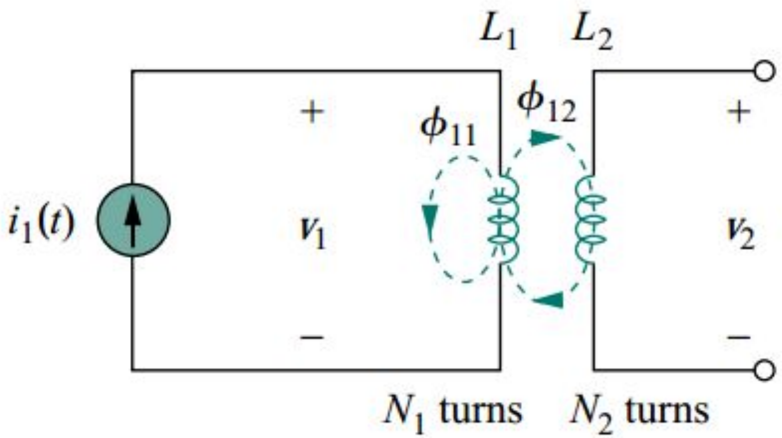
MUTUAL INDUCTANCE

Mutual inductance is the ability of one inductor to induced voltage across a neighboring inductor, measured in henrys (H).

- Now consider two coils with self-inductances L1 and L2 that are in close proximity with each other.



- For the sake of simplicity, assume that the second inductor carries no current.
- The magnetic flux $\varphi 1$ emanating from coil 1 has two components:

$$\phi_1 = \phi_{11} + \phi_{12}$$

MUTUAL INDUCTANCE

- Although the two coils are physically separated, they are said to be *magnetically coupled.*

Since the entire flux φ_1 links coil 1, the voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

Only flux φ_{12} links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

Again, as the fluxes are caused by current i_1 flowing in coil 1

$$v_1 = N_1 \frac{d\phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

where $L_1 = N_1\, d\phi_1/di_1$ is the self-inductance of coil 1

Similarly,

$$v_2 = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

Where $\boxed{M_{21} = N_2 \frac{d\phi_{12}}{di_1}}$

MUTUAL INDUCTANCE

- M_{21} is known as the *mutual inductance* of coil 2 with respect to coil 1.
- Subscript 21 indicates that the inductance M_{21} relates the voltage induced in coil 2 to the current in coil 1.

$$M_{12} = M_{21} = M$$

we refer to M as the mutual inductance between the two coils

Keep in mind that mutual coupling only exists when the inductors or coils are in close proximity, and the circuits are driven by time-varying sources.

DOT CONVENTION

- Although **mutual inductance *M* is always a positive quantity**, **the mutual voltage *M di/dt* may be negative or positive**, just like the self induced voltage *L di/dt*.

- The polarity of mutual voltage *M di/dt* is not easy to determine, because **four terminals are involved**.

- It is made by examining the orientation or particular way in which both coils are physically wound and applying Lenz's law in conjunction with the right-hand rule.

- Since it is inconvenient to show the construction details of coils on a circuit schematic, we apply the *dot convention* in circuit analysis.

