Wye-Delta Transformations

- Situations often arise in circuit analysis when the ***resistors are neither in parallel nor in series*** as shown in Fig. here.

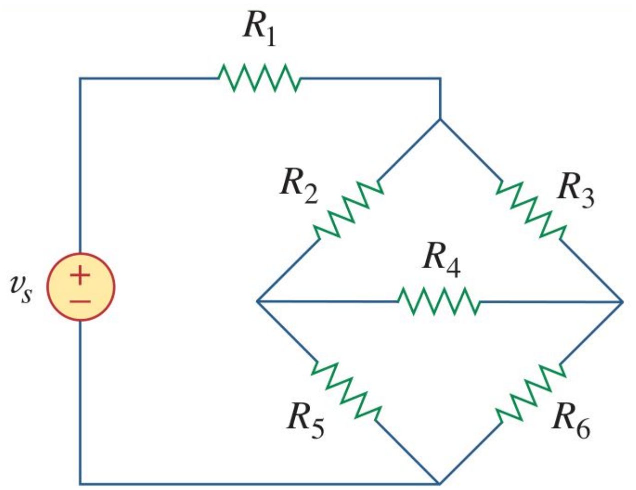


Fig. The bridge network.

Many circuits of this type can be simplified by using three-terminal equivalent networks known as the wye (Y) and the delta (Δ) networks.

Wye-Delta Transformations

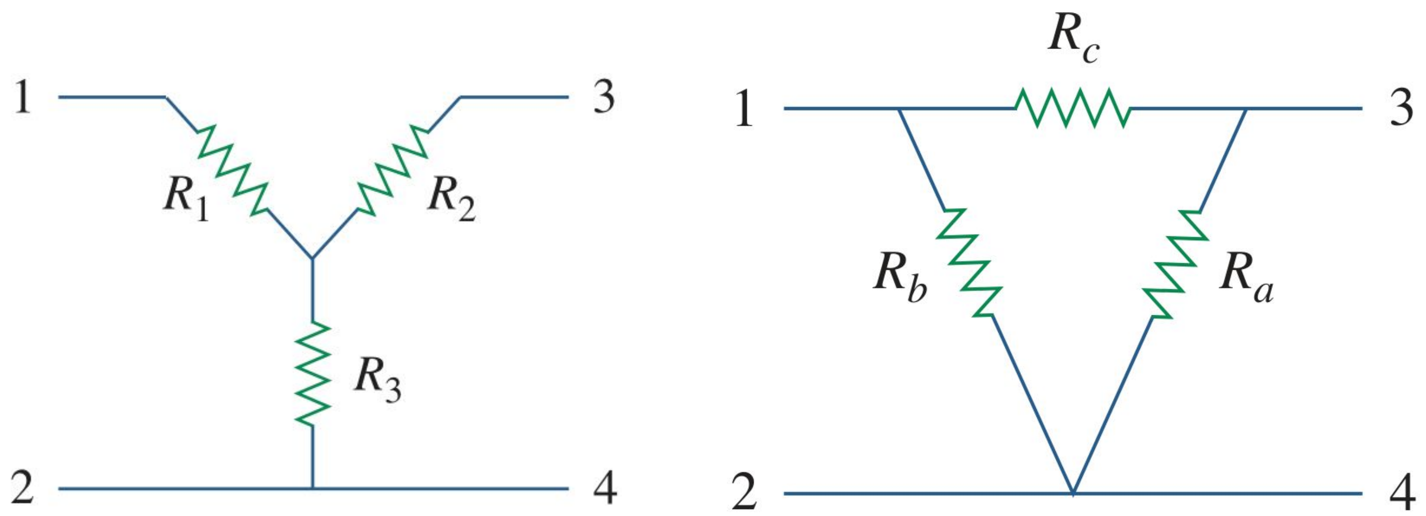


Figure: The wye and delta networks

Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. Then the existing delta network will be transformed into an wye network.

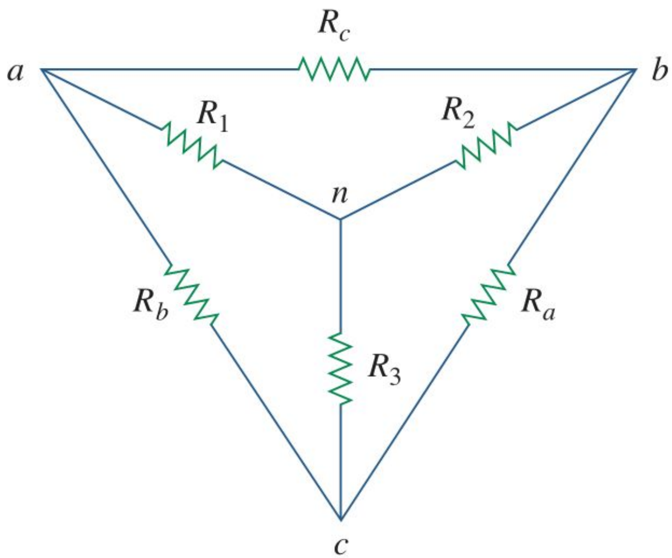


Fig: Superposition of Y and Δ networks

The equivalent resistances in the wye network are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Wye to Delta Conversion

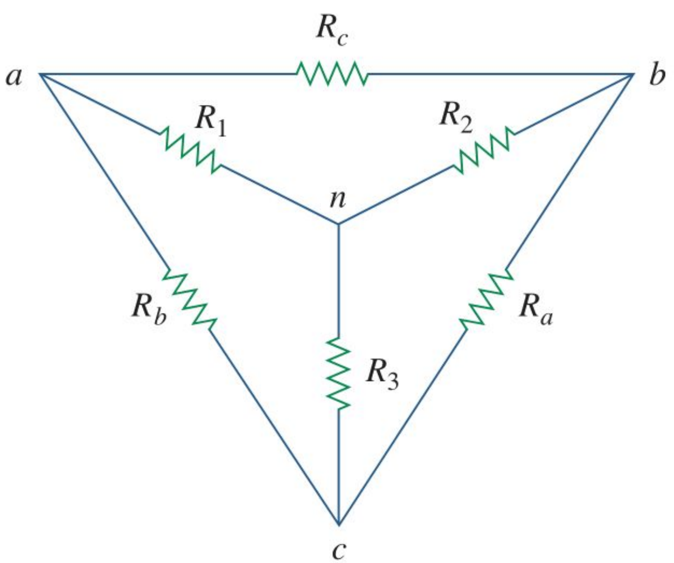


Fig: Superposition of Y and Δ networks

The equivalent resistances in the delta network can be estimated as

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

The Y and Δ networks are said to be balanced when $R_1 = R_2 = R_3 = R_Y$, $R_a = R_b = R_c = R_\Delta$. Under these conditions, $R_Y = \frac{R_\Delta}{3}$ and $R_\Delta = 3R_Y$

