Electrical Networks
CSEE 102 S1

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Syllabus

Books

- ***Engineering Circuit Analysis*** by William H. Hayt
- ***Fundamentals of Electric Circuits*** by Charles K. Alexander
- ***Electrical circuits*** by Edminister Joseph A., Schaum's outline series, McGraw hill, 2nd edition, 1983.

Introduction

- In electrical engineering we often find ourselves talking about specific currents, voltages, or powers, we call them ***quantities***.

- In electrical engineering, we are often interested in transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an ***electric circuit***, and each component of the circuit is known as an ***element***.

An electric circuit is an interconnection of electrical elements

Simple Circuit

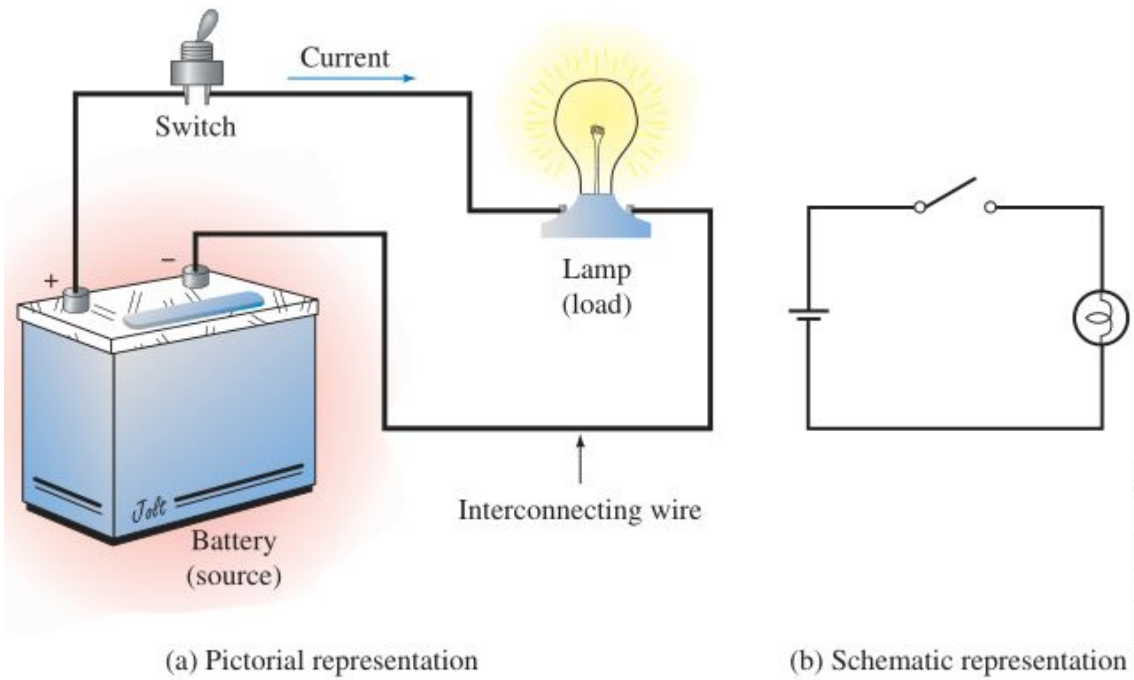


Fig.1. shows a simple electric circuit consisting of a source of electrical energy (battery), a switch, a load, and inter-connecting wire.

SI Units

- In order to state the value of some measurable quantity, we must give a number. In addition to a numerical value, we must include a unit.
- The system of units most commonly used in electrical engineering is the SI.
- Quite frequently, however, the SI units yield numbers that are either too large or too small for convenient use.
- To handle these, engineering notation and a set of **standard prefixes** have been developed.

The basic units in the SI system are listed with their symbols, in Table.

Quantity	Unit
Length	meter, m
Mass	kilogram, kg
Time	second, s
Electric Current	ampere, A
Temperature	kelvin, K
Luminous Intensity	candela, cd
Amount of Substance	mole, mol
Charge	coulomb, C

Derived SI units use combinations of basic units and there are many of them. Two examples are: Velocity and Acceleration

Standard Prefixes

- To describe quantities that occur in large multiples or small fractions of a unit, standard prefixes are used to denote powers of 10 of SI (and derived) units.

Prefix	Name	Meaning
M	mega	Multiply by 1 000 000 (i.e. $\times 10^6$)
k	kilo	Multiply by 1 000 (i.e. $\times 10^3$)
m	milli	Divide by 1000
μ	micro	Divide by 1 000 000 (i.e. $\times 10^{-6}$)
n	nano	Divide by 1 000 000 000 (i.e. $\times 10^{-9}$)
p	pico	Divide by 1 000 000 000 000 (i.e. $\times 10^{-12}$)

Charge, Current, Voltage and Power

- The most basic quantity in an electric circuit is the electric charge.
- Charge is an electrical property of the atomic particles of which matter consists, **measured in coulombs (C)**.
- All matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons.
- Charge ***e*** on an electron is negative and equal in magnitude to 1.602×10^{-19}C.

Charge

- In 1 C of charge, there are $1/1.602\times 10^{-19} = 6.24\times 10^{19}$ electrons.
- The ***law of conservation of charge*** states that charge can neither be created nor destroyed, only transferred.
- Electric charge or electricity is mobile. It can transferred from one place to another, where it can be converted to another form of energy.

Current

When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction.

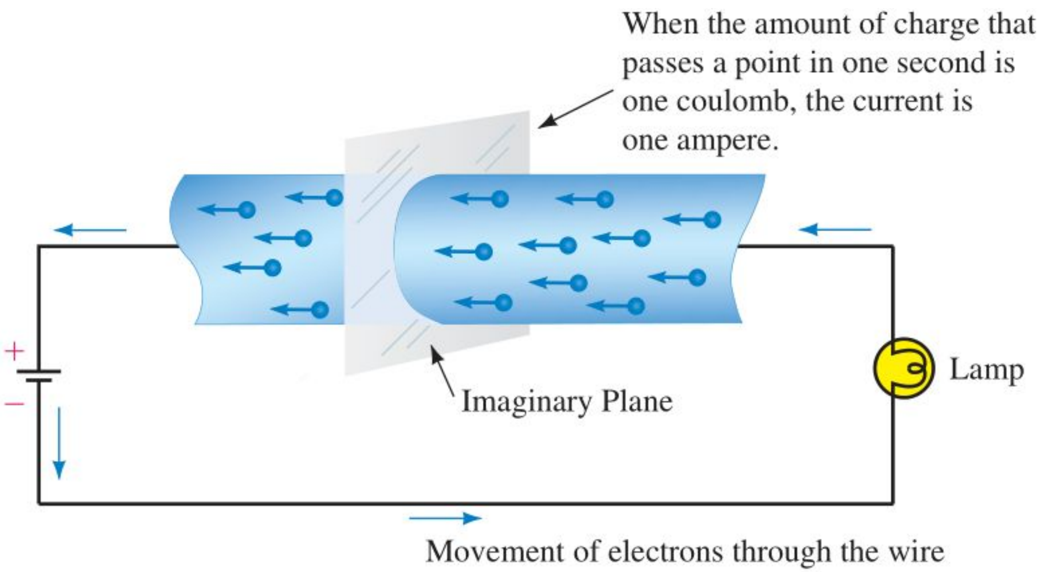


Fig. Electron flow in a conductor.

- Electrons (-) are attracted to the positive (+) pole of the battery. As electrons move around the circuit, they are replenished at the negative pole of the battery. **This flow of charge is called an electric current.**

Current

- It is conventional to take the current flow as the movement of positive charges. That is, opposite to the flow of negative charges.

- The current present in a discrete path, such as a metallic wire, has both a numerical value and a direction associated with it.

- It is a measure of the rate at which charge is moving past a given reference point in a specified direction.

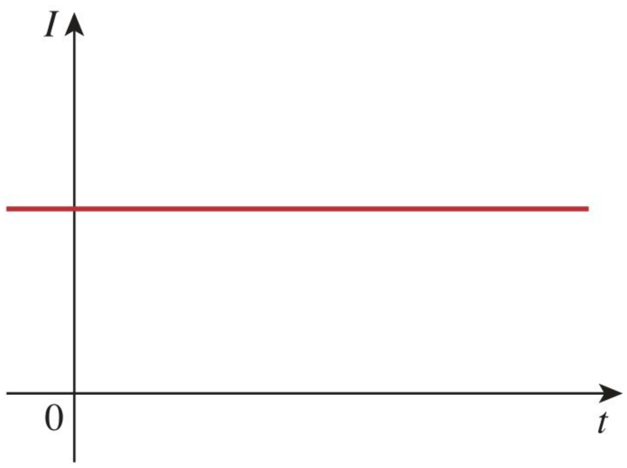
- Mathematically, the relationship between current *i*, charge *q* and time *t* is $i = \mathrm{dq/dt}$.

- The unit of current is the ampere (A), named after Andre-Marie Ampère, a French physicist. **One ampere equals 1 coulomb per second.**

Current

There can be several types of current; that is, charge can vary with time in several ways.

- If the current does not change with time, but remains constant, we call it a ***direct current (dc)*** as shown below



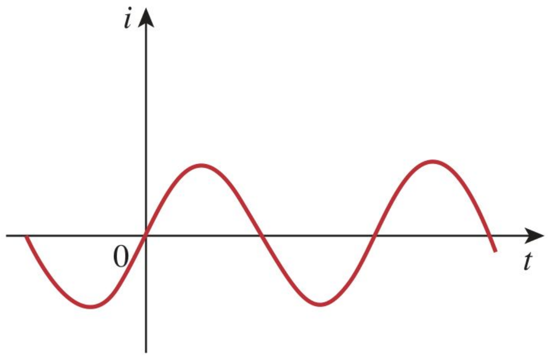
A direct current (dc) is a current that remains constant with time.

The symbol ***I*** (capital letter) is used to represent such a constant current.

A time-varying current is represented by the symbol *i*.

Current

- A common form of time-varying current is the sinusoidal current or alternating current (ac).



An **alternating current (ac)** is a current that varies sinusoidally with time. This form are present in normal household circuits . the AC electricity delivered by the power company has a frequency of 50 cycles/s, or 50Hz.

Direct current and alternating current are **the two most common** types of current.

Direction of Current

As mentioned earlier, the direction of current flow is conventionally taken as the direction of positive charge movement.

We create a graphical symbol for current by placing an arrow next to the conductor.

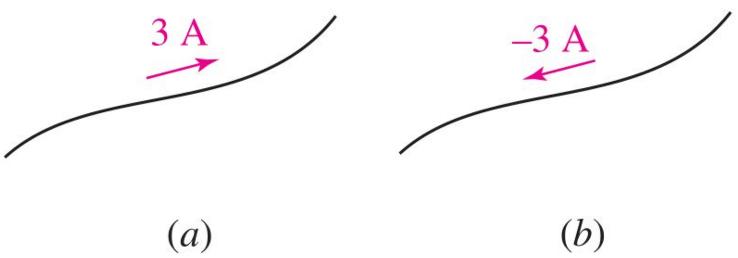


FIGURE: Two methods of representation for the exact same current.

Based on this convention, a current of 3A may be represented positively or negatively as shown in Fig.

Direction of Current

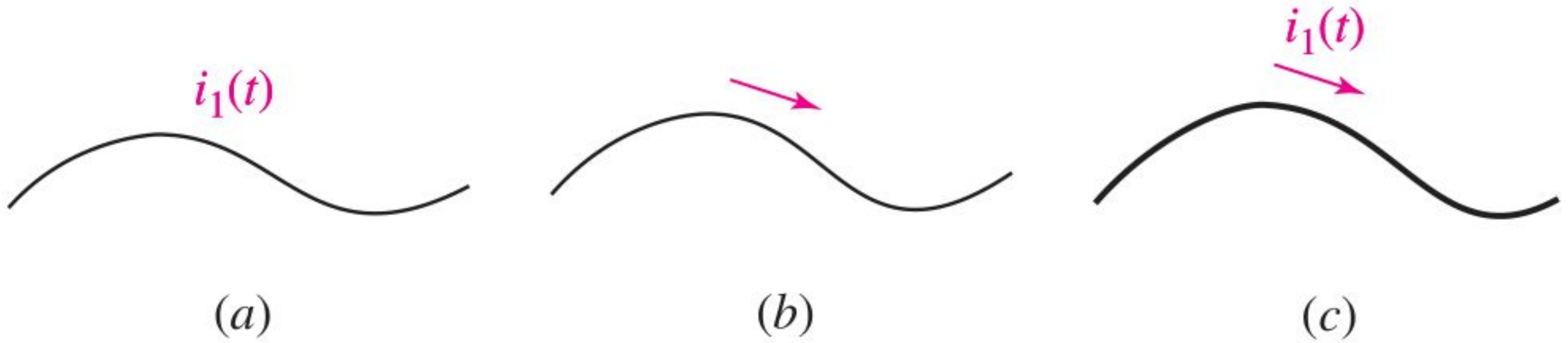


FIGURE: (a, b) Incomplete, improper, and incorrect definitions of a current. (c) The correct definition of $i_1(t)$.

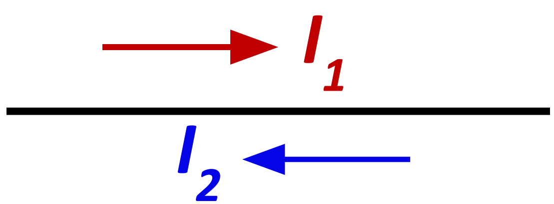


Fig.

In the wire of Fig., electrons are moving left to right to create a current of 1 mA. Determine I_1 and I_2.

Voltage

- To move the electron in a conductor in a particular direction requires some work or energy transfer.
- Voltage (or **potential difference**) is the energy required to move a unit charge through an element.
- The voltage between two points is **one volt** if it requires **one joule of energy** to move **one coulomb** of charge from one point to the other.
- This work is performed by an external electromotive force (emf), typically represented by the battery.
- This emf is also known as voltage or potential difference.

Voltage

- A voltage can exist between a pair of electrical terminals whether a current is flowing or not.
- An automobile battery, for example, has a voltage of 12 V across its terminals even if nothing whatsoever is connected to the terminals.
- The voltage between two points ***a*** and ***b*** in an electric circuit is the energy (or work) needed to move a unit charge from ***a*** to ***b***; mathematically

$$v_{ab} = \frac{\mathrm{d}w}{\mathrm{d}q}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage or simply v is measured in

Voltage

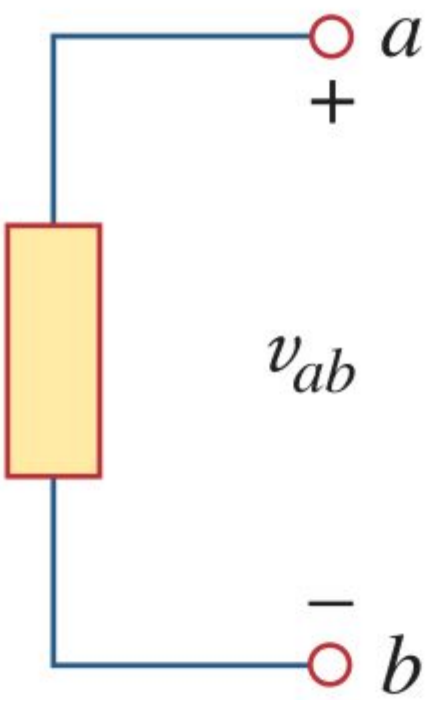


Fig: Polarity of voltage v_{ab}
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The plus (+) and minus (-) signs are used to define reference direction or voltage polarity.

The v_{ab} can be interpreted in two ways: (1) Point ***a*** is at a potential of v_{ab} volts higher than point ***b***, or (2) the potential at point ***a*** with respect to point ***b*** is v_{ab}.

Logically $v_{ab} = -v_{ba}$

NOTE: Electric current is always through an element and that electric voltage is always across the element or between two points.