DOT CONVENTION

- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

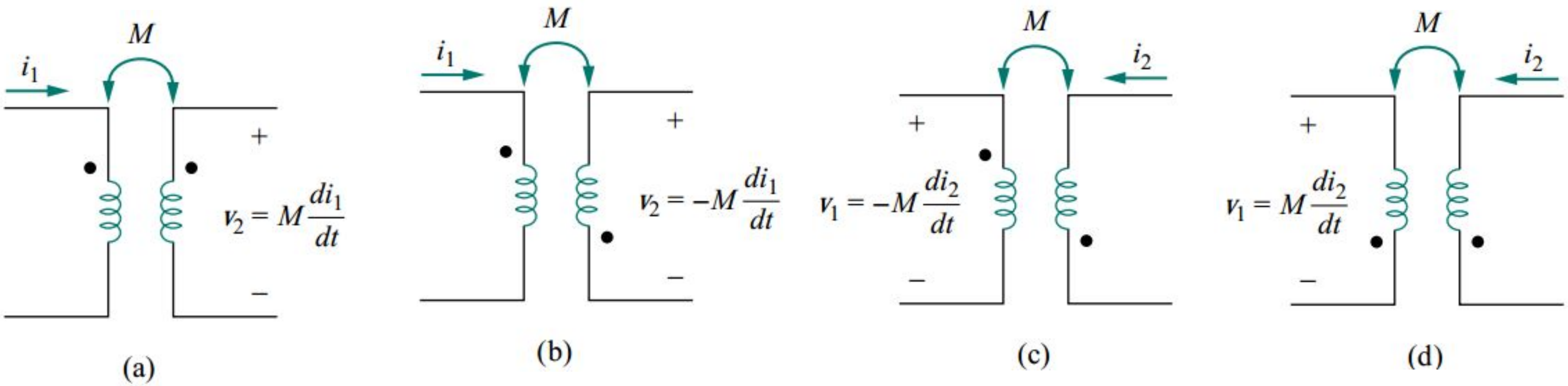
OR

- If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.

Thus, the reference polarity of the mutual voltage depends on the reference direction of the inducing current and the dots on the coupled coils.

DOT CONVENTION

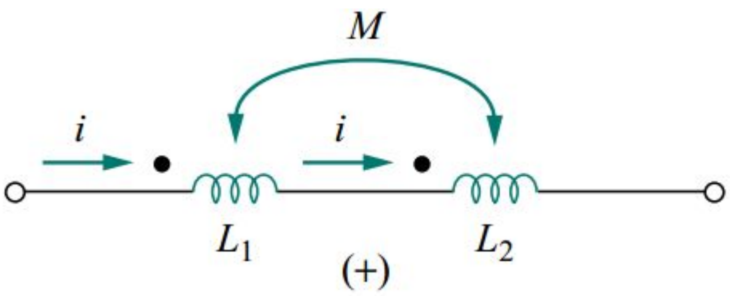
Application of the dot convention is illustrated in the four pairs of mutually coupled coils as follows.



	In Fig a	In Fig b	In Fig c	In Fig d
At dotted terminal of the coil 1	i_1 enters	i_1 enters	v_1 is positive	v_1 is negative
At dotted terminal of the coil 2	v_2 is positive	v_2 is negative	i_2 leaves	i_2 leaves
The mutual voltage	$+M di_1/dt$	$-M\, di_1/dt$.	$-M\, di_2/dt$.	$+M di_2/dt$

DOT CONVENTION

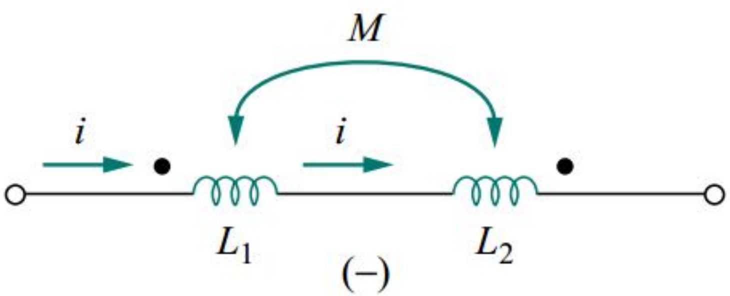
- The dot convention for coupled coils in series.



(a)

the total inductance is

$$\boxed{L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}}$$



(b)

$$\boxed{L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}}$$

Fig: the sign indicates the polarity of the mutual voltage: (a) series-aiding connection, (b) series-opposing connection.

