon \\ \alpha_{norm}(x) + \alpha_{risk}(x) = 1 \quad \forall x \in X \\ \alpha_{norm}(x) \geq 0 \quad \forall x \in X \\ \alpha_{risk}(x) \geq 0 \quad \forall x \in X \end{array} \right.$$

Neyman-Pearson approach

$p_1 \cdot x_1 + \dots + p_n \cdot x_n \rightarrow \max$	$b_1 \cdot y_1 + \dots + b_m \cdot y_m \rightarrow \min$
$a_{11} \cdot x_1 + \dots + a_{1n} \cdot x_n \leq b_1$	$y_1 \geq 0$
$a_{k1} \cdot x_1 + \dots + a_{kn} \cdot x_n \leq b_k$	$y_k \geq 0$
$a_{k+1,1} \cdot x_1 + \dots + a_{k+1,n} \cdot x_n = b_{k+1}$	$y_{k+1} - \text{св. перем.}$
$a_{m1} \cdot x_1 + \dots + a_{mn} \cdot x_n = b_m$	$y_m - \text{св. перем.}$
$x_1 \geq 0$	$a_{11} \cdot y_1 + \dots + a_{m1} \cdot y_m \geq p_1$
$x_s \geq 0$	$a_{1s} \cdot y_1 + \dots + a_{ms} \cdot y_m \geq p_s$
$x_{s+1} - \text{св. перем.}$	$a_{1,s+1} \cdot y_1 + \dots + a_{m,s+1} \cdot y_m = p_{s+1}$
$x_n - \text{св. перем.}$	$a_{1n} \cdot y_1 + \dots + a_{mn} \cdot y_m = p_n$

Neyman-Pearson approach

$p_1 \cdot x_1 + \dots + p_n \cdot x_n \rightarrow \max$	$b_1 \cdot y_1 + \dots + b_m \cdot y_m \rightarrow \min$
$a_{11} \cdot x_1 + \dots + a_{1n} \cdot x_n \leq b_1$	$y_1 \geq 0$
$a_{k1} \cdot x_1 + \dots + a_{kn} \cdot x_n \leq b_k$	$y_k \geq 0$
$a_{k+1,1} \cdot x_1 + \dots + a_{k+1,n} \cdot x_n = b_{k+1}$	$y_{k+1} - \text{св. перем.}$
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$x_1 \geq 0$	$a_{11} \cdot y_1 + \dots + a_{m1} \cdot y_m \geq p_1$
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$x_{s+1} - \text{св. перем.}$	$a_{1,s+1} \cdot y_1 + \dots + a_{m,s+1} \cdot y_m = p_{s+1}$
$x_n - \text{св. перем.}$	$a_{1n} \cdot y_1 + \dots + a_{mn} \cdot y_m = p_n$

$$(a_{11} \cdot x_1 + \dots + a_{1n} \cdot x_n - b_1) \cdot y_1 = 0$$

$$(a_{11} \cdot y_1 + \dots + a_{m1} \cdot y_m - p_1) \cdot x_1 = 0$$

Neyman-Pearson approach

Solution

$$\left\{ \begin{array}{l} \sum_{x \in X} \alpha_{risk}(x) \cdot p(x|norm) \rightarrow \min \\ - \sum_{x \in X} \alpha_{norm}(x) \cdot p(x|norm) \geq -\varepsilon \\ \alpha_{norm}(x) + \alpha_{risk}(x) = 1 \quad \forall x \in X \\ \alpha_{norm}(x) \geq 0 \quad \forall x \in X \\ \alpha_{risk}(x) \geq 0 \quad \forall x \in X \end{array} \right.$$

$$\left\{ \begin{array}{l} -\varepsilon \cdot \tau + 1 \cdot t(x_1) + \dots + 1 \cdot t(x_i) \rightarrow \max \\ \tau \geq 0 \\ t(x) \text{ free var} \\ -p(x|risk) \cdot \tau + 1 \cdot t(x) \leq 0 \\ 1 \cdot t(x) \leq p(x|norm) \end{array} \right.$$

Neyman-Pearson approach

Solution

$$\begin{aligned} x \in X_{norm} &\Rightarrow \alpha_{norm}(x) = 1, \alpha_{risk}(x) = 0 &\Rightarrow &\begin{cases} -p(x|risk) \cdot \tau + 1 \cdot t(x) = 0 \\ 1 \cdot t(x) < p(x|norm) \end{cases} &\Rightarrow \\ \Rightarrow &p(x|risk) \cdot \tau \leq p(x|norm) \end{aligned}$$

$$\begin{aligned} x \in X_{risk} &\Rightarrow \alpha_{norm}(x) = 0, \alpha_{risk}(x) = 1 &\Rightarrow &\begin{cases} -p(x|risk) \cdot \tau + 1 \cdot t(x) \leq 0 \\ 1 \cdot t(x) = p(x|norm) \end{cases} &\Rightarrow \\ \Rightarrow &p(x|risk) \cdot \tau \geq p(x|norm) \end{aligned}$$

Neyman-Pearson approach

Solution

$$\frac{p(x|norm)}{p(x|risk)} \underset{risk}{\overset{norm}{\geq}} \tau$$

Neyman-Pearson approach

Some Variations: two **risky** states

$$\sum_{x \in X_{risk}} p(x|norm) \rightarrow min$$

$$\sum_{x \in X_{norm}} p(x|risk1) \leq \varepsilon$$

$$\sum_{x \in X_{norm}} p(x|risk2) \leq \varepsilon$$

Neyman-Pearson approach

Some Variations: two **normal** states

$$\sum_{x \in X_{risk}} p(x|norm1) + \sum_{x \in X_{risk}} p(x|norm2) \rightarrow \min$$

$$\sum_{x \in X_{norm}} p(x|risk) \leq \varepsilon$$

Neyman-Pearson approach

Minimax problem

$$\max \left\{ \sum_{x \notin X_1} p(x|1) , \sum_{x \notin X_2} p(x|2) \dots, \sum_{x \notin X_K} p(x|K) \right\} \rightarrow \min$$