Lecture 10
Dynamics

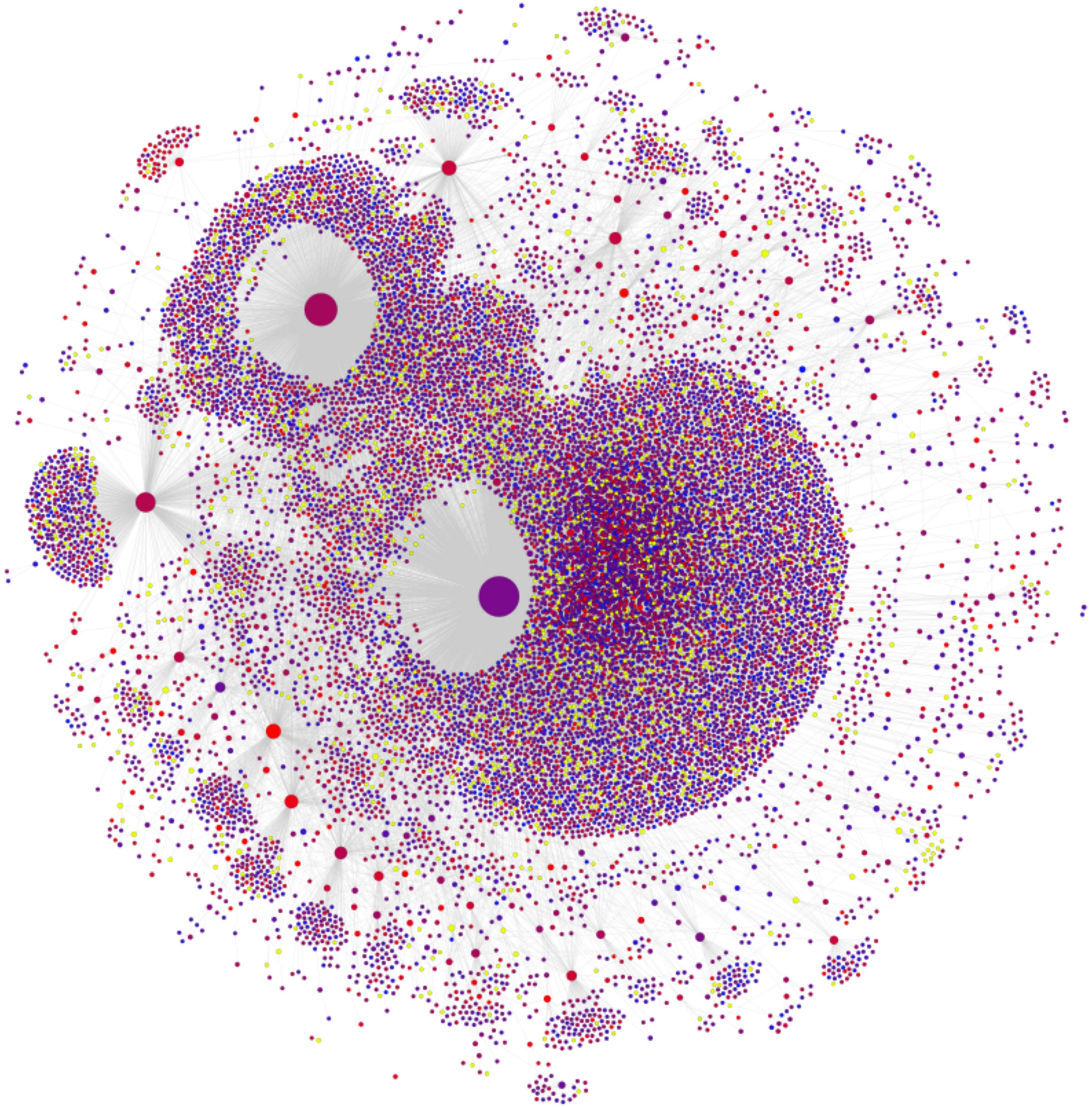
Outline

- Ideas, information, influence
- Epidemic spreading
- Opinion dynamics

Example: fake news spreading

A fake news article spreading via Twitter during 2016 US presidential campaign

Red nodes are likely bots



Dynamics on networks

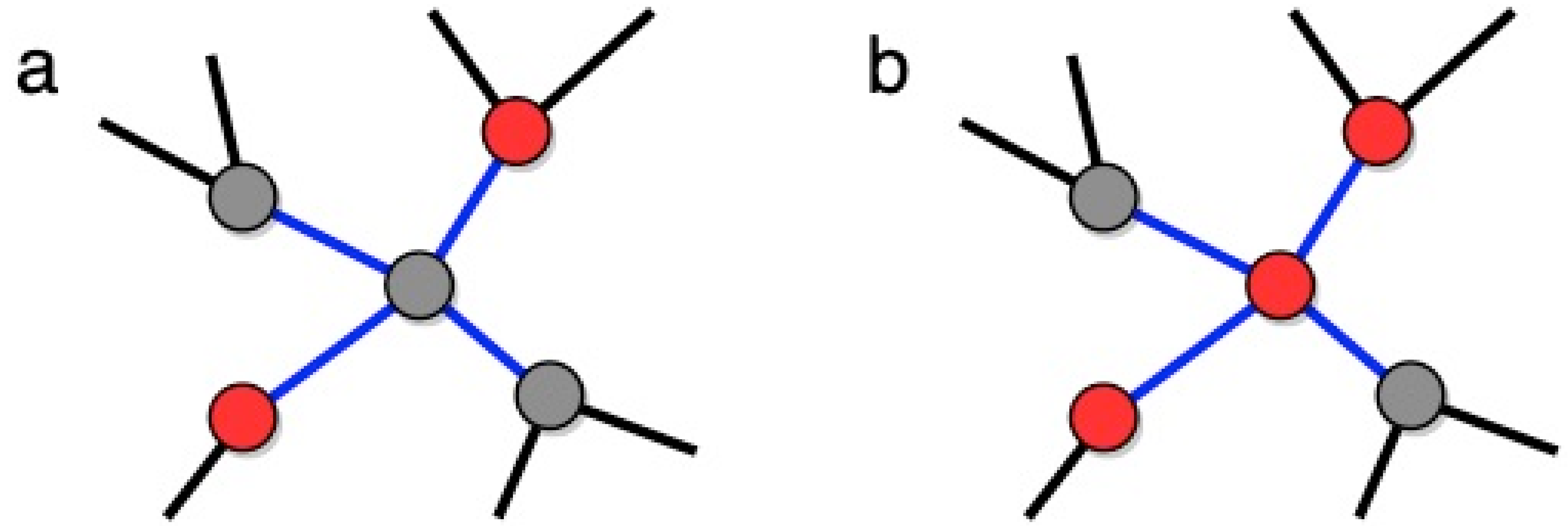
- Misinformation diffusion is an example of dynamics on networks: nodes carry features that can be modified or copied via/by those of their neighbors!
- **Examples:**
  - **Information diffusion**
  - **Epidemic spreading**
  - **Opinion dynamics**
  - **Network search**