DOT CONVENTION

EXAMPLE:(a)

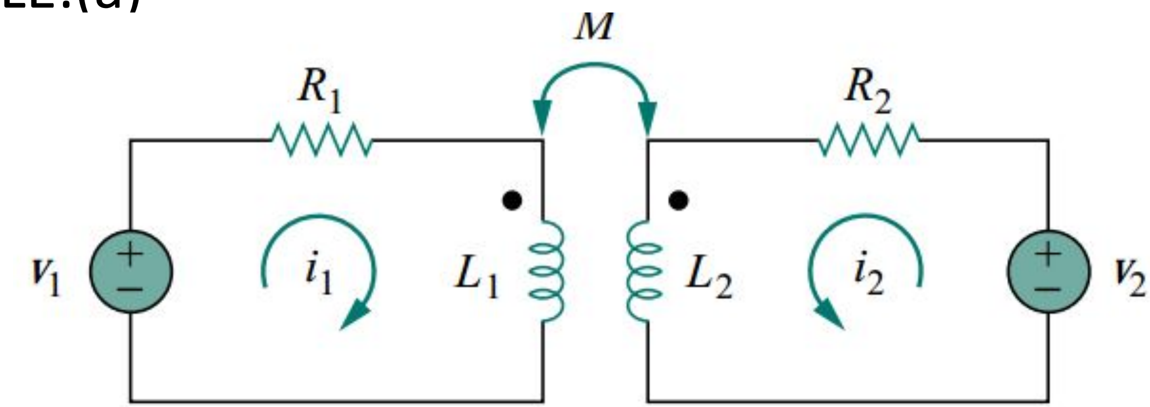


Fig: Time-domain analysis of a circuit containing coupled coils.

	Time-domain analysis	Frequency-domain analysis
For coil 1,KVL		
For coil 2, KVL		

DOT CONVENTION

EXAMPLE (b)

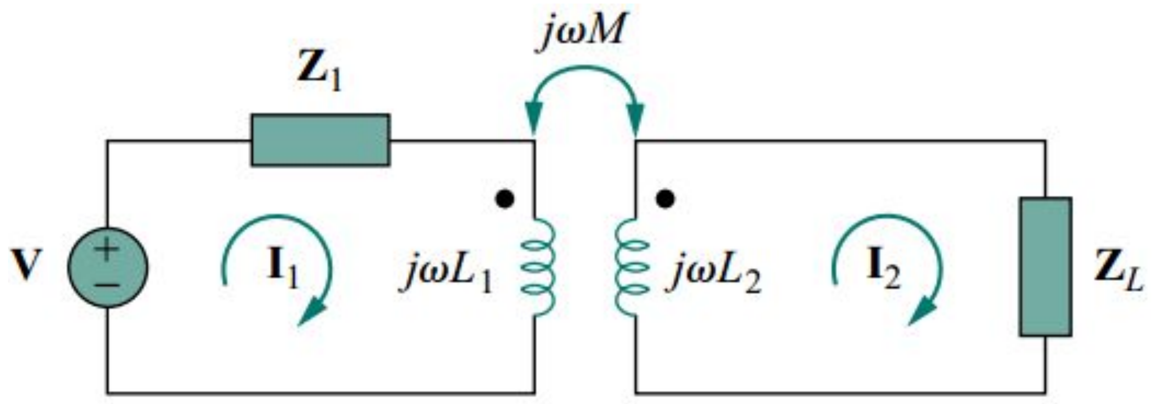


Fig: Frequency-domain analysis of a circuit containing coupled coils

	Time-domain analysis	Frequency-domain analysis
For coil 1,KVL		
For coil 2, KVL		

ENERGY IN A COUPLED CIRCUIT

The energy stored in an inductor is given by

$$w = \frac{1}{2}Li^2$$

- We assume that currents i_1 and i_2 are zero initially, so that the energy stored in the coils is zero.

CASE 1:

- If we let i_1 increase from zero to I_1 while maintaining $i_2 = 0$, the power in coil 1 is

$$p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt}$$

- The energy stored in the circuit is

$$\boxed{w_1 = \int p_1\, dt = L_1 \int_0^{I_1} i_1\, di_1 = \frac{1}{2} L_1 I_1^2}$$

