1 School Funding, Again (6 points)

In HW2, we used a Bayesian hierarchical model for the state-level average funding gap, treating the state-level variance as known. In this question, we'll instead examine a hierarchical model for the state-level variances, treating the means μ_i as known and fixed:

$$y_{ij} \mid \sigma_i \overset{\text{iid}}{\sim} \mathcal{N}(\mu_i, \sigma_i^2)$$
$$\sigma_i^2 \sim \text{InverseGamma}(\alpha, \beta)$$

The **inverse Gamma** distribution is the conjugate prior for the variance of a normal distribution when the mean is known. *Hint: information about the parameters of the inverse Gamma and the posterior distribution is provided on the reference sheet, but you should be able to answer this entire question without doing any computation.*

In this question, we'll compare three approaches:

(1) A frequentist model where we treat each σ_i as fixed but unknown, and estimate each one separately from the data for the corresponding state using maximum likelihood estimation (MLE).

(2) A Bayesian model where α and β are chosen using empirical Bayes.

(3) A fully hierarchical Bayesian model where we treat α and β as random variables, with exponential priors for each one. Note that the exponential distribution is not the conjugate prior for either parameter of the inverse gamma distribution.

(a) [3 Pts] **For this part, only consider models (1) and (2)**. Which of the following statements are true? Select all answers that apply. No work or explanations will be graded for this question.

- ■ **For large states (e.g., California), the MLE frequentist estimates from model (1) and MAP Bayesian estimates from model (2) will be similar.**
- ☐ The MLE frequentist estimates from model (1) for small states will depend on the choice of α and β.
- ■ **Model (1) is an example of an approach with no pooling.**

(b) [3 Pts] **For this part, only consider model (3)**. Which of the following statements are true? Select all answers that apply. No work or explanations will be graded for this question.

- ■ **Let $Y = y_{11}, \ldots, y_{nm}$ be the collection of all observed funding gaps for all districts in all states. Then the posterior distribution is $p(\alpha, \beta, \sigma_1, \ldots, \sigma_n \mid Y)$.**
- ☐ The posterior distribution can be computed exactly using numerical techniques: no approximation is necessary.
- ■ **The posterior distribution for α and β represents nation-wide information about the variation in funding gaps.**
- ☐ Model (3) is an example of complete pooling.