

* Statistics →

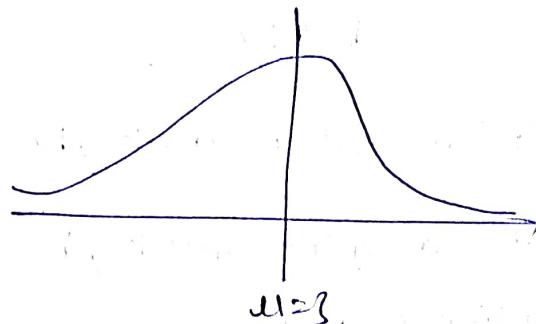
understanding normal distribution

Eg:

1, 2, 3, 4, 5

mean = 3

SD =



$$S.D(\sigma) = \sqrt{\frac{\sum_{i=1}^n (x_i - \mu)^2}{n}} = \sqrt{2} \approx 1.41.$$

→ S.D says how close it is to mean.

So, total data lies in $(\bar{x} \pm 1.41)$

which gives $(4.58 - 4.41)$

→ But 1, 5 lie outside the range.

→ If we take

$\bar{x} \pm 2S.D.$

Total data $\rightarrow (\bar{x} \pm 2 \times 1.41)$

$\rightarrow [0.18 - 5.32]$

now P(15%) per cent

Note

→ one thing we can understand is,

with

$\bar{x} \pm S.D$

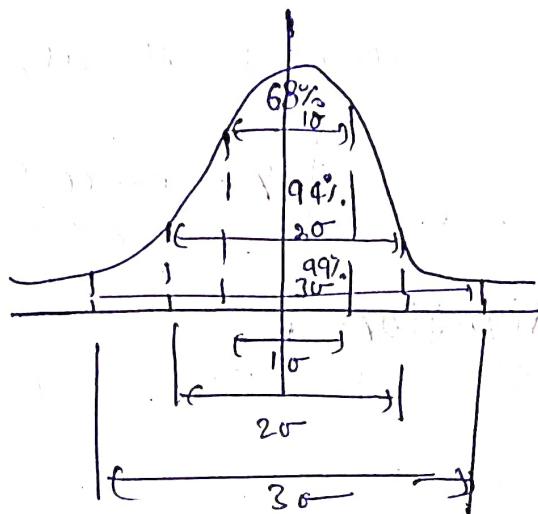
we are not getting 100% data

but only 68%.

$\mu \pm \sigma$ → 68% of data → All the data is not available here.

$\mu \pm 2\sigma$ → 94% of data

$\mu \pm 3\sigma$ → 99% of data



Standard normal distribution

→ data with $\mu=0$ & $\sigma=1$

That means complete data will do $\mu \pm \sigma$.

Ex: $[-1, 1]$ → real data. 0 ± 1

converting normal data to SND

1, 2, 3, 4, 5 →

$$\sigma = 1.41$$

$$z\text{ value} = \frac{x - \mu}{\sigma}$$

$$\left\{ \begin{array}{l} z_1, z_2, z_3, z_4, z_5 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ \left(\frac{1-3}{1.41} \right) \left(\frac{2-3}{1.41} \right) \left(\frac{3-3}{1.41} \right) \left(\frac{4-3}{1.41} \right) \left(\frac{5-3}{1.41} \right) \end{array} \right.$$

So,

for the new data.

$$-1.41, -0.70, 0, 0.70, 1.41$$

$\mu = 0$

$S.D = 1$

→ so, do bring them like this,
we convert it all,

so, converting this is called

$$Z = \frac{x - \mu}{\sigma}$$

"standardization"

notes

1) normalization — scaling the data.

to bring close to mean ...

2) standardization — making $\mu=0, S.D=1$.

Converting data to $[0, 1]$

$$Z = \frac{x - \mu}{\sigma}$$



a) The time reqd to build a computer is normally distributed with mean of 50 min & a SD of 10 min.

What is probability that a computer is assembled in a time b/w us & 60 min?

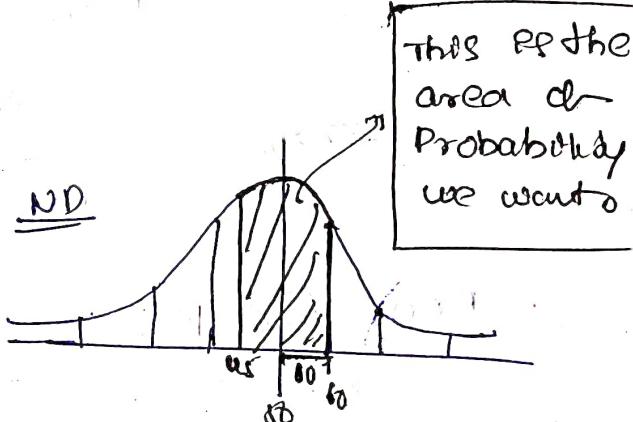
Sol

We can't fit this in Poisson / binomial.

Avg time = 50 min }
SD = 10 min } to build a
computer

us min Probability \leq 60 min.

?



But, we can calculate the area if we convert it to

SND - standard normal distribution

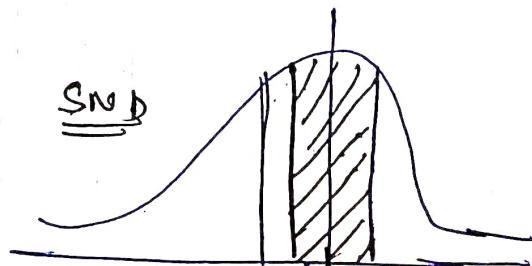
- But, in normal distribution we have random mean & SD
- So, difficult to calculate that area.

