Simulating electoral system changes and how voters might respond

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October 14, 2024

Abstract

A crucial determinant of electoral system choice is how many seats a specific political party would win under some alternative electoral system, but this quantity has long been estimated using a simple approach that is known to have multiple major flaws. I address these problems by developing a computational formal model that estimates how the seat totals in a particular Single-Member District Plurality election might have been different if the election had been held under another electoral system, explicitly simulating how strategic voters might have voted differently. The model is falsifiable, and outperforms the widespread simple approach in estimating seat counts after New Zealand's electoral system change. I apply the model to Canada and Britain, simulating elections under Borda count and Mixed-Member Proportional systems. The results suggest that parties do not consistently support the systems that would win them the most seats.

*Massachusetts Institute of Technology, sbaltz@umich.edu. Walter R. Mebane, Jr. provided an enormous volume of helpful feedback and suggestions on this work, as did Allen Hicken, Iain Osgood, and Scott E. Page. The work also benefited at various stages from comments by Rob Franzese, Jean-François Godbout, Mai Hassan, Jonathan Katz, Michael Lerner, Lawrence LeDuc, James D. Morrow, Lucia Motolinia, Lori Thorlakson, Michael Thompson-Brusstar, George Tsebelis, Fabricio Vasselai, and Patrick Y. Wu. Work on this paper was facilitated by the Rackham Predoctoral Fellowship and the Michigan Institute for Computational Discovery and Engineering Fellowship, and it builds on models that were created under NSF award SES 1523355.

1 Introduction

A democracy's fundamental characteristics are shaped by its choice of electoral system, since the rules that turn votes into winners and losers will determine who governs the country (Taagepera, 2007). When stable democracies switch electoral systems, it is often because self-interested political parties conclude that a different system would improve their electoral performance (Ahmed, 2012, Benoit, 2004, Boix, 1999). The "empirical question" of how well a given party would perform under a different electoral system is therefore central to understanding how and why these institutions change (Cochrane, 2017, p. 42). And yet, there is no consensus for how to estimate the number of seats that a *specific* party might win under a *specific* alternative electoral system.

Parties support or oppose alternative electoral systems according to "a virtual iron law of political self-preservation" (Nagel, 1994b, p. 525), for example by evaluating their "expected seat shares under each electoral system alternative" (Benoit, 2004). But when it comes to estimating the precise effects of alternative electoral systems, Taagepera (2007, p. 36) gives voice to the conventional wisdom: "no one can predict their actual workings — and you kid yourself if you think that you can". Without any consensus for how to approach this problem, academics, journalists, and policymakers have relied for decades on a simple approach: they take the votes cast in an election under one electoral system, assume that *the only change* is the rule for turning votes into winners and losers, and simply apply a different rule to the votes that were cast. This ubiquitous approach has never been assessed empirically, and in fact it has serious and well-known problems: it cannot be applied to electoral system changes which require more information than just the votes cast in a previous election, and it incorrectly assumes that people vote the same way under different electoral systems (Blais et al., 2012).

I introduce a simulation-based method that resolves both major problems with the simple approach, building on work that has demonstrated that computational models

of elections can generate results that resemble real vote counts (Bendor et al., 2011, ch. 7, Laver and Sergenti, 2012, ch. 11, Siegel, 2018, p. 748 – 749). I show that this computational formal model is a more accurate way to estimate how many seats a specific political party would win under a different electoral system. The model combines public opinion data, game theoretic models of vote choice, and local-level election results to estimate how a specific party would have performed in a specific Single-Member District Plurality (SMDP) election if that election had instead occurred under either Borda count or Mixed-Member Proportional election systems. I derive and explicitly model strategic voting under those different systems, and I vary how many people vote strategically. The model is falsifiable and accurate: its results under SMDP closely match the results of Canadian and British elections, and when applied to the 1996 electoral system change in New Zealand, it assigns 115 out of 120 seats to the correct parties, whereas the simple approach only assigns 104 seats correctly.

I apply the model to Canada and Britain, where recent failed attempts to change the national legislative electoral system illustrate the lack of shared expectations about seat shares under alternative systems. A particularly dramatic episode played out in Canada around 2015: the historically dominant Liberal Party of Canada was reeling from its worst election ever, and its leader promised that the upcoming election would be "the last federal election using" SMDP (Kohut, 2016). The Liberals thereby joined two other left-of-centre parties, the New Democratic Party (NDP) and the Green Party, who had both long supported replacing SMDP. But even after these three left-of-centre parties together won a supermajority in parliament, they could not agree on *which* alternative system to adopt. The Liberal Party insisted on a ranked voting system, but the NDP and Greens would only support proportional representation (Small, 2020). Does this mean no electoral system would mutually benefit these parties? Or, did they overlook a system that would boost the left at the expense of the right?

If Canada's left-of-centre parties had precise expectations about the effects of different systems, they do not seem to match the expectations evinced by several British parties during a 2011 referendum on Instant Runoff Voting (in that context called the

Alternative Vote). Like Canada's NDP and Greens, Britain's Liberal Democrats and the Scottish National Party (SNP) had long supported proportional representation. However, unlike the NDP and Greens, they both supported ranked voting over SMDP. And whereas the Liberal Party of Canada would only support Instant Runoff, Britain's Labour Party was dramatically divided on the issue.

These cases illustrate a deep problem in one of the fundamental questions about how democracy works. Because we lack shared expectations about the practical effects of electoral systems for those who have the power to adopt them, it is difficult to understand which parties will support a change and why. Indeed, parties often appear to support changes that would be bad for them or oppose changes that would be good for them (Evci and Kaminski, 2020). Of course, parties' motivations are undoubtedly not *just* a matter of short-term seat-maximization (Ahmed, 2012, p. 69); parties may judge electoral systems for many other reasons, including ideological commitments, possible coalition dynamics, public opinion, or the fate of a specific political entrepreneur. However, a party's expected seat count under different alternative systems is commonly thought to be the most important factor in understanding electoral system choice in stable democracies (Cusack et al., 2007).

This article makes three main contributions. First, I identify a simple method that has quietly dominated our understanding of how a change in electoral systems would affect the results of a country's elections. This method has been used for decades by academics, journalists, and even policymakers, who have applied it to countries around the world. This is the first paper to not only identify the ubiquity of this method, but to point out that its users almost universally note exactly the same problems with it. I then develop and implement solutions to those problems, I show that the result is a more accurate method, and I use that more accurate method to draw substantive conclusions about electoral system choice.

These substantive conclusions are the paper's second contribution: I estimate how the major Canadian and British parties would perform under either a type of ranked voting (Borda count) or a type of proportional representation (Mixed-Member Propor-

tional), including voters' strategic responses to the new electoral system. In Canada the conventional wisdom, shaped by the simple method, says that if "ranked ballots" were used in Canada, the centre-left Liberals "would be guaranteed a place in government forever", while the only hope for the parties to their left is proportional representation (Dutil, 2017). However, the model results suggest that at least one ranked ballot system — Borda count — might be much better for those smaller parties than the *status quo*, while Mixed-Member Proportional may be better for the two largest parties than expected. In Britain, the simulated results under both alternative systems are strikingly similar to the real SMDP results.

To assess how generalizable these results might be to other electoral systems, I compare the Borda count results to simulations of another ranked voting system (Instant Runoff Voting), and the Mixed-Member Proportional results to another proportional system (Multi-Member Districts). Methodological limitations make these models more speculative, but the results are broadly similar. I also offer general takeaways for the study of electoral systems beyond those two countries, including a suggestion that strategic voting under Borda count may be especially beneficial to small but broadly palatable parties.

The article's third contribution is to the formal theory of vote choice. I introduce a computational modeling infrastructure that samples real preferences reported in a survey, assigns them to simulated voters, and then uses game theoretic models of strategic vote choice to conduct elections in which varying proportions of voters make their vote choices strategically. Eggers and Nowacki (2023) have used similar ideas to estimate features of voting in different electoral systems; I hold the number of strategic voters fixed and instead focus on the outcome of parties' seat counts under different systems. I introduce two types of empirical comparisons that demonstrate the accuracy of such models. I also supplement precisely strategic models with models based on a simple voting heuristic, and I demonstrate that the results do not depend on heroic assumptions of voter rationality, since the heuristic and the precisely strategic models of voting in SMDP produce almost identical results. Together these contributions form

a computer modeling framework that can be used to estimate party seat counts after a switch away from SMDP.

