Lecture 24 EEP118

Limited Dependent Variable

1. Overview- Limited Dependent (Y) variables
2. Linear Probability model
3. Logit

Study chapter 7.5 and the first section of 17.1

Posted all remaining DA and solutions

Soon to be posted - practice final and previous year final

Quiz 6 ungraded posted - causality - impact analysis

Limited Depedent Variable Y

The basic context of this set of lectures is when Y is not continuous

Y=0 or 1, Y is binary. YES/NO

Other cases

Y categorical variable, e.g., income ranges

Y ordered variable,

Y truncated, e.g. lots of zeros and then Y>0 also

Y is a count variable , e.g. number of kids, 0,1,2,...

Y categorical and not ordered, e.g. major of graduates, transportation modes

The example we will use Y=1 if respondent is in labor force, =0 otherwise

Use a Data set on Women labor force participation

Source: MROZ.RAW in Wooldridge. T.A. Mroz (1987), “The Sensitivity of an Empirical Model of Married Women’s Hours of Work to Economic and Statistical Assumptions,” Econometrica 55, 765-799.

Y= 1 or 0 column called inlf (short for in labor force)

Obs: N=753

inlf byte %9.0g inlf=1 if in labor force, 1975, inlf=0 otherwise

age byte %9.0g woman's age in years

educ byte %9.0g years of schooling

kidslt6 byte %9.0g # kids < 6 years

kidsge6 byte %9.0g # kids 6-18

nwifeinc float %9.0g (faminc - wage*hours)/1000

hushrs int %9.0g hours worked by husband, 1975

husage byte %9.0g husband's age

huseduc byte %9.0g husband's years of schooling

huswage float %9.0g husband's hourly wage, 1975

city byte %9.0g =1 if live in SMSA

In [1]:
```r
#Lecture24.R
#Lecture 24



# Load the 'pacman' package
library(pacman)
#packages to use load them now using the pacman "manager"
p_load(dplyr, haven, readr)
#Another great feature of p_load(): if you try to load a package that is not
p_load(ggplot2)


pacman::p_load(lfe, lmtest, haven, sandwich, tidyverse)
# lfe for running fixed effects regression
# lmtest for displaying robust SE in output table
# haven for loading in dta files
# sandwich for producing robust Var-Cov matrix
# tidyverse for manipulating data and producing plots


#The big difference with Stata that appears here is lm() by default
#doesn't compute robust SE - we have to use additional packages/functions
#to compute it. felm does allow for multi-way clustering by default though
#which is nice.

#I added an alternate version of the first plots to show that we can
#change the color of the points according to whether the prediction
#is in [0,1] or outside of it. You can also specify factor(inlf) for
#the latter plots of actual vs. predicted to only have the values 0 or 1 on
#the x-axis.
```

```
pacman::p_load(lfe, lmtest, haven, sandwich, tidyverse)
# lfe for running fixed effects regression
# lmtest for displaying robust SE in output table
# haven for loading in dta files
# sandwich for producing robust Var-Cov matrix
# tidyverse for manipulating data and producing plots
install.packages(sandwich)
install.packages(lfe)
install.packages(lmtest)
install.packages(tidyverse)
library(sandwich)
library(lfe)
library(lmtest)
library(tidyverse)


# alternate plot theme for ggplot
theme_ed <- theme(
  legend.position = "bottom",
  panel.background = element_rect(fill = NA),
  # panel.border = element_rect(fill = NA, color = "grey75"),
  axis.ticks = element_line(color = "grey95", size = 0.3),
  panel.grid.major = element_line(color = "grey95", size = 0.3),
  panel.grid.minor = element_line(color = "grey95", size = 0.3),
  legend.key = element_blank())
```

```
Installing package into ‘/srv/r’
(as ‘lib’ is unspecified)

Error in as.character(x): cannot coerce type 'closure' to vector of type 'ch
aracter'
Traceback:

1. install.packages(sandwich)
2. grepl("[.]tar[.](gz|bz2|xz)$", pkgs)
```

