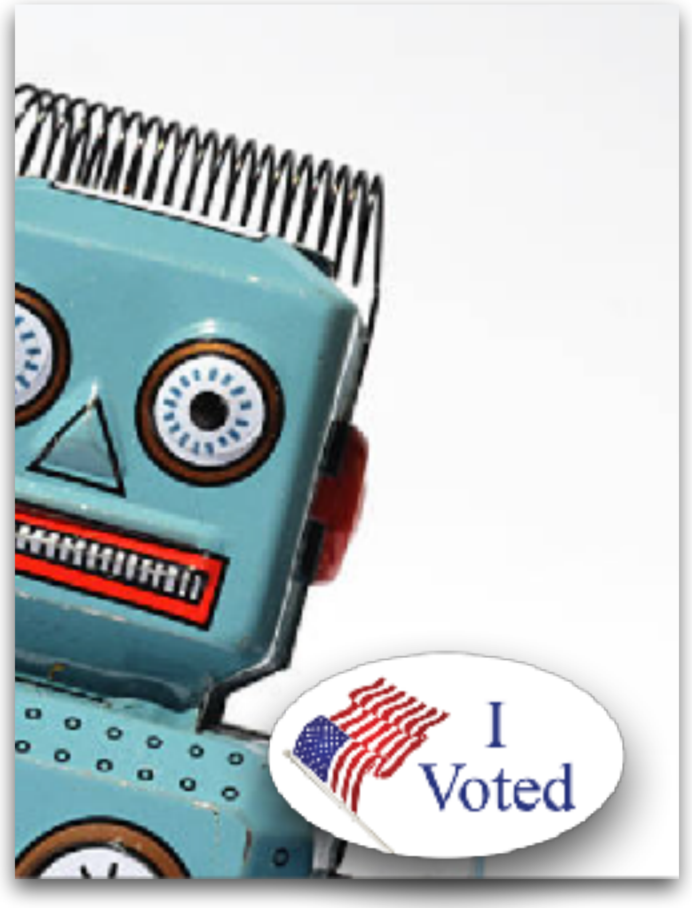
Modeling Democracy

Lecture 7 - **The spatial hypothesis**



Moon Duchin, Winter 2026 — Wed Jan 28

An intriguing framework: Metric embeddings

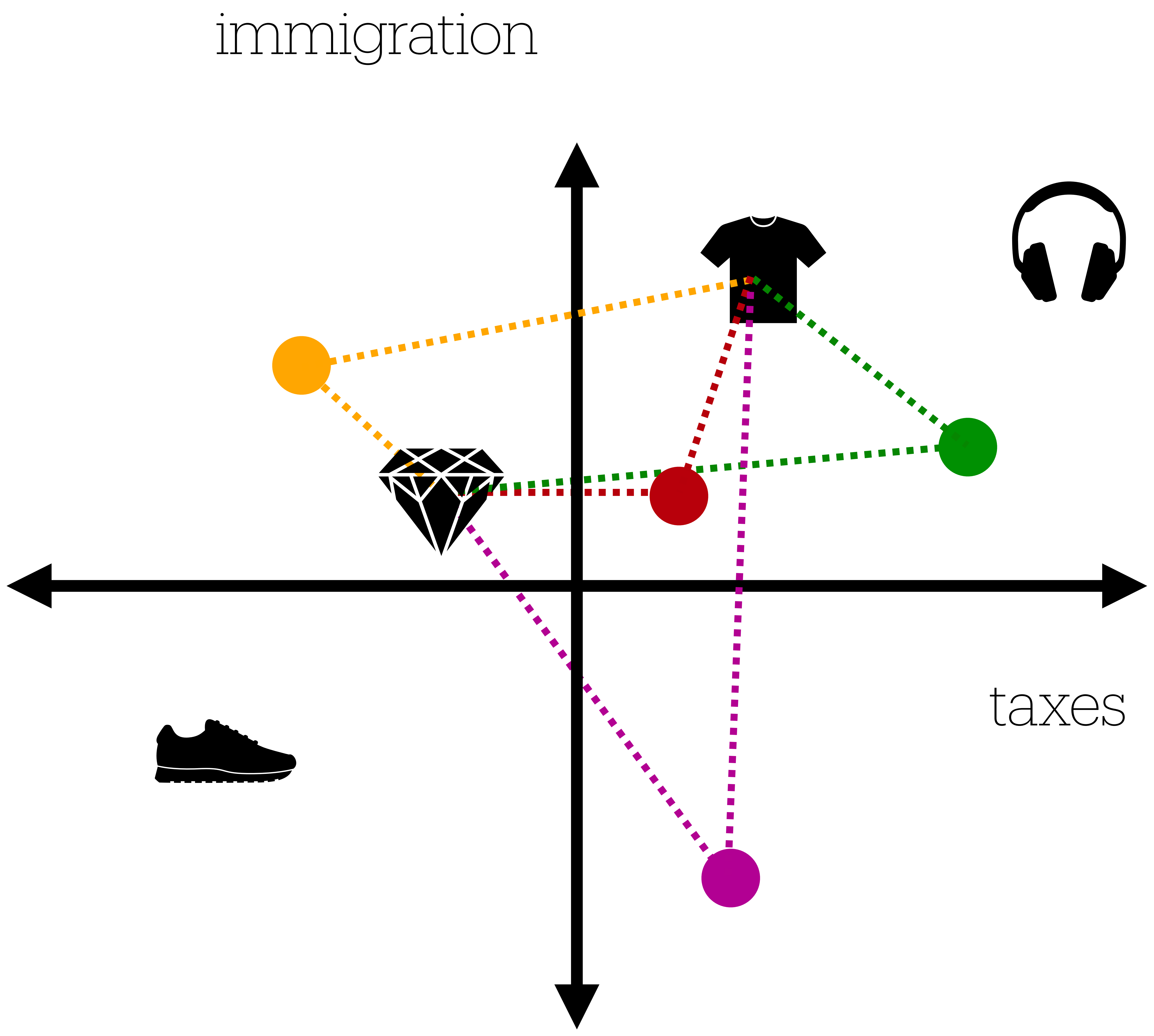
- Let's adopt a hypothesis (of questionable realism) and see where it takes us.
- Suppose first that issue positions can be measured along a **line**.
- Then a collection of mutually orthogonal issues can be thought of as spanning a coordinate **space**, and both candidates and voters can be located in that space.

