Lecture 26 EEP118

Time Series

1. How does Time series differ from cross sectional data
2. Basic Static model
3. Regression between two variables that have a trend – spurious correlation
4. First differencing

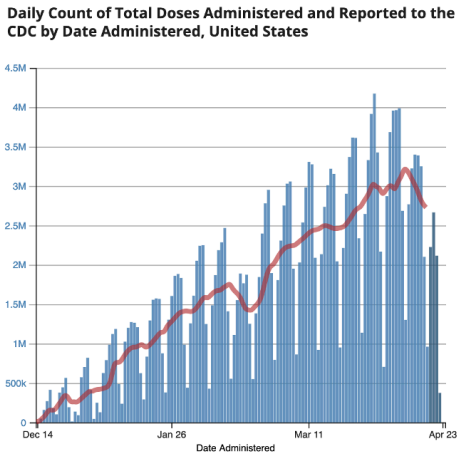
Study ch 10

Practice final and solutions posted

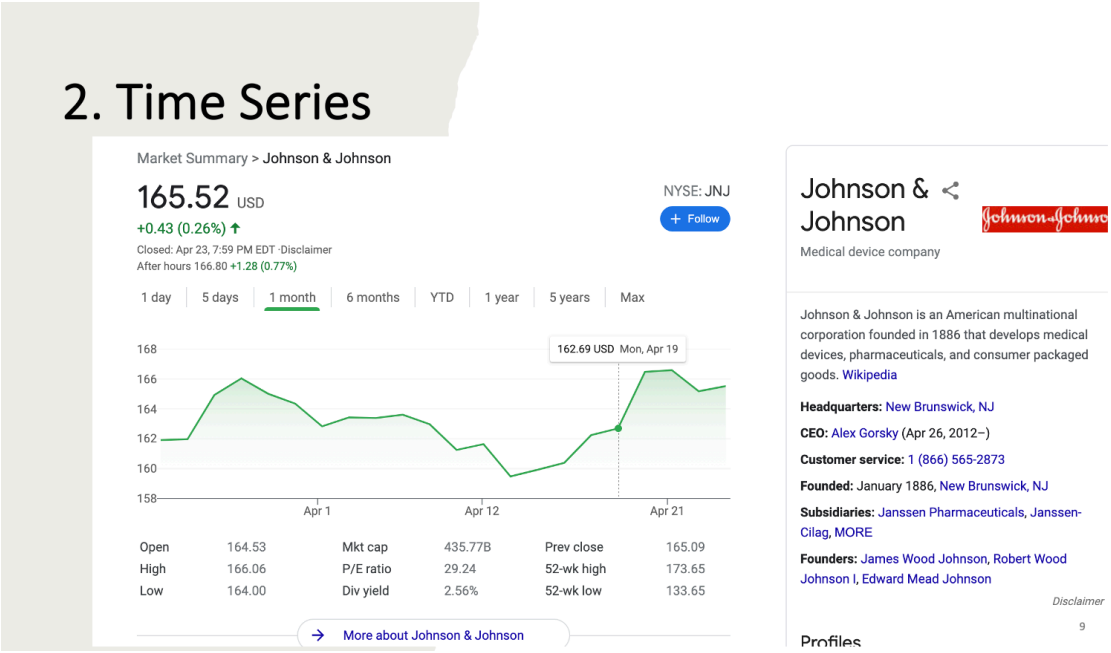
Examples of Time Series Data we see every day

2. Time Series

- Some recent examples of time series data we see every day



Data Table for Trends in Number of COVID-19 Vaccinations in the US



How does Time Series Data differ from crosss sectional data?

Population is infinite and we only see one realization

MLR2 does not apply because successive time periods are not independent (y1,y2, ... yt, ... yT).

Need to replace MLR2 with something equivalent

Basic Static Model

All the effects take place within the same t

$wage_t = \beta_0 + \beta_1 productivity_t + u_t$

Typical time series question: “Are wages related to labor productivity?”

Let us use a time series dataset from 1947-1987

In [1]:
```
#Lecture 26


# Load the 'pacman' package
library(pacman)
#packages to use load them now using the pacman "manager"
p_load(dplyr, haven, readr)
#Another great feature of p_load(): if you try to load a package that is not
p_load(ggplot2,margins)
```

```
pacman::p_load(lfe, lmtest, haven, sandwich, tidyverse)
# lfe for running fixed effects regression
# lmtest for displaying robust SE in output table
# haven for loading in dta files
# sandwich for producing robust Var-Cov matrix
# tidyverse for manipulating data and producing plots
```

Source: EARNS in Wooldridge. Economic Report of the President, 1989, Table B-47. The data are for the nonfarm business sector.