In [3]:

```
#load data
mydata<- read_dta("Lecture24MROZ.DTA")
#Summary stats inlf age educ kidslt6 kidsge6 nwifeinc hushrs husage huseduc
summary(mydata)
```

```
     inlf             hours            kidslt6          kidsge6     
Min.   :0.0000   Min.   :   0.0   Min.   :0.0000   Min.   :0.000  
1st Qu.:0.0000   1st Qu.:   0.0   1st Qu.:0.0000   1st Qu.:0.000  
Median :1.0000   Median : 288.0   Median :0.0000   Median :1.000  
Mean   :0.5684   Mean   : 740.6   Mean   :0.2377   Mean   :1.353  
3rd Qu.:1.0000   3rd Qu.:1516.0   3rd Qu.:0.0000   3rd Qu.:2.000  
Max.   :1.0000   Max.   :4950.0   Max.   :3.0000   Max.   :8.000  

     age              educ            wage            repwage      
Min.   :30.00   Min.   : 5.00   Min.   : 0.1282   Min.   :0.00  
1st Qu.:36.00   1st Qu.:12.00   1st Qu.: 2.2626   1st Qu.:0.00  
Median :43.00   Median :12.00   Median : 3.4819   Median :0.00  
Mean   :42.54   Mean   :12.29   Mean   : 4.1777   Mean   :1.85  
3rd Qu.:49.00   3rd Qu.:13.00   3rd Qu.: 4.9708   3rd Qu.:3.58  
Max.   :60.00   Max.   :17.00   Max.   :25.0000   Max.   :9.98  
                                NA's   :325                     
    hushrs          husage          huseduc         huswage      
Min.   : 175   Min.   :30.00   Min.   : 3.00   Min.   : 0.4121  
1st Qu.:1928   1st Qu.:38.00   1st Qu.:11.00   1st Qu.: 4.7883  
Median :2164   Median :46.00   Median :12.00   Median : 6.9758  
Mean   :2267   Mean   :45.12   Mean   :12.49   Mean   : 7.4822  
3rd Qu.:2553   3rd Qu.:52.00   3rd Qu.:15.00   3rd Qu.: 9.1667  
Max.   :5010   Max.   :60.00   Max.   :17.00   Max.   :40.5090  

    faminc            mtr             motheduc         fatheduc     
Min.   : 1500   Min.   :0.4415   Min.   : 0.000   Min.   : 0.000  
1st Qu.:15428   1st Qu.:0.6215   1st Qu.: 7.000   1st Qu.: 7.000  
Median :20880   Median :0.6915   Median :10.000   Median : 7.000  
Mean   :23081   Mean   :0.6789   Mean   : 9.251   Mean   : 8.809  
3rd Qu.:28200   3rd Qu.:0.7215   3rd Qu.:12.000   3rd Qu.:12.000  
Max.   :96000   Max.   :0.9415   Max.   :17.000   Max.   :17.000  

     unem             city            exper          nwifeinc      
Min.   : 3.000   Min.   :0.0000   Min.   : 0.00   Min.   :-0.02906  
1st Qu.: 7.500   1st Qu.:0.0000   1st Qu.: 4.00   1st Qu.:13.02504  
Median : 7.500   Median :1.0000   Median : 9.00   Median :17.70000  
Mean   : 8.624   Mean   :0.6428   Mean   :10.63   Mean   :20.12896  
3rd Qu.:11.000   3rd Qu.:1.0000   3rd Qu.:15.00   3rd Qu.:24.46600  
Max.   :14.000   Max.   :1.0000   Max.   :45.00   Max.   :96.00000  

    lwage            expersq      
Min.   :-2.0542   Min.   :   0  
1st Qu.: 0.8165   1st Qu.:  16  
Median : 1.2476   Median :  81  
Mean   : 1.1902   Mean   : 178  
3rd Qu.: 1.6036   3rd Qu.: 225  
Max.   : 3.2189   Max.   :2025  
NA's   :325                     
```

Two problems linear prob model

Problem 1: We violate homoscedasticity: Var(y|x)=p(x)(1-p(x)). Depends on x!!!

