HW 1

due Mon Jan 12

Modeling Democracy
Duchin, Winter 2026

General instructions: Please TeX your solutions. Ask for help with math or TeX or anything else if needed!

Problem 1. Practice all of the voting systems we discussed on this simple reduced preference schedule. (That is: Plurality, PWC, Borda, Condo-Borda, Top-Two, IRV, Sequential, STV, Coombs, Secondality, Smith, Beatpath, Dodgson, Kemeny, Dictatorship, Ranked Pairs, and any others of your choice.) For some of these you will have to specify ancillary information, like a sequential order or a tiebreaking protocol.

Explain your answers, of course.

1	2	3	1
A	*B*	*D*	*A*
B	*C*	*A*	*C*
C	*A*	*C*	*B*
D	*D*	*B*	*D*

Problem 2. Show that domsets are nested. (That is, if X and Y are both dominating sets for a given preference profile, then $X \subseteq Y$ or $Y \subseteq X$.)

Problem 3. Show that the winner of a sequential election is always "strong" (i.e., belongs to the Smith set).

Problem 4. Show that a Condorcet candidate can be a losing spoiler.

Problem 5.

(a) Show that beatpath elimination ($\triangleright$) is transitive.

(b) Conclude that the beatpath method is well-defined (same winner(s) no matter what order you consider candidates in) and that $|\mathcal{W}| \geq 1$.

(c) Show that the beatpath method has the unanimity property.

(d) Show that beatpath is Smith-fair.