Examples

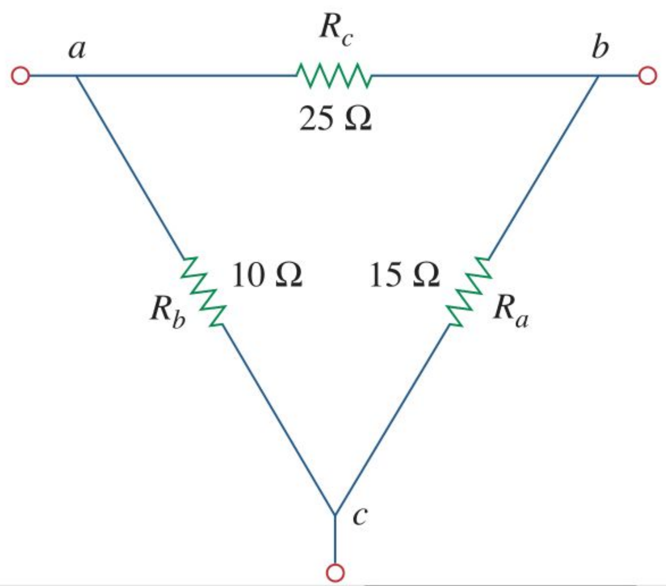


Fig.1: Convert the Δ network to Y network

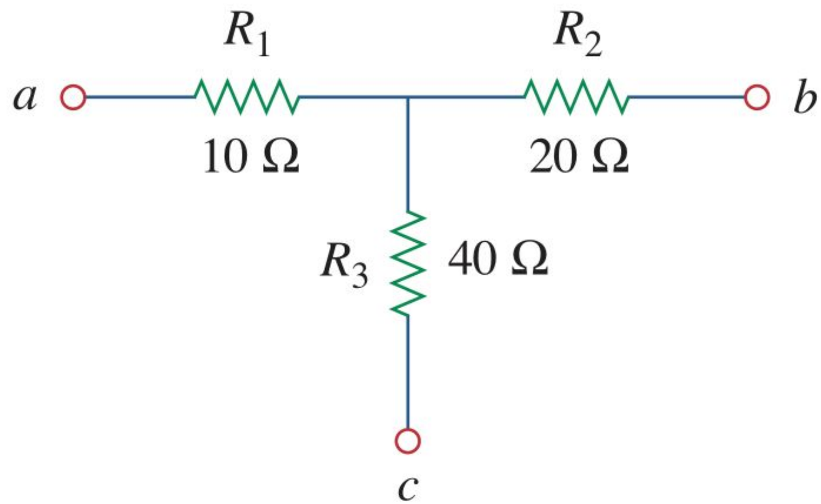


Fig.2: Convert the Y network to Δ network

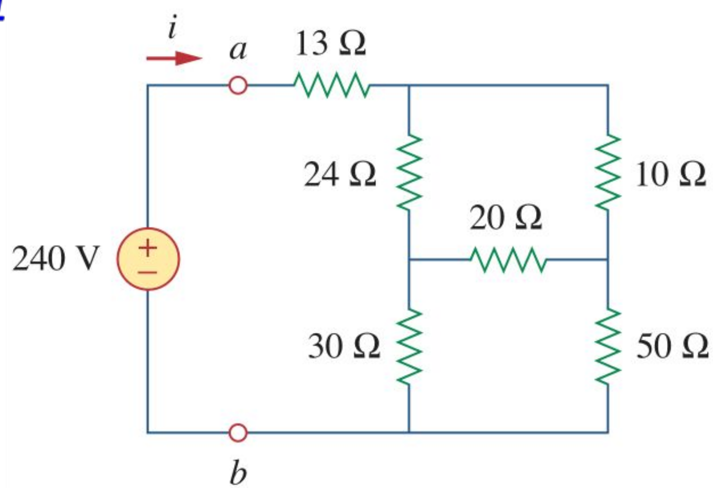


Fig.3: Calculate R_{ab} and I .

Mesh and Nodal Analysis

Nodal analysis and mesh analysis—both of which allow us to investigate many different circuits with a consistent, methodical approach.

Nodal analysis is based on a systematic application of Kirchhoff’s current law (KCL), and mesh analysis is based on a systematic application of Kirchhoff’s voltage law (KVL).

Nodal analysis

- Nodal analysis provides a general procedure for analysing circuits using ***node voltages*** as the ***circuit variables***.
- In nodal analysis, we are ***interested*** in finding the ***node voltages***. Given a circuit with ***n*** nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.
- Circuits that contain voltage sources will be analysed later.

Steps to Determine Node Voltages

1. **Select** a node as the reference **node**. Assign voltages $v_1, v_2, \ldots\ v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. **Apply KCL** to each of the $n-1$ **non-reference** nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Reference Node

The reference node is commonly called ***the ground*** since it is ***assumed*** to have ***zero potential.*** However, it is important to remember that ***any node can be*** designated as the ***reference node***. Thus, the reference node is at zero volts with respect to the other defined nodal voltages, and not necessarily with respect to ***earth ground***.

A reference node is indicated by any of the three symbols shown in below Fig.

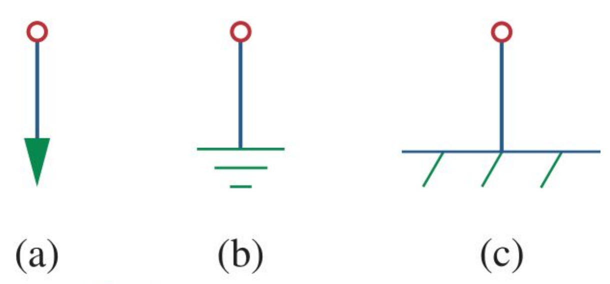


Figure: Common symbols for indicating a reference node, (a) common ground, (b) ground, (c) chassis ground.

Nodal Analysis

- Consider the circuit shown below. Node 0 is the reference node while nodes 1 and 2 are assigned voltages $v1$ and $v2$ and respectively.

Figure: Typical circuit for nodal analysis.

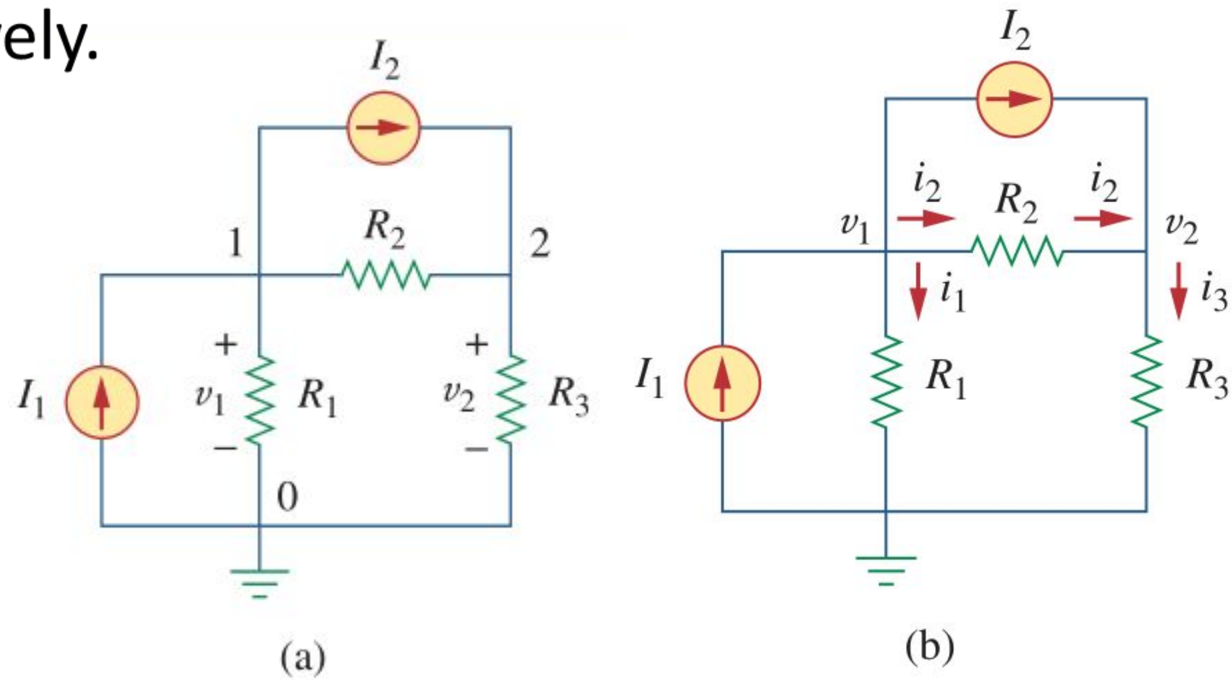


Fig. (a) is redrawn in Fig. (b), where the currents through resistors R_1, R_2 and R_3 are indicated as i_1, i_2 and i_3 respectively.

Nodal Analysis

- ***As the second step***, apply KCL to each non-reference node in the circuit.

At node 1:

$$I_1 = I_2 + i_1 + i_2$$

At node 2:

$$I_2 + i_2 = i_3$$

Now apply Ohm's law to express the unknown currents and in terms of node voltages.

$$i_1 = \frac{v_1 - 0}{R_1}, i_2 = \frac{v_1 - v_2}{R_2}, i_3 = \frac{v_2 - 0}{R_3}$$

<u>Note:</u> ***Current flows from a higher potential to a lower potential in a resistor.***

Nodal Analysis

- At node 1: $I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$

At node 2: $I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$

In terms of the conductances,

$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2)$$
$$I_2 + G_2(v_1 - v_2) = G_3 v_2$$