As P(S) area of C.I always.

SND

$$\frac{50-50}{10} \leq P \leq \frac{60-50}{10} \Rightarrow -0.5 \leq P \leq 1$$

we need area b/w 60-50



Estimation Theory

○ Estimation by P

1) Point Estimation

2) Interval Estimation

$$\text{Estimation Formula} \rightarrow \bar{x} \pm \frac{z_{\alpha/2}}{2} \left(\frac{\sigma}{\sqrt{n}} \right)$$

→ It is the

Probability distribution for $(1-\alpha)$ confidence intervals.

where, α - level of significance.

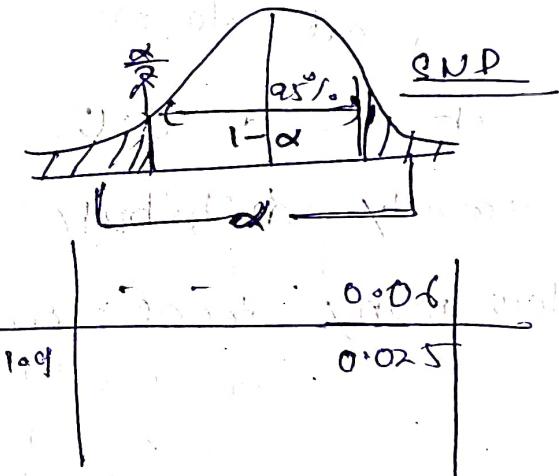
→ Let's take 95%

$$1-\alpha = 0.95$$

$$\alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

→ And here $\frac{\alpha}{2}$ is the area value in Z -table.



→ Go on revenue, find for the $\frac{\alpha}{2}$ area & see the z -value in Z -table.

Hence, here for 0.025 area we have 1.96 .

→ As the C.I. the range gets wider.

Notes:

Notes

- 1) As C.L. then range "gets wider."
- 2) For low C.L, Risk of failure is less.
For high C.L, Risk of failure is High.

(Q.) A lumber company must estimate the mean dia of trees to determine whether or not there is suff. lumber to harvest in an area of forest. They need to estimate this to within 9%

1 inch at a C.L of 99%. The tree dia are normally distributed with a SD of 5 inches. How many trees need to be sampled

Soln

We have,

$$1-\alpha = 0.99$$

$$\alpha' = 0.01 \rightarrow \frac{\alpha}{2} = 0.005 \quad z_{\frac{\alpha}{2}} = 2.575$$

$$\text{Formula} \rightarrow \bar{x} \pm \left(z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

→ width is 1 inch,

$$\text{i.e. } z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 1 \rightarrow 2.575 \cdot \frac{5}{\sqrt{n}} = 1$$

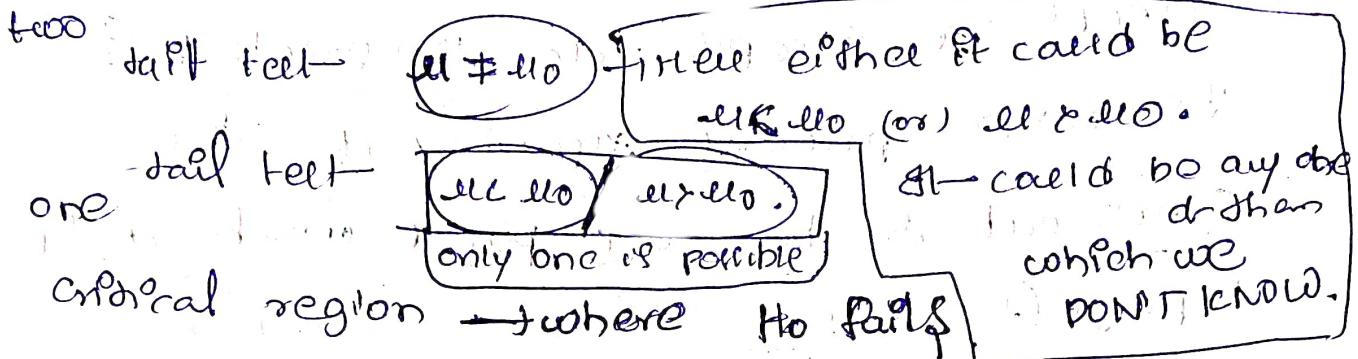
n=229 we need at least 229 trees.

* Hypothesis

$$H_0: \mu = \mu_0$$

$$H_1: \mu \neq \mu_0$$

\rightarrow if we disapprove that, then automatically H_0 is true.



\rightarrow it could be only left/right side.
or it could also be on both sides

confusion matrix

		Expected	
		T	F
Predicted	T	TP	FP
	F	FN	TN

TYPE I error
Actually false, but predicted as true.

TYPE II error.

TYPE I error - Person is not guilty yet

Jury says he is convicted.

TYPE II error - Person is guilty yet

Jury says he is not

(Q) A manufacturer of printer cartridge claims that a certain cartridge manufacture by him has a mean printing capacity of at least 500 pages. A wholesale purchaser selects a sample of 100 printers & tests them. The mean printing capacity of the sample came out to be 490 pages with a SD of 30 pages.

Should the purchaser reject the claim of manufacturer at a 5% level?