I emphatically do not claim to perfectly predict the results of electoral system changes. These events are notoriously challenging objects of study, and are famously multi-causal with other features of a country's politics (Benoit, 2007, Colomer, 2004). But the fact that electoral systems are complicated and highly multi-causal should not prevent us from improving the methods that shape how we understand them. Instead, I follow Franzese's (2009, p. 67) recommendation: "context matters, so model it!" While I model how voters might respond to electoral system changes, I do not address how larger institutions like party systems might shift or the resulting changes in voters' preferences, so the model only estimates election results *shortly* after an electoral system change. It is also only a model of switches away from SMDP in legislative elections, applied to three changes in three Westminster-style democracies. Within that scope, however, the model a) is accurate, b) is more accurate than the existing method, and c) is a necessary first step towards more sophisticated models with looser scope conditions.

2 The preferences problem and fixed voters

Questions about the specific effects of electoral systems are notoriously hard to answer, since established democracies have overhauled their electoral systems only a few dozen times in modern history (Colomer, 2004, p. 54). Nevertheless, researchers have developed a number of empirical approaches to understand the connection between electoral systems and important features of a country's politics. With methods like comparing a country's politics before and after an electoral system change (Nagel, 1994b), comparing countries that use different systems (Norris, 1997), experiments in which people are asked to vote under varying rules (Blais et al., 2012, Blumenau et al., 2017), and natural experiments (Bechtel et al., 2016, Eggers, 2015, Ponattu, 2018, Sanz, 2017), researchers have identified connections between electoral systems and the number of

viable parties (Duverger, 1951, Book 2, Ch. 1, §1, Lijphart, 1994, ch. 5, Shugart and Taagepera, 2017, ch. 4), voter turnout (Cox, 2015, Eggers, 2015, Gosnell, 1930), and representation (Krook, 2018, Lublin and Bowler, 2018).

However, a method that estimates how an electoral system change might affect (for example) the party system provides a somewhat roundabout answer to the question of how each *specific* party's fortunes might change, and this is the crucial quantity that determines whether party leaders — and even voters (Riambau et al., 2021) — decide whether or not to support a reform.

To be sure, some scholars have productively used this approach. For example, Vowles et al. (1995, ch. 11) used Nagel's (1994a) estimate of how *many* parties would survive New Zealand's system change to estimate how many seats each *specific* party might win. Most authors, however, have pursued a much simpler and more direct idea: simply take the votes that were cast in a previous election, apply a different electoral system to those votes, and treat the resulting number of seats as an estimate of election results under that system. So, for example, if Canada's New Democratic Party received 30% of the votes in an election, a researcher simulating a perfectly proportional system using the simple approach would assign it 30% of the seats.

It is difficult to overstate how widely used this simple approach has become — it is the main way that researchers make conjectures about how an electoral system change might affect a country's parties — without ever receiving serious theoretical scrutiny. Appendix §1 lists examples of the simple approach, and shows that it has been used for decades, applied across nearly every continent, and appeared in academic articles, policy analyses, and public commentaries.

Unfortunately, the simple approach has a major limitation that can make it difficult or impossible to use, and even when it is possible, it requires a crucial assumption that is known to be wrong. The first problem is what I call the "preferences problem": some major alternatives to SMDP, like ranked voting systems, ask voters for more information than just their top vote choice. To use votes in a single-vote system to estimate votes in a multi-vote system, we need more information about voters'

preferences.

A second problem, which affects every alternative system, is what I call the "fixed voters assumption". If we apply the rules of one electoral system to votes that were cast under a different electoral system, we treat voters as though they were rigid constants unaware of the context in which they vote. This assumption is flatly contradicted by the evidence (Blais et al., 2012),[1] and those who use the simple approach consistently describe the fixed voters assumption as a necessary evil.

The authors of analyses that have used the simple approach, many of which are listed in Appendix §1, consistently identify both problems. LeDuc (1999, p. 76) assumes "that votes cast [after a reform would be] similar" to those cast in a real election, Grenier (2015) assumes "that voters do not behave much differently" under different systems, and Jansen (2015) assumes that if SMDP voters had instead voted in a ranked system, they "would have cast the same first-preference votes", as do Miljan and Jackson (2016, p. 53 – 55) and Fournier (2019). None of these authors defend the fixed voters assumption, and many criticize it. LeDuc (1999, p. 76), after invoking the fixed voters assumption, writes that "[o]f course, such an assumption is not entirely reasonable." Blumenau and Hix (2015) go further: to them the fixed voters assumption is "completely unreasonable!"

Previous authors have even identified the same core theoretical threat to the fixed voters assumption: some electoral system reforms are "certain [...] to encourage strategic voting" (LeDuc, 1999, p. 76). Miljan and Jackson (2016) write that "we cannot know how elections in the past may have been affected by [...] strategic decisions", while Brandenburg (2019) argues that "the most obvious flaw" of the fixed voters assumption is that, under a different system, there are "very different needs for tactical voting". What is the connection between the fixed voters assumption and strategic voting? Non-strategic voters will always choose their sincere top preference, so only strategic voters should be systematically affected by changes in electoral systems. By

[1]More precisely, the assumption is of "fixed votes", because voters could be exactly exchanging votes without changing the aggregate vote totals. However, the assumption is usually framed as voters not changing their votes when the system changes.

modifying the probability that a party will win a seat if it receives a certain number of votes, a change in electoral systems also changes the expected utility of a voter's possible vote choices.[2] Of course, not everyone votes strategically, but some people do; so, not everyone would change how they vote if the electoral system changed, but some people would.

There is, of course, an analog to the fixed voters assumption that must be acknowledged: the simple method does not just hold *voters* fixed, in fact it holds *everything* fixed. So why should we single out the "fixed voters assumption"? Are we not actually dealing with a "fixed everything assumption"? I will give two reasons to focus on the fixed voters assumption specifically.

The first reason is that votes are the outcome of interest, and other events can only affect election results *by changing peoples' votes.* As Vowles et al. (1995, p. 13) explain, second-order effects of system changes, like the creation of new major parties, depend "ultimately on the behaviour of voters and on the institutions which determine how votes are cast and counted". Indeed, these indirect changes might not even occur; while considering proportional representation in Canada, Stephenson (2016) notes that "so many parties already exist in Canada, the question is not so much whether more options will crop up under a new [more proportional] system, but whether voters will shift their support to some of the parties that are already available under different electoral rules." Challenging the fixed voters assumption and seriously examining the direct connection between electoral systems and vote choice is the natural first step in understanding the larger multi-causal system.

Second, however, there is already a long history of researchers studying the most important indirect changes that would affect vote choice, including simulations of how parties might adapt to electoral system changes (Baker and Scheiner, 2007). By focusing on voter adaptation, this article studies the neglected mirror image of that better-studied question. Even so, recognizing that the connection between electoral

[2]By strategic voting, I mean any process in which voters consult the behaviour of other voters to try to cast an optimal vote.

systems and vote choice is just one small portion of an interconnected system of multi-causal changes that might transpire after an electoral system change, I emphasize a simple scope constraint: this article's model of how voters might respond to an electoral system change should be understood as a model of what might happen *soon* after that change is introduced. Voters can react to such a change quickly, but if parties have sufficient time to form, collapse, or jockey for position in a reshaped party system, they could drag the election results away from the model's short-term expectations.[3]

How short exactly is the short-term? One way to think about these models is as the jumping-off point for broader institutional drift. Party leaders might want to know how people are likely to vote under the new system, before adjusting their strategies in ways that restructure the party system. These models provide that initial expectation. They are, however, not *just* a thought experiment, or fodder for elite expectations: after introducing a modeling framework in the next section, I will show empirically that it can at least provide good expectations about the results of the first election under the new system.

Even such a short-term model may fly in the face of conventional wisdom, which has typically held that one simply cannot predict the specific effects of a complicated new system, "and you kid yourself if you think that you can" (Taagepera, 2007, p. 36). But there are two major problems with such a critique: one practical, and one theoretical. The practical issue is that the decades-old simple method, with the preferences problem unsolved and the fixed voters assumption always regretfully invoked, continues to be applied in academic, public policy, and journalistic venues around the world. If we simply declare that it will never be possible to improve on it, then expectations about the specific effects of electoral system changes on party seat shares will continue to be shaped by a method that is known to have multiple major flaws.