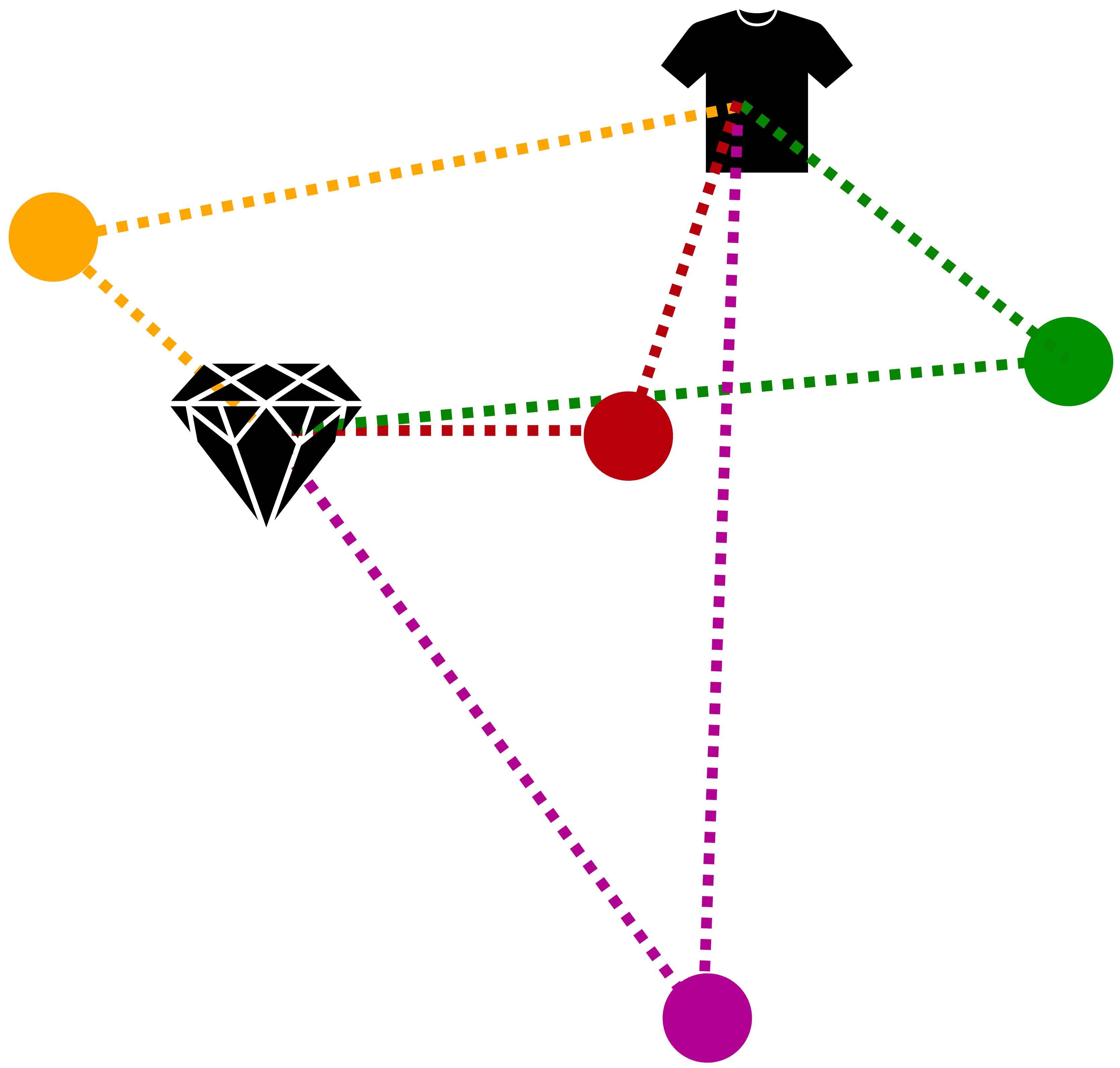
spatial voting

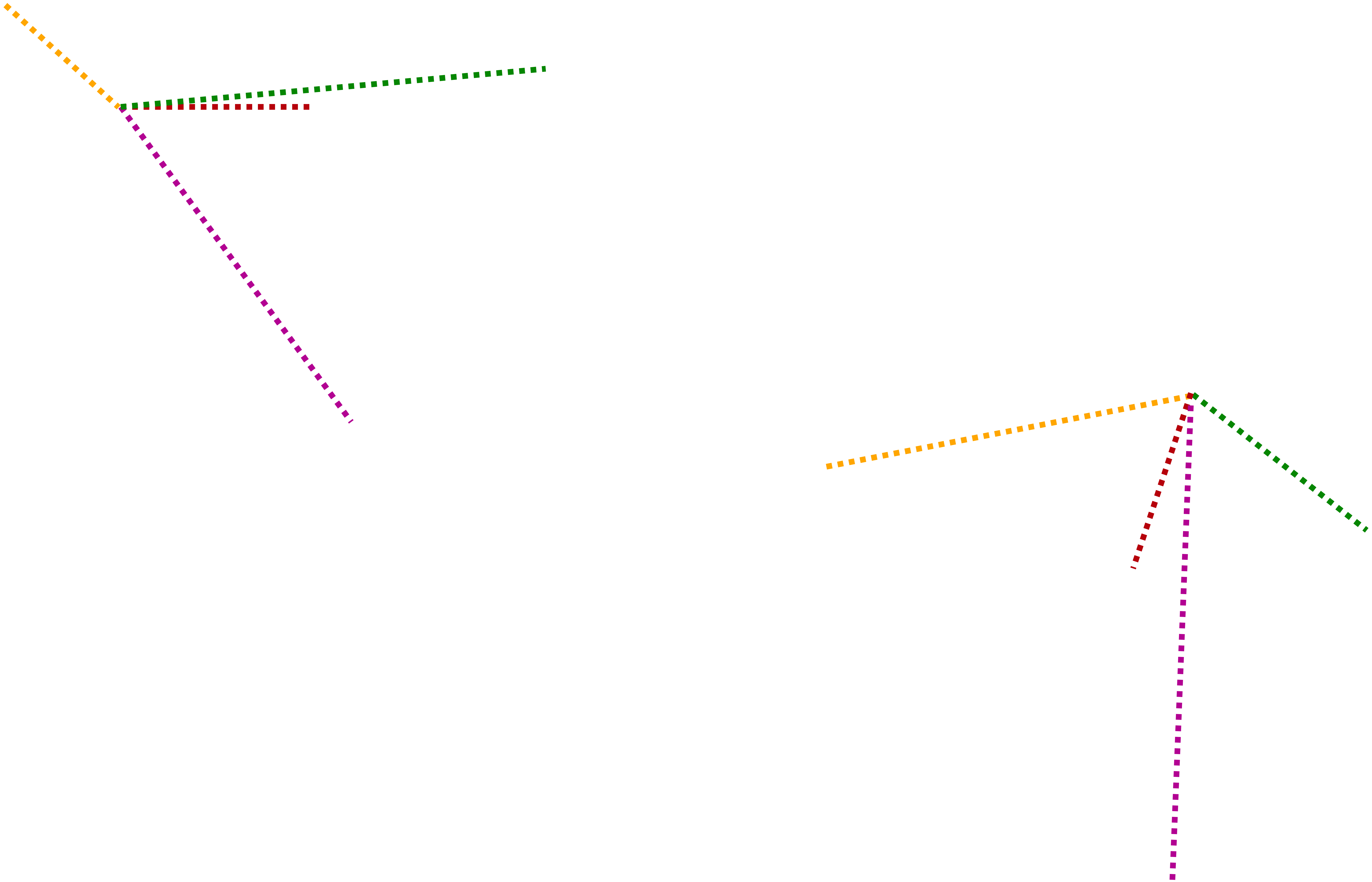
voters

"**cost**": sum of distances to voters

optimal winner has lowest cost



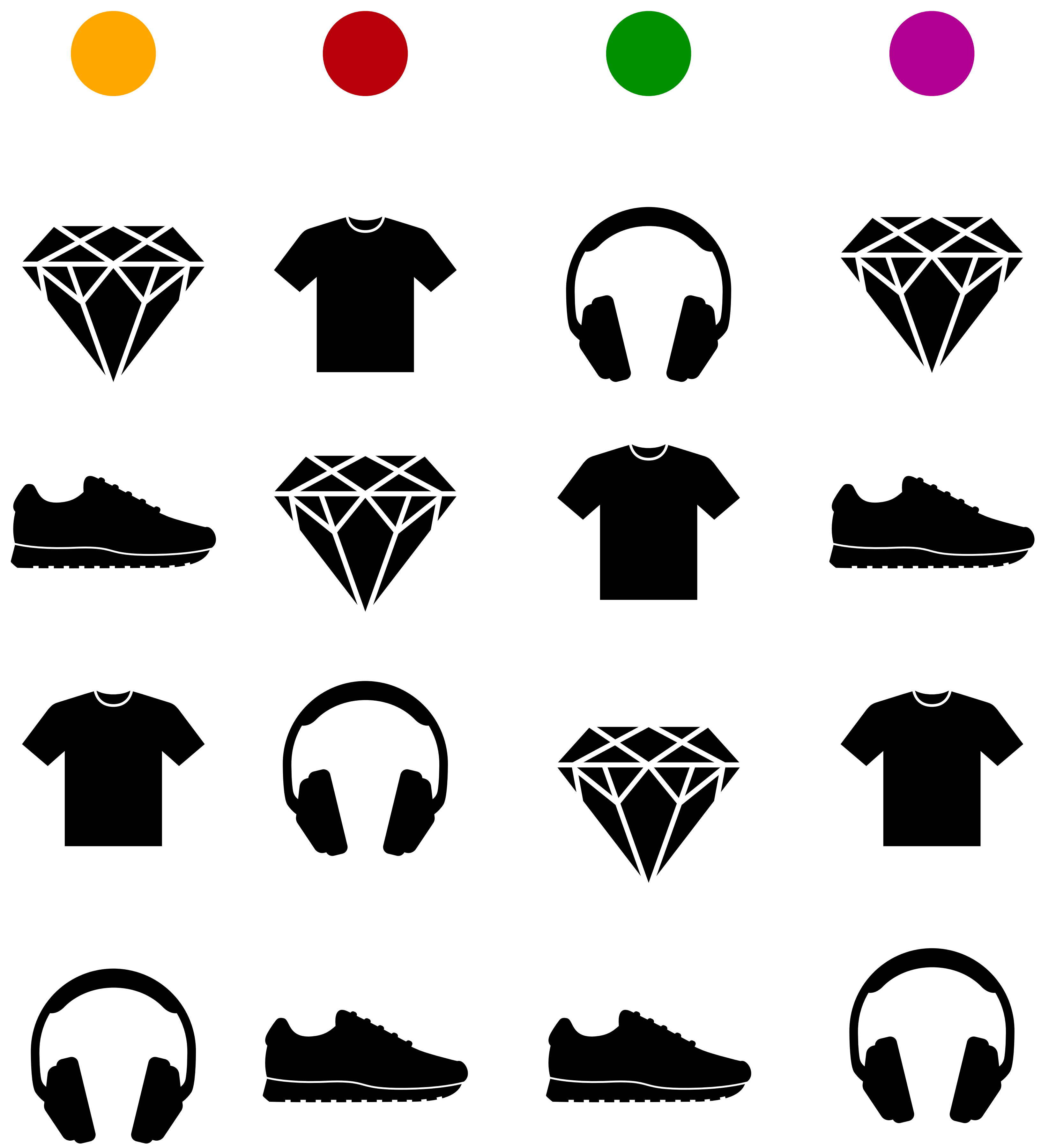


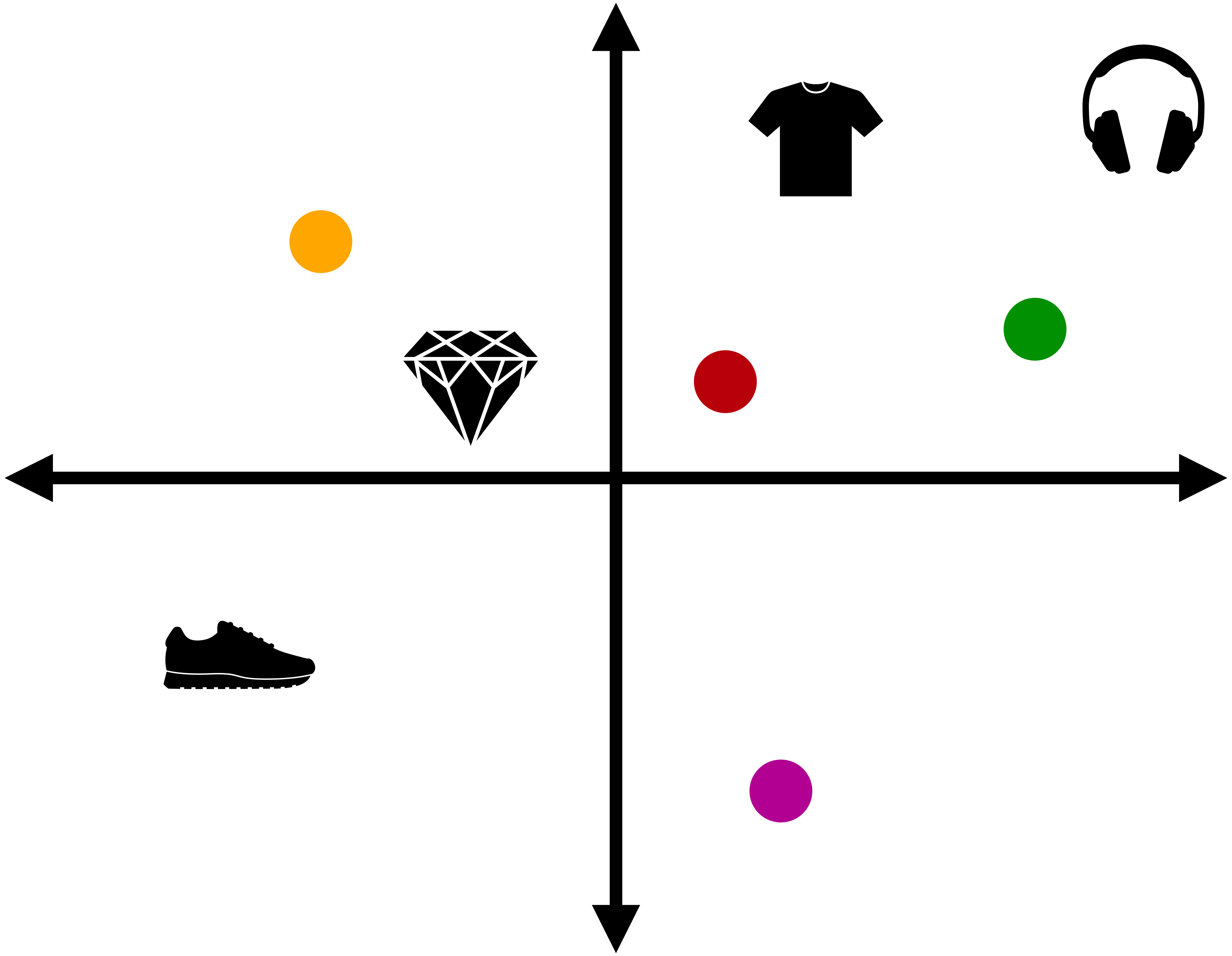


cost ratio 1.0975

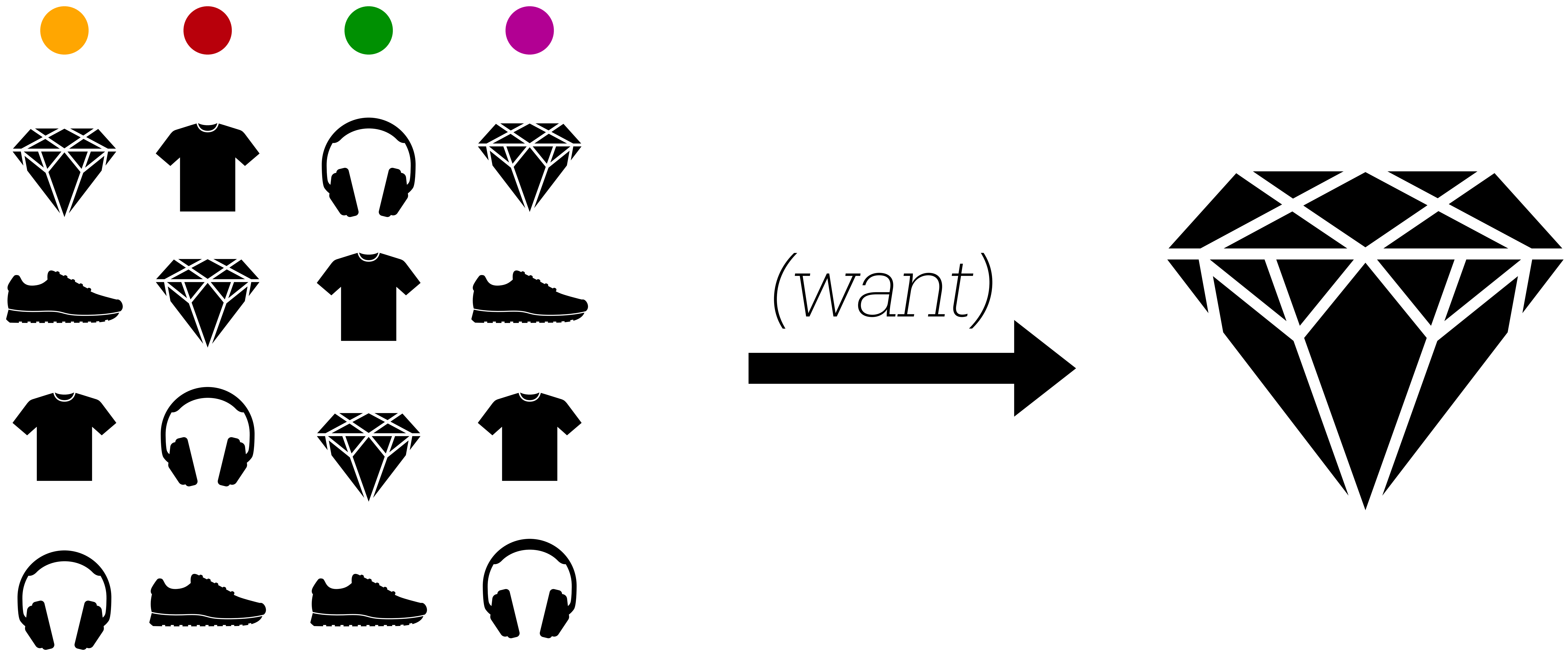
spatial voting

voters

