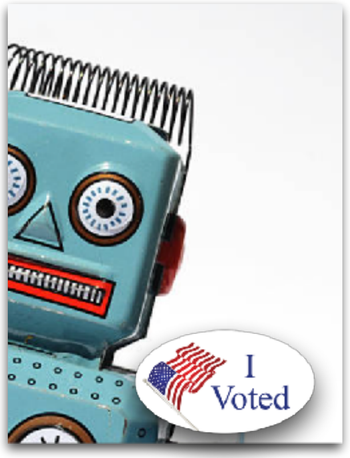
# Modeling Democracy

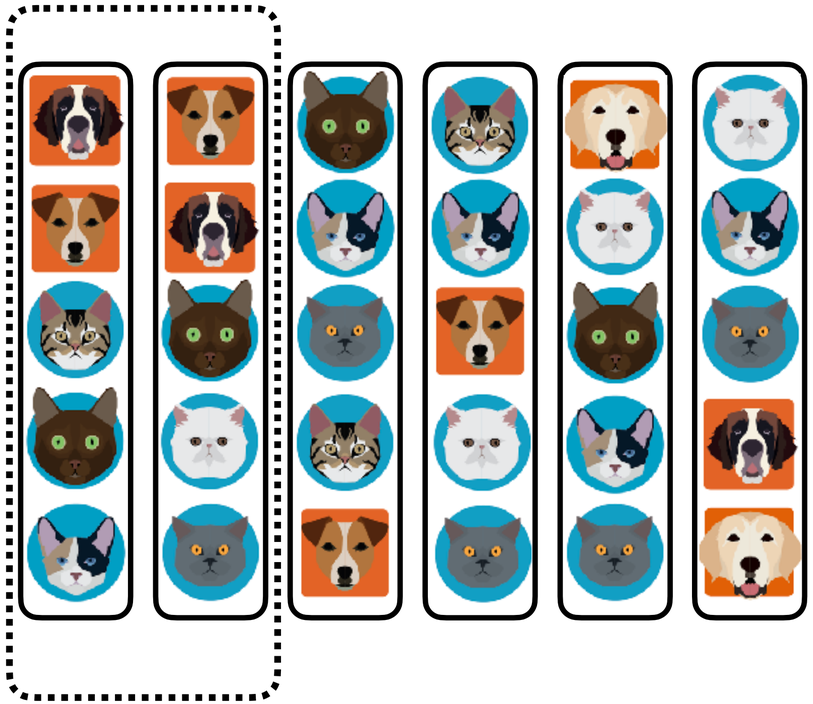
## Lecture 9 - **Proportionality à la COMSOC**




Moon Duchin, Winter 2026 — Wed Feb 4

# What might we mean by proportionality?

## *Proportionality for solid coalitions* (Dummett)



Say that voters $S \subseteq \mathscr{V}$ are a **solid bloc** for candidates $T \subseteq \mathscr{C}$ if S voters rank all T above all not-T candidates.

(Hare quota)

**Axiom (PSC):** If any $j \cdot N/k$ voters are a solid bloc for a set $T$ of candidates, then $\geq j$ of those candidates must be elected (or all are elected, if $|T| \leq j$).

**Theorem:** Single Transferable Vote (STV) satisfies PSC.

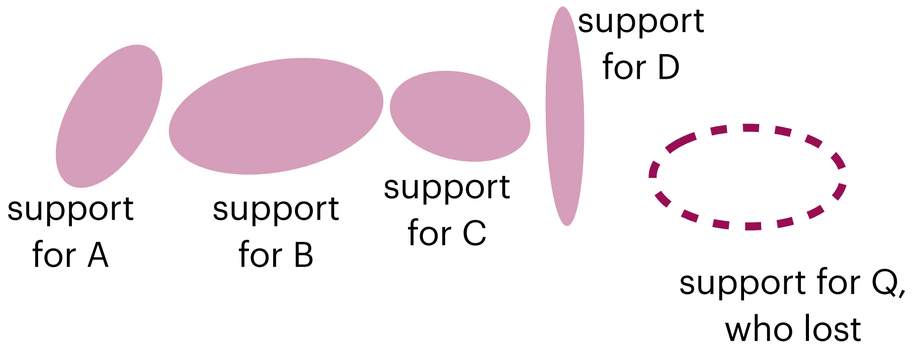
In this profile, 1/3 of the voters have cohesive preferences at the top of their ballots for a particular pair of dogs — suppose there are m=3 seats

