When Nash Meets Stackelberg - Instance Description

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This is a description of Carvalho et al. [2019] attached to the GitHub repository https://github.com/ds4dm/EPECInstances.

1 Computations

Governments act as Stackelberg leaders by trading energy, intending to minimize their emissions, and eventually to maximize tax incomes. Within each country, energy producers act as Stackelberg followers and play a Nash game between themselves, aiming to maximize their profits. Each country is interested in imposing a tax that is not preventing profitable domestic production, as it is constrained to keep the domestic energy price less than a predetermined threshold. We present the optimization problems of the players formally below. For ease of understanding, the quantities in red are parameters, i.e., inputs to the model. And the quantities in blue are decision variables, decided by the country or of the energy producers in the same country. Quantities in green are variables of a *different* country influencing the country's problem. Each country C solves the following problem.

$$
\begin{aligned}
\min_{\mathbf{q}^p, \mathbf{t}^p, \mathbf{q}_{\text{imp}}, \mathbf{q}^C_{\text{exp}}} \quad &: \quad \left(\sum_{p \in \mathscr{P}} \mathbf{C}^p_{\text{emmision}} \mathbf{q}^p - b \mathbf{t}^p \mathbf{q}^p \right) + \sum_{C' \in \mathscr{C} \setminus C} \pi^{C'} \mathbf{q}^{C' \to C}_{\text{imp}} - \pi^C \mathbf{q}^C_{\text{exp}} && \text{(1a)} \\
\text{subject to} \quad & \mathbf{t}^p \quad \leq \quad \overline{\mathbf{t}^p} && \text{(1b)} \\
& \alpha^C - \beta^C \left(\sum_{p \in \mathscr{P}} \mathbf{q}^p + \mathbf{q}^C_{\text{imp}} - \mathbf{q}^C_{\text{exp}} \right) \quad \geq \quad \underline{\pi^C} && \text{(1c)} \\
& \sum_{C' \in \mathscr{C}} \mathbf{q}^{C' \to C}_{\text{imp}} \quad = \quad \mathbf{q}^C_{\text{imp}} && \text{(1d)} \\
& \mathbf{q}^p \quad \in \quad \text{SOL(Lower Level Nash Game)} && \text{(1e)}
\end{aligned}
$$

$\mathbf{C}^p_{\text{emmision}}$ is the dollar value of the emission caused by producer p while producing a unit quantity of energy. This number is the product of cost incurred due to the emission of one unit of greenhouse gases (GHG), some times referred to as the *social cost of carbon* and the quantity of GHG emitted for each unit of energy produced by the producer p, called as the *emission factor*. b dictates whether the objective should include the tax revenue earned by the government or not. $\mathbf{q}^p$ is the quantity of energy produced by the producer $p \in \mathscr{P}$, $\mathbf{q}^C_{\text{imp}}, \mathbf{q}^C_{\text{exp}}$ are respectively import and export quantities, and α^C, β^C are the intercept and the slope of the demand curve. The domestic price, for each country, is given by $\alpha^C - \beta^C Q$, where Q is the quantity of energy available domestically. Finally, $\pi^{C'}$ is the price at which the country can import energy from other countries, hence the variable linking the optimization problems of different countries. Optionally for some countries, we introduce a *carbon tax* paradigm, where the tax imposed on the followers is proportional to the emissions they cause. i.e., there is a constraint $\mathbf{t}^p = \mathbf{C}^p_{\text{emmision}} \mathbf{t}^{\text{GHG}}$, where the government decides the tax payable per unit emission. Furthermore, note that if b is non-zero, the objective is no longer linear. In such a case, we replace the product term with a McCormick relaxation. Finally, $\overline{\mathbf{t}^p}$, and $\underline{\pi^C}$ are the tax cap and price cap, respectively.

