Poly-phase circuits

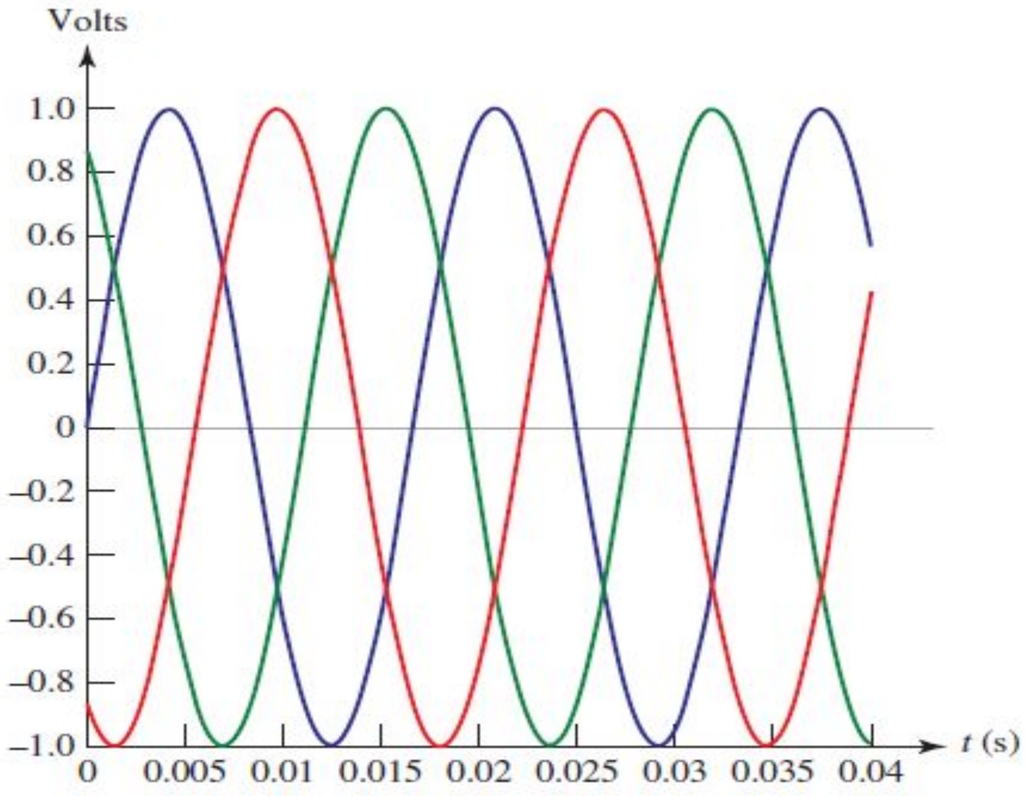
Advantages of 3-phase system

- Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less) and in turn reduces construction and maintenance costs.
- The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.
- Three-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.
- In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry.

- The number of **phase voltages** that can be produced by a polyphaser generator is not limited to three. Any number of phases can be obtained by spacing the windings for each phase at the proper angular position around the stator.

Balanced 3-phase system

- The source has three terminals (not counting a ***neutral*** or ***ground*** connection), and voltmeter measurements will show that sinusoidal voltages of equal amplitude are present between any two terminals.
- However, these voltages are not in phase; each of the three voltages is 120° out of phase with each of the other two, the sign of the phase angle depending on the sense of the voltages