Information diffusion

- Networks are critical to the diffusion of information:
  - We may buy a new phone because a friend has bought one
  - We may discover a piece of news because friends forward it to us
- **Social contagion:** how individuals' opinions, beliefs, ideas influence or are influenced by their neighbors in social networks

Information diffusion

- Standard setup of models of influence/information spreading:
  - A certain number of nodes (**influencers**) are initially activated (e.g., some individuals adopt an idea, innovation, behavior)
  - Each inactive node is activated (or not) according to some rule that depends on the presence of active neighbors and other factors
- Outcome: **influence cascades**, the activation in sequence of a subset of the nodes in the network
- Options: cascades may involve a handful of nodes or a major proportion of the network (**global cascades**)



Threshold models

- **Principle:** a node can be activated only if the influence exerted on it by its active neighbors exceeds a given value (**threshold**)

- **Linear threshold model:** the influence on a node is defined as a sum over its active neighbors, in which the contribution of each neighbor is given by the weight of the link joining it to the node:

$$I(i) = \sum_{j:\,active} w_{ji}$$

- w_{ji} = weight of the link from j to i

Threshold models

- **Activation condition:** $$I(i) \geq \theta_i$$

  where θ_i is the threshold of node *i*, indicating its tendency to be influenced

- **On unweighted networks:** $$I(i) = n_i^{on} \geq \theta_i$$

  where n_i^{on} is the number of active neighbors of node *i*

- If all nodes have the same threshold θ: $$I(i) = n_i^{on} \geq \theta$$

Threshold models

- **Model dynamics:**
  - Initially some nodes are activated, for instance a random number or a random proportion
  - All active nodes remain active
  - Each inactive node is activated if the number of active neighbors is at or above its threshold
- **Two different ways of updating the node states:**
  - **Asynchronous update:** the new activation state of each node depends on the activation states of the nodes in the current (changing) configuration
  - **Synchronous update:** the new activation state of each node depends on the activation states of the nodes in the configuration at the end of the previous iteration

Fractional threshold model

- **Principle:** instead of the number of active neighbors, we consider the **fraction**

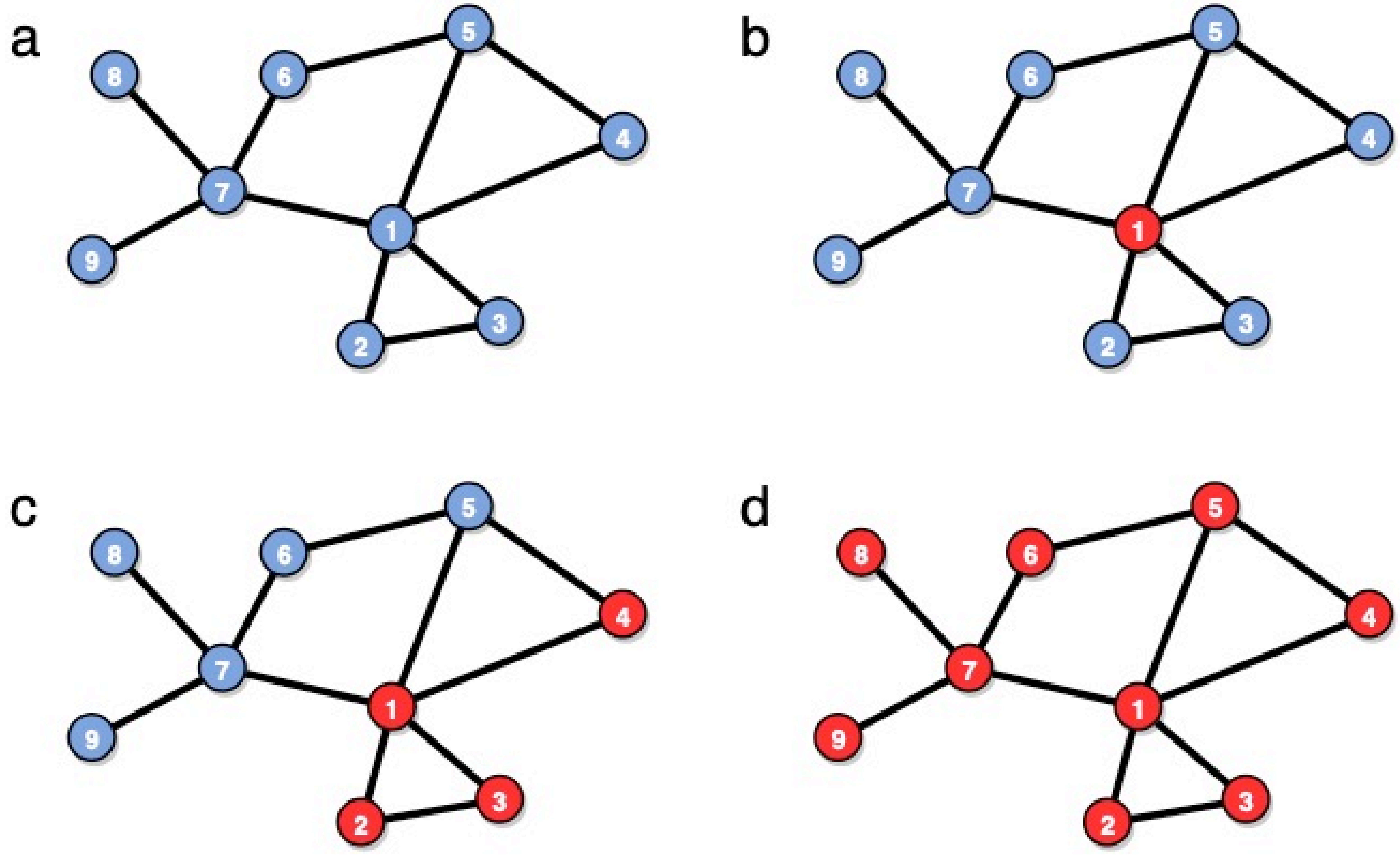
- **Example:** if θ=1/2, at least half of the neighbors must be active!