Voltage

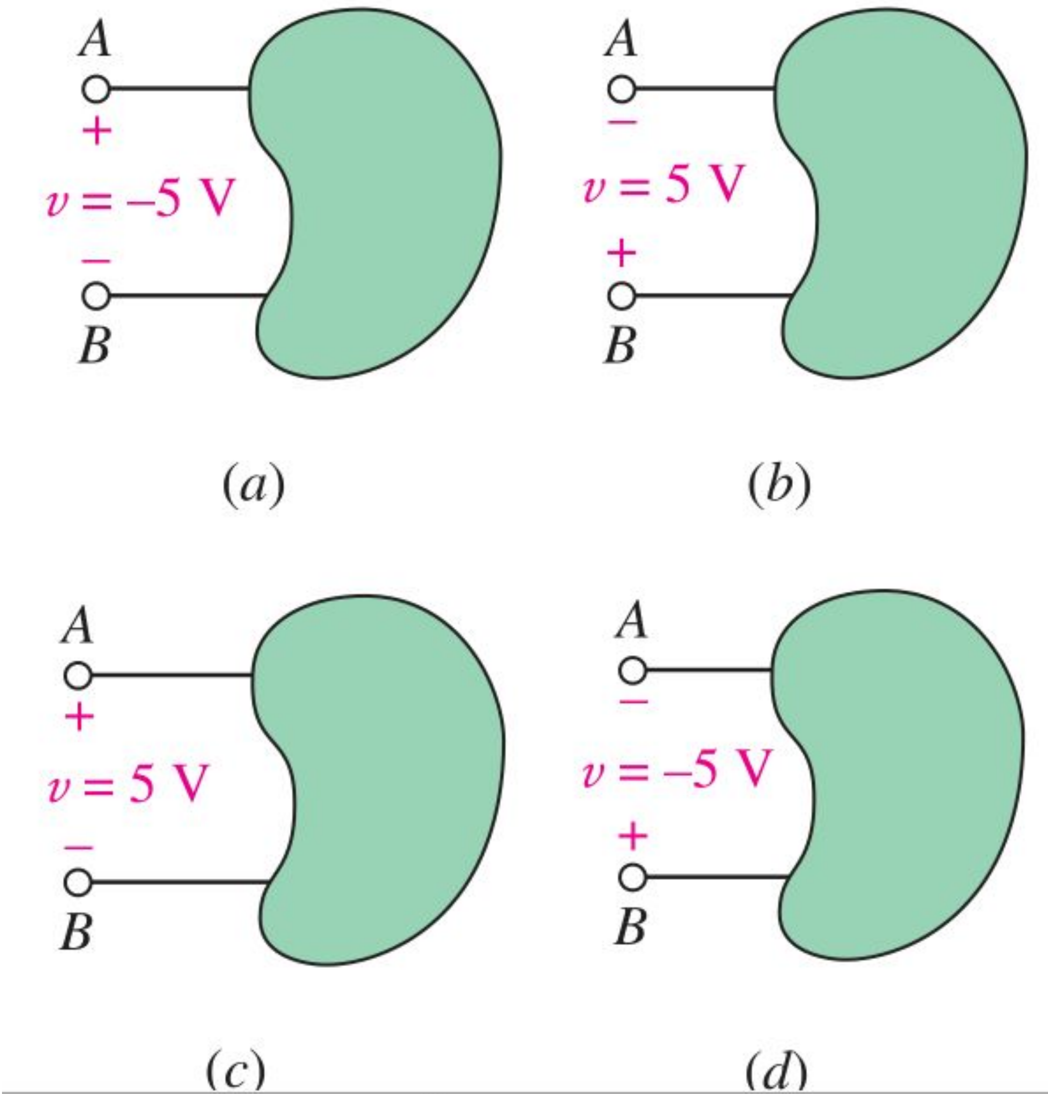


FIGURE : (a, b) Terminal B is 5 V positive with respect to terminal A; (c, d) terminal A is 5 V positive with respect to terminal B.

Voltage

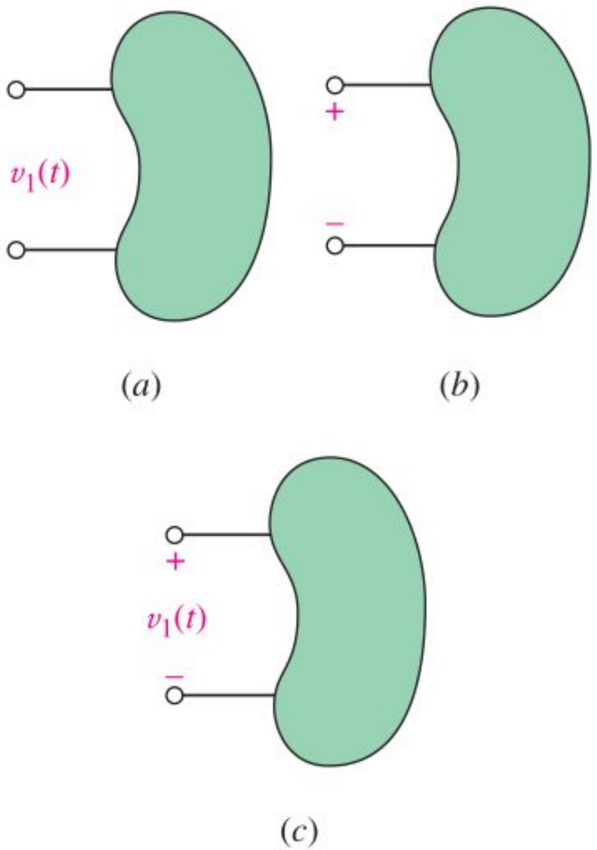


FIGURE: (a, b) These are inadequate definitions of a voltage. (c) A correct definition includes both a symbol for the variable and a plus-minus symbol pair.

The definition of any voltage must include a plus-minus sign pair.

A constant voltage is called a **dc voltage** and is represented by V.

A sinusoidally time-varying voltage is called an **ac voltage** and is represented by *v*.

A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

Potential Difference

- The difference in the potentials of two charged bodies is called potential difference.
- If two bodies have different electric potentials, a potential difference exists between the bodies.
- If the two bodies are joined through a conductor, then electrons will flow from one body to another.
- The potential difference between any two points in the circuit is the energy used by one coulomb in moving from one point to another.

Power

- Power is the time rate of expending or absorbing energy, measured in watts (W).

$$p = \frac{\mathrm{d}w}{\mathrm{d}t}$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s).

$$p = \frac{dw}{dt} = \frac{dw}{dq} . \frac{dq}{dt} = vi$$

Thus, the **power absorbed or supplied** by an element is the **product** of the **voltage across** the element and the

Power

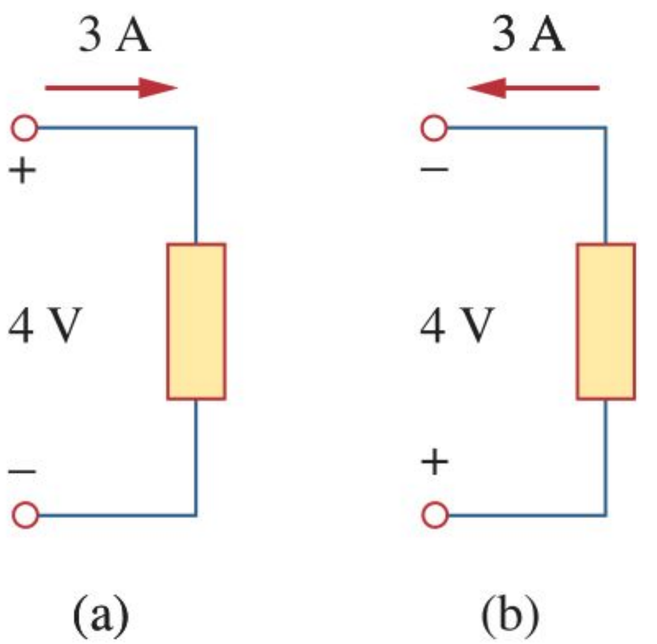


Figure: Two cases of an element with an absorbing power of 12 W:

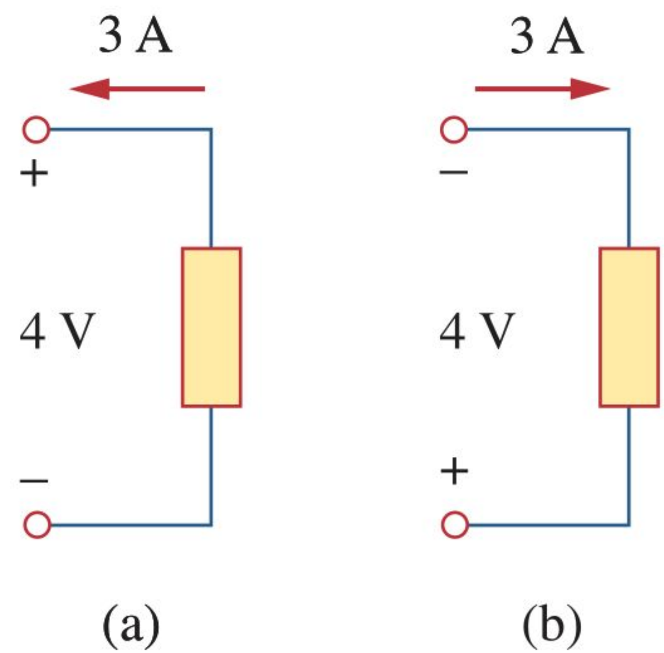


Figure: Two cases of an element with an supplying power of 12 W:

Passive Sign Convention

- Current direction and voltage polarity play a major role in determining the sign of power.
- ***Passive sign convention*** is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

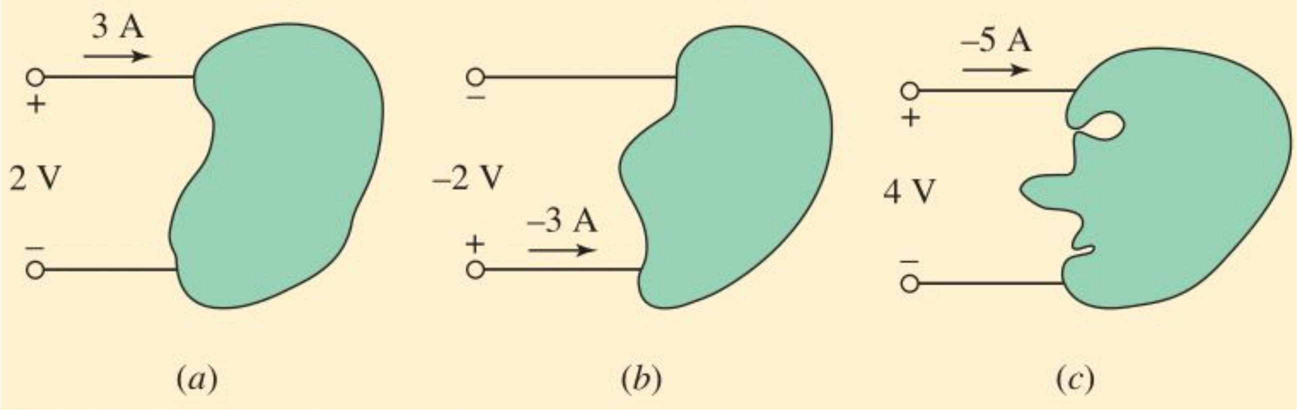


Fig : Compute the power absorbed by each part

Power

- The law of conservation of energy must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\sum p = 0$$

- The total power supplied to the circuit must balance the total power absorbed.

Energy

- Energy is the capacity to do work, measured in joules (J).

- The energy absorbed or supplied by an element from time t_0 to time t is

Circuit Elements

- ✔ There are two types of elements found in electric circuits: ***passive elements*** and ***active elements***.
- ✔ An active element is capable of generating energy while a passive element is not.
- ✔ Examples of passive elements are ***resistors, capacitors, and inductors.***
- ✔ The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them.
- ✔ There are two kinds of sources: **independent and dependent sources.**

VOLTAGE AND CURRENT SOURCES

- An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.
- An ideal independent voltage source is characterized by a ***terminal voltage*** which is completely ***independent of*** the current through it.
- It delivers whatever current is necessary to maintain its terminal voltage.

Independent Voltage Sources

- Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources.
- An automobile storage battery, for example, has a 12 V terminal voltage that remains essentially constant as long as the current through it does not exceed a few amperes.
- An ordinary household electrical outlet also approximates an independent voltage source,

Independent Voltage Sources

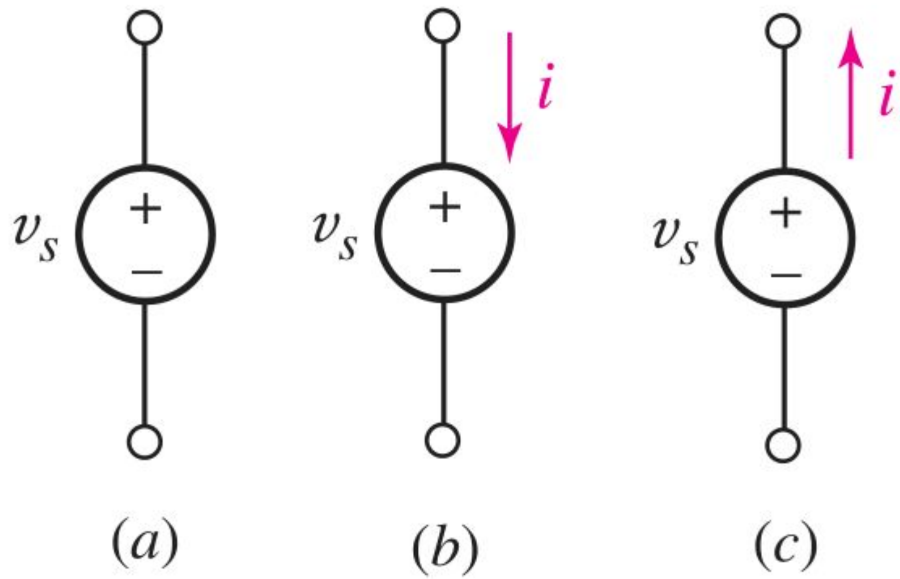


Fig. Circuit symbol of the independent voltage source with constant or time-varying voltage.

Independent Voltage Sources

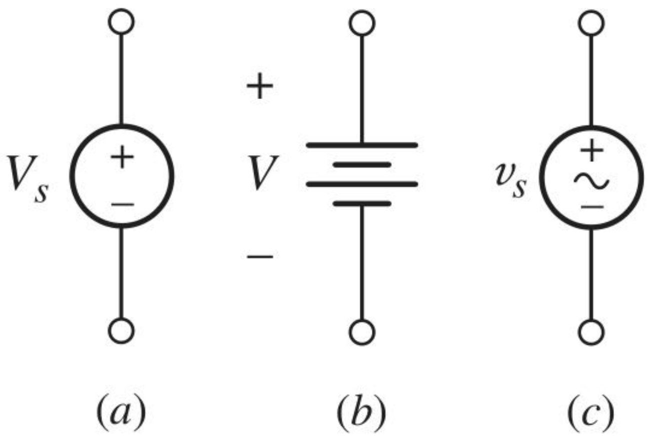
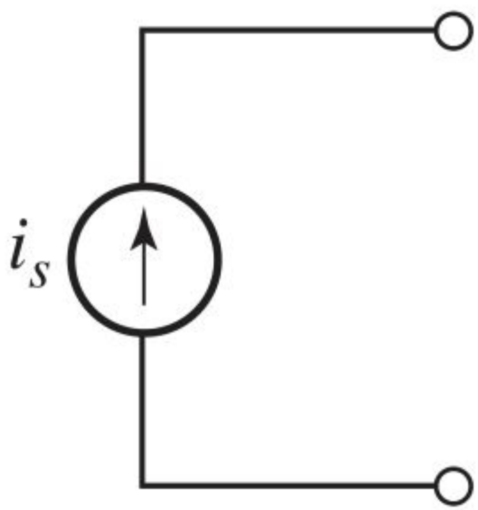


Fig. (a) DC voltage source symbol; (b) battery symbol; (c) ac voltage source symbol.