Complementing this practical issue is a theoretical rationale: it is a common methodological recommendation that extremely multi-causal political processes should be

[3]Polga-Hecimovich and Siavelis (2015, p. 272-273) call this "a fundamental limitation of electoral design": "although institutions can be manipulated to benefit one group in the short-term, longer-term behavior is much more unpredictable".

modeled. Electoral system changes are what Franzese (2020, p. 580) would call omni-causal ("just about everything causes just about everything else"), and modeling how voters might respond to them is a first step in following his instructions for how to understand such complicated situations: "context matters, so model it!" (Franzese, 2009, p. 67). Cochrane (2017) too writes that "we need to model" the "imaginary worlds of electoral system reform", precisely because "[t]he complexity is overwhelming". The following methodology takes up those authors' recommendations.

3 Methodology

In order to address the preferences problem and the fixed voters assumption, three pieces are required. First, we need to estimate every voter's full preference ordering over the relevant political parties. Second, we need a strategic decision rule that voters can use to turn those preferences into votes. Third, we need to be able to tune the level of strategic voting so that it is empirically reasonable. We will also need tools to assess how accurate the model is, and to choose specific systems to simulate.

The modeling framework is not restricted to any specific alternative electoral system: I will describe a method for simulating changes from SMDP to any other electoral system. However, in this article I will primarily consider two alternative systems (Borda count and Mixed-Member Proportional), and briefly touch on two others (Instant Runoff Voting and Multi-Member Districts). Borda count is a type of ranked voting in which a voter's first choice gets some number of points, their second choice gets fewer points, and so on, and whoever receives the most points in a district is elected. In Mixed-Member Proportional, voters vote for a local representative in a geographic district (their "constituency vote") and for a party (their "party vote"). The candidate in each district who received a plurality of constituency votes is elected. Then, additional seats are assigned to parties such that the proportion of seats resembles the proportion of party votes. Instant Runoff Voting also uses ranked ballots, but iteratively eliminates the lowest vote-getters, and redistributes peoples' votes until one

candidate remains. Finally, in Multi-Member Districts using Single-Nontransferable Votes, candidates compete for multiple seats in a geographic district, but each voter only casts one vote, and then multiple candidates are elected in proportion to their party's vote share.

3.1 Model structure

For every electoral district, the model first generates a list of preferences, representing the preferences that real people hold in that district. If everyone votes sincerely, then the simulated election results are just the number of times that someone ranks each party first. But if there is any voter who strategizes about how to vote, they begin by consulting the results of the previous election in their district, and selecting their highest utility vote choice.[4] This provides new information about the expected vote choices, so voters continue to iteratively signal their intended vote choice, update that vote choice according to what every voter signaled, and so on until they reach a type of equilibrium (the stopping condition is discussed in Appendix §2). This builds on work that has recently studied this type of myopic strategic voting in a system with iterative polling, in both computational and game theoretic models (Baltz, 2022a, Eggers and Nowacki, 2023, Kloeckner, 2020, Mebane et al., 2019, Vasselai, 2022), and is analogous to the process of consulting opinion polls during an election to judge which parties are locally competitive.

Setting the proportion of strategic voters to some realistic value will generate expected election results under SMDP, at which point the model's accuracy can be checked: the vote total at this stage should be close to the real election results. Finally, an electoral system change is introduced by modifying how strategic voters judge the competitiveness of the parties. Figure 1 summarizes the model's structure.

[4]They may instead begin by responding to all other voters' sincere preferences as the initial expected vote choices; in this paper's applications, those approaches are similarly accurate.

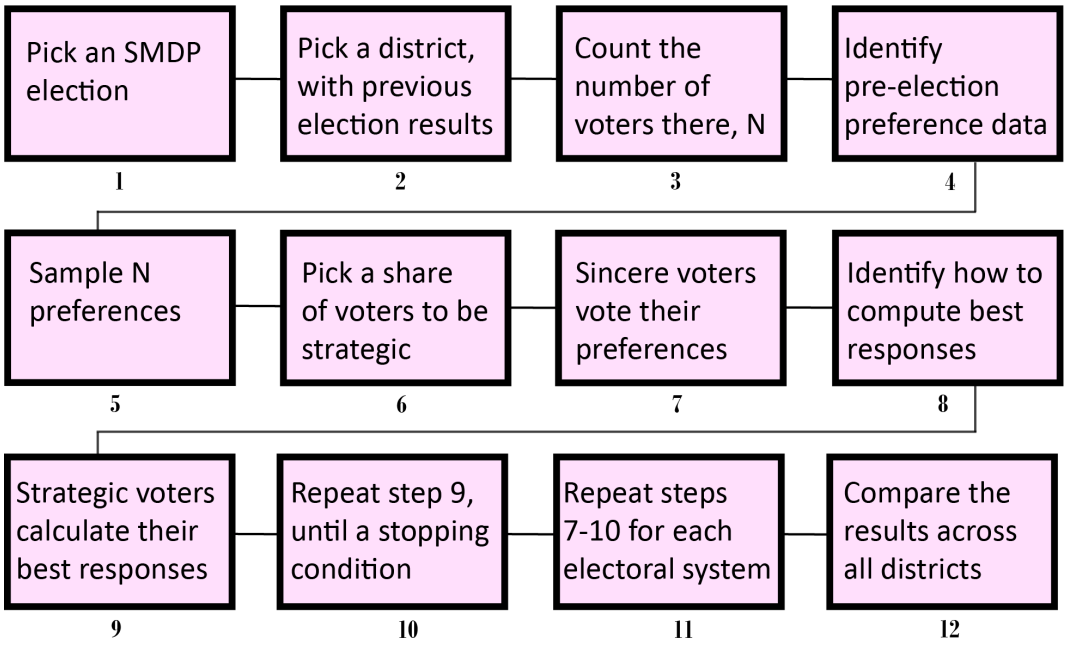


Figure 1: The model's structure.

3.2 Addressing preferences and fixed voters

Some studies that use the simple approach have assumed that, in an ordinal system, people would rank whichever candidate they voted for in SMDP first, and then used survey data to impute what they would do second (see Appendix §1).[5] However, this approach quickly runs into the intransitivity and incompleteness that exhaustive pairwise comparisons invite. Instead, I turn to the long history of using respondents' numerical ratings of parties to infer their preference orderings (Eggers and Nowacki, 2023, p. 9 – 10). Each complete set of responses to these thermometers provides an actual person's preference ordering over the parties, which I then convert into a utility using a conventional normalization. A simulated electorate is formed by sampling with replacement from the preferences reported in each district. I describe the procedure and the survey data in Appendix §3.

The fixed voters assumption presents a more serious challenge. As half a century of empirical paradoxes and red-hot debate have shown, there is no single correct way to

[5]Ndegwa (1997) has studied how to use vote choice in this way when survey data is not even available.

model strategic voting. How, then, should we choose which model of strategic voting to apply?

There is one core constraint, which I will call the "constancy requirement": the type of strategic voting logic *must be held constant across electoral systems*. The problem is as follows. Imagine if we posited that in SMDP voters use party ID to form their vote choice, but in Borda count they use candidate policy positions, and then we ran a model that shows very different results between SMDP and Borda count. We would have to ask: are they different because Borda count would actually produce very different outcomes from SMDP, or are they different because those differing heuristics produce different vote choices?

Instead, I propose to hold the *underlying logic* constant. This means that we should focus on a type of strategic voting that can plausibly occur in several different systems. This constancy requirement virtually rules out heuristic-based rulesets. As Laslier (2010, p. 322) writes, any pair of heuristics that aim to represent the same voting style under different election rules would not be "the only ones"; the best they can be is "derived from the same idea".[6] In contrast, rational choice voting styles can satisfy the constancy requirement, while also being theoretically well-understood, and supported by a consensus that *at least some* voters are responsive to strategic incentives in forming their vote choice (Aldrich et al., 2018, Eggers et al., 2022, Eggers and Vivyan, 2020). Voters in different electoral systems will face different probabilities of casting a pivotal vote, which means that the rules of the electoral system can be incorporated into the process of finding the vote choice with the largest expected utility.[7]

One type of strategic voting that satisfies all these conditions is wasted vote logic. While wasted vote logic is an especially important consideration in majoritarian systems, there are also signals of wasted vote reasoning in certain highly proportional systems (Aldrich et al., 2018, p. 17). A classic example is the constituency votes of Mixed-Member Proportional systems, which (in contrast to the party votes) appear to

[6] In Appendix §4 I consider some counterarguments.
[7] A fact already used by Eggers and Nowacki (2023).

often be the result of substantial wasted vote reasoning (Gschwend, 2006). There is less research on strategic voting in ranked voting systems, including Borda count and Instant Runoff, but it seems plausible that wasted vote reasoning affects vote choice in these systems too (Simmons et al., 2022). Wasted vote logic is also particularly well-understood, and Mebane et al. (2019) have already implemented computer simulations of single-vote plurality elections based on the Cox (1994) wasted vote model.