Problem 2: For certain values of x we can have y hat < 0 or > 1

Fixing Problem 1 - correct Standard errors for heteroskedacitity

In [4]:
```
######## Fixing Problem 1 of Heteroskedasticity ------------
reg1 <- lm(inlf ~ nwifeinc + educ + age + kidslt6 + kidsge6, mydata)
summary(reg1)

# note that lm() uses regular standarde errors by default so we won't use su
# to get robust, we can use the following combo of coeftest() with vcovHC()
# HC1 is the Stata default for ",robust" but we could specify many alternate

#correct standard errors
coeftest(reg1, vcov = vcovHC(reg1, "HC1"))
```

```
Call:
lm(formula = inlf ~ nwifeinc + educ + age + kidslt6 + kidsge6,
    data = mydata)

Residuals:
    Min      1Q  Median      3Q     Max
-0.9345 -0.4762  0.1910  0.3808  0.9379

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.644779   0.160657   4.013 6.59e-05 ***
nwifeinc    -0.006844   0.001515  -4.518 7.25e-06 ***
educ         0.052459   0.007802   6.724 3.51e-11 ***
age         -0.011679   0.002542  -4.594 5.10e-06 ***
kidslt6     -0.296804   0.035789  -8.293 5.12e-16 ***
kidsge6     -0.011692   0.013970  -0.837    0.403
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4594 on 747 degrees of freedom
Multiple R-squared:  0.1466,    Adjusted R-squared:  0.1409
F-statistic: 25.66 on 5 and 747 DF,  p-value: < 2.2e-16
t test of coefficients:

              Estimate Std. Error t value  Pr(>|t|)