year : 1947 to 1987

outphr : output per labor hour

hrwage : average real earnings / hour

In [2]:
```
# read in data
mydata <- read_dta("Lecture26EARNS.DTA")

# summarize data
summary(mydata)
```

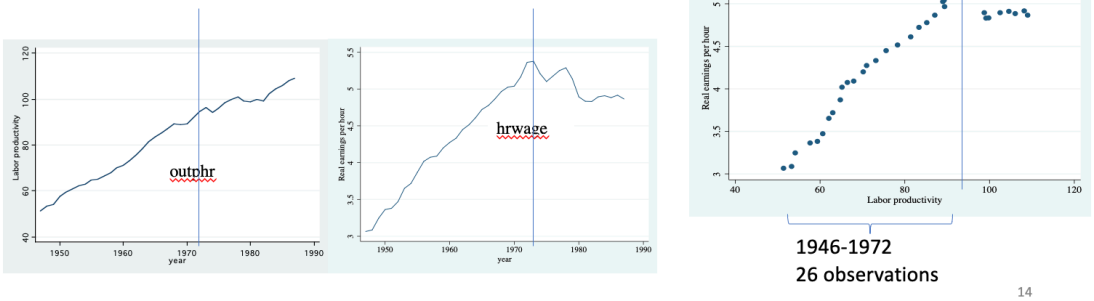
```
      year         wkearns         wkhours         outphr           hrwage
 Min.   :1947   Min.   :123.4   Min.   :34.8   Min.   : 51.40   Min.   :3.06
5
 1st Qu.:1957   1st Qu.:157.9   1st Qu.:36.0   1st Qu.: 66.50   1st Qu.:4.07
3
 Median :1967   Median :171.3   Median :38.0   Median : 87.10   Median :4.83
0
 Mean   :1967   Mean   :168.6   Mean   :37.6   Mean   : 82.91   Mean   :4.50
9
 3rd Qu.:1977   3rd Qu.:184.8   3rd Qu.:39.0   3rd Qu.: 98.80   3rd Qu.:5.02
5
 Max.   :1987   Max.   :198.4   Max.   :40.3   Max.   :109.00   Max.   :5.37
5

    lhrwage         loutphr             t         ghrwage
 Min.   :1.120   Min.   :3.940   Min.   : 1   Min.   :-0.048669
 1st Qu.:1.404   1st Qu.:4.197   1st Qu.:11   1st Qu.: 0.003043
 Median :1.575   Median :4.467   Median :21   Median : 0.013589
 Mean   :1.494   Mean   :4.394   Mean   :21   Mean   : 0.011547
 3rd Qu.:1.614   3rd Qu.:4.593   3rd Qu.:31   3rd Qu.: 0.024275
 Max.   :1.682   Max.   :4.691   Max.   :41   Max.   : 0.050727
                                              NA's   :1
    goutphr            ghrwge_1            goutph_1           goutph_2
 Min.   :-0.02203   Min.   :-0.048669   Min.   :-0.02203   Min.   :-0.02203
 1st Qu.: 0.01102   1st Qu.: 0.003421   1st Qu.: 0.01278   1st Qu.: 0.01206
 Median : 0.02082   Median : 0.013690   Median : 0.02135   Median : 0.02150
 Mean   : 0.01879   Mean   : 0.012113   Mean   : 0.01909   Mean   : 0.01907
 3rd Qu.: 0.02878   3rd Qu.: 0.024413   3rd Qu.: 0.02887   3rd Qu.: 0.02895
 Max.   : 0.06258   Max.   : 0.050727   Max.   : 0.06258   Max.   : 0.06258
 NA's   :1          NA's   :2           NA's   :2          NA's   :3
    lwkhours
 Min.   :3.550
 1st Qu.:3.584
 Median :3.638
 Mean   :3.626
 3rd Qu.:3.664
 Max.   :3.696
```

Regression between two variables that have a trend: spurious correlation

Looking at observations before 1972, for productivity and for wages over time, on the left, both series in the two left graphs have a regular trend until 1970-80.

If you represent a scatter diagram hourly wage against labor productivity), you may have a spurious correlation:

- Both look very similar until about 1972 with a linear trend



In [3]:
```
#regress hourly wage on productivity using only data up to 1972 including

reg1<-lm(hrwage~ outphr,mydata[mydata$year<=1972,])
summary(reg1)
```

```
Call:
lm(formula = hrwage ~ outphr, data = mydata[mydata$year <= 1972,
    ])

Residuals:
     Min       1Q   Median       3Q      Max
-0.15208 -0.07159 -0.02181  0.08053  0.18871

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.492528   0.110093   4.474 0.000158 ***
outphr      0.051174   0.001494  34.253  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.09886 on 24 degrees of freedom
Multiple R-squared:   0.98,     Adjusted R-squared:  0.9791
F-statistic:  1173 on 1 and 24 DF,  p-value: < 2.2e-16
```

```
reg1<-lm(hrwage~ outphr,mydata[mydata$year<=1972,])
Summary(reg1)
Call:
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```

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We are estimating a positive correlation and R squared of 98% !!!

a spurious correlation because both wage and output per hour have a linear trend as we saw in the picture

In [ ]:
```
#using all data now

reg2<-lm(hrwage~ outphr,mydata)
summary(reg2)
```

Even for the whole sample

Dominated by the overall trend!

We have an omitted variable Time t