- **Activation condition:**

$$\frac{n_i^{on}}{k_i} \geq \theta_i$$

where k_i is the degree of node i

Fractional threshold model



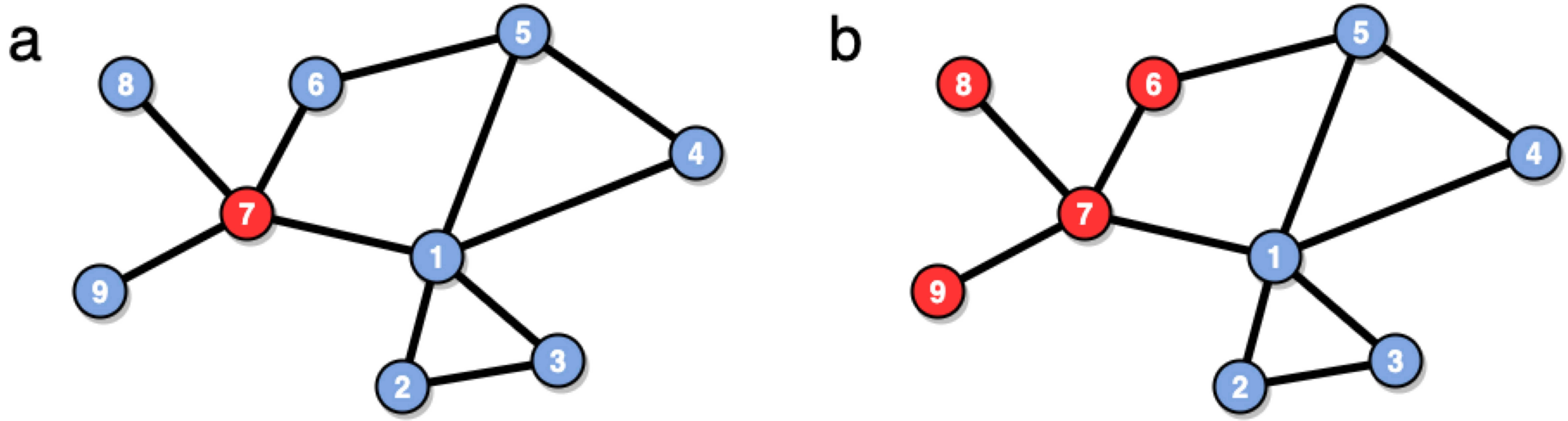
Fractional threshold model

- **Remark:** if the network is sparse, whether or not a cascade is triggered depends on its structure
- Key drivers: **vulnerable nodes**, i.e., the nodes that can be activated by a single active neighbor
- **Condition for a node to be vulnerable:** $k_i \leq \dfrac{1}{\theta_i}$
- To have global cascades, the number of vulnerable nodes must be sufficiently large!

Fractional threshold model

- Hubs are *usually* very effective influencers
- **Caveat:** being a hub does *not necessarily* imply being a good influencer
- **Importance of active node position:** being in the core of the network makes global cascades more likely than if the node is in the periphery
- **Importance of community structure:** the spread is easier within dense communities than across communities! In fact, to influence another community it is necessary for (some of) its nodes to be activated, which is not easy because they tend to have few neighbors in the activated community

Fractional threshold model



Fractional threshold model

- **Cascade control:** knowing the structure of the network allows us to control the evolution of a cascade. Sometimes small cascades can become large by activating a few carefully chosen nodes
- **Detecting key influencers:** critical for the success of a product or idea!
- **Viral marketing:** social networks are used to promote products

Independent cascade models

- Principle of threshold models: **peer pressure**, the more people try to persuade you, the more likely they will succeed

- **Remark:** social influence often works **one-to-one**, we may be persuaded by a single passionate individual

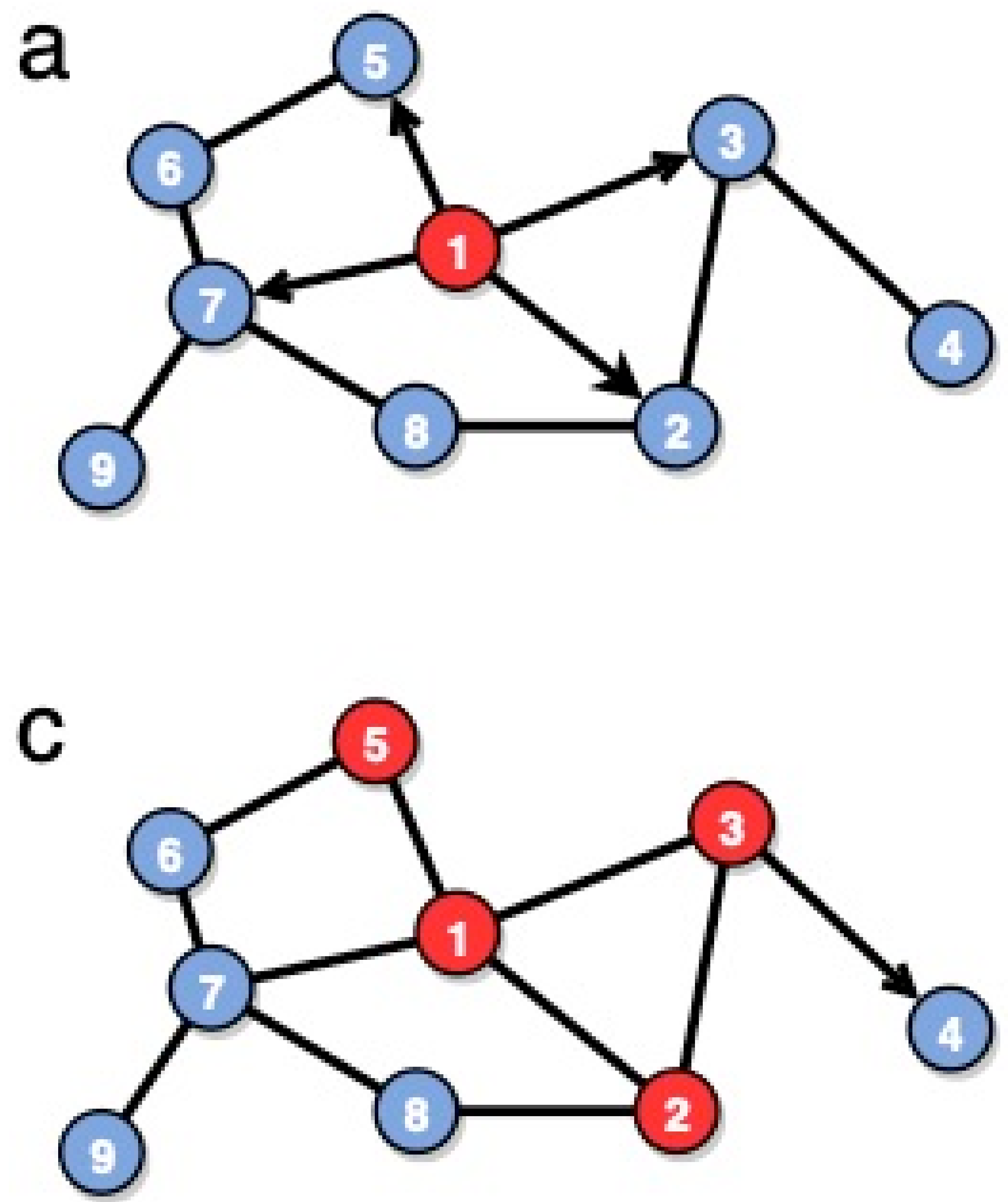
- **Alternative principle:** each of our contacts has their own influence