In Appendix §4 I motivate this choice further, and I compare the utility-maximizing rules developed in this section to a dramatically simpler heuristic, and show that the results are quite similar under SMDP. The constancy requirement means that we must use the rational choice ruleset in the main analysis, but the fact that under SMDP wasted vote logic produces very similar results to an extremely simple heuristic should provide reassurance that the model's results are not driven by some peculiarity of rational choice.

My model begins by adapting the SMDP simulations of Mebane et al. (2019) which in turn are based on Cox (1994). In their framework, the utility of voting for some candidate i is the probability of breaking a first-place tie between i and some other candidate j, so long as i would not have won a tie-breaker against j, plus the probability of raising i into a first-place tie, so long as i goes on to win the tie-breaker. So,

$$u(i) = \frac{1}{2} \cdot \sum_{j \in \mathbf{C} \setminus \{i\}} \Big[\mathbb{P}(v_i = v_j \wedge v_j > v_k) \cdot (u_i - u_j) + \mathbb{P}(v_j - v_i = 1 \wedge v_j > v_k) \cdot (u_i - u_j) \Big]$$

for all other candidates k, where u_c is the utility obtained from the victory of candidate c and v_c is the vote total of candidate c, and $\mathbf{C}$ is the set of candidates, assuming that tie-breaking is fair, and ties between more than two candidates are ignored.[8]

Mebane et al. (2019) set up their simulation as a Poisson voting game among a finite number of voters (Myerson, 2000), so that the difference between vote totals is a difference between Poisson random variables, which in turn is Skellam distributed.

To implement Mixed-Member Proportional elections, I follow the classic result that

[8] Work by Vasselai (2022) suggests that these are only relevant in the edge case where nearly everyone abstains from voting.

party votes are not strongly affected by wasted vote calculations, but constituency votes are (Harfst et al., 2018, Karp et al., 2002). So, a sincere voter in this model will cast both their constituency vote and their party vote for their sincerely most-preferred party. A strategic voter will likewise cast a party vote for their sincerely most-preferred party, but their constituency vote will go to their highest-utility vote choice.[9]

Modeling multi-vote elections is more complicated. I will focus on Borda count, for methodological reasons discussed in Appendix §5. Appendix §5.1 derives (and §5.2 gives a worked example of) the following expression[10] for the utility u a voter can expect to obtain from casting a ballot β in a Borda count election:

$$u(\beta) = \sum_{i \in \mathbf{B}} \left(\sum_{j \in \mathbf{J}_i} \left[\left\{ u_i - u_j \right\} \cdot \left\{ \frac{1}{2} \cdot \mathbb{P}\big((v_j = v_i) \wedge (v_j + t_j > v_k + t_k)\big) + \sum_{\rho=1}^{t_i - t_j - 1} \mathbb{P}\big((v_j - v_i = \rho) \wedge (v_j + t_j > v_k + t_k)\big) + \frac{1}{2} \cdot \mathbb{P}\big((v_j - v_i = t_i - t_j) \wedge (v_j + t_j > v_k + t_k)\big) \right\} \right] \right)$$

where $\mathbf{B}$ is the set of candidates on the ballot β, $\mathbf{J}_i$ is the set of candidates j for which $t_j < t_i$, u_c is the utility obtained from the victory of candidate c, v_c is the number of points c will obtain from all other voters' ballots, t_c is the number of points that the voter assigns to c, and k is every candidate other than candidates i and j.

Every strategic voter uses the appropriate equation to update its vote choice in each iteration of the model. In Appendix §6 I specify an entire run of the model in pseudocode. Appendix §7 takes up the question of how dangerous it is to only model *voters* and not consider abstention. I argue that this necessary simplification is not likely to change the results of the model by more than a few percent of the seats in the

[9] Because seats are assigned proportional to party votes, does this mean that our Mixed-Member Proportional results will be totally driven by how we impute preferences? The strategic voting model matters for two reasons. First, strategic constituency votes will shape *where* certain parties are represented. Second, constituency seats can affect the overall distribution of seats through disproportionate "overhang seats", where one party wins such a large share of constituencies that there are not enough party seats to bring all other parties up to their proportional allocation.

[10] A special case of the result by Myerson and Weber (1993).

legislature.

3.3 Validation and application methods

To assess the model's accuracy, I follow Whicker and Sigelman's (1991, p. 66) description of "event validity": if the simulation uses data from a real-world event, it should respond "in the same way as the real-world system". We have two opportunities to test the simulation's event validity.

First, we can apply it to a real electoral system change. New Zealand switched from SMDP in 1993 to Mixed-Member Proportional in 1996. Can the model beat the simple approach if we use data from *before* the 1996 election to predict the results *of* the 1996 election? Second, we can also ensure that the model is accurate when applied to the specific cases under consideration: if we use the model to simulate Canada's and Britain's 2019 SMDP elections, does it come close to the real results?

I use three tools to assess model performance. In SMDP, we can check the number of districts that the model assigns to the correct party. I will both report the number of correct ridings and show maps of the results. This district-based evaluation, however, does not cleanly apply to proportional systems. So in addition, I will report the sum of differences between the seats assigned to each party by the model and the seats that party won in reality:

$$a \equiv S - \frac{1}{2} \cdot \sum_{i \in \mathbf{T}} |s_i - r_i|$$

where $\mathbf{T}$ is the set of parties, s_i is the simulated seat total of party i, r_i is that party's real seat total, S is the number of seats in parliament, and the factor of $\frac{1}{2}$ avoids double-counting. I use "seat share accuracy" to refer to the percent of seats assigned correctly, $100 \cdot \frac{a}{S}\%$.

I will evaluate the model's accuracy using the 1996 New Zealand election, and then

apply it to the 2019 Canadian and British elections.[11] In New Zealand I rely on the 1996 pre-election study (Vowles et al., 1996), but substantial work was required to reconstruct all the necessary data, which makes this a particularly challenging validation. I discuss these problems in Appendix §8. The results of the previous Canadian and British elections are obtained from official government reports (Elections Canada, 2015, UK Parliament, 2019). Preferences are from the national election studies (Fieldhouse et al., 2019, Stephenson et al., 2020), which had an average number of complete party thermometers per district ranging from 50 (in Wales) to over 80 (in Canada's non-Québec districts). I divide the number of voters by 250 for the sake of runtime. More information about these data are available in Appendix §3.2, where I also show that dividing the population does not appear to affect the results.

The type of Borda count I simulate has voters rank three parties (because the smallest number of parties in the jurisdictions I simulate is four, so this means there is always at least one party left off the ballot), with the top getting three points, the second two points, and the third one point. For the Mixed-Member Proportional simulations, we must decide how many constituency seats and how many party seats there should be. The most prominent proposal for Mixed-Member Proportional in Canada, made by a government advisory commission, recommended assigning two-thirds of seats by constituency and one third by party vote (Law Commission of Canada, 2023, p. 104), and in Britain several regional assemblies use a similar system with similar seat breakdowns (Dunleavy, 2018, p. 57). Following New Zealand, I add the party seats by increasing the size of parliament, and I do not vary the number of seats as a function of election results. So, the simulated Mixed-Member Proportional parliament in Canada will have 507 seats (338 constituency, 169 party), and Britain's will have 948 seats (632 constituency, since we set aside Northern Ireland, and 316 party).

One last consideration is what share of voters should vote strategically. Because there is no one true number of people who vote strategically in every election, in every case I will present three results: the model with only sincere voters, the model with only

[11] Setting aside Northern Ireland because of its distinct party system.

strategic voters, and the model with a *realistic* proportion of strategic voters. Inspired by recent estimates, I set this proportion to 30% in each country (Eggers et al., 2022, Eggers and Vivyan, 2020). The model with 30% of people voting strategically is the most substantively reasonable, but the fully sincere and fully strategic models show the direction and possible extent of strategic pressures. Ideally, the number of seats that each party won in the election should lie between the sincere and strategic model results, and closest to the realistic proportion.