(Intercept)  0.6447792  0.1555271  4.1458 3.775e-05 ***
nwifeinc    -0.0068444  0.0015309 -4.4708 9.001e-06 ***
educ         0.0524590  0.0072904  7.1956 1.516e-12 ***
age         -0.0116789  0.0024414 -4.7837 2.074e-06 ***
kidslt6     -0.2968039  0.0329886 -8.9972 < 2.2e-16 ***
kidsge6     -0.0116920  0.0143619 -0.8141    0.4158
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

In [5]:
```
# get predictions (adding them into our df)
mydata <- mutate(mydata, reg_fit = reg1$fitted.values)
```

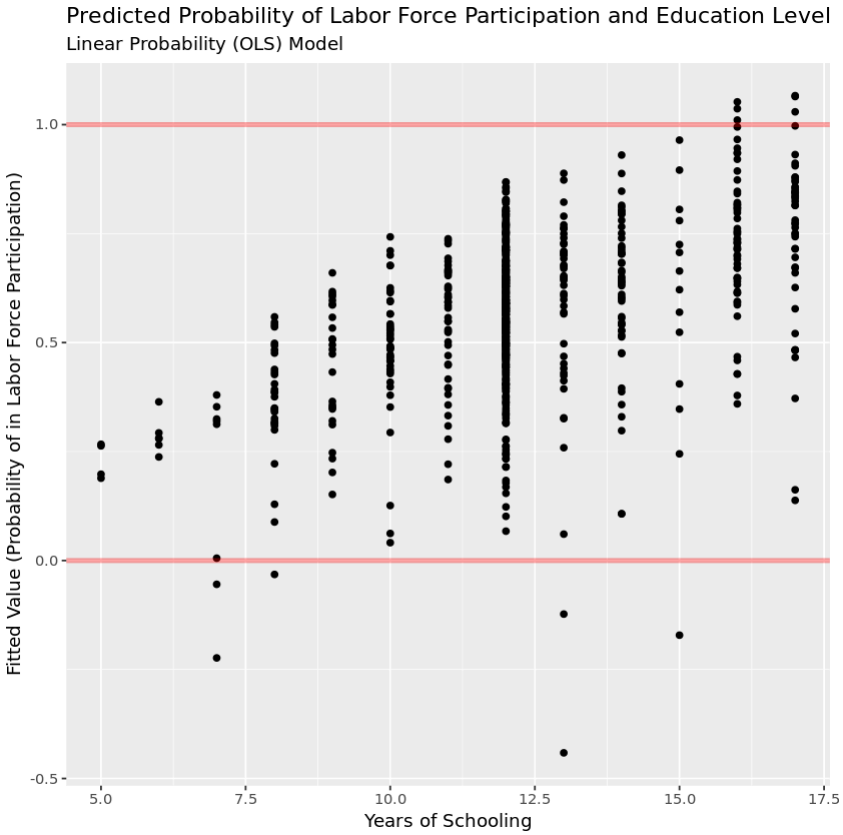
```
#to plot them next
```

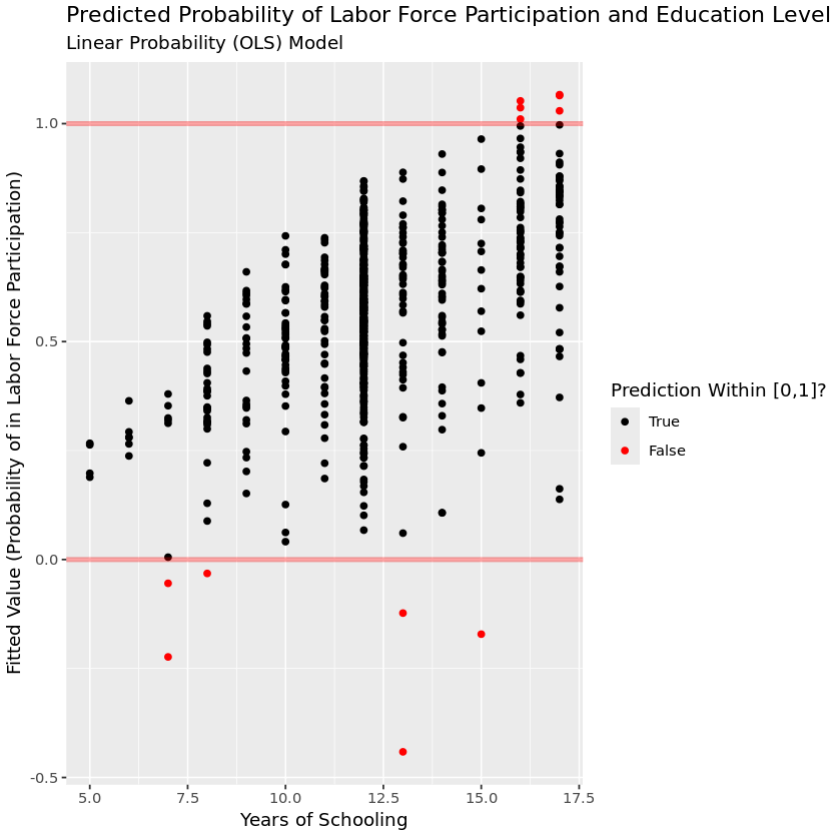
Showing Problem 2 that some predictions are negative and some greater than 1

In [6]:
```
# plot predicted values and education
# set data and aesthetics (x and y vars here since the same for all elements
ggplot(mydata, aes(x = educ, y = reg_fit)) +
  # First add points, color determined by whether in or out of [0,1]
  geom_point() + # add points
  # add horizontal lines, width slightly wider, making partially transparent
  geom_hline(yintercept=0, size = 1.4, alpha = 0.35, color = "red") + # add
  geom_hline(yintercept=1, size = 1.4, alpha = 0.35, color = "red") + # add
  # generate labels
  labs(title = "Predicted Probability of Labor Force Participation and Educa
       subtitle = "Linear Probability (OLS) Model",
       x = "Years of Schooling",
       y = "Fitted Value (Probability of in Labor Force Participation)")