In matrix form

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Example

Calculate the node voltages in the circuit shown here

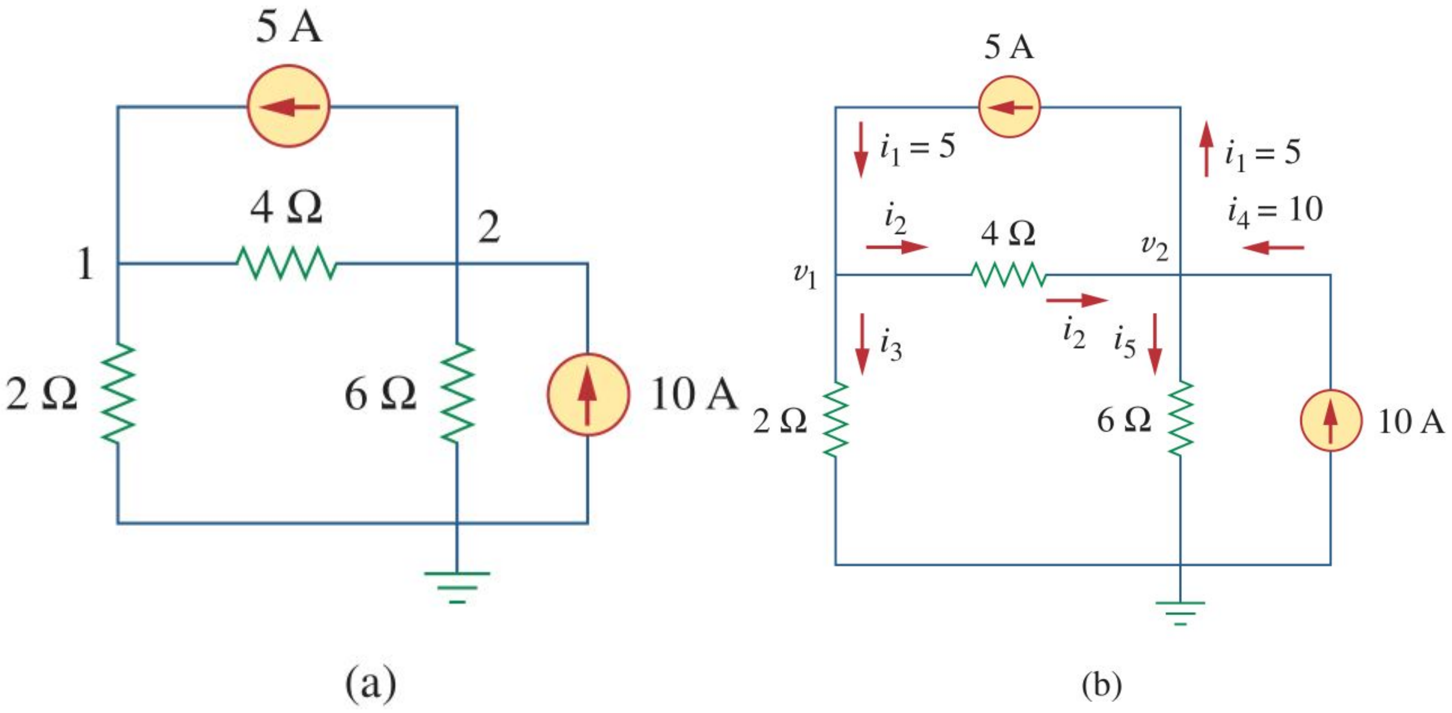
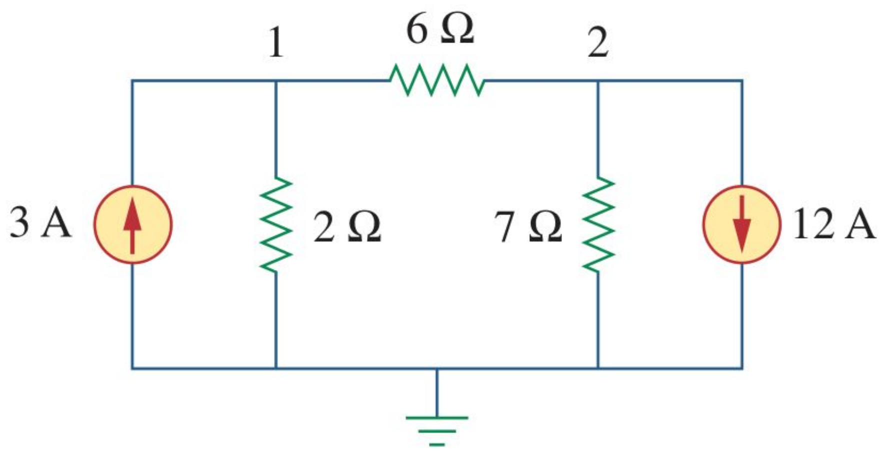
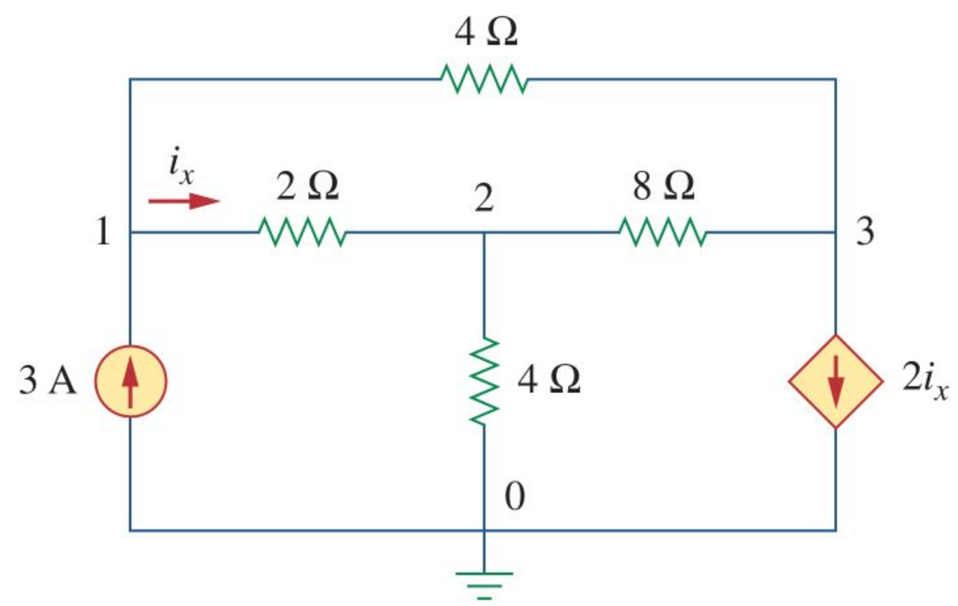


Figure: (a) original circuit, (b) circuit for analysis.

Examples

Obtain the node voltages in the circuits shown below.





Nodal Analysis with Voltage Sources

Consider the circuit shown here for illustration.

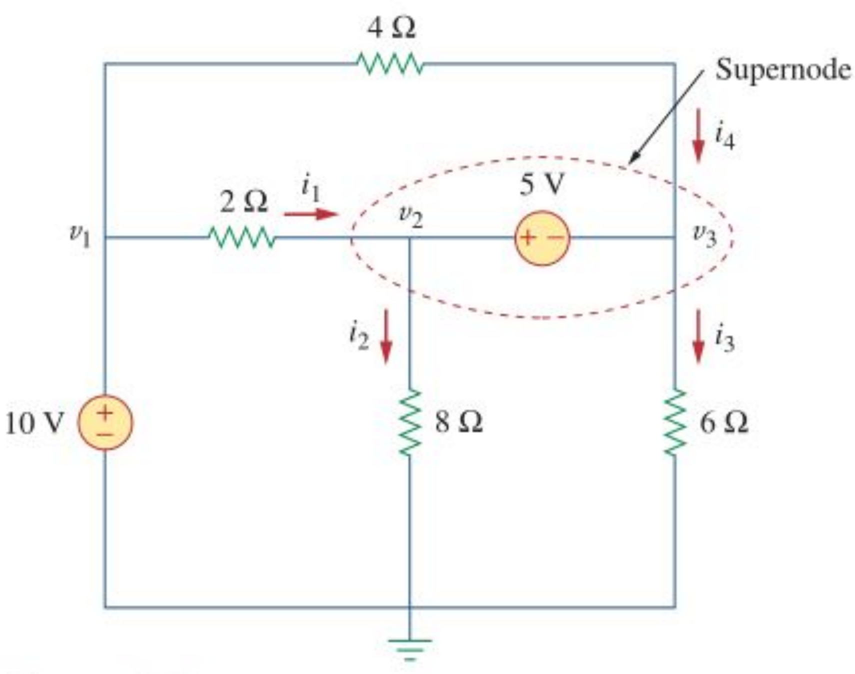


Figure: A circuit with a supernode.

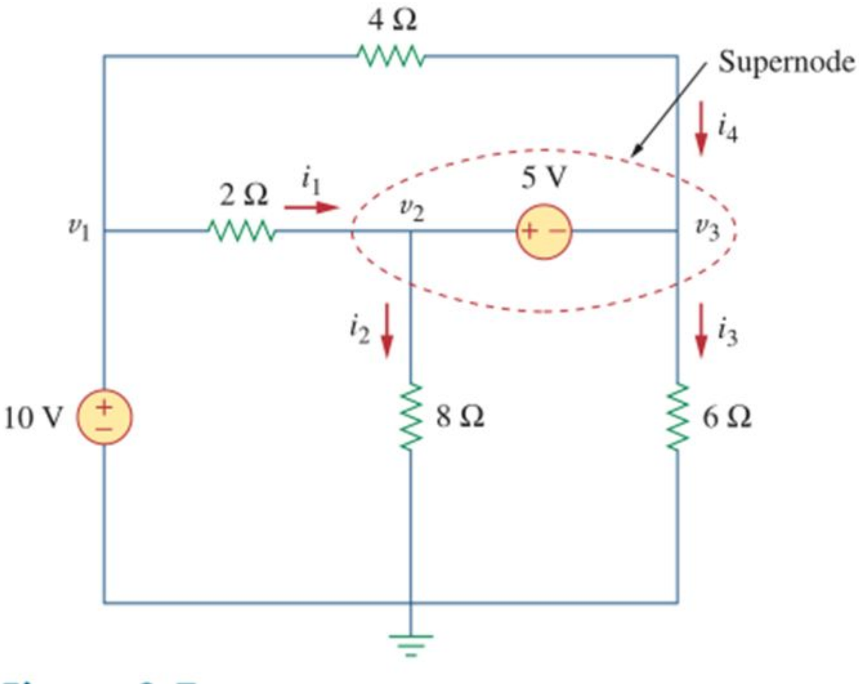
CASE 1

If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source.

