Small World Property

Materials Part II

Class0210_partI.pdf (this presentation)

watts-collective_dynamics-nature_1998.pdf

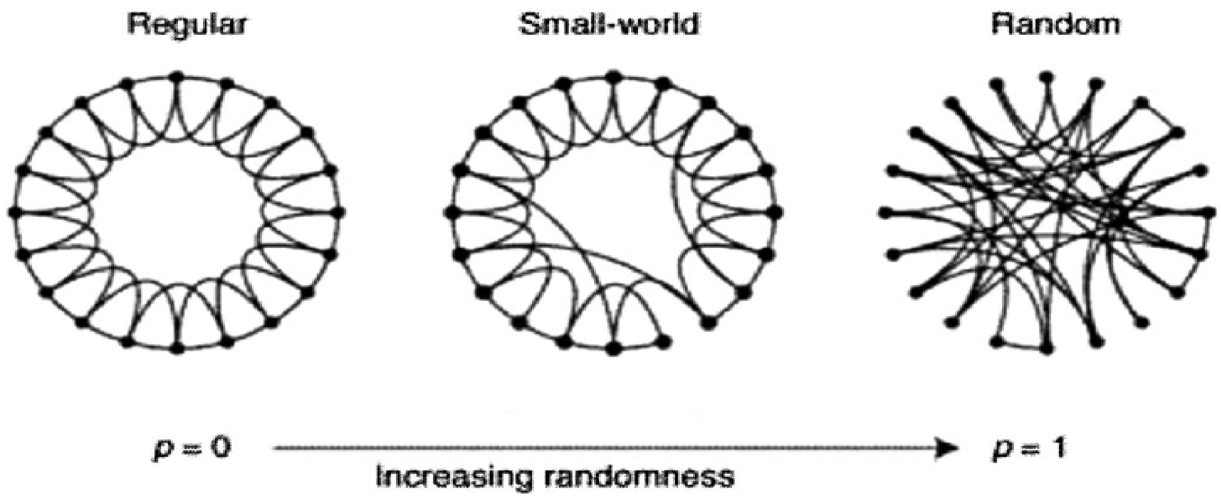
Reading_SocialNetwork.ipynb

SchoolEdges.csv

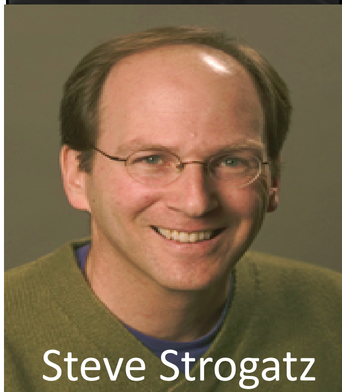
SchoolNodes.csv

Collective dynamics of 'small-world' networks

Duncan J. Watts* & Steven H. Strogatz



Duncan Watts



Steve Strogatz

Small world phenomenon:
applicable to other kinds of networks

Same pattern:

high clustering

low average shortest path

$$C_{\text{network}} >> C_{\text{random graph}}$$

$$l_{\text{network}} \approx \ln(N)$$

Small world phenomenon:
Milgram’s experiment

Instructions:
Given a target individual (stockbroker in Boston), pass the message to a person you correspond with who is “closest” to the target.

Outcome:

20% of initiated chains reached target
average chain length = 6.5

- “Six degrees of separation”

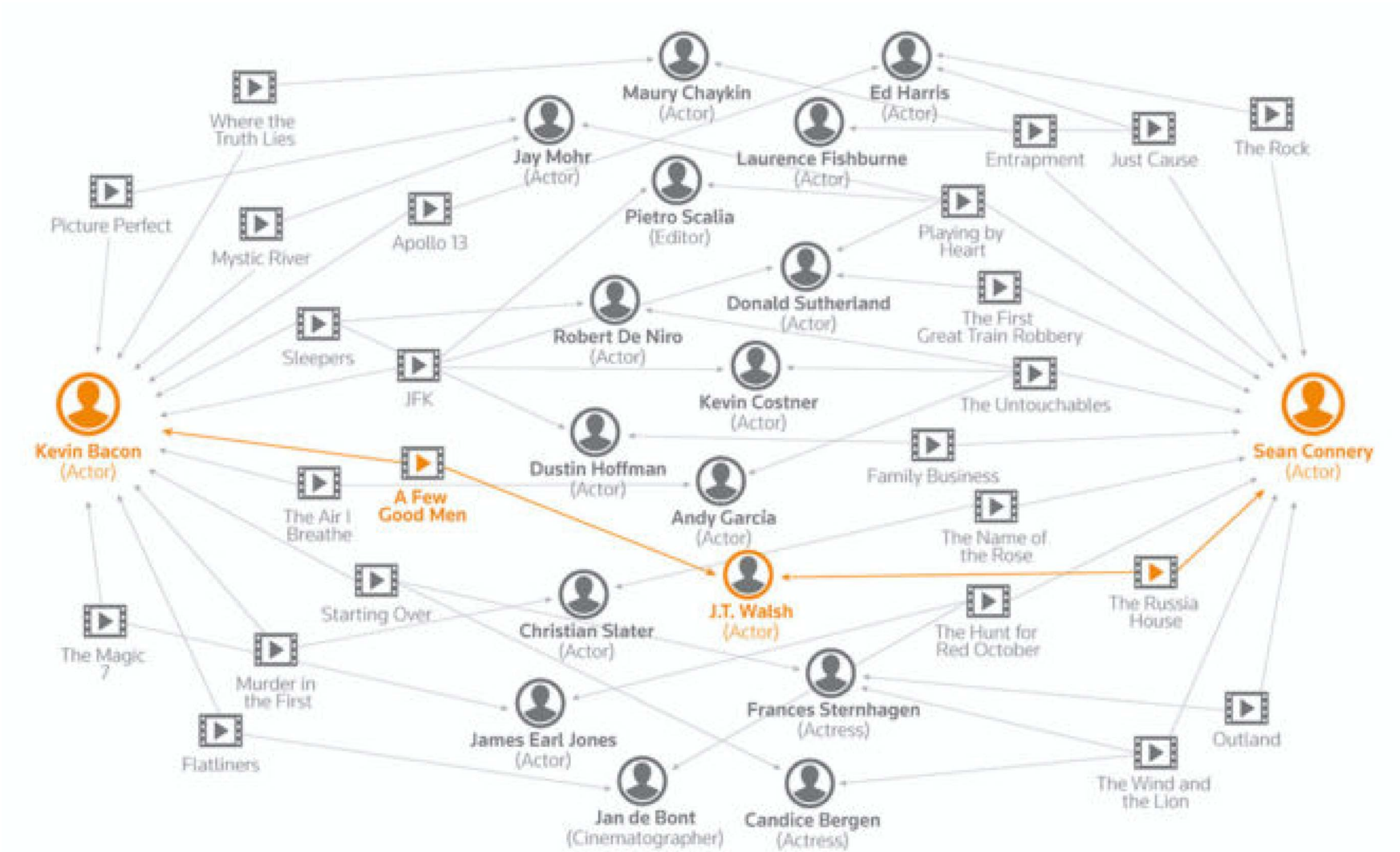
Collective dynamics of 'small-world' networks