An independent voltage source with a constant terminal voltage is often termed an independent dc voltage source and can be represented by either of the symbols shown in Fig. a and b.

Independent Current Sources

- Another ideal source which we will need is the independent current source. Here, **the current through** the element is completely **independent** of the **voltage across** it.



If $\boldsymbol{i_s}$ constant, we call the source an independent dc current source.

Fig: Circuit symbol for the independent current source.

Dependent Sources

- An ideal dependent (or controlled) source is an active element in which the source quantity is controlled (or determined) by a voltage or current existing at some other location in the system being analysed.

There are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

Dependent Sources

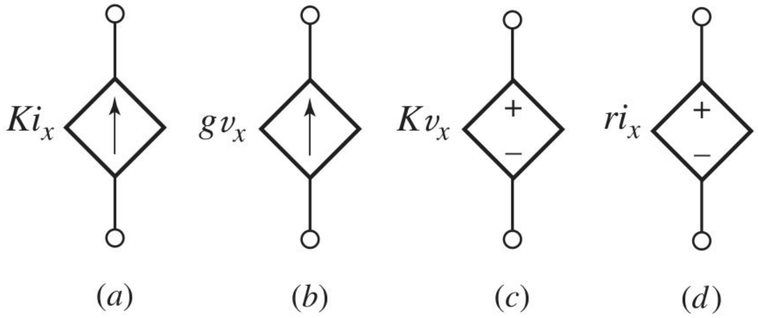
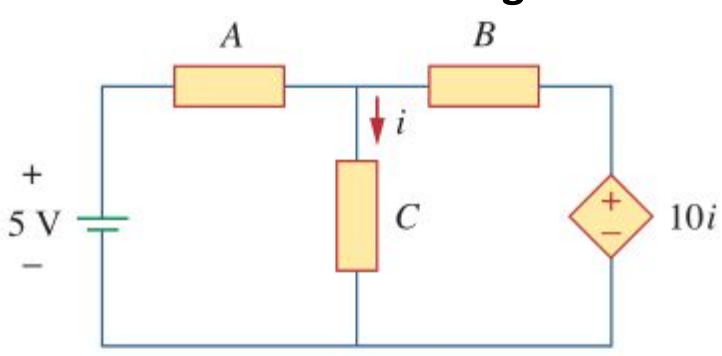


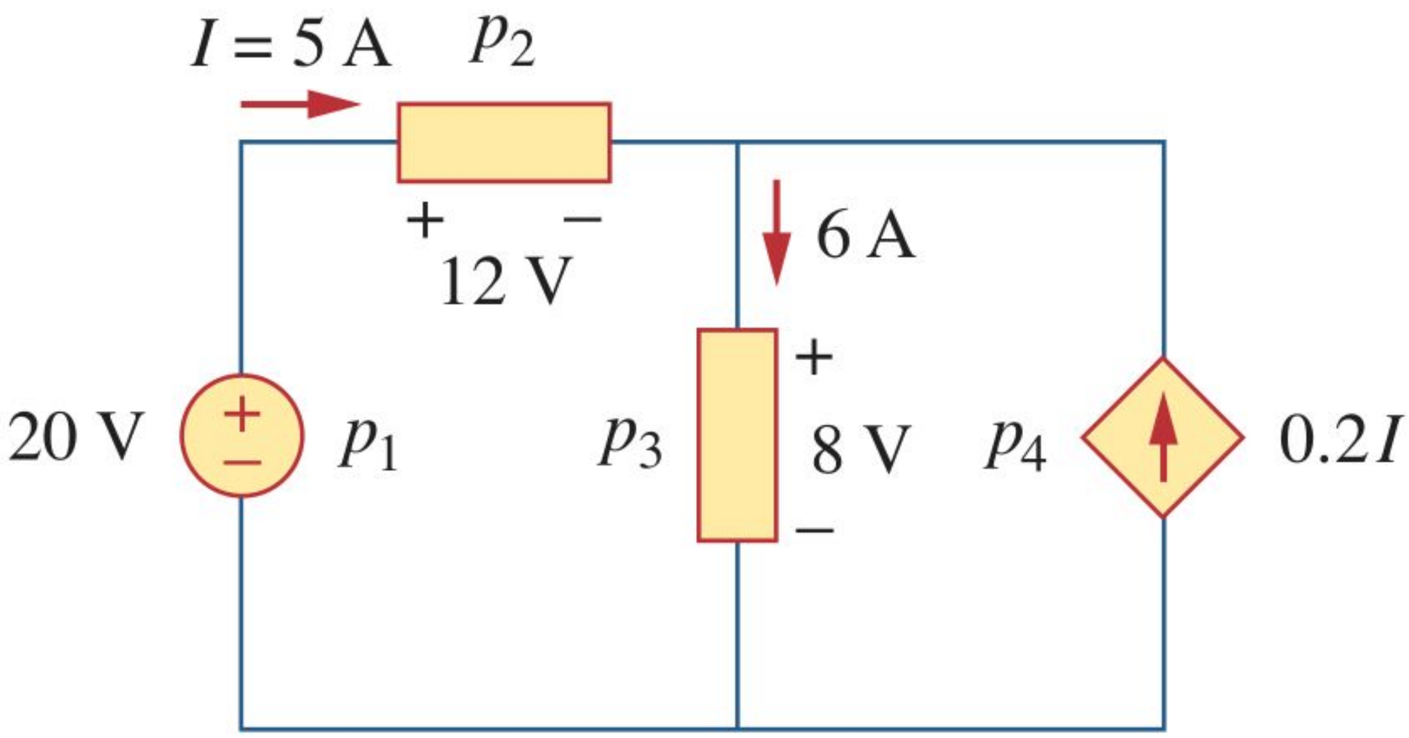
FIGURE :The four different types of dependent sources: (a) current-controlled current source; (b) voltage-controlled current source; (c) voltage-controlled voltage source; (d) current-controlled voltage source.



Dependent sources are useful in modelling elements such as transistors, operational amplifiers, and integrated circuits.

In Fig. 2 a and c, ***K*** is a dimensionless scaling constant. In Fig. b, '***g***' is a scaling factor with units of A/V; in Fig. d, '***r***' is a scaling factor with units of V/A.

Example: Calculate the power supplied or absorbed by each element



RESISTOR

- When electric current flows through a metal wire or through other circuit elements, it encounters a certain amount of resistance, the magnitude of which depends on the electrical properties of the material.
- This physical property, or ***ability to resist current***, is known as resistance and is represented by the symbol R.
- The resistance of any material with a uniform cross-sectional area A depends on **A** and its length *l*.

RESISTOR

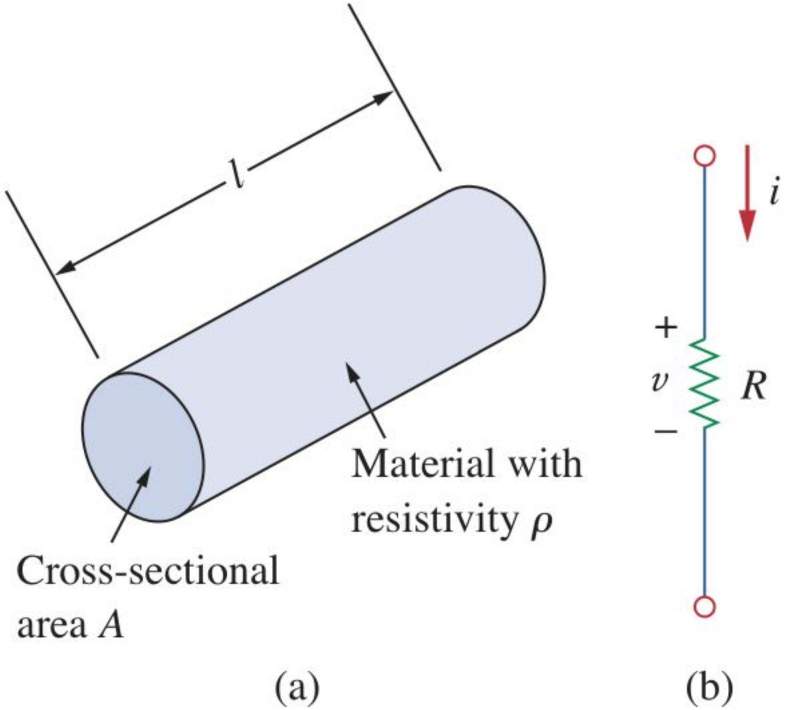


Figure: (a) Resistor, (b) Circuit symbol for resistance.

We can represent resistance in mathematical form,

$$R = \rho \frac{l}{A}$$

Where $\boldsymbol{\rho}$ is known as the resistivity of the material in ohm-meters.

Good conductors, such as copper and aluminium, have **low resistivities**, while **insulators**, such as mica and paper, have **high resistivities**.

OHM'S LAW

Ohm's law states that the voltage across conducting materials is directly proportional to the current flowing through the material.

$$v = Ri$$

where the constant of proportionality ***R*** is called the resistance. The unit of resistance is the **ohm**, which is 1 V/A and customarily abbreviated by a capital omega, Ω.

Not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as **a linear resistor**. Its *i-v* graph is a straight line passing through the origin.

A nonlinear resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in

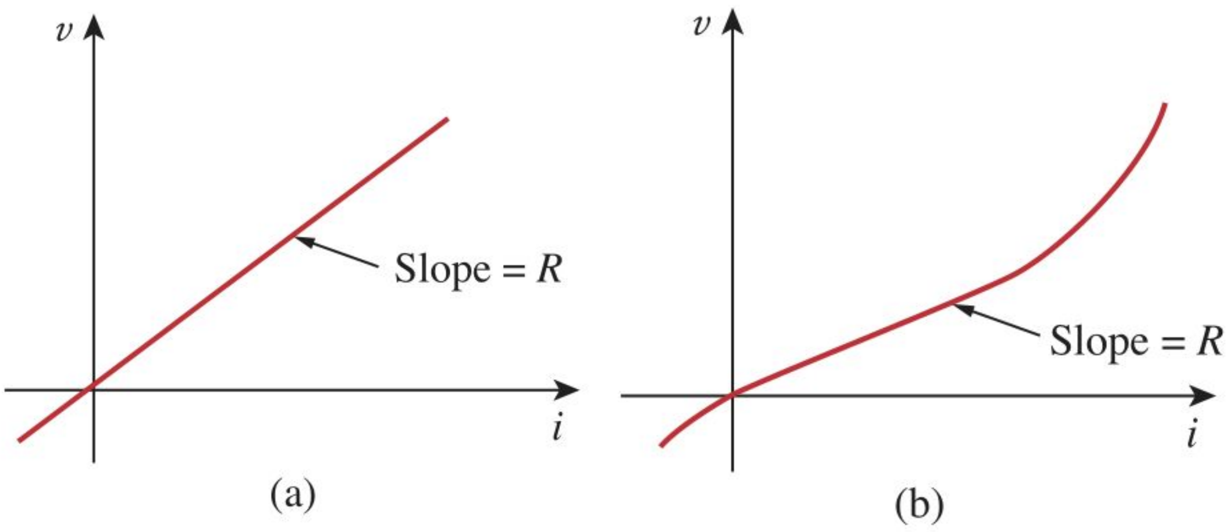


Figure: The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

Short Circuit and Open Circuit

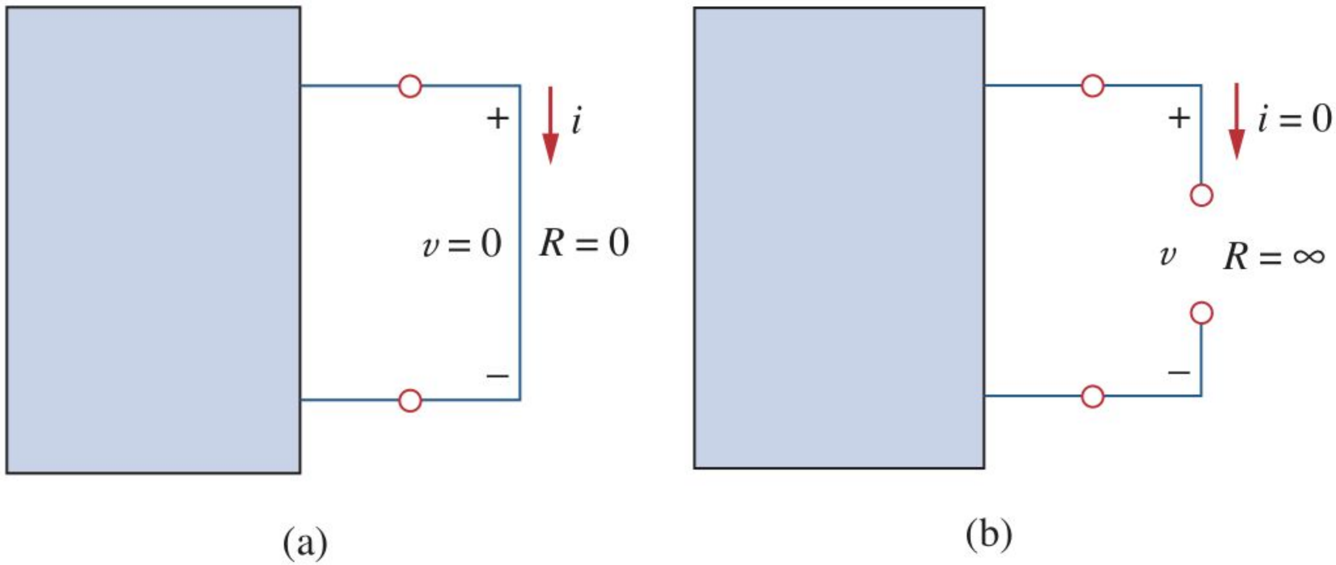


Figure :(a) Short circuit (R=0), (b) Open circuit (R=∞).

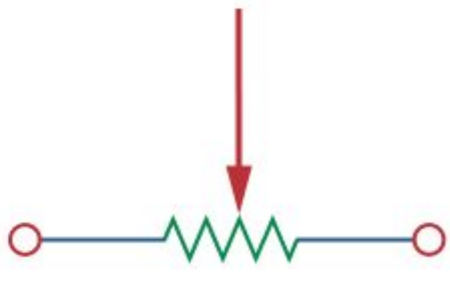
A short circuit is a circuit element with resistance approaching zero.

An open circuit is a circuit element with resistance approaching infinity.

Variable Resistors

- A resistor is either ***fixed or variable***. Most resistors are of the fixed type, meaning their resistance remains constant.
- Variable resistors have adjustable resistance. The symbol for a variable resistor is shown in Fig. (a).
- A common variable resistor is known as a potentiometer or pot for short, with the symbol shown in Fig. (b).

(a)



(b)

Figure: Circuit symbol for: (a) a variable resistor in general, (b) a potentiometer.