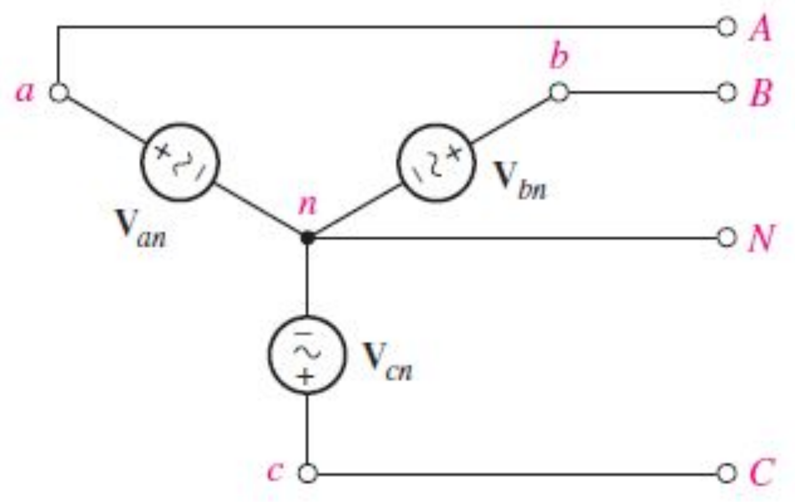


Connections

- In a 3-phase system, two types of connections are possible:
  - Star Connection
  - Delta Connection

3-Phase Star (Y)- Connection

- Three-phase sources have three terminals, called the *line* terminals, and they may or may not have a fourth terminal, the *neutral* connection.
- We will consider only balanced three-phase sources, which may be defined as having



$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

- These three voltages, each existing between one line and the neutral, are called ***phase voltages.*** If we arbitrarily choose **V***an* as the reference, or define

$$\mathbf{V}_{an} = V_p \underline{/0^\circ}$$

- where we will consistently use *Vp* to represent the rms *amplitude* of any of the phase voltages, then the definition of the three-phase source is indicated as

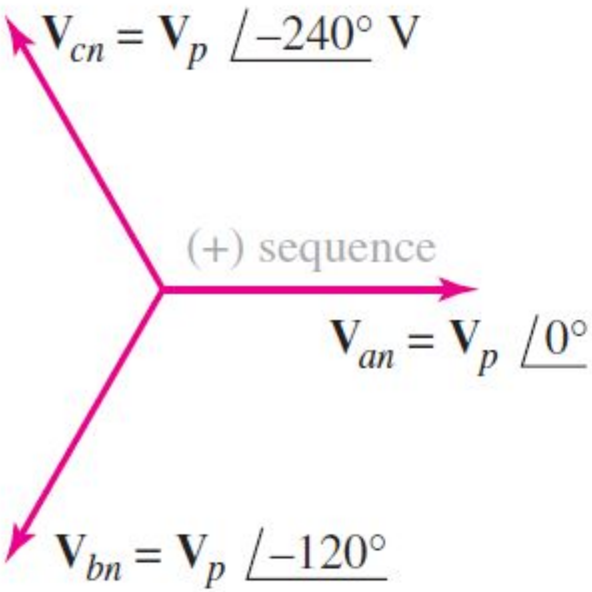
$$\mathbf{V}_{bn} = V_p \underline{/-120^\circ} \qquad \text{and} \qquad \mathbf{V}_{cn} = V_p \underline{/-240^\circ}$$

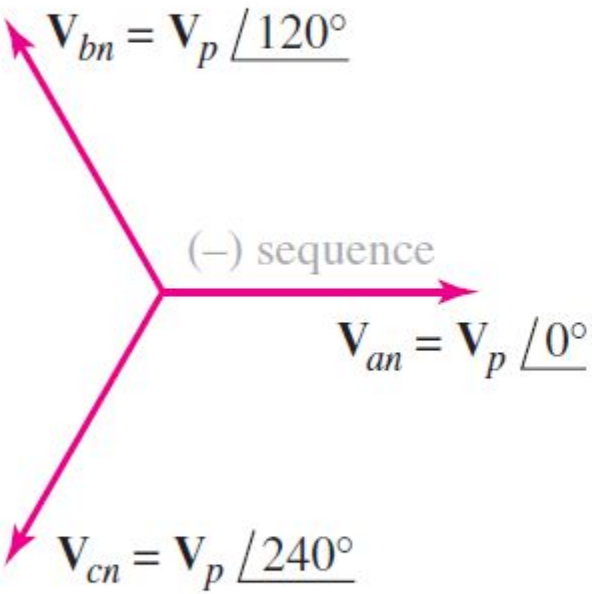
- This sequence is called as **Positive Phase sequence.**