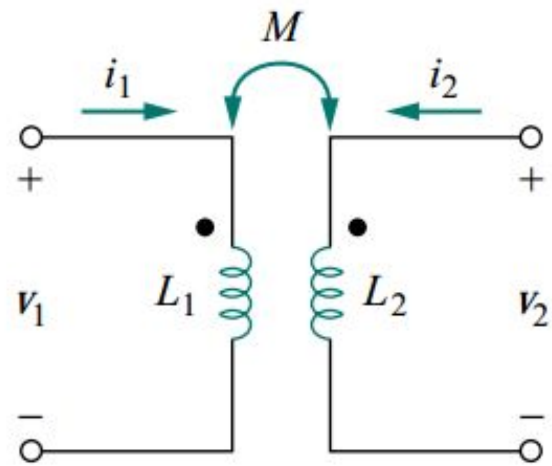


Fig: The circuit for deriving energy stored in a coupled circuit

ENERGY IN A COUPLED CIRCUIT

CASE 2:

- If we now maintain $i_1 = I_1$ and increase i_2 from zero to I_2, The mutual voltage induced in coil 1 is $M_{12} = di_2/dt$, while the mutual voltage induced in coil 2 is zero, since i_1 does not change(M di/dt =0).

- The power in the coils is now

$$p_2(t) = i_1 M_{12} \frac{di_2}{dt} + i_2 v_2 = I_1 M_{12} \frac{di_2}{dt} + i_2 L_2 \frac{di_2}{dt}$$

- The energy stored in the circuit is

$$w_2 = \int p_2\, dt = M_{12} I_1 \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2\, di_2 \quad \boxed{= M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2}$$

- The total energy stored in the coils when both i_1 and i_2 have reached constant values is

$$\boxed{w = w_1 + w_2 = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2}$$

ENERGY IN A COUPLED CIRCUIT

- If we reverse the order by which the currents reach their final values, then M_{12} become M_{21}, the total energy stored in the coils is

$$w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{21} I_1 I_2$$

- The total energy stored should be the same regardless of how we reach the final conditions. Therefore

$$\boxed{M_{12} = M_{21} = M}$$

$$\boxed{w = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2}$$

- This equation was derived based on the assumption that the coil currents both entered the dotted terminals.

ENERGY IN A COUPLED CIRCUIT

- If one current enters one dotted terminal while the other current leaves the other dotted terminal, the mutual voltage is negative, so that the mutual energy MI_1I_2 is also negative.

  In that case,

$$w = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 - MI_1I_2$$

- The general expression for the instantaneous energy stored in the circuit is

$$\boxed{w = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 \pm Mi_1i_2}$$

UPPER LIMIT FOR THE MUTUAL INDUCTANCE *M*

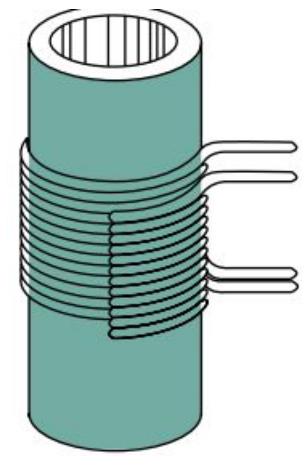
- The energy stored in the circuit cannot be negative because the circuit is passive.
- This means that the quantity $$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0$$
- To complete the square, we both add and subtract the term $i_1 i_2 \sqrt{L_1 L_2}$, and obtain

$$\frac{1}{2}(i_1\sqrt{L_1} - i_2\sqrt{L_2})^2 + i_1 i_2(\sqrt{L_1 L_2} - M) \geq 0$$

- The squared term is never negative; Therefore, the second term on the right-hand side of above Eq. must be greater than zero; that is, $$\boxed{M \leq \sqrt{L_1 L_2}}$$
- Thus, the mutual inductance cannot be greater than the geometric mean of the self-inductances of the coils.

COUPLING COEFFICIENT

- The degree to which *M* approaches its maximum value is described by the ***coupling coefficient.***

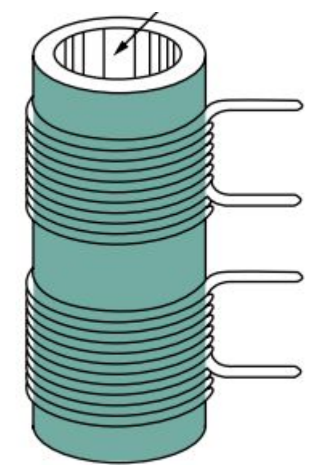


(a)Tightly coupled

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}} \quad \text{OR} \quad \boxed{M = k\sqrt{L_1 L_2}}$$

where $0 \le k \le 1$ or equivalently $0 \le M \le \sqrt{L_1 L_2}$.