$$v_1 = 10\ V$$

Nodal Analysis with Voltage Sources

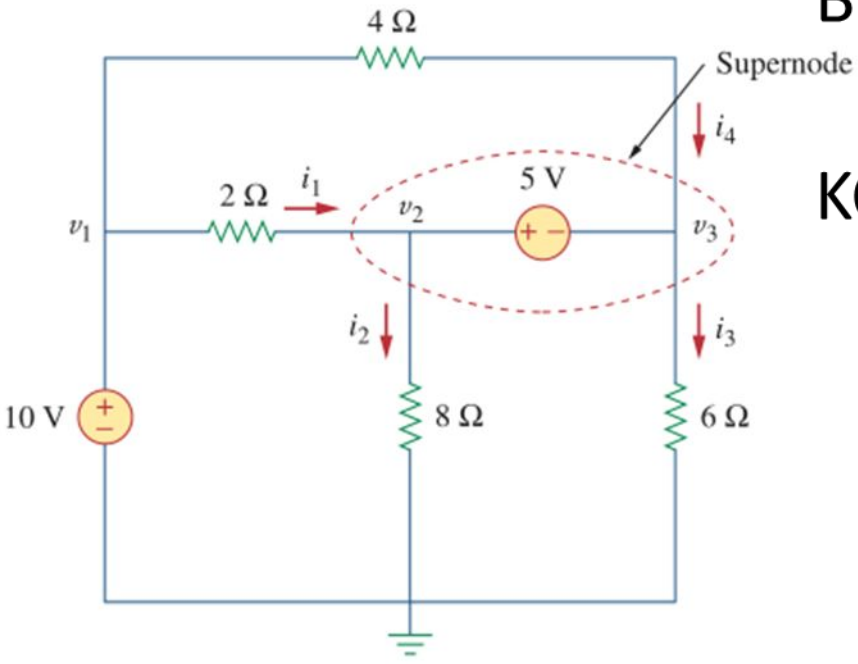
CASE2: If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a generalized node or supernode;



To treat this case, node 2, node 3, and the voltage source together form a supernode and apply KCL to both nodes at the same time. KCL must be satisfied at a supernode like any other node.

Nodal Analysis with Voltage Sources

As we have three nonrefernce nodes, we need three equations to obtain nodal voltages.



At node 1: $v_1 = 10\,V$

Between nodes 2 and 3 :

$$v_2 - v_3 = 5\,V$$

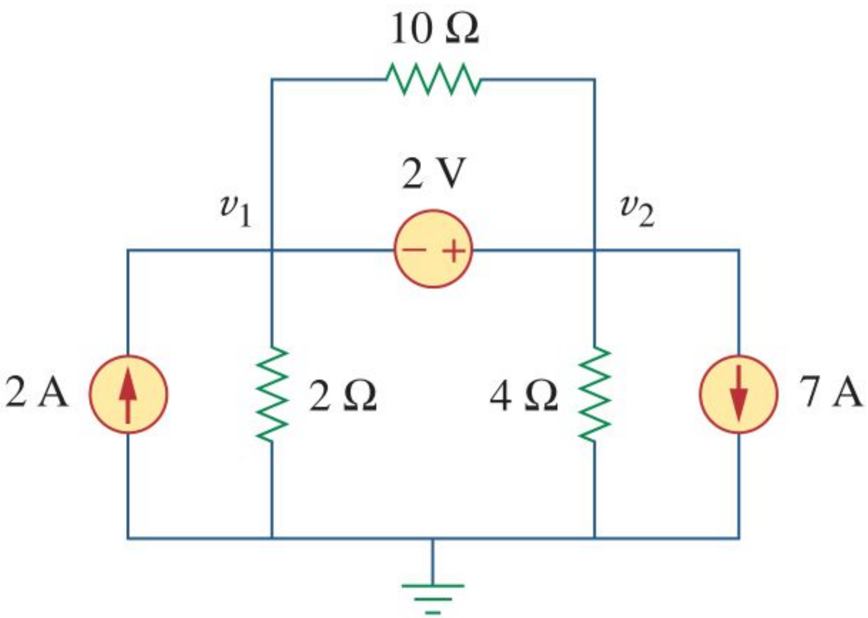
KCL at Super node:

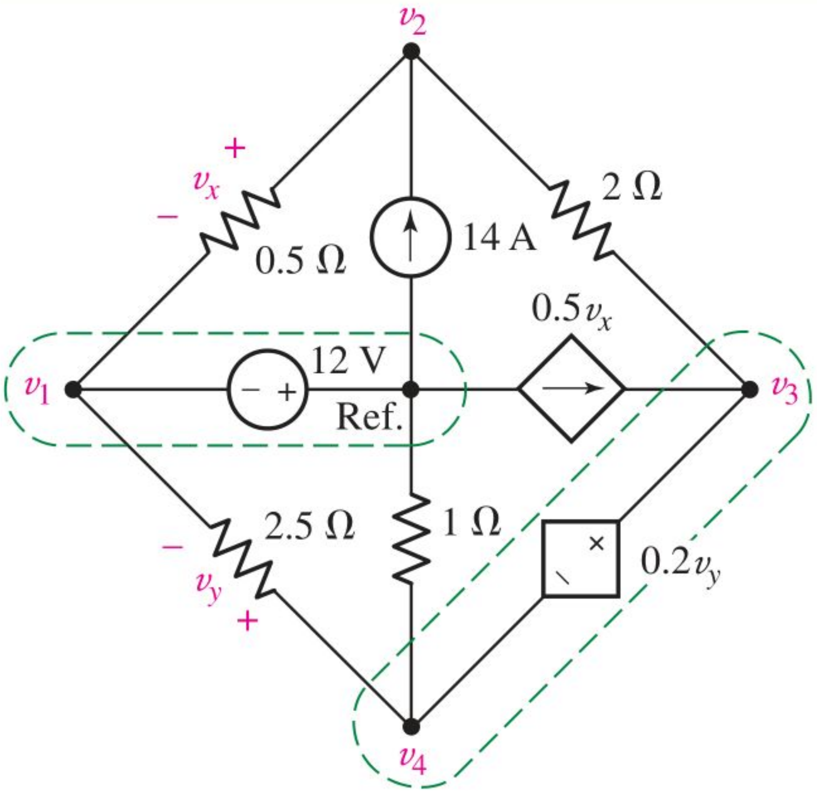
$$i_1 + i_4 = i_2 + i_3$$

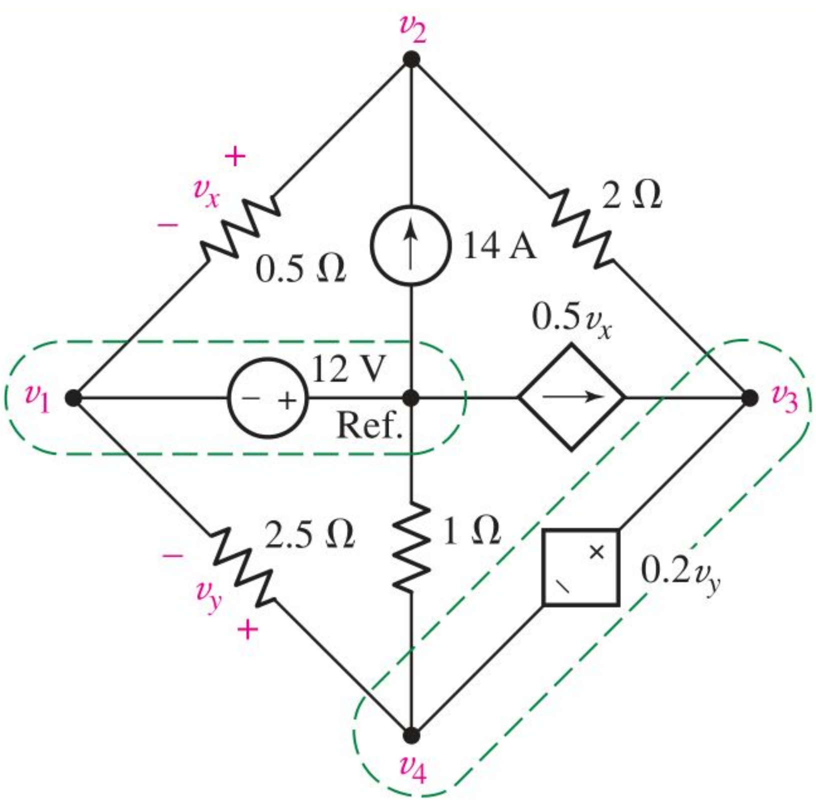
$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

