Midterm 2 Reference Sheet

Algorithm 1 The Benjamini-Hochberg Procedure

input: FDR level α, set of n p-values $P_1, \dots, P_n$

Sort the p-values $P_1, \dots, P_n$ in non-decreasing order $P_{(1)} \le P_{(2)} \le \cdots \le P_{(n)}$

Find $K = \max\{i \in \{1, \dots, n\} : P_{(i)} \le \frac{\alpha}{n} i\}$

Reject the null hypotheses (declare discoveries) corresponding to $P_{(1)}, \dots, P_{(K)}$

Useful Distributions:

Distribution	Support	PDF/PMF	Mean	Variance	Mode
$X \sim \text{Poisson}(\lambda)$	$k = 0, 1, 2, \dots$	$\frac{\lambda^k e^{-\lambda}}{k!}$	λ	λ	$\lfloor \lambda \rfloor$
$X \sim \text{Gamma}(\alpha, \beta)$	$x \ge 0$	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$\frac{\alpha-1}{\beta}$
$X \sim \mathcal{N}(\mu, \sigma^2)$	$x \in \mathbb{R}$	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	μ	σ^2	μ
$X \sim \text{Exponential}(\lambda)$	$x \ge 0$	$\lambda \exp(-\lambda x)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	0

Conjugate Priors: For observations x_i, $i = 1, \dots, n$:

Likelihood	Prior	Posterior
$x_i \vert \theta \sim \text{Bernoulli}(\theta)$	$\theta \sim \text{Beta}(\alpha, \beta)$	$\theta \vert x_{1:n} \sim \text{Beta}\left(\alpha + \sum_i x_i, \beta + \sum_i (1 - x_i)\right)$
$x_i \vert \mu \sim \mathcal{N}(\mu, \sigma^2)$	$\mu \sim \mathcal{N}(\mu_0, 1)$	$\mu \vert x_{1:n} \sim \mathcal{N}\left(\frac{\sigma^2}{\sigma^2+n}\left(\mu_0 + \frac{1}{\sigma^2}\sum_i x_i\right), \frac{\sigma^2}{\sigma^2+n}\right)$
$x_i \vert \lambda \sim \text{Exponential}(\lambda)$	$\lambda \sim \text{Gamma}(\alpha, \beta)$	$\lambda \vert x_{1:n} \sim \text{Gamma}\left(\alpha + n, \beta + \sum_i x_i\right)$

Generalized Linear Models

Regression	Inverse link function	Likelihood
Linear	identity	Gaussian
Logistic	sigmoid	Bernoulli
Poisson	exponential	Poisson
Negative binomial	exponential	Negative binomial

Sigmoid function: $\sigma(x) = \dfrac{1}{1 + e^{-x}}$