- **Independent cascade models** are based on node-node interactions!

Independent cascade models

- **Model dynamics:**
  - An active node i has a probability p_{ij} to convince its inactive neighbor j ($p_{ij} \neq p_{ji}$, in general)
  - All active nodes are considered in sequence: the inactive neighbor j of the active node i is activated with probability p_{ij}. All inactive neighbors of i have one chance to be persuaded by i
  - If a node j is activated, it has only one chance to activate its inactive neighbors

Independent cascade models



Independent cascade models

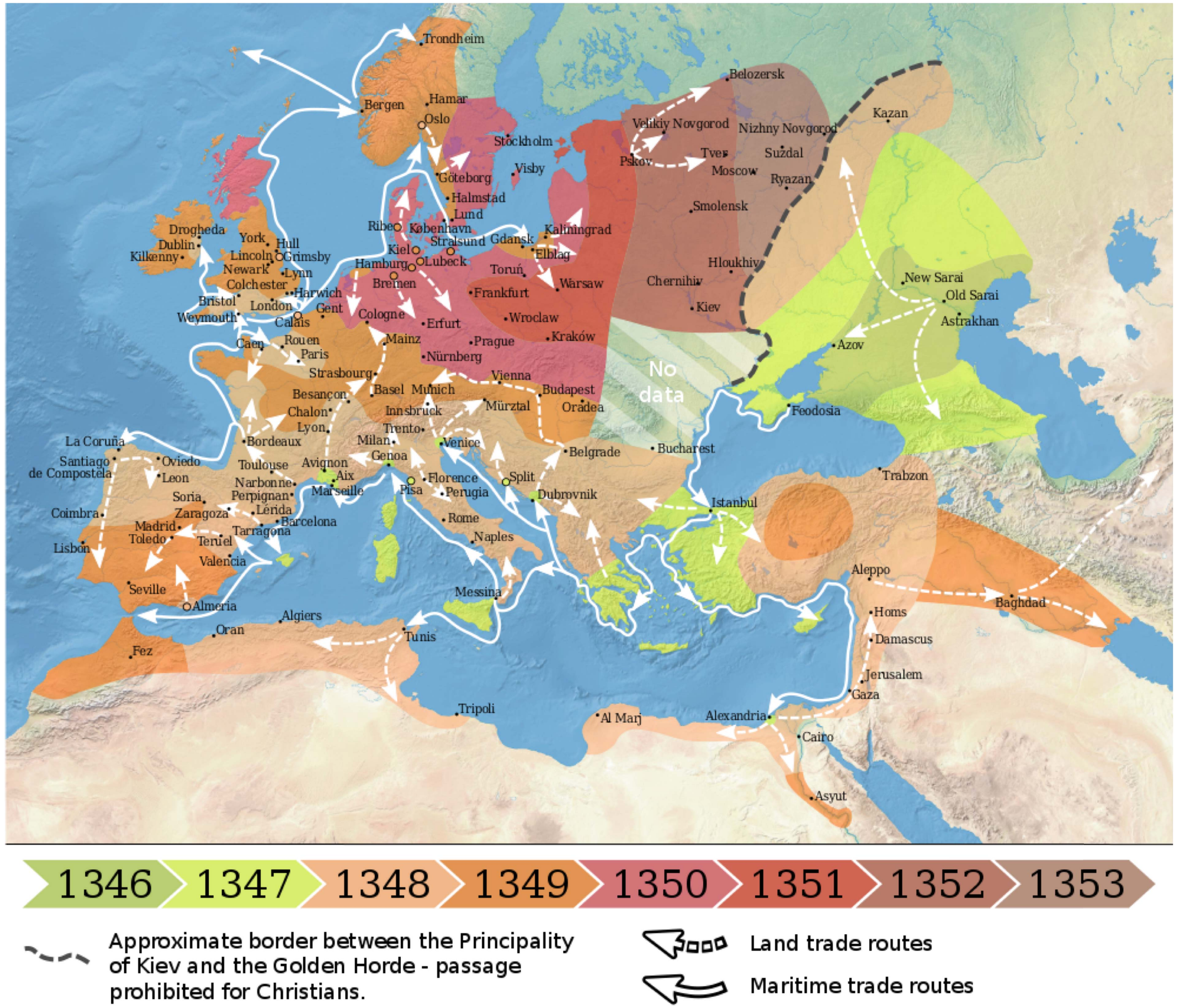
- **Remark**: the more active neighbors, the more likely a node will be activated
- **Independent cascade versus threshold models**:
  - Threshold models focus on the inactive nodes, independent cascade models on the active ones
  - Threshold models are (usually) **deterministic**: the dynamics depends on whether the threshold condition is satisfied or not
  - Independent cascade models are **probabilistic**: nodes are activated with a given probability —> it is more **difficult to control a cascade!**

Information diffusion

- **Problem**: models are too simple to be realistic
- **Solution**: more sophisticated variants!
- **Example**:
  - Probabilistic version of threshold model, in which the chance of being activated grows with the number of active neighbors (instead of the usual yes/no dynamics)
  - Similar to independent cascade model, except that the active neighbors **do not exert influence independently of each other!**
- **Complex contagion:** each new person exposing us to a new idea or product has greater influence than the previous ones!