- If the sequence is considered as

$$\mathbf{V}_{an} = V_p \angle 0^\circ \qquad \mathbf{V}_{bn} = V_p \angle 120^\circ \qquad \text{and} \qquad \mathbf{V}_{cn} = V_p \angle 240^\circ$$

- This considered as **Negative phase sequence.**





Line-to-Line Voltages

- the line-to-line voltages (often simply called the ***line voltages***) is the voltage between two line terminals.
- From the Phasor diagram these can be calculated as:

$$\mathbf{V}_{ab} = \sqrt{3}V_p\underline{/30^\circ}$$
$$\mathbf{V}_{bc} = \sqrt{3}V_p\underline{/-90^\circ}$$
$$\mathbf{V}_{ca} = \sqrt{3}V_p\underline{/-210^\circ}$$

- sum of these three line voltages is also zero, in case of the balanced voltages

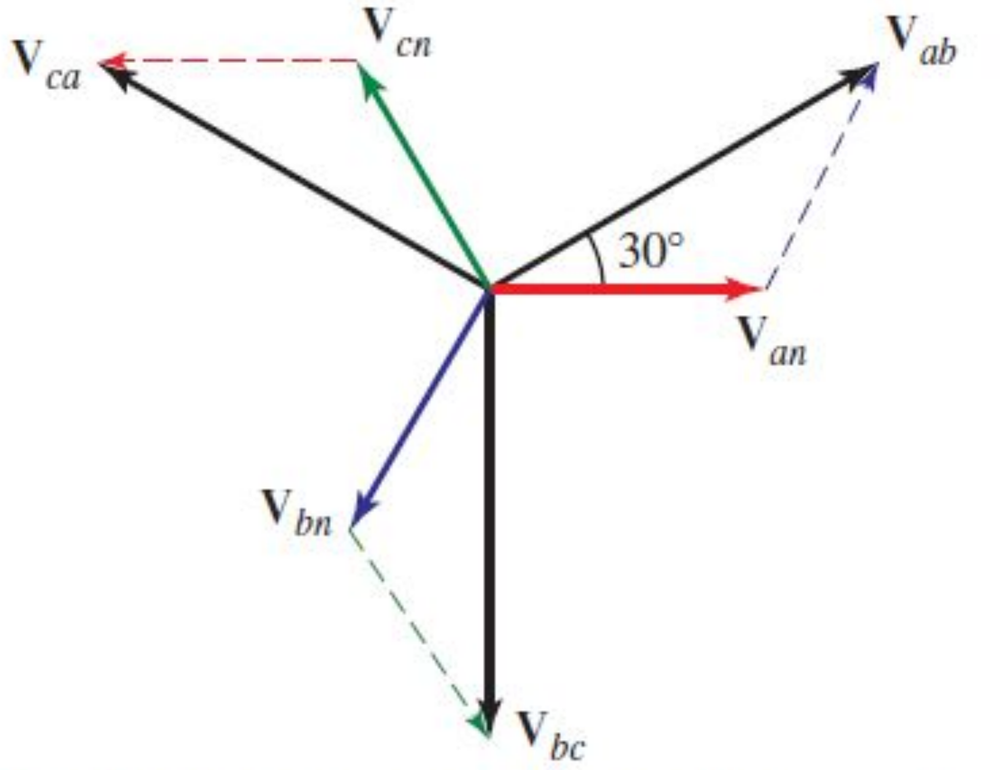


FIGURE 12.13 A phasor diagram which is used to determine the line voltages from the given phase voltages. Or, algebraically, $\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\underline{/0^\circ} - V_p\underline{/-120^\circ} = V_p - V_p\cos(-120^\circ) - jV_p\sin(-120^\circ) = V_p(1 + \frac{1}{2} + j\sqrt{3}/2) = \sqrt{3}V_p\underline{/30^\circ}$.