Duncan J. Watts* **& Steven H. Strogatz**

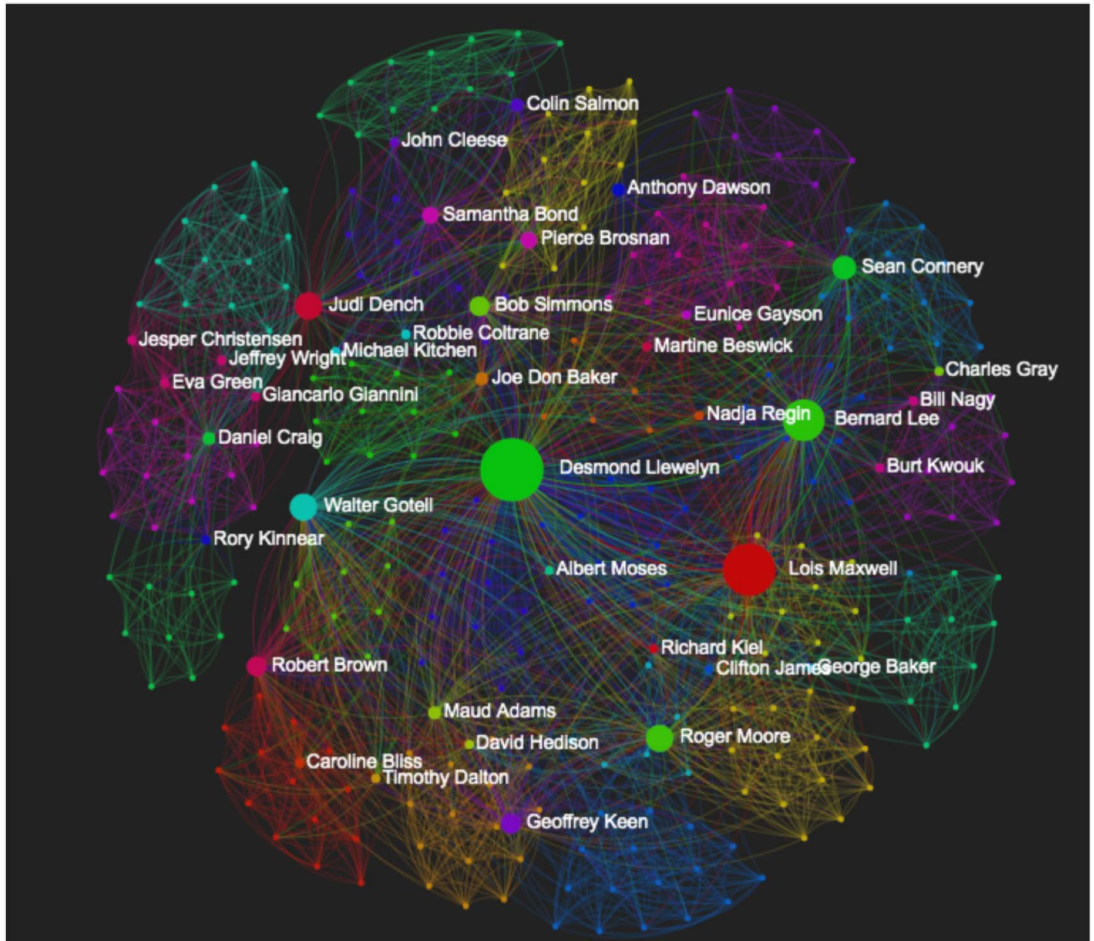
Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

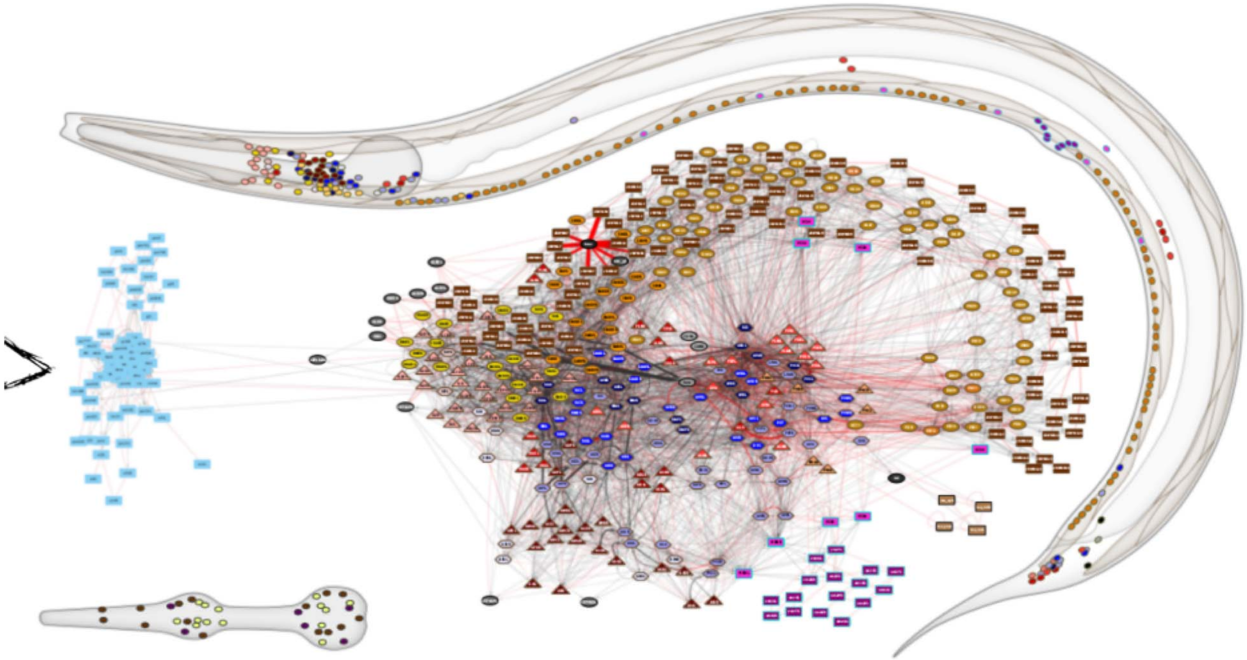
Table 1 Empirical examples of small-world networks