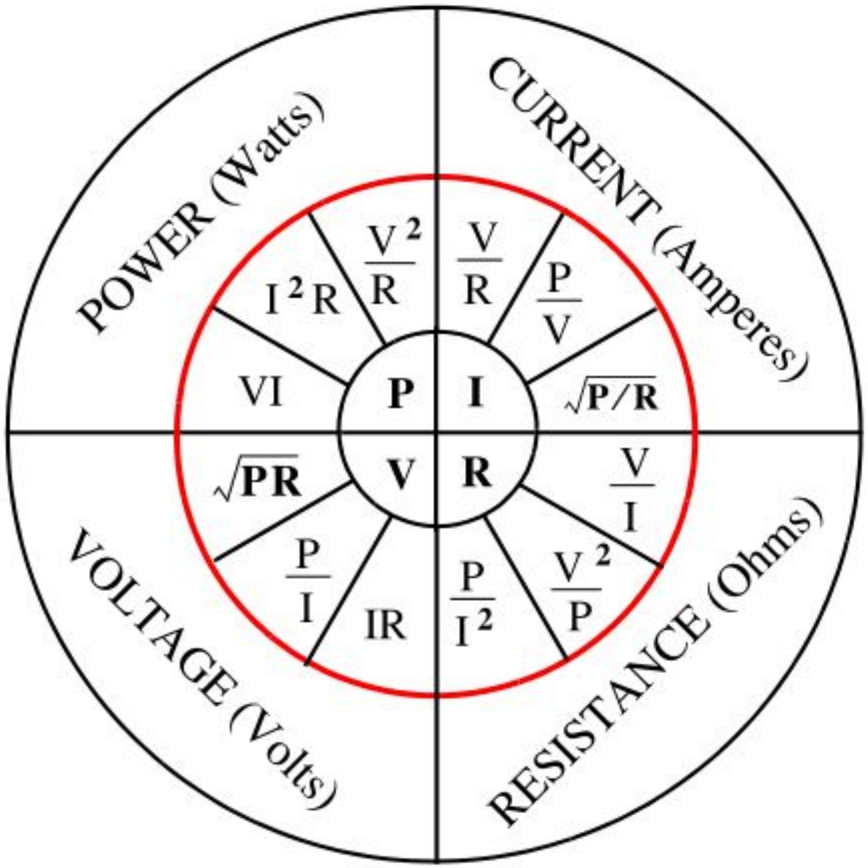
Conductance

- Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).
- It is a measure of how well an element will conduct electric current.
- It is the reciprocal of resistance R, known and denoted by G.

$$G = \frac{1}{R} = \frac{i}{v}; \qquad p = vi = \frac{i^2}{G} = v^2 G$$

Since R and G are positive quantities, the power dissipated in a resistor is always positive. It makes the resistor a passive element

Power Absorbed by a Resistor



VOLTAGE AND CURRENT LAWS

Network and Circuit

- The interconnection of two or more simple circuit elements forms an electrical network.

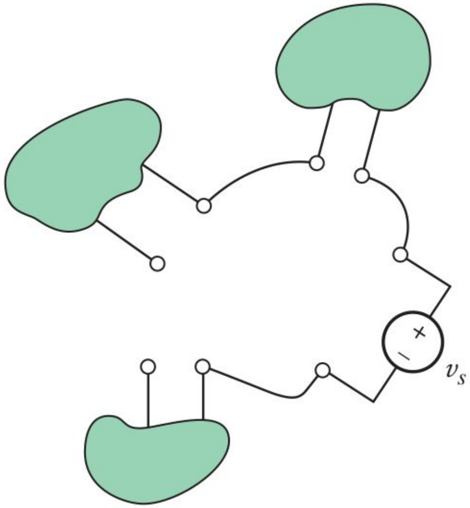


Fig. Electrical Network

- If the network contains at least one closed path, it is also an electric circuit.

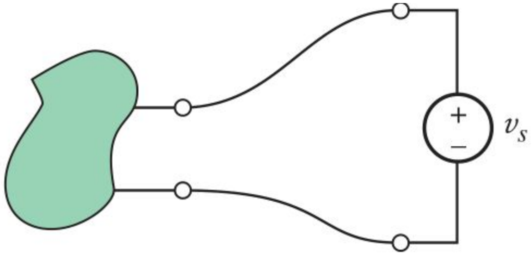


Fig. Electric Circuit

- Every circuit is a network, but not all networks are circuits

Network and Circuit

- A network that contains at least one active element, such as an independent voltage or current source, is an active network.
- A network that does not contain any active elements is a passive network.

NODES, LOOPS, AND BRANCHES

- **A branch** represents a single element such as a voltage source or a resistor. In other words a branch represents any two-terminal element.
- **A node** is the common point at which two or more devices (passive or active) are connected.
- A node is usually indicated by **a dot** in a circuit.

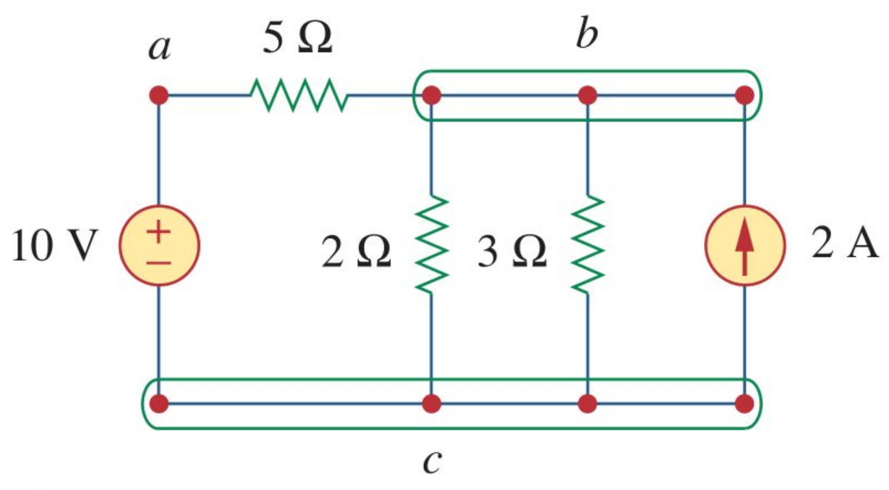


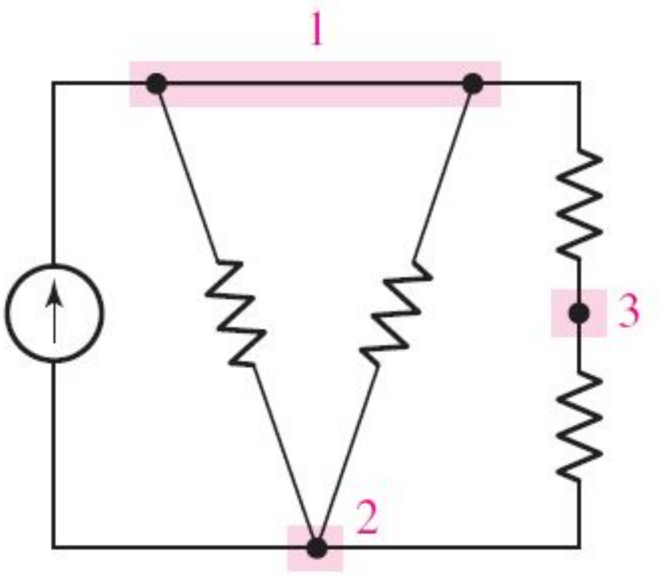
Figure: Illustration of Nodes, branches, and loops.

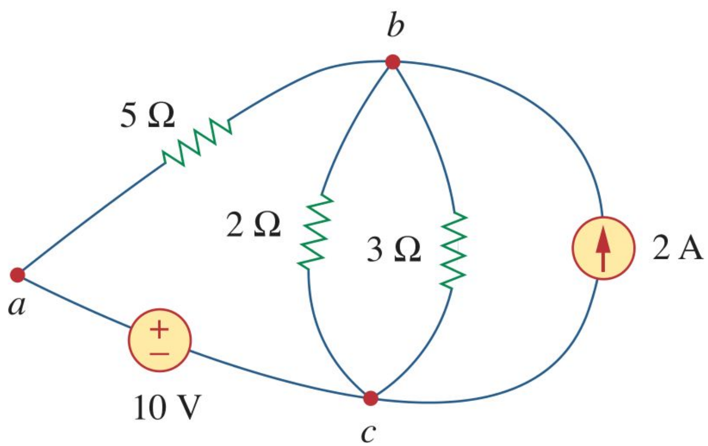
Loop

- A loop is any closed path in a circuit.
- A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- A loop is said to be ***independent*** if it contains at least one branch which is not a part of any other independent loop.
- Independent loops or paths result in independent sets of equations.

Path, Loop

- If we move from node 2 through the current source to node 1, and then through the upper right resistor to node 3, we have established a path; since we have not continued on to node 2 again, we have not made a loop.
- If we proceeded from node 2 through the current source to node 1, down through the left resistor to node 2, and then up through the central resistor to node 1 again, we do not have a path, since a node (actually two nodes) was encountered more than once; we also do not have a loop, because a loop must be a path.
- Another term whose use will prove convenient is branch.





The three-node circuit of previous Fig. is redrawn.

Independent loops:

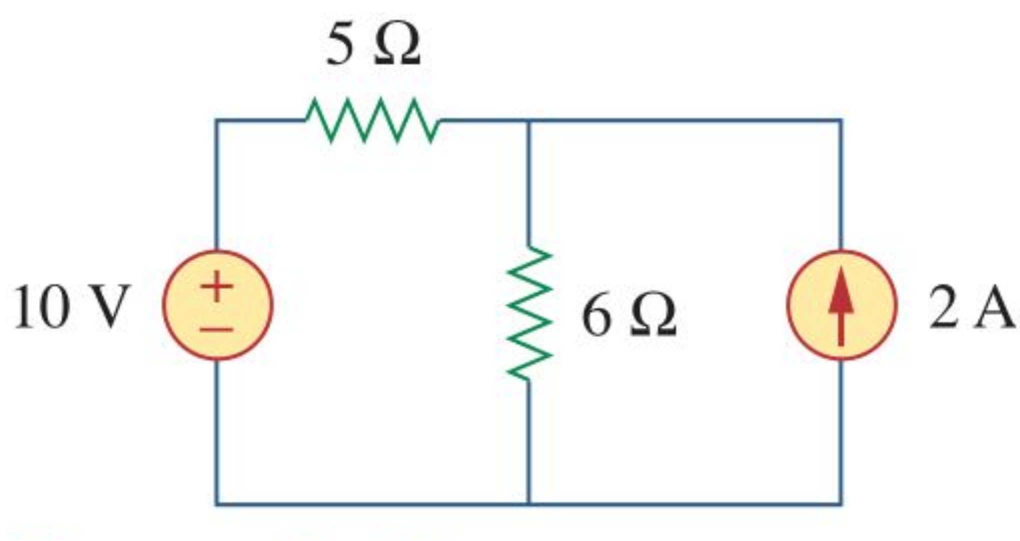
1. ***abca*** with the resistor 2Ω is independent.
2. ***abca*** with the resistor 3Ω is independent.
3. A loop with the resistor 3Ω and the current source is independent.

Two or more elements are in series if they exclusively share a single node and consequently carry the same current.

Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them

Example

- Determine the number of branches and nodes in the circuit shown in below Fig. Identify which elements are in series and which are in parallel.



Kirchhoff's Current Law (KCL)

- KCL states that the algebraic sum of all currents leaving (or entering) a node is equal to zero.
- ***A node is not a circuit element***, and it certainly cannot store, destroy, or generate charge. Hence, the currents must sum to zero.
- If we assign a plus (+) sign to the currents leaving the node, we must assign a minus (-) sign to the currents entering the node.

KCL

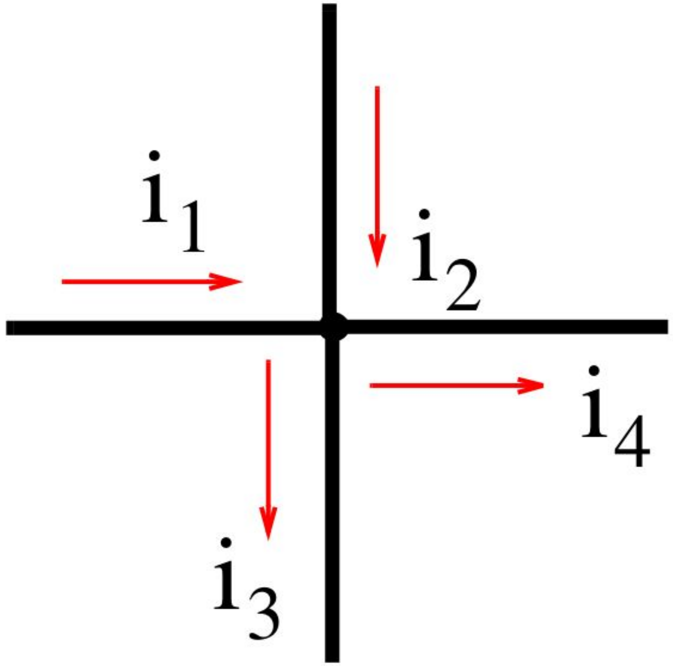


Figure. Node to illustrate KCL

$$-i_1 - i_2 + i_3 + i_4 = 0$$

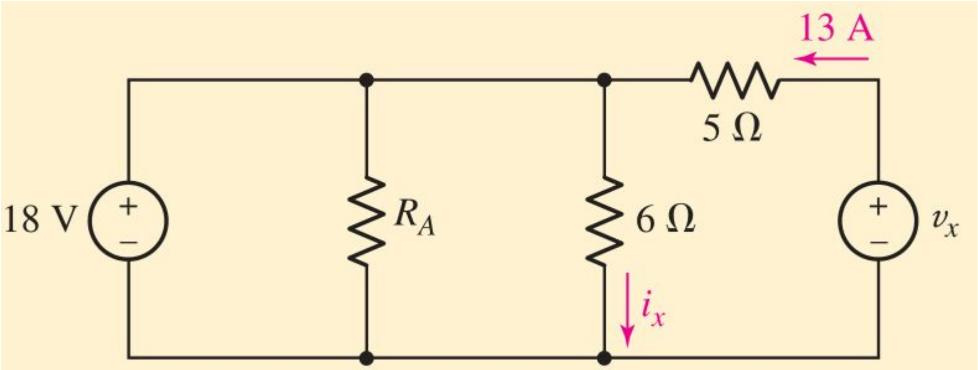
$$\Downarrow$$

$$i_1 + i_2 = i_3 + i_4$$

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Example

Example



If i_x = 3 A and the 18 V source delivers 8 A of current, what is the value of R_A?

Kirchhoff’s voltage law (KVL)

- KVL states that the algebraic sum of the voltage drops (voltages from + to -) or voltage rises (voltages from - to +) around any closed path (mesh or loop) in a circuit is equal to zero.
- If we assign a (+) sign to the voltage drops, we must assign a (-) sign to the voltage rises.
- KVL can be applied in two ways: by taking either a clockwise or a counter clockwise trip around the loop. Either way, the algebraic sum of voltages around the loop is zero.

KVL

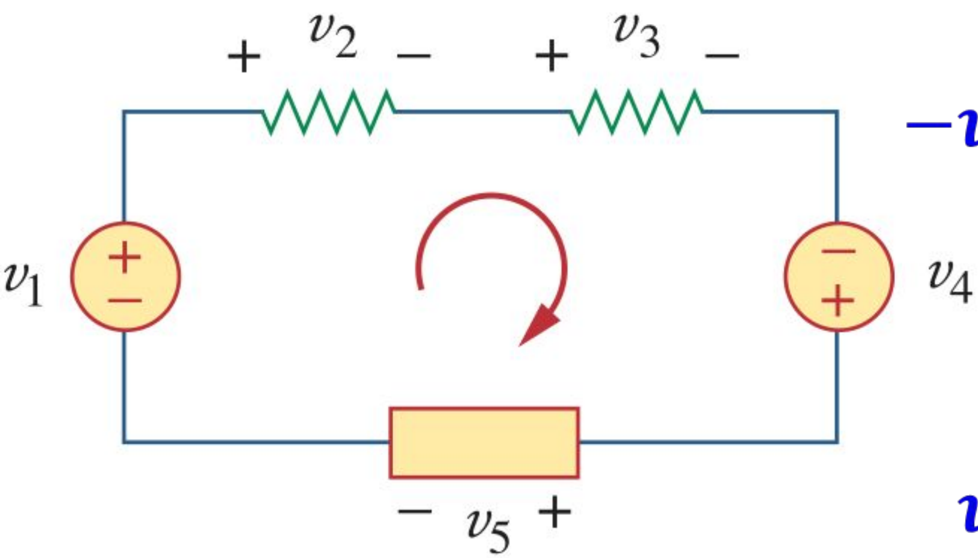


Fig. A single-loop circuit illustrating KVL

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$v_1 + v_4 = v_2 + v_3 + v_5 = 0$$

Sum of voltage drops = Sum of voltage rises

KVL

- When voltage sources are connected in series, KVL can be applied to obtain the total voltage.