Epidemic spreading: the Black Death

Probably originated in Central Asia, it spread throughout all of Europe between 1346 and 1353. The Black Death is estimated to have killed 30-60% of Europe's population

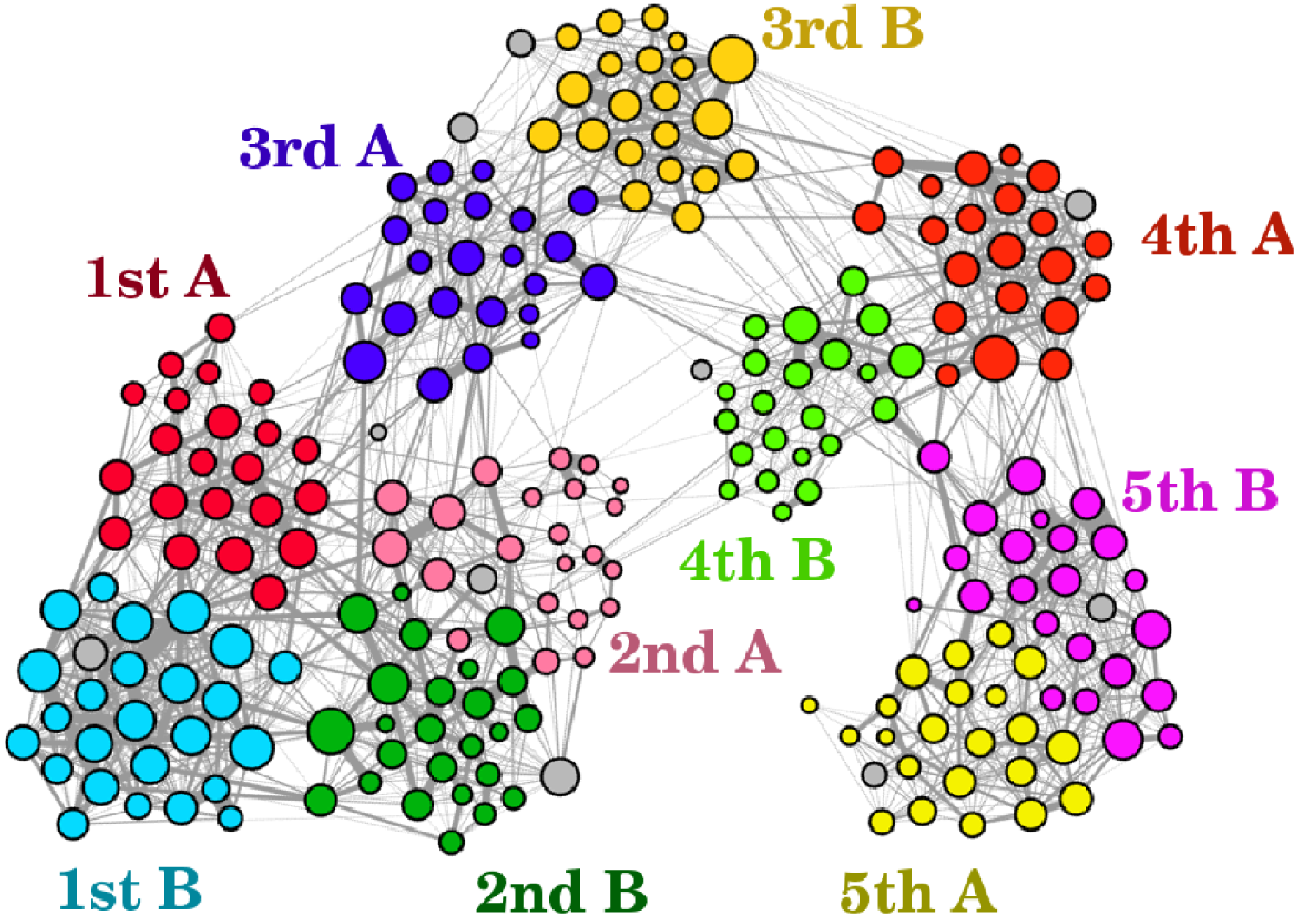


Epidemic spreading

- **Problems**:
  - Nowadays the speed of epidemic spreading has increased enormously due to advances in transportation: someone contracting Ebola in Africa can travel to Europe, America and Asia and spread the disease before being aware of it
  - Technology has created new types of epidemics: computer viruses & malware spread over the Internet. Mobile phone viruses spread via Bluetooth or MMS. Misinformation spreads through social media, etc.

Contact networks

Epidemics spread on **contact networks**, such as networks of physical contacts, transportation, the Internet, email, online social networks, and mobile phone communication