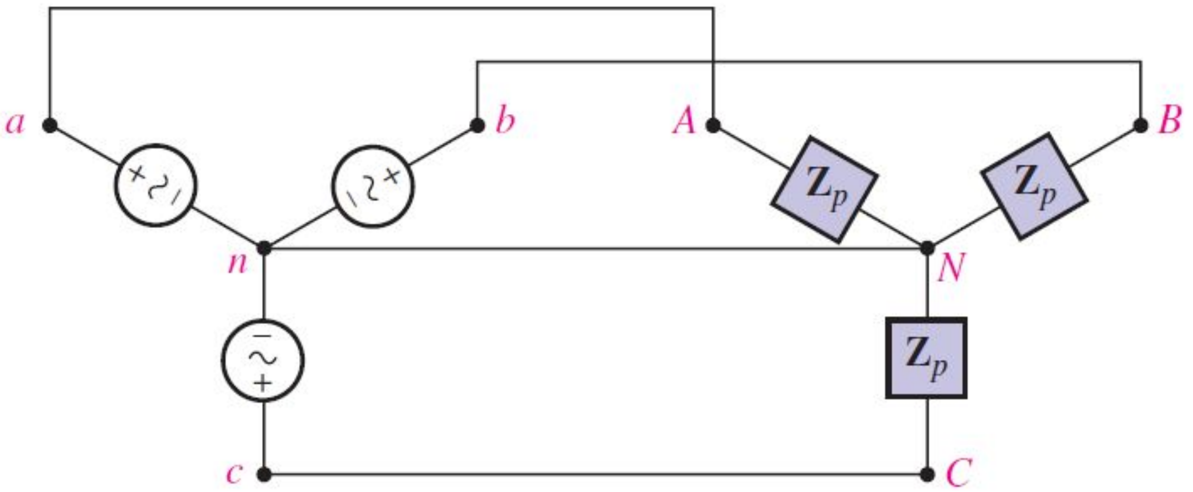
- If the ***rms*** amplitude of any of the line voltages is denoted by V_L, then one of the important characteristics of the ***Y-connected three-phase*** source may be expressed as

$$\boxed{V_L = \sqrt{3}V_p}$$

- Note that with ***positive phase sequence***, **V***an* leads **V***bn* and **V***bn* leads **V***cn*, in each case by 120◦, and also that **V***ab* leads **V***bc* and **V***bc* leads **V***ca* , again by 120◦.
- The statement is true for ***negative phase sequence*** if "lags'' is substituted for "leads.''

Three phase Y-Y Connection

- The load is represented by an impedance **Z***p* between each line and the neutral. The three line currents are calculated as:



$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p}$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_p} = \frac{\mathbf{V}_{an}\underline{/-120^\circ}}{\mathbf{Z}_p} = \mathbf{I}_{aA}\underline{/-120^\circ}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\underline{/-240^\circ}$$

and therefore

$$\mathbf{I}_{Nn} = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} = 0$$

Thus, the neutral carries no current if the source and load are both balanced.

For the circuit of Fig. 12.15, find both the phase and line currents, and the phase and line voltages throughout the circuit; then calculate the total power dissipated in the load.