S.P

H_0 : mean page > 500

H_1 : mean < 500

→ one-tail test.

$\alpha = 5\%$

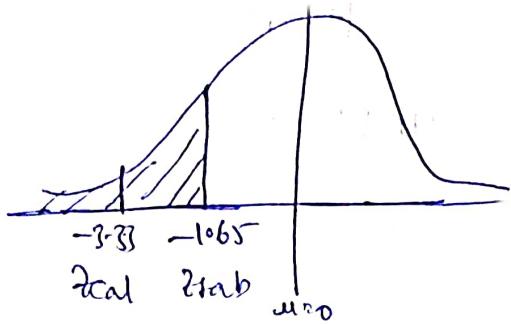
$\mu = 500$, $\bar{x} = 490$, $n = 100$, $S = 30$

$$t_{\text{value}} = \frac{\bar{x} - \mu}{SE} = \frac{\text{Sample mean} - \text{Pop. mean}}{\text{Std. Error}}$$

$$\frac{30}{\sqrt{100}} = \frac{SD}{\sqrt{n}} \quad \begin{matrix} \text{Sample} \\ \text{no. of sample case.} \end{matrix}$$

$$t_{\text{value}} = \frac{490 - 500}{30} = -3.33$$

$$t_{\text{tab}} = t_{0.05\%} = 1.65$$



→ we can see z_{cal} is in critical region.
So, Reject H_0 .

- mean is less than hypothesized.

Note:

If variance of population is not given.
then we will go for t-test & normal distribution.

Sample Size	Population variance	Normality of sample	Sample variance	Type of test
large (>30)	known	normal / non-normal	-	z-test
large (>30)	unknown	normal	use this to calculate t-score	t-test
large (>30)	unknown	unknown	use this to calculate z-score	z-test
small (<30)	known	normal	-	z-test
small (<30)	unknown	normal	use this to calculate t-score	t-test

Z-test

→ Z-test when normality, SD of Popl. is not known & we know SD of sample.

Then,

$$z_{\text{val}} = \frac{\bar{x}_m - \mu_m}{\left(\frac{\sigma}{\sqrt{n}}\right)} \rightarrow \text{std. error.}$$

→ z-test for TWO DIFFERENT POPULATIONS

$$z_{\text{val}} = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \rightarrow \frac{\text{sample mean of } 1 - \text{sample mean of } 2}{\sqrt{\frac{(\text{SD}_1)^2}{n_1} + \frac{(\text{SD}_2)^2}{n_2}}}$$

1) A company used a specific brand of tube lights, on the part which has an avg. life of 1000 hours. A new brand has approached the company with new tube lights with same power at a lower price.

A sample of 120 light bulbs were taken for testing which yielded an avg. of 1100 hours with SD of 90 hours. Should the company give the contract to this new company at a 1% L.O.S. Also find CI.

504

claim

one tail test.

$H_0: \text{price} \leq \text{original price}$

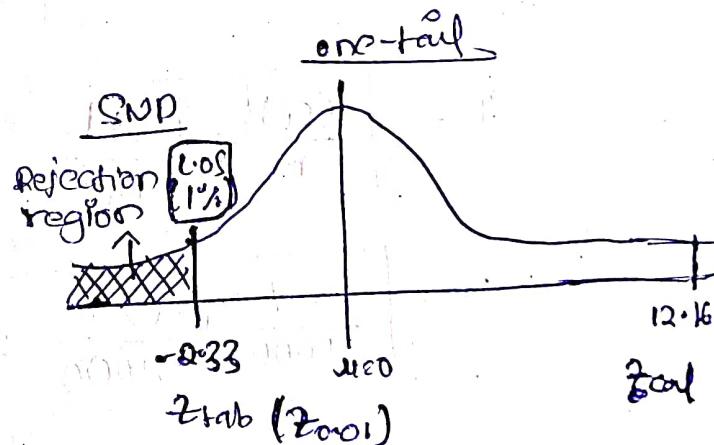
$H_1: \text{price} > \text{original price}$

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1100 - 1000}{\frac{90}{\sqrt{120}}}$$

$$\left(\frac{s}{\sqrt{n}} \right)$$

$$z_{\text{cal}} = \frac{100}{8.22} = 12.16$$

$z_{\text{tab}} @ 0.01 \text{ L.O.S.}$
which is -2.33



→ we can clearly see,

Rejection region

Acceptance region
i.e. P_{α} = acceptance region

So, accept H_0 i.e.,

what they claim is true.

they offer with same power at low price

Accept Project

Q) In two samples of men from diff states A & B, the height of 1000 men & 2000 men respectively are 76.5 & 77 inches. If population SD for both states is same & is 7 inches, can we assume mean heights at both states can be regarded same at 5% L.O.S.