4 Empirical assessments

Figure 2 compares the simulation results to an implementation of the simple approach, marked with a square for each party. Figure 2 provides strong support for the model, which is able to very closely anticipate the result of the first Mixed-Member Proportional election. On average across model runs, the simulations correctly assign 115 of the 120 seats in parliament. The simple approach only correctly assigns 104 seats.

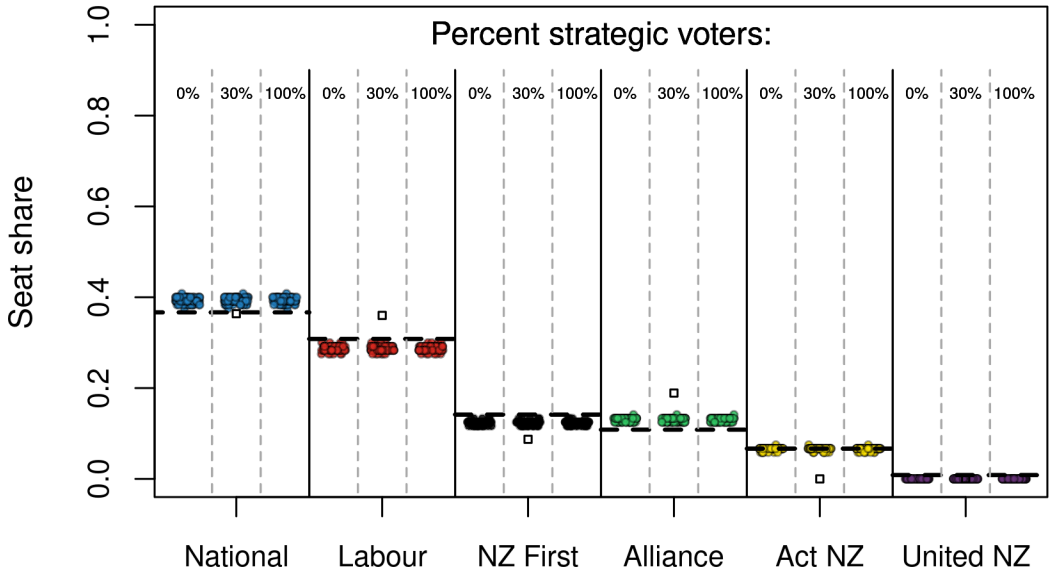


Figure 2: 100 runs of fully sincere and strategic simulations of New Zealand's 1996 election, with each party's seats in a run marked with a circle. The real results are marked with horizontal lines, and the results of the simple approach are marked with a square.

This success suggests that preference data from before an electoral system change

can provide good expectations about the results of the first election under the new system. It also supports the idea that the model provides accurate short-term expectations of the results of electoral system changes, where the short-term can extend as far as the first election under the new system. However, because the system is so proportional, this validation was not useful for assessing the contribution of the strategic model. For that, we must turn to the other type of validation, and simulate the 2019 Canadian and British SMDP elections. These simulations are shown in Figure 3.[12]

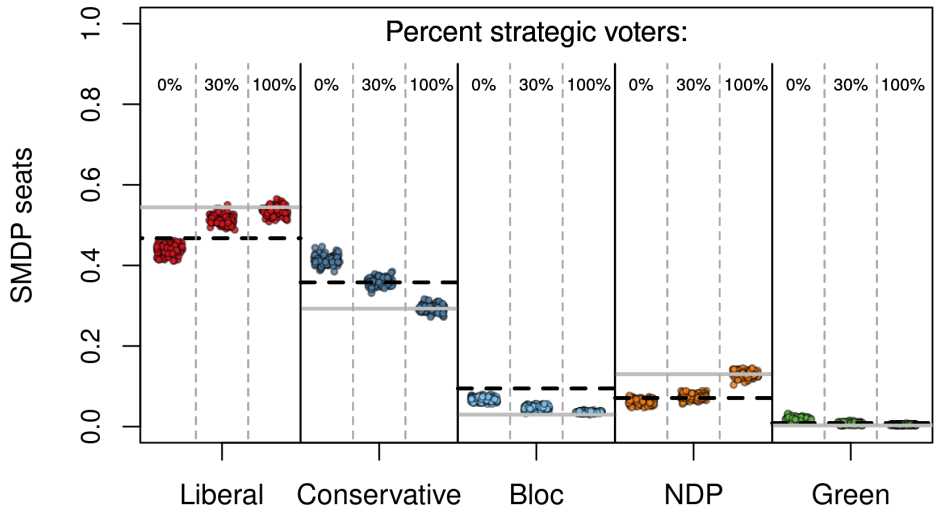
For every Canadian party except the Bloc, the real results lie between the sincere and strategic models.[13] The model with realistic levels of strategic voting almost exactly reproduces the real seat total of the Conservative, NDP, and Green Parties, while slightly overestimating the Liberals at the expense of the Bloc.

In Britain, a larger share of respondents to the British Election Study reported most-preferring the Conservative party than that party's share of the vote. Consequently, the the real results are not between the sincere and strategic model for either of the two biggest parties. However, the overall accuracy of the models are similar between the two countries. Table 1 shows the seat share accuracy of each model, alongside the percentage of electoral districts that each model type assigns to the correct party. The models always sign more than 77% of districts correctly, with the most accurate model assigning 88% to the right party, while the seat share accuracy ranges from 86.7% to 94.3%. Interestingly, neither the entirely sincere model nor the entirely strategic model consistently performs better than the other.

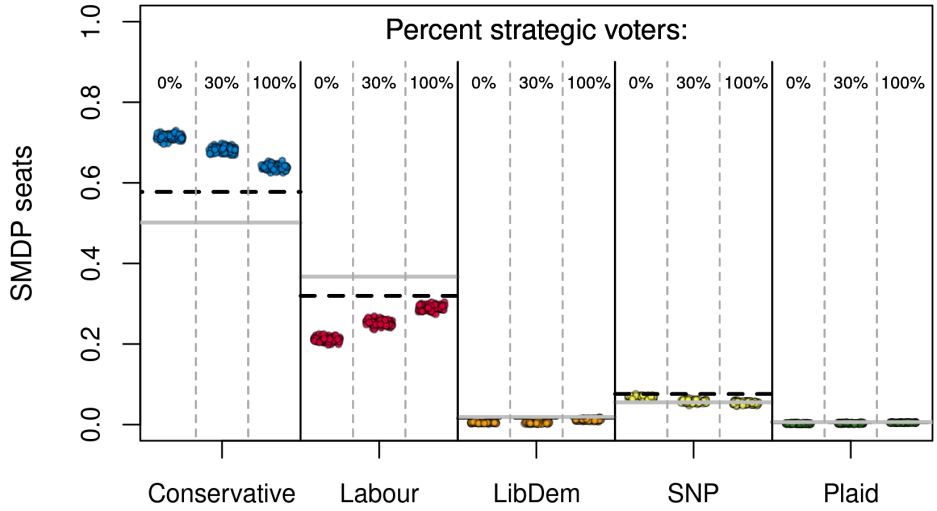
I also visualize the fully sincere and fully strategic models' success rates for specific districts in Figure 4. The colour of each district in those maps is the party that won the district in the real 2019 election, and districts are opaque in proportion to the share of model runs that assigned them to the correct party, so that the model was more consistently correct about more opaque districts and more consistently incorrect about more transparent districts.

[12] Appendix §10 shows all Canadian and British model results in boxplot form.

[13] Why not the Bloc? The district-by-district public opinion data underestimates them by about 10 seats, and they did worse in 2015 than in 2019, so strategic voting further reduces their seat totals.