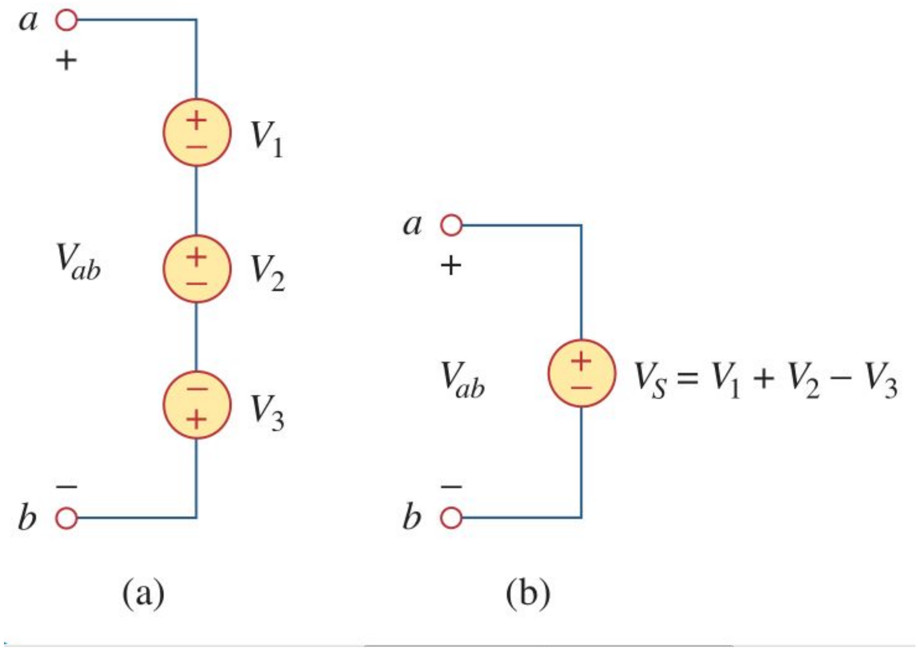


Figure: Voltage sources in series: (a) original circuit, (b) equivalent circuit.

KVL Example

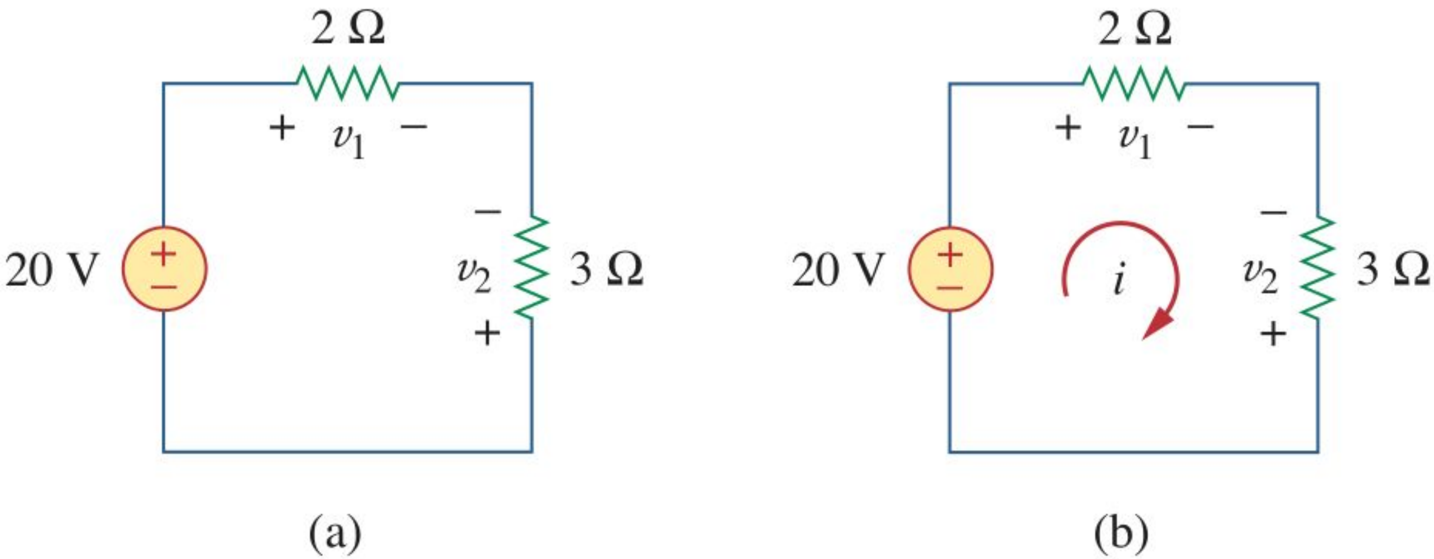


Fig: Find voltages $v_1$and v_2.

Series Resistors and Voltage Division

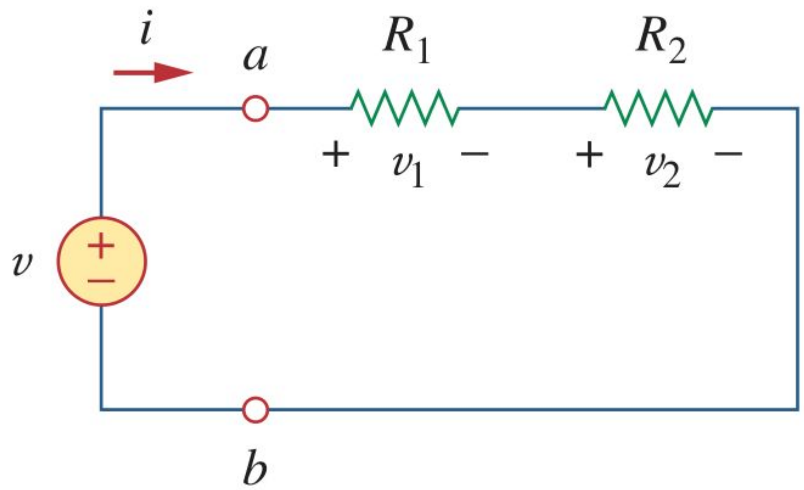


Fig: A single-loop circuit with two resistors in series.

Applying Ohm's law to each of the resistors, we have

$$v_1 = iR_1 \quad v_2 = iR_2$$

Apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0$$
$$v = v_1 + v_2 = i(R_1 + R_2)$$

Series Resistors and Voltage Division

$$v = iR_{eq}\ ;\text{where } R_{eq} = R_1 + R_2$$

Using above equation, an equivalent circuit for the previous circuit can be drawn as shown below where single equivalent resistance R_{eq} replaces the two series resistors.

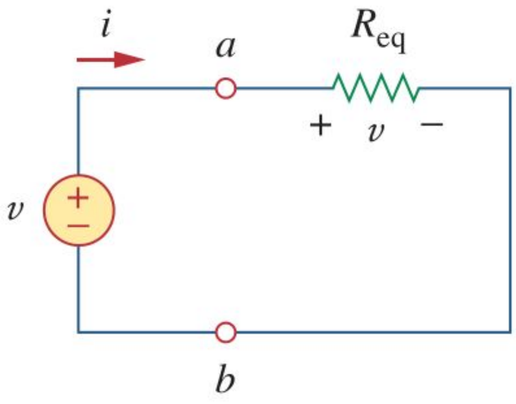
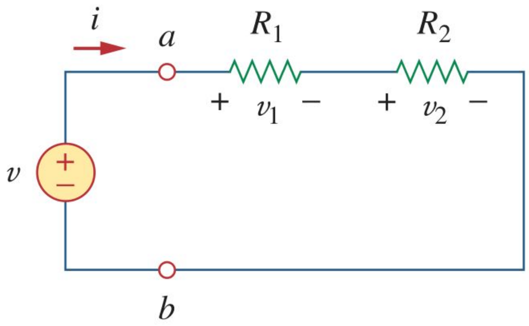


Figure: Equivalent circuit

- Resistors in series behave as a single resistor whose resistance is equal to the sum of the resistances of the individual resistors.

Series Resistors and Voltage Division

- The voltage each resistor can be determined as follows:



$$v_1 = iR_1 = \frac{v}{R_1 + R_2} R_1$$

$$v_2 = iR_2 = \frac{v}{R_1 + R_2} R_2$$

Here, the source voltage *v* is ***divided among the resistors*** in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the ***principle of voltage division*** and that circuit is called ***voltage divider***

Series Resistors and Voltage Division

- If a voltage divider has N resistors in series with the source voltage v, the n^{th} resistor $(R_1, R_2, R_3, \ldots R_N)$ will have a voltage drop of

$$v_n = \frac{R_n}{R_1 + R_2 + R_3 + \cdots + R_N} v$$

Parallel Resistors and Current Division

Consider the circuit in Fig. below, where two resistors are connected in parallel and therefore have the same voltage across them.

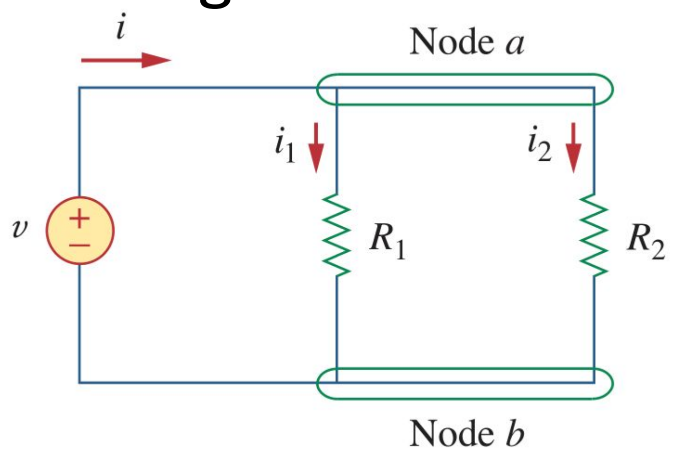


Fig: Two resistors in parallel.

- From Ohm's law,

$$v = i_1 R_1 = i_2 R_2$$

And $i = i_1 + i_2$

$$i = \frac{v}{R_1} + \frac{v}{R2} = v\left[\frac{R_1 + R_2}{R_1 R_2}\right]$$

$$\frac{v}{i} = \frac{R_1 R_2}{R_1 + R_2} = R_{eq}$$

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

Parallel Resistors and Current Division

- The current through the resistor R_1 can be obtained as

$$v = i\frac{R_1 R_2}{R_1 + R_2} = i_1 R_1$$

$$i_1 = i\frac{R_2}{R_1 + R_2}$$

$$i_2 = i\frac{R_1}{R_1 + R_2}$$

The total current i is being ***shared by*** the resistors in ***inverse proportion*** to their resistances. This is known as the ***principle of current division***, and **the circuit** in

Parallel Resistors: Special Cases

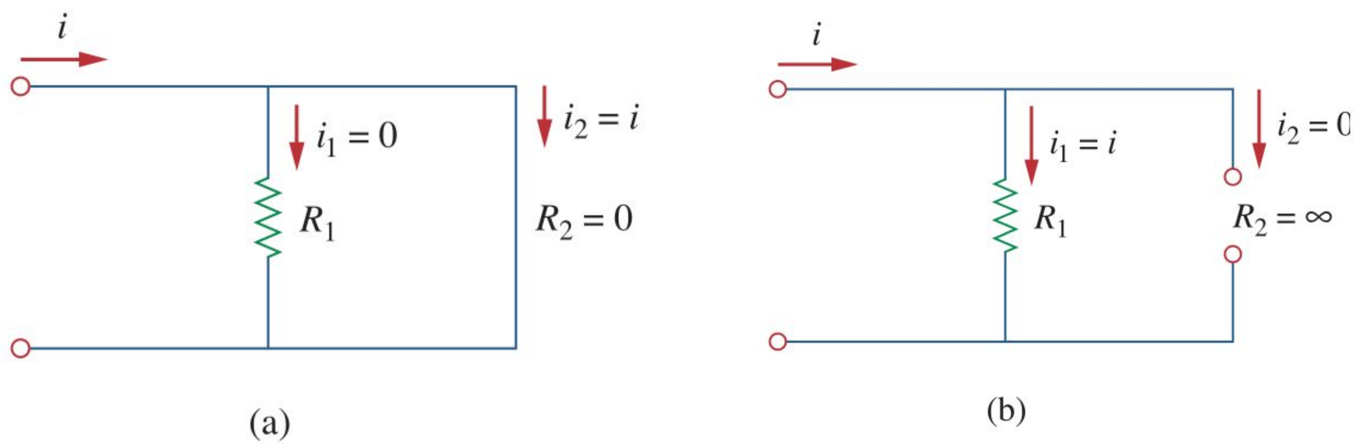


Figure: (a) A shorted circuit, (b) an open circuit.

In an extreme case where one of the resistors in above Fig. is zero (i.e. R_2=0). Then we have $R_{eq} = \frac{R_1 R_2}{R_1+R_2} = 0, i_1 = i\frac{R_2}{R_1+R_2} = 0$ and $i_2 = i\frac{R_1}{R_1+R_2} = i$.

The entire current *i* bypasses and flows through **the short circuit**, the path of least resistance.

Parallel Resistors: Special Cases

- Consider another extreme case, where $R_2 = \infty$, that is, R_2 is an open circuit, as shown in previous Fig.

Then $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = R_1$, $i_1 = i$ and $i_2 = 0$.

The current still flows through the path of least resistance.

Parallel Resistors

- In case of a circuit with N resistors in parallel, the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

Note that is always smaller than the resistance of the smallest resistor in the parallel combination.

If $R_1 = R_2 = R_3 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N}$$

It is often more convenient to use conductance rather than

Parallel Resistors

- The equivalent conductance for N resistors in parallel is

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

where, $G_{eq} = \frac{1}{R_{eq}}, G_1 = \frac{1}{R_1}, G_2 = \frac{1}{R_2}, \ldots\ldots G_N = \frac{1}{R_N}$

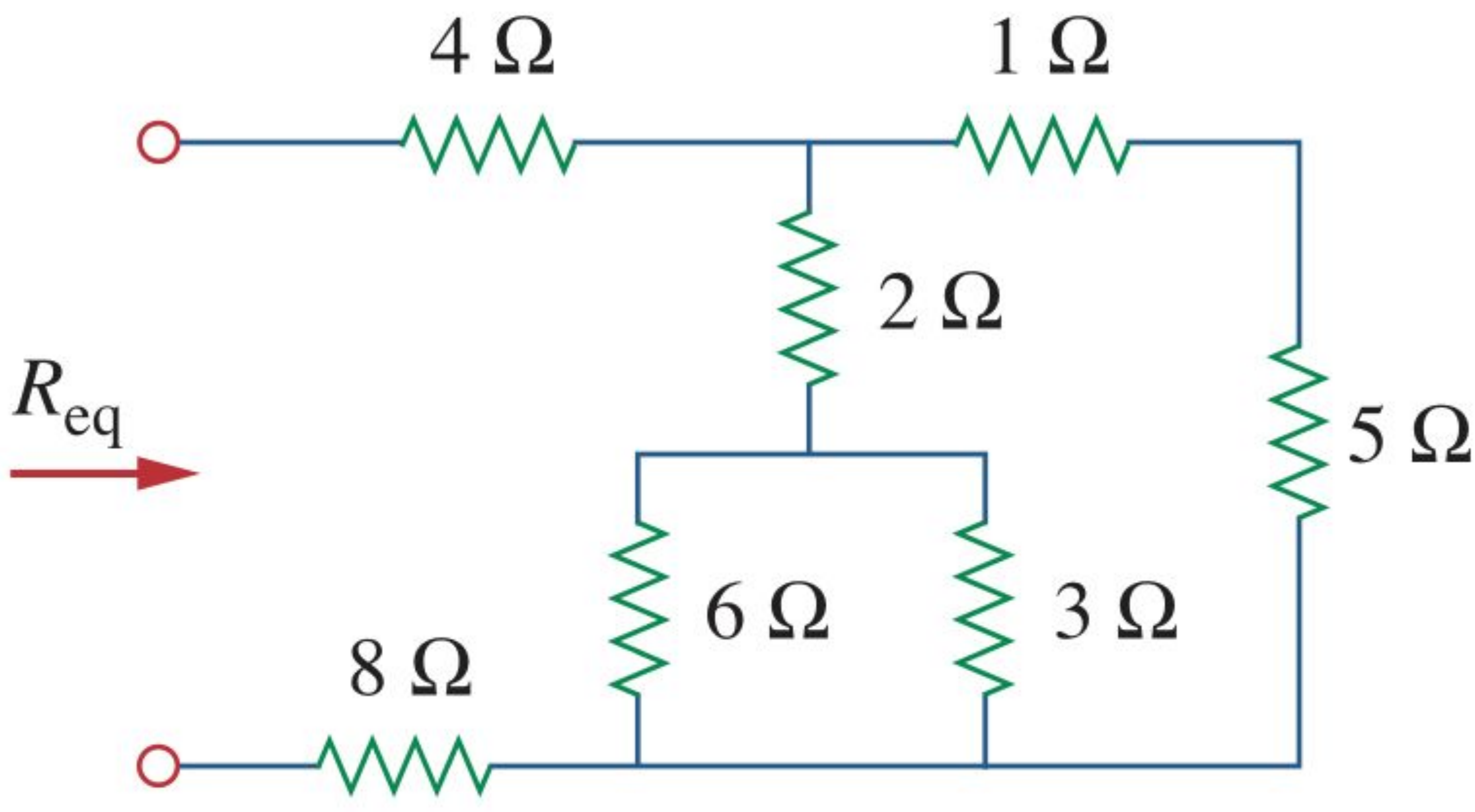
The equivalent conductance of parallel resistors is obtained the same way as the equivalent resistance of series resistors.