Sol: 1000 popl. sum → Here what they want is to check whether they are same. So, we have to check on both samp.

$$\begin{aligned} H_0: \mu_1 = \mu_2 & \quad (\text{Both means equal}) \\ H_1: \mu_1 \neq \mu_2 & \quad (\text{not equal}) \end{aligned} \quad \left. \begin{array}{l} \text{two-tail test} \\ \text{we want} \\ \text{stab at } 5\% \text{ L.O.S.} \\ \text{for two tail.} \end{array} \right\}$$

$$SE = \sqrt{\frac{(S_1)^2}{n_1} + \frac{(S_2)^2}{n_2}}$$

$$SE = \sqrt{\frac{(7)^2}{1000} + \frac{(7)^2}{2000}} = 0.27$$

$$t_{\text{cal}} = \frac{\bar{x}_1 - \bar{x}_2}{SE}$$

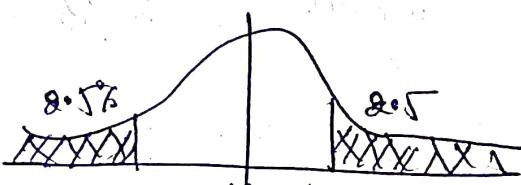
$$t_{\text{cal}} = \frac{76.5 - 77}{0.27}$$

$$t_{\text{cal}} = -1.085$$

$$Z_{0.05} = -1.64$$

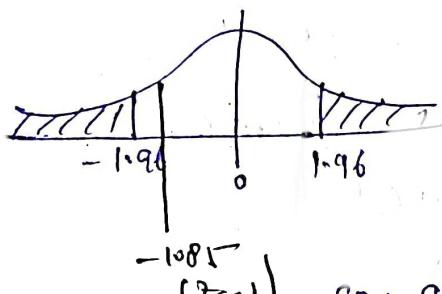
→ But, here as P.L is two tail. we got to divide the 5% for both tail.

i.e., 2.5% (left) + 2.5% (right)



$$\begin{aligned} z_{0.25} &= +1.96 \\ z_{0.25} &= -1.96 \end{aligned} \quad \left. \begin{array}{l} \text{So, for the } Z_{\text{tab}} \text{ at 5% L.O.S} \\ \text{for two-tail is} \end{array} \right\} \quad \begin{array}{l} \text{So, for the } Z_{\text{tab}} \text{ at 5% L.O.S} \\ \text{for two-tail is} \end{array}$$

$Z = 1.96.$



(tical) so, accept H_0

if it is not in
critical region

so both means are equal.

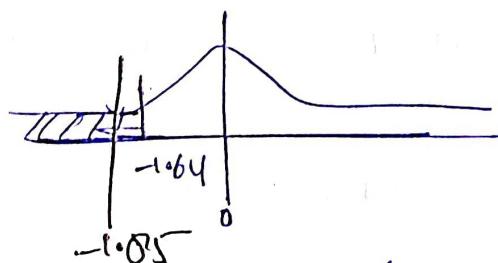


Note

1) make sure whether it is two-tail / one-tail.

If you calculate the above sum it is one-tail.
then,

$Z_{\text{tab}} @ 5\% \text{ L.O.S} \text{ (for one-tail)} = z_{0.05} = -1.64.$

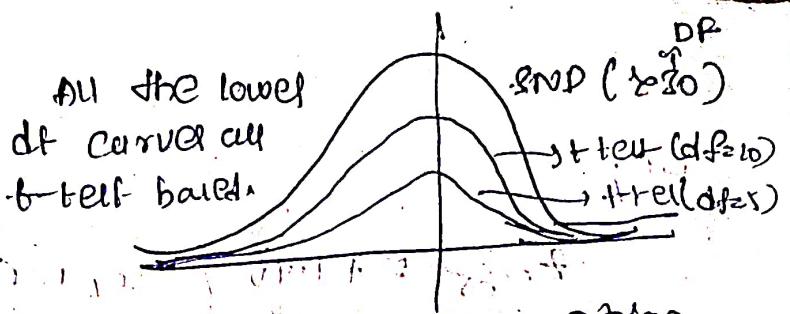


(tical) \rightarrow (reject H₀)

So, if it is one-tail,
we would have rejected H₀.

$Z_{\text{tab}} @ 5\% \text{ L.O.S}$
(for one-tail) = $z_{0.025} = 1.96$

T-test



→ when the sample is small, the assumption of normal distribution doesn't hold good & as such t-test will not be appropriate.

Instead another test known as t-test' is used
Procedure of all the sample creeps up
with the t-table values.

$$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sigma / \sqrt{n}}$$

* For two diff populations,

$$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_e^2}{n_1} + \frac{\sigma_e^2}{n_2}}}$$

$$\sigma_e (\text{common s.d.}) = \sqrt{\frac{(s_1)^2 + (s_2)^2}{n_1 + n_2 - 2}}$$

Prob'l

1) A tyre manufacturer claims that the avg. life of a particular category of P.T. II tyre is 18000 km when used under normal driving cond.
 A random sample of 16 tyres was tested.
 The mean & SD of life of tyres in the sample were 20,000 km & 6000 km respectively.
 Assuming that the life of the tyres is normally distributed, test the claim of manufacturer at 1% L.O.S.

Sol'l

→ claims P.T. is 18000. They are not taking about less or more. Hence, two-tail.

