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Exercise 3.1.2

Autocorrelation for a MA(q) process with lag k:

$$\rho_k = \frac{-\theta_k + \sum_{j=1}^{q-k} \theta_j \theta_{j+k}}{1 + \sum_{j=1}^{q} \theta_j^2}, \quad k = 1, 2, \cdots, q.$$

$$= 0, \quad k > q$$

MA(2) process:

$$x(t) = a(t) + 0.2\, a(t-1) - 0.48\, a(t-2)$$

$$\theta_1 = -0.2$$
$$\theta_2 = +0.48$$

$q=2$ & $k=0$ $\quad \rho_0 = \frac{-\theta_0 + \sum_{j=1}^{2} \theta_j \theta_j}{1 + \sum_{j=1}^{2} \theta_j^2} = \boxed{1}$

$q=2$ & $k=1$ $\quad \rho_1 = \frac{-\theta_1 + \sum_{j=1}^{1} \theta_j \theta_{j+1}}{1 + \sum_{j=1}^{2} \theta_j^2} = \frac{0.2 + [(-0.2)(0.48)]}{1 + (-0.2)^2 + (0.48)^2}$

$$= \frac{.104}{1.2704} = \boxed{.08186}$$

$q=2$ & $k=2$ $\quad \rho_2 = \frac{-\theta_2 + \sum_{j=1}^{0} \theta_j \theta_{j+2}}{1 + \sum_{j=1}^{2} \theta_j^2} = \frac{-0.48 + 0}{1 + (-0.2)^2 + (0.48)^2}$

$$= \frac{-0.48}{1.2704} = \boxed{-.3778}$$

$q=2$ & $k=3$ $\quad \boxed{\rho_3 = 0}$ because $k > q$