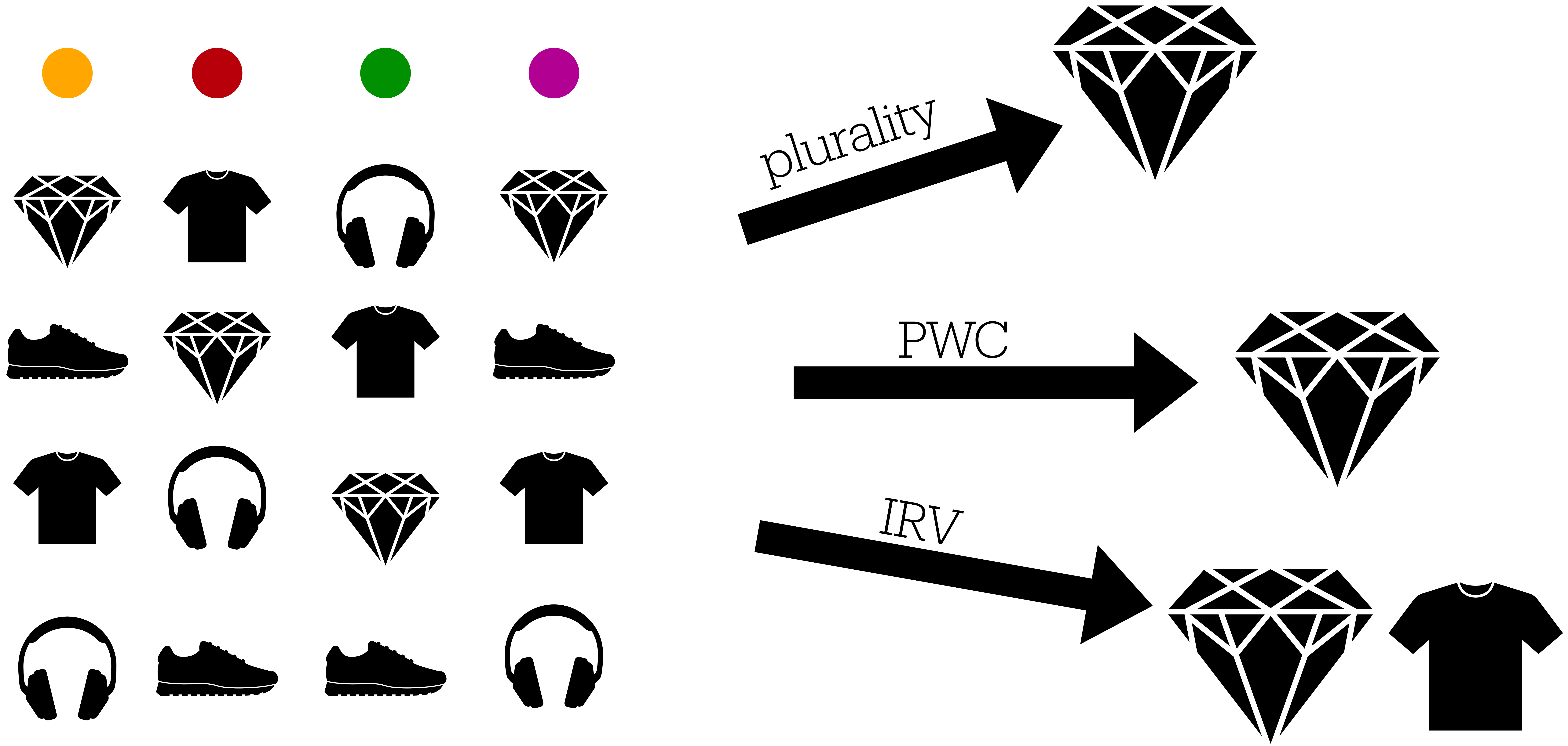




can we come up with a ranking-based rule that finds the (spatially) optimal winner?



metric distortion: if you can't guarantee finding optimal winner, can you guarantee bounded cost ratio?

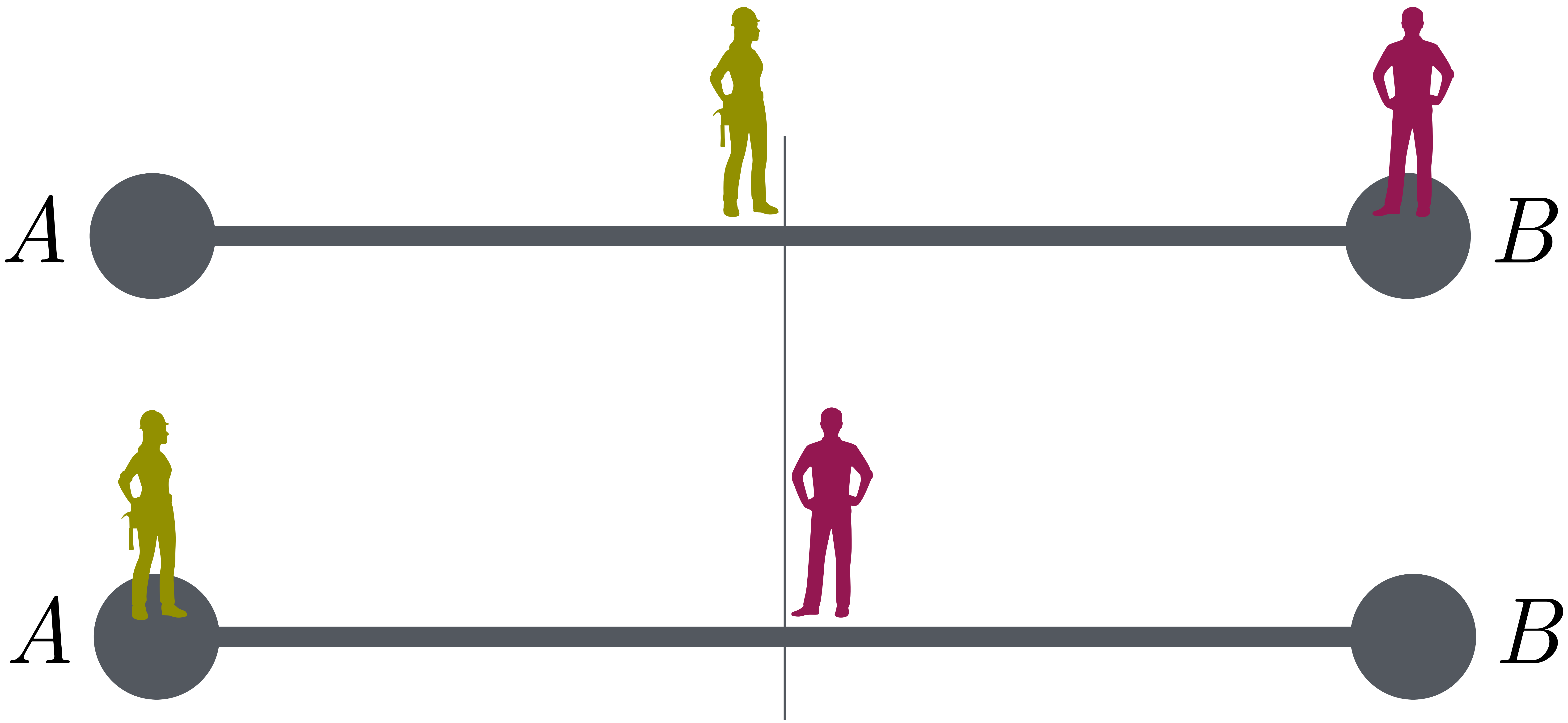


$$f\left(\begin{array}{|c|c|}\hline 1 & 1 \\ \hline A & B \\ \hline B & A \\ \hline \end{array}\right) = ?$$

P

Def: ***(metric) distortion*** of f is the worst-case cost ratio in any metric embedding

Observation: all deterministic rules have distortion ≥ 3. To see, consider N=m=2 example.



if f chose A, this is a bad scenario compatible with P: cost ratio is 3

if f chose B, this likewise fits P and has cost ratio 3

Utility framework

- Metric distortion relates interestingly to a general framework of **utility** distortion.

- Suppose voters have a truth of the matter in their preferences that is **cardinal** rather than ordinal: they have scores for each candidate, normalized to add to 10, say. Up to tiebreaking, this is strictly more information than an **ordinal** ballot (ranking), just like the metric embedding was.