# alternate version of the figure where we change the color of fitted values
# first create logical variable of whether fitted value is within [0,1]
mydata <- mutate(mydata, reg_outside = reg_fit < 0 | reg_fit > 1)
# now use this variable to determine color of points
# set data and aesthetics (x and y vars here since the same for all elements
ggplot(mydata, aes(x = educ, y = reg_fit)) +
  # First add points, color determined by whether in or out of [0,1]
  geom_point(aes(colour = factor(reg_outside))) +
  # add horizontal lines, width slightly wider, making partially transparent
  geom_hline(yintercept=0, size = 1.4, alpha = 0.35, color = "red") + # add
  geom_hline(yintercept=1, size = 1.4, alpha = 0.35, color = "red") + # add
  # generate labels
  labs(title = "Predicted Probability of Labor Force Participation and Educa
       subtitle = "Linear Probability (OLS) Model",
       x = "Years of Schooling",
       y = "Fitted Value (Probability of in Labor Force Participation)") +
  # Manually set the color scale to the title we want and the colors
  scale_colour_manual(name = "Prediction Within [0,1]?",
                      values = c("black", "red"),
                      labels = c("True", "False"))
```

```
Warning message:
“Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
i Please use `linewidth` instead.”
```





what do we see?

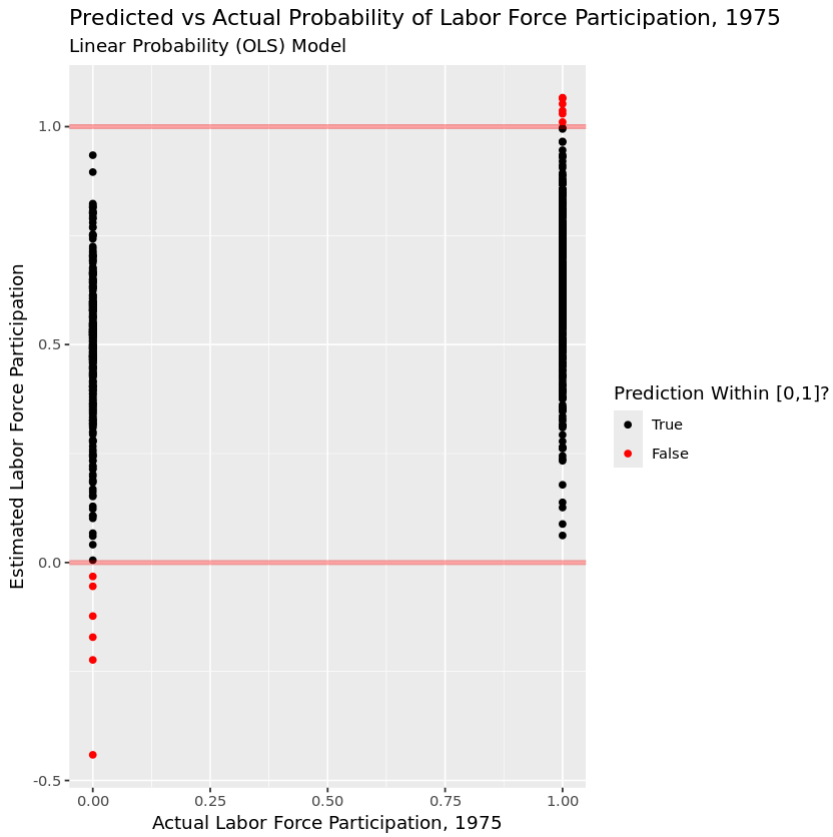
for example

A woman with 3 kids, age 31 nwifeinc=73.6 is observed not working and we predict for her Y hat < 0

see the red dot in the above graph

In [7]:
```
# Plot 2 ------------
# Produce second plot for actual y
# set data and aesthetics (x and y vars here since the same for all elements
ggplot(mydata, aes(x = inlf, y = reg_fit)) +
  # First add points, color determined by whether in or out of [0,1]
  geom_point(aes(colour = factor(reg_outside))) +
  # add horizontal lines, width slightly wider, making partially transparent
  geom_hline(yintercept=0, size = 1.4, alpha = 0.35, color = "red") + # add
  geom_hline(yintercept=1, size = 1.4, alpha = 0.35, color = "red") + # add
  # generate labels
  labs(title = "Predicted vs Actual Probability of Labor Force Participation
       subtitle = "Linear Probability (OLS) Model",
       x = "Actual Labor Force Participation, 1975",
       y = "Estimated Labor Force Participation") +
  # Manually set the color scale to the title we want and the colors
  scale_colour_manual(name = "Prediction Within [0,1]?",
                      values = c("black", "red"),
                      labels = c("True", "False"))