Examples

- Determine the node-to-reference voltages in the circuits shown below







v_2
+
v_x
−
0.5 Ω
14 A
2 Ω
0.5v_x
12 V
v_1
Ref.
v_3
2.5 Ω
1 Ω
−
v_y
+
0.2v_y
v_4

Mesh Analysis-Introduction

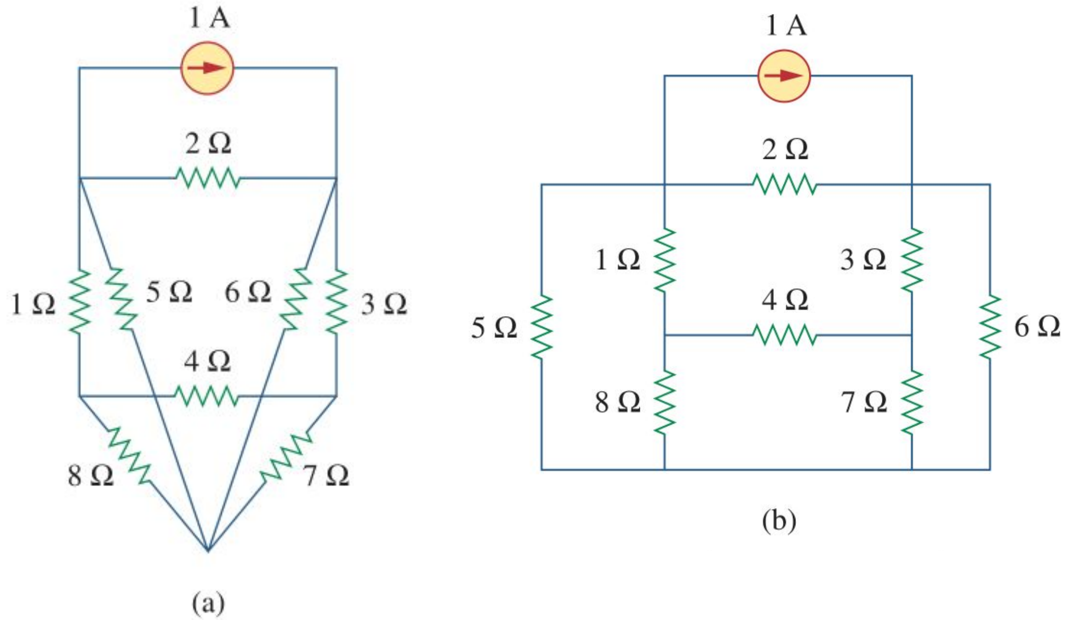
- Mesh analysis provides another general procedure for analysing circuits, using ***mesh currents as the circuit variables.***
- Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.
- Mesh analysis ***applies KVL*** to find unknown currents.
- Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar.

Mesh Analysis

A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.

A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches.

Figure:(a) A planar circuit with crossing branches,
(b) the same circuit redrawn with no crossing branches.



Mesh Analysis

The circuit shown below is nonplanar, because there is no way to redraw it and avoid the branches crossing. Nonplanar circuits can be handled using nodal analysis

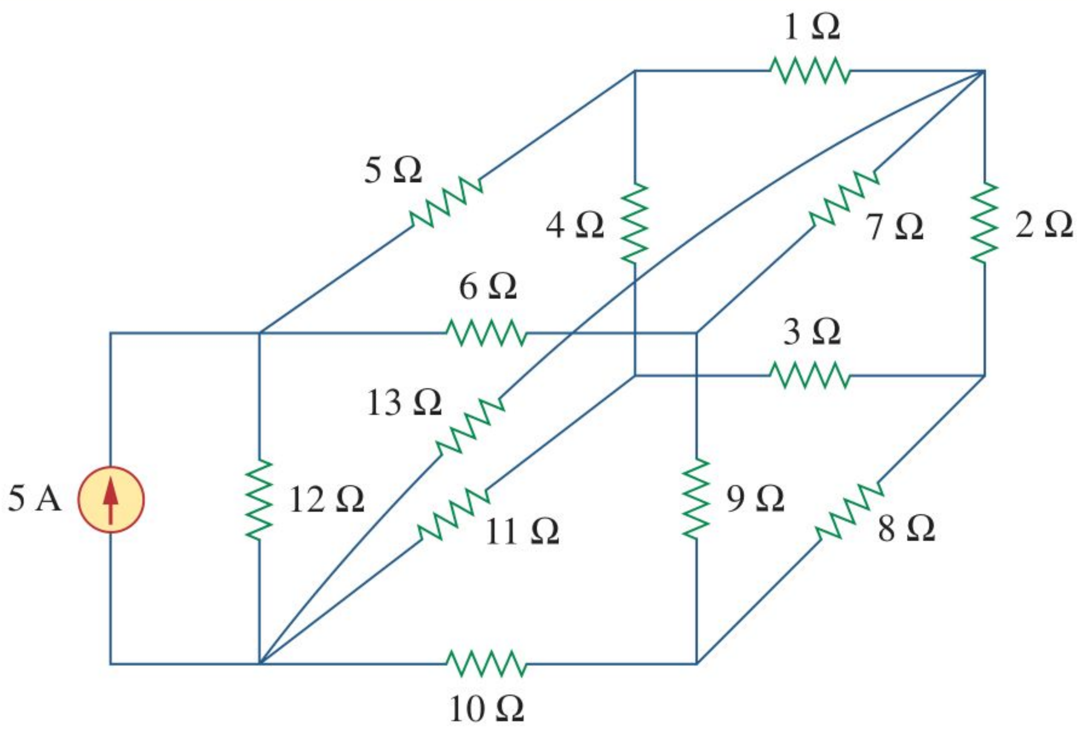


Figure: A nonplanar circuit.

Mesh Analysis

A mesh is a ***loop*** which does not contain any other loops within it.

abefa and ***bcdeb*** are meshes, but path ***abcdefa*** is not a mesh. The current through a mesh is known as mesh current.

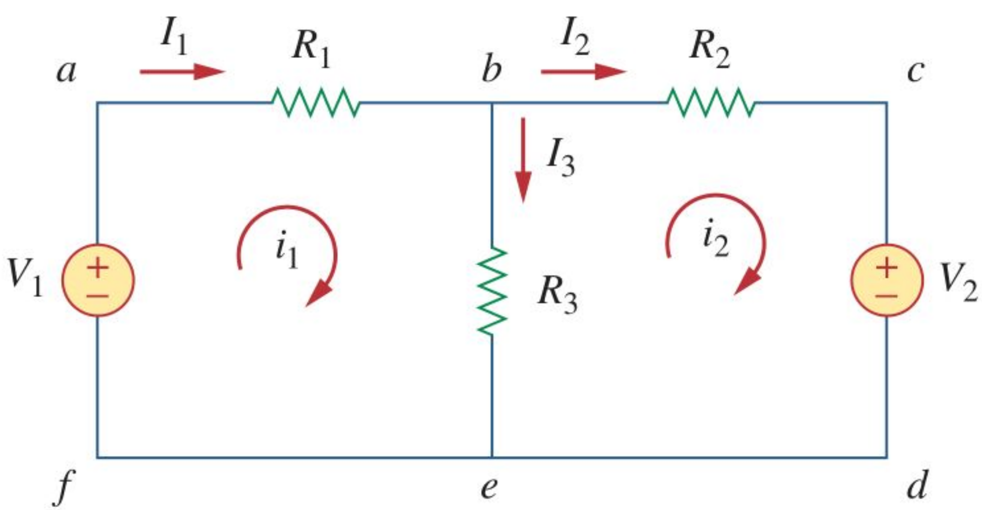


Figure: A circuit with two meshes.

Although path ***abcdefa*** is a loop and not a mesh, KVL still holds.

Steps to Determine Mesh Currents

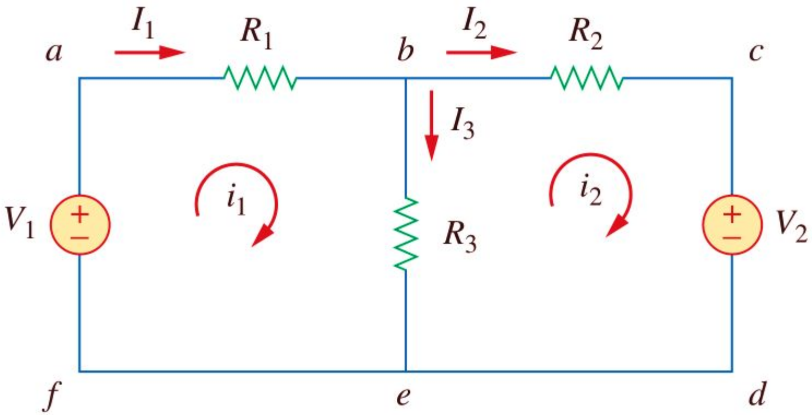
1. Assign mesh currents $i_1, i_2, \ldots\ i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

The ***direction*** of the ***mesh current*** is ***arbitrary***—(clockwise or counterclockwise)—and does not affect the validity of the solution. It is ***conventional*** to ***assume*** that each mesh current flows ***clockwise***.

Mesh Analysis

- To illustrate the steps, consider the circuit shown here:

Step 1: Mesh currents i_1 and i_2 are assigned to meshes 1 and 2.