	2	8	3	2
	0	1	3	1
	3	0	1	3
	5	1	3	4

utility ratio

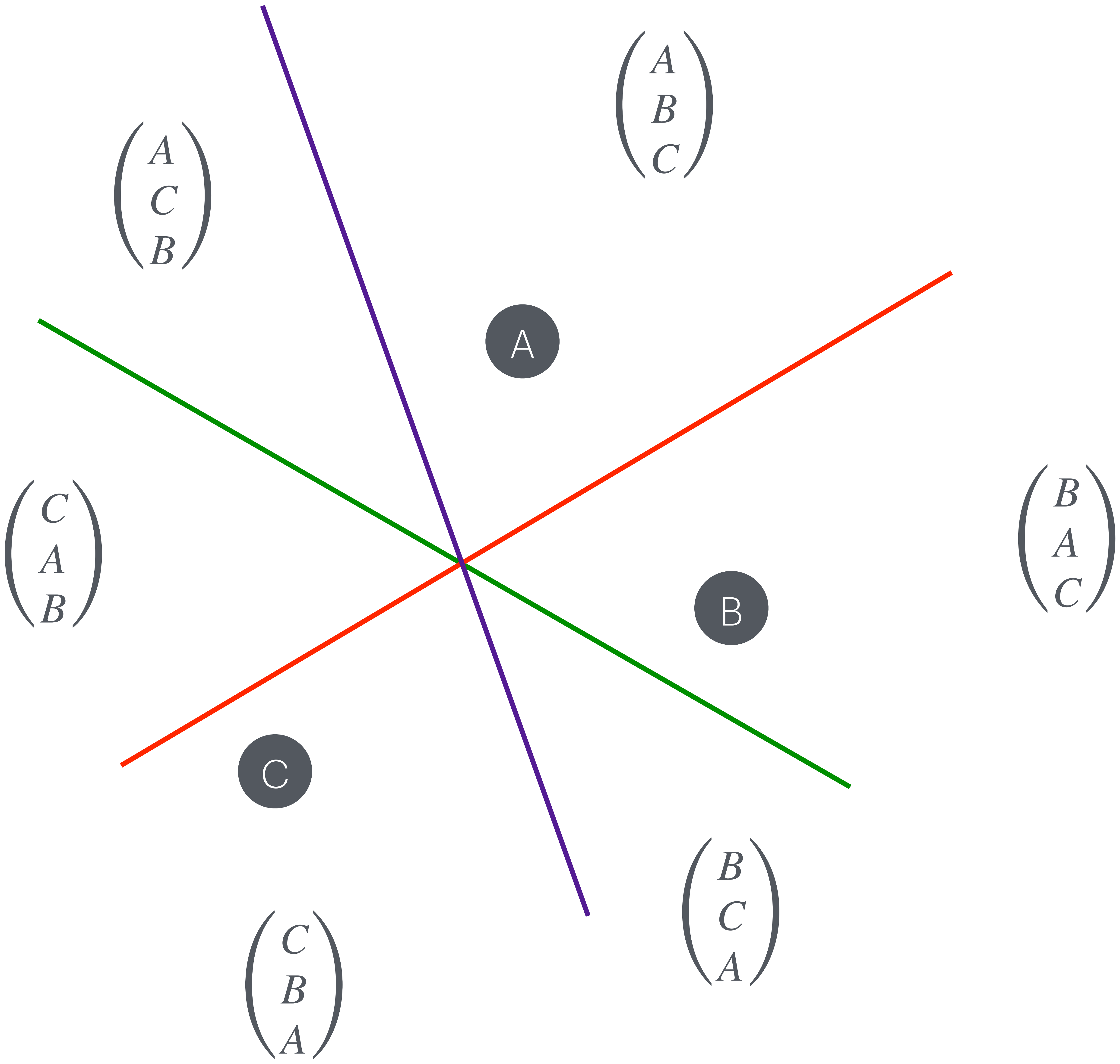
$15/13 \approx 1.15$

- Here too we can seek a rule that minimizes (worst-case) utility loss relative to the optimal winner.

- **Plurality** has distortion $O(m^2)$ where $|\mathscr{C}| = m$, and this is best possible!

Relationship between frameworks

- My pictures have been Euclidean plane, but metric framework doesn’t assume that.
- **Metric** defined by three properties: symmetric, positive-definite, triangle inequality.
- This is fairly restrictive! If candidate locations are placed, a voter being close to A forces a certain approximate distance from B.
- Is the political preference world this consistent?
- Another difference: utilities typically normalized
    - Does this account for “stakes”?



Proposition: For any profile on three candidates, there is a *planar* metric embedding that realizes it.

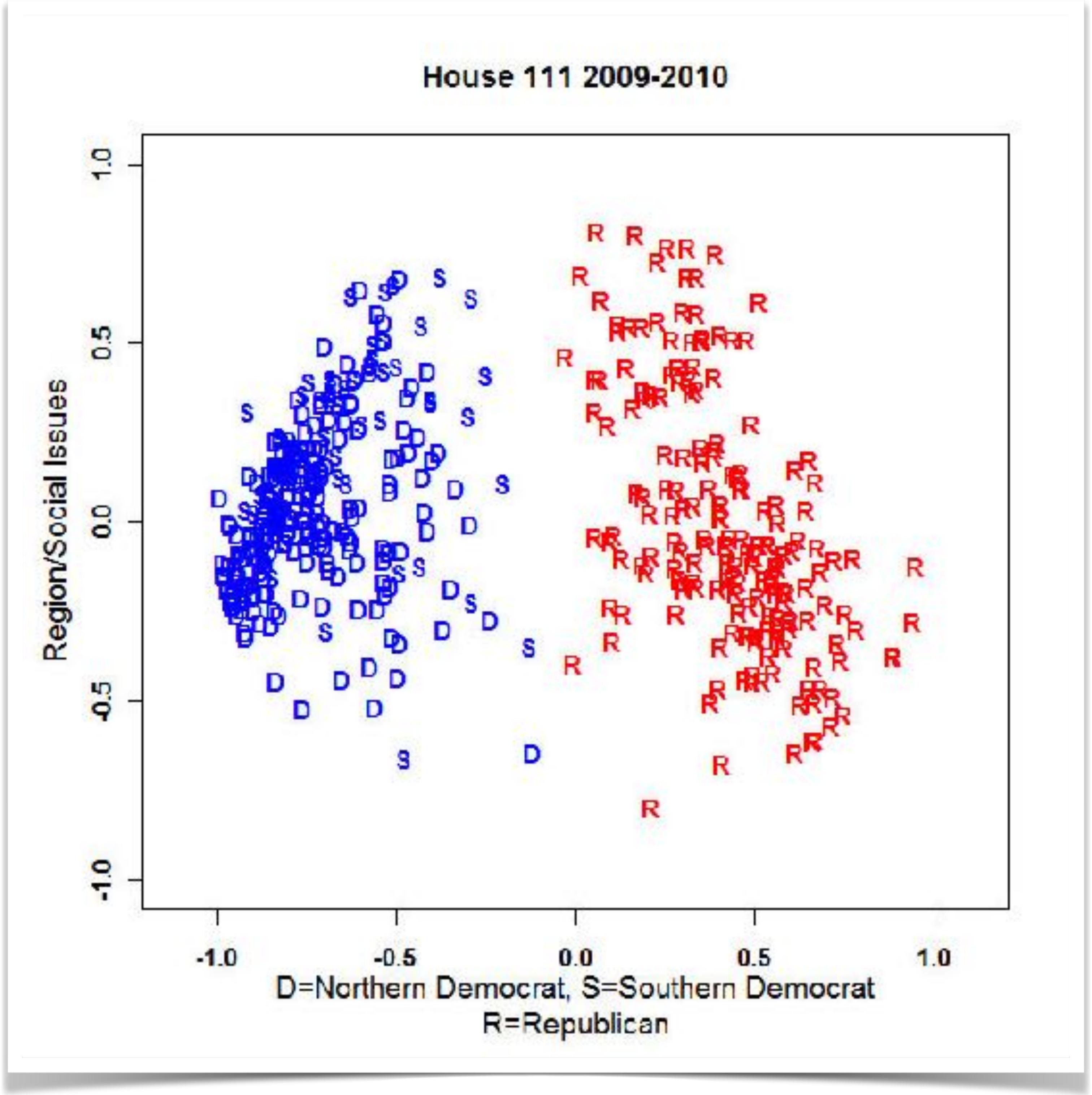
Proof: construction.