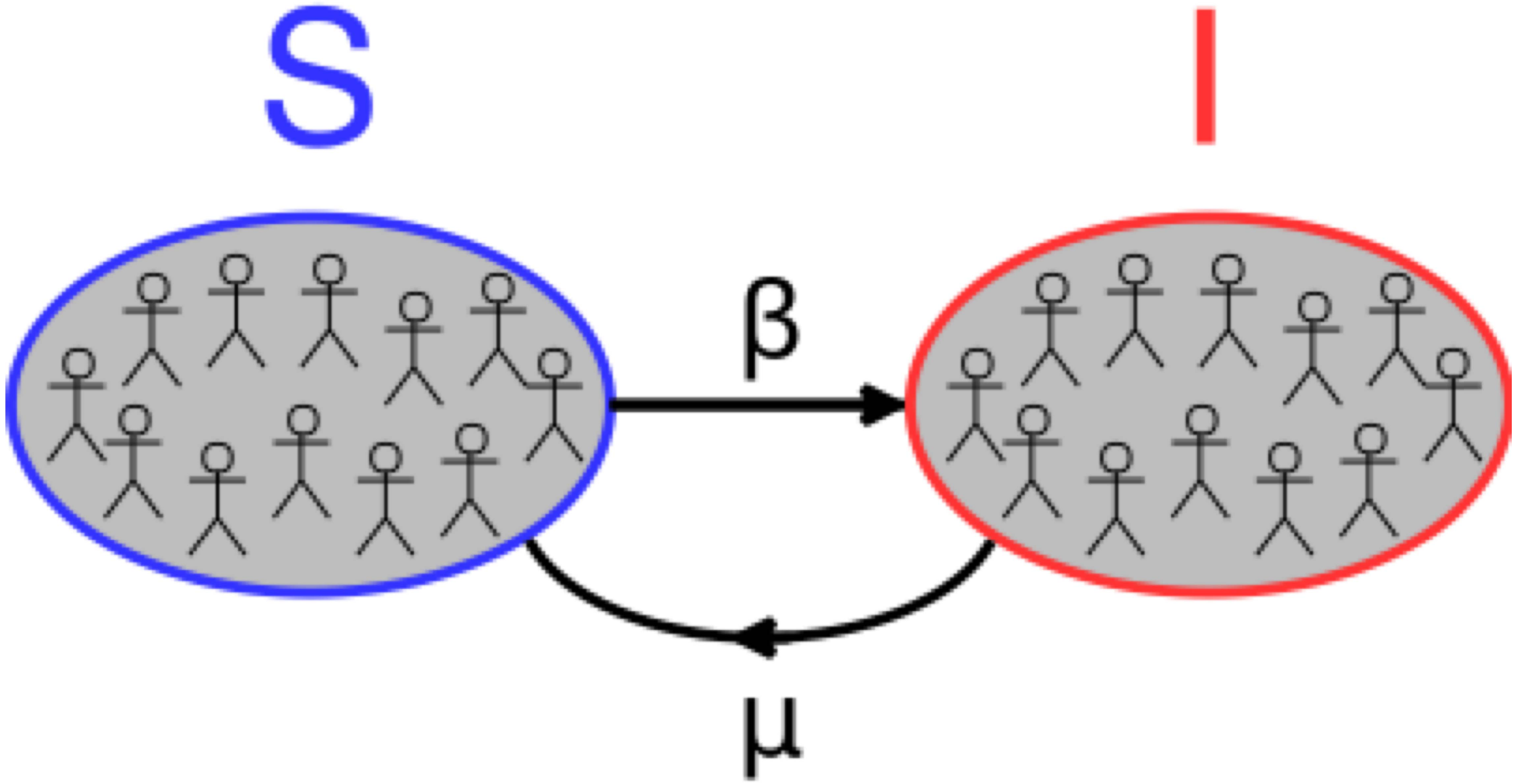
Epidemic models

- Classic epidemic models divide the population into **compartments**, corresponding to different stages of the disease
- **Key compartments**:
  - **Susceptible (S)**: individuals who can contract the disease
  - **Infected (I)**: individuals who have contracted the disease and can transmit it to susceptible individuals
  - **Recovered (R)**: individuals who recovered from the disease and cannot be infected anymore

The SIS model

- Just **two compartments**: Susceptible (S) and Infected (I)
- **Dynamics**:
  - A susceptible individual gets infected with a probability β (**infection rate**)
  - An infected individual recovers and becomes susceptible again with a probability μ (**recovery rate**)
- The model applies to diseases that do not confer long-lasting immunity (e.g., common cold)

The SIS model

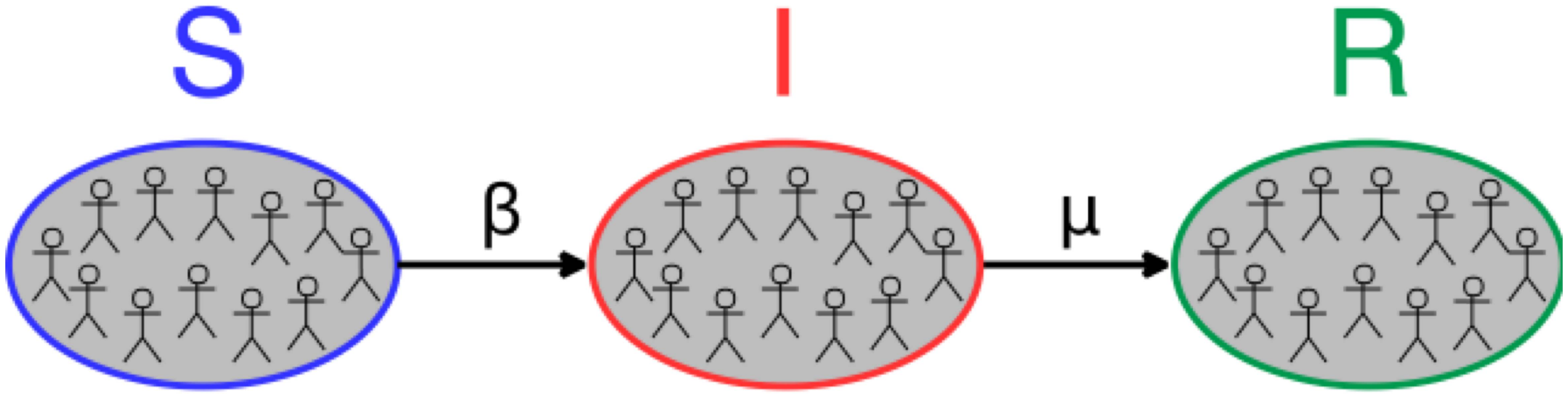


The SIS model

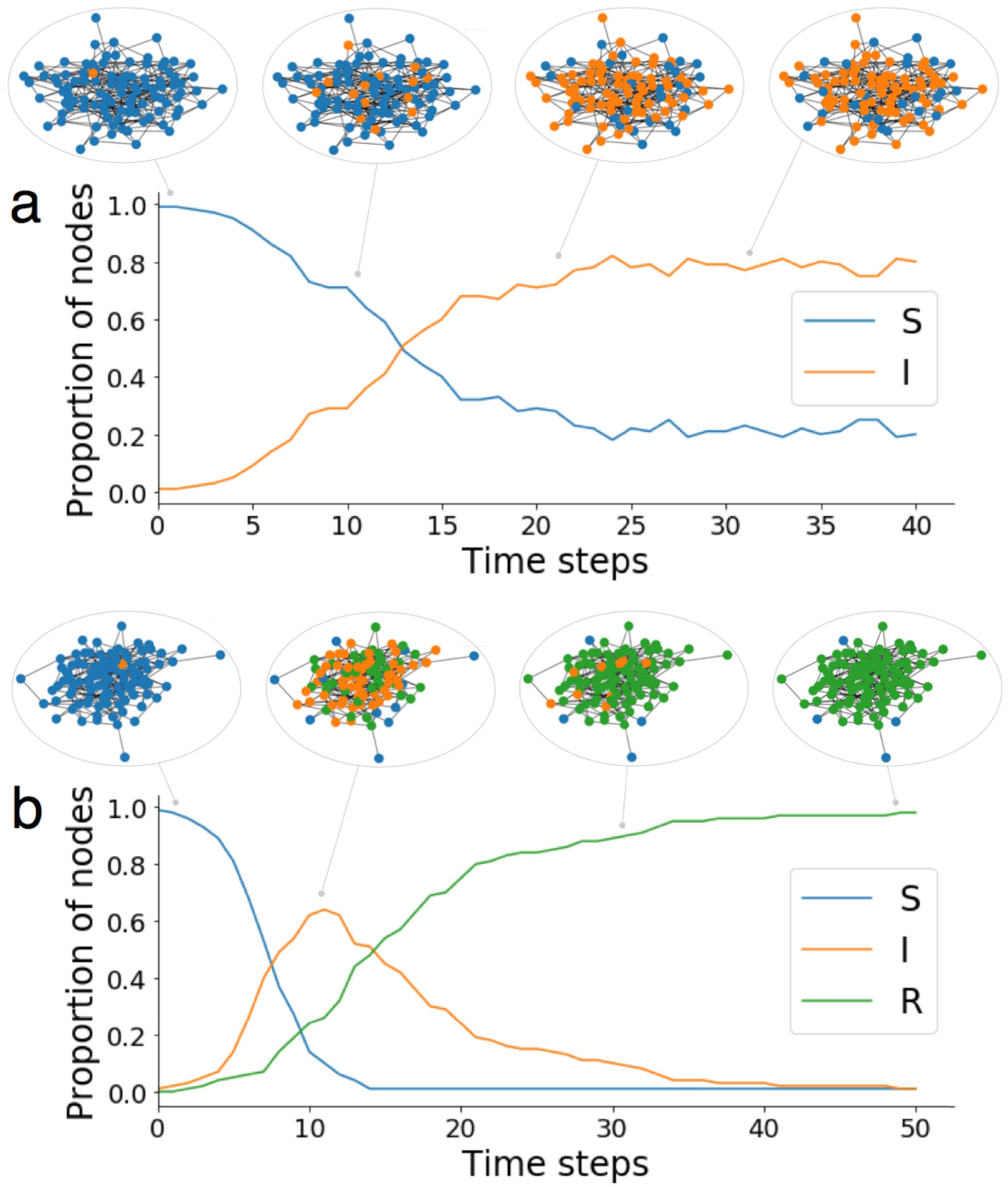
- **Simulation of SIS dynamics on networks:**
  - Take a network (e.g., a random network or a real contact network)
  - A number (fraction) of the nodes are infected (e.g., at random), all others are susceptible
  - All nodes are visited in sequence
  - For each node *i*:
    - If *i* is susceptible, loop over its neighbors: for each infected neighbor, *i* becomes infected with probability β
    - If *i* is infected, it becomes susceptible with probability μ

