

# Data 88: Economic Models

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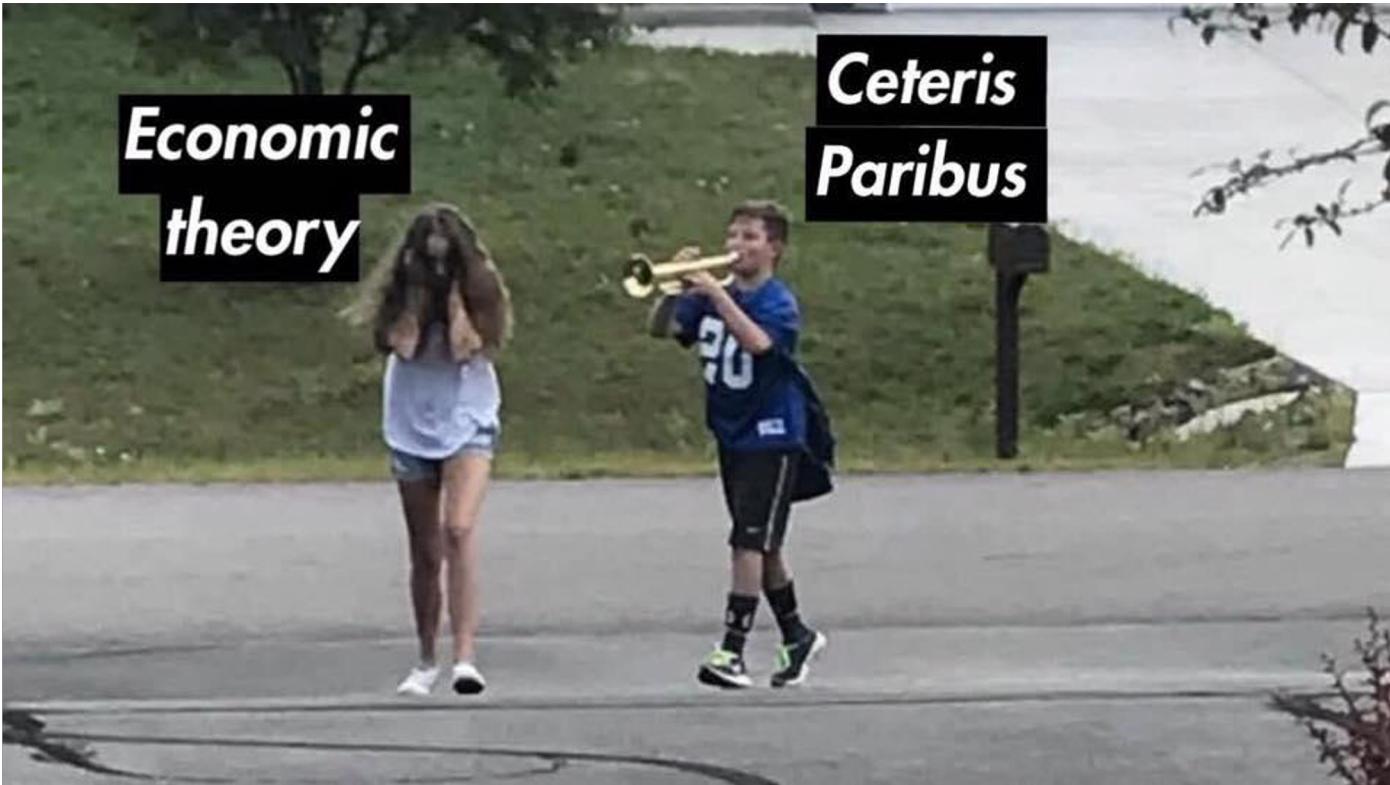
## Lecture 4: Production

# Announcements - Project 2

- Project 2 will be released today
- It will be coding-heavy (Data 8 Project 1-esque), but Lab 4 will greatly help you start
- Whole project due (all 3 parts) 2 weeks from now:  
October 5 @ 11:59pm
  - Office hours are there for you to get help!
  - We will schedule additional office hours if there is demand
  - Recommended: Finish first 2 parts by Sunday, last part next week