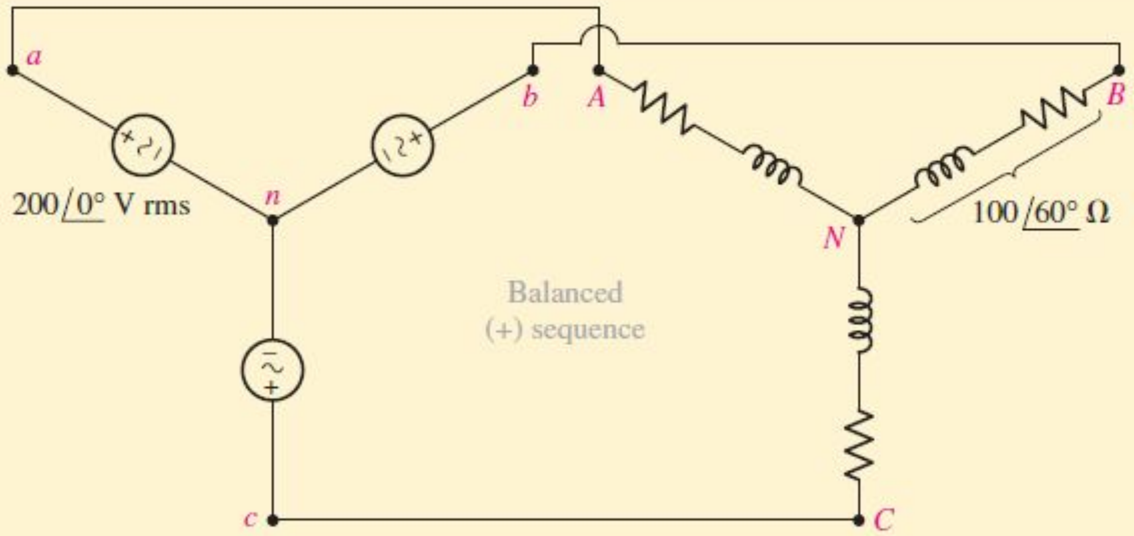
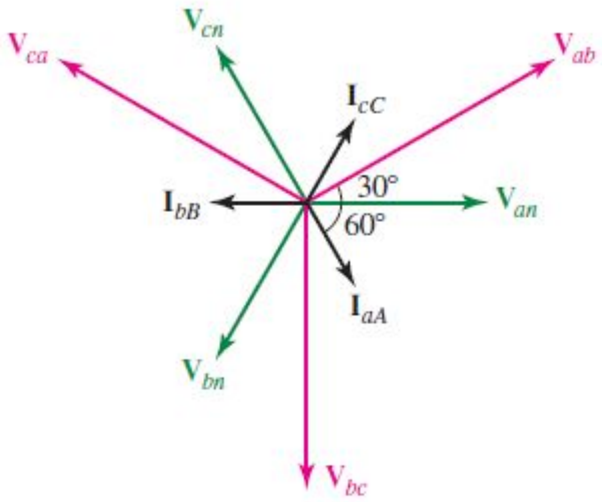


FIGURE 12.15 A balanced three-phase three-wire Y-Y connected system.



Since one of the source phase voltages is given and we are told to use the positive phase sequence, the three phase voltages are:

$$\mathbf{V}_{an} = 200\underline{/0^\circ}\text{ V} \qquad \mathbf{V}_{bn} = 200\underline{/-120^\circ}\text{ V} \qquad \mathbf{V}_{cn} = 200\underline{/-240^\circ}\text{ V}$$

The line voltage is $200\sqrt{3} = 346$ V; the phase angle of each line voltage can be determined by constructing a phasor diagram, as we did in Fig. 12.13 (as a matter of fact, the phasor diagram of Fig. 12.13 is applicable), subtracting the phase voltages using a scientific calculator, or by invoking Eqs. [1] to [3]. We find that $\mathbf{V}_{ab}$ is $346\underline{/30^\circ}$ V, $\mathbf{V}_{bc} = 346\underline{/-90^\circ}$ V, and $\mathbf{V}_{ca} = 346\underline{/-210^\circ}$ V.