```
reg2<-lm(hrwage~ outphr,mydata)
summary(reg2)
Call:
lm(formula = hrwage ~ outphr, data = mydata)

Residuals:
   Min     1Q Median     3Q    Max
-0.5827 -0.2402  0.1217  0.1929  0.4295

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.528445   0.208223   7.34 7.34e-09 ***
outphr      0.035950   0.002458  14.62  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2726 on 39 degrees of freedom
Multiple R-squared:  0.8458,  Adjusted R-squared:  0.8418
F-statistic: 213.9 on 1 and 39 DF,  p-value: < 2.2e-16
```

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We are estimating a positive correlation and R squared of 84% even using all data even after 1972!!!

a spurious correlation because both wage and output per hour have a linear trend as we saw in the picture

Review for Final: OVB

- Full model: $wage_t = \beta_0 + \beta_1\, productivity_t + \beta_2\, t + u_t$

- Restricted model we just estimated:
- $\widetilde{wage}_t = \widetilde{\beta_0} + \widetilde{\beta_1}\, productivity_t + \widetilde{u_t}$

$\widetilde{\beta_1} = \beta_1$ +corr(productivity, t) corr(wage, t)
Given that corr(wage, t) = β_2 >0 from the graph wage and time are positively correlated, and given that corr(productivity, t) > 0 also see graph, then

By Omitting time "t" from the regression, we will overestimate beta 1, an upper bias is present, and so $\widetilde{\beta_1} > \beta_1$

We have an omitted variable, time t !

By Omitting time "t" from the regression, we will overestimate beta 1, an upper bias is present,

and so β_1 tilde > β_1

In [4]:
```
#Method 1 : Controlling for time t
reg3<-lm(hrwage~outphr+t,mydata[mydata$year<=1972,])
summary(reg3)
```

```
Call:
lm(formula = hrwage ~ outphr + t, data = mydata[mydata$year <=
    1972, ])

Residuals:
      Min        1Q    Median        3Q       Max
-0.116596 -0.026215  0.004611  0.025773  0.111055

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.536654   0.391560   9.032 5.03e-09 ***
outphr      -0.010874   0.007931  -1.371    0.184
t            0.107908   0.013724   7.863 5.76e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05259 on 23 degrees of freedom
Multiple R-squared:  0.9946,    Adjusted R-squared:  0.9941
F-statistic:  2104 on 2 and 23 DF,  p-value: < 2.2e-16
```

Before omitting t beta1_hat= β_1 hat= 0.0511744 (see slide 8)

By adding t beta1 hat will drop relative to beta1 hat tilde.

Change slide and see for yourself, now

Beta1hat is in the next slide -0.0108 < 0.0511 indeed

Method 1 with dealing with Omitted variable time

reg3<-lm(hrwage~outphr+t, mydata[mydata$year <= 1972,])

- You could add the omitted time variable directly to the regression

```
lm(formula = hrwage ~ outphr + t, data = mydata[mydata$year <= 1972, ])

Residuals:
     Min       1Q   Median       3Q      Max
-0.116596 -0.026215  0.004611  0.025773  0.111055

Coefficients:
             Estimate  Std. Error  t value   Pr(>|t|)
(Intercept)  3.536654   0.391560    9.032    5.03e-09 ***
outphr      -0.010874   0.007931   -1.371    0.184
t            0.107908   0.013724    7.863    5.76e-08 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05259 on 23 degrees of freedom
Multiple R-squared:  0.9946,  Adjusted R-squared:  0.9941
F-statistic:  2104 on 2 and 23 DF,  p-value: < 2.2e-16
```

t = [
1
2
3
4
5
6
7
8
…]

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Method 1 with dealing with Omitted variable time

- **After adding t no relationship left between productivity and wages**

```
lm(formula = hrwage ~ outphr + t, data = mydata[mydata$year <= 1972, ])

Residuals:
     Min       1Q   Median       3Q      Max
-0.116596 -0.026215  0.004611  0.025773  0.111055