	L_{actual}	L_{random}	C_{actual}	C_{random}
Film actors	3.65	2.99	0.79	0.00027
Power grid	18.7	12.4	0.080	0.005
C. elegans	2.65	2.25	0.28	0.05



Maury Chaykin (Actor)
Ed Harris (Actor)
Where the Truth Lies
Jay Mohr (Actor)
Laurence Fishburne (Actor)
Entrapment
Just Cause
The Rock
Picture Perfect
Mystic River
Apollo 13
Pietro Scalia (Editor)
Playing by Heart
Donald Sutherland (Actor)
The First Great Train Robbery
Sleepers
Robert De Niro (Actor)
JFK
Kevin Costner (Actor)
The Untouchables
Kevin Bacon (Actor)
Sean Connery (Actor)
Dustin Hoffman (Actor)
Family Business
A Few Good Men
The Air I Breathe
Andy Garcia (Actor)
The Name of the Rose
J.T. Walsh (Actor)
The Russia House
Starting Over
Christian Slater (Actor)
The Hunt for Red October
The Magic 7
Murder in the First
Frances Sternhagen (Actress)
Outland
James Earl Jones (Actor)
Flatliners
The Wind and the Lion
Jan de Bont (Cinematographer)
Candice Bergen (Actress)


Colin Salmon
John Cleese
Anthony Dawson
Samantha Bond
Pierce Brosnan
Sean Connery
Judi Dench
Bob Simmons
Eunice Gayson
Jesper Christensen
Robbie Coltrane
Jeffrey Wright
Michael Kitchen
Martine Beswick
Charles Gray
Eva Green
Joe Don Baker
Giancarlo Giannini
Bill Nagy
Nadja Regin
Bernard Lee
Daniel Craig
Desmond Llewelyn
Burt Kwouk
Walter Gotell
Rory Kinnear
Albert Moses
Lois Maxwell
Richard Kiel
Clifton James
George Baker
Robert Brown
Maud Adams
Roger Moore
David Hedison
Caroline Bliss
Timothy Dalton
Geoffrey Keen



Reconstructed biological neural network

systemsbiology.usu.edu/TRN.php

Microsoft Release... Yahoo Used Nordictrack... Survey Voices Clearer sky in Beiji... cccc Applying for a stu... HotSpot

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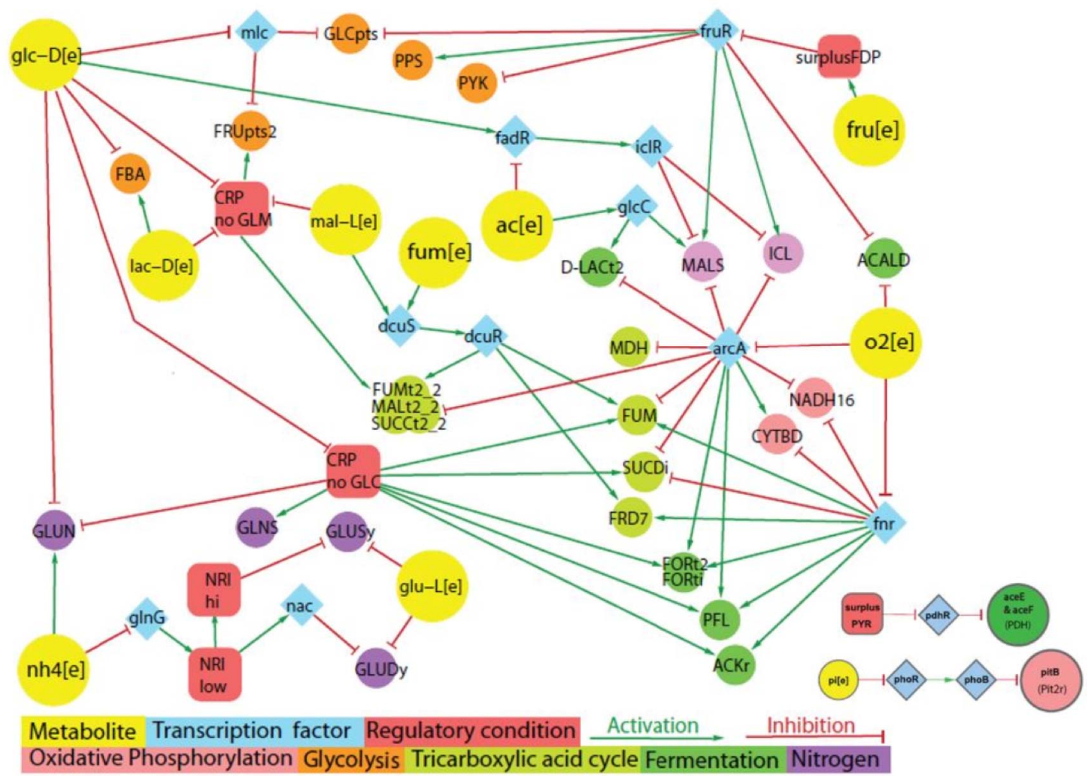
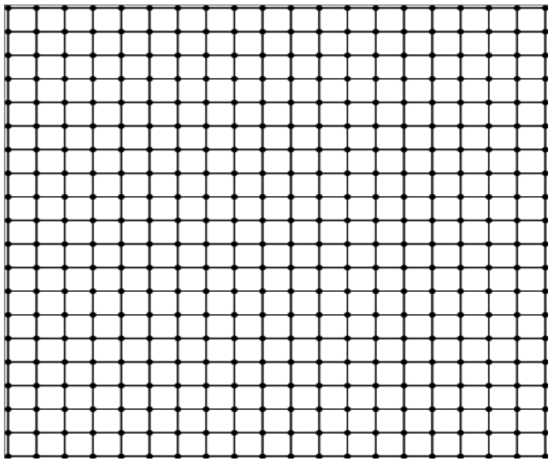
Transcriptional Regulatory Networks