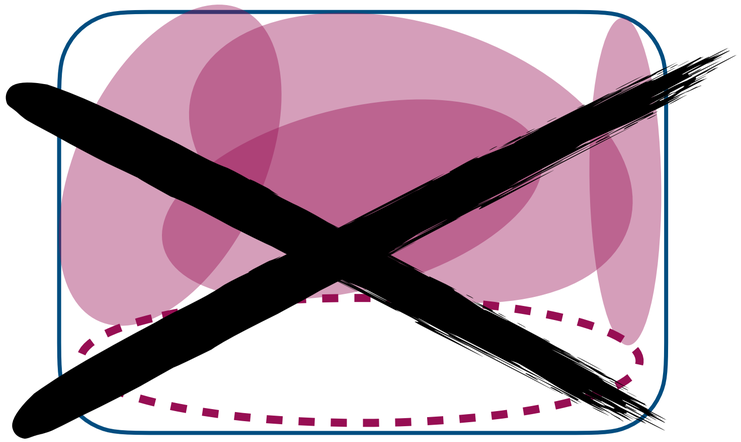
apply axiom with j=1: to qualify for PSC, a system must guarantee one of these two dogs is elected

# Can we prove theorems?

Aziz et al.,
Sánchez-Fernández et al.,
Brill–Peters, …

## *Justified representation*



Voters' approvals (overlapping)

**Axiom (JR):** If a bloc of $N/k$ of the voters all approve of a common candidate, then they can't be totally shut out—i.e., at least one of them must approve of at least one winner

**Theorem:** Proportional Approval Voting (PAV) satisfies JR.

**Variants**: EJR (for every $\ell$-cohesive group, some member must approve at least $\ell$), PJR (some $\ell$ winners must be approved by at least one bloc member), rank-PJR, EJR+, .....

**JR** - someone gets one

**EJR** - someone gets $\ell$

**PJR** - group gets $\ell$

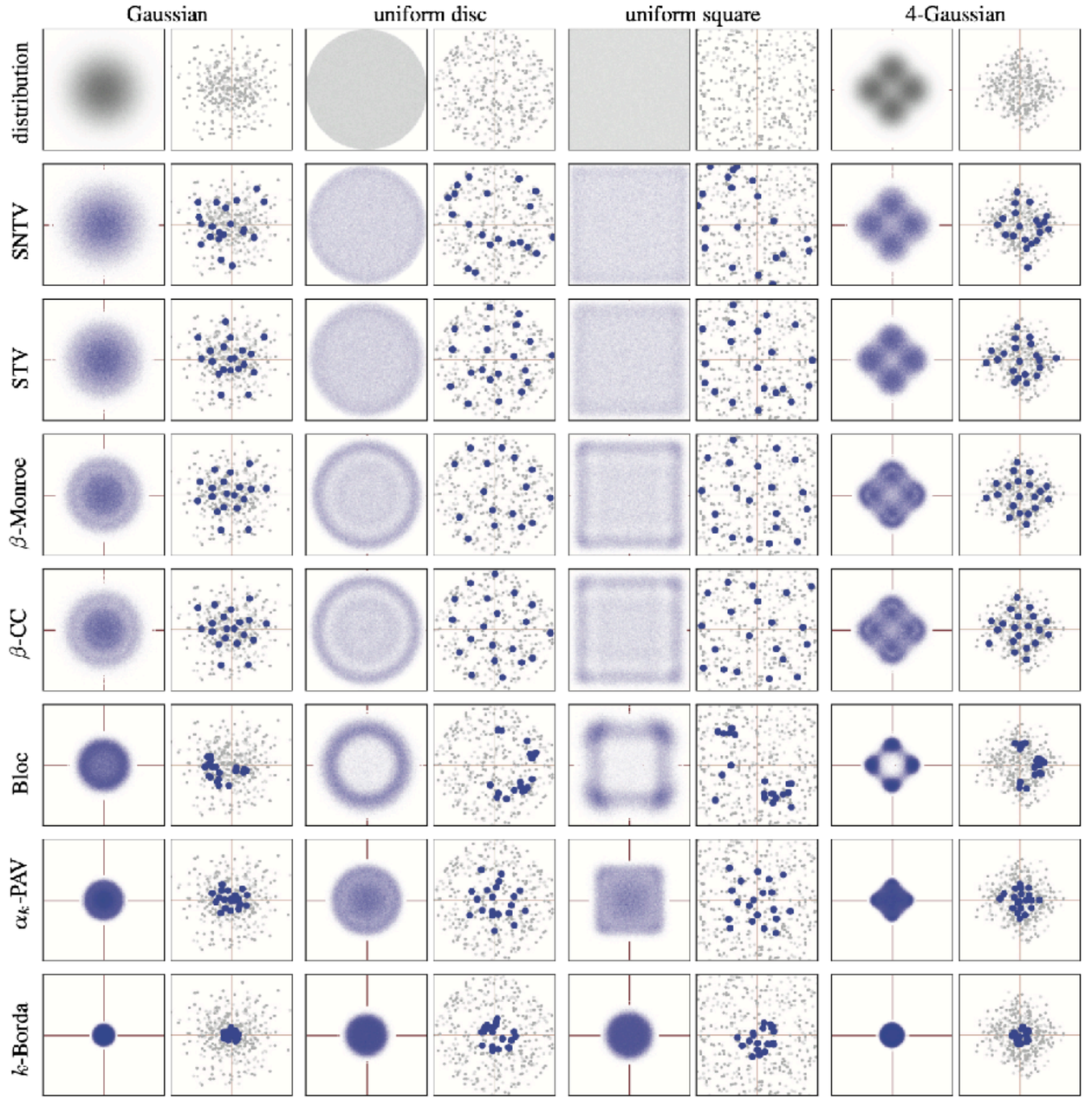
to make statements about
**typical profiles**,
we need probabilistic models