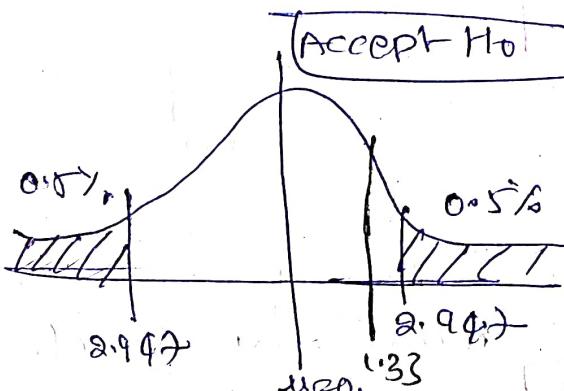
$$H_0: \mu = 18,000 \quad \left\{ \begin{array}{l} \text{small sample of 16 L.S.O.} \\ \text{D.O.F} = n-1 = 16-1 = 15, \end{array} \right.$$

$$H_1: \mu \neq 18,000.$$

$$t_{\text{cal}} = \frac{\bar{x} - \mu_0}{S_E} \rightarrow \frac{\sigma}{\sqrt{n}} = \frac{6000}{\sqrt{16}} = 1500$$

$$t_{\text{cal}} = \frac{20,000 - 18,000}{1500}$$

$$\boxed{t_{\text{cal}} = 1.33}$$



t_{tab} @ 1% L.O.S

(for two-tail)

(or) t_{tab} @ 0.5% L.O.S

(for one-tail)

i.e., (ii) → one tail

0.5% left + 0.5% right

so, t_{tab} at 1% L.O.S (two-tail)

$$\boxed{t_{\text{tab}} = 2.947.}$$

a) the means of two random samples of size 10 & 8 from two normal popl. is 210.4 & 208.92 . The sum of squared of deviations from their means is 26.94 & 24.50 respectively. Assuming populations with equal variances, can we consider the normal populations have eq. mean? ($H_0: \mu_1 = \mu_2$)

Soln

$$\text{S.E. [common S.D.]} = \sqrt{\frac{s_1^2 + s_2^2}{n_1+n_2-2}} = \sqrt{\frac{26.94 + 24.50}{10+8-2}} = 2.79.$$

$$S.E. = \sqrt{\frac{1.79^2}{10} + \frac{1.79^2}{8}} = 0.84.$$

$H_0: \mu_1 = \mu_2$	two-tail
$H_1: \mu_1 \neq \mu_2$	

$$t_{\text{cal}} = \frac{210.4 - 208.92}{0.84}$$

$$t_{\text{cal}} = 1.76$$

$t_{0.05}$ at 5% D.O.F

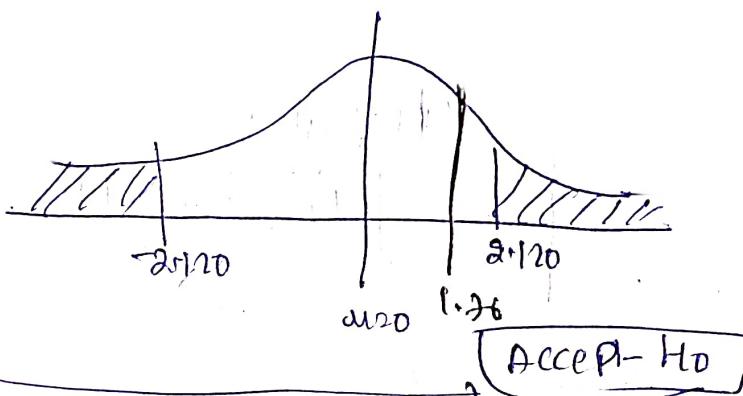
for two-tail

$t_{0.05}$ at (10+8-2)

16 D.O.F

$= 2.120$

$$(t_{0.05} = 2.120)$$



mean are equal

Accept H_0

(χ^2) chi-squared test



- It is a non-parametric test which means that it doesn't need any parameters to test, like mean, median etc..
- And it is widely used to test the hypothesis involving "categorical" data mainly 'Nominal'.
- It highly depends on the 'degree of freedom'.
- For a smaller D.O.F it gets 'skewed', mostly skewed distribution.
As the DF approaches or ≥ 30 the df becomes normal distribution.
- Right skewed, χ^2 + non-negative.

Tests that can be performed

- ① Tests the hypothesis about the 'variance' & SD of a population.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$
- ② Tests the hypothesis for more than two categories called 'Goodness of fit test'.

$$\chi^2 = \sum \frac{(\text{observed} - \text{Expected})^2}{\text{Expected}}$$
- ③ Tests the hypothesis about contingency tables called ' χ^2 test of independence' or homogeneity test/contingency table'.

$\text{Expected} = \frac{(RT)(CT)}{GT}$	$\text{DF} = (r-1)(c-1)$
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Problem 8

model ① - HT of variance

i) the variance of a certain size of towel

Produced by a machine is 7.2 over a long period of time. A random sample of 20 towels gave a variance of 8. You need to check if the variability for towel has increased at 5% L.O.S, assuming a normally distributed sample.

Soln

$$\text{Population variance } (\sigma^2) = 7.2$$

$$\text{Sample size } (n) = 20, \text{ sample variance } (s^2) = 8$$

- need to check whether sample derived from pop.
- You would check something when you have a 'doubt' on pt. And there, pt is not what you want.

$$H_0: \text{var} < 7.2$$

$$H_1: \text{var} > 7.2$$

$$\chi^2_{\text{tab}} \text{ at } 5\% \text{ L.O.S}$$

$$\text{For one-tail test } 20-1 = 19 \text{ DF}$$

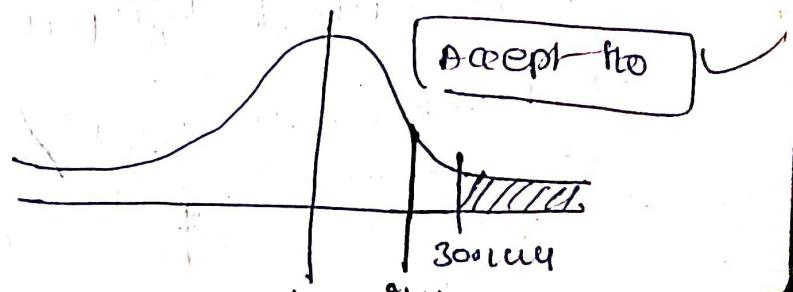
$$\chi^2_{\text{cal}} = \frac{(n-1)s^2}{\sigma^2}$$