The SIR model

- **Difference from SIS model**: when infected individuals recover, they do not become susceptible again, but they are moved to the compartment R and play no further role in the dynamics
- The model applies to diseases that confer long-lasting immunity (e.g., measles, mumps, rubella, etc.)



Epidemic spreading



- Three characteristic stages of the dynamics:
  - **Initial stage**: just a few people are infected, and the diffusion of the epidemic is irregular and slow
  - **Ramp-up phase of exponential growth**, that can quickly affect a large number of people
  - **Stationary state**, in which the disease is either **endemic**, i.e. it affects a stable fraction of the population over time, or **eradicated**

Homogeneous mixing

- **Hypothesis**: every individual is in contact with every other
- **Consequence**: all individuals in the same compartment have identical behavior and only the relative proportions of people in the various compartments matter for the model dynamics
- Justified for a small population, e.g., the inhabitants of a little village where all people are in touch with each other.
- In real large-scale epidemics, individuals can only be infected by the people they come in contact with. In this case it is **necessary to reconstruct the actual network of contacts**

SIS & SIR models on networks

- **Start**: homogeneous contact network, with all nodes having degree approximately equal to $\langle k \rangle$
- **Early stage**: few people are infected, so we can assume that every infected individual is in contact with mostly susceptible individuals
- Each infected individual can transmit the disease to about $\langle k \rangle$ people at each iteration —> the expected number of people infected by a single person after one iteration is $\beta\langle k \rangle$
- If there are I infected individuals, we expect to have $I_{sec} = \beta\langle k \rangle I$ new infected people after one iteration and $I_{rec} = \mu I$ recovered people

SIS & SIR models on networks

- **Threshold condition** for epidemic spreading: $I_{sec} > I_{rec}$

$$\beta\langle k\rangle I > \mu I \implies R_0 = \frac{\beta}{\mu}\langle k\rangle > 1$$

- $R_0 = \beta\langle k\rangle/\mu$ is the **basic reproduction number**

- If $R_0 < 1$, the **initial outbreak dies out in a short time**, affecting only a few individuals

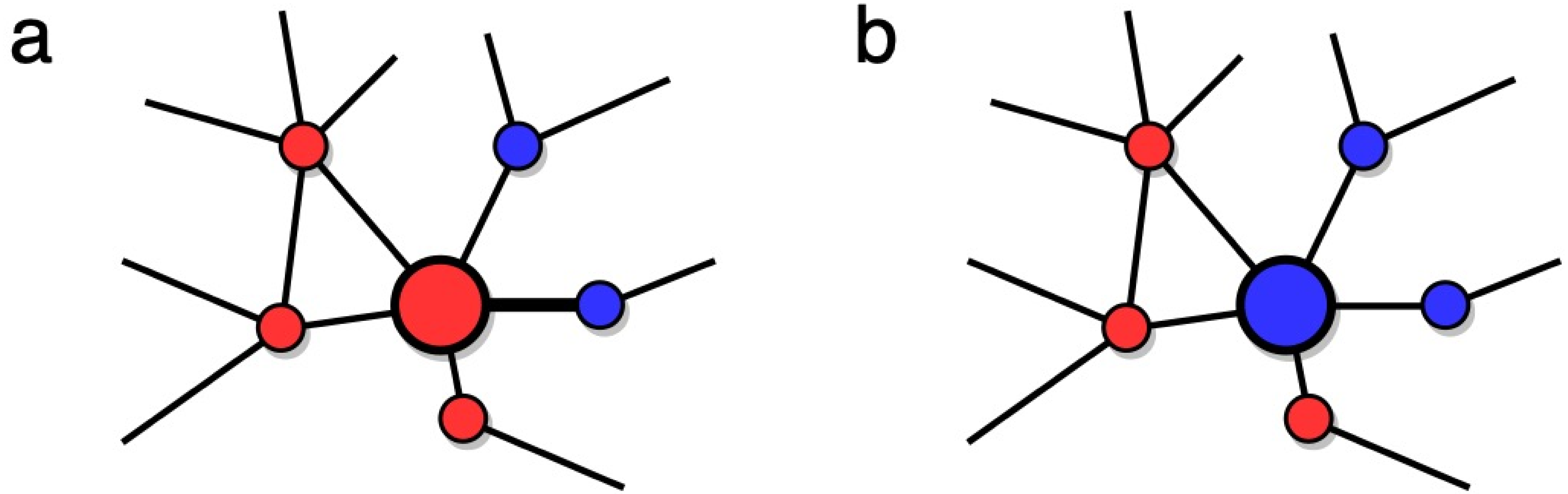
- If $R_0 > 1$, the **epidemic keeps spreading**

SIS & SIR models on networks

- **Problem:** real contact networks are not homogeneous
- Hubs drastically change the scenario. **On contact networks with hubs there is effectively no epidemic threshold —>** even diseases with low infection rate and/or high recovery rate may end up affecting a sizable fraction of the population!
- **Reason**: even if the infection rate is low, the process is likely to eventually infect a hub, via one of its many contacts; the hub can in turn infect a large number of susceptible individuals, including possibly other hubs, and so on
- Effective disease containment strategies should aim at **isolating/vaccinating individuals with many contacts**. The latter can be identified by picking the endpoints of randomly selected links, as this increases the chance to bump into hubs. So, don't vaccinate a random sample of the population: **vaccinate their friends**!