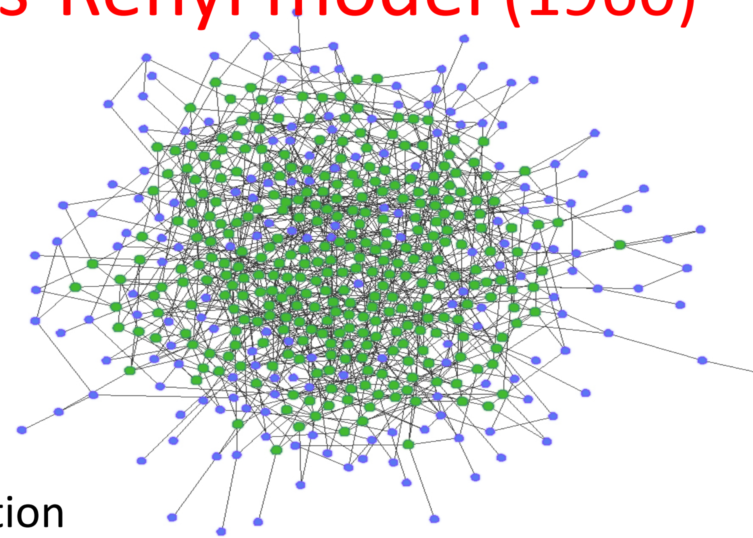


WATTS & STROGATZ

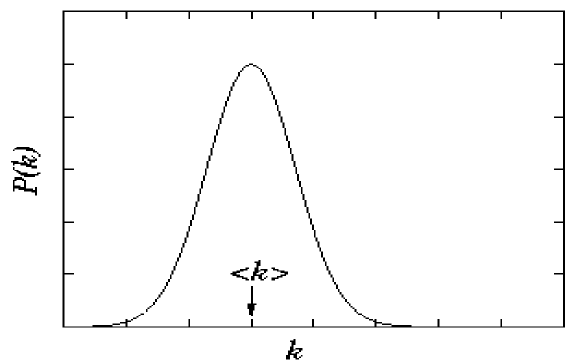
Lattice



Erdös-Rényi model (1960)



Poisson distribution



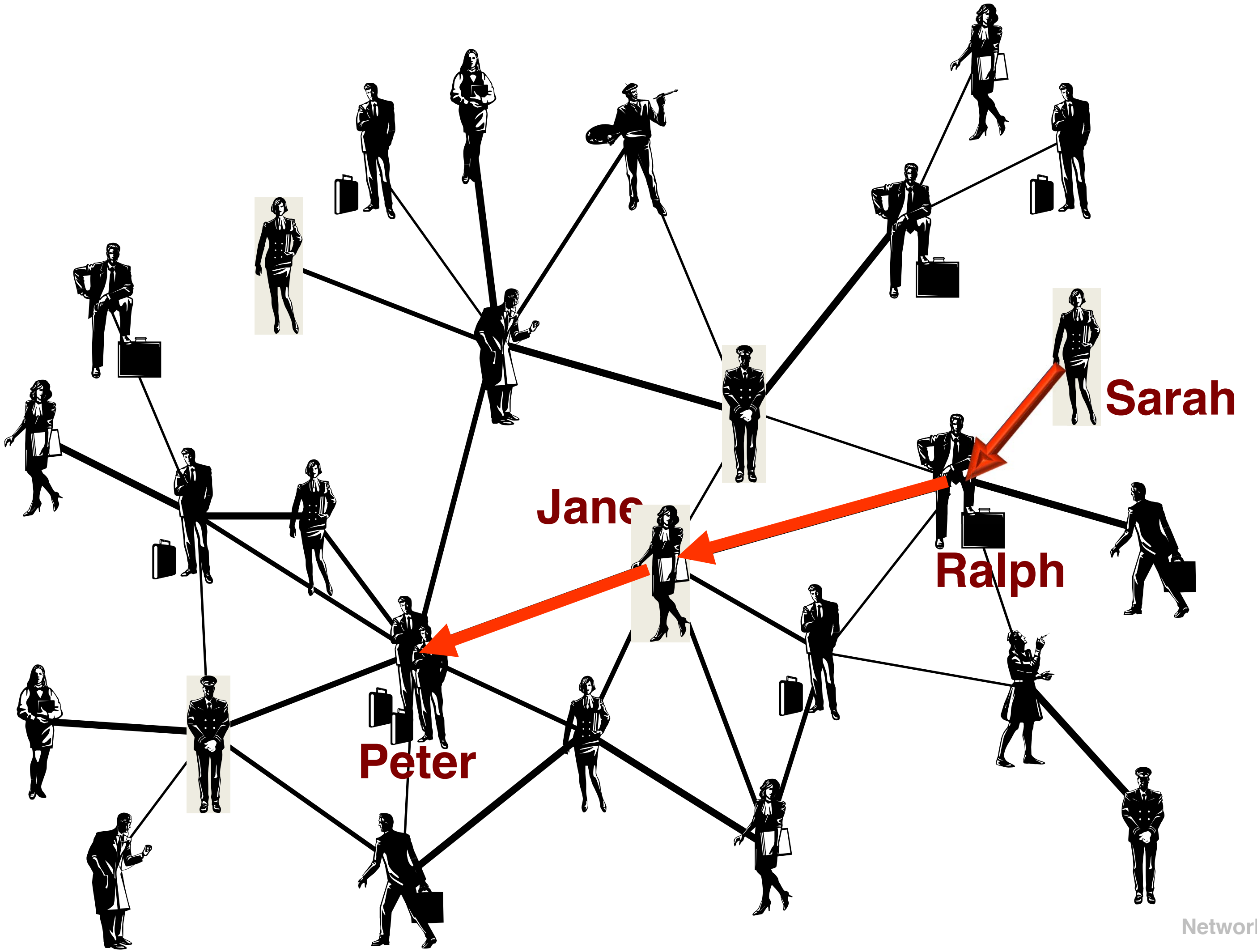
Comparison with “random graph” used to determine whether real-world network is “small world”

Network	size	av. shortest path	Shortest path in fitted random graph	Clustering (averaged over vertices)	Clustering in random graph
Film actors	225,226	3.65	2.99	0.79	0.00027
MEDLINE co-authorship	1,520,251	4.6	4.91	0.56	1.8×10^{-4}
E.Coli substrate graph	282	2.9	3.04	0.32	0.026
C.Elegans	282	2.65	2.25	0.28	0.05

SIX DEGREES

1967: Stanley Milgram

SIX DEGREES small worlds



Frigyes Karinthy, 1929
Stanley Milgram, 1967

In the social sciences, the word **"clique"** is used to describe a group of 2 to 12 (averaging 5 or 6) "persons who interact with each other more regularly and intensely than others in the same setting."