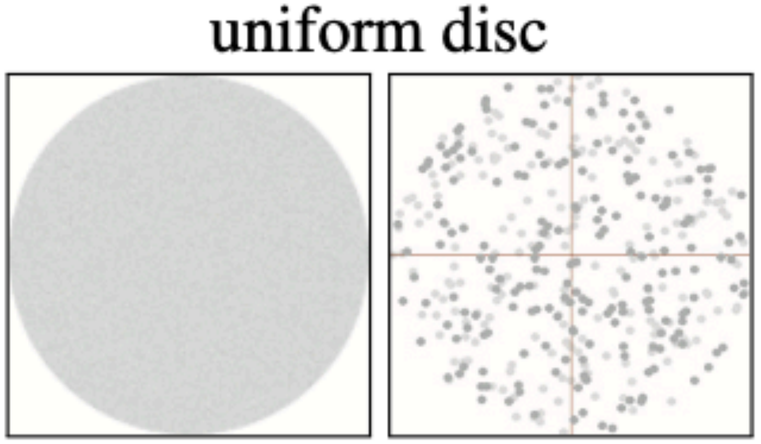
i.e., distributions over profiles

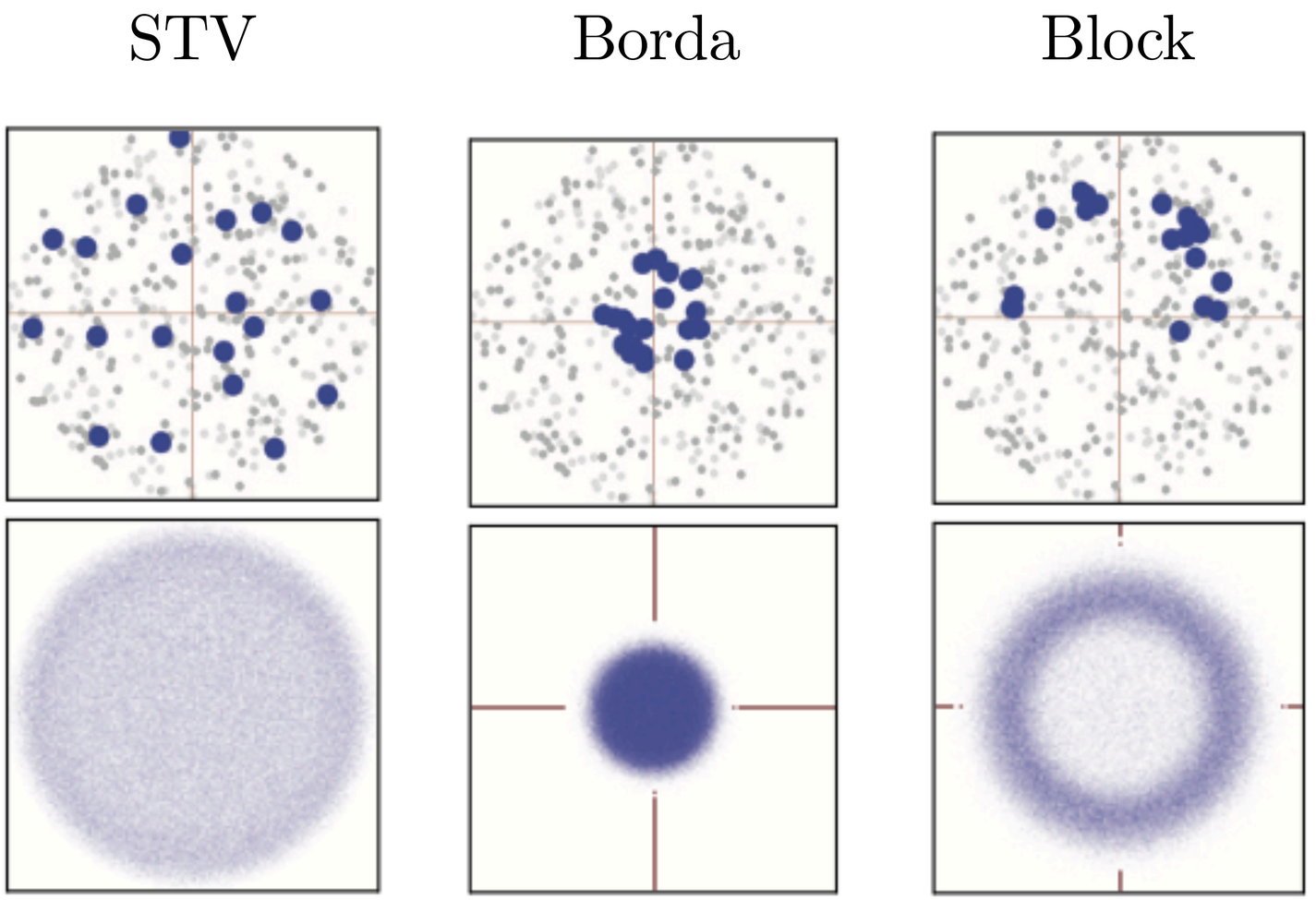
in COMSOC these are called “statistical cultures”

# spatial models as generative models

Elkind et al.





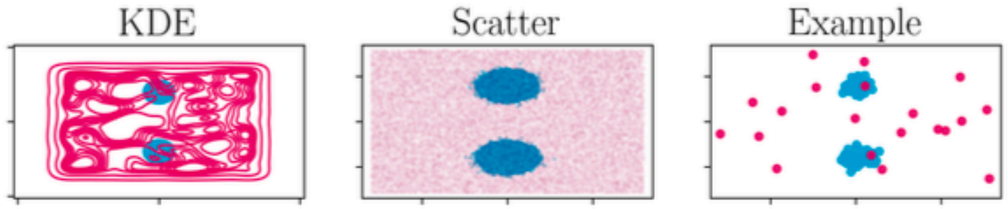




(Visually) justifies statements like...

- STV gives proportional representation
- Borda elects centrists
- Block voting is anti-centrist

# Putting ideas together for a different account of fairness

Duchin-Quinn




- Let's use the metric framework!
- Define **inefficiency** for a particular group that "deserves" $\ell$ winners as the cost ratio of its favorite $\ell$ winners over its favorite $\ell$ candidates
- Elkind et al. style experiment design