If a current divider has N conductors $(G_1, G_2, G_3, \ldots G_N)$ in parallel with the source current i, the nth conductor (G_n) will have current

$$i_n = \frac{G_n}{G_1 + G_2 + G_3 + \cdots + G_N} i$$

Example

- ***Find R_{eq}***



Wye-Delta Transformations

- Situations often arise in circuit analysis when the resistors are neither in parallel nor in series as shown in Fig. here.

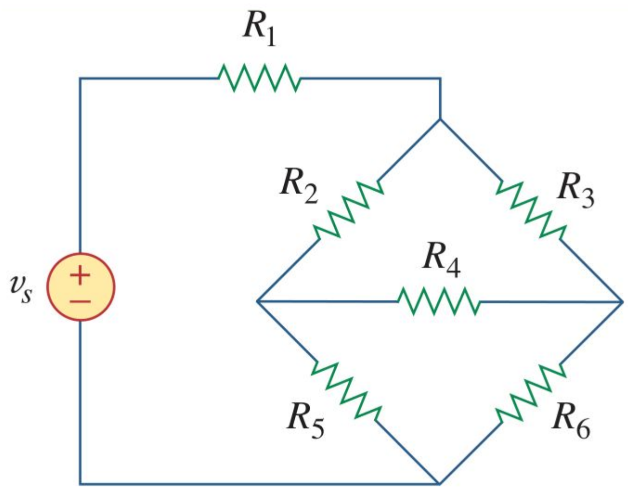


Fig. The bridge network.

Many circuits of this type can be simplified by using three-terminal equivalent networks known as the wye (Y) and the delta (Δ) networks.

Wye-Delta Transformations

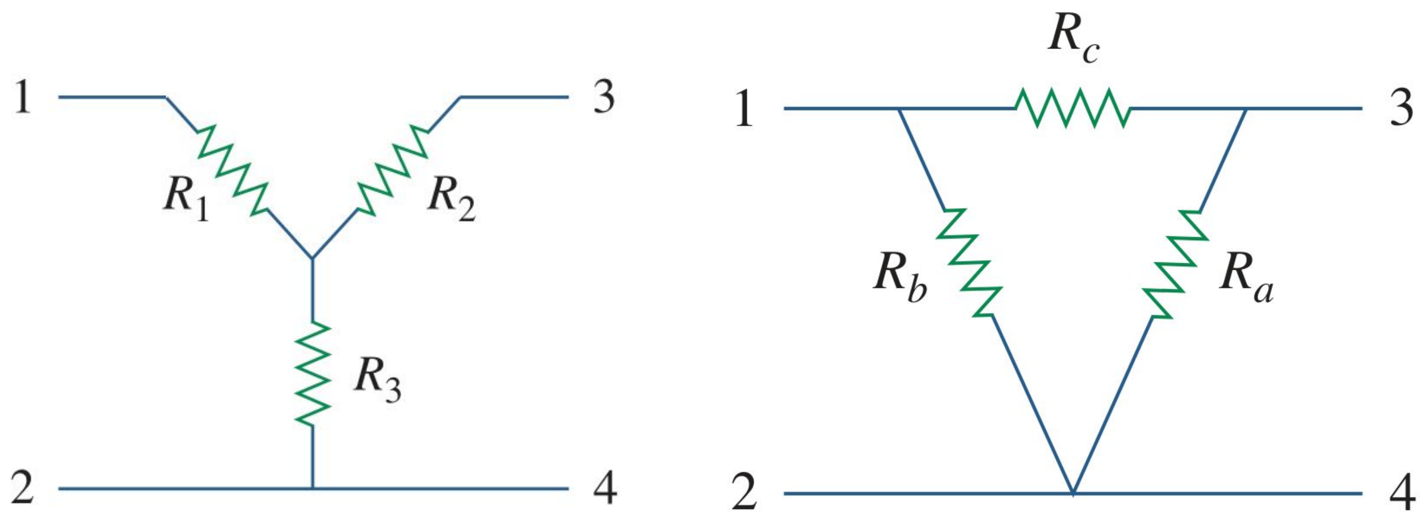


Figure: The wye and delta networks

Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. Then the existing delta network will be transformed into an wye network.

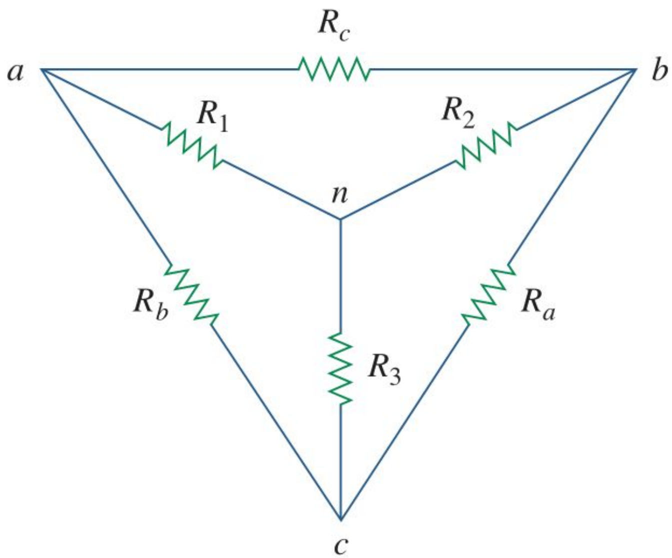


Fig: Superposition of Y and Δ networks

The equivalent resistances in the wye network are

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Wye to Delta Conversion

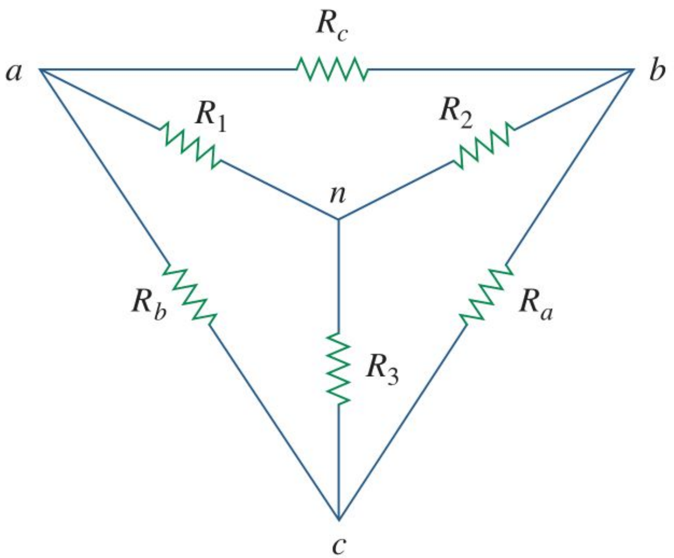


Fig: Superposition of Y and Δ networks

The equivalent resistances in the delta network can be estimated as

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

The Y and Δ networks are said to be balanced when $R_1 = R_2 = R_3 = R_Y$, $R_a = R_b = R_c = R_\Delta$. Under these conditions, $R_Y = \frac{R_\Delta}{3}$ and $R_\Delta = 3R_Y$

Examples

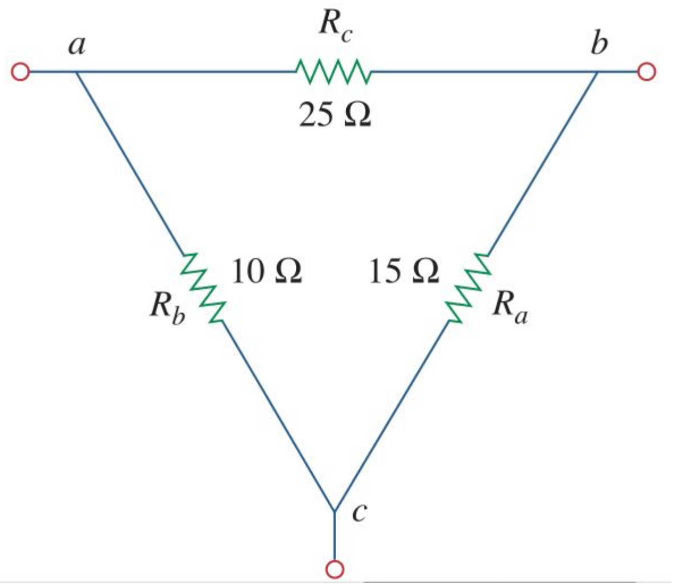


Fig.1: Convert the Δ network to Y network

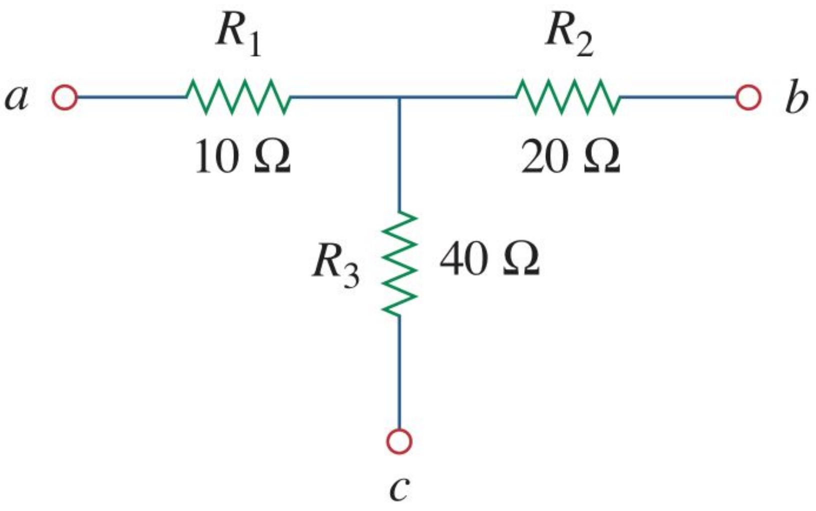


Fig.2: Convert the Y network to Δ network

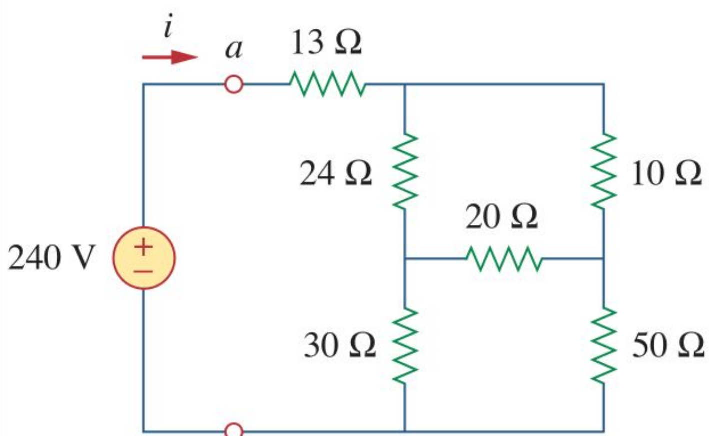
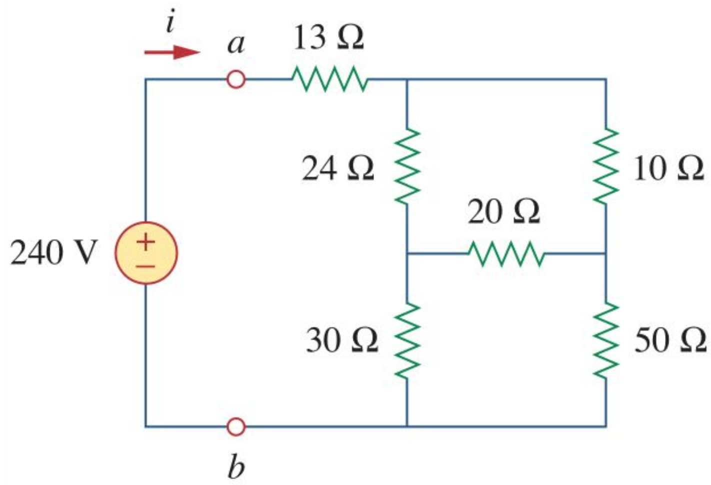


Fig.3: Calculate R_{ab} and I .



i
a
13 Ω
24 Ω
10 Ω
20 Ω
240 V
+
−
30 Ω
50 Ω
b

Mesh and Nodal Analysis

Nodal analysis and mesh analysis—both of which allow us to investigate many different circuits with a consistent, methodical approach.

Nodal analysis is based on a systematic application of Kirchhoff’s current law (KCL), and mesh analysis is based on a systematic application of Kirchhoff’s voltage law (KVL).

Nodal analysis

- Nodal analysis provides a general procedure for analysing circuits using ***node voltages*** as the ***circuit variables***.
- In nodal analysis, we are ***interested*** in finding the ***node voltages***. Given a circuit with ***n*** nodes <u>without</u> <u>voltage sources</u>, the nodal analysis of the circuit involves taking the following three steps.
- Circuits that contain voltage sources will be analysed later.

Steps to Determine Node Voltages

1. **Select** a node as the reference **node**. Assign voltages $v_1, v_2, \ldots\ v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.
2. **Apply KCL** to each of the $n-1$ **non-reference** nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Reference Node

The reference node is commonly called ***the ground*** since it is ***assumed*** to have ***zero potential.*** However, it is important to remember that ***any node can be*** designated as the ***reference node***. Thus, the reference node is at zero volts with respect to the other defined nodal voltages, and not necessarily with respect to ***earth ground***.

A reference node is indicated by any of the three symbols shown in below Fig.

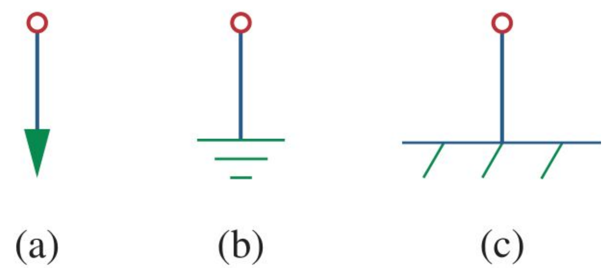


Figure: Common symbols for indicating a reference node, (a) common ground, (b) ground, (c) chassis ground.

Nodal Analysis

- Consider the circuit in Fig. Node 0 is the reference node while nodes 1 and 2 are assigned voltages $v1$ and $v2$ and respectively.

Figure: Typical circuit for nodal analysis.