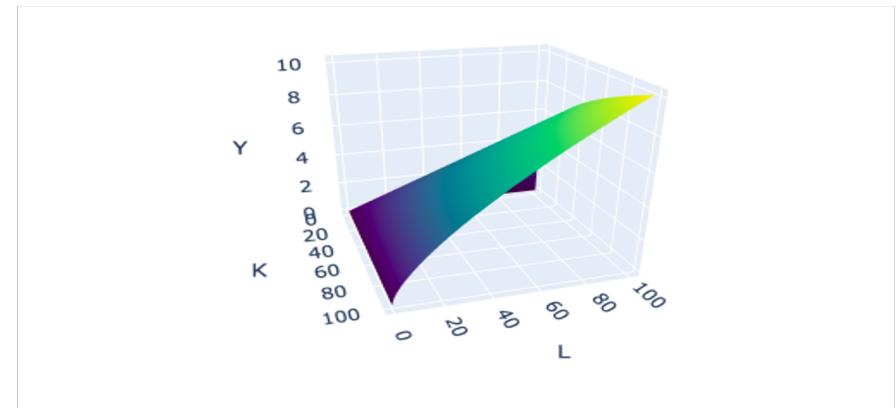


- Graduated in May 2020 with degrees in Economics and Data Science
- Taught this course (Data 88), Data 8 and Econ 100B
- Data Scientist with a Singapore-based company
- If you have any questions about Data Science in industry, please feel free to email me at [manikui@berkeley.edu](mailto:manikui@berkeley.edu)



# Lecture 4 Outline

- Overall
  - Factors of Production
  - Total Factor Productivity
  - Cobb-Douglas Production Functions (3D SURFACES!!!!)
  - Cross-Country Analysis (this is what the project is)
- Derivation
  - Cobb Douglas
  - Log of Equation
  - Solve for one variable
- Intuition
  - Shifts in A (SLIDERS!!!!!!)
  - What does  $\alpha$  mean?
  - Returns to scale
- Lab time



# Name all of the macroeconomic indicators you know

~~GDP~~ ( $/\text{capita}$ )