The line current for phase A is

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p} = \frac{200\underline{/0^\circ}}{100\underline{/60^\circ}} = 2\underline{/-60^\circ}\text{ A}$$

Since we know this is a balanced three-phase system, we may write the remaining line currents based on $\mathbf{I}_{aA}$:

$$\mathbf{I}_{bB} = 2\underline{/(-60^\circ - 120^\circ)} = 2\underline{/-180^\circ}\text{ A}$$
$$\mathbf{I}_{cC} = 2\underline{/(-60^\circ - 240^\circ)} = 2\underline{/-300^\circ}\text{ A}$$

Finally, the average power absorbed by phase A is $\text{Re}\{\mathbf{V}_{an}\mathbf{I}^*_{aA}\}$, or

$$P_{AN} = 200(2)\cos(0^\circ + 60^\circ) = 200\text{ W}$$

Thus, the total average power drawn by the three-phase load is 600 W.

A balanced three-phase system with a line voltage of 300 V is supplying a balanced Y-connected load with 1200 W at a leading PF of 0.8. Find the line current and the per-phase load impedance.

The phase voltage is $300/\sqrt{3}$ V and the per-phase power is $1200/3 = 400$ W. Thus the line current may be found from the power relationship

$$400 = \frac{300}{\sqrt{3}}(I_L)(0.8)$$

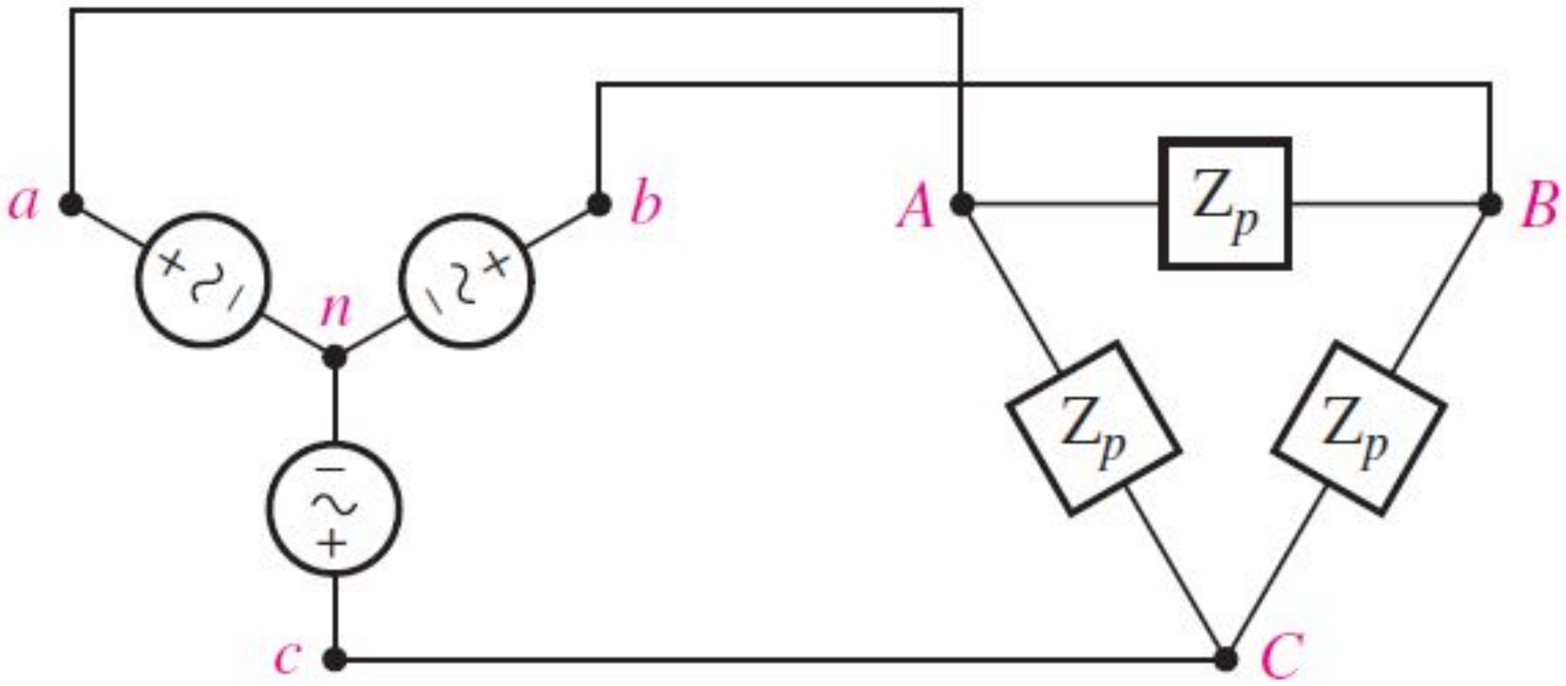
and the line current is therefore 2.89 A. The phase impedance magnitude is given by