Small world phenomenon:
applicable to other kinds of networks

Same pattern:

high clustering

low average shortest path

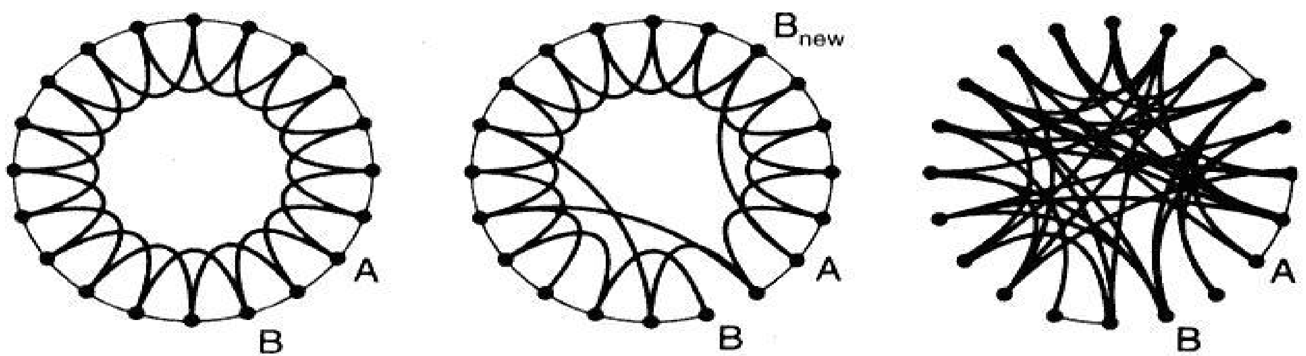
$$C_{\text{network}} >> C_{\text{random graph}}$$

$$l_{\text{network}} \approx \ln(N)$$

Small world phenomenon:
Watts/Strogatz model

Reconciling two observations:

- **High clustering:** my friends' friends tend to be my friends
- **Short average paths**



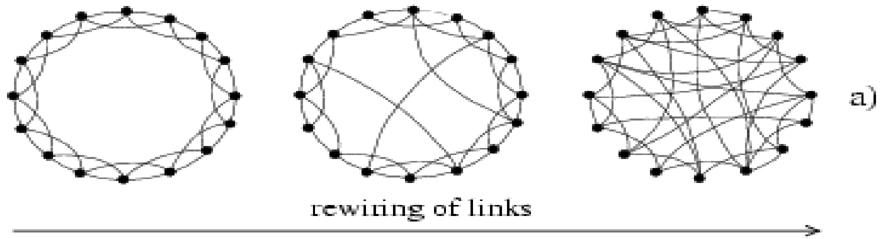
Source: Watts, D.J., Strogatz, S.H.(1998) Collective dynamics of 'small-world' networks. Nature 393:440-442.

Watts-Strogatz model:
Generating small world graphs

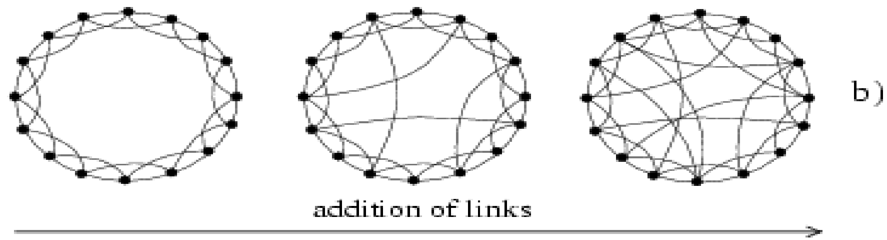
- Each node has K>=4 nearest neighbors (local)
- tunable: vary the probability *p* of rewiring any given edge
- small *p*: regular lattice
- large *p*: classical random graph

Watts-Strogatz model:

Generating small world graphs



Select a fraction p of edges
Reposition on of their endpoints



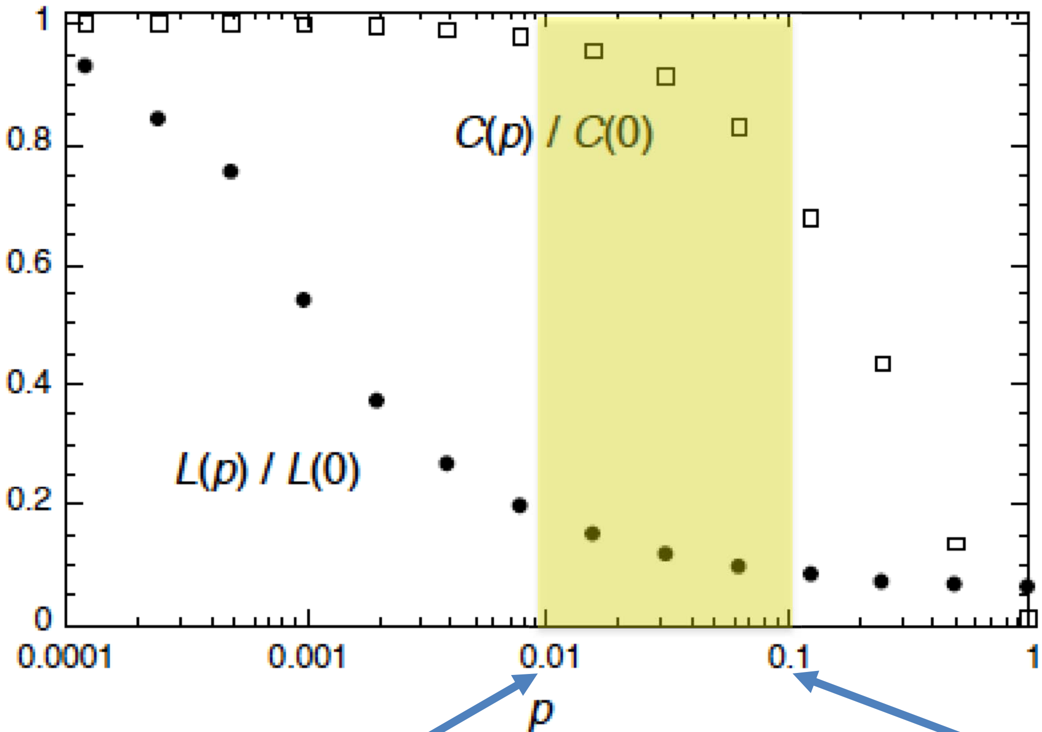
Add a fraction p of additional edges leaving underlying lattice intact

- As in many network generating algorithms
  - Disallow self-edges
  - Disallow multiple edges

Source: Watts, D.J., Strogatz, S.H.(1998) Collective dynamics of 'small-world' networks. Nature 393:440-442.

Watts/Strogatz model:
What happens in between?