U

m1, m2

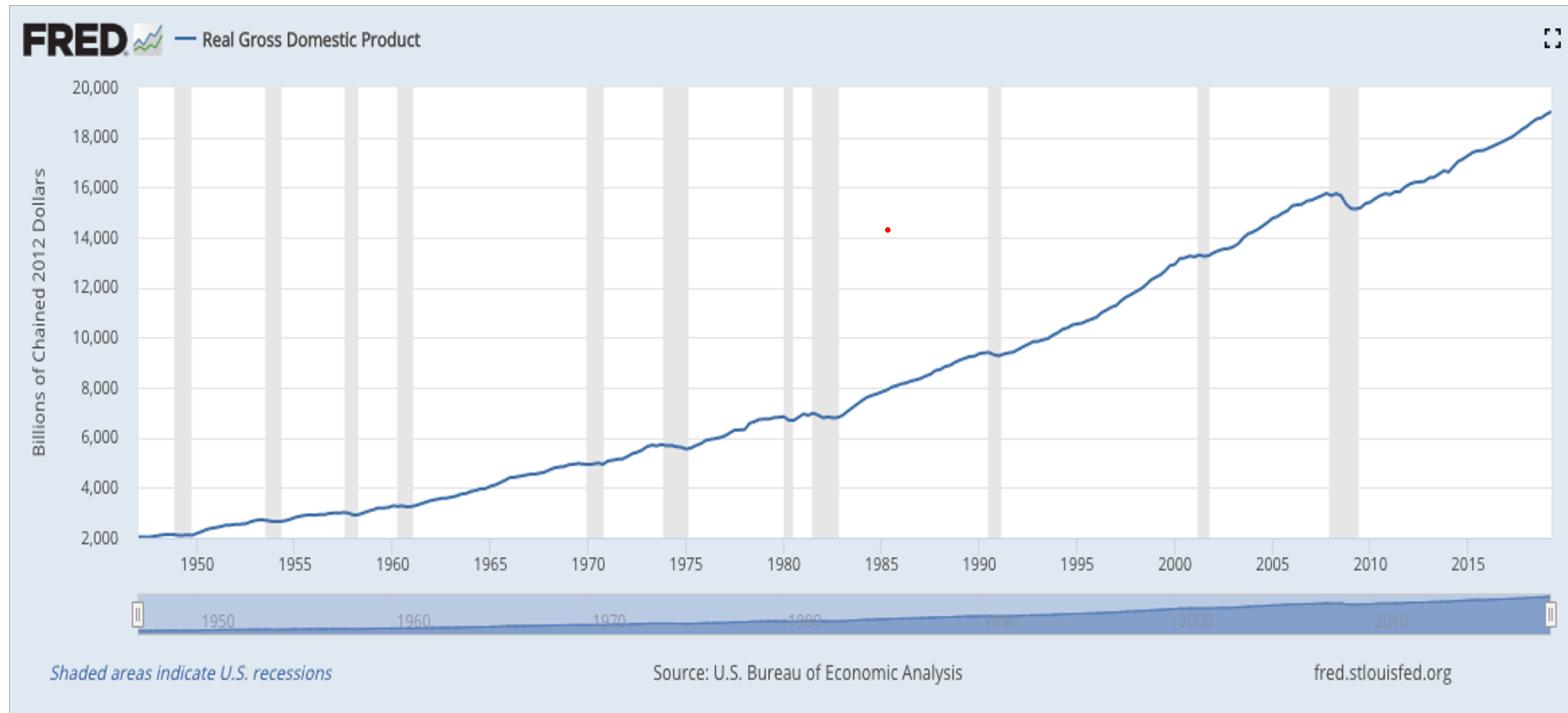
π (inflation), CPI

r

IM/EX

CO<sub>2</sub>

# What is GDP?



## How do we calculate it?

# Ways to Calculate GDP

- Income approach

rent, wages,  $R$ , profits

- Expenditure approach  $\rightarrow C + I + G + Ex - Im$



- Production approach

# Production in the Economy

- What is production?

inputs →  → output  
(GDP)

- How do we quantify a country's production?

inputs · Prices

- Why would it be important to calculate this?

- what sectors need help



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# Inputs (or Factors) of Production

Let's make a list of **all** the resources that go into producing and selling an iPhone:

metals (K)  
labour (slave) (L)  
silicon (K)  
R&D (A)  
shipping (K, L)  
sales (A)  
I.P. (A)

packaging (K, L)  
stores (K)  
factories (K)  
education (A)

K - capital  
L - labour  
A - TFP (Tech.)



Image source: [https://store.storeimages.cdn-apple.com/4982/assets.apple.com/is/refurb-iphoneX-spacegray\\_AV1?wid=1144&hei=1144&fmt=jpeg&qlt=80&op\\_usm=0.5,0.5&.v=1548459945536](https://store.storeimages.cdn-apple.com/4982/assets.apple.com/is/refurb-iphoneX-spacegray_AV1?wid=1144&hei=1144&fmt=jpeg&qlt=80&op_usm=0.5,0.5&.v=1548459945536)

A nation's output is a function of the amount of the factors of production that are utilized in its economy; that is to say output is a function of labor and capital, scaled by some value A.

$$f(K, L)$$

.  $\uparrow$

TFP

$$Y = A \cdot f(K, L)$$

What could A be? Why is it on the outside of the production function?

# Total Factor Productivity (TFP) (A)

- Technology or research and development
- A country with a high TFP can produce far more goods and services than another country with a lower TFP but the same amount of capital and labor

$$A=2 \quad \text{vs.} \quad A=1$$

- Differences between TFP and other factors of production
    - “Scales” production by some factor A - creates proportional increase in output
    - Technology is non-rivalrous (*more than one person can use it*)
    - Technology is non-excludable (*cannot prevent other people from using it*)
    - Ordinal measure
- ↳ comparison*

# How the equation was formulated

Weighted avg.

Based on the graphs to the right, what would  $F(K, L)$  look like?

What assumptions can we make about  $Y$ ,  $K$  and  $L$ ?

$$Y = F(K, L) \cdot A$$

$$\ln(Y) = \underline{\text{int}} + \underline{\alpha} \cdot \ln(K) + \underline{\beta} \cdot \ln(L)$$

(Cobb and Douglas did these calculations without computers 😮)

“A” 
$$Y = e^{\underline{\text{int}}} \cdot (K^{\underline{\alpha}} \cdot L^{\underline{\beta}})$$