Coefficients:
             Estimate  Std. Error  t value   Pr(>|t|)
(Intercept)  3.536654   0.391560    9.032    5.03e-09 ***
outphr      -0.010874   0.007931   -1.371    0.184
t            0.107908   0.013724    7.863    5.76e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05259 on 23 degrees of freedom
Multiple R-squared:  0.9946,  Adjusted R-squared:  0.9941
F-statistic:  2104 on 2 and 23 DF,  p-value: < 2.2e-16
```

t=[
1
2
3
4
5
6
7
8
…]

20

Method 2 First Differencing

The general method is to use first differencing

This method works for any other trend, any other common variation over time that is not linear

That is why we only used years up to 1972 in method 1, only before 1972 was there a linear trend and therefore it would make sense to add t

The general method is to use first differencing

in R see if sorted t

generate a first difference

mydata$dwage < -mydata$hrwage-lag(mydata$hrwage)

1 missing value generated

mydata$doutphr = mydata$outphr-lag(mydata$outphr)

also 1 missing val generated

And then run reg on the first differences instead of on the levels

In [6]:
```r
#Method 2: first differencing


#sort t

#generate a first difference
mydata$dwage<-mydata$hrwage-lag(mydata$hrwage)
#one missing value generated


mydata$doutphr=mydata$outphr-lag(mydata$outphr)
#also for output per hour one missing value generated
#because the first period 1 has no period 0 to difference out


#see the data
head(mydata)

# and see that first observation of dwage is NA as is for doutphr
```

A ti

year	wkearns	wkhours	outphr	hrwage	lhrwage	loutphr	t	ghr
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<
1947	123.52	40.3	51.4	3.065012	1.120052	3.939638	1	
1948	123.43	40.0	53.3	3.085750	1.126795	3.975936	2	0.00674
1949	127.84	39.4	54.2	3.244670	1.177014	3.992681	3	0.05021
1950	133.83	39.8	57.7	3.362563	1.212703	4.055257	4	0.03568
1951	134.87	39.9	59.4	3.380200	1.217935	4.084294	5	0.00523
1952	138.47	39.9	60.7	3.470426	1.244277	4.105944	6	0.02634

In [7]:
```
#regress the first differences using only data up to 1972
reg4<-lm(dwage~doutphr,mydata[mydata$year<=1972,])
summary(reg4)
```

```
Call:
lm(formula = dwage ~ doutphr, data = mydata[mydata$year <= 1972,
    ])

Residuals:
      Min        1Q    Median        3Q       Max
-0.073834 -0.038284 -0.003681  0.017143  0.093104

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.06886    0.02200   3.131  0.00469 **
doutphr      0.01330    0.01126   1.181  0.24966
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.05079 on 23 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.05718,   Adjusted R-squared:  0.01618
F-statistic: 1.395 on 1 and 23 DF,  p-value: 0.2497
```

The general method is to use first differencing

INDEED up to 1972 nothing relating changes in wages to changes in productivity

Method 2: first differencing

- The general method is to use first differencing

INDEED up to 1972 nothing relating changes in wages to changes in productivity

```
lm(formula = dwage ~ doutphr, data = mydata[mydata$year <= 1972, ])
Residuals:
     Min       1Q   Median       3Q      Max
-0.073834 -0.038284 -0.003681  0.017143  0.093104

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.06886    0.02200   3.131  0.00469 **
doutphr      0.01330    0.01126   1.181  0.24966
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.05079 on 23 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.05718,    Adjusted R-squared:  0.01618
F-statistic: 1.395 on 1 and 23 DF,  p-value: 0.2497
```


In [8]:
```
#regress the first differences all years
reg5<-lm(dwage~doutphr,mydata)
summary(reg5)
```

```
Call:
lm(formula = dwage ~ doutphr, data = mydata)

Residuals:
      Min        1Q    Median        3Q       Max
-0.199185 -0.044477  0.004589  0.042674  0.152055

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.021709   0.018509  -1.173    0.248
doutphr      0.046314   0.009854   4.700 3.37e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.07516 on 38 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.3676,    Adjusted R-squared:  0.351
F-statistic: 22.09 on 1 and 38 DF,  p-value: 3.373e-05
```

Method 2: first differencing

Using all the data we see that labor productivity increases by one point then wages increase significantly by 4.63 cents

```
lm(formula = dwage ~ doutphr, data = mydata)

Residuals:
     Min       1Q   Median       3Q      Max
-0.199185 -0.044477  0.004589  0.042674  0.152055

Coefficients:
             Estimate   Std. Error  t value  Pr(>|t|)
(Intercept) -0.021709   0.018509   -1.173    0.248
doutphr      0.046314   0.009854    4.700    3.37e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.07516 on 38 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.3676,  Adjusted R-squared:  0.351
F-statistic: 22.09 on 1 and 38 DF,  p-value: 3.373e-05
```


Not covered in EEP 118

Assumptions of Time Series Reg Models and properties of OLS

Test whether you need to first difference or not

Serial correlation