$$\chi^2_{\text{cal}} = \frac{(20-1)(8)}{7.2}$$

$$\chi^2_{\text{cal}} = 21.011$$

$$\boxed{\chi^2_{\text{tab}} = 30.144}$$

$$\chi^2_{\text{cal}} < \chi^2_{\text{tab}}$$



- 2) A survey conducted by a pet food company determined that 60% of dog owners have 1 dog, 28% have 2 dogs, 12% have 3 dogs or more, you were not convinced by survey & decided to conduct your own survey & have collected data, out of 129 dog owned, 73 - 1 dog, 38 - 2 dogs. Determine whether your data supports the results of the survey by the pet. use a 5% L.O.S.

soln

H_0 & Survey justify the pet Survey (Both are same) convinced

H_1 : Both are not same.

This is model ② + Goodman or Art by

	observed	Expected
1)	73	$60\% \text{ of } 129 = 77.4$
2)	38	$28\% \text{ of } 129 = 36.12$
3)	18	$12\% \text{ of } 129 = 15.48$

χ^2 tab at 5% L.O.S

Two-tail but P doesn't matter.

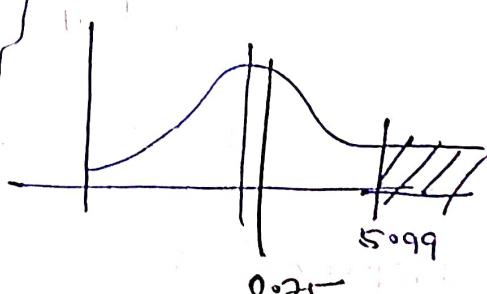
χ^2 - always right skewed

$$D.O.F = 3 - 1 = 2$$

$$\chi^2_{cal} = \frac{(73 - 77.4)^2}{77.4} + \frac{(38 - 36.12)^2}{36.12} + \frac{(18 - 15.48)^2}{15.48}$$

$$\chi^2_{tab} = 5.99$$

$$\chi^2_{cal} = 0.7533$$



convinced with
the pet Survey

[Accept H_0] ✓

③ out of 8000 graduate in a town 800
are female & out of 1600 graduate
employed 120 are female. Use χ^2 to
determine if any distribution is made
in appointment on basis of sex.

use S.Y. U.O.S.

Q. 11)

	unemployed graduates	employed graduates	row total
male	5720	1480	7200
female	680	120	800
column total	6400	1600	8000

$$D.O.P = (r-1)(c-1)$$

$$DP = (2-1)(2-1)$$

$$DP = 1$$

H_0 : Employment depends on sex.

H_1 : Doesn't depend.

Formula for

Expected =

Observed	Expected	$(O_i - E_i)^2 / E_i$
5720	5760	0.277
1480	1440	0.111
680	640	2.5
120	160	10

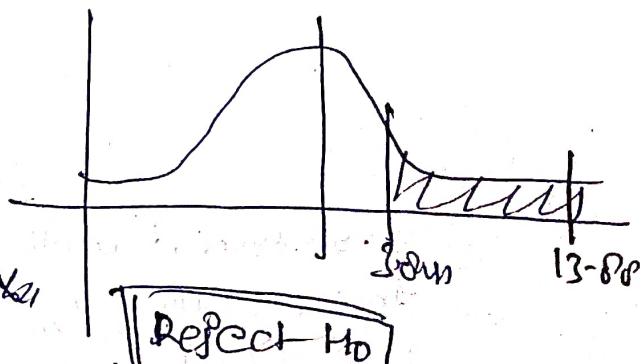
$$\frac{(\text{row total})(\text{column total})}{\text{grand total}}$$

grand total

$\chi^2_{\text{tab}} \text{ at } 1 \text{ D.F., S.Y.U.O.S}$

$$= 3.841$$

$$\chi^2_{\text{cal}} = 13.088$$



\therefore Employment
doesn't depend on sex.

Reject H_0

F-test

$$F = \frac{\sigma_1^2}{\sigma_2^2}$$

$$\gamma_1 \rightarrow n_1 - 1, \gamma_2 \rightarrow n_2 - 1 \quad (\underline{\gamma_1 > \gamma_2})$$

It is designed to test if two population variances are equal.

→ It is simply the ratio of two variances.

(larger one on numerator)

→ we can use F-test ...

i) To test the overall significance for a Regression model.

ii) To compare the R² of diff models.

iii) To test specific regression terms.

iv) To test the equality of diff means.

→ mainly used in ANOVA

→ we choose F-test when we only have f.d.

$$F = \frac{\sigma_1^2}{\sigma_2^2}$$

$$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$$

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$$

$$s_1^2 = \frac{(\bar{x}_1)^2}{n} - \left(\frac{\bar{x}_1}{n} \right)^2$$

$$s_2^2 = \frac{(\bar{x}_2)^2}{n} - \left(\frac{\bar{x}_2}{n} \right)^2$$

$$s_1 > s_2$$

must

ANOVA (Analysis of variance)

- It is basically you're testing groups to see if there's a diff b/w them.
- observing the dependency of groups based on one/more variable.