The lower level problem that each producer p solves is formulated as follow:

$$\min_{\mathbf{q}^p} \quad : \quad \mathbf{C}_p\mathbf{q}^p + \frac{1}{2}\mathbf{D}_p{\mathbf{q}^p}^2 + \mathbf{t}^p\mathbf{q}^p - \left(\alpha^C - \beta^C\left(\sum_{p'\in\mathscr{P}}\mathbf{q}^{p'} + \mathbf{q}^C_{\text{imp}} - \mathbf{q}^C_{\text{exp}}\right)\right)\mathbf{q}^p \tag{2a}$$

$$\text{subject to} \quad \mathbf{q}^p \quad \geq \quad 0 \tag{2b}$$

$$\mathbf{q}^p \leq \overline{\mathbf{q}^\mathbf{p}} \tag{2c}$$

The first two terms in the objective correspond to the energy producer's cost, while the third term is the tax expense. The parenthesis results in the revenue of p, which is the product of domestic price and the quantity produced. Further, the producer is constrained by their capacity limits. Note that the product of variables ($\mathbf{t}^p\mathbf{q}^p$) in the objective does not pose any additional difficulty to the problem. This is because the follower's problem is still convex quadratic for a *fixed* value of $\mathbf{t}^p$, and the KKT conditions give complementarity constraints with only linear terms.

1.1 Instance sets

We generated three instances sets for our computations. (i) *InstanceSetA* contains 149 instances where there are 3 to 5 countries (ii) *InstanceSetB* contains 50 instances with strictly 7 countries. These instances were selected if the full enumeration algorithm was not able to solve them within 10 second on a single core machine. (iii) *InstanceSetInsights* contains 50 instances with 2 countries with 3 followers each. Such instances are useful to derive managerial insights from our model. The specific parameters for all these instances are described in table 1 and are available in our open-source GitHub repository. All our tests run on a 8-cores Intel(R) Xeon Gold 6142, with 32GB of RAM and *Gurobi* 9.0.

Parameter	Distribution	Notes
Capacities	50, 100, 130, 170, 200, 1000, 1050, 20000	Each follower's capacity is randomly drawn from these values.
Emission Costs	25, 50, 100, 200, 300, 500, 550, 600	The first two values are reserved for green producers. The following two for averagely-polluting producers, while the remaining three for highly-polluting ones.
Linear Costs	150, 200, 220, 250, 275, 290, 300	Linear costs are generally inverse-proportional to the emission cost. For instance, a producer with a 50 emission cost will generally have a linear cost around 290.
Quadratic Costs	0, 0.1, 0.2, 0.3, 0.5, 0.55, 0.6	Quadratic costs are generally inverse-proportional to the emission cost. Same rationale as linear costs.
Tax Caps	0, 50, 100, 150, 200, 250, 275, 300	Tax caps are assigned following the same rational of emission costs. The lower the emission cost of a given producer, the lower the maximum tax applicable to it.
Demand Alpha	275, 300, 325, 350, 375, 450	Each country alpha is randomly drawn from this set.
Demand Beta	0.5, 0.6, 0.7, 0.75, 0.8, 0.9	Each country beta is randomly drawn from this set.
Price Cap	0.8, 0.85, 0.90, 0.95	Each country price-limit is randomly drawn from this set. The final price-limit is made of the product of this value and the country's demand alpha parameter.
Tax Paradigm	Standard, Single, Carbon	A country tax scheme can be: (i.) Standard, where followers are taxed at different levels per unit-energy, or (ii.) Single, where all the followers are taxed with the same level per unit-energy (iii.) Carbon, where all the followers are taxed with the same level per unit-emission

Table 1: Description of the parameters for our instances.

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1.2 Folder Organization

The folders in the GitHub repository reflect the organization of the paper. Therefore, there are 4 main folders:

1. InstanceSetA contains the related instances of A.
2. InstanceSetB contains the related instances of B.
3. InstanceSet_Insights contains the insights instances.
4. ChileArgentina contains the Chile Argentina data and instance generator.

Bibliography

Margarida Carvalho, Gabriele Dragotto, Felipe Feijoo, Andrea Lodi, and Sriram Sankaranarayanan. When Nash Meets Stackelberg. *ArXiv preprint*, 2019. URL https://arxiv.org/abs/1910.06452.