- If k>0.5 then we can say the coils which are physically closer as shown in Fig(a) and if k<0.5 then the coils are loosely coupled as shown in Fig(b).



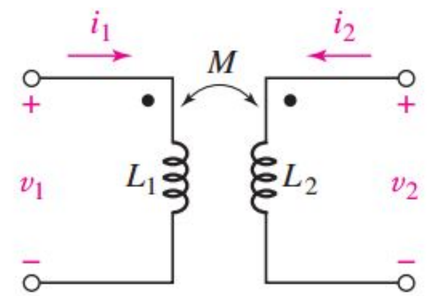
(b) Loosely coupled

- If the entire flux produced by one coil links another coil, then *k* = 1(100 percent coupling or *perfectly coupled)*
- So, the coupling coefficient is the fraction of the total flux emanating from one coil that links the other coil.

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{12}}{\phi_{11} + \phi_{12}} \quad \text{OR} \quad k = \frac{\phi_{21}}{\phi_2} = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

PROBLEM:

In Fig., let L_1 = 0.4 H, L_2 = 2.5 H, k = 0.6, and i_1 = $4i_2$ = 20 cos (500t − 20◦) mA. Determine both $v_1(0)$ and the total energy stored in the system at t = 0.



SOLUTION

To determine the value of v_1, apply KVL and also paying attention to the dot convention

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

To evaluate this quantity, we require a value for *M*

$$M = k\sqrt{L_1 L_2} = 0.6\sqrt{(0.4)(2.5)} = 0.6 \text{ H}$$

$$\text{Thus, } v_1(0) = 0.4[-10 \sin(-20°)] + 0.6[-2.5 \sin(-20°)] = 1.881 \text{ V}.$$

The total energy is found by summing the energy stored in each inductor, which has three separate components since the two coils are known to be magnetically coupled. Since both currents enter a "dotted'' terminal,

$$w(t) = \frac{1}{2}L_1[i_1(t)]^2 + \frac{1}{2}L_2[i_2(t)]^2 + M[i_1(t)]\,[i_2(t)]$$

Current through coil 1 at $t = 0$

$$i_1(0) = 20 \cos(-20^\circ) = 18.79 \text{ mA}$$

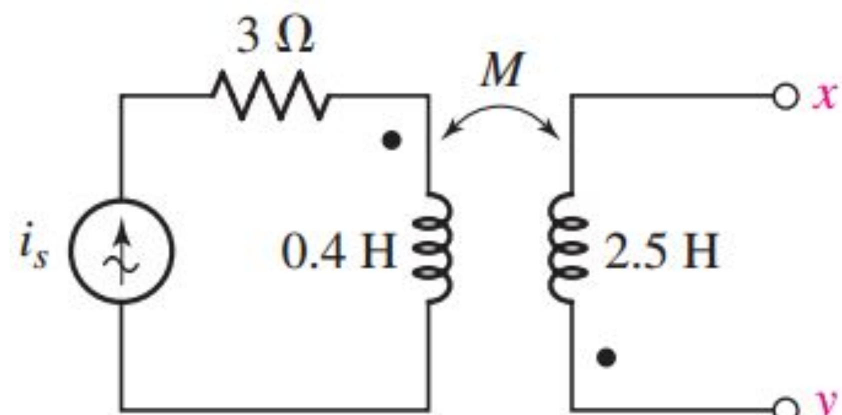
Current through coil 2 at $t = 0$

$$i_2(0) = 0.25 i_1(0) = 4.698 \text{ mA}$$

The total energy stored in the two coils at $t = 0$, is 151.2 μJ

PRACTICE PROBLEM1:

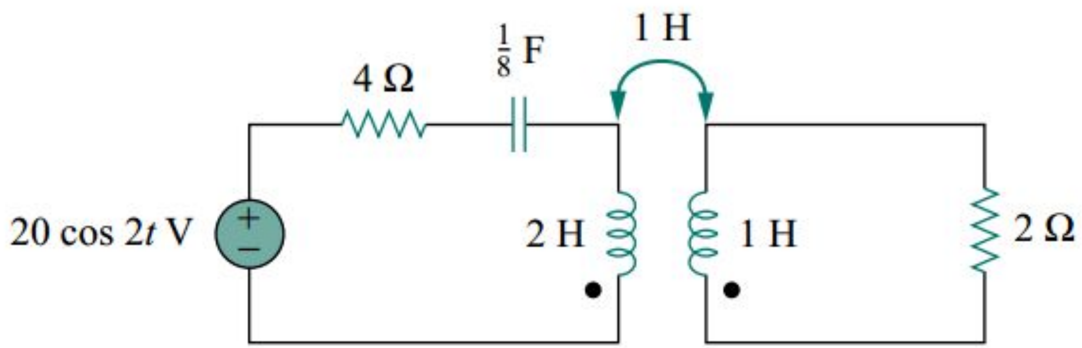
Let I_s = 2cos(10*t)* A in the circuit of Fig., and find the total energy stored in the passive network at t = 0 if k = 0.6 and terminals x and y are (*a*) left open-circuited; (*b*) short-circuited.



Ans: 0.8 J; 0.512 J.

PRACTICE PROBLEM2:

For the circuit in Fig., determine the coupling coefficient and the energy stored in the coupled inductors at t = 1.5 s.



Answer: 0.7071, 9.85 J.

PROBLEM:

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig.

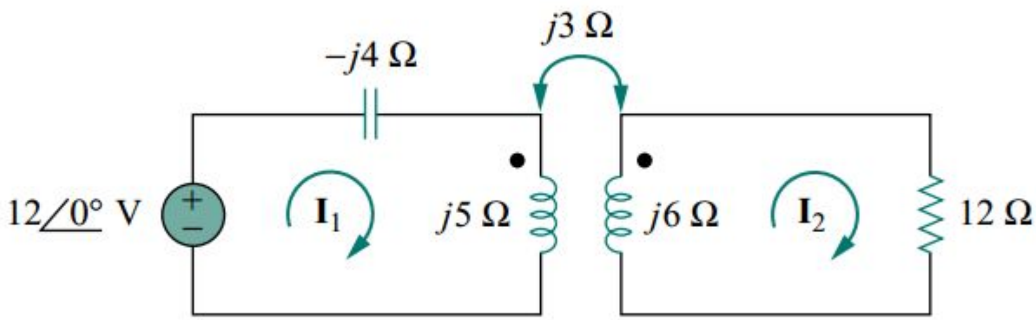


Fig.

SOLUTION

For coil 1

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

$$\boxed{j\mathbf{I}_1 - j3\mathbf{I}_2 = 12}$$

For coil 2

$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

$$\boxed{\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2}$$

By using Substitution method,

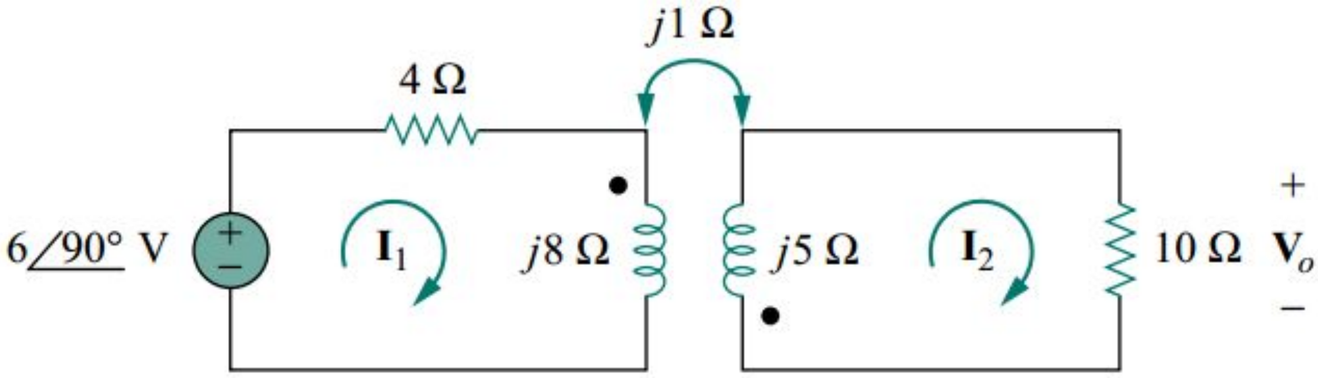
$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$$