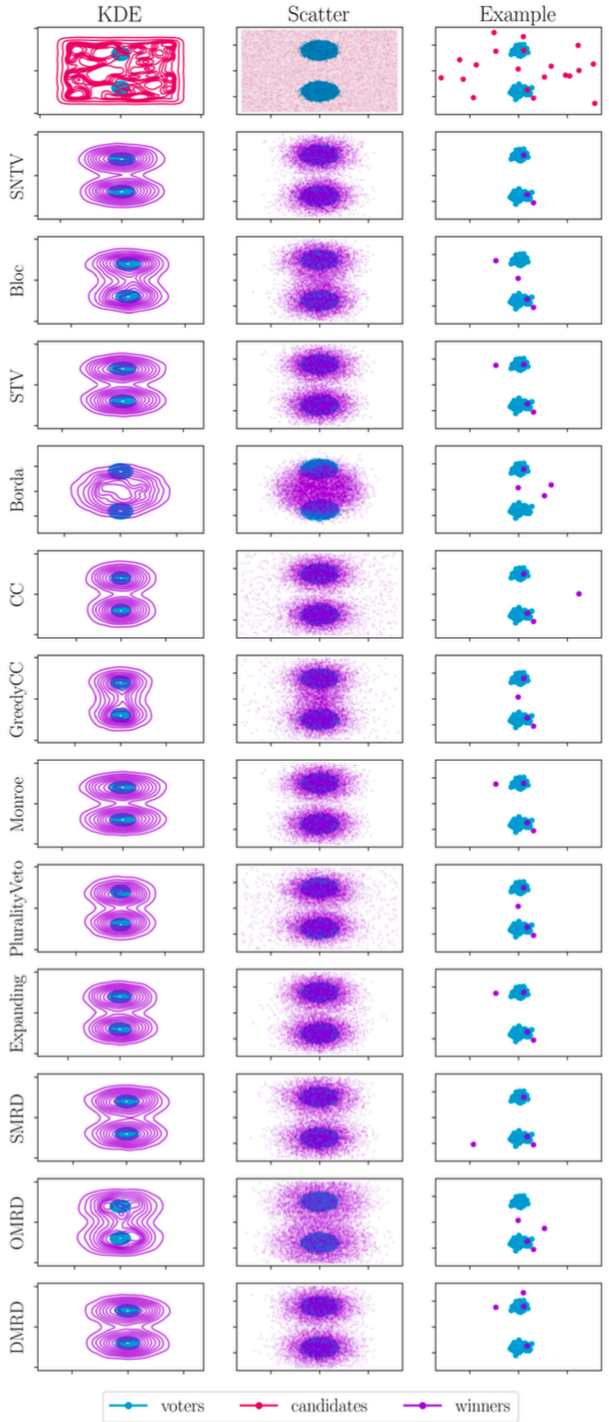
# Putting ideas together for a different account of fairness