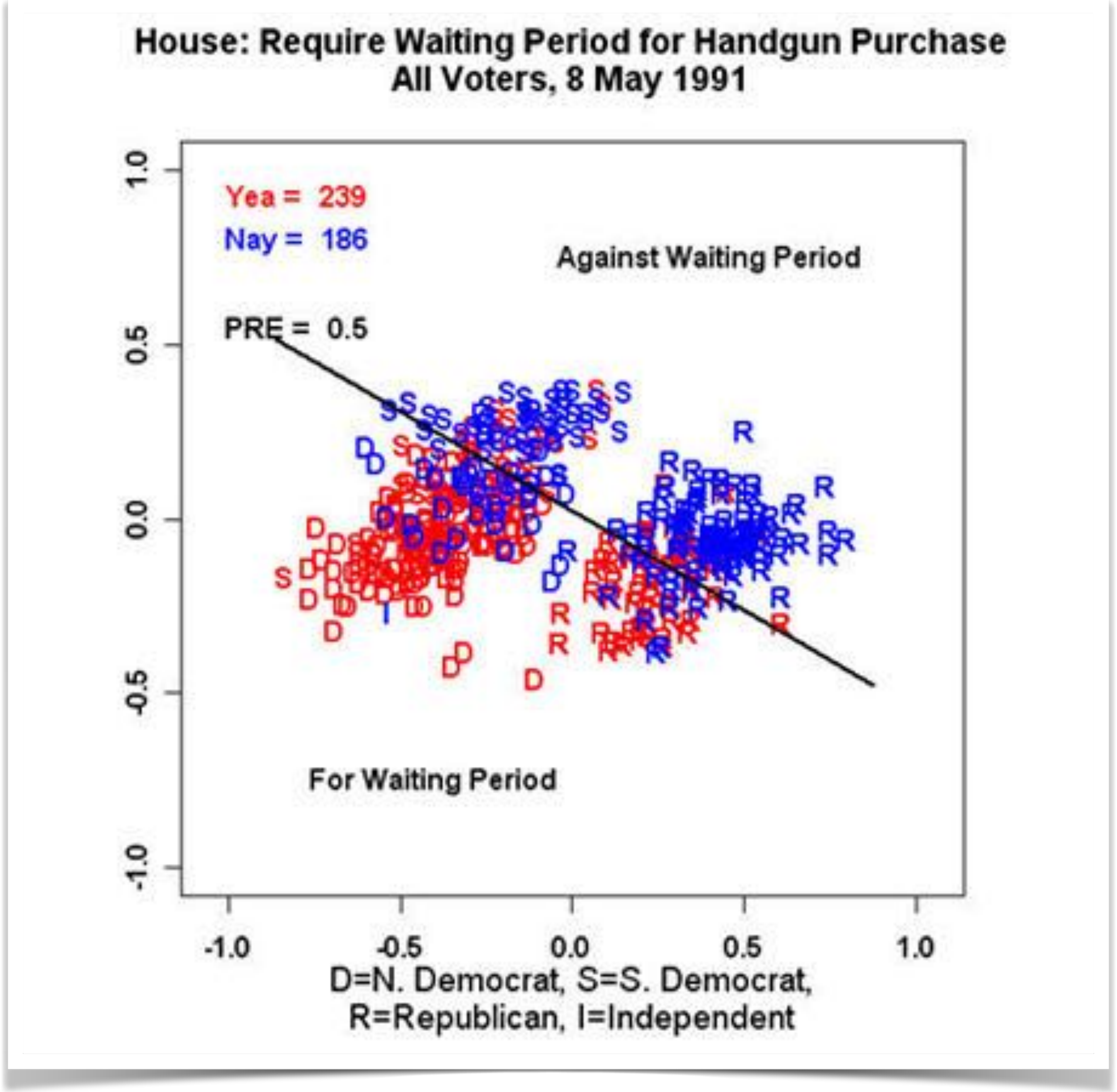
Question:

given a preference profile, can you find an “efficient” metric embedding?

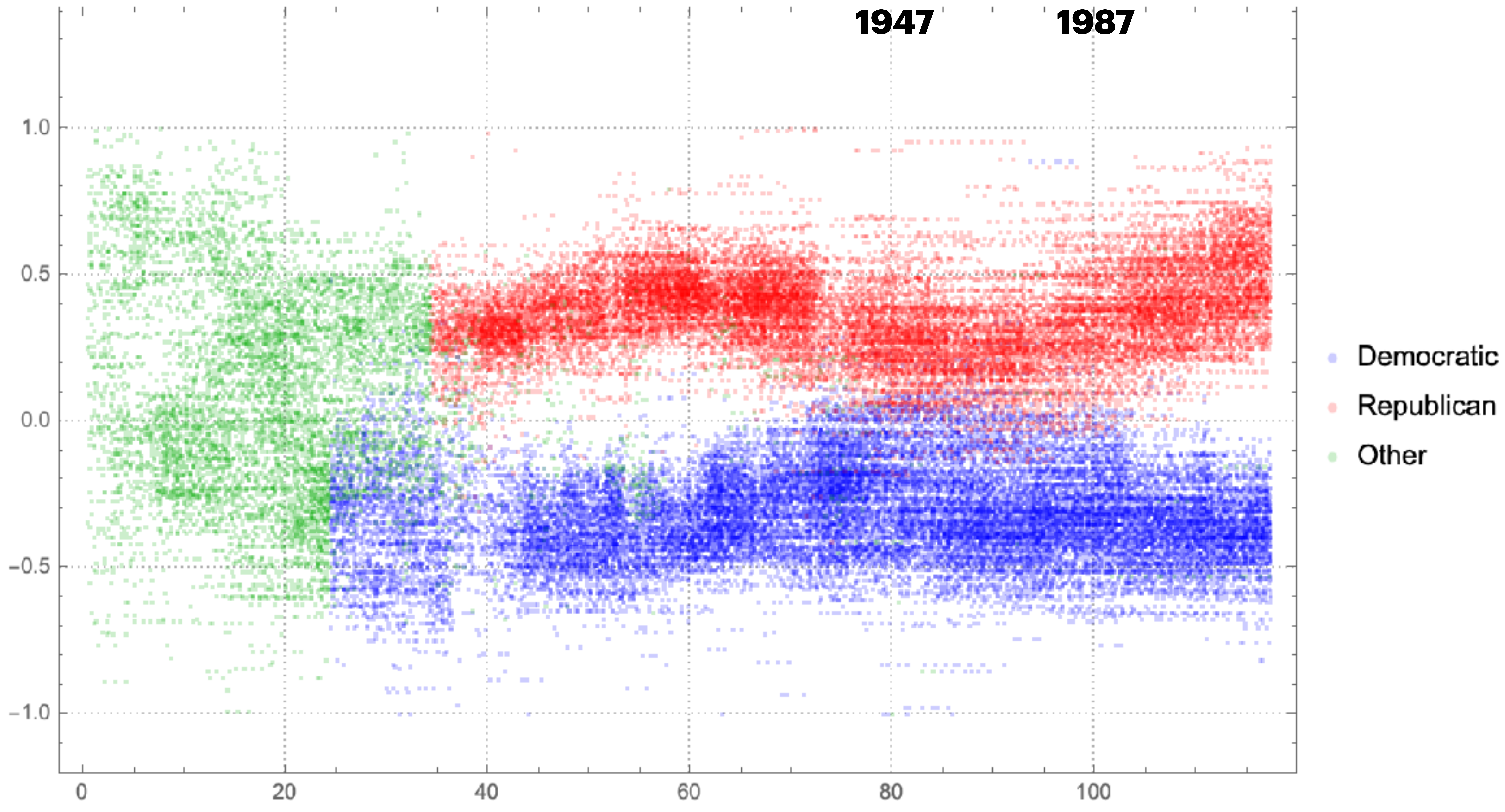
And the truth?