$$|\mathbf{Z}_p| = \frac{V_p}{I_L} = \frac{300/\sqrt{3}}{2.89} = 60\ \Omega$$

Since the PF is 0.8 leading, the impedance phase angle is $-36.9°$; thus $\mathbf{Z}_p = 60\underline{/-36.9°}\ \Omega$.

THE DELTA (Δ) CONNECTION

- This type of configuration is very common, and ***does not possess a neutral connection***



Let us consider a balanced Δ-connected load which consists of an impedance $\mathbf{Z}_p$ inserted between each pair of lines. With reference to Fig. 12.18, let us assume known line voltages

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

or known phase voltages

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

where

$$V_L = \sqrt{3}V_p \qquad \text{and} \qquad \mathbf{V}_{ab} = \sqrt{3}V_p\underline{/30^\circ}$$

as we found previously. Because the voltage across each branch of the Δ is known, the *phase currents* are easily found:

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_p} \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_p} \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_p}$$

and their differences provide us with the line currents, such as

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

Since we are working with a balanced system, the three phase currents are of equal amplitude:

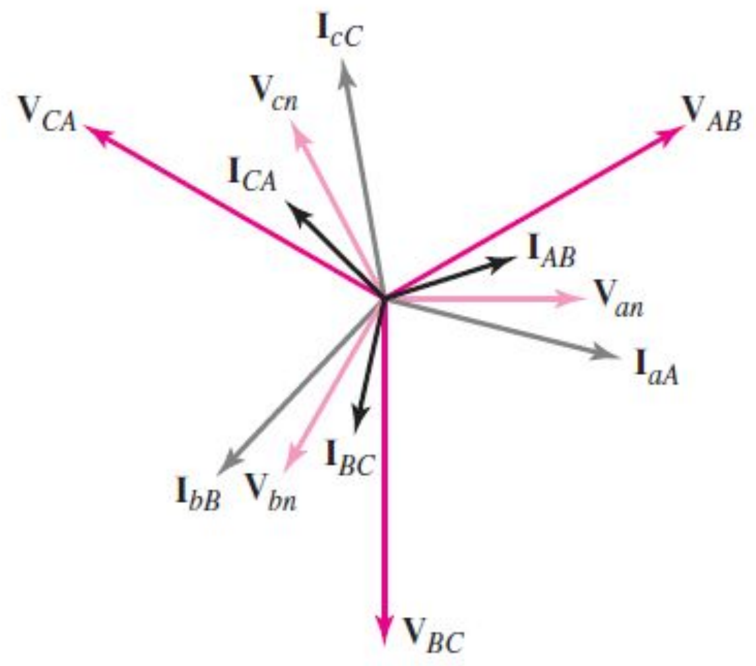
$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

The line currents are also equal in amplitude; the symmetry is apparent from the phasor diagram of Fig. 12.19. We thus have

$$I_L = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|$$

and

$$I_L = \sqrt{3}I_p$$



If the load is Delta-connected, then the phase voltage and the line voltage are Same, but the line current is larger than the phase current by a factor of √3; with a Y-connected load, however, the phase current and the line current refer to the same current, and the line voltage is greater than the phase voltage by a factor of √3.