- Small shortest path means small clustering?
- Large shortest path means large clustering?
- Through numerical simulation
  - As we increase p from 0 to 1
    - Fast decrease of mean distance
    - Slow decrease in clustering



C(p) / C(0)
L(p) / L(0)
0
0.2
0.4
0.6
0.8
1
0.0001
0.001
0.01
0.1
1
p

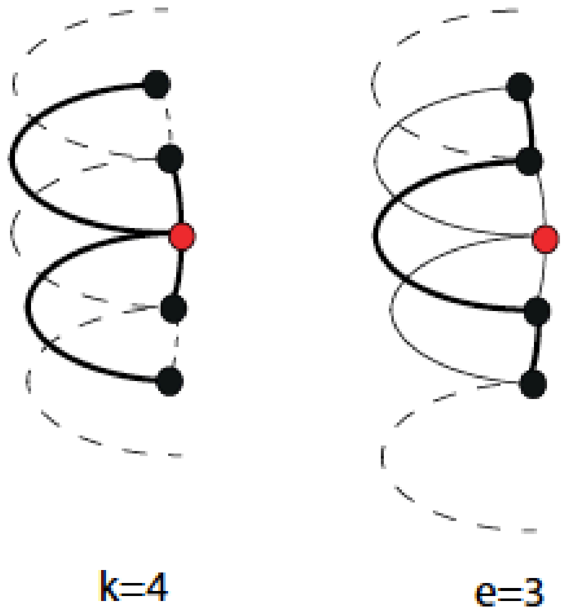
`clustering`(*G, nodes=None, weight=None*) [source]

Compute the clustering coefficient for nodes.

For unweighted graphs, the clustering of a node u is the fraction of possible triangles through that node that exist,

$$c_u = \frac{2T(u)}{deg(u)(deg(u) - 1)},$$

where $T(u)$ is the number of triangles through node u and $deg(u)$ is the degree of u.



- Degree per node, $k = 4$
- Edges among neighbors, $e = 3$
- Clustering coefficient, $C = 3/6 = 0.5$

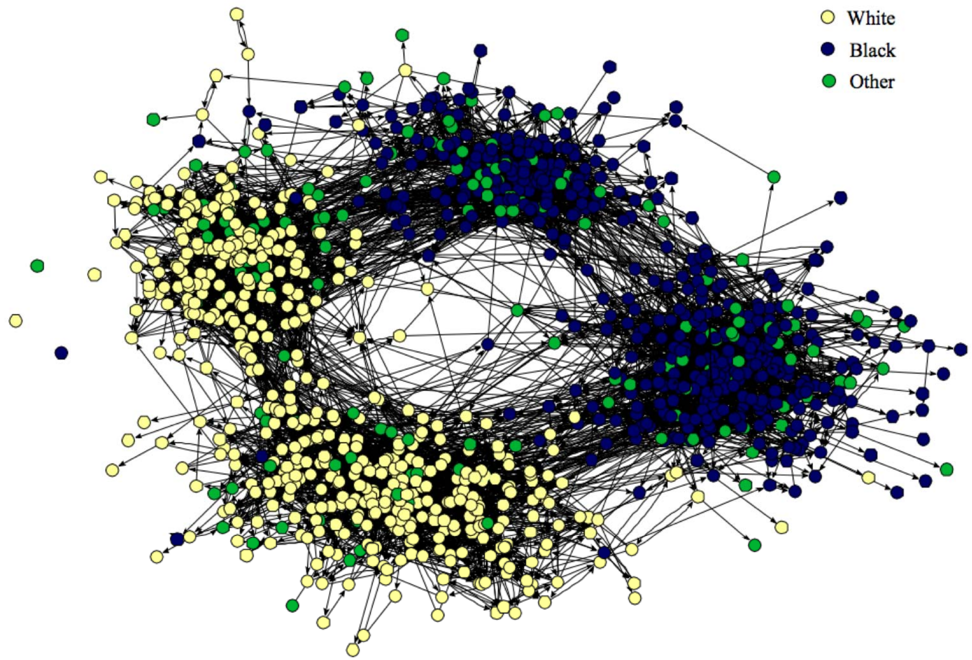


Fig. 3.4 *Friendship network of children in a U.S. school. Friendships are determined by asking the participants, and hence are directed, since A may say that B is their friend but not vice versa. Vertices are color coded according to race, as marked, and the split from left to right in the figure is clearly primarily along lines of race. The split from top to bottom is between middle school and high school, i.e., between younger and older children. Picture courtesy of James Moody.*

In the SchoolEdges.csv and SchoolNodes.csv you find the result of a survey on friendships in a school.

Every student was given a paper-and-pencil questionnaire and a copy of a list with every schoolmate. Weighted dyadic links were generated based on the number of sheared activities.

Weights were in the range from 1, meaning the student nominated the friend without reporting any activity, to 6 meaning that the student nominated the friend and reported participating in five activities with (him/her). Answer the following questions

~/CE88/Lec1/Powerlaw — less SchoolNodes.csv

```
Id Label "Nodes " Sex Race Grade Scode totalnoms
1 1 1 1 1 10 1 0
2 2 2 2 1 12 1 0
3 3 3 1 5 8 1 3
4 4 4 2 1 12 1 4
5 5 5 2 3 12 1 10
6 6 6 2 1 12 1 10
7 7 7 1 1 7 1 9
8 8 8 2 1 11 1 8
```

Lab

- Report # of Students of each Race
- Report # of Students in each Grace

Assignment1: Properties of Empirical Networks

Published Edit ⋮

Assignmet here: 2020_Assignmet1_Part1.pdf

Reading: future_of_urban_science_report_for_nsf_lobo_et_al_1132020.pdf

Highly recommended: Finish Part I by Feb 10 before class.