Eg1

- 1) students from diff. college take the same exam, you want to see if one college outperforms the others.
 - 2) A manufacturer has two diff. processes to make light-bulbs. They want to know if one is better than the other.
 - 3) A group of psychiatric patients are trying 3 diff. therapies: counseling, medication, & bio feedback. You want to see if one better than the other.
- It uses F-test.
- If they depend on one variable, one-way test
two-variables & two-way test
many-variables & MANOVA.

Q.) In a survey conducted to test the knowledge of mathematics among 4 diff schools in city.

School	maths marks						
School ①	8	6	7	5	9		
School ②	6	4	6	5	6	7	
School ③	6	5	5	6	7	8	5
School ④	5	6	6	7	6	7	

Sol:

there is a procedure for ANOVA.

Here it is one way ANOVA as it only depends on maths.

H_0 : mean of all schools is same.

H_1 : not same.

Procedure:

School (1)	S_2	S_3	S_4	$(S_i - \bar{S}_1(\text{mean}))^2$	$(S_2 - \bar{S}_2)^2$	$(S_3 - \bar{S}_3)^2$	$(S_4 - \bar{S}_4)^2$
8	6	6	5	1	0.0111	0	1.36
6	4	5	6	1	2.777	1	0.02
7	6	5	6	0	0.0111	1	0.02
5	5	6	7	4	0.944	0	0.69
9	6	7	6	4	0.441	1	0.02
	7	8	7	4	1.777	4	0.69
		5				4	
Total	35	34	42	37	10	5.333	8
Mean	7	5.67	6	6.167			

$$\boxed{\text{Grand mean} = 6.208}$$

① Sum Squared between ($SS_{B\text{tw}}$) = $\sum n_i (\text{data}_i - \text{mean} - \text{Grand mean})^2$

$$SS_{B\text{tw}} = 5(7 - 6.208)^2 + 6(5.67 - 6.208)^2 + 7(6 - 6.208)^2$$

$$SS_{B\text{tw}} = 4.99$$

$$6(6.16 - 6.208)^2$$

② Mean sum squared $B\text{tw} = \frac{SS_{B\text{tw}}}{(\text{no. of groups} - 1)} = \frac{4.99}{4 - 1} = 1.666$

(ii) Sum of Squares within = $E(S_i - \bar{S}_i)^2$

$$= 10 + 5 \cdot 33 + 8 + 2 \cdot 83 \\ = 26 \cdot 16$$

(iii) mean sum of square within = $\frac{\text{SS within}}{(\text{no. of observations} - \text{no. of groups})}$

$$= \frac{26 \cdot 16}{24 - 4} = \frac{26 \cdot 16}{20} = 1.308.$$

And by F-test,

$$F_{\text{cal}} = \frac{\text{MSST}}{\text{MSE}}$$

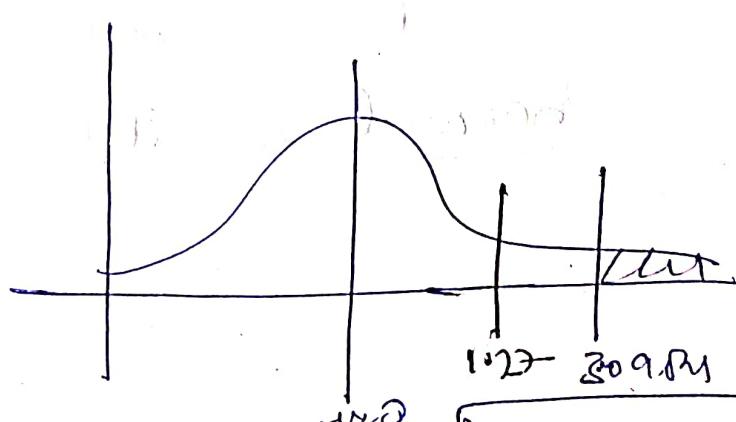
$$F_{\text{cal}} = \frac{1.666}{1.308}$$

$$F_{\text{cal}} = 1.27$$

$F_{\text{tab}} @ 8, f_1, f_2$

at 0.05

$$F_{\text{tab}} = 3.09 \text{ R.H.}$$



$P_{\text{accept}} \rightarrow H_0$

* Central Limit Theorem (CLT)

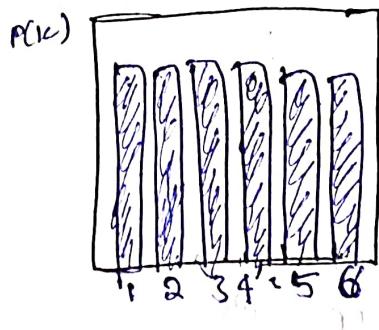
- The CLT states that the sampling distribution of the sample "means" approaches a normal distribution as the sample size gets larger.
- * no matter what the shape of popl distribution.

[Sample Size]

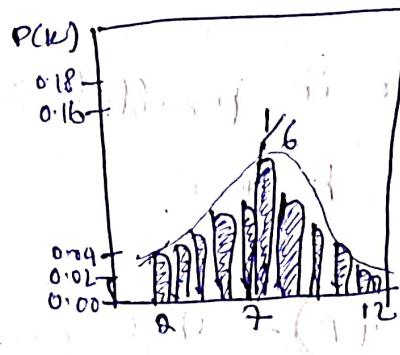
In other words,

- You take more samples especially larger ones. Your graph of the "sample means" will look more like a normal distribution.
- Here's what the CLT is saying, graphically. Rolling a fair die. The more times you roll the die, the more likely the shape of the distribution of the means tends to look like a normal distribution graph!