(a) Canada



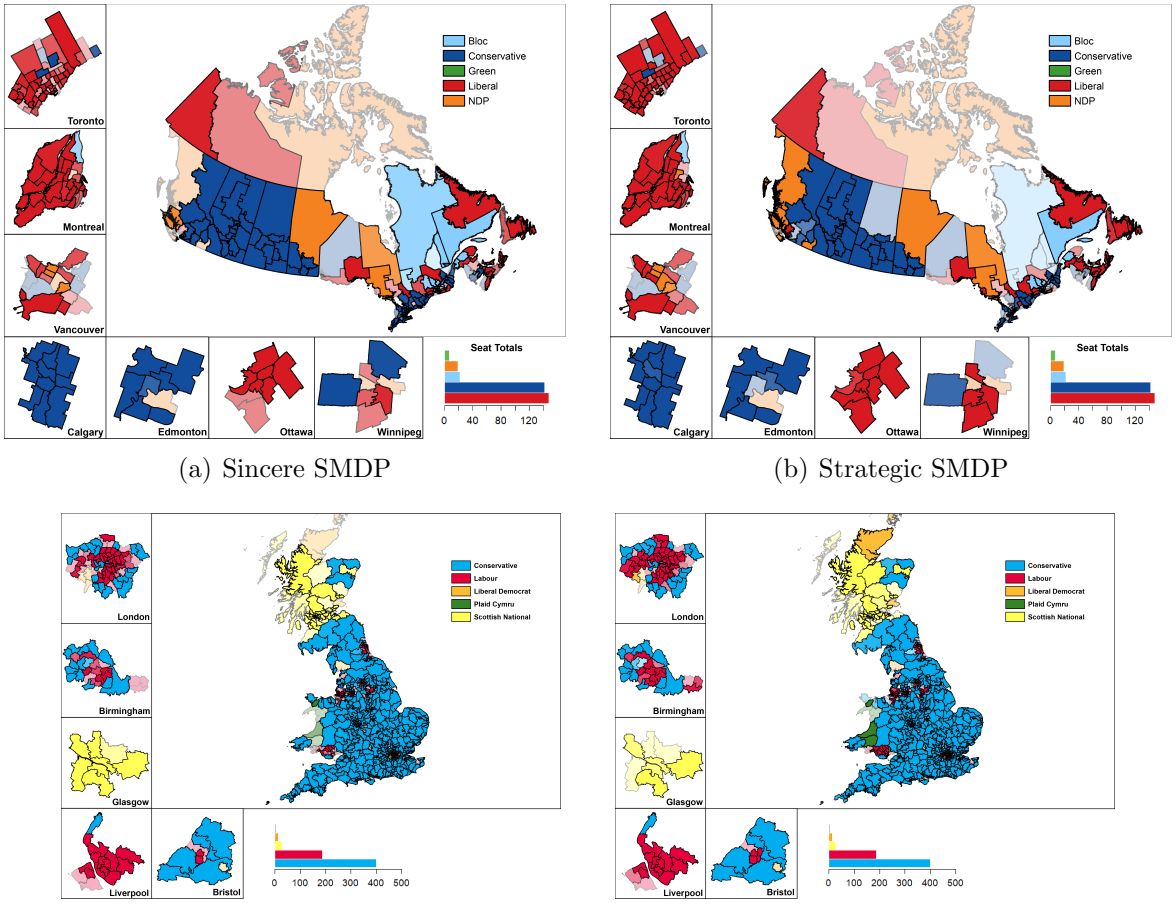
(b) Britain

Figure 3: The results of the sincere and strategic simulations under SMDP. The model is run 100 times with each strategic proportion. The real 2019 election results are represented with a black horizontal line, and the previous election's results with a grey line.

The accuracy is robust in two important ways. First, I have said that to believe the model, one does not need to believe that pivotal voting logic is a literal description of reality. We saw from Appendix §4 that the results are very similar with simulated voters who use much simpler heuristic voting logic. The only reason the heuristic cannot replace best response logic when we model electoral system changes is that it fails the constancy requirement, but its similarity to the strategic voting model should

Country	Voting rule	Seat share accuracy	Districts correct
Canada	Sincere	93.3%	78.8%
Canada	30% strategic	94.3%	81.2%
Canada	100% strategic	87.7%	77.6%
Britain	Sincere	86.7%	84.0%
Britain	30% strategic	89.2%	86.2%
Britain	100% strategic	93.9%	88.4%

Table 1: The accuracy of the model, in terms of both seat share accuracy and the percent of districts assigned to the correct party, on average over all runs.



(a) Sincere SMDP

(b) Strategic SMDP

(c) Sincere SMDP

(d) Strategic SMDP

Figure 4: The real 2019 election results in Canada and Britain, where opacity represents the model's accuracy: more opaque districts were assigned to the real election winner in a larger share of model runs. The barplots are average model results.

mitigate concerns about the realism of rational choice models. The models' event validity, and the fact that the sincere model is not clearly better than the strategic

model in Table 1, should also mitigate those concerns. Even if voters do not literally calculate the expected utility of their vote choice, these models appear to accurately capture the outcome of real peoples' strategic reasoning. Second, if voters take every voter's sincerely most-preferred party as their initial expected vote choice rather than consulting the previous election's results, the models have seat share accuracies ranging from 86.8% up to 93.4%, very similar to the model as presented in Table 1. That assumption makes the models more accurate in these cases, but it is far from essential.

5 Results

The model has demonstrated event validity and can now be used to simulate Canadian and British elections under Borda count and Mixed-Member Proportional. I will compare the simulation results to the conventional wisdom about how these systems would affect parties in those countries.

5.1 Borda count, Canada

The traditional motivation for ordinal voting systems is to ensure that representatives are broadly palatable (de Borda, 1781), and these systems are widely understood to usually "favor politicians that are more centrist" (Tolbert and Kuznetsova, 2021, p. 266). In Canadian politics, this has cemented into the belief that "under such a system the Liberals would be guaranteed a place in government forever" (Dutil, 2017). Appendix §1 suggests that this consensus has been bolstered by the simple approach.

We should therefore expect that, if Canada adopted Borda count, the Liberal seat share would surge at the expense of every other party. But does this conventional wisdom hold true when we use voters' real preferences, and account for strategic voting? The result of that simulation, in Figure 5, shows a very different pattern.

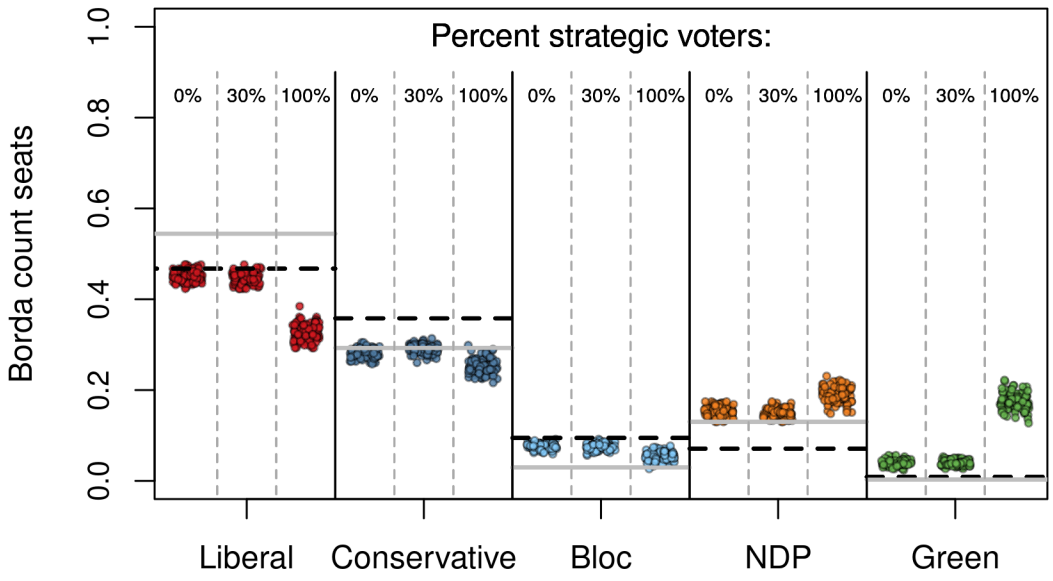


Figure 5: A hypothetical Canadian Borda count election with varying levels of strategic voting. The electoral districts are the same as Canada's real electoral districts, and voters can rank three candidates. The model is run 100 times with each strategic proportion. The real 2019 election results are represented with a black horizontal line, and the previous election's results with a grey line.

In the sincere and partially strategic models, the Liberals win almost exactly as many seats as they won in reality. The Conservatives win slightly fewer, but the NDP and Greens both win more. It is hardly obvious that this would be a good situation for the Liberals. There is no sign of the anticipated Liberal surge — even in the sincere model, where the only ingredient is public opinion data — and while the major competitor to their right is slightly weakened, the competitors to their left are suddenly stronger.