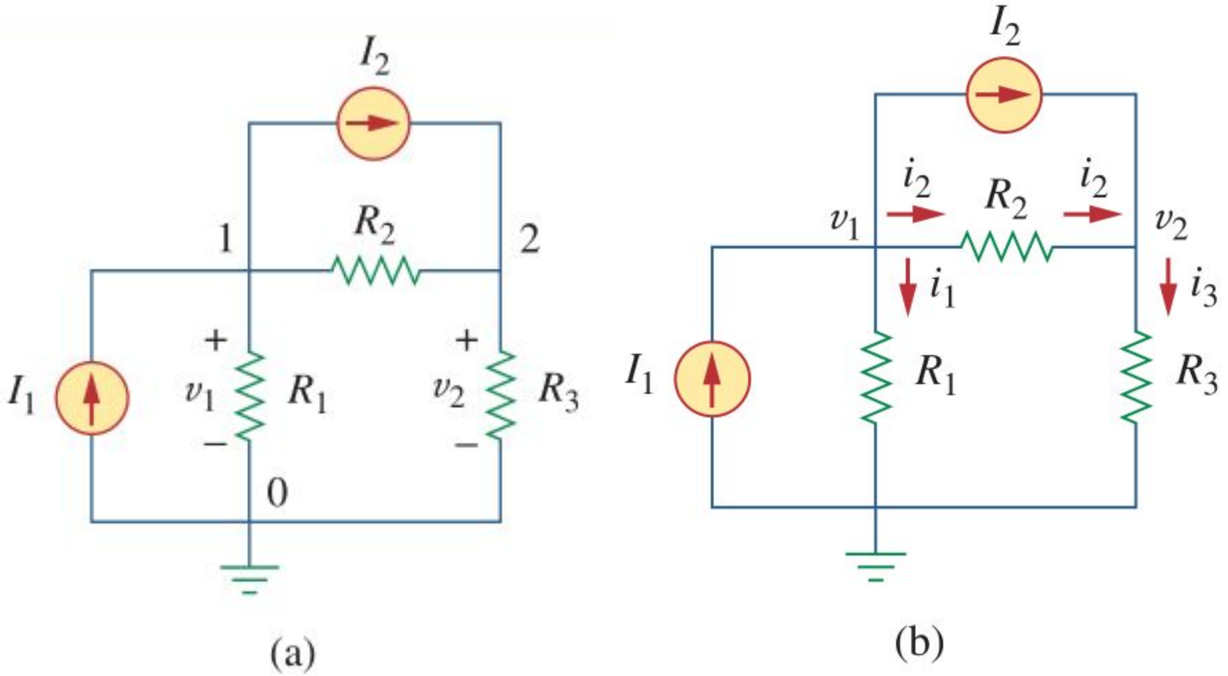


Fig. (a) is redrawn in Fig. (b), where the currents through resistors R_1, R_2 and R_3 are indicated as i_1, i_2 and i_3 respectively.

Nodal Analysis

- As the second step, apply KCL to each non-reference node in the circuit.

At node 1:

$$I_1 = I_2 + i_1 + i_2$$

At node 2:

$$I_2 + i_2 = i_3$$

Now apply Ohm's law to express the unknown currents and in terms of node voltages.

$$i_1 = \frac{v_1 - 0}{R_1}, i_2 = \frac{v_1 - v_2}{R_2}, i_3 = \frac{v_2 - 0}{R_3}$$

Note: ***Current flows from a higher potential to a lower potential in a resistor.***

Nodal Analysis

- At node 1: $I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2}$

At node 2: $I_2 + \frac{v_1 - v_2}{R_2} = \frac{v_2}{R_3}$

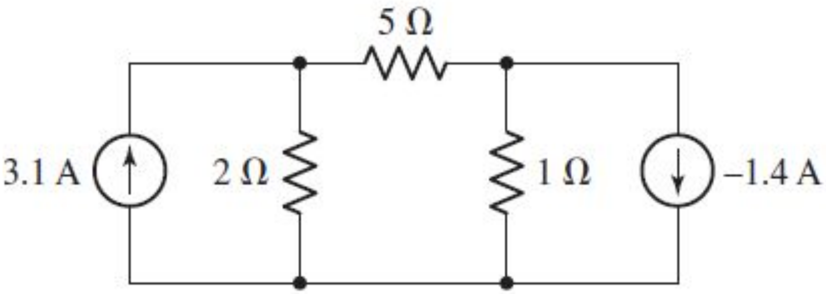
In terms of the conductance's,

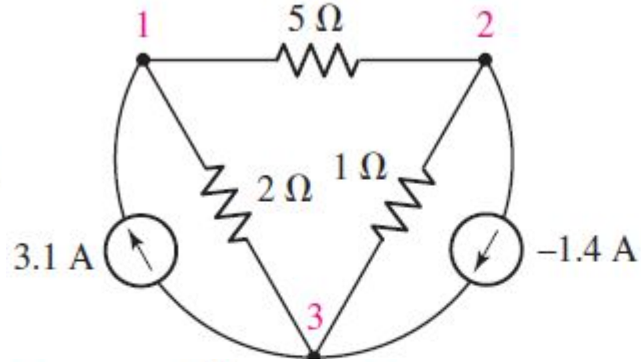
$$I_1 = I_2 + G_1 v_1 + G_2(v_1 - v_2)$$
$$I_2 + G_2(v_1 - v_2) = G_3 v_2$$

In matrix form

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

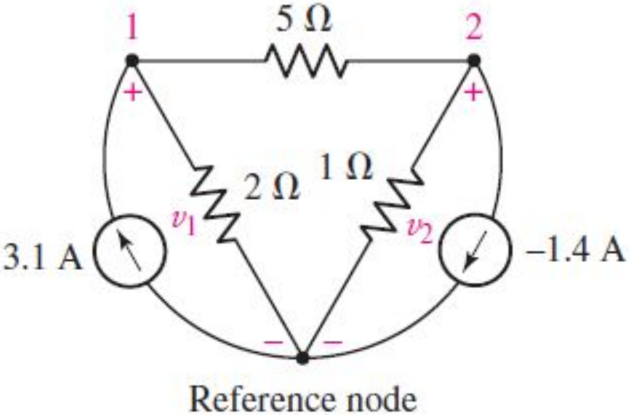
Example

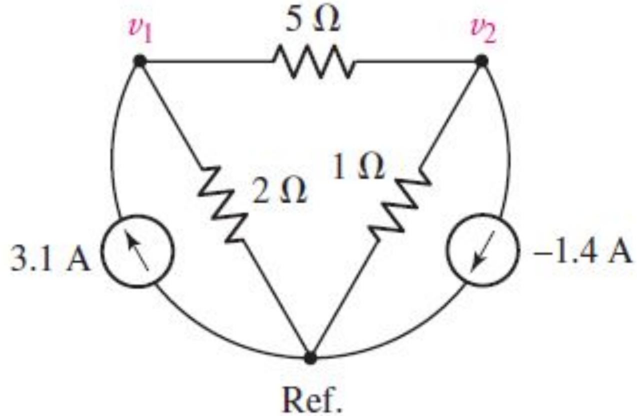




$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1$$

$$0.7v_1 - 0.2v_2 = 3.1$$





$$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4)$$

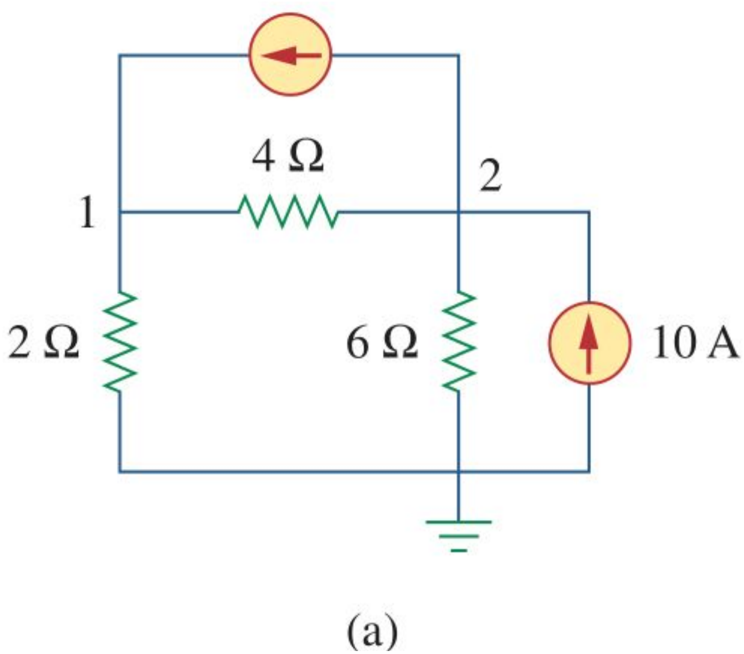
$$-0.2v_1 + 1.2v_2 = 1.4$$

$$v_1 = 5 \text{ V and } v_2 = 2 \text{ V.}$$

From this, it is straightforward to determine the voltage across the 5 Ohm resistor: $v_5 = v_1 - v_2 = 3$ V.

Example

Calculate the node voltages in the circuit shown here



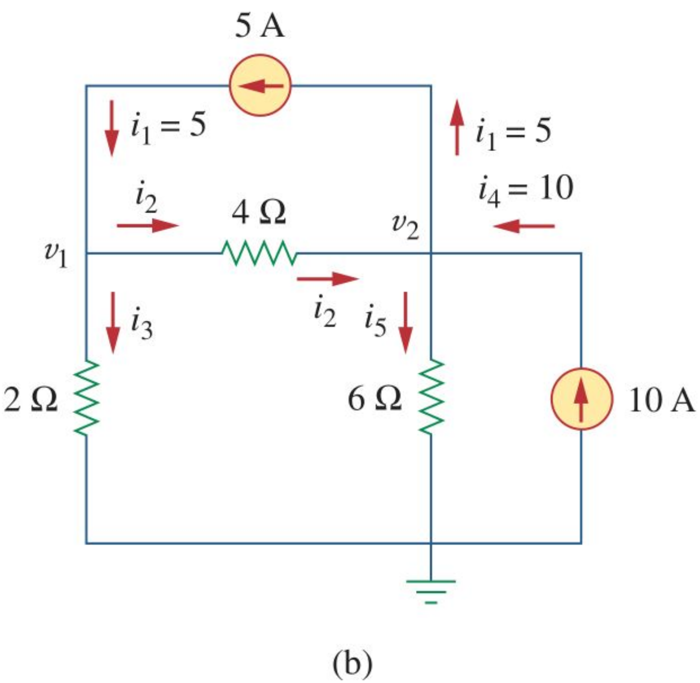
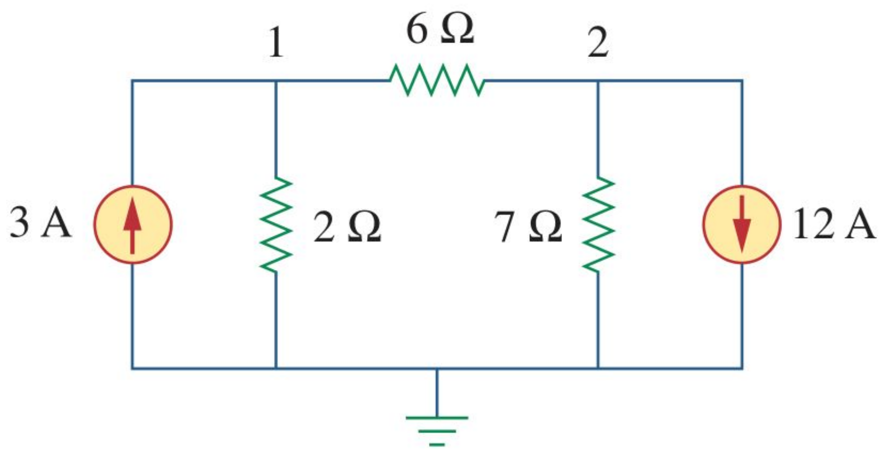
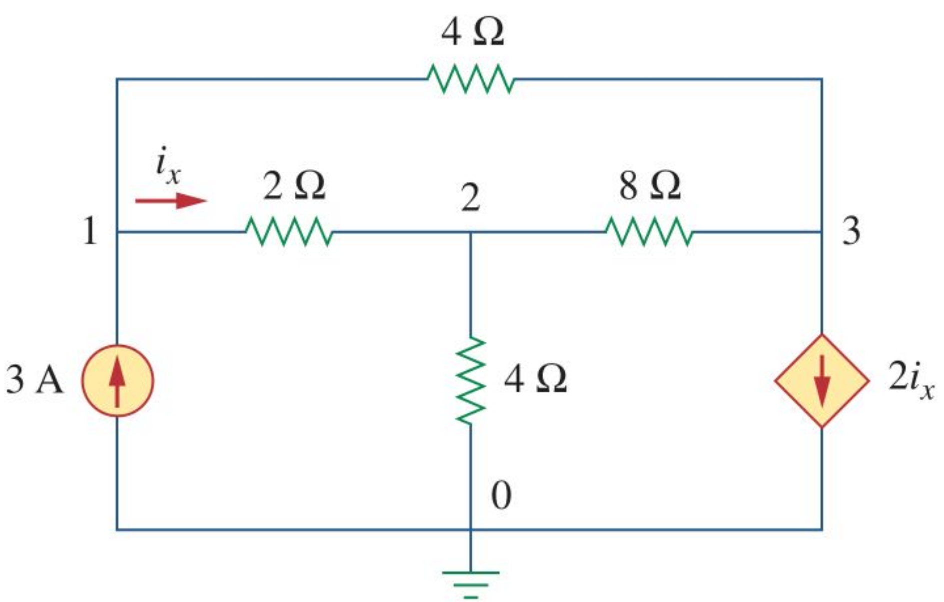


Figure: (a) original circuit, (b) circuit for analysis.

Examples

Obtain the node voltages in the circuits shown below.





Nodal Analysis with Voltage Sources

Consider the circuit shown here for illustration.

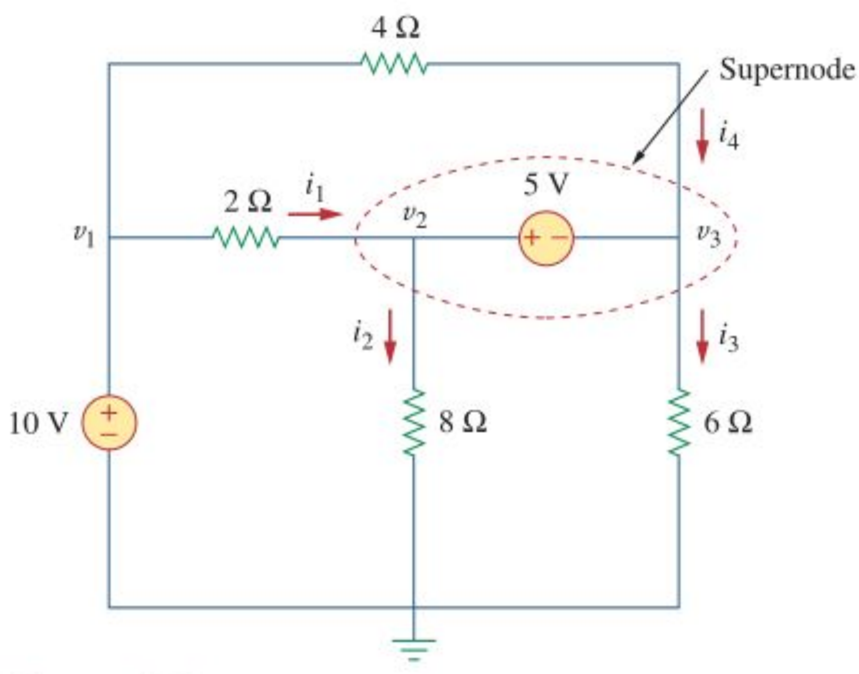


Figure: A circuit with a supernode.