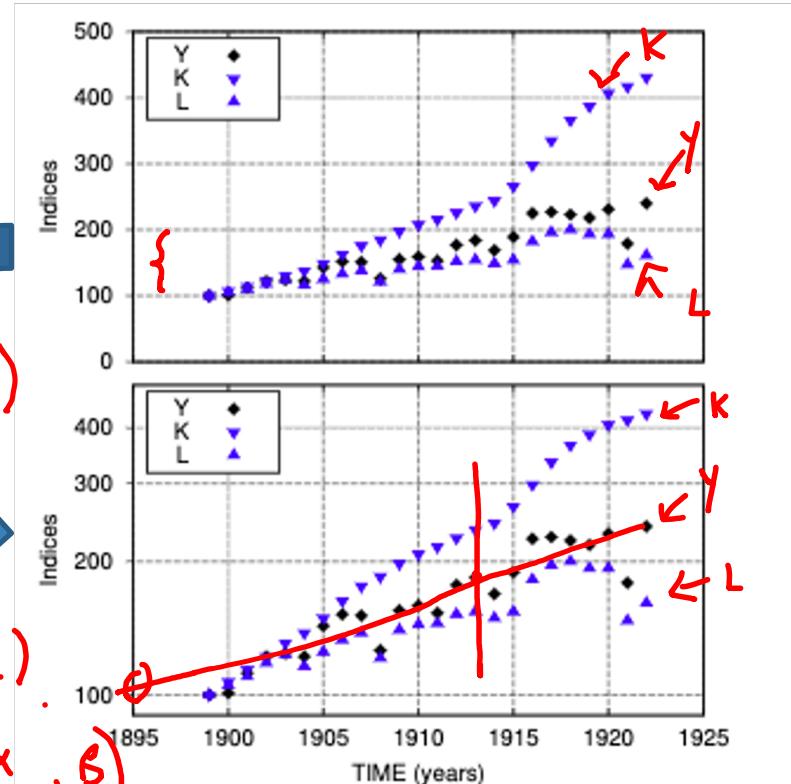


Image source: Hawkins' 100B; Lecture 4, Slide 9

# Cobb-Douglas Production Functions

$$f(K, L) = K^\alpha L^\beta$$
$$\Rightarrow \underline{Y} = A \cdot f(K, L) = \underline{A} \underline{K}^\alpha \underline{L}^\beta$$

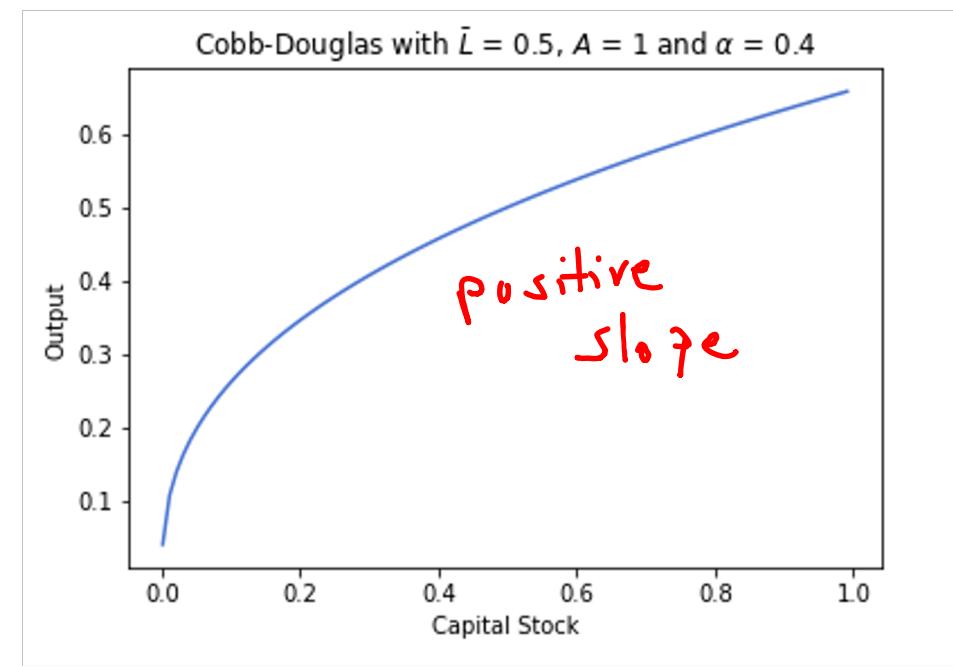
- For simplicity sake, we will assume constant returns to scale. That is,  $\alpha + \beta = 1$ . This allows us to rewrite the function in the following way:
- Where else will we see this function?