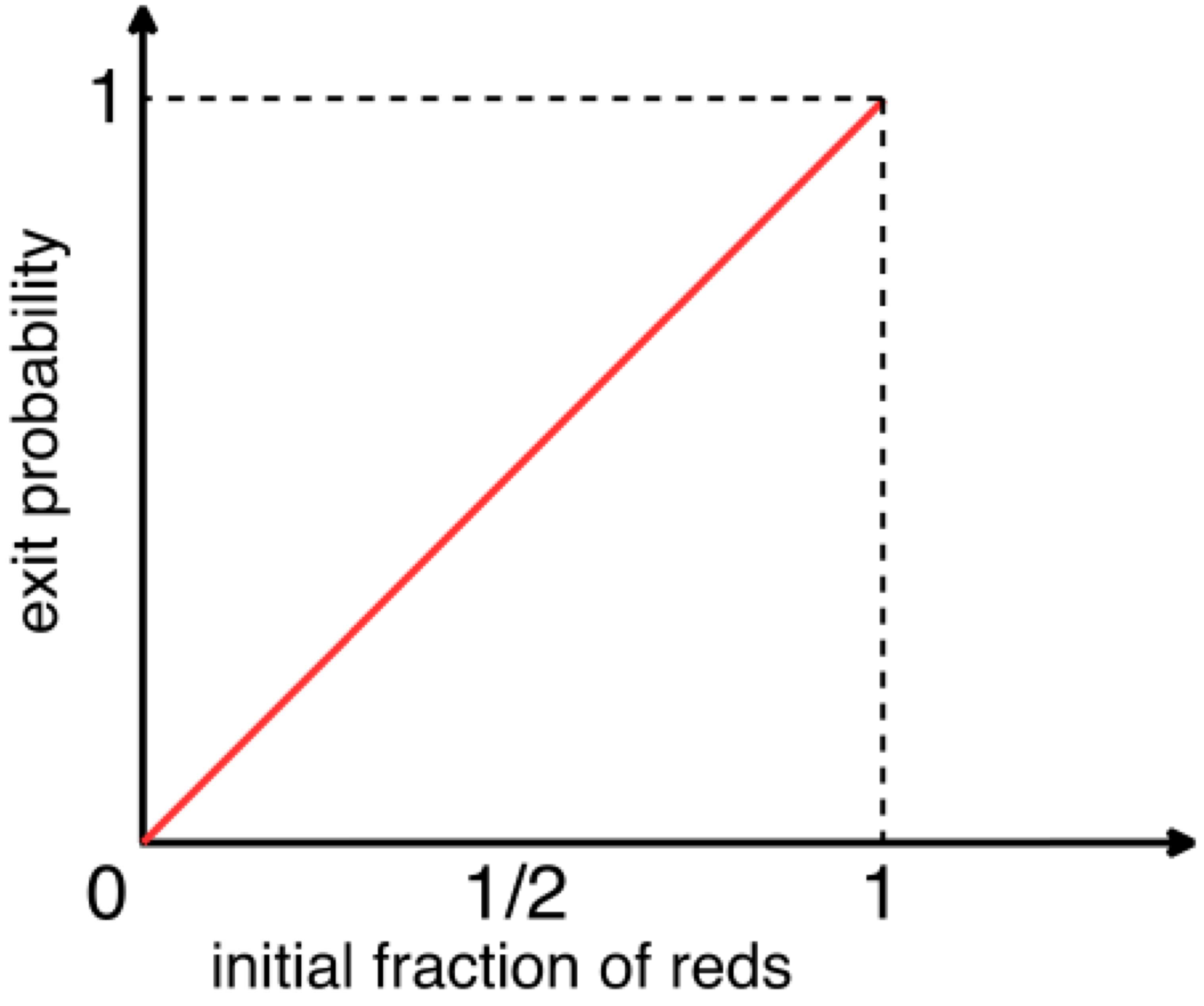
Voter model

- **Rule**: each node takes the opinion of a randomly chosen neighbor
- Consensus is the only stable state of the dynamics and **is reached on any network**
- In any state with different opinions, neighbors with different opinions can always influence each other



Voter model

- The exit probability of the voter model coincides with the fraction of initial nodes with opinion one: **diagonal function**
- Here, even if we start from an initial configuration with a majority of opinion one (zero) we may end up with consensus at opinion zero (one)
- **Example:** if 30% of the nodes have opinion one in the initial configuration we expect that in 30% of the runs all nodes will end up having opinion one



Voter model: variants

- Several variants of the voter model exist, obtained by adding features and/or making suitable modifications to the basic version. These include:
  - The presence of **zealots**, nodes who never change their opinion. If they all have the same opinion, they will favor consensus around that opinion, otherwise consensus is never reached
  - Considering **more than two opinion states**, with interactions constrained to occur only among nodes with sufficiently close opinions. For example one could have three opinions (1, 2 and 3) such that only neighboring opinions can interact (1 and 2, 2 and 3, but not 1 and 3)
  - The possibility for nodes to **change their opinion spontaneously**, for example with a certain probability at each iteration, on top of the voter dynamics

Further Readings

- Epidemic Modeling 101: Or why your CoVID-19exponential fits are wrong
- Epidemic Modeling 102: All CoVID-19 models are wrong,but some are useful
- Modeling of COVID-19 epidemic in the United States and Estimates: https://www.gleamproject.org/covid-19#mode

Refs: Modeling of COVID-19 epidemic in the United States

Balcan, Duygu, et al. "Multiscale mobility networks and the spatial spreading of infectious diseases." *Proceedings of the National Academy of Sciences* 106.51 (2009): 21484-21489.

After Class

- Go through Dynamics_on_Networksv3.ipynb
- Go through Assignment 2 solution
- Questions and Answers about Papers, and Class Material