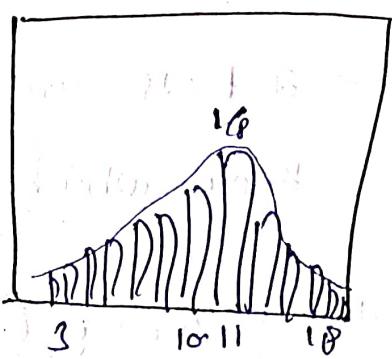
n=1



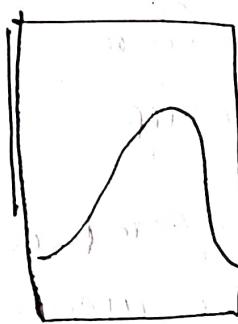
n=2



n=3



n=4



for n=2

possibilities

1	1	1	2	1	1.5	3	1	0	4	1	2.5	5	1	3	6	1	3.5
1	2	1.5	2	2	2	3	2	2.5	4	2	3	5	2	3.5	6	2	4
1	3	2	2	3	2.5	3	3	3	4	3	3.5	6	3	4	6	3	4.5
1	4	2.5	2	4	3	3	3	3	4	3	3.5	5	3	4	6	4	5
1	5	3	2	5	3.5	3	4	3.5	4	4	4	5	4	5	6	3	4.5
1	6	3.5	2	6	4	3.5	4	4	4	4.5	5	5	5	5	6	4	5
						3.5	4	4.5	4.5	5	5.5	6	5	5.5	6	5.5	6

"sample mean"

note

gives normal distribution

a) An essential component of CLT is that

Avg. of your sample means will be the actual population mean.

Similarly for SD.

P-value

→ P-value is the smallest level of significance at which a null hypothesis can be rejected. That's why many tests now a days give P-value & it is more preferred since it gives out more information than the critical value.

For right-tailed test

$$P\text{-value} = P \left[\begin{array}{l} \text{test statistic} \geq \\ \text{observed value} \\ \text{or test statistic} \end{array} \right]$$

For left-tailed test

$$P\text{-value} = P \left[\begin{array}{l} \text{test statistic} \leq \\ \text{observed value} \\ \text{or test statistic} \end{array} \right]$$

For two-tailed test

$$P\text{-value} = 2 * P \left[\begin{array}{l} \text{test statistic} \geq \\ \text{observed value} \\ \text{or test statistic} \end{array} \right]$$

Decision making with p-value

→ The p-value is compared to the significance level (α) for decision making on H_0 .

(1) If $p\text{-value} > \alpha$, Accept H_0

(2) If $p\text{-value} < \alpha$, Reject H_0

Note:

→ P-value is the actual area.

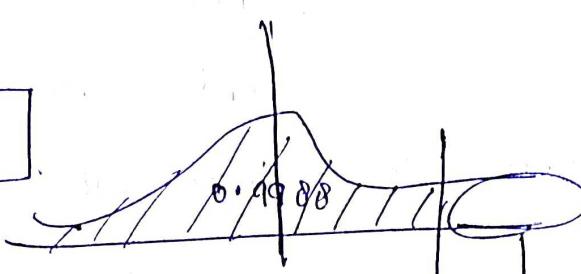
Find p-value

i) Find z-standard value

Eg 1

$$z = -2.97$$

$$P_{z \geq -2.97} = 0.9988$$



$$1 - 0.9988 = 0.0014.$$

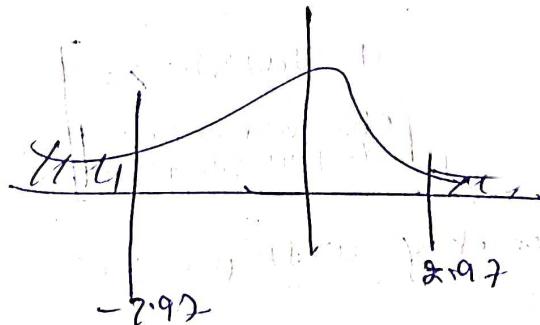
$$\text{ii) } P\text{-value} = 0.14\%$$

But we want
this value.

iii) If two-tail

$$P\text{-value} = 2 \times 0.0014$$

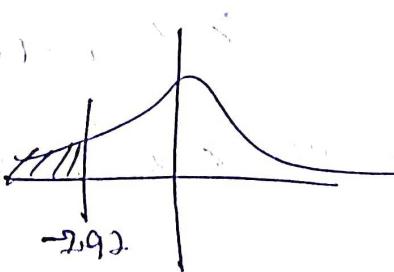
$$P\text{-value} = 0.28\%$$



iv) If $z = -2.97$

$$P_{z \leq -2.97} = 0.00014$$

$$\text{P-value} = 0.14\%$$



For left tail,
no need of subtraces

Notes

P-value

The probability that the test statistic is that the value or more extreme than does the alternate hypothesis.

④ Area of critical region
that is

Test statistic

A measurement of how far a sample statistic is from the assumed parameter if H_0 is true.

A/B Testing

→ 2 tests are run in parallel to know which is better.

- 1) treatment Group (Group A) - This group is exposed to new web page
 - 2) control Group (Group B) - No change from current setup.
- the goal of the A/B test is then to compare the 'conversion rate' of two groups using statistical inference.