$$Y = AK^\alpha L^{1-\alpha}$$

utility

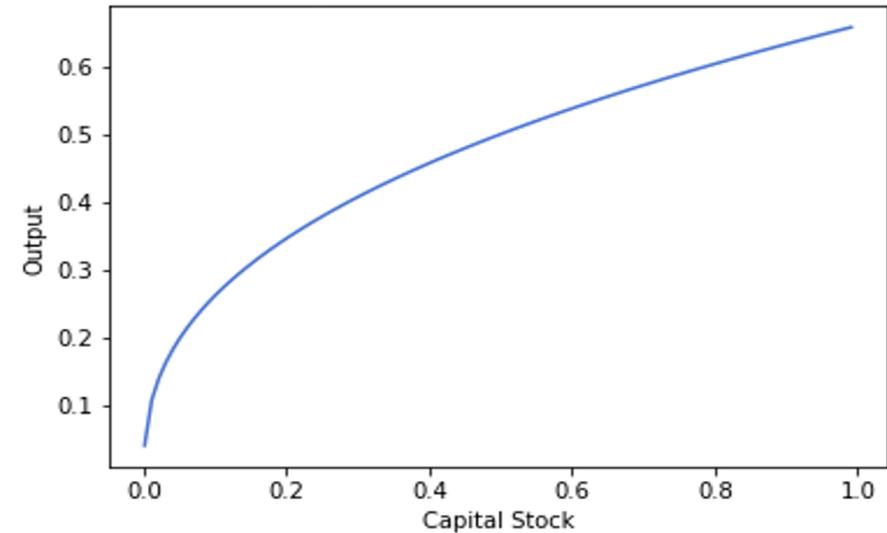
A monetary value of the stock or value of all productive, physical assets in an economy (**not bonds, stocks or other financial products**)

Increasing Returns to Capital  
(What does that mean?)



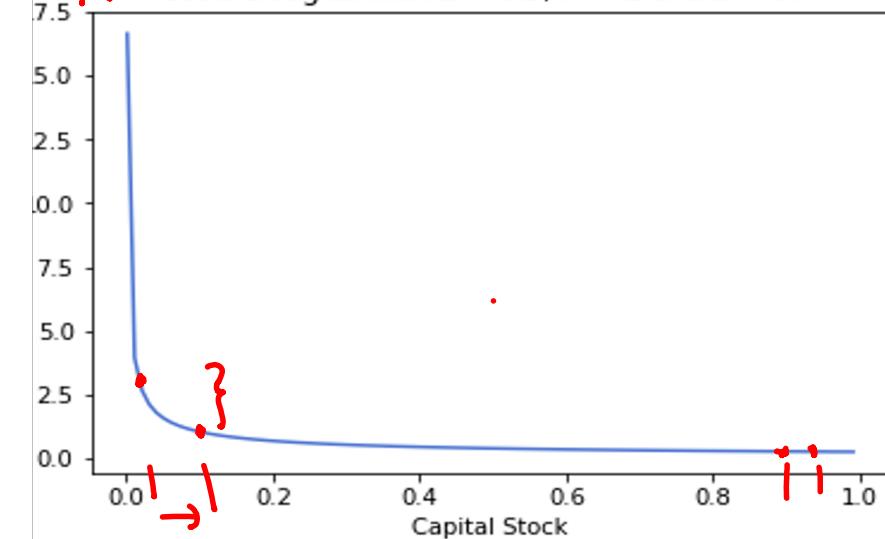
$$Y = AK^\alpha L^{1-\alpha}$$

Cobb-Douglas with  $\bar{L} = 0.5$ ,  $A = 1$  and  $\alpha = 0.4$



MPK

Cobb-Douglas with  $\bar{L} = 0.5$ ,  $A = 1$  and  $\alpha = 0.4$



Diminishing marginal returns to capital

What is the intuition behind this?

