

# Perspective Projections Project

## Background

Consider an image  $P(X,Y)$  that lies on the following 3D plane:

$$Z = aX + bY + c, \quad (1)$$

The perspective projection of  $P(X,Y)$  is the image  $Q(u,v)$ . In this project you are asked to compute  $Q$  from  $P$ , the camera calibration parameters  $f, u_0, v_0$ , and the plane parameters  $a, b, c$ . It can be shown that the inverse transformation is given by the following equations:

$$X = \frac{c(u - u_0)}{f - a(u - u_0) - b(v - v_0)} = \frac{cx}{f - ax - by}, \quad Y = \frac{c(v - v_0)}{f - a(u - u_0) - b(v - v_0)} = \frac{cy}{f - ax - by}. \quad (2)$$

## Part I

The function calculates the  $3 \times 3$  matrix of a perspective transform so that:

$$\begin{bmatrix} t_i x'_i \\ t_i y'_i \\ t_i \end{bmatrix} = \text{map\_matrix} \cdot \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

where

$$\text{dst}(i) = (x'_i, y'_i), \text{src}(i) = (x_i, y_i), i = 0, 1, 2, 3$$

Write a program (OpenCV, python) that gets as input  $P, f, u_0, v_0$ , and  $a, b, c$  from the command line and displays  $Q$ . It should be executed as follows:

```
python3 mynetid1.py image f u0 v0 a b c
```

First, we want to get the  $Q$ . We have two ways.

- get  $u, v$  from (2)
- simply, change the value in `map_matrix` below, and project it

In this formula,  $x' = u, y' = v$  in  $Q(u,v)$   $x = X, y = Y$  in  $P(X,Y)$   $t = Z$  in (1)

`map_matrix`

```
[[c1,c2,c3],
```

```
[c4,c5,c6],
```

```
[c7,c8,c9]]
```

$$u = x' = (c1x+c2y+c3)/t \quad v = y' = (c4x+c5y+c6)/t \quad Z = t = (c7x+c8y+c9)$$

c1~c6: camera parameter c1 c5: focus parameter c3,c6: u0,v0

c7 = a, c8 = b, c9 = c

The map\_matrix will be set as  $\begin{bmatrix} f & 0 & u_0 \\ 0 & f & -v_0 \\ a & b & c \end{bmatrix}$

## Part II

Write a program (OpenCV, python) that gets as input the image P and c from the command line.  
(All other arguments are hardcoded.)

python3 mynetid2.py image c

The two images to be displayed are defined as follows:

$$Z = aX + bY + c$$

### 1. The left side of the image has a smaller Z value than the right side.

We need to create a plan that slop is positive along the axis x.

Set a = 0.001

map\_matrix

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.001 & 0 & c \end{bmatrix}$

### 2. The bottom side of the image has a smaller Z value than the top side.

We need to create a plan that slop is negative along the axis y.

Set b = -0.001

map\_matrix

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -0.001 & c \end{bmatrix}$