The fully strategic voting model is more different still. In these simulations, there is a surge for the Green Party. In Canada's real 2019 election, the Green Party won less than 1% of the seats in parliament. In the fully sincere and partially strategic Borda count models, the Greens win about 4% of the seats. But in a typical run of the fully strategic Borda count model, they capture more than 15% of the seats in parliament.

Why would the Green Party in particular receive such a large boost? Let us consider an example from a run of the model: a voter considering how to vote in the district

London North Center.[14] The Green Party was once highly competitive there, and while it is very unlikely to win the district, many people there still rank it as their second-most preferred party; this voter has the preference ordering `Conservative` $\succ$ `Green` $\succ$ `Liberal` $\succ$ `NDP`. Checking the expected vote choices of the other simulated voters, they find that the Liberals are in a tight race with the NDP. After computing the pivotal probability of every ballot they could cast, they find that they expect the highest utility from assigning the Liberals 3 points (to prevent the NDP from winning), the Conservatives 2 points, and the Greens 1 point. Enough voters cast similar ballots that in the next iteration, the Conservatives are actually expected to receive almost as many points as the Liberals. The voter prefers the Conservatives over the Liberals, so the next ballot they signal ranks the Conservatives, then the Greens, and then the Liberals. But Liberal supporters do the opposite, with many of them listing the Liberals first and the Greens second. Now so many supporters of both major parties are giving the Green Party 2 points that the Greens *actually surpass* both of the major parties, and are suddenly boosted into first place. In this model run, voters in several dozen districts similarly coalesce around the Green Party as a compromise against the more polarizing parties.

This particular story depended on the mass adoption of precise rational choice reasoning, but a more muted version is easy to imagine. The Green Party is such a common second-place preference across the country that voters under Borda count might quickly notice its surprising strength in the polls, and realize that ranking it second on their ballot is not necessarily a wasted vote. If enough people do this, it could give the Greens a chance in districts they cannot seriously contest in SMDP. By addressing the preferences problem and challenging the fixed voters assumption, the strategic voting model has revealed a plausible dynamic that previously lay hidden in the preference structure of the Canadian electorate.

Before turning to the British case, we should return to the question of how reflective Borda count simulations may be of the proposals currently on the table, which

[14] What follows is an actual example of a simulated voter in one run of the model.

mainly concern Instant Runoff. The same kinds of methods that we have used to simulate Borda count can, in principle, be used to simulate Instant Runoff Voting. The problem is that Instant Runoff is a dramatically more complicated electoral system, and the literature on strategic voting has only very recently begun to grapple with its complexity. In Appendix §5.3 I use a computer model that was recently developed by Baltz (2022b) to simulate Canadian elections under Instant Runoff, and I compare the results of that simulation to the Borda count results in this section. The two models similarly contradict the conventional wisdom that ordinal voting systems would dramatically help the Liberal Party. However, the Instant Runoff models do not have the same pattern of strategically boosting the Green Party, which appears to be a peculiar quirk of strategic voting under Borda count.

5.2 Borda count, Britain

A referendum that proposed switching British elections to Instant Runoff Voting (in that setting called the Alternative Vote) produced divisions and mixed reactions within nearly every major party (Laycock, 2013, Stamp, 2011). The leaders of both the Conservative and Labour parties supported the change, but legions of their Members of Parliament opposed it. The Liberal Democrats enthusiastically supported it, while the SNP and Plaid Cymru also endorsed it, viewing it as not the most desirable alternative system but still better than the *status quo*.

Research at the time was mixed, with competing predictions about just how different British elections would be under Instant Runoff (Lundberg and Steven, 2013, p. 21). The results of my simulations of Britain's 2019 election under Borda count, shown in Figure 6, add to the narrative that the parties did not know what to expect from that different ordinal system: in Borda count in 2019, even when strategic voting is accounted for, the parties would hardly perform any better or worse than they did in the real SMDP election.

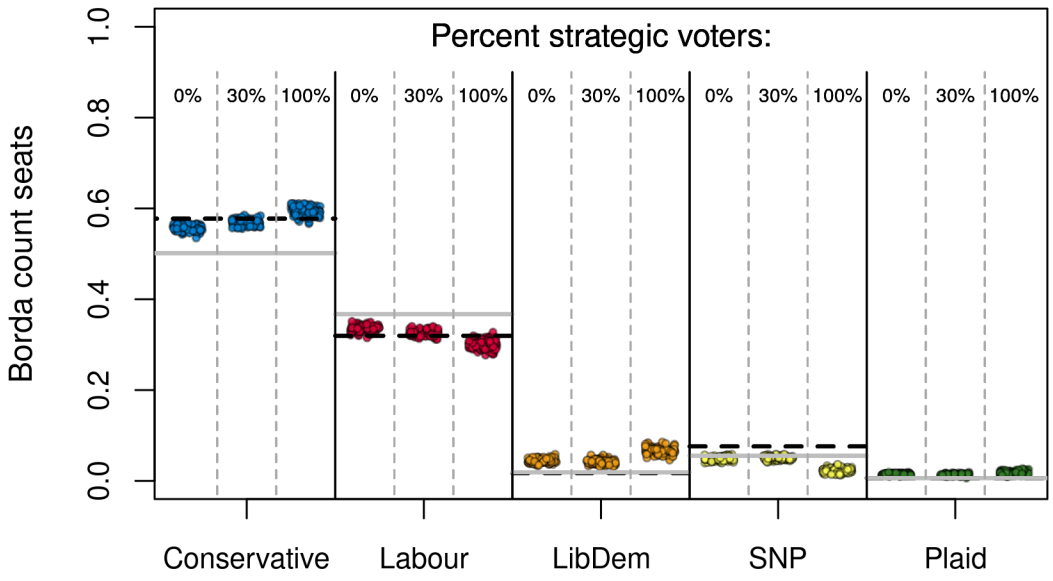


Figure 6: A hypothetical British Borda count election with varying levels of strategic voting. The electoral districts are the same as Canada's real electoral districts, and voters can rank three candidates. The model is run 100 times with each strategic proportion. The real 2019 election results are represented with a black horizontal line, and the previous election's results with a grey line.

These results might help to explain why Britain's parties were so messily divided on the question of electoral system change. If the effect of changing systems on each parties' number of seats would actually be very small, it may be especially challenging for the party members to anticipate the direction of those changes, which could drive them to take different positions on the issue.

5.3 Mixed-Member Proportional, Canada

The Canadian parties' positions on Mixed-Member Proportional (the NDP and Greens support it, while the Liberals, Conservatives, and Bloc oppose it) are easily explained: the parties that support this highly proportional system are the parties which typically have larger vote shares than seat shares.

The model lets us dig beneath this surface in two ways. First, because we have addressed the preferences problem, we can estimate specifically how good or bad the system would be for each party, using preferences rather than the previous election's

vote counts. Second, because we have removed the fixed voters assumption, we can estimate the potential impact of strategic voting in the constituency vote on election results overall, and in particular, estimate how many overhang seats may arise due to strategic behaviour in the electorate level. Figure 7 shows the results of the simulation.

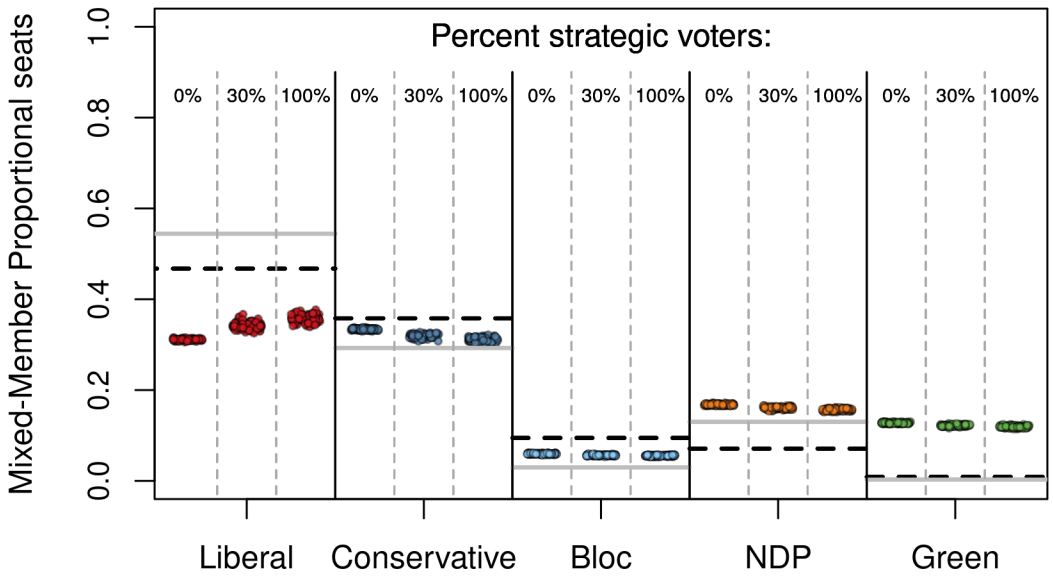


Figure 7: A hypothetical Canadian Mixed-Member Proportional election. Strategic voters use wasted vote logic at the constituency level, while party votes are sincere. The constituencies are the same as Canada's real electoral districts, supplemented by half as many party seats as constituencies. The model is run 100 times with each strategic proportion. The real 2019 election results are represented with a black horizontal line, and the previous election's results with a grey line.