Determine the amplitude of the line current in a three-phase system with a line voltage of 300 V that supplies 1200 W to a Δ-connected load at a lagging PF of 0.8; then find the phase impedance.

Let us again consider a single phase. It draws 400 W, 0.8 lagging PF, at a 300 V line voltage. Thus,

$$400 = 300(I_p)(0.8)$$

and

$$I_p = 1.667 \text{ A}$$

and the relationship between phase currents and line currents yields

$$I_L = \sqrt{3}(1.667) = 2.89 \text{ A}$$

Next, the phase angle of the load is $\cos^{-1}(0.8) = 36.9^\circ$, and therefore the impedance in each phase must be

$$\mathbf{Z}_p = \frac{300}{1.667}\underline{/36.9^\circ} = 180\underline{/36.9^\circ}\ \Omega$$

Comparison of Y- and Δ-Connected Three-Phase Loads. V_p Is the Voltage Magnitude of Each Y-Connected *Source* Phase

Load	Phase Voltage	Line Voltage	Phase Current	Line Current	Power per Phase
Y	$\mathbf{V}_{AN} = V_p\angle 0^\circ$ $\mathbf{V}_{BN} = V_p\angle -120^\circ$ $\mathbf{V}_{CN} = V_p\angle -240^\circ$	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{AN} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{BN} = \sqrt{3}V_p\angle -90^\circ$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = (\sqrt{3}\angle 30^\circ)\mathbf{V}_{CN} = \sqrt{3}V_p\angle -210^\circ$	$\mathbf{I}_{aA} = \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = \mathbf{I}_{BN} = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = \mathbf{I}_{CN} = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_p}$	$\mathbf{I}_{aA} = \mathbf{I}_{AN} = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = \mathbf{I}_{BN} = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = \mathbf{I}_{CN} = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_p}$	$\sqrt{3}V_L I_L \cos\theta$ where $\cos\theta =$ power factor of the load
Δ	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = \sqrt{3}V_p\angle -210^\circ$	$\mathbf{V}_{AB} = \mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{BC} = \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ$ $\mathbf{V}_{CA} = \mathbf{V}_{ca} = \sqrt{3}V_p\angle -210^\circ$	$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_p}$ $\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_p}$ $\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_p}$	$\mathbf{I}_{aA} = (\sqrt{3}\angle -30^\circ)\frac{\mathbf{V}_{AB}}{\mathbf{Z}_p}$ $\mathbf{I}_{bB} = (\sqrt{3}\angle -30^\circ)\frac{\mathbf{V}_{BC}}{\mathbf{Z}_p}$ $\mathbf{I}_{cC} = (\sqrt{3}\angle -30^\circ)\frac{\mathbf{V}_{CA}}{\mathbf{Z}_p}$	$\sqrt{3}V_L I_L \cos\theta$ where $\cos\theta =$ power factor of the load

Watt-Meter

- In electrical systems, the power is measured by using a device called Watt-meter
- a wattmeter that contains two separate coils. One of these coils is made of heavy wire, having a **very low resistance**, and is called the ***current coil***
- the second coil is composed of a much greater number of turns of fine wire, with relatively **high resistance**, and is termed the ***potential coil,*** or ***voltage coil***.
- The torque applied to the moving system and the pointer is proportional to the instantaneous product of the currents flowing in the two coils.
- The mechanical inertia of the moving system, however, causes a deflection that is proportional to the ***average*** **value** of this torque.

- The wattmeter is used by connecting it into a network in such a way that the current flowing in the current coil is the current flowing into the network and the voltage across the potential coil is the voltage across the two terminals of the network.
- The current in the potential coil is thus the input voltage divided by the resistance of the potential coil.