# Specifying x as factor simplifies the plot to only show 0 or 1 for x-axis
# set data and aesthetics (x and y vars here since the same for all elements
ggplot(mydata, aes(x = factor(inlf), y = reg_fit)) +
  # First add points, color determined by whether in or out of [0,1]
  geom_point(aes(colour = factor(reg_outside))) +
  # add horizontal lines, width slightly wider, making partially transparent
  geom_hline(yintercept=0, size = 1.4, alpha = 0.35, color = "red") + # add
  geom_hline(yintercept=1, size = 1.4, alpha = 0.35, color = "red") + # add
  # generate labels
  labs(title = "Predicted vs Actual Probability of Labor Force Participation
       subtitle = "Linear Probability (OLS) Model",
       x = "Actual Labor Force Participation, 1975",
       y = "Estimated Labor Force Participation") +
  # Manually set the color scale to the title we want and the colors
  scale_colour_manual(name = "Prediction Within [0,1]?",
                      values = c("black", "red"),
                      labels = c("True", "False"))
```



What do we see?

we predict some above 1 and some below 0

Fixing Problem 2, make sure predictions are between 0 and 1

Solution Problem 1 –

use a functional for for the probability as a function G( ) of the xs that stays between 0 and 1

e.g., the Logit Model!

the ratio of exponents in the logit below is always between 0 and 1

Prob [Y=1 | x] = $G(\beta_0 + \beta_1\ x_1 + \beta_2\ x_2 + \cdots + \beta_3\ x_k)$

Get a G that stays between 0 and 1, and **the Logit is**

Prob [Y=1 | x] = $G(\beta_0 + \ldots + \beta_k\ \ X_k) = \frac{e^{\beta_0+\beta_1\ x_1+\beta_2\ x_2+\cdots+\beta_3\ x_k}}{1+e^{\beta_0+\beta_1\ x_1+\beta_2\ x_2+\cdots+\beta_3\ x_k}}$

Prob[Y=1| X]= $\Lambda\ (\beta_0 + \beta_1\ x_1 + \beta_2\ x_2 + \cdots + \beta_3\ x_k)$

This ratio of exponentials is always between 0 and 1 no matter the betas and xs

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In [8]:
```
###### Fixing Problem 2 so that predicted Y hats are less than 1 and greater
# In R, use the glm(formula, data, family = binomial(link = "logit")) functi

logit <- glm(inlf ~ nwifeinc + educ + age + kidslt6 + kidsge6, mydata, famil
summary(logit)
```

```
Call:
glm(formula = inlf ~ nwifeinc + educ + age + kidslt6 + kidsge6,
    family = binomial(link = "logit"), data = mydata)

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept)  0.722993   0.788698   0.917    0.359
nwifeinc    -0.034891   0.007884  -4.426 9.62e-06 ***
educ         0.257965   0.040744   6.331 2.43e-10 ***
age         -0.057553   0.012737  -4.519 6.23e-06 ***
kidslt6     -1.484437   0.198013  -7.497 6.55e-14 ***
kidsge6     -0.066363   0.067856  -0.978    0.328
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 1029.75  on 752  degrees of freedom
Residual deviance:  908.37  on 747  degrees of freedom
AIC: 920.37