Assignmet here: Spring20_Assignment1_partII.pdf

Highly recommended: Have most of this part ready by Feb 24 before class

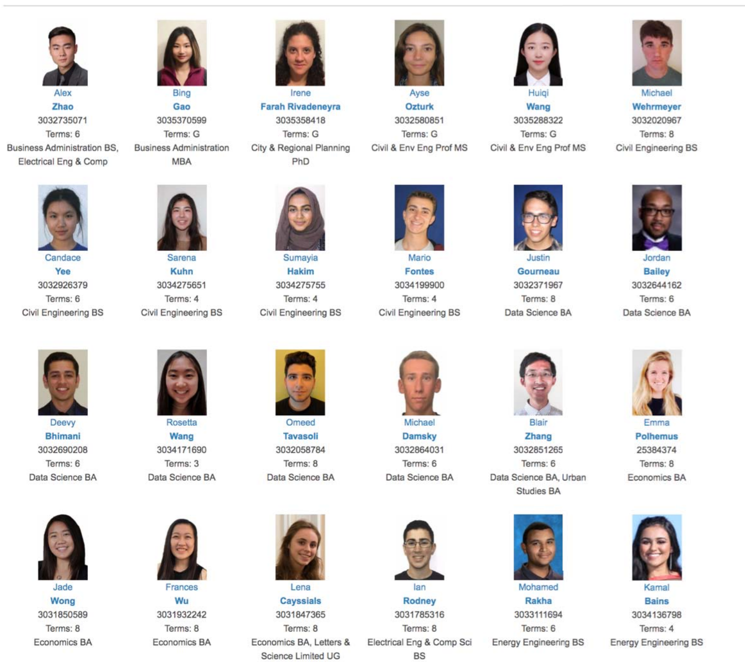
Deadline: March 2nd by midnight

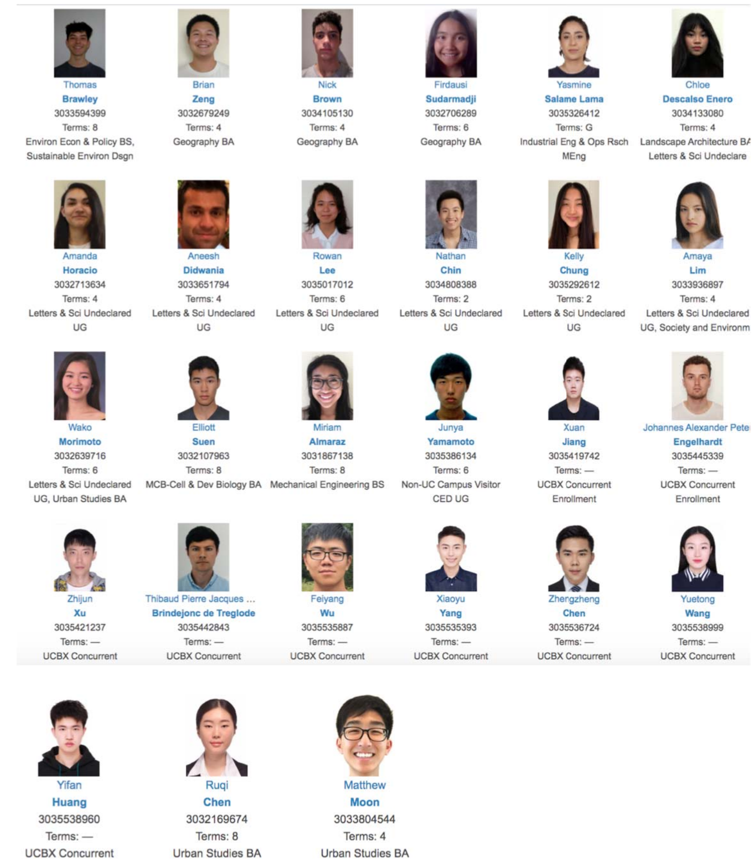
Points 15

Submitting a file upload

Due	For	Available from	Until
Mar 2	Everyone	Feb 3 at 9am	-

+ Rubric


Alex Zhao 3032735071 Terms: 6 Business Administration BS, Electrical Eng & Comp
Bing Gao 3035370599 Terms: G Business Administration MBA
Irene Farah Rivadeneyra 3035358418 Terms: G City & Regional Planning PhD
Ayse Ozturk 3032580851 Terms: G Civil & Env Eng Prof MS
Huiqi Wang 3035288322 Terms: G Civil & Env Eng Prof MS
Michael Wehrmeyer 3032020967 Terms: 8 Civil Engineering BS
Candace Yee 3032926379 Terms: 6 Civil Engineering BS
Sarena Kuhn 3034275651 Terms: 4 Civil Engineering BS
Sumayia Hakim 3034275755 Terms: 4 Civil Engineering BS
Mario Fontes 3034199900 Terms: 4 Civil Engineering BS
Justin Gourneau 3032371967 Terms: 8 Data Science BA
Jordan Bailey 3032644162 Terms: 6 Data Science BA
Deevy Bhimani 3032690208 Terms: 6 Data Science BA
Rosetta Wang 3034171690 Terms: 3 Data Science BA
Omeed Tavasoli 3032058784 Terms: 8 Data Science BA
Michael Damsky 3032864031 Terms: 6 Data Science BA
Blair Zhang 3032851265 Terms: 6 Data Science BA, Urban Studies BA
Emma Polhemus 25384374 Terms: 8 Economics BA
Jade Wong 3031850589 Terms: 8 Economics BA
Frances Wu 3031932242 Terms: 8 Economics BA
Lena Cayssials 3031847365 Terms: 8 Economics BA, Letters & Science Limited UG
Ian Rodney 3031785316 Terms: 8 Electrical Eng & Comp Sci BS
Mohamed Rakha 3033111694 Terms: 6 Energy Engineering BS
Kamal Bains 3034136798 Terms: 4 Energy Engineering BS


Thomas Brawley 3033594399 Terms: 8 Environ Econ & Policy BS, Sustainable Environ Dsgn
Brian Zeng 3032679249 Terms: 4 Geography BA
Nick Brown 3034105130 Terms: 4 Geography BA
Firdausi Sudarmadji 3032706289 Terms: 6 Geography BA
Yasmine Salame Lama 3035326412 Terms: G Industrial Eng & Ops Rsch MEng
Chloe Descalso Enero 3034133080 Terms: 4 Landscape Architecture BA Letters & Sci Undeclare
Amanda Horacio 3032713634 Terms: 4 Letters & Sci Undeclared UG
Aneesh Didwania 3033651794 Terms: 4 Letters & Sci Undeclared UG
Rowan Lee 3035017012 Terms: 6 Letters & Sci Undeclared UG
Nathan Chin 3034808388 Terms: 2 Letters & Sci Undeclared UG
Kelly Chung 3035292612 Terms: 2 Letters & Sci Undeclared UG
Amaya Lim 3033936897 Terms: 4 Letters & Sci Undeclared UG, Society and Environm
Wako Morimoto 3032639716 Terms: 6 Letters & Sci Undeclared UG, Urban Studies BA
Elliott Suen 3032107963 Terms: 8 MCB-Cell & Dev Biology BA
Miriam Almaraz 3031867138 Terms: 8 Mechanical Engineering BS
Junya Yamamoto 3035386134 Terms: 6 Non-UC Campus Visitor CED UG
Xuan Jiang 3035419742 Terms: — UCBX Concurrent Enrollment
Johannes Alexander Pete Engelhardt 3035445339 Terms: — UCBX Concurrent Enrollment
Zhijun Xu 3035421237 Terms: — UCBX Concurrent
Thibaud Pierre Jacques ... Brindejonc de Treglode 3035442843 Terms: — UCBX Concurrent
Feiyang Wu 3035535887 Terms: — UCBX Concurrent
Xiaoyu Yang 3035535393 Terms: — UCBX Concurrent
Zhengzheng Chen 3035536724 Terms: — UCBX Concurrent
Yuetong Wang 3035538999 Terms: — UCBX Concurrent
Yifan Huang 3035538960 Terms: — UCBX Concurrent
Ruqi Chen 3032169674 Terms: 8 Urban Studies BA
Matthew Moon 3033804544 Terms: 4 Urban Studies BA