Step 2: Apply KVL to each mesh.

$$-V_1 + i_1 R_1 + (i_1 - i_2)R_2 = 0$$

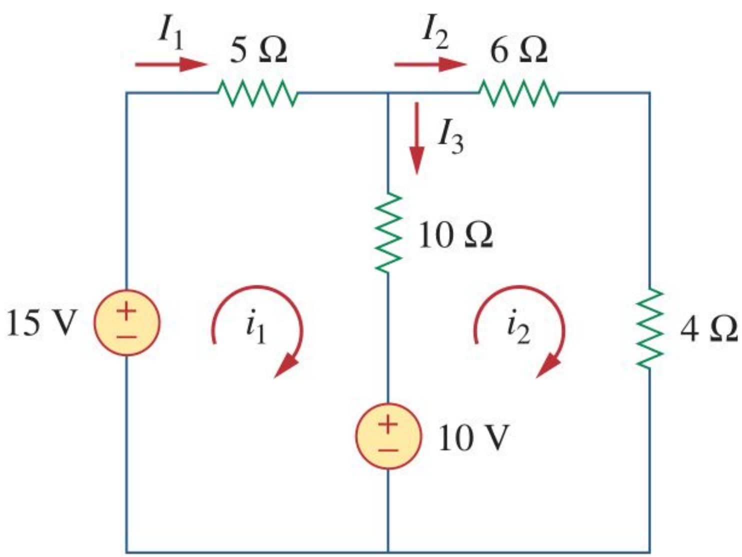
$$V_2 + (i_2 - i_1)R_3 + i_2 R_2 = 0$$

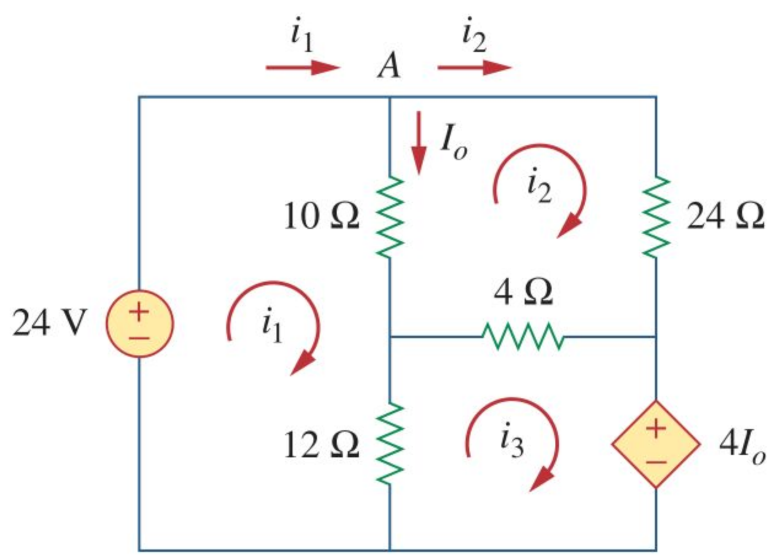
Step 3: Solve above equations for i_1 and i_2

Using i_1 and i_2, branch currents can be obtained as $I_1 = i_1$, $I_2 = i_2$, and $I_3 = i_1 - i_3$

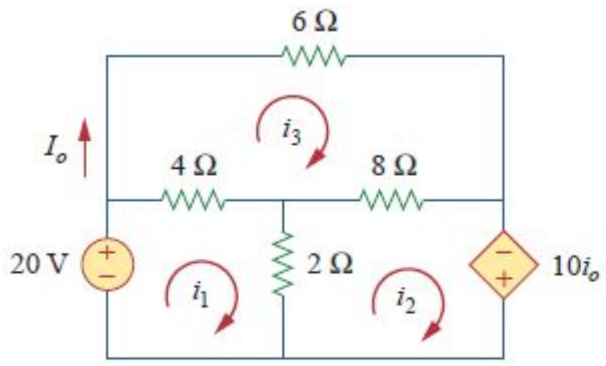
Example

For the circuits shown below, find the branch currents and using mesh analysis.





Using mesh analysis find Io.



Ans: Io = -5 A

Mesh Analysis with Current Sources

- ***CASE 1:*** When a current source exists only in one mesh.

For the circuit in shown here, we set $i_2 = -5\,A$ and write a mesh equation for the other mesh in the usual way; that is,

$$10V = 4i_1 + 6(i_1 - i_2),\ i_2 = -5\,A$$

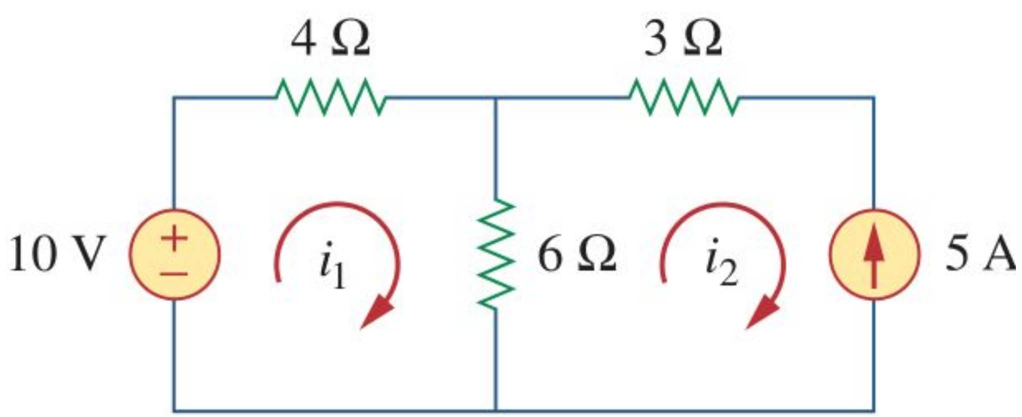


Figure: A circuit with a current source.

Mesh Analysis with Current Sources

CASE 2: When a current source exists between two meshes: We create a supermesh by excluding the current source and any elements connected in series with it, as shown below.

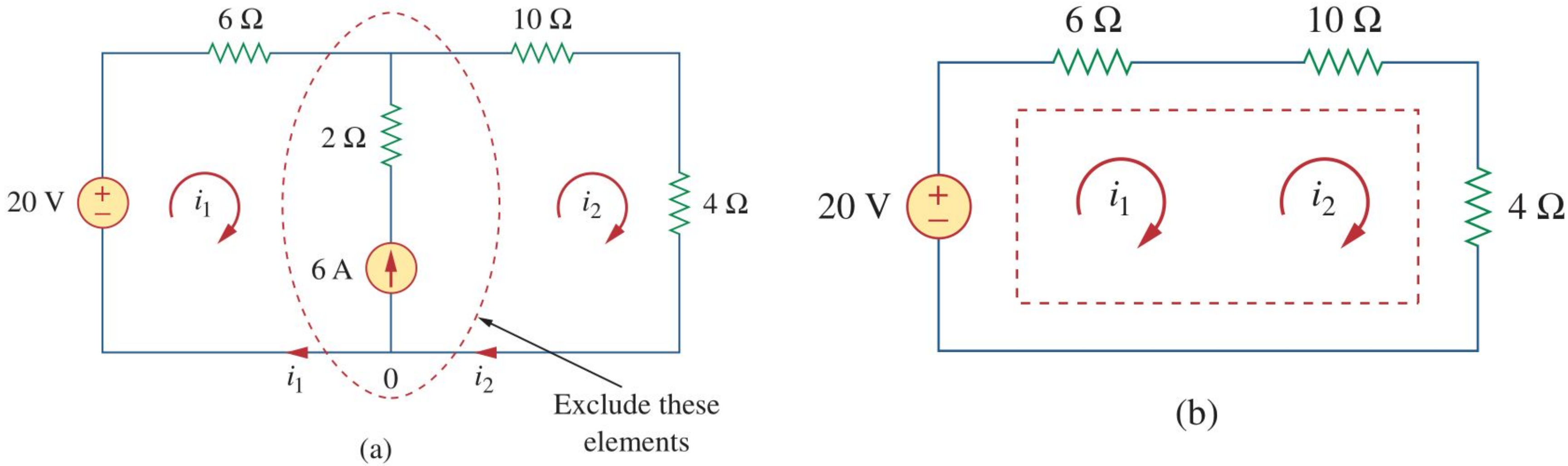


Figure: (a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

Mesh Analysis with Current Sources

A supermesh must satisfy KVL like any other mesh. Therefore, applying KVL to the supermesh gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20$$

Applying KCL to node 0 gives,

$$i_1 + 6 = i_2$$

Solving above equation gives

$$i_1 = -3.2\ A, \qquad i_2 = 2.8\ A$$

Examples

For the circuit in Fig. below, find i_1 to i_4 using mesh analysis.

Note: Meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source in common.