How do we get the price of capital?

$$\text{MPK} = \frac{\partial Y}{\partial K} = A^\alpha K^{\alpha-1} L^{1-\alpha}$$

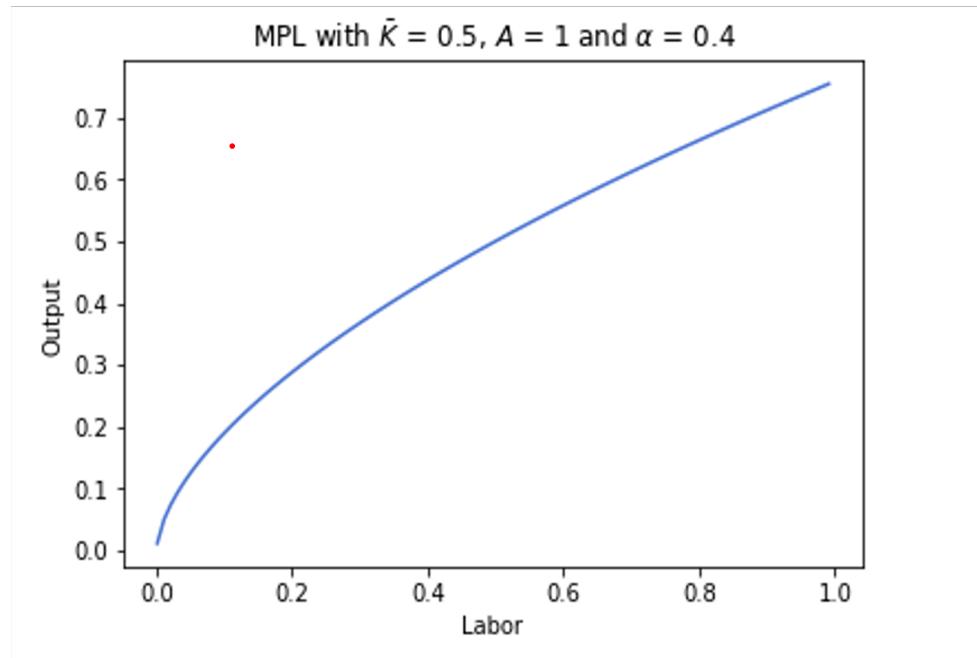
$$= A^\alpha \left(\frac{L}{K}\right)^{1-\alpha}$$

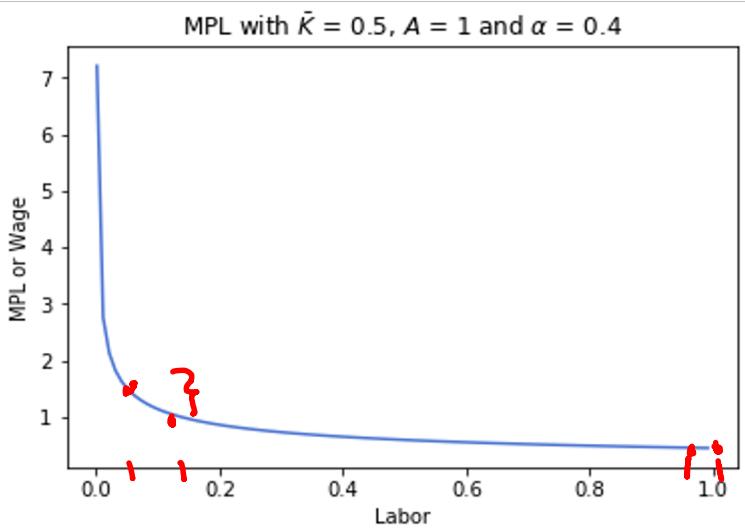
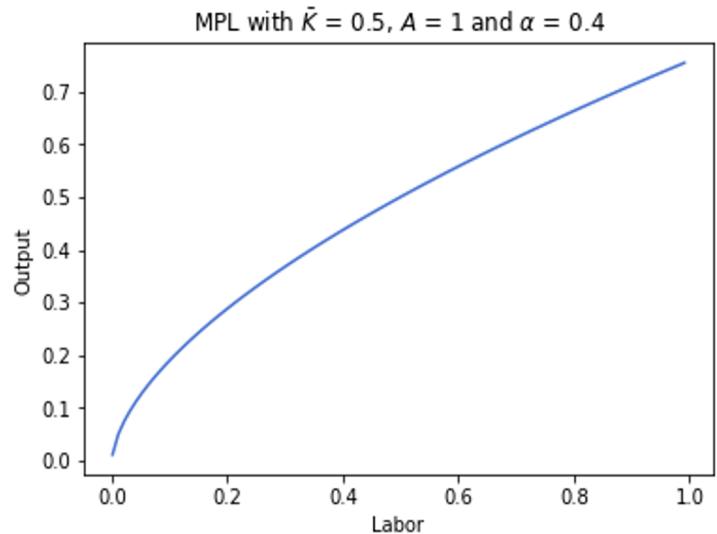
$$\text{MR} \geq \text{MC}$$

$$\text{MR} = \text{MC} \Rightarrow P \cdot \underline{\text{MPK}} = r$$

The number of hours worked by all individuals who are willing and able to work within a country for a given year

Increasing Returns to Labor  
(What does that mean?)