On the first question, the distribution of seats suggested by voters' reported preferences, we should turn our attention to the fully sincere voting model. As expected, the Liberal Party and the Bloc suffer dramatic losses, with the Greens and NDP rising commensurately. The biggest surprise, however, is the Conservative Party, which hardly fares any worse in the sincere Mixed-Member Proportional election than it did in the real SMDP election.[15] In fact, the party does much better in relative votes: it

[15]This may partly be an artifact of the public opinion data showing stronger support for the Conservatives compared to the votes they received in the real election, but this is not necessarily a false signal: it may be that the reported preferences accurately reflected the party votes that people would have cast under Mixed-Member Proportional, but strategic considerations intervened to reduce the Conservative Party's vote share in the real SMDP election.

goes from a distant second place in reality to winning a plurality of seats in Mixed-Member Proportional. The party's firm opposition to proportional representation may be a matter of the expected left-of-centre coalitions that could form with this distribution of seats. However, the sincere simulation introduces the nuance that the Conservatives could form a minority government under Mixed-Member Proportional if the other parties fail to form a coalition.

On the second question, overhang seats in the strategic model make a crucial difference in the distribution of seats in parliament. The Liberals strongly benefit from wasted vote logic at the constituency level — for example, a voter who most-prefers the NDP will report that in their party vote, but if their local constituency only has two competitive candidates, a Liberal and a Conservative, they may nevertheless vote for the Liberal rather than the NDP in their constituency, causing the Liberal to win over the Conservative. With a parliament of 338 constituency seats and half as many party seats, there are not enough party seats to counter-balance the Liberals' gains in constituency seats. The number of overhang seats they win tips them into being the plurality seat-getters, which could change their incentives to pursue a coalition agreement. We might imagine, therefore, that strategic voting in Mixed-Member Proportional could noticeably affect the governments that may be formed under that system.

In Appendix §9, I supplement this study of Mixed-Member Proportional by also examining another common proposal for achieving proportional representation in Canada: Multi-Member Districts. The results suggest that the NDP and Greens would benefit from such a system, so long as there is not a very high level of strategic voting, but that strategic pressures could reduce their gains from that more proportional system.

5.4 Mixed-Member Proportional, Britain

Similar to the breakdown of Canadian party support for proportional representation, such a system is supported by the smaller British parties and opposed by the largest. Both the SNP and Plaid Cymru support proportional representation above other alter-

native electoral systems, while the Liberal Democrats supported Instant Runoff Voting more. Figure 8 shows the results of the Mixed-Member Proportional simulations in Britain.

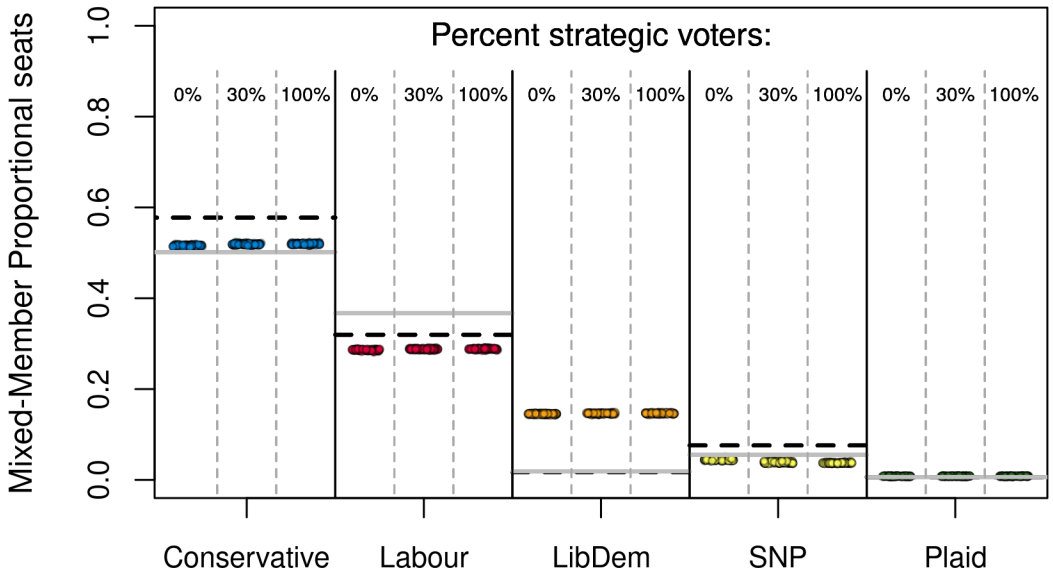


Figure 8: A hypothetical British Mixed-Member Proportional election. Strategic voters use wasted vote logic at the constituency level, while party votes are sincere. The constituencies are the same as Britain's real electoral districts, supplemented by half as many party seats as constituencies. The model is run 100 times with each strategic proportion. The real 2019 election results are represented with a black horizontal line, and the previous election's results with a grey line.

As with Borda count, perhaps the most striking feature of the figure is its similarity to the real SMDP election results, except the jump in Liberal Democrat seats. The Conservatives and Labour receive only slightly diminished seat shares in the Mixed-Member Proportional simulation compared to their shares in SMDP. Perhaps more surprising, though, is the performance of Plaid Cymru and the SNP in Mixed-Member Proportional. Both these parties support adopting a proportional system, and yet in the simulation Plaid Cymru does almost identically to its SMDP results, and SNP does even worse. This is not consistent with the parties' positions, if they are engaging in short-term seat maximization. In the Multi-Member District simulations in Appendix §9, there is a similar pattern: the Liberal Democrats benefit strongly so long as there

is not a very high level of strategic voting, but overall the results are quite similar to the real SMDP election results.

6 Conclusion

The question of how many seats a *specific* political party would receive under a *specific* alternative electoral system has been quietly dominated by a simple approach that has two major problems, which I call the "preferences problem" and the "fixed voters assumption". This article introduced a computational formal model that addresses both issues. Applying the model to Canada and Britain suggested that parties may not consistently support the system that would net them the most seats. The paper's main contribution, however, is a computational modeling framework that can be adapted to other cases, to falsifiably and accurately generate positive numerical conjectures about how a specific party would perform after an electoral system change.

Applications for the estimates may go beyond formal theory and predictions about electoral system changes. For example, surveys or experiments which aim to gauge peoples' reactions to electoral system changes might use the seat counts from these simulations when presenting respondents with hypothetical seat distributions under counterfactual election rules. The simulation results may also have a role in practical politics, since parties appear to often lack conscious and consistent approaches to estimating how well they would do if they agreed to some electoral system change. These simulations could help to inform those expectations.

I have argued that simulating voter strategy is a necessary and important *first* step to understanding the larger multi-causal system of why electoral systems do or do not change. Many pieces of that puzzle remain open for study. I have only studied wasted vote logic, but voters engage in many other types of strategic reasoning (Kedar, 2014, p. 13). Party strategies may also be less surprising when viewed from an angle other than short-term seat maximization, such as the expected coalitions under different systems, or the parties' ideological commitments to a certain kind of representation

(Ahmed, 2012, p. 69). New methods are also needed to quantify the uncertainty in the estimates of election results under counterfactual rules. One possibility would be to combine models of voter strategy with models of party strategy, and to model how voters' preferences evolve alongside the party system.

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