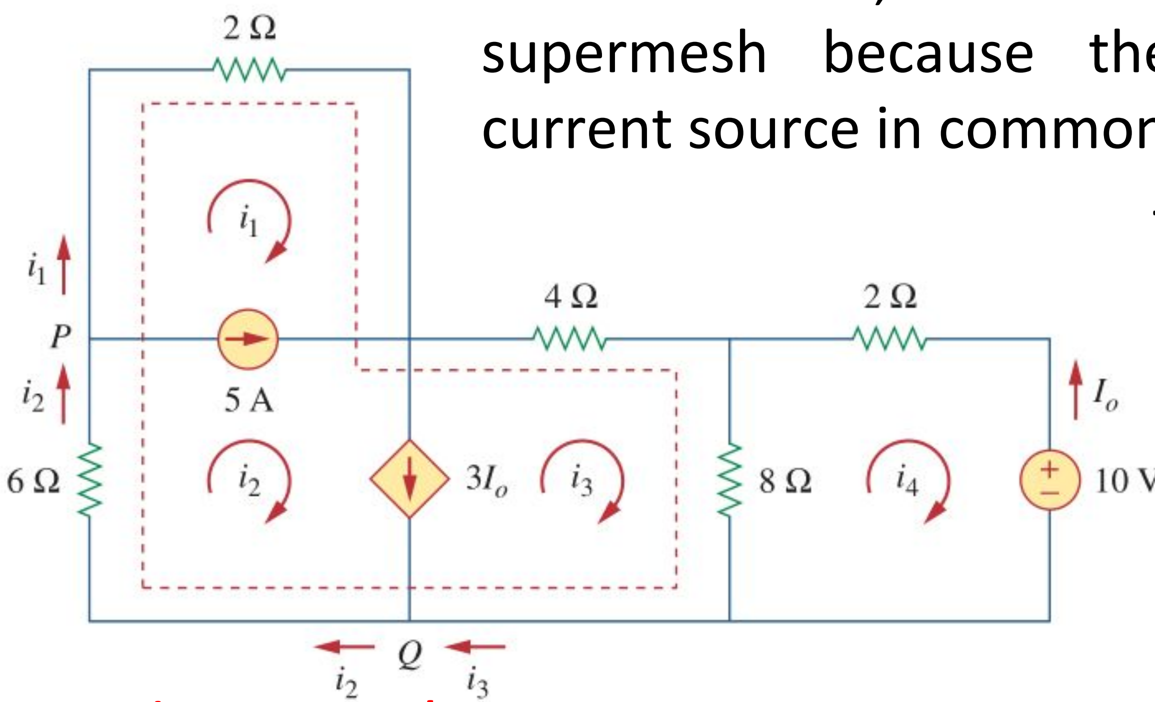
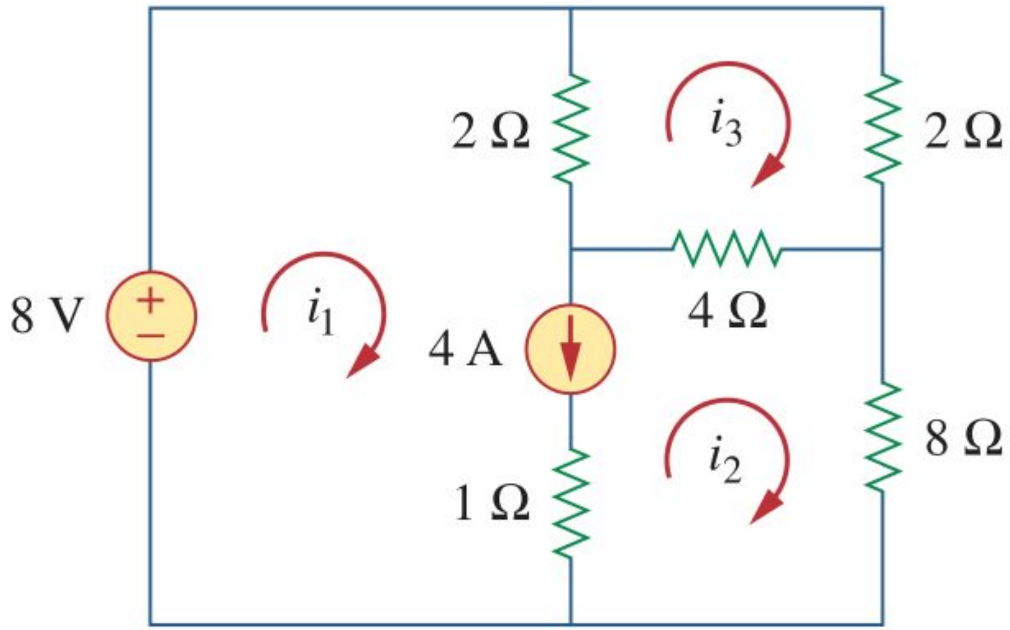


Figure : Example.

The two supermeshes intersect and form a larger supermesh as shown.

Practice Problem

- Use mesh analysis to determine i_1, i_2 and i_3.

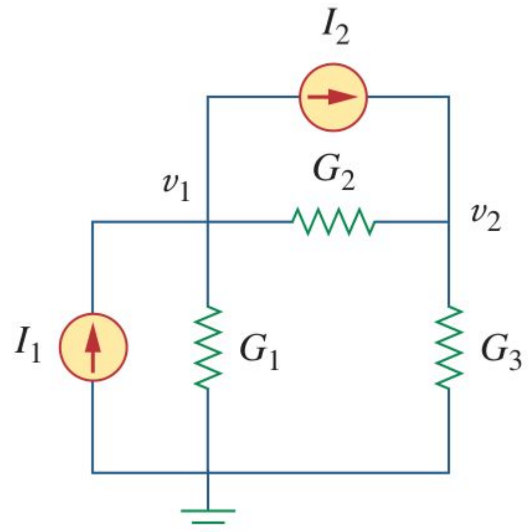


Nodal Analysis by Inspection

1. It is a shortcut approach based on mere inspection of a circuit.
2. ***When all sources in a circuit are independent current sources***, we do not need to apply KCL to each node to obtain the node-voltage equations as we did before. We can obtain the equations by mere inspection of the circuit.

For the given circuit, node-voltage equations are

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$



Nodal Analysis by Inspection

- Each of the ***diagonal terms*** of conductance matrix is the sum of the ***conductances connected directly*** to node 1 or 2, while the ***off-diagonal terms*** are the negatives of the conductances connected between the nodes.

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

- Also, each term on the right-hand side of above Eq. is the algebraic sum of the currents entering the node.

Nodal Analysis by Inspection

In general, if a circuit with independent current sources has N non-reference nodes, the node-voltage equations can be written in terms of the conductances as

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

where

G_{kk} = **Sum of the conductances connected to node** k

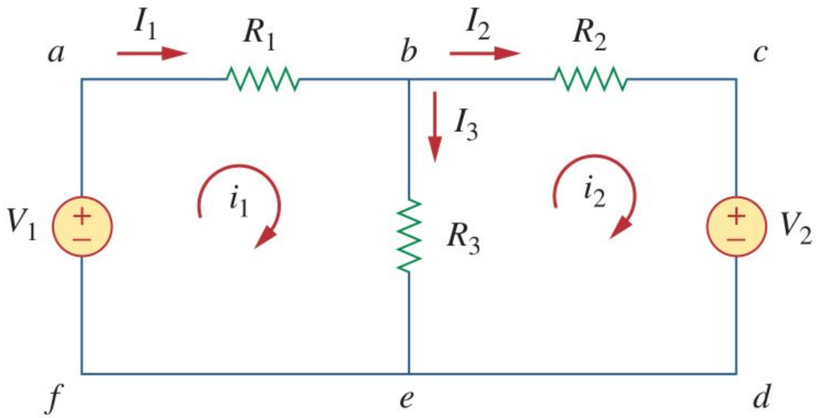
$G_{jk} = G_{kj}$ = **Negative of the sum of the conductances directly connecting nodes** k **and** j

v_k = **Unknown voltage at node** k

i_k = **Sum of all independent current sources directly connected to node** k**, with currents entering the node treated as positive**

Mesh Analysis by Inspection

We can also obtain mesh-current equations by inspection when a linear resistive circuit ***has only independent voltage sources***.



For the circuit shown here, the mesh-current equations are

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

<u>Observations:</u>

Diagonal terms = the sum of the resistances in the related mesh.

The off-diagonal terms = the negative of the resistance common to meshes 1 and 2.

Each term on the right-hand side of Eq. is the algebraic sum taken clockwise of all independent voltage sources in the related mesh.

Mesh Analysis by Inspection

- In general, if the circuit has N meshes, the mesh-current equations can be expressed in terms of the resistances as

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

where

$\boldsymbol{R_{kk}} =$ **Sum of the resistances connected in mesh $\boldsymbol{k}$**

$\boldsymbol{R_{jk} = R_{kj}} =$**Negative of the sum of the resistances in common with meshes $\boldsymbol{k}$ and $\boldsymbol{j, k \neq j}$**

$\boldsymbol{i_k} =$ **Unknown mesh current for mesh $\boldsymbol{k}$ in the clockwise direction**

$\boldsymbol{v_k} =$**Sum taken clockwise of all independent voltage sources in mesh $\boldsymbol{k}$, with voltage rise treated as positive**

Practice Problems

1. By inspection, obtain the node-voltage equations for the circuit in Fig. 1.
2. By inspection, write the mesh-current equations for the circuit in Fig. 2.