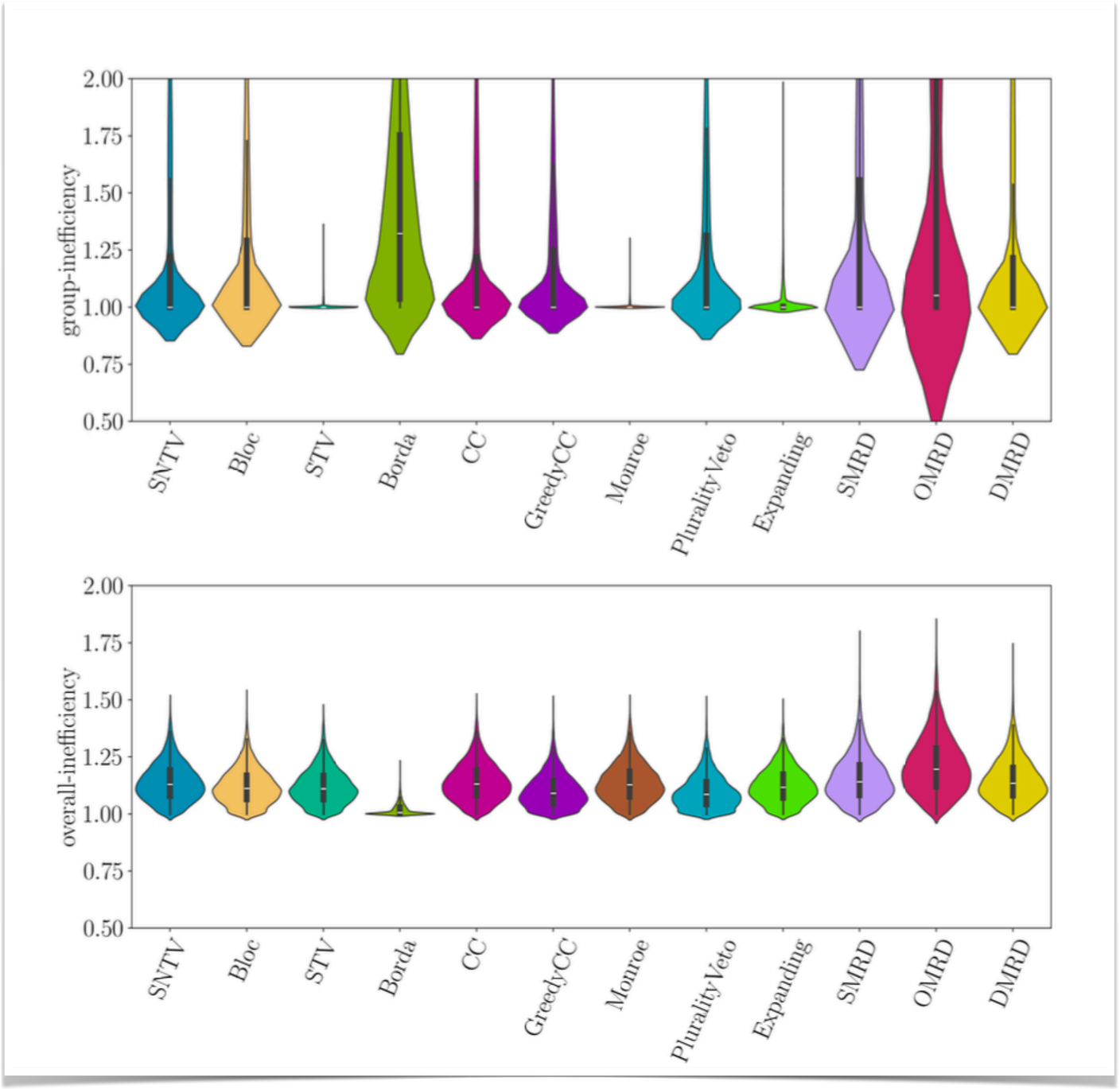
- Let's use the metric framework!
- Define **inefficiency** for a particular group that "deserves" $\ell$ winners as the cost ratio of its favorite $\ell$ winners over its favorite $\ell$ candidates
- Elkind et al. experiment design
  - SNTV, Block, STV, Borda
  - other popular COMSOC systems (Chamberlain-Courant, Greedy CC, Monroe, Plurality Veto, Expanding Approvals)
  - Random Dictator variants (Sequential, Discounted, One-Shot)

**Chamberlin-Courant:** This rule elects the group of $k$ candidates that maximizes the sum over voters of the highest Borda score they award to any winner.

**Monroe:** This is nearly identical to Chamberlin-Courant, but we add an additional constraint on the number of voters that a single candidate can represent to be between $\lfloor \frac{n}{k} \rfloor$ and $\lceil \frac{n}{k} \rceil$.

**Expanding approvals** Another recently designed multi-round election (Kalayci, Kempe, and Kher 2024; Aziz and Lee 2020), where at each round $i$ all voters are queried in a randomized order for their $i$th candidate preference. As soon as a candidate $c$ is named $\lceil \frac{n}{k} \rceil$ times, they are immediately elected and all voters who previously voted for $c$ are removed. This repeats until ballots have been exhausted and we have elected $k$ candidates.