Social Network Analysis (Indegree, Outdegree and Weights)
In the file ce88_nodes.xlsx in:
https://drive.google.com/file/d/1Cs4tNnbKzGybdckreCk1T5G1W6AXmCXZ/view?usp=sharing
You find a node ID together with other attributes:
Name I Course I Majors I Terms in Attendance

Go to ce88_edges.xlsx in:
Select from 1 to 4 students in the roster and add a weight.

Weight 5 denotes that you know the person and have worked together
Weight 4 denotes that you know the person but do not have worked together
Weight 3 denotes that you would like to work with the person based on major
Weight 2 denotes that you would like to work with the person based on Term
Weight 1 denotes just a random selection

From tomorrow 02/11/2020, download the files to analyze the resulting network:
https://docs.google.com/spreadsheets/d/1MDYVQvtgjhvwRPO-4Tr_p7_bXj1AjD7k/edit#gid=1227182835

1) How many nodes and links are there? **(0.5 pts.)**
2) What is the average degree of the network? **(0.5 pts.)**
3) Calculate the in-degree of each node. Who is the student with the largest in-degree in this network? **(1 pt.)**
4) Report the different Majors and # Students in each Major? **(1 pt.)**
5) Report the different Terms and the Number of Students in each Term? **(1 pt.)**
6) Calculate the average weight of each node (taking into account incoming and outgoing links) **(1pt.)**

Problem 2: Check if the CEE social Network has Small World property (5 pts)

1) Fill the following table with the properties of the network **(2pts),** Average Clustering Coefficient, average degree, average shortest path, number of nodes and number of links
2) In the same Figure, Plot the in-degree and out-degree of histograms of this **(3pts)**

Network	<C>	<K>	<L>	#nodes	#Links
CEE_88 Network					

Submit separately your ipynb code and the answers to each question in a pdf file format.

Special Issue Complex Systems and Networks

INTRODUCTION TO SPECIAL ISSUE

Connections

BY BARBARA R. JASNY, LAURA M. ZAHN, ELIOT MARSHALL

SCIENCE | 24 JUL 2009 : 405

Full Text PDF

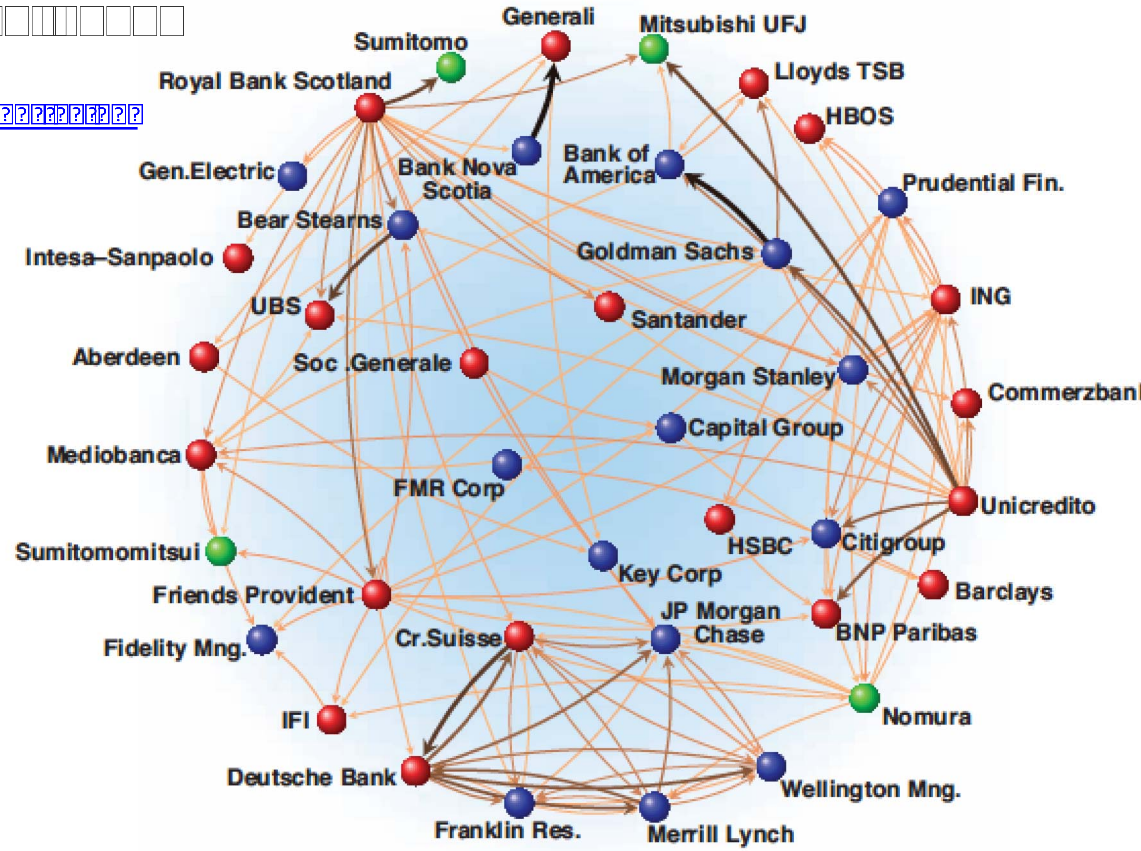


Fig. 2. A sample of the international financial network, where the nodes represent major financial institutions and the links are both directed and weighted and represent the strongest existing relations among them. Node colors express different geographical areas: European Union members (red), North America (blue), other countries (green). Even with the reduced number of links displayed in the figure, relative to the true world economy, the network shows a high connectivity among the financial institutions that have mutual share-holdings and closed loops involving several nodes. This indicates that the financial sector is strongly interdependent, which may affect market competition and systemic risk and make the network vulnerable to instability.

Predicting the Behavior of Techno-Social Systems

Alessandro Vespignani