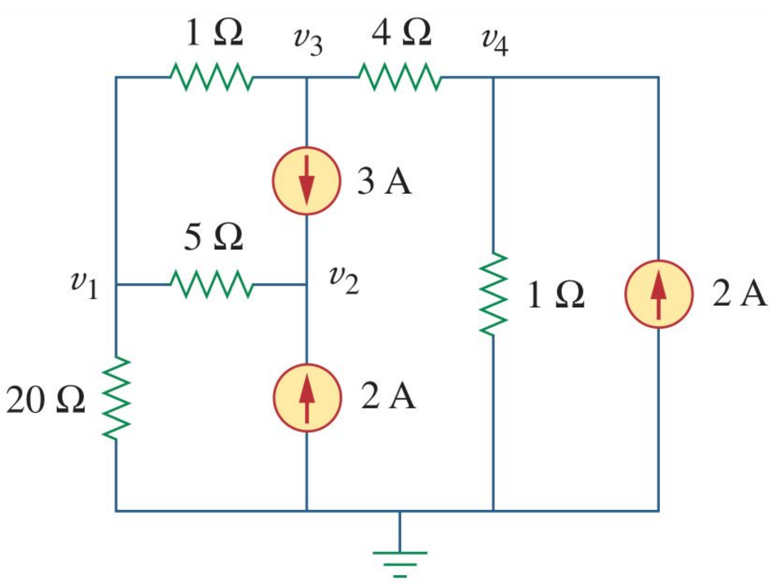


Fig. 1.

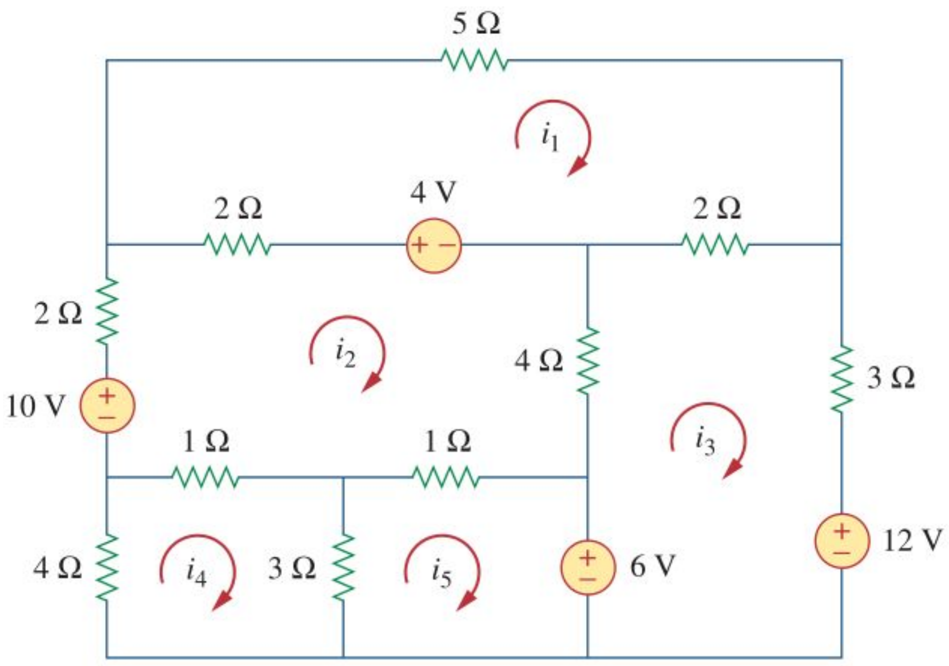
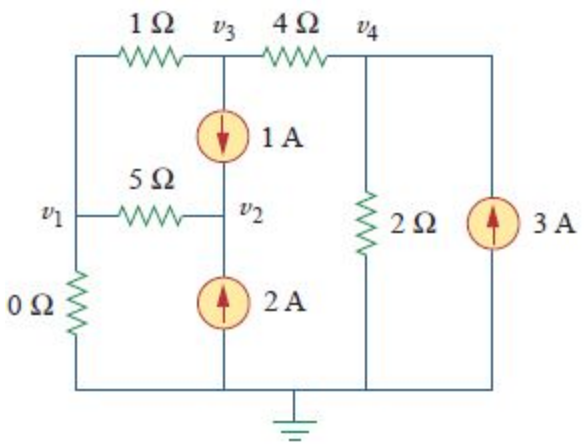
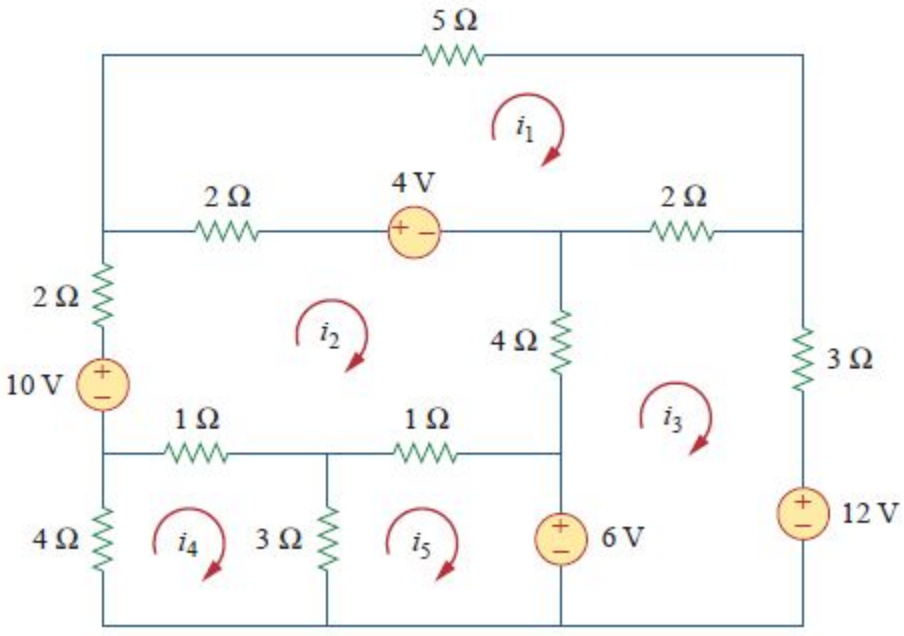


Fig. 2.

Solution

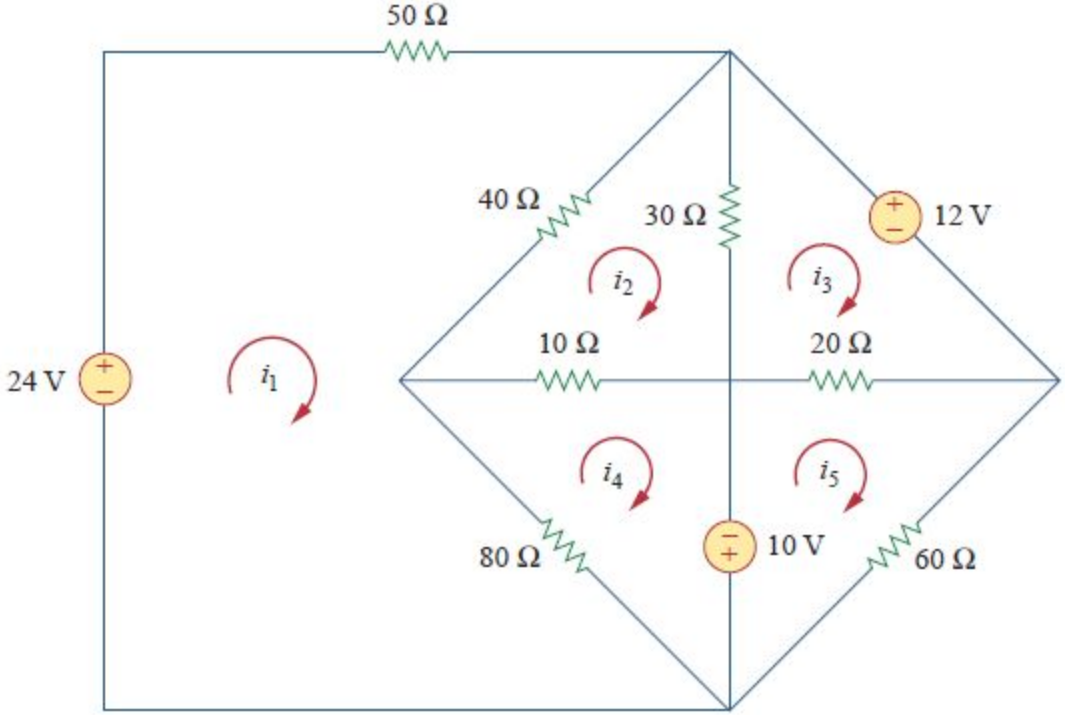




$$\begin{bmatrix} 1.3 & -0.2 & -1 & 0 \\ -0.2 & 0.2 & 0 & 0 \\ -1 & 0 & 1.25 & -0.25 \\ 0 & 0 & -0.25 & 0.75 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

By inspection, obtain the mesh-current equations for the circuit in Fig.



$$\begin{bmatrix} 170 & -40 & 0 & -80 & 0 \\ -40 & 80 & -30 & -10 & 0 \\ 0 & -30 & 50 & 0 & -20 \\ -80 & -10 & 0 & 90 & 0 \\ 0 & 0 & -20 & 0 & 80 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -12 \\ 10 \\ -10 \end{bmatrix}$$