★ Diminishing Marginal Returns to Labor

What is the intuition behind this?

How do we get the price of labor?

$$MPL = \frac{\partial Y}{\partial L} = A(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$$

$$MR = MC$$

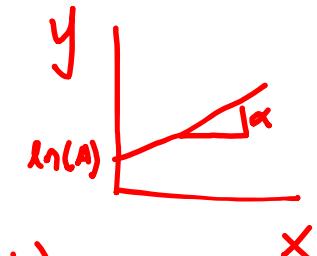
$$P \cdot MPL = W$$

$$Y = A K^{\alpha} L^{\beta}$$

Cobb and Douglas found that a country's output can be modeled very well as a weighted average of the log of capital and labor.

How can we rearrange the function to only be in terms of one variable?

$$Y = AK^\alpha L^{1-\alpha}$$



$$\begin{aligned}\ln(Y) &= \ln(A) + \alpha \ln(K) + (1-\alpha) \ln(L) \\ &= \ln(A) + \underline{\alpha \ln(K)} + \underline{\ln(L)} - \underline{\alpha \ln(L)}\end{aligned}$$

$$\begin{aligned}\ln(Y) - \ln(L) &= \ln(A) + \alpha [\ln(K) - \ln(L)] \\ \ln\left(\frac{Y}{L}\right) &= \ln(A) + \alpha \ln\left(\frac{K}{L}\right)\end{aligned}$$

# Cross-Country Comparisons (cont.)



$$\ln\left(\frac{Y}{L}\right) = \underline{\ln(A)} + \underline{\alpha} \ln\left(\frac{K}{L}\right)$$



$$Y = A K^{\underline{\alpha}} L^{\underline{\beta}}$$

$\alpha = 0.7$   
 $\beta = 0.3$

How can we use the above function to compare countries?

- ①  $\alpha$  changes  $\rightarrow$   $\beta$  changes  $\rightarrow$  Capital vs. labor intensive
- ②  $A$  differences  $\rightarrow$  technology

What are possible problems with the function?

$\underline{\alpha}, \underline{A}$  fixed

Romer  $\Rightarrow \Delta A$  over time

$$Y = A K^{\underline{\alpha}} L^{\underline{\beta}}$$

- Supply or Technology shocks

- Natural environment ← COVID

- Energy prices war

- Significant financial crises

- What would happen to output?

Δ A?

→ govt intervention

- Can anyone think of any real-world examples?

- OPEC oil crises

- subsidizing technology

# What does alpha mean?

Output elasticities of capital and labor: measure the responsiveness of output to a change in the levels of either labor or capital, holding all else constant.

$$Y = AK^\alpha L^{1-\alpha}$$

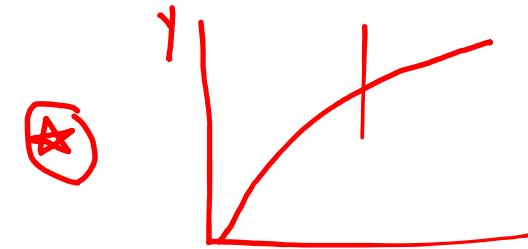
Let us assume constant returns to scale. What does  $\alpha > 0.5$  mean?

labour  
intensive

# Returns to scale

Constant Returns to Scale:  $\alpha + \beta = 1$

$$\underline{c} \cdot Y = f(\underline{c} \cdot K, \underline{c} \cdot L)$$



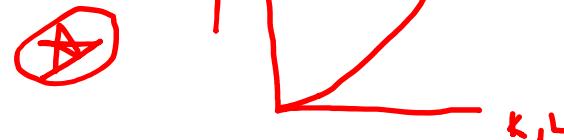
Decreasing Returns to Scale:  $\alpha + \beta < 1$

$$\underline{c}Y < f(cK, cL)$$



Increasing Returns to Scale:  $\alpha + \beta > 1$

$$\underline{c}Y > f(cK, cL)$$



What would the last two look like if their production functions were plotted?

# Lab Time!

Open up Lab 4 and work through the notebook. We'll be here to help.

The questions will be **very** helpful in preparing you for Project 2!