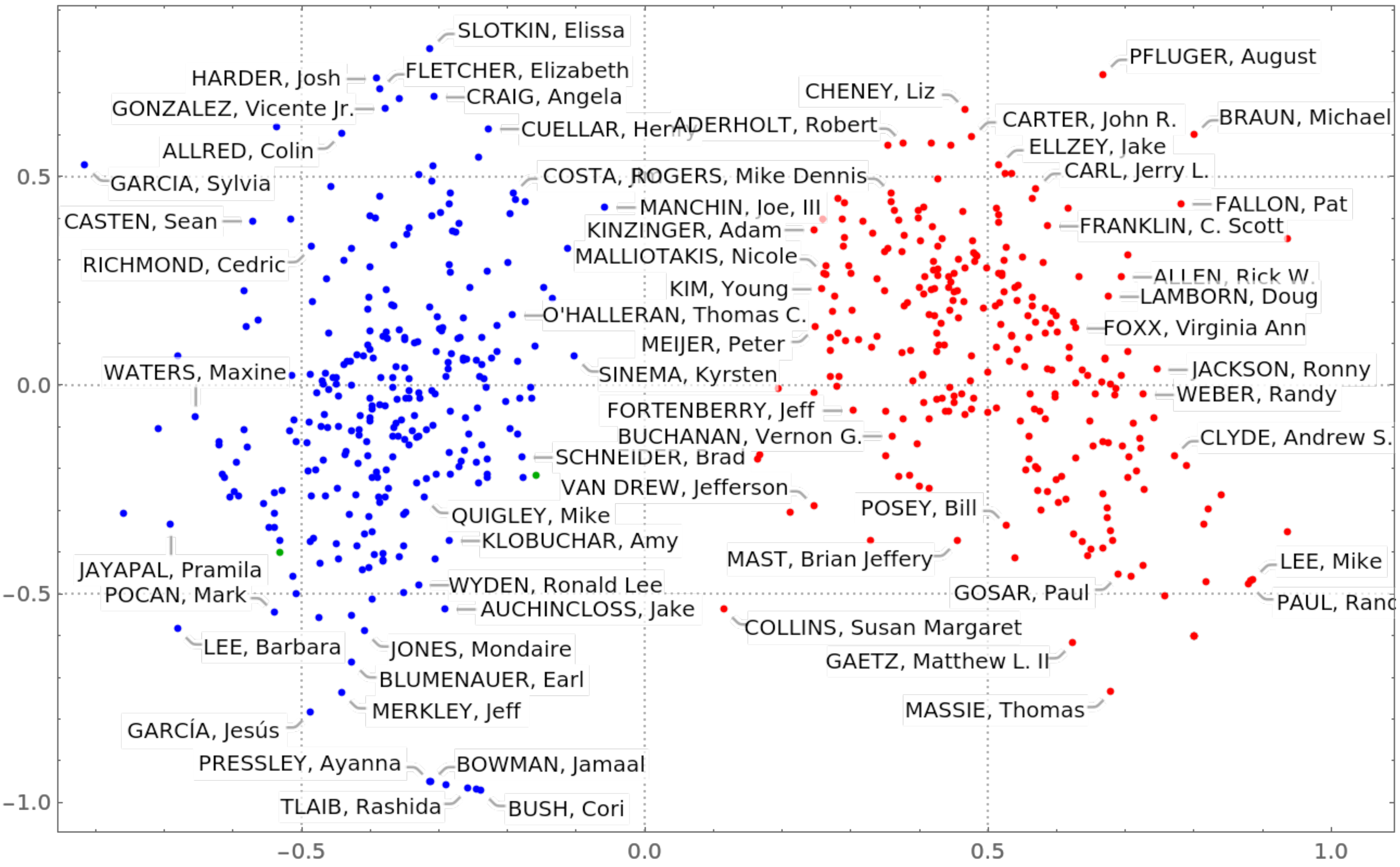
- Which is a **better** model to guide us in our search for good voting rules?





DW-NOMINATE


1947
1987
1.0
0.5
0.0
−0.5
−1.0
0
20
40
60
80
100
Democratic
Republican
Other


SLOTKIN, Elissa
HARDER, Josh
FLETCHER, Elizabeth
GONZALEZ, Vicente Jr.
CRAIG, Angela
CUELLAR, Her
ALLRED, Colin
GARCIA, Sylvia
COSTA,
CASTEN, Sean
RICHMOND, Cedric
O'HALLERAN, Thomas C.
WATERS, Maxine
SINEMA, Kyrsten
SCHNEIDER, Brad
VAN DREW, Jefferson
QUIGLEY, Mike
KLOBUCHAR, Amy
JAYAPAL, Pramila
POCAN, Mark
WYDEN, Ronald Lee
AUCHINCLOSS, Jake
LEE, Barbara
JONES, Mondaire
BLUMENAUER, Earl
MERKLEY, Jeff
GARCÍA, Jesús
PRESSLEY, Ayanna
BOWMAN, Jamaal
TLAIB, Rashida
BUSH, Cori
PFLUGER, August
CHENEY, Liz
ADERHOLT, Robert
CARTER, John R.
BRAUN, Michael
ELLZEY, Jake
CARL, Jerry L.
ROGERS, Mike Dennis
MANCHIN, Joe, III
FALLON, Pat
KINZINGER, Adam
FRANKLIN, C. Scott
MALLIOTAKIS, Nicole
ALLEN, Rick W.
KIM, Young
LAMBORN, Doug
FOXX, Virginia Ann
MEIJER, Peter
JACKSON, Ronny
WEBER, Randy
FORTENBERRY, Jeff
BUCHANAN, Vernon G.
CLYDE, Andrew S.
POSEY, Bill
MAST, Brian Jeffery
LEE, Mike
GOSAR, Paul
PAUL, Ranc
COLLINS, Susan Margaret
GAETZ, Matthew L. II
MASSIE, Thomas
0.5
0.0
−0.5
−1.0
−0.5
0.0
0.5
1.0

A lesson from history of science: seek parsimony, be willing to fundamentally update your models. (cf. *epicycles*)