Number of Fisher Scoring iterations: 4
```

Cannot easily interpret parameters here,

next class estimate implied marginal effects given the above estimated Logit parameters

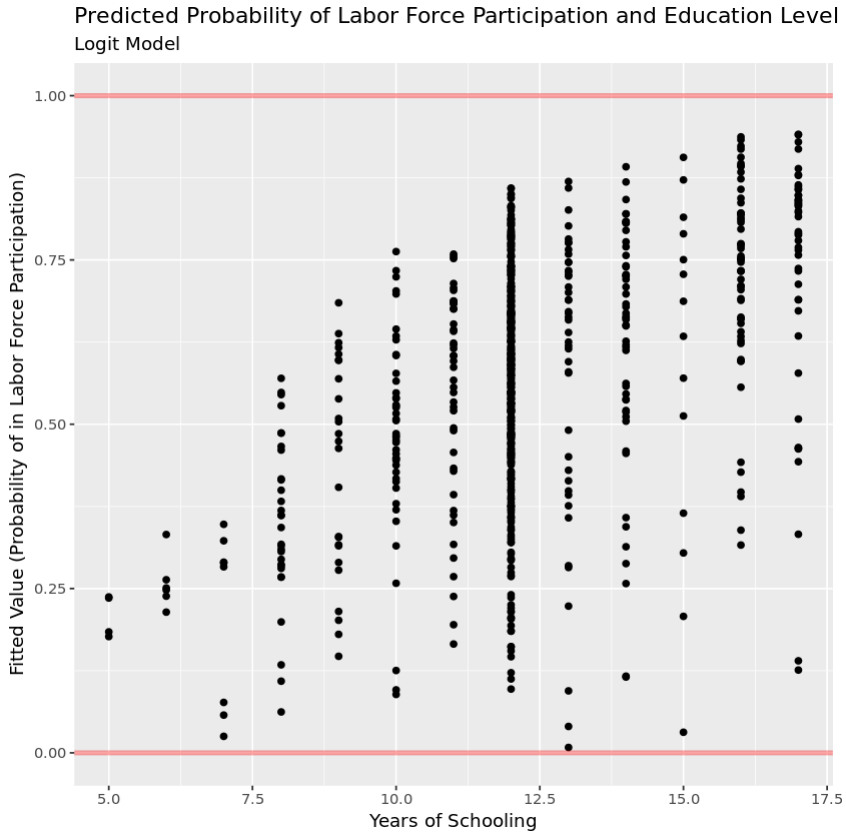
. . . . . More on the logit model next class

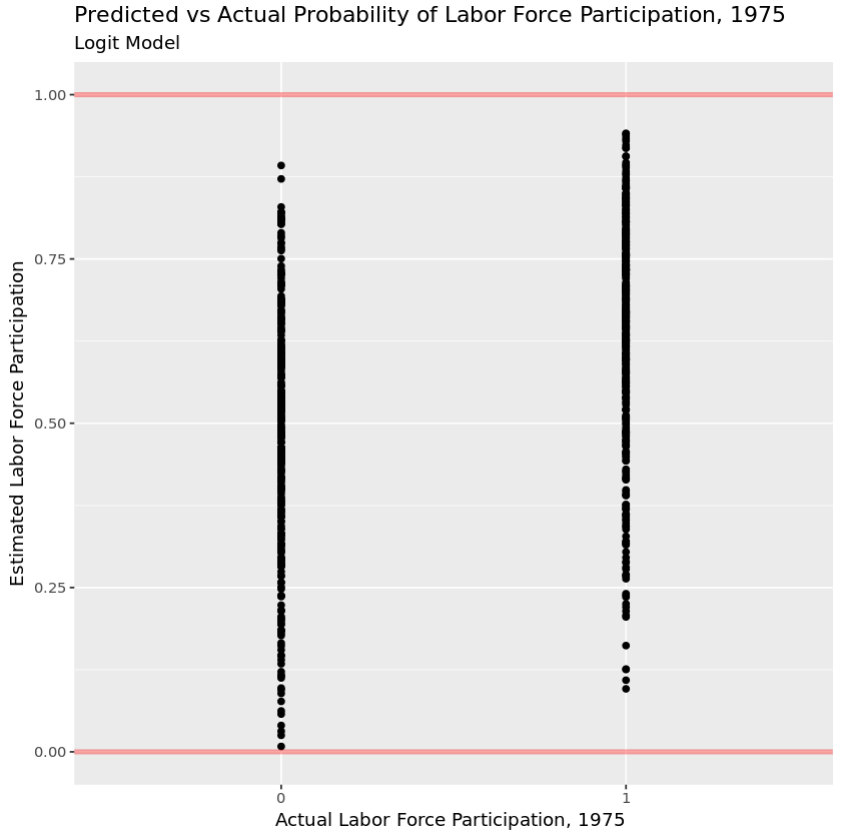
In [9]:
```r
#generate predictions
mydata <- mutate(mydata, log_fit = logit$fitted.values) # add in the logit f
```

In [10]:
```r
# Reproduce figures for logit
# no need to use the second approach as we're always within [0,1] with logit
# set data and aesthetics (x and y vars here since the same for all elements
ggplot(mydata, aes(x = educ, y = log_fit)) +
  # First add points, color determined by whether in or out of [0,1]
  geom_point() + # add points
  # add horizontal lines, width slightly wider, making partially transparent
  geom_hline(yintercept=0, size = 1.4, alpha = 0.35, color = "red") + # add
  geom_hline(yintercept=1, size = 1.4, alpha = 0.35, color = "red") + # add
  # generate labels
  labs(title = "Predicted Probability of Labor Force Participation and Educa
       subtitle = "Logit Model",
       x = "Years of Schooling",
       y = "Fitted Value (Probability of in Labor Force Participation)")

# actual vs predicted
ggplot(mydata, aes(x = factor(inlf), y = log_fit)) +
  # First add points, color determined by whether in or out of [0,1]
  geom_point() +
  # add horizontal lines, width slightly wider, making partially transparent
  geom_hline(yintercept=0, size = 1.4, alpha = 0.35, color = "red") + # add
  geom_hline(yintercept=1, size = 1.4, alpha = 0.35, color = "red") + # add
```

```
  # generate labels
  labs(title = "Predicted vs Actual Probability of Labor Force Participation
       subtitle = "Logit Model",
       x = "Actual Labor Force Participation, 1975 ",
       y = "Estimated Labor Force Participation")
```





Looking at the graph above, all predictions now are between 0 and 1

more in Lecture 25