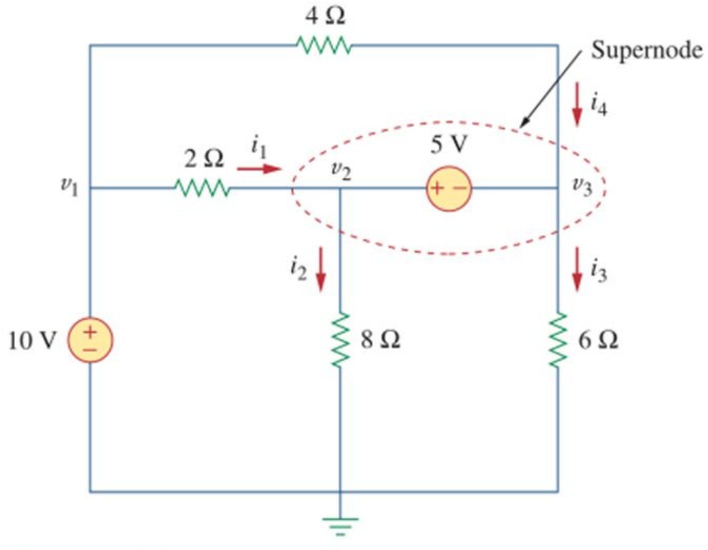
CASE 1

If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source.

$$v_1 = 10\ V$$

Nodal Analysis with Voltage Sources

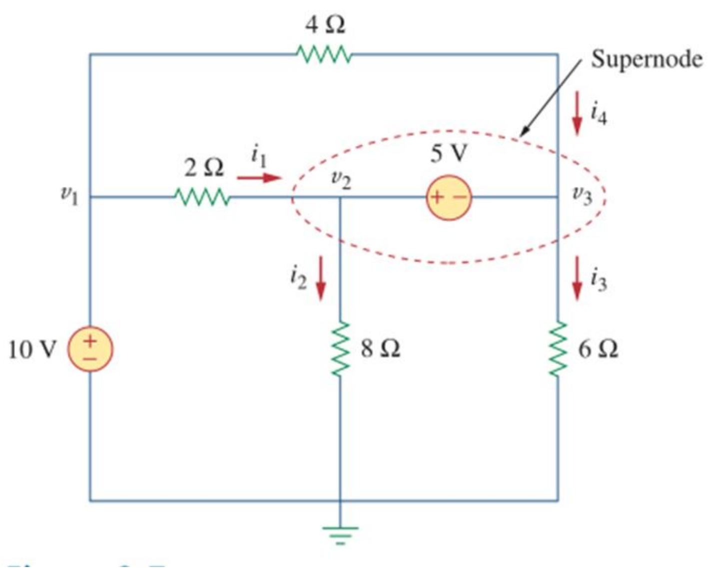
CASE2: If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a generalized node or supernode;



To treat this case, node 2, node 3, and the voltage source together form a supernode and apply KCL to both nodes at the same time. KCL must be satisfied at a supernode like any other node.

Nodal Analysis with Voltage Sources

As we have three nonrefernce nodes, we need three equations to obtain nodal voltages.



At node 1: $v_1 = 10\,V$

Between nodes 2 and 3 :

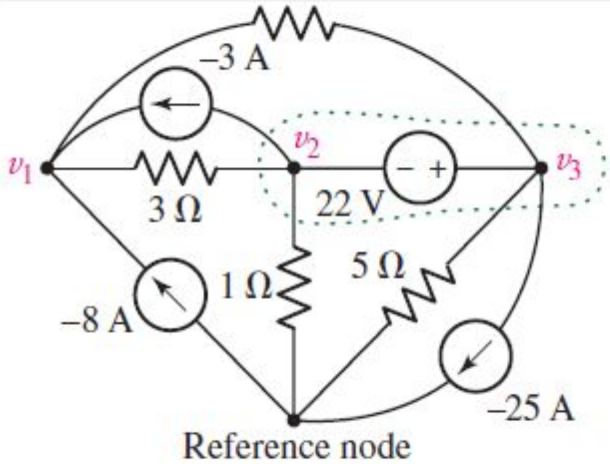
$$v_2 - v_3 = 5\,V$$

KCL at Super node:

$$i_1 + i_4 = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

Example



Determine the value of the unknown node voltage *v*1 in the circuit

$$-8 - 3 = \frac{v_1 - v_2}{3} + \frac{v_1 - v_3}{4}$$

$$0.5833v_1 - 0.3333v_2 - 0.2500v_3 = -11$$

Now, we consider the 2-3 supernode. Two current sources are connected, and four resistors. Thus,

$$3 + 25 = \frac{v_2 - v_1}{3} + \frac{v_3 - v_1}{4} + \frac{v_3}{5} + \frac{v_2}{1}$$

$$-0.5833v_1 + 1.3333v_2 + 0.45v_3 = 28$$

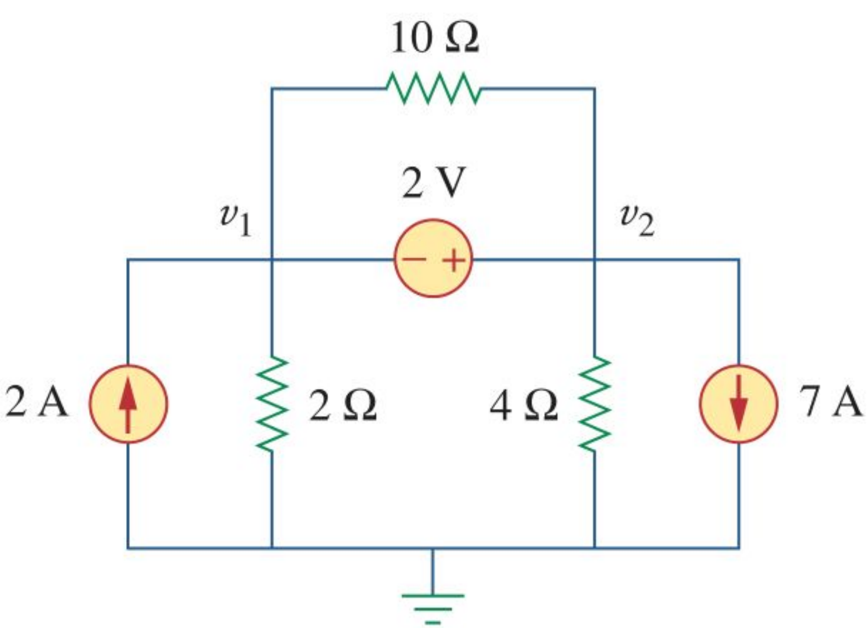
Since we have three unknowns, we need one additional equation, and it must utilize the fact that there is a 22 V voltage source between nodes 2 and 3:

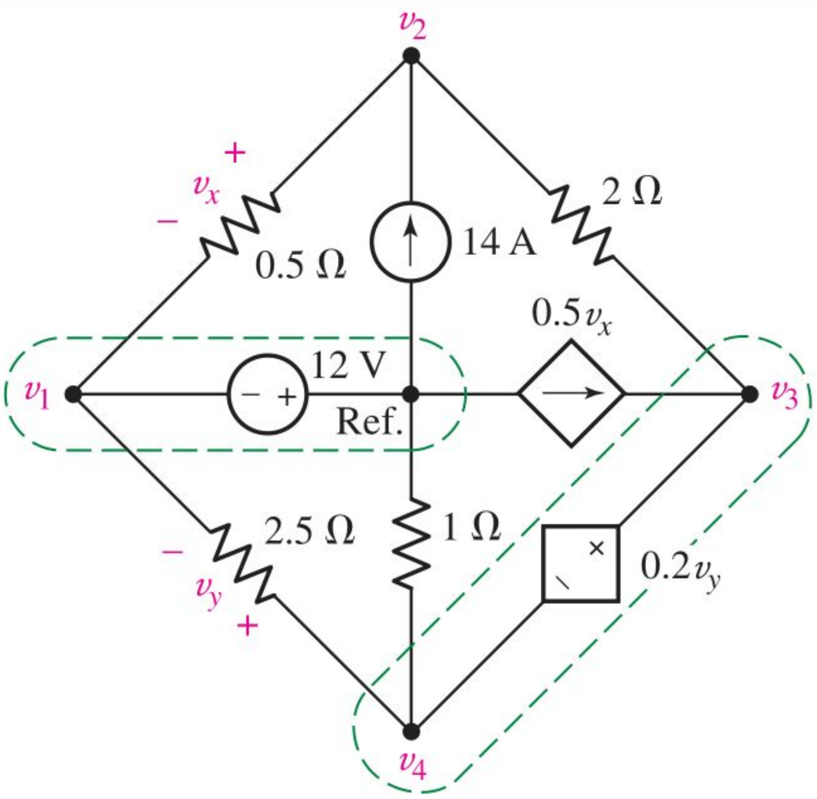
$$v_2 - v_3 = -22$$

From the above, The solution for v_1 is 1.071 V

Examples

- Determine the node-to-reference voltages in the circuits shown below





Mesh Analysis-Introduction

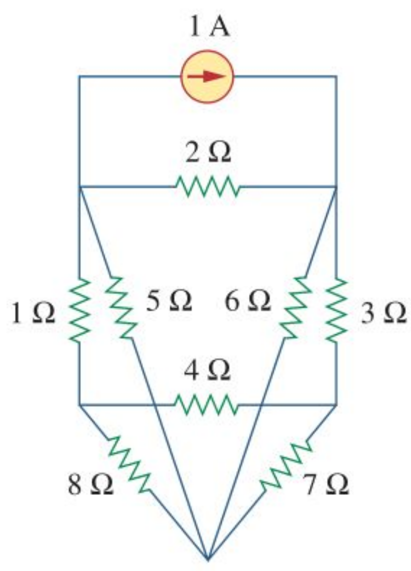
- Mesh analysis provides another general procedure for analysing circuits, using mesh currents as the circuit variables.
- Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.
- Mesh analysis ***applies KVL*** to find unknown currents.
- Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar.

Mesh Analysis

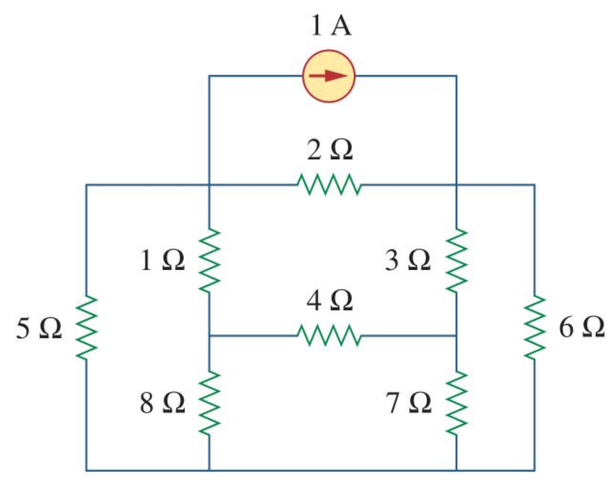
A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.

A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches.

Figure:(a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.



(a)



(b)

Mesh Analysis

The circuit shown below is nonplanar, because there is no way to redraw it and avoid the branches crossing. Nonplanar circuits can be handled using nodal analysis

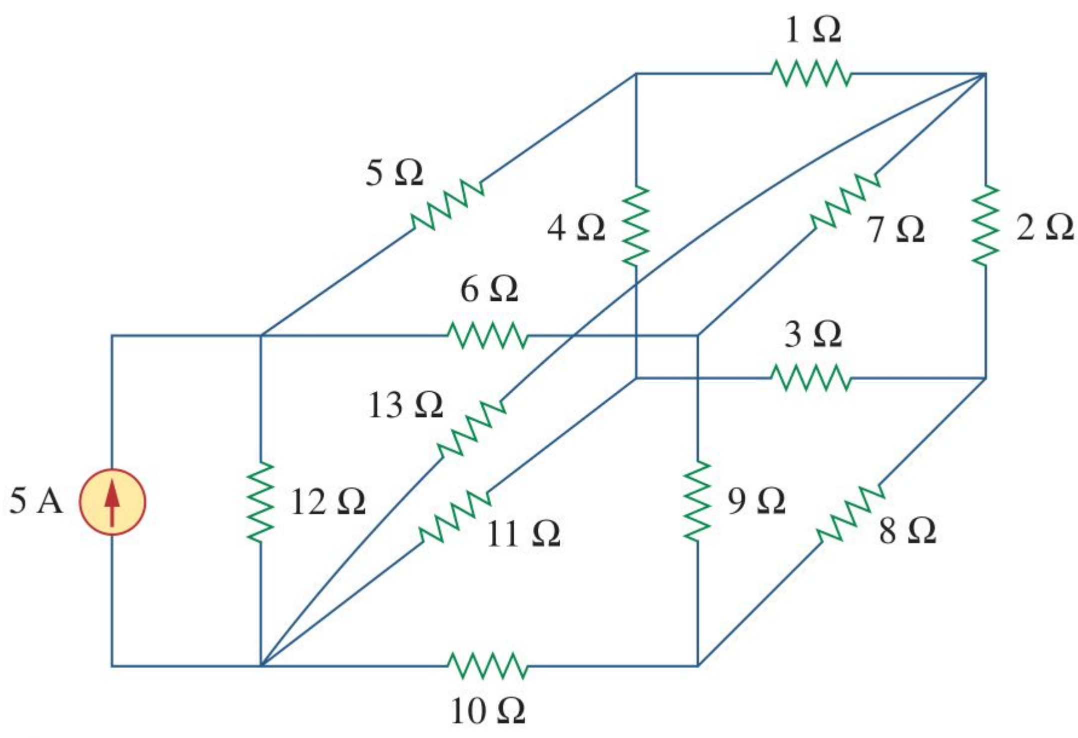


Figure: A nonplanar circuit.

A mesh is a loop which does not contain any other loops within it.

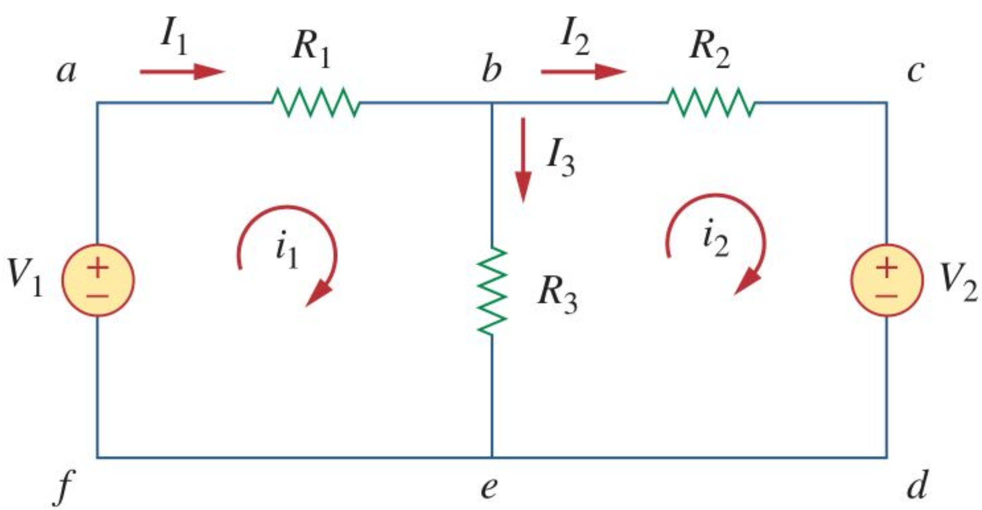


Figure: A circuit with two meshes.

abefa and ***bcdeb*** are meshes, but path ***abcdefa*** is not a mesh. The current through a mesh is known as mesh current.

Although path ***abcdefa*** is a loop and not a mesh, KVL still holds.

Steps to Determine Mesh Currents

1. Assign mesh currents $i_1, i_2, \ldots\ i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

The ***direction*** of the ***mesh current*** is ***arbitrary***—(clockwise or counterclockwise)—and does not affect the validity of the solution. It is ***conventional*** to ***assume*** that each mesh current flows ***clockwise***.

Mesh Analysis

To illustrate the steps, consider the previous circuit

First step requires that mesh currents and are assigned to meshes 1 and 2.