group-inefficiency
2.00
1.75
1.50
1.25
1.00
0.75
0.50
SNTV
Bloc
STV
Borda
CC
GreedyCC
Monroe
PluralityVeto
Expanding
SMRD
OMRD
DMRD
overall-inefficiency
2.00
1.75
1.50
1.25
1.00
0.75
0.50
SNTV
Bloc
STV
Borda
CC
GreedyCC
Monroe
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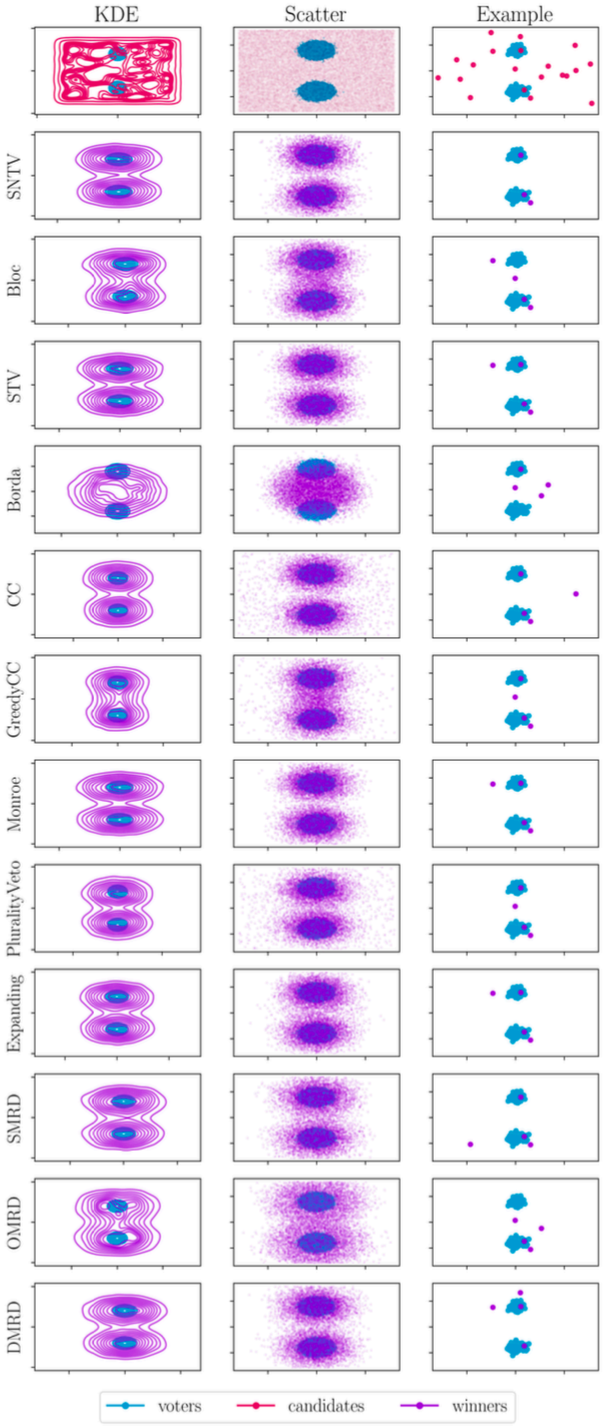
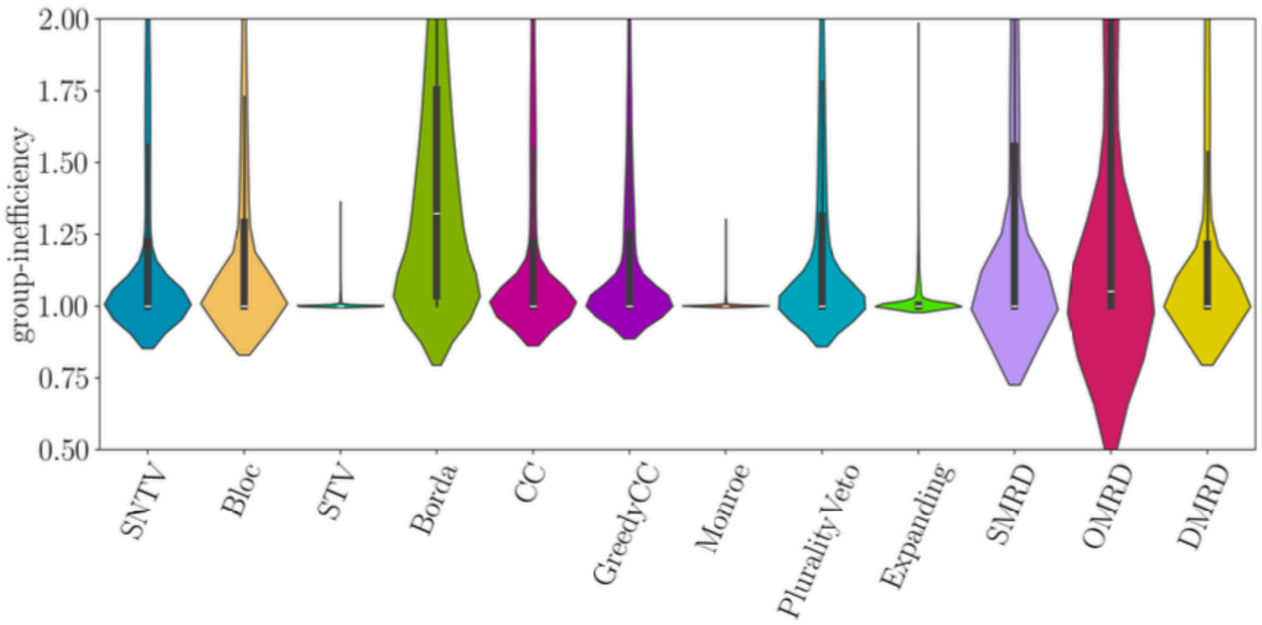
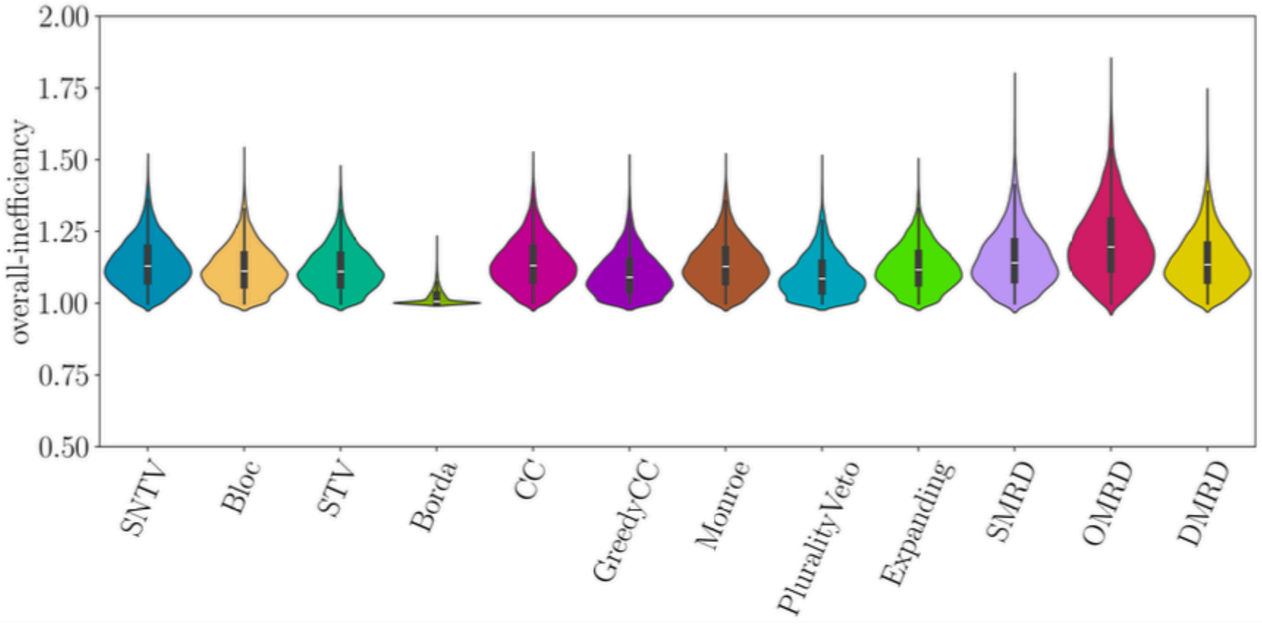
SMRD
OMRD
DMRD


KDE
Scatter
Example
SNTV
Bloc
STV
Borda
CC
GreedyCC
Monroe
PluralityVeto
Expanding
SMRD
OMRD
DMRD
voters
candidates
winners




Best from **group**-based view:
STV, Monroe, Expanding approvals

Worst: Borda and OMRD

**Overall** view hides this and valorizes (only) Borda!

(so worst-case defs will fail as well)

# How does group fairness depend on the size of the bloc?

For groups of 25-40%, Block and OMRD are *ghastly* and Borda is quite bad — in the group view

The overall view not only hides this but likes Block and Borda *best* in the same range