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\underline{/14.04^\circ} \text{ A}$$

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\underline{/-63.43^\circ})(2.91\underline{/14.04^\circ})$$
$$= 13.01\underline{/-49.39^\circ} \text{ A}$$

PRACTICE PROBLEM.1:

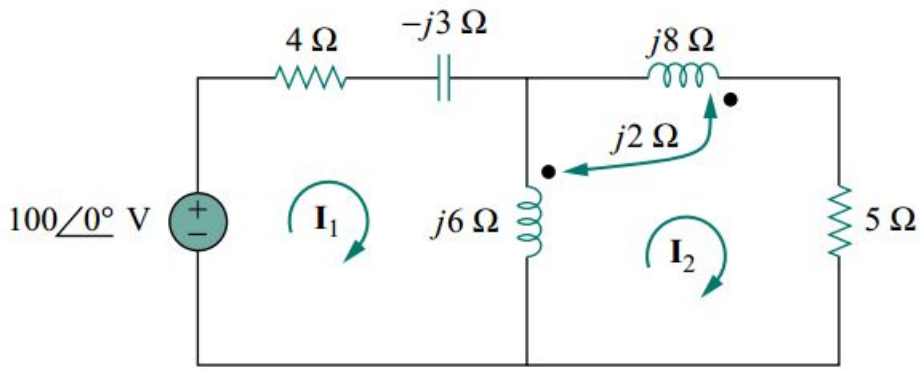
Determine the voltage **V***o* in the circuit of Fig



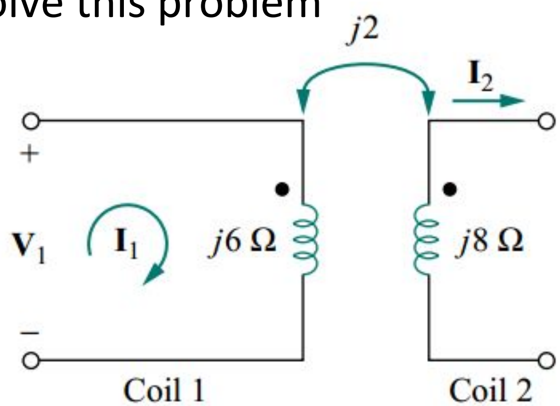
Fig

PRACTICE PROBLEM.2:

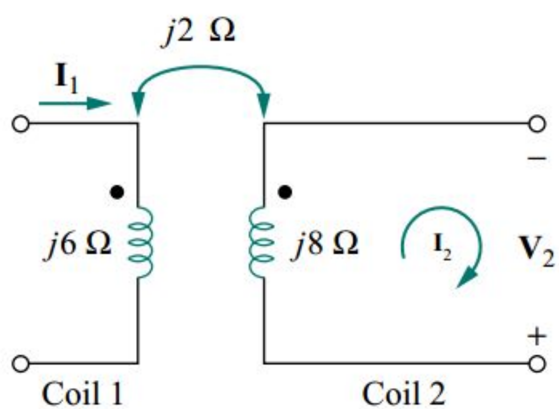
Calculate the mesh currents in the circuit of following fig.



Clue to solve this problem



(a) $\mathbf{V}_1 = -2j\mathbf{I}_2$



(b) $\mathbf{V}_